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CS20BTECH11063

Data Science Analysis Assignment 7

In [4]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as stats
from scipy import optimize
import astroML
from astroML.stats import sigmaG
import pandas as pd
import seaborn as sns
np.random.seed(0)
import emcee
import corner
from sklearn.neighbors import KernelDensity
from IPython.display import display, Math
import dynesty
import nestle
```

Q1

$$f_{gas} = f_0(1 + f_1 z)$$

In [14]:
```
df = pd.read_csv('q1.txt', sep=' ')
z = df['z']
fgas = df['fgas']
fgas_error = df['fgas_error']

def log_prior(theta):
    f0, f1 = theta
    if 0 < f0 < 0.5 and -0.5 < f1 < 0.5:
        return 0.0
    return -np.inf
```

```python
def log_likelihood(theta, x, y, sigma_y):
    f0, f1 = theta
    model = f0 * (1 + f1 * x)
    chi2 = np.sum(np.log(2 * np.pi * sigma_y ** 2) + ((y - model) ** 2 / sigma_y ** 2))
    return -0.5 * chi2

def log_likelihood2(theta, x, y, sigma_y):
    f0, f1 = theta
    model = f0 * (1 + f1 * x)
    chi2 = np.sum(np.log(2 * np.pi * sigma_y ** 2) + ((y - model) ** 2 / sigma_y ** 2))
    return 0.5 * chi2

def log_probability(theta, x, y, sigma_y):
    lp = log_prior(theta)
    if not np.isfinite(lp):
        return -np.inf
    return lp + log_likelihood(theta, x, y, sigma_y)

# Solution from scipy.optimize.minimize
optimized_sol = optimize.fmin(log_likelihood2, [0.2, 0.0], args=(z, fgas, fgas_error))

# # plot the results
# f0, f1 = optimized_sol
# print(f0, f1)
# x = np.linspace(0, 1.5, 1000)
# y = f0 * (1 + f1 * x)
# plt.figure(figsize=(15, 5))
# plt.errorbar(z, fgas, yerr=fgas_error, fmt='.', color='black', capsize=5, ecolor='gray', label='Data')
# plt.plot(x, y, color='red')
# plt.xlabel('z')
# plt.ylabel('fgas')
# plt.ylim(0.06, 0.21)
# plt.xlim(0, 1.5)
# plt.grid()
# plt.show()

# Solution from emcee
ndim, nwalkers = 2, 64
# f0_random = np.random.uniform(0, 0.5, nwalkers)
# f1_random = np.random.uniform(-0.5, 0.5, nwalkers)
# pos = np.array([f0_random, f1_random]).T #optimized_sol.x + 1e-4 * np.random.randn(nwalkers, ndim)
pos = optimized_sol + 1e-4 * np.random.randn(nwalkers, ndim)
sampler = emcee.EnsembleSampler(nwalkers, ndim, log_probability, args=(z, fgas, fgas_error))

nburn = 1000
nsteps = 2000
```

```python
sampler.run_mcmc(pos, nsteps, progress=True)
samples = sampler.get_chain(flat=True, discard=nburn)
# print(samples.shape)

f0_median, f1_median = np.median(samples, axis=0)
f0_68, f1_68 = np.percentile(samples, [16, 84], axis=0)
f0_90, f1_90 = np.percentile(samples, [5, 95], axis=0)

# print the results using math display
display(Math(r'f_0 = {0:.3f} +{1:.3f} -{2:.3f} (68\% CI)'.format(f0_median, f0_68[1] - f0_median, f0_median - f0_68[0])))
display(Math(r'f_1 = {0:.3f} +{1:.3f} -{2:.3f} (68\% CI)'.format(f1_median, f1_68[1] - f1_median, f1_median - f1_68[0])))
display(Math(r'f_0 = {0:.3f} +{1:.3f} -{2:.3f} (90\% CI)'.format(f0_median, f0_90[1] - f0_median, f0_median - f0_90[0])))
display(Math(r'f_1 = {0:.3f} +{1:.3f} -{2:.3f} (90\% CI)'.format(f1_median, f1_90[1] - f1_median, f1_median - f1_90[0])))
# print("f0 = {0:.3f} +{1:.3f} -{2:.3f} (68% CI)".format(f0_median, f0_68[1] - f0_median, f0_median - f0_68[0]))
# print("f1 = {0:.3f} +{1:.3f} -{2:.3f} (68% CI)".format(f1_median, f1_68[1] - f1_median, f1_median - f1_68[0]))
# print("f0 = {0:.3f} +{1:.3f} -{2:.3f} (90% CI)".format(f0_median, f0_90[1] - f0_median, f0_median - f0_90[0]))
# print("f1 = {0:.3f} +{1:.3f} -{2:.3f} (90% CI)".format(f1_median, f1_90[1] - f1_median, f1_median - f1_90[0]))


