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CS20BTECH11063

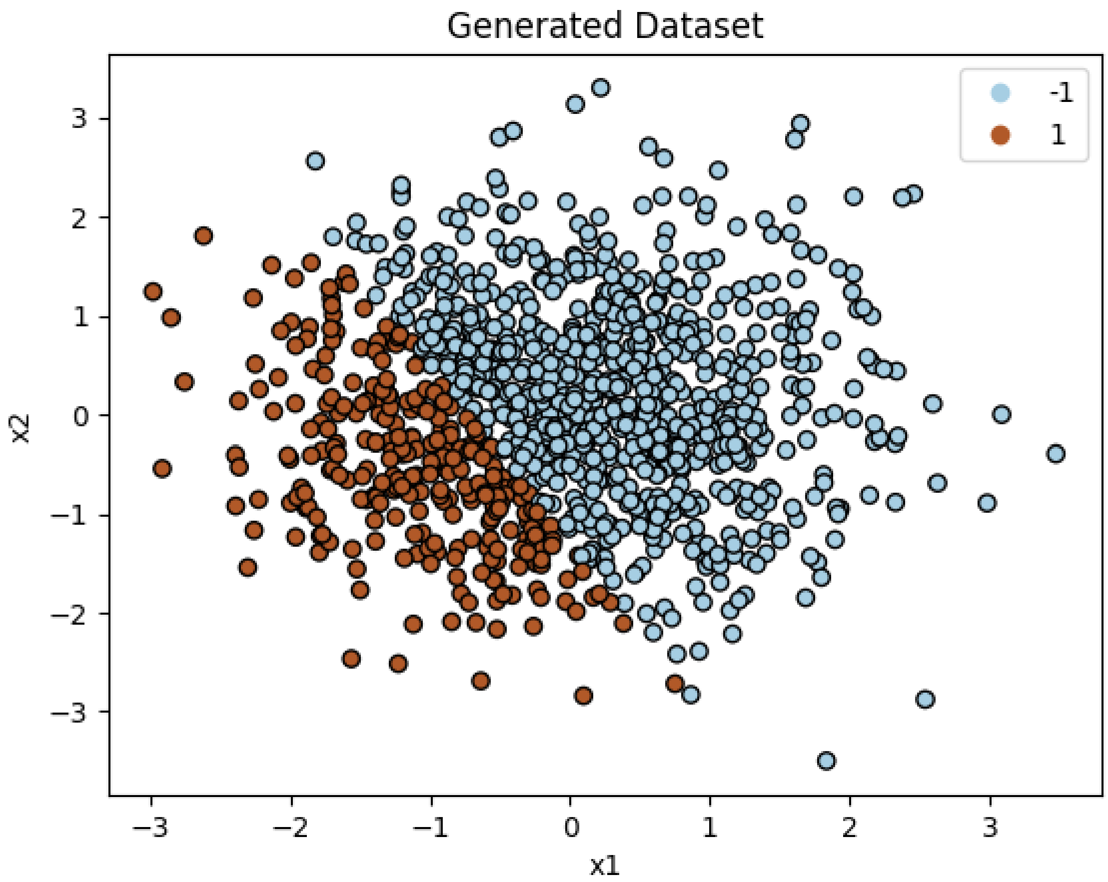
Deep Learning Assignment 1

In [ ]:
```
import torch
import numpy as np
import matplotlib.pyplot as plt
import math
import torch.functional as F
import torch.nn as nn
```

Q1

In [ ]:
```
# Create a Linearly separable 2D dataset
def create_dataset(n=100, gamma=0.1):
    x = torch.randn(n, 2)
    y = torch.zeros(n)

    # for i in range(n):
    #     y[i] = 1 if x[i, 0] + gamma * x[i, 1] > 0 else -1
    # return x, y

    # Random initialize the weights and bias
    w = torch.randn(2)
    b = torch.randn(1)

    # Create the Dataset
    y = torch.sign(torch.matmul(x, w) + b)
    y[np.random.rand(n) < gamma] *= -1
    return x, y


# Plot the scatter plot with legend
```

```
x, y = create_dataset(1000, 1)
scatter = plt.scatter(x[:, 0], x[:, 1], c=y, cmap=plt.cm.Paired, edgecolors='k')
xmin, xmax, ymin, ymax = plt.axis()
plt.title("Generated Dataset")
plt.xlabel("x1")
plt.ylabel("x2")
plt.legend(handles=scatter.legend_elements()[0], labels=["-1", "1"])
plt.show()


# print frequency percentage of labels
print("Percentage of -1 labels: ", (y == -1).sum().item() / len(y))
print("Percentage of 1 labels: ", (y == 1).sum().item() / len(y))
```




```
Percentage of -1 labels:  0.731
Percentage of 1 labels:  0.269
```

In [ ]:
```
# split dataset into train and test
def split_dataset(x, y, train_ratio=0.8):
    n = len(x)
    train_size = int(train_ratio * n)
    x_train = x[:train_size]
    y_train = y[:train_size]
    x_test = x[train_size:]
    y_test = y[train_size:]
    return x_train, y_train, x_test, y_test

x_train, y_train, x_test, y_test = split_dataset(x, y, 0.8)
```

