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## CS20BTECH11063

### Data Science Analysis Assignment 7

In [4]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as stats
from scipy import optimize
import astroML
from astroML.stats import sigmaG
import pandas as pd
import seaborn as sns
np.random.seed(0)
import emcee
import corner
from sklearn.neighbors import KernelDensity
from IPython.display import display, Math
import dynesty
import nestle
```

# Q1

$$f_{gas} = f_0(1 + f_1 z)$$

In [14]:
```
df = pd.read_csv('q1.txt', sep=' ')
z = df['z']
fgas = df['fgas']
fgas_error = df['fgas_error']

def log_prior(theta):
    f0, f1 = theta
    if 0 < f0 < 0.5 and -0.5 < f1 < 0.5:
        return 0.0
    return -np.inf
```

```python
def log_likelihood(theta, x, y, sigma_y):
    f0, f1 = theta
    model = f0 * (1 + f1 * x)
    chi2 = np.sum(np.log(2 * np.pi * sigma_y ** 2) + ((y - model) ** 2 / sigma_y ** 2))
    return -0.5 * chi2

def log_likelihood2(theta, x, y, sigma_y):
    f0, f1 = theta
    model = f0 * (1 + f1 * x)
    chi2 = np.sum(np.log(2 * np.pi * sigma_y ** 2) + ((y - model) ** 2 / sigma_y ** 2))
    return 0.5 * chi2

def log_probability(theta, x, y, sigma_y):
    lp = log_prior(theta)
    if not np.isfinite(lp):
        return -np.inf
    return lp + log_likelihood(theta, x, y, sigma_y)

# Solution from scipy.optimize.minimize
optimized_sol = optimize.fmin(log_likelihood2, [0.2, 0.0], args=(z, fgas, fgas_error))

# # plot the results
# f0, f1 = optimized_sol
# print(f0, f1)
# x = np.linspace(0, 1.5, 1000)
# y = f0 * (1 + f1 * x)
# plt.figure(figsize=(15, 5))
# plt.errorbar(z, fgas, yerr=fgas_error, fmt='.', color='black', capsize=5, ecolor='gray', label='Data')
# plt.plot(x, y, color='red')
# plt.xlabel('z')
# plt.ylabel('fgas')
# plt.ylim(0.06, 0.21)
# plt.xlim(0, 1.5)
# plt.grid()
# plt.show()

# Solution from emcee
ndim, nwalkers = 2, 64
# f0_random = np.random.uniform(0, 0.5, nwalkers)
# f1_random = np.random.uniform(-0.5, 0.5, nwalkers)
# pos = np.array([f0_random, f1_random]).T #optimized_sol.x + 1e-4 * np.random.randn(nwalkers, ndim)
pos = optimized_sol + 1e-4 * np.random.randn(nwalkers, ndim)
sampler = emcee.EnsembleSampler(nwalkers, ndim, log_probability, args=(z, fgas, fgas_error))

nburn = 1000
nsteps = 2000
```

```python
sampler.run_mcmc(pos, nsteps, progress=True)
samples = sampler.get_chain(flat=True, discard=nburn)
# print(samples.shape)

f0_median, f1_median = np.median(samples, axis=0)
f0_68, f1_68 = np.percentile(samples, [16, 84], axis=0)
f0_90, f1_90 = np.percentile(samples, [5, 95], axis=0)

# print the results using math display
display(Math(r'f_0 = {0:.3f} +{1:.3f} -{2:.3f} (68\% CI)'.format(f0_median, f0_68[1] - f0_median, f0_median - f0_68[0])))
display(Math(r'f_1 = {0:.3f} +{1:.3f} -{2:.3f} (68\% CI)'.format(f1_median, f1_68[1] - f1_median, f1_median - f1_68[0])))
display(Math(r'f_0 = {0:.3f} +{1:.3f} -{2:.3f} (90\% CI)'.format(f0_median, f0_90[1] - f0_median, f0_median - f0_90[0])))
display(Math(r'f_1 = {0:.3f} +{1:.3f} -{2:.3f} (90\% CI)'.format(f1_median, f1_90[1] - f1_median, f1_median - f1_90[0])))
# print("f0 = {0:.3f} +{1:.3f} -{2:.3f} (68% CI)".format(f0_median, f0_68[1] - f0_median, f0_median - f0_68[0]))
# print("f1 = {0:.3f} +{1:.3f} -{2:.3f} (68% CI)".format(f1_median, f1_68[1] - f1_median, f1_median - f1_68[0]))
# print("f0 = {0:.3f} +{1:.3f} -{2:.3f} (90% CI)".format(f0_median, f0_90[1] - f0_median, f0_median - f0_90[0]))
# print("f1 = {0:.3f} +{1:.3f} -{2:.3f} (90% CI)".format(f1_median, f1_90[1] - f1_median, f1_median - f1_90[0]))


