

Assignment 4

Tanmay Goyal - AI20BTECH11021

Download all python codes from

<https://github.com/tanmaygoyal258/AI1103---Probability/blob/main/Assignment4/code.py>

and latex-tikz codes from

<https://github.com/tanmaygoyal258/AI1103---Probability/blob/main/Assignment4/main.tex>

1 PROBLEM

The number N of persons getting injured in a bomb blast at a busy market place is a random variable having a Poisson Distribution with parameter $\lambda (\geq 1)$. A person injured in the explosion may either suffer a minor injury requiring first aid or suffer a major injury requiring hospitalisation. Let the number of persons with minor injury be N_1 and the conditional distribution of N_1 given N is

$$\Pr(N_1 = i | N) = \frac{1}{N} \quad (1.0.1)$$

Find the expected number of persons requiring hospitalisation.

2 SOLUTION

Let N_2 be the number of persons hospitalised.

Let $N = a$, and $N_1 = i (i \leq a)$, then, $N_2 = a - i$

We know,

$$\Pr(A|B) = \frac{\Pr(A, B)}{\Pr(B)} \quad (2.0.1)$$

Then, from (1.0.1) :

$$\frac{\Pr(N_1 = i, N = a)}{\Pr(N = a)} = \frac{1}{a} \quad (2.0.2)$$

$$\Pr(N_1 = i, N = a) = \frac{1}{a} \frac{e^{-\lambda} \lambda^a}{a!} \quad (2.0.3)$$

$$\Pr(N_2 = a - i, N = a) = \frac{1}{a} \frac{e^{-\lambda} \lambda^a}{a!} \quad (2.0.4)$$

using the fact that, for a Poisson Distribution:

$$\Pr(N = x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad (2.0.5)$$

where λ is the parameter.

To obtain p.d.f of N_2 , we sum over all possible values of N :

$$\Pr(N_2 = a - i) = \sum_{a=0}^{\infty} \frac{1}{a} \frac{e^{-\lambda} \lambda^a}{a!} \quad (2.0.6)$$

Thus,

$$E(N_2) = \sum_{a=0}^{\infty} \sum_{i=1}^a (a - i) \times \frac{1}{a} \frac{e^{-\lambda} \lambda^a}{a!} \quad (2.0.7)$$

$$= \sum_{a=0}^{\infty} \frac{e^{-\lambda} \lambda^a}{a!} \sum_{i=1}^a \frac{a - i}{a} \quad (2.0.8)$$

$$= \sum_{a=0}^{\infty} \frac{e^{-\lambda} \lambda^a}{a!} \left(a - \frac{(a + 1)}{2} \right) \quad (2.0.9)$$

$$= \sum_{a=0}^{\infty} \frac{e^{-\lambda} \lambda^a}{a!} \frac{a - 1}{2} \quad (2.0.10)$$

$$= \frac{e^{-\lambda}}{2} \left[\sum_{a=0}^{\infty} \frac{a \lambda^a}{a!} - \sum_{a=0}^{\infty} \frac{\lambda^a}{a!} \right] \quad (2.0.11)$$

$$= \frac{e^{-\lambda}}{2} \left[\lambda \sum_{a=1}^{\infty} \frac{\lambda^{a-1}}{(a-1)!} - \sum_{a=0}^{\infty} \frac{\lambda^a}{a!} \right] \quad (2.0.12)$$

$$= \frac{e^{-\lambda}}{2} [\lambda e^{\lambda} - e^{\lambda}] \quad (2.0.13)$$

$$= \frac{\lambda - 1}{2} \quad (2.0.14)$$