In [ ]:
```
# Append 1 to x for bias
x_train = torch.cat((x_train, torch.ones(x_train.shape[0], 1)), dim=1)
print("Shape of Training x: ", x_train.shape)
print("Shape of Training y: ", y_train.shape)
# print(x)
```

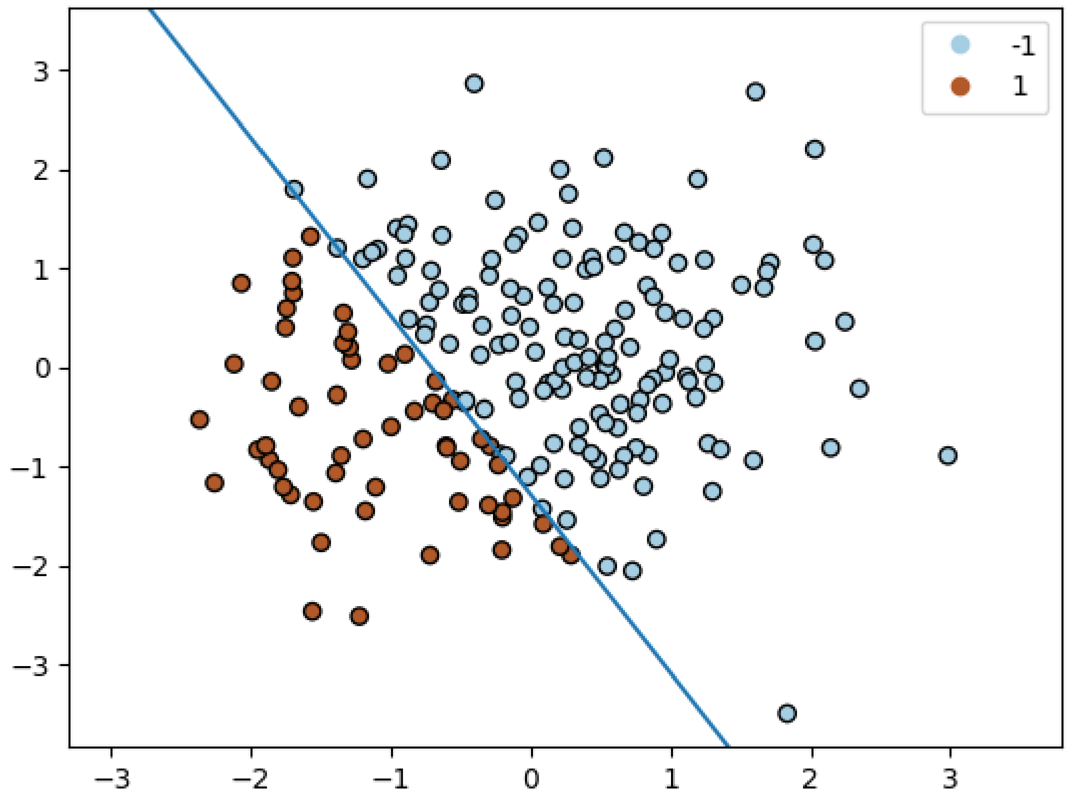
```
Shape of Training x:  torch.Size([800, 3])
Shape of Training y:  torch.Size([800])
```

In [ ]:
```
# Perceptron Training Algorithm
def perceptron_train(x, y, max_epochs=100):
    w = torch.zeros(3)
    k = 0
    for epoch in range(max_epochs):
        nb_changes = 0
        for i in range(x.size(0)):
            if x[i].dot(w) * y[i] <= 0:
                w = w + y[i] * x[i]
                nb_changes = nb_changes + 1
        if nb_changes == 0:
            # print('Stopping at Epoch: ', epoch)
            break
        k = k + 1
    # print('Number of changes: ', nb_changes)

    # return the weights and number of epochs
    return w, k
```

In [ ]:
```
w, max_epochs_run = perceptron_train(x_train, y_train, 100)
print("W = ", w, " Max Epochs Run = ", max_epochs_run)
```

```
W =  tensor([-16.7979,  -9.3326, -12.0000])  Max Epochs Run =  19
```