# Corner plot for 68% and 90% confidence intervals
# fig = plt.figure(figsize=(10, 10))
labels = [r"$f_0$", r"$f_1$"]
fig = corner.corner(samples, labels=labels, levels=[0.68, 0.9], show_titles=True, title_kwargs={"fontsize": 12})
plt.show()

# plot the best fit line and the data
samples_reshaped = samples.reshape((-1, ndim)).T
# print(samples_reshaped.shape)
x = np.linspace(0, 1.5, 1000)
f0, f1 = samples_reshaped[:2]
y = f0[:, None] * (1 + f1[:, None] * x)
# print(y.shape)
mu = np.mean(y, axis=0)
sig = 2 * y.std(0)
upper_bound = mu + sig
lower_bound = mu - sig

plt.figure(figsize=(15, 5))
plt.plot(x, mu, color='black', label='Best fit line')
plt.errorbar(z, fgas, yerr=fgas_error, fmt='.', color='black', capsize=5, ecolor='gray', label='Data')
plt.fill_between(x, lower_bound, upper_bound, color='grey', alpha=0.2, label='Confidence Interval Region')
plt.xlim(0, 1.5)
plt.ylim(0.06, 0.21)
plt.grid()
plt.legend()
plt.show()
```

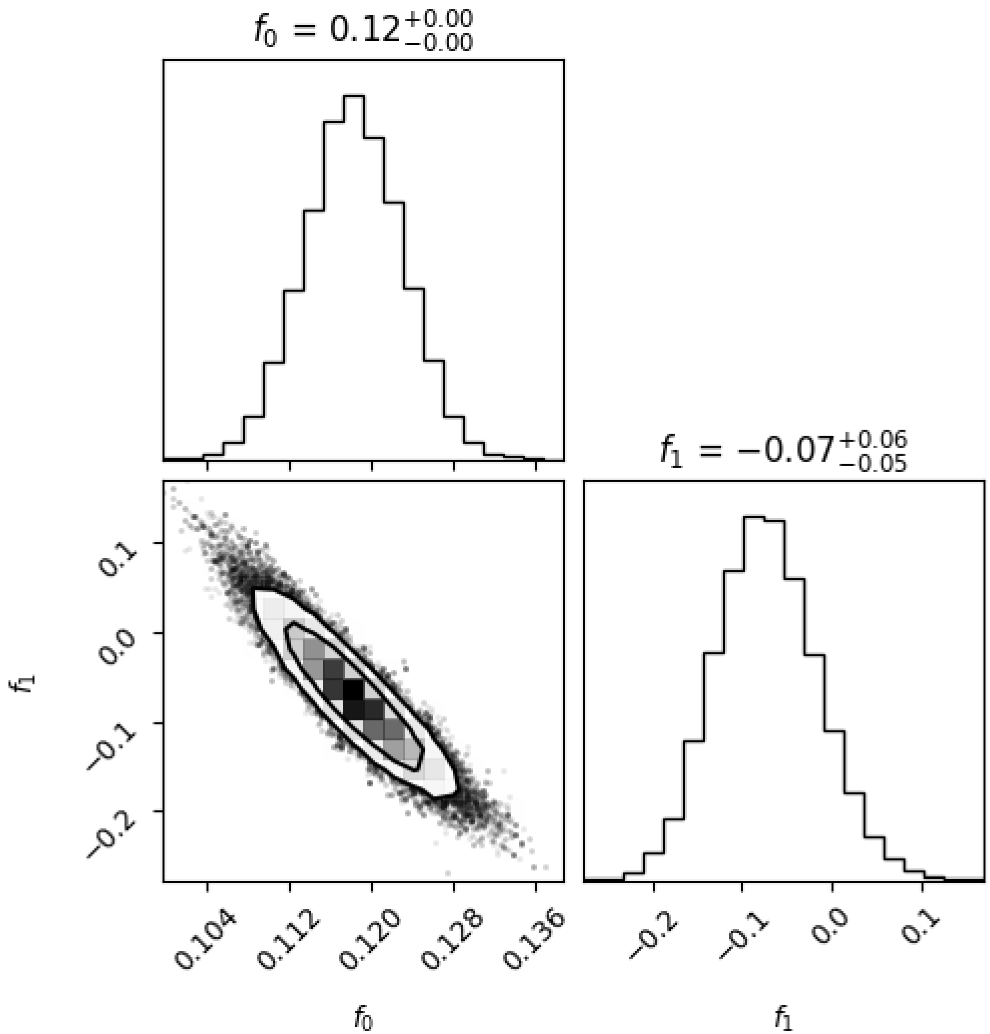
```
Optimization terminated successfully.
         Current function value: -200.732528
         Iterations: 60