# Corner plot for 68% and 90% confidence intervals
# fig = plt.figure(figsize=(10, 10))
labels = [r"$f_0$", r"$f_1$"]
fig = corner.corner(samples, labels=labels, levels=[0.68, 0.9], show_titles=True, title_kwargs={"fontsize": 12})
plt.show()

# plot the best fit line and the data
samples_reshaped = samples.reshape((-1, ndim)).T
# print(samples_reshaped.shape)
x = np.linspace(0, 1.5, 1000)
f0, f1 = samples_reshaped[:2]
y = f0[:, None] * (1 + f1[:, None] * x)
# print(y.shape)
mu = np.mean(y, axis=0)
sig = 2 * y.std(0)
upper_bound = mu + sig
lower_bound = mu - sig

plt.figure(figsize=(15, 5))
plt.plot(x, mu, color='black', label='Best fit line')
plt.errorbar(z, fgas, yerr=fgas_error, fmt='.', color='black', capsize=5, ecolor='gray', label='Data')
plt.fill_between(x, lower_bound, upper_bound, color='grey', alpha=0.2, label='Confidence Interval Region')
plt.xlim(0, 1.5)
plt.ylim(0.06, 0.21)
plt.grid()
plt.legend()
plt.show()
```

```
Optimization terminated successfully.
         Current function value: -200.732528
         Iterations: 60
         Function evaluations: 114
```

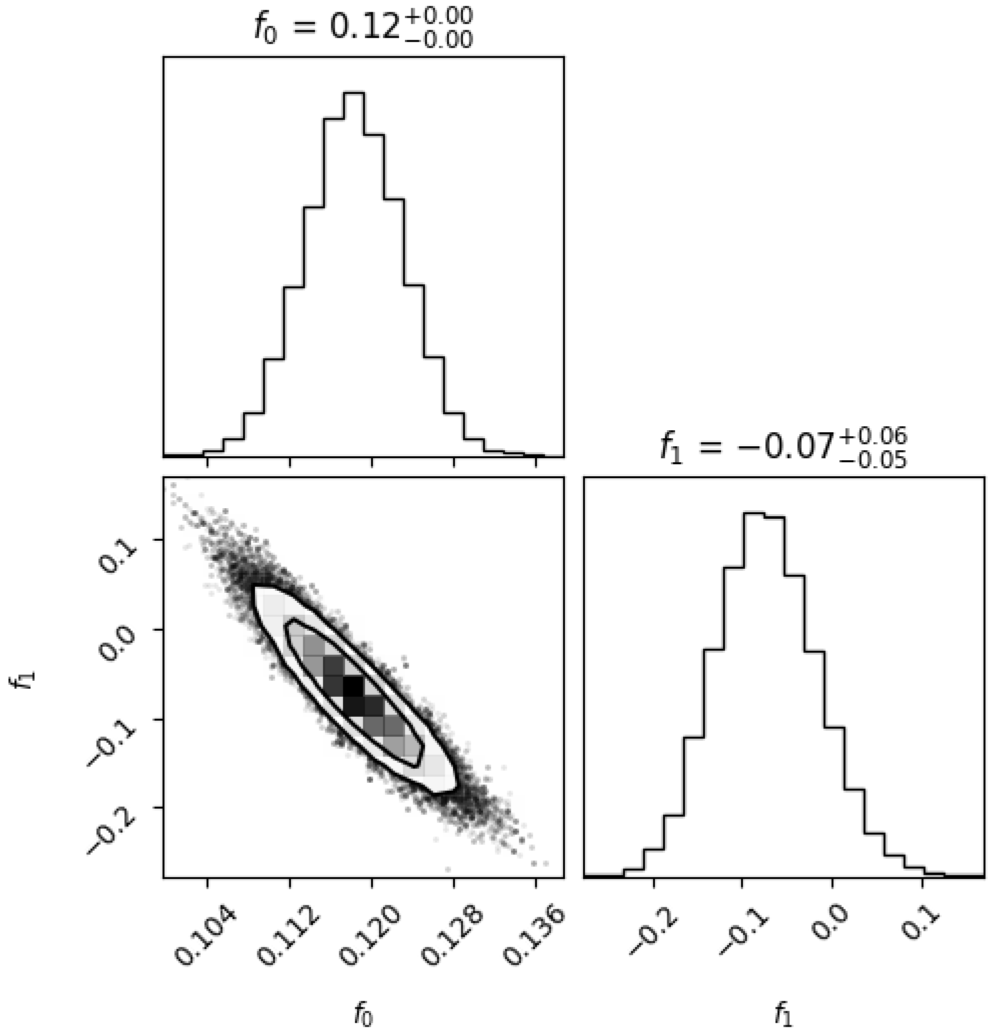
```
100%|██████████| 2000/2000 [01:53<00:00, 17.63it/s]
```

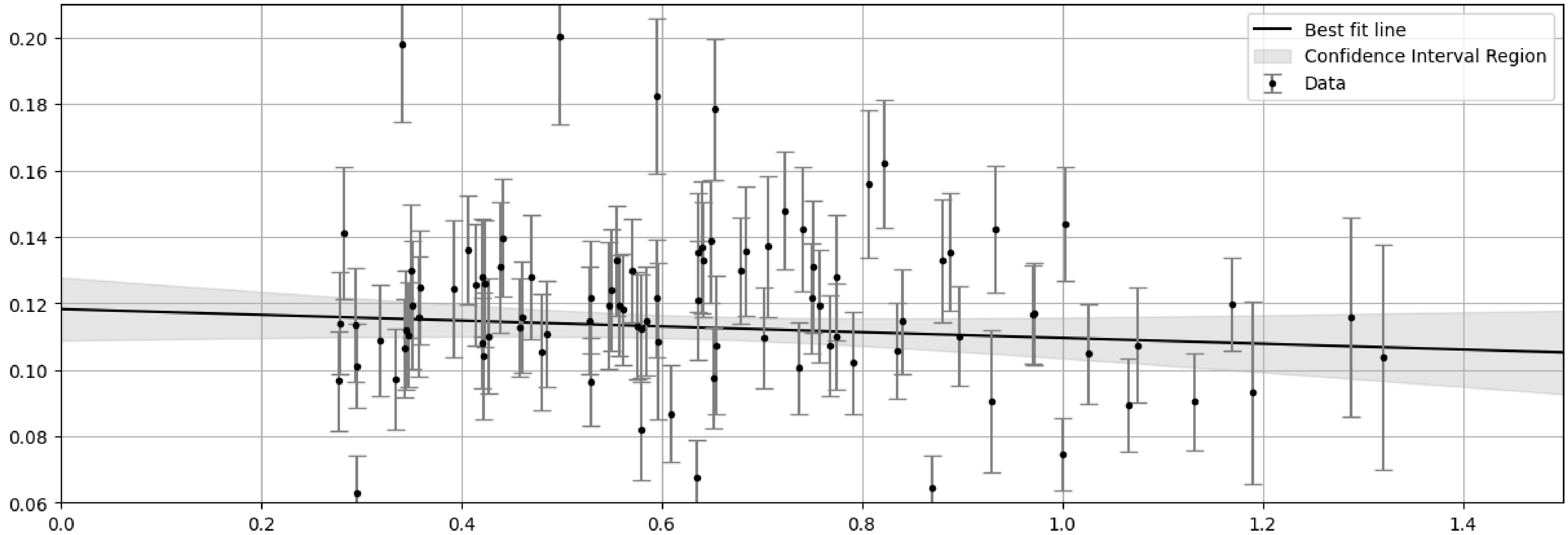
$f_0 = 0.118 + -0.246 - 0.005 (68\% CI)$

$f_1 = -0.074 + 0.059 - -0.197 (68\% CI)$

$f_0 = 0.118 + -0.279 - 0.008 (90\% CI)$

$f_1 = -0.074 + 0.098 - -0.200 (90\% CI)$






Reference: http://jakevdp.github.io/blog/2014/06/14/frequentism-and-bayesianism-4-bayesian-in-python/

# Q2

In [32]:

```
import numpy as np
data = np.array([[ 0.42,  0.72,  0.  ,  0.3 ,  0.15,
                   0.09,  0.19,  0.35,  0.4 ,  0.54,
                   0.42,  0.69,  0.2 ,  0.88,  0.03,
                   0.67,  0.42,  0.56,  0.14,  0.2  ],
                 [ 0.33,  0.41, -0.22,  0.01, -0.05,
                  -0.05, -0.12,  0.26,  0.29,  0.39,
                   0.31,  0.42, -0.01,  0.58, -0.2 ,
                   0.52,  0.15,  0.32, -0.13, -0.09 ],
                 [ 0.1 ,  0.1 ,  0.1 ,  0.1 ,  0.1 ,
                   0.1 ,  0.1 ,  0.1 ,  0.1 ,  0.1 ,
                   0.1 ,  0.1 ,  0.1 ,  0.1 ,  0.1 ,
                   0.1 ,  0.1 ,  0.1 ,  0.1 ,  0.1  ]])
x, y, sigma_y = data

# def polynomial_fit(theta, x):
#     val = 0
#     for i in range(len(theta)):
```

```python
#         val += theta[i] * x**i
#     return val

def polynomial_fit(theta, x):
    """Polynomial model of degree (len(theta) - 1)"""
    return sum(t * x ** n for (n, t) in enumerate(theta))

def log_likelihood(theta, x=x, y=y, sigma_y=sigma_y):
    model = polynomial_fit(theta, x)
    # return -0.5 * (np.sum(np.log((y - model)**2 / sigma_y**2)) - np.sum(np.log(2 * np.pi * sigma_y**2)))
    return -0.5 * np.sum(np.log(2 * np.pi * sigma_y ** 2) + (y - model) ** 2 / sigma_y ** 2)

# def prior_transform(theta):
#     return theta

def prior_transform_linear(theta):
    # m = theta[0]*20 - 10
    # c = theta[1]*20 - 10
    m = theta[0]*10 - 5
    c = theta[1]*20 - 10
    return np.array([m, c])

def prior_transform_quadratic(theta):
    # a = theta[0]*20 - 10
    # b = theta[1]*20 - 10
    # c = theta[2]*20 - 20
    a = theta[0]*10 - 5
    b = theta[1]*10 - 5
    c = theta[2]*20 - 20

    return np.array([a, b, c])

nlive = 1024
bound = 'multi'
ndims = 2
sample = 'unif'
tol = 0.1

sampler_linear = dynesty.NestedSampler(log_likelihood, prior_transform_linear, ndims, nlive=nlive, bound=bound, sample=sample)
samples_linear = sampler_linear.run_nested(dlogz=tol, print_progress=True)
results_linear = sampler_linear.results
log_z_linear = results_linear.logz[-1]
print(results_linear.summary())

nlive = 1024
bound = 'multi'
```

```python
ndims = 3
sample = 'unif'
tol = 0.1

sampler_quadratic = dynesty.NestedSampler(log_likelihood, prior_transform_quadratic, ndims, nlive=nlive, bound=bound, sample=sample)
samples_quadratic = sampler_quadratic.run_nested(dlogz=tol, print_progress=True)
results_quadratic = sampler_quadratic.results
log_z_quadratic = results_quadratic.logz[-1]
print(results_quadratic.summary())


print("Bayes Factor Quad/Linear: ", np.exp(log_z_quadratic - log_z_linear))
print("Bayes Factor Linear/Quad: ", np.exp(log_z_linear - log_z_quadratic))
```