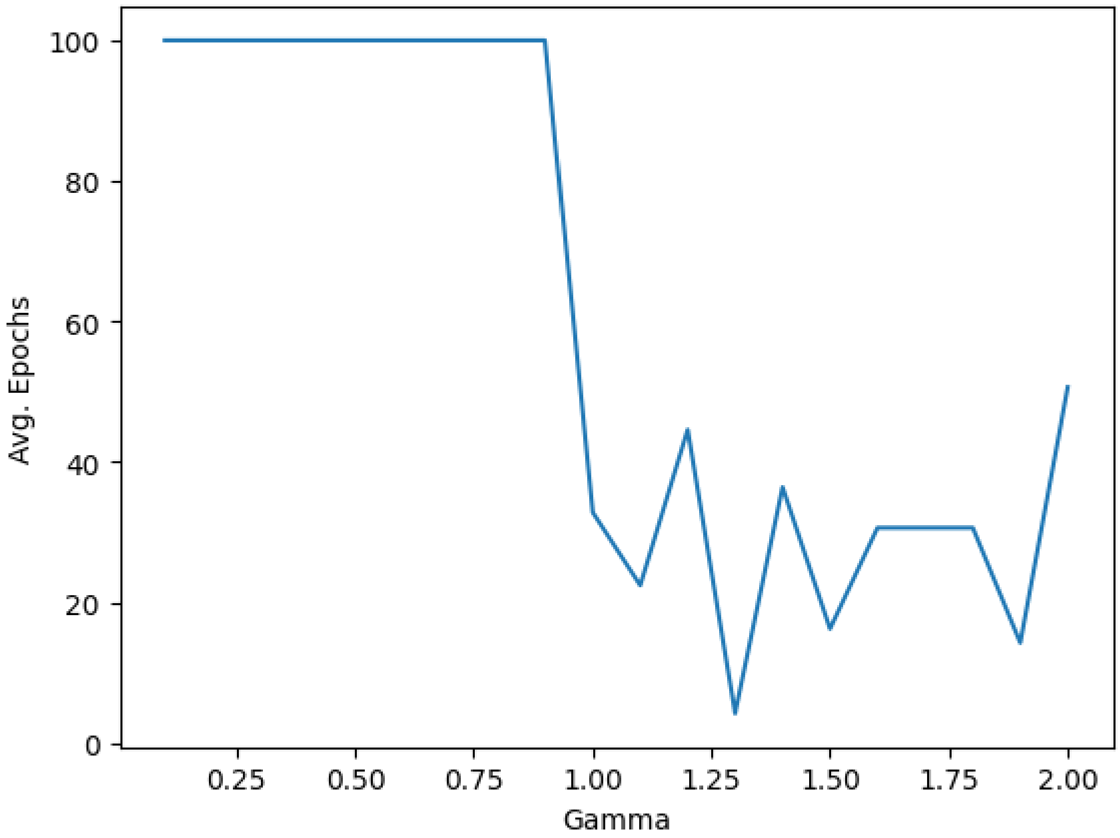
In [ ]:
```python
# plot the decision boundary using test data
x1 = np.linspace(-5, 5, 100)
x2 = -(w[0] * x1 + w[2]) / w[1]
plt.plot(x1, x2)
scatter = plt.scatter(x_test[:, 0], x_test[:, 1], c=y_test, cmap=plt.cm.Paired, edgecolors='k')
plt.ylim(ymin, ymax)
plt.xlim(xmin, xmax)
plt.legend(handles=scatter.legend_elements()[0], labels=["-1", "1"])
plt.show()

# print accuracy on test data
x_test = torch.cat((x_test, torch.ones(x_test.shape[0], 1)), dim=1)
y_pred = torch.sign(x_test @ w)
print("Accuracy = ", (torch.sum(y_pred == y_test) / y_test.shape[0]).item() * 100, "%")
```




```
Accuracy =  99.50000047683716 %
```

In [ ]:
```
# Running the perceptron training algorithm for different values of gamma for multiple trials
num_trials = 5
gamma_val = []
k_val = []
for gamma in np.linspace(0.1, 2, 20):
    acc = 0
    k_avg = 0
    for i in range(num_trials):
        x, y = create_dataset(1000, gamma)

        # split dataset into train and test
        x_train, y_train, x_test, y_test = split_dataset(x, y, 0.8)
        # x = x_train
        # y = y_train

        x_train = torch.cat((x_train, torch.ones(x_train.shape[0], 1)), dim=1)
        w, k = perceptron_train(x_train, y_train, 100)
        k_avg = k_avg + k

        x_test = torch.cat((x_test, torch.ones(x_test.shape[0], 1)), dim=1)
        y_pred = torch.sign(x_test @ w)
        acc = acc + torch.sum(y_pred == y_test) / y_test.shape[0]
        # print('----------------------------------------------')
    k_avg = k_avg / num_trials
    gamma_val.append(gamma)
    k_val.append(k_avg)
    print('Gamma: %f, Accuracy: %f, Avg. Epochs: %f' % (gamma, acc / num_trials, k_avg))
    # print('----------------------------------------------')

plt.plot(gamma_val, k_val)
plt.xlabel('Gamma')
plt.ylabel('Avg. Epochs')
plt.show()
```

```
Gamma: 0.100000, Accuracy: 0.821000, Avg. Epochs: 100.000000
Gamma: 0.200000, Accuracy: 0.613000, Avg. Epochs: 100.000000
Gamma: 0.300000, Accuracy: 0.525000, Avg. Epochs: 100.000000
Gamma: 0.400000, Accuracy: 0.502000, Avg. Epochs: 100.000000
Gamma: 0.500000, Accuracy: 0.505000, Avg. Epochs: 100.000000
Gamma: 0.600000, Accuracy: 0.510000, Avg. Epochs: 100.000000
Gamma: 0.700000, Accuracy: 0.595000, Avg. Epochs: 100.000000
Gamma: 0.800000, Accuracy: 0.701000, Avg. Epochs: 100.000000
Gamma: 0.900000, Accuracy: 0.779000, Avg. Epochs: 100.000000
Gamma: 1.000000, Accuracy: 0.995000, Avg. Epochs: 32.800000
Gamma: 1.100000, Accuracy: 0.999000, Avg. Epochs: 22.400000
Gamma: 1.200000, Accuracy: 1.000000, Avg. Epochs: 44.600000
Gamma: 1.300000, Accuracy: 0.996000, Avg. Epochs: 4.200000
Gamma: 1.400000, Accuracy: 0.999000, Avg. Epochs: 36.400000
Gamma: 1.500000, Accuracy: 0.999000, Avg. Epochs: 16.200000
Gamma: 1.600000, Accuracy: 0.998000, Avg. Epochs: 30.600000
Gamma: 1.700000, Accuracy: 0.998000, Avg. Epochs: 30.600000
Gamma: 1.800000, Accuracy: 0.996000, Avg. Epochs: 30.600000
Gamma: 1.900000, Accuracy: 0.995000, Avg. Epochs: 14.200000
Gamma: 2.000000, Accuracy: 0.995000, Avg. Epochs: 50.600000
```



