Assignment 4

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Download all python codes from

https://github.com/tanmaygoyal258/AI1103---Probability/blob/main/Assignment4/code.py

and latex-tikz codes from

https://github.com/tanmaygoyal258/AI1103---Probability/blob/main/Assignment4/main.tex

1 Problem

The number N of persons getting injured in a bomb blast at a busy market place is a random variable having a Poisson Distribution with parameter $\lambda(\geq 1)$. A person injured in the explosion may either suffer a minor injury requiring first aid or suffer a major injury requiring hospitalisation. Let the number of persons with minor injury be N_1 and the conditional distribution of N_1 given N is

$$\Pr(N_1 = i|N) = \frac{1}{N} \tag{1.0.1}$$

Find the expected number of persons requiring hospitalisation.

2 Solution

We know,

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} \tag{2.0.1}$$

Also, for a Poisson Distribution:

$$\Pr(N = x) = \frac{e^{-\lambda}\lambda^x}{x!} \tag{2.0.2}$$

where λ is the parameter
Let N_2 be the number of persons hospitalised.
Let $N = a$, and $N_1 = i(i \leq a)$, then, $N_2 = a - i$
Then, from (1.0.1) :

$$\frac{\Pr(N_1 = i \cap N = a)}{\Pr(N = a)} = \frac{1}{a} \tag{2.0.3}$$

$$\Pr(N_1 = i \cap N = a) = \frac{1}{a}\frac{e^{-\lambda}\lambda^a}{a!} \tag{2.0.4}$$

$$\Pr(N_2 = a - i) = \Pr(N_1 = i) \tag{2.0.5}$$

$$= \frac{1}{a}\frac{e^{-\lambda}\lambda^a}{a!} \tag{2.0.6}$$

Thus,

$$E(N_2) = \sum_{a=0}^{\infty}\sum_{i=0}^{a}(a - i) \times \frac{1}{a}\frac{e^{-\lambda}\lambda^a}{a!} \tag{2.0.7}$$

$$= \sum_{a=0}^{\infty}\frac{e^{-\lambda}\lambda^a}{a!}\sum_{i=0}^{a}\frac{a-i}{a} \tag{2.0.8}$$

$$= \sum_{a=0}^{\infty}\frac{e^{-\lambda}\lambda^a}{a!}\left(a - \frac{(a+1)}{2}\right) \tag{2.0.9}$$

$$= \sum_{a=0}^{\infty}\frac{e^{-\lambda}\lambda^a}{a!}\frac{a-1}{2} \tag{2.0.10}$$

$$= \frac{e^{-\lambda}}{2}\left[\sum_{a=0}^{\infty}\frac{a\lambda^a}{a!} - \sum_{a=0}^{\infty}\frac{\lambda^a}{a!}\right] \tag{2.0.11}$$

$$= \frac{e^{-\lambda}}{2}\left[\lambda\sum_{a=1}^{\infty}\frac{\lambda^{a-1}}{(a-1)!} - \sum_{a=0}^{\infty}\frac{\lambda^a}{a!}\right] \tag{2.0.12}$$

$$= \frac{e^{-\lambda}}{2}\left[\lambda e^{\lambda} - e^{\lambda}\right] \tag{2.0.13}$$

$$= \frac{\lambda - 1}{2} \tag{2.0.14}$$