```
12130it [00:17, 681.47it/s, +1024 | bound: 10 | nc: 1 | ncall: 51632 | eff(%): 25.992 | loglstar:   -inf < 22.011 <    inf | logz: 12.519 +/-  0.091 | dlogz:  0.000 >  0.100]
Summary
=======
nlive: 1024
niter: 12130
ncall: 50608
eff(%): 25.992
logz: 12.519 +/-  0.116
None
14622it [00:22, 648.18it/s, +1024 | bound: 16 | nc: 1 | ncall: 61719 | eff(%): 25.778 | loglstar:   -inf < 22.941 <    inf | logz: 11.009 +/-  0.103 | dlogz:  0.000 >  0.100]
Summary
=======
nlive: 1024
niter: 14622
ncall: 60695
eff(%): 25.778
logz: 11.009 +/-  0.129
None
Bayes Factor Quad/Linear:  0.2207967800740394
Bayes Factor Linear/Quad:  4.529051554396181
```

According to the above sampling using `dynesty` package, we can see that linear model is being favoured more than the quadratic model, and this does not match with the value given in the blog

Reference:

http://jakevdp.github.io/blog/2015/08/07/frequentism-and-bayesianism-5-model-selection/

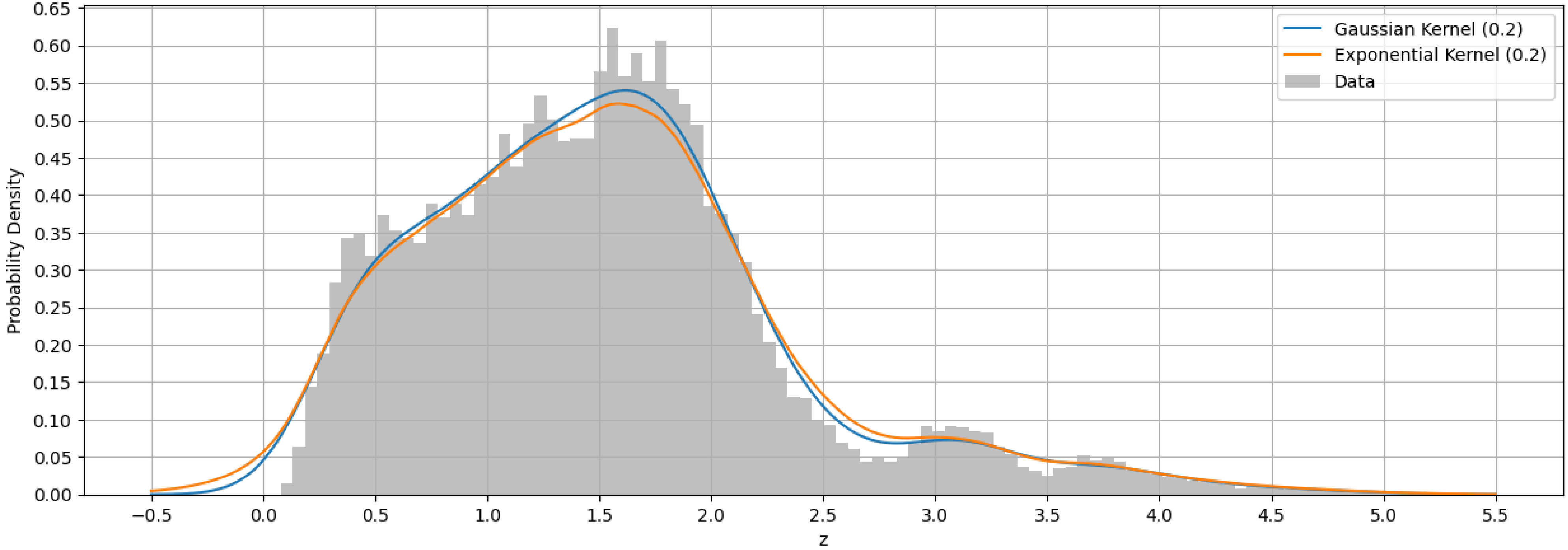
# Q3

In [3]:
```
df = pd.read_csv('q3.csv', sep='\t')
z = df['z']
# Plot KDE estimate for z using gaussian and exponential kernels (with bandwidth=0.2)

plt.figure(figsize=(15, 5))
plt.xticks(np.arange(-1, 6, 0.5))
plt.yticks(np.arange(0, 1.1, 0.05))

x = np.linspace(-0.5, 5.5, 1000)
kde = KernelDensity(kernel='gaussian', bandwidth=0.2).fit(z.to_numpy().reshape(-1, 1))
log_dens = kde.score_samples(x.reshape(-1,1))

plt.plot(x, np.exp(log_dens), label='Gaussian Kernel (0.2)')
kde = KernelDensity(kernel='exponential', bandwidth=0.2).fit(z.to_numpy().reshape(-1, 1))
log_dens = kde.score_samples(x.reshape(-1,1))
plt.plot(x, np.exp(log_dens), label='Exponential Kernel (0.2)')
plt.hist(z, bins='auto', density=True, color='grey', alpha=0.5, label='Data')
plt.xlabel('z')
plt.ylabel('Probability Density')
plt.legend()
plt.grid()
plt.show()
```



Gaussian Kernel (0.2)
Exponential Kernel (0.2)
Data
Probability Density
z
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5