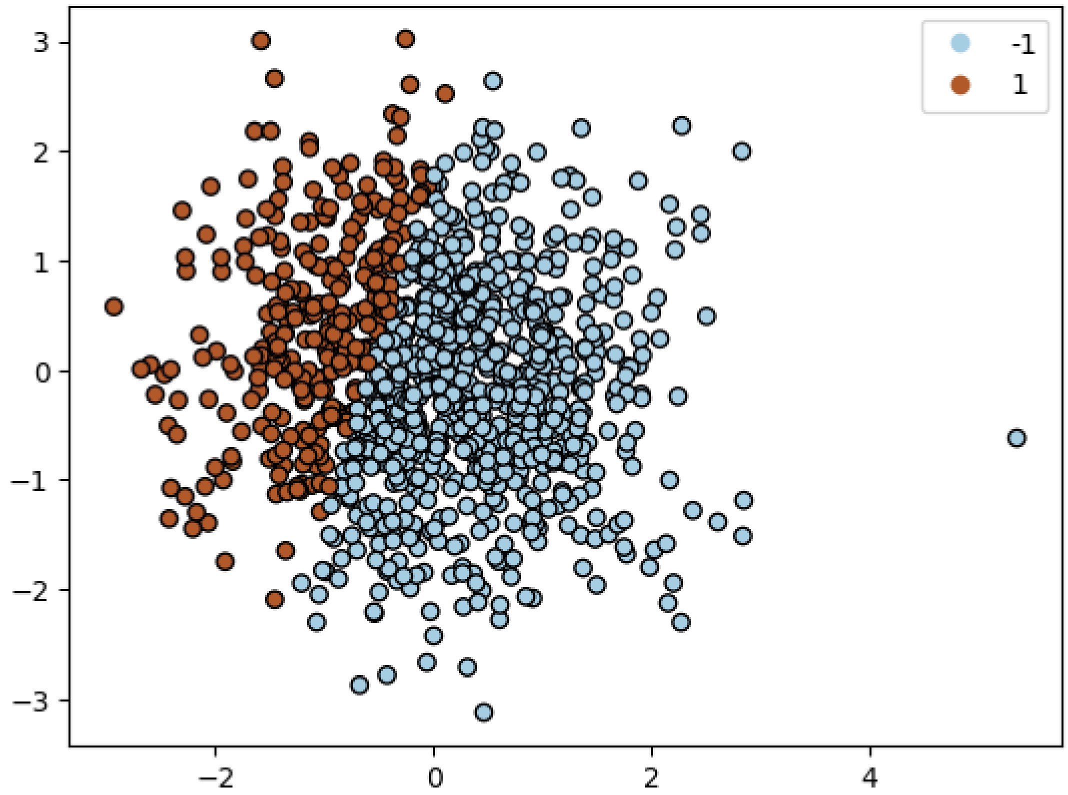
We can see that as the value of γ increases, the average number of epochs required to converge decreases. This is because as γ increases, the data becomes more and more linearly separable and hence the number of epochs required to converge decreases.

Q2

We will be using Hinge loss function with scratch implementation of gradient descent algorithm

In [ ]:
```
# Create a Linearly separable 2D dataset
x, y = create_dataset(1000, 1)
scatter = plt.scatter(x[:, 0], x[:, 1], c=y, cmap=plt.cm.Paired, edgecolors='k')
xmin, xmax, ymin, ymax = plt.axis()
plt.legend(handles=scatter.legend_elements()[0], labels=["-1", "1"])
plt.show()
```

```
print("Percentage of -1 labels: ", (y == -1).sum().item() / len(y))
print("Percentage of 1 labels: ", (y == 1).sum().item() / len(y))
```




```
Percentage of -1 labels:  0.718
Percentage of 1 labels:  0.282
```

In [ ]:

```
x_train, y_train, x_test, y_test = split_dataset(x, y, 0.8)

# Append 1 to x for bias

x_train = torch.cat((x_train, torch.ones(x_train.shape[0], 1)), dim=1)
print("Shape of Training x: ", x_train.shape)
print("Shape of Training y: ", y_train.shape)
```

```
Shape of Training x:  torch.Size([800, 3])
Shape of Training y:  torch.Size([800])
```

