#### 1

# Assignment 4

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## Download all python codes from

https://github.com/tanmaygoyal258/AI1103---Probability/blob/main/Assignment4/code.py

and latex-tikz codes from

https://github.com/tanmaygoyal258/AI1103---Probability/blob/main/Assignment4/main.tex

#### 1 Problem

The number N of persons getting injured in a bomb blast at a busy market place is a random variable having a Poisson Distribution with parameter  $\lambda(\geq 1)$ . A person injured in the explosion may either suffer a minor injury requiring first aid or suffer a major injury requiring hospitalisation. Let the number of persons with minor injury be  $N_1$  and the conditional distribution of  $N_1$  given N is

$$\Pr(N_1 = i|N) = \frac{1}{N}$$
 (1.0.1)

Find the expected number of persons requiring hospitalisation.

### 2 Solution

Let  $N_2$  be the number of persons hospitalised. Let N=a, and  $N_1=i(i \le a)$ , then,  $N_2=a-i$ We know,

$$Pr(A|B) = \frac{Pr(A,B)}{Pr(B)}$$
 (2.0.1)

Then, from (1.0.1):

$$\frac{\Pr(N_1 = i, N = a)}{\Pr(N = a)} = \frac{1}{a}$$
 (2.0.2)

$$\Pr(N_1 = i, N = a) = \frac{1}{a} \frac{e^{-\lambda} \lambda^a}{a!}$$
 (2.0.3)

$$\Pr(N_2 = a - i, N = a) = \frac{1}{a} \frac{e^{-\lambda} \lambda^a}{a!}$$
 (2.0.4)

Thus,

$$E(N_2) = \sum_{a=0}^{\infty} \sum_{i=0}^{a} (a-i) \times \frac{1}{a} \frac{e^{-\lambda} \lambda^a}{a!}$$
 (2.0.5)

$$= \sum_{a=0}^{\infty} \frac{e^{-\lambda} \lambda^a}{a!} \sum_{i=0}^{a} \frac{a-i}{a}$$
 (2.0.6)

$$= \sum_{a=0}^{\infty} \frac{e^{-\lambda} \lambda^a}{a!} \left( a - \frac{(a+1)}{2} \right)$$
 (2.0.7)

$$= \sum_{a=0}^{\infty} \frac{e^{-\lambda} \lambda^a}{a!} \frac{a-1}{2}$$
 (2.0.8)

$$=\frac{e^{-\lambda}}{2}\left[\sum_{a=0}^{\infty}\frac{a\lambda^a}{a!}-\sum_{a=0}^{\infty}\frac{\lambda^a}{a!}\right]$$
(2.0.9)

$$= \frac{e^{-\lambda}}{2} \left[ \lambda \sum_{a=1}^{\infty} \frac{\lambda^{a-1}}{(a-1)!} - \sum_{a=0}^{\infty} \frac{\lambda^a}{a!} \right]$$
 (2.0.10)

$$=\frac{e^{-\lambda}}{2}\left[\lambda e^{\lambda}-e^{\lambda}\right] \qquad (2.0.11)$$

$$=\frac{\lambda-1}{2}\tag{2.0.12}$$