         Function evaluations: 114
100%|██████████| 2000/2000 [01:53<00:00, 17.63it/s]
```

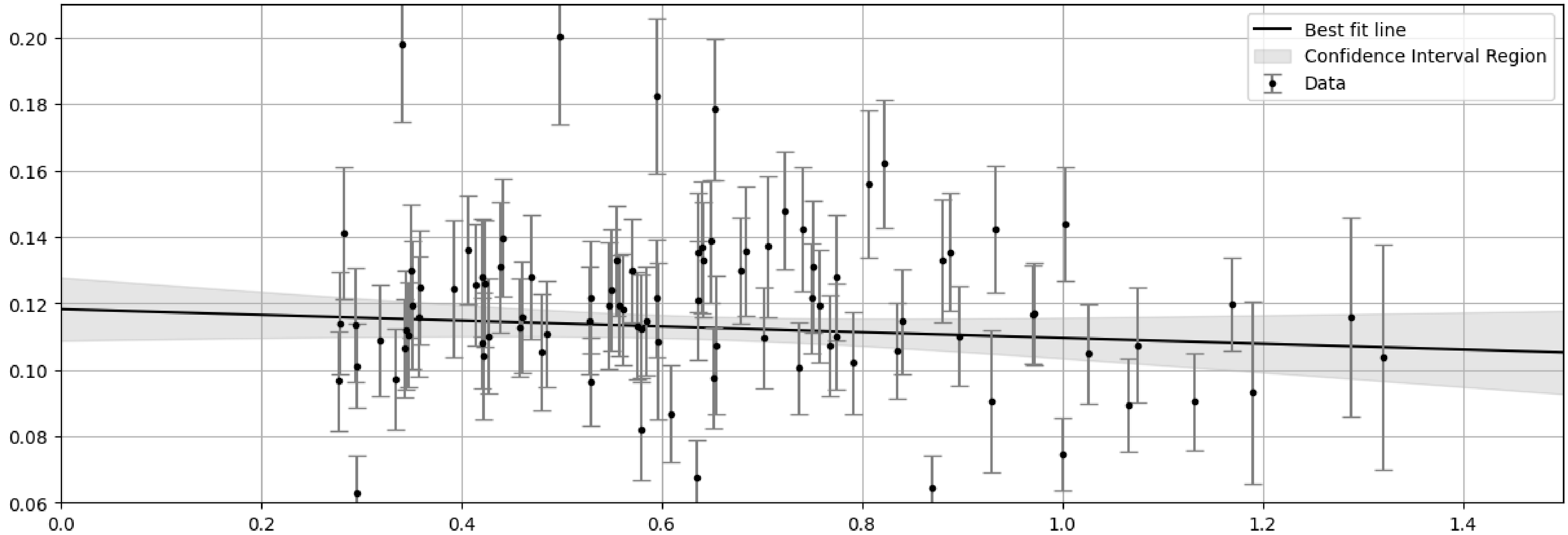
$$f_0 = 0.118 + -0.246 - 0.005 (68\% CI)$$

$$f_1 = -0.074 + 0.059 - -0.197 (68\% CI)$$

$$f_0 = 0.118 + -0.279 - 0.008 (90\% CI)$$

$$f_1 = -0.074 + 0.098 - -0.200 (90\% CI)$$





Reference: http://jakevdp.github.io/blog/2014/06/14/frequentism-and-bayesianism-4-bayesian-in-python/

Q2

In [27]:

```
import numpy as np
data = np.array([[ 0.42,  0.72,  0.  ,  0.3 ,  0.15,
                   0.09,  0.19,  0.35,  0.4 ,  0.54,
                   0.42,  0.69,  0.2 ,  0.88,  0.03,
                   0.67,  0.42,  0.56,  0.14,  0.2  ],
                 [ 0.33,  0.41, -0.22,  0.01, -0.05,
                  -0.05, -0.12,  0.26,  0.29,  0.39,
                   0.31,  0.42, -0.01,  0.58, -0.2 ,
                   0.52,  0.15,  0.32, -0.13, -0.09 ],
                 [ 0.1 ,  0.1 ,  0.1 ,  0.1 ,  0.1 ,
                   0.1 ,  0.1 ,  0.1 ,  0.1 ,  0.1 ,
                   0.1 ,  0.1 ,  0.1 ,  0.1 ,  0.1 ,
                   0.1 ,  0.1 ,  0.1 ,  0.1 ,  0.1  ]])
x, y, sigma_y = data