In [ ]:

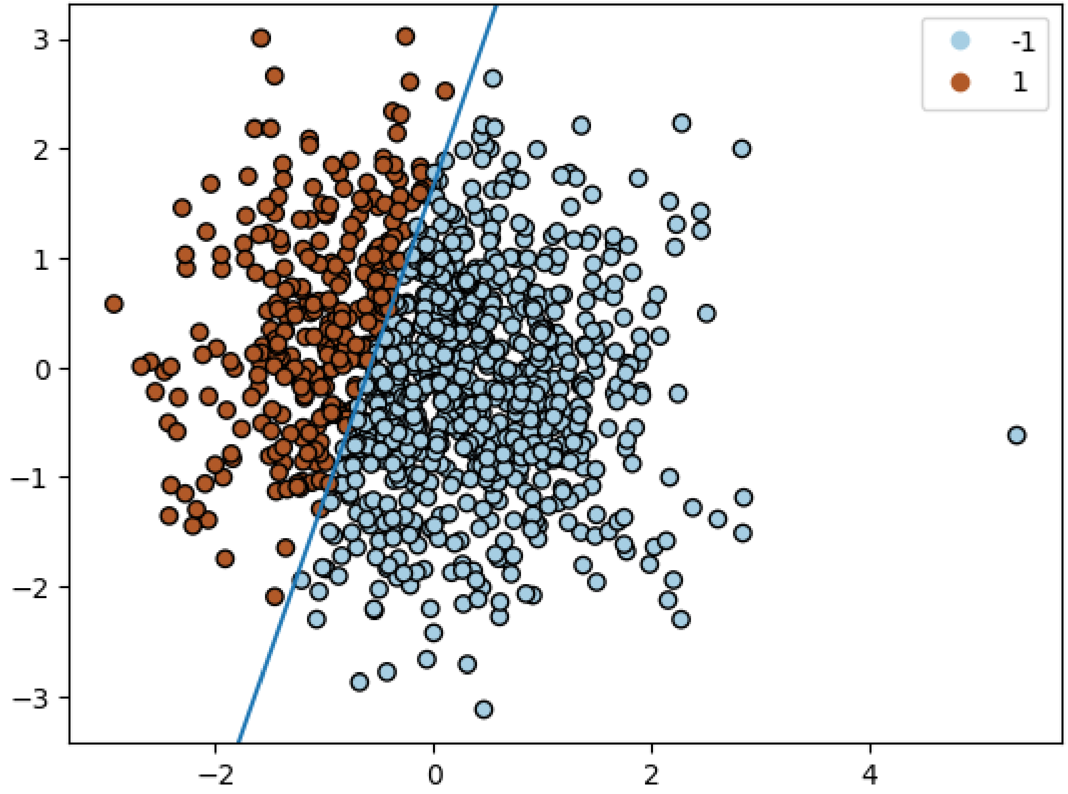
```
# Gradient Descent Algorithm for Hinge Loss
def hinge_gradient_descent(x, y, lr=0.1, max_epochs=100):
    w = torch.zeros(3)
```

```python
    k = 0
    for epoch in range(max_epochs):
        nb_changes = 0
        for i in range(x.size(0)):
            if x[i].dot(w) * y[i] < 1:
                w = w + lr * (y[i]*x[i])
                nb_changes = nb_changes + 1
        if nb_changes == 0:
            # print('Stopping at Epoch: ', epoch)
            break
        k = k + 1
    # print('Number of changes: ', nb_changes)

    # return the weights and number of epochs
    return w, k
```

In [ ]:
```python
w, max_epochs = hinge_gradient_descent(x_train, y_train, 0.1, 100)
print("W = ", w, " Max Epochs Run = ", max_epochs)
```

```
W =  tensor([-17.2833,   6.0684, -10.1000])  Max Epochs Run =  100
```

In [ ]:
```python
# plot the decision boundary
x1 = np.linspace(-4, 4, 100)
x2 = -(w[0] * x1 + w[2]) / w[1]
plt.plot(x1, x2)
scatter = plt.scatter(x[:, 0], x[:, 1], c=y, cmap=plt.cm.Paired, edgecolors='k')
plt.xlim(xmin, xmax)
plt.ylim(ymin, ymax)
plt.legend(handles=scatter.legend_elements()[0], labels=["-1", "1"])
plt.show()

# print accuracy
x_test = torch.cat((x_test, torch.ones(x_test.shape[0], 1)), dim=1)
y_pred = torch.sign(x_test @ w)
print("Accuracy = ", (torch.sum(y_pred == y_test) / y_test.shape[0]).item() * 100, "%")
```

