

Assignment 6

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Download all python codes from

<https://github.com/tanmaygoyal258/AI1103---Probability/tree/main/Assignment6/code.py>

and latex-tikz codes from

<https://github.com/tanmaygoyal258/AI1103---Probability/blob/main/Assignment6/main.tex>

Converting X to a standard normal random variable using :

$$Z = \frac{X - \mu_x}{\sigma_x} \quad (2.0.5)$$

$$\Pr(X \leq 1) \quad (2.0.6)$$

$$= \Pr\left(\frac{X - 2}{2\sqrt{2}} \leq \frac{1 - 2}{2\sqrt{2}}\right) \quad (2.0.7)$$

$$= \Pr\left(Z \leq \frac{-1}{2\sqrt{2}}\right) \quad (2.0.8)$$

1 PROBLEM

$X(t)$ is a random process with a constant mean value of 2 and the autocorrelation function:

$$R_x(\tau) = 4[e^{-0.2|\tau|} + 1] \quad (1.0.1)$$

Let X be the Gaussian Random Variable obtained by sampling the process at $t = t_i$, and let

$$Q(\alpha) = \int_{\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \quad (1.0.2)$$

Find $\Pr(X \leq 1)$

- (A) $1 - Q(0.5)$ (B) $Q(0.5)$
 (C) $Q(\frac{1}{2\sqrt{2}})$ (D) $1 - Q(\frac{1}{2\sqrt{2}})$

2 SOLUTION

X is a normal random variable defined by

$$X \sim N(2, \sigma_x^2) \quad (2.0.1)$$

Thus, from (1.0.1):

$$\text{Var}(X) = \sigma_x^2 = R_x(0) \quad (2.0.2)$$

$$\sigma_x^2 = 8 \quad (2.0.3)$$

$$\sigma_x = 2\sqrt{2} \quad (2.0.4)$$

where Z is a standard normal random variable defined by $Z \sim N(0, 1)$

Due to symmetry of the bell curve graph:

$$\Pr\left(Z \leq \frac{-1}{2\sqrt{2}}\right) = \Pr\left(Z \geq \frac{1}{2\sqrt{2}}\right) \quad (2.0.9)$$

From (1.0.2),

$$Q(\alpha) = \int_{\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = \Pr(Z \geq \alpha) \quad (2.0.10)$$

Thus,

$$\Pr\left(Z \geq \frac{1}{2\sqrt{2}}\right) = Q\left(\frac{1}{2\sqrt{2}}\right) \quad (2.0.11)$$