# def polynomial_fit(theta, x):
#     val = 0
#     for i in range(len(theta)):
```

```python
#         val += theta[i] * x**i
#     return val

def polynomial_fit(theta, x):
    """Polynomial model of degree (len(theta) - 1)"""
    return sum(t * x ** n for (n, t) in enumerate(theta))

def log_likelihood(theta, x=x, y=y, sigma_y=sigma_y):
    model = polynomial_fit(theta, x)
    # return -0.5 * (np.sum(np.log((y - model)**2 / sigma_y**2)) - np.sum(np.log(2 * np.pi * sigma_y**2)))
    return -0.5 * np.sum(np.log(2 * np.pi * sigma_y ** 2) + (y - model) ** 2 / sigma_y ** 2)

# def prior_transform(theta):
#     return theta

def prior_transform_linear(theta):
    m = theta[0]*200 - 100
    c = theta[1]*200 - 100
    return np.array([m, c])

def prior_transform_quadratic(theta):
    a = theta[0]*200 - 100
    b = theta[1]*200 - 100
    c = theta[2]*200 - 200
    return np.array([a, b, c])

nlive = 512
bound = 'multi'
ndims = 2
sample = 'unif'
tol = 0.1

sampler_linear = dynesty.NestedSampler(log_likelihood, prior_transform_linear, ndims, nlive=nlive, bound=bound, sample=sample)
samples_linear = sampler_linear.run_nested(dlogz=tol, print_progress=True)
results_linear = sampler_linear.results
log_z_linear = results_linear.logz[-1]
print(results_linear.summary())

nlive = 512
bound = 'multi'
ndims = 3
sample = 'unif'
tol = 0.1

sampler_quadratic = dynesty.NestedSampler(log_likelihood, prior_transform_quadratic, ndims, nlive=nlive, bound=bound, sample=sample)
samples_quadratic = sampler_quadratic.run_nested(dlogz=tol, print_progress=True)
```

```
results_quadratic = sampler_quadratic.results
log_z_quadratic = results_quadratic.logz[-1]
print(results_quadratic.summary())


print("Bayes Factor: ", np.exp(log_z_quadratic - log_z_linear))
print("Bayes Factor: ", np.exp(log_z_linear - log_z_quadratic))
```

```
8792it [00:14, 623.98it/s, +512 | bound: 17 | nc: 1 | ncall: 31019 | eff(%): 30.498 | loglstar:   -inf < 22.010 <    inf | logz:  7.204 +/-  0.164 | dlogz:  0.000 >  0.100]
Summary
=======
nlive: 512
niter: 8792
ncall: 30507
eff(%): 30.498
logz:  7.204 +/-  0.203
None
11617it [00:17, 671.05it/s, +512 | bound: 31 | nc: 1 | ncall: 42146 | eff(%): 29.132 | loglstar:   -inf < 22.937 <    inf | logz:  2.612 +/-  0.193 | dlogz:  0.000 >  0.100]
Summary
=======
nlive: 512
niter: 11617
ncall: 41634
eff(%): 29.132
logz:  2.612 +/-  0.237
None
Bayes Factor:  0.010133800655170794
Bayes Factor:  98.6796596881692
```

Q3

In [3]:

```
df = pd.read_csv('q3.csv', sep='\t')
z = df['z']
# Plot KDE estimate for z using gaussian and exponential kernels (with bandwidth=0.2)

plt.figure(figsize=(15, 5))
plt.xticks(np.arange(-1, 6, 0.5))
plt.yticks(np.arange(0, 1.1, 0.05))

x = np.linspace(-0.5, 5.5, 1000)
kde = KernelDensity(kernel='gaussian', bandwidth=0.2).fit(z.to_numpy().reshape(-1, 1))
```

```
log_dens = kde.score_samples(x.reshape(-1,1))

plt.plot(x, np.exp(log_dens), label='Gaussian Kernel (0.2)')
kde = KernelDensity(kernel='exponential', bandwidth=0.2).fit(z.to_numpy().reshape(-1, 1))
log_dens = kde.score_samples(x.reshape(-1,1))
plt.plot(x, np.exp(log_dens), label='Exponential Kernel (0.2)')
plt.hist(z, bins='auto', density=True, color='grey', alpha=0.5, label='Data')
plt.xlabel('z')
plt.ylabel('Probability Density')
plt.legend()
plt.grid()
plt.show()
```