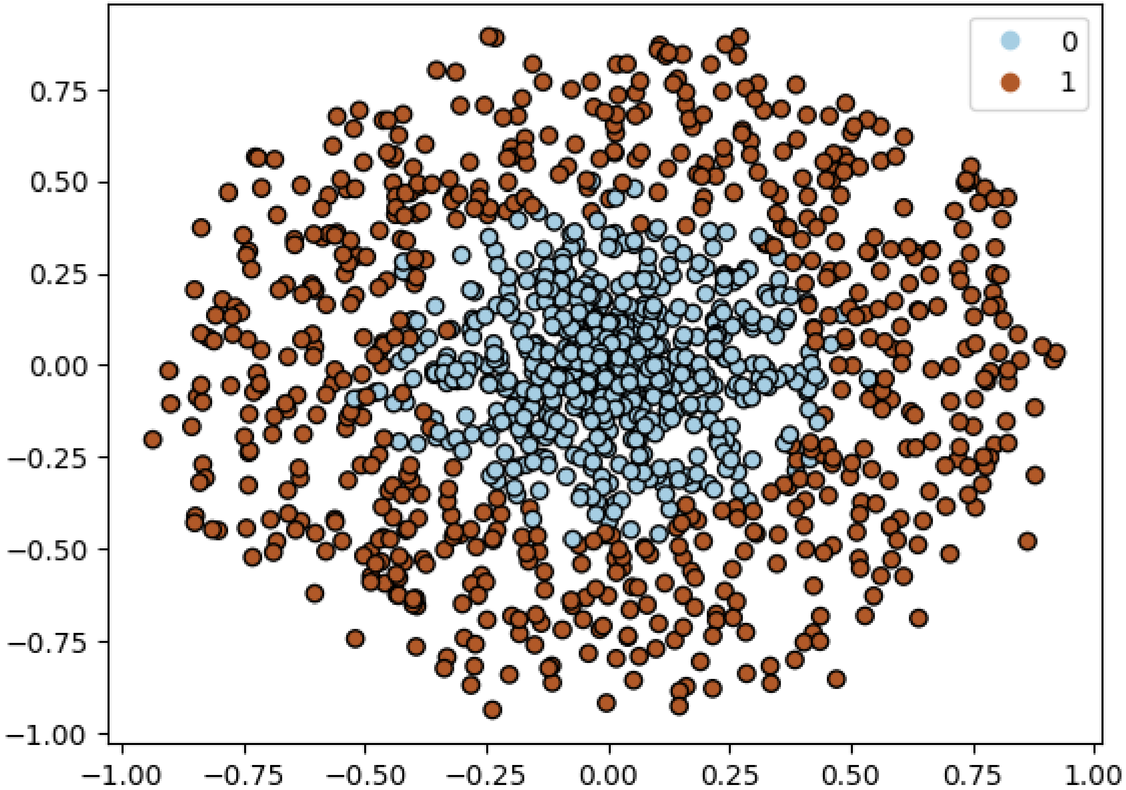



```
Accuracy =  99.00000095367432 %
```

Q3

In [ ]:
```
# create dataset with concentric circles
# import sklearn.datasets as skdata
from sklearn.datasets import make_circles

def create_concentric_dataset(n_samples, factor=0.9, noise=0.05):
    radius = np.random.rand(n_samples) * factor
    angle = np.random.rand(n_samples) * 2 * np.pi

    dataset = np.column_stack((radius * np.cos(angle), radius * np.sin(angle))) + np.random.normal(0, noise, (n_samples, 2))

    labels = np.zeros(n_samples)
    labels[radius > factor / 2] = 1
```

```python
    return dataset, labels

x, y = create_concentric_dataset(n_samples=1250, factor=0.9, noise=0.05)
scatter = plt.scatter(x[:, 0], x[:, 1], c=y, cmap=plt.cm.Paired, edgecolors='k')
plt.legend(handles=scatter.legend_elements()[0], labels=["0", "1"])
plt.show()

print("Percentage of 0 labels: ", (y == 0).sum().item() / len(y))
print("Percentage of 1 labels: ", (y == 1).sum().item() / len(y))
```




```
Percentage of 0 labels:  0.4872
Percentage of 1 labels:  0.5128
```

In [ ]:

```python
print(x.shape)
print(y.shape)
x = torch.from_numpy(x).float()
y = torch.from_numpy(y).float()

x_train, y_train, x_test, y_test = split_dataset(x, y, 0.8)
```

```
print("Shape of Training x: ", x_train.shape)
print("Shape of Training y: ", y_train.shape)
```

```
(1250, 2)
(1250,)
Shape of Training x:  torch.Size([1000, 2])
Shape of Training y:  torch.Size([1000])
```

