

# Computational Number Theory

## Programming HW 2

Due Date: 11/02/2024

You may reuse code/functions from your first assignment.

1. **Arithmetic in  $\mathbb{Z}_n$**  The input is a text file, with each line having three comma-separated integers:  $n, a, b$ , with  $n > 1, 0 < a < n < 10^9, 0 < b < 10^9$ .

For each triple  $(n, a, b)$ , print the value of  $a^b$  in  $\mathbb{Z}_n$ , followed by true if  $a \in \mathbb{Z}_n^*$  and false otherwise, followed by  $a^{-1}$  in  $\mathbb{Z}_n$  (if the previous value was true, otherwise print nothing).

Output for the sample input file (testinput-Zn.txt):

```
10,true,14
0,false
37,true,53
367482574,false
85976398,true,147977274
```

2. **Chinese Remainder Theorem** Write a function that accepts four integers  $a, m, b, n$  with  $0 < m, n < 10^9, 0 \leq |a|, |b| < 10^9$  and returns the least non-negative integer  $x$  such that  $x$  simultaneously satisfies:  $x \equiv a \pmod{m}$  and  $x \equiv b \pmod{n}$ . If  $\gcd(m, n) \neq 1$ , your function outputs -1 (although a solution may exist).

Output for the sample input file (testinput-crt.txt):

```
14
34
-1
1001
3834241559682
```