

Common Tutorial 6

All vectors ~~each row~~ are of the form $\begin{bmatrix} \text{iPad} \\ \text{iPhone} \\ \text{iPod} \end{bmatrix}$

Ideal Productable = $\begin{bmatrix} 6000 \\ 5000 \\ 3000 \end{bmatrix}$ Profit = $\begin{bmatrix} 4 \\ 6 \\ 10 \end{bmatrix}$
(ideal weekday produce)

Storage = $\begin{bmatrix} 40/1000 \\ 45/1000 \\ 20/1000 \end{bmatrix}$ Min Delivered = $\begin{bmatrix} 5000 \\ 0 \\ 4000 \end{bmatrix}$
(per unit)

Max Demanded = $\begin{bmatrix} 10000 \\ 15000 \\ 8000 \end{bmatrix}$

- (a) let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be the vector to represent the proportion of time for each product
we have assumed that no. of produced items of each product is proportional to time taken to produce each item.

Our objective $f^* = \max (S \cdot X^T \cdot (\text{diag}(\text{Profit}) \cdot \text{IdealProductable}))$
 $= \max (S \cdot X^T \cdot 120000x_1 + 150000x_2 + 150000x_3)$

Our constraints = $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \geq X \geq \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (1 \geq x_1, x_2, x_3 \geq 0)$
 $X^T \begin{bmatrix} 1 \\ 1 \end{bmatrix} \leq 1 \quad (x_1 + x_2 + x_3 \leq 1)$

$S \cdot X^T \cdot (\text{diag}(\text{Storage}) \cdot \text{IdealProductable}) \leq 6000$
 $(1200x_1 + 1125x_2 + 3150x_3 \leq 6000)$

Max Demanded $\Rightarrow S \cdot X^T \cdot \text{diag}(\text{IdealProductable}) X \geq \text{Min Delivered}$

$\begin{pmatrix} 0000 \\ 15000 \\ 8000 \end{pmatrix} \geq \begin{pmatrix} 30000x_1 \\ 25000x_2 \\ 15000x_3 \end{pmatrix} \geq \begin{pmatrix} 5000 \\ 0 \\ 4000 \end{pmatrix}$

- (b) let $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ be the vector to represent the amount of each product produced

$\text{IdealProductable} \cdot \text{Min Delivered} = \text{Min Delivered} \cdot \text{IdealProductable}$
 $I_{PH} = \begin{bmatrix} 116000 \\ 115000 \\ 113000 \end{bmatrix}$

Our objective $f^* = \max (Y^T \text{ Profits})$
 $= \max (4y_1 + 6y_2 + 10y_3)$

Our constraints =

$$y \geq 0$$

$$y_1, y_2, y_3 \geq 0$$

$$\text{Max Demand} \geq y \geq \text{Min Delivered}$$

$$Y^T \text{ storage} \leq 6000$$

$$Y^T \text{ Ipp} \leq 5$$

$$\frac{40y_1}{1000} + \frac{45y_2}{1000} + \frac{210y_3}{1000} \leq 6000$$

$$\frac{y_1}{6000} + \frac{y_2}{5000} + \frac{y_3}{3000} \leq 5$$

maximum produce in a week

(c) let $Z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$ be the vector to represent the amount of hrs spent on each product

$$P_{\text{Kaw}} \text{ Productivity} = \frac{\text{Total Productivity}}{8} = \begin{pmatrix} 750 \\ 625 \\ 375 \end{pmatrix}$$

Our objective $f^* = \max \left(\frac{1}{1000} Z^T (\text{diag}(\text{Profits}) P_{\text{Kaw}}) \right)$

$$= \max (3z_1 + 3.75z_2 + 3.75z_3)$$

Our constraints =

$$\begin{pmatrix} 40 \\ 40 \\ 40 \end{pmatrix} \geq Z \geq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (40 \geq z_1, z_2, z_3 \geq 0)$$

$$Z^T \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \leq 40 \quad (z_1 + z_2 + z_3 \leq 40)$$

$$Z^T (\text{diag}(\text{Storage}) P_{\text{Kaw}}) \leq 6000$$

$$(30z_1 + 28.125z_2 + 78.75z_3 \leq 6000)$$

$$\text{Max Demand} \geq \text{diag}(P_{\text{Kaw}}) Z \geq \text{Min Delivered}$$

$$\begin{pmatrix} 10000 \\ 15000 \\ 8000 \end{pmatrix} \geq \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} \geq \begin{pmatrix} 5000 \\ 0 \\ 4000 \end{pmatrix}$$

(d) The relationship b/w (a) & (c) is that (a) represents the proportion of time spent while (c) represents the time spent $\therefore z_1 = 40x_1, z_2 = 40x_2, z_3 = 40x_3$ as total 40 hrs in a week

$$\therefore \underline{Z = 40X}$$