In [ ]:

```
# Create MLP with 1 hidden layer from scratch
class MLP:
    def __init__(self, x, y, hidden_size=4, lr=0.1) -> None:
        self.x = x
        self.y = y
        self.input_size = x.shape[1]
        self.hidden_size = hidden_size
        self.output_size = 1
        self.lr = lr

        # Weights and Biases
        self.w1 = torch.randn(self.input_size, self.hidden_size)
        # print("w1: ",self.w1.shape)
        self.b1 = torch.randn(1) * torch.randn(self.hidden_size)
        # print("b1: ",self.b1.shape)
        self.w2 = torch.randn(self.hidden_size, self.output_size)
        # print("w2: ",self.w2.shape)
        self.b2 = torch.randn(1)


    # Signmoid Activation Function
    def sigmoid(self, x):
        return 1 / (1 + torch.exp(-x))

    # Signmoid Derivative
    def sigmoid_derivative(self, x):
        return x * (1 - x)

    # Hinge Loss Function
    def hinge_loss(self, y_pred, y):
        # print(y_pred.shape)
        # print(y.shape)
        # print((y_pred * y).shape)
        return torch.max(torch.zeros_like(y_pred), 1 - y_pred * y)

    # Square Loss Function
```

```python
    def square_loss(self, y_pred, y):
        return (y_pred - y) ** 2

    # Hinge Loss Derivative
    def hinge_loss_derivative(self, y_pred, y):
        # print(y_pred.shape)
        # print(y.shape)
        return -y * (y * y_pred < 1)

    # Square Loss Derivative
    def square_loss_derivative(self, y_pred, y):
        return 2 * (y_pred - y)

    # Binary Cross Entropy Loss Function
    def binary_cross_entropy(self, y_pred, y):
        return - y * torch.log(y_pred) - (1 - y) * torch.log(1 - y_pred)

    # Binary Cross Entropy Loss Derivative
    def binary_cross_entropy_derivative(self, y_pred, y):
        return  -(y / y_pred) + ((1 - y) / (1 - y_pred))

    # Forward Propagation
    def forward(self, x):
        self.z1 = x @ self.w1 + torch.Tensor.repeat(self.b1, x.shape[0], 1)
        # print("z1: ",self.z1.shape)

        self.a1 = self.sigmoid(self.z1)
        # print("a1: ",self.a1.shape)

        self.z2 = self.a1 @ self.w2 + self.b2
        # print("z2: ",self.z2)

        self.a2 = self.sigmoid(self.z2)
        # print("a2: ",self.a2)

        # print("w2: ",self.w2)
        # print("w1: ",self.w1)
        return self.a2

    # Backward Propagation
    def backward(self, x, y):
        y = y.reshape(-1, 1)
        # print("x shape: ", x.shape)
```

```
        self.loss_a2 = self.binary_cross_entropy(self.a2, y)
        # print("Loss_a2 shape: ", self.loss_a2.shape)

        # Calculate Gradients
        self.dL_d_a2 = self.binary_cross_entropy_derivative(self.a2, y)
        # print("dL_d_a2 shape: ", self.dL_d_a2.shape)

        self.da2_dz2 = self.sigmoid_derivative(self.a2)
        # print("da2_dz2 shape: ", self.da2_dz2.shape)

        self.dz2_d_w2 = self.a1.T
        # print("dz2_d_w2 shape: ", self.dz2_d_w2.shape)

        self.dz2_d_b2 = torch.ones_like(self.z2)
        # print("dz2_d_b2 shape: ", self.dz2_d_b2.shape)

        self.dL_d_w2 = self.dz2_d_w2 @ (self.dL_d_a2 * self.da2_dz2)
        # print("dL_d_w2 shape: ", self.dL_d_w2.shape)

        # self.dL_d_b2 = (self.dz2_d_b2 * (self.dL_d_a2.T @ self.da2_dz2)).reshape(-1)
        self.dL_d_b2 = ((self.dL_d_a2 * self.da2_dz2).T @ self.dz2_d_b2).reshape(-1)
        # print("dL_d_b2 shape: ", self.dL_d_b2.shape)

        self.da1_dz1 = self.sigmoid_derivative(self.a1)
        # print("da1_dz1 shape: ", self.da1_dz1.shape)

        self.dz1_dw1 = x
        # print("dz1_dw1 shape: ", self.dz1_dw1.shape)

        self.dz1_x = self.w1

        self.dz2_d_a1 = self.w2

        # print("dz2_d_a1 shape: ", self.dz2_d_a1.shape)
        # print("dz1_x shape: ", self.dz1_x.shape)
        # self.dz1_d_b1 = torch.ones_like(self.a1)
        # print("dz1_d_b1 shape: ", self.dz1_d_b1.shape)

        # print("ter: ", ((self.dL_d_a2 * self.da2_dz2) @ self.w2.T).shape)
        self.dL_dw1 =  self.dz1_dw1.T @ (((self.dL_d_a2 * self.da2_dz2) @ self.w2.T) * self.da1_dz1)

        # print("dL_dw1 shape: ", self.dL_dw1.shape)
        # print("ter ",(((self.dL_d_a2 * self.da2_dz2) @ self.w2.T) @ self.da1_dz1.T).shape)
        # print((((self.dL_d_a2 * self.da2_dz2).T @ (self.da1_dz1 * self.dz1_d_b1))).T.shape)
```

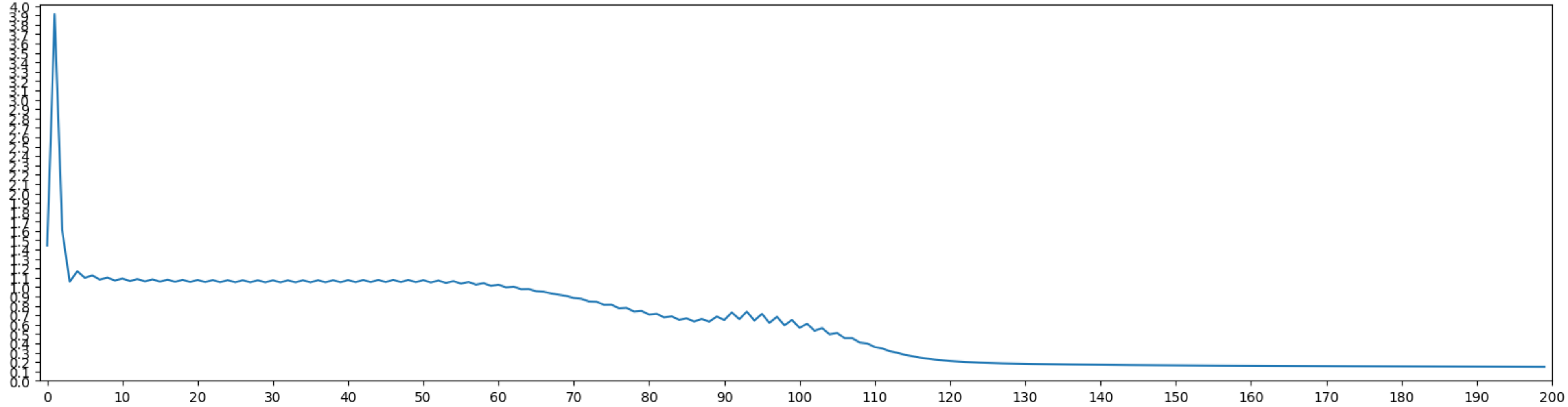
```
        self.dL_db1 = (((self.dL_d_a2 * self.da2_dz2) @ self.w2.T) * self.da1_dz1).sum(axis=0)

        # print("dL_db1 shape: ", self.dL_db1.shape)

        # Updating Weights and Biases
        self.w2 -= self.lr * self.dL_d_w2
        self.b2 -= self.lr * self.dL_d_b2
        self.w1 -= self.lr * self.dL_dw1
        self.b1 -= self.lr * self.dL_db1


    # Training the Model
    def train(self, epochs=100):
        loss = []
        for epoch in range(epochs):
            self.forward(self.x)
            # print("W1 in forward: ", self.w1)
            # print("W2 in forward: ", self.w2)
            # print("B2 in forward: ", self.b2)
            # print("A2 in forward: ", self.a2)
            self.backward(self.x, self.y)
            # print("W1 in backward: ", self.w1)
            # print("W2 in backward: ", self.w2)
            # print("B2 in backward: ", self.b2)

            loss.append(self.loss_a2.mean())
            if epoch % 10 == 0:
                print("Epoch: ", epoch, " Loss: ", self.loss_a2.mean())
        # print("Final Predicted: ", self.forward(self.x))
        # print("Ground Truth: ", self.y)
        # print("z2: ", self.z2)

        # Plotting the Loss Curve
        plt.figure(figsize=(20, 5))
        plt.xticks(np.arange(0, epochs + 10, 10))
        plt.plot(loss)
        plt.xlim(-1, epochs)
        plt.ylim(min(loss), max(loss) + 0.1)
        plt.yticks(np.arange(0, max(loss) + 0.1, 0.1))
        plt.show()
        return

    def predict(self, x, threshold=0.5):
```

```python
        predicted = self.forward(x)
        # print(torch.round(predicted))
        # apply threshold of 0.6 for label
        # print(predicted)

        return predicted > threshold


    def accuracy(self, x, y):
        # print(self.predict(x).shape)
        y = y.reshape(-1, 1)
        return torch.sum(self.predict(x) == y) / y.shape[0]
```

In [ ]:

```python
# create MLP
mlp = MLP(x_train, y_train, hidden_size=4, lr=0.01)
mlp.train(epochs=200)

print("Accuracy on Test set: ", mlp.accuracy(x_test, y_test).item() * 100, "%")

# plot the predicted labels on the test set
scatetr = plt.scatter(x_test[:, 0], x_test[:, 1], c=mlp.predict(x_test).detach().numpy(),  cmap=plt.cm.Paired, edgecolors='k')
plt.legend(handles=scatetr.legend_elements()[0], labels=['0', '1'])
plt.show()
```

```
Epoch:  0  Loss:  tensor(1.4430)
Epoch:  10  Loss:  tensor(1.0920)
Epoch:  20  Loss:  tensor(1.0762)
Epoch:  30  Loss:  tensor(1.0729)
Epoch:  40  Loss:  tensor(1.0751)
Epoch:  50  Loss:  tensor(1.0746)
Epoch:  60  Loss:  tensor(1.0250)
Epoch:  70  Loss:  tensor(0.8837)
Epoch:  80  Loss:  tensor(0.7067)
Epoch:  90  Loss:  tensor(0.6489)
Epoch:  100  Loss:  tensor(0.5653)
Epoch:  110  Loss:  tensor(0.3599)
Epoch:  120  Loss:  tensor(0.2105)
Epoch:  130  Loss:  tensor(0.1802)
Epoch:  140  Loss:  tensor(0.1703)
Epoch:  150  Loss:  tensor(0.1643)
Epoch:  160  Loss:  tensor(0.1599)
Epoch:  170  Loss:  tensor(0.1563)
Epoch:  180  Loss:  tensor(0.1533)
Epoch:  190  Loss:  tensor(0.1506)
```




```
Accuracy on Test set:  96.39999866485596 %
```