Assignment 4

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Download all python codes from

https://github.com/tanmaygoyal258/AI1103---Probability/blob/main/Assignment4/code.py

and latex-tikz codes from

https://github.com/tanmaygoyal258/AI1103---Probability/blob/main/Assignment4/main.tex

1 Problem

The number N of persons getting injured in a bomb blast at a busy market place is a random variable having a Poisson Distribution with parameter $\lambda(\geq 1)$. A person injured in the explosion may either suffer a minor injury requiring first aid or suffer a major injury requiring hospitalisation. Let the number of persons with minor injury be N_1 and the conditional distribution of N_1 given N is

$$\Pr\left(N_1 = i|N\right) = \frac{1}{N} \tag{1.0.1}$$

Find the expected number of persons requiring hospitalisation.

2 Solution

Let N_2 be the number of persons hospitalised.
Let $N = a$, and $N_1 = i (i \leq a)$, then, $N_2 = a - i$
We know,

$$\Pr\left(A|B\right) = \frac{\Pr\left(A, B\right)}{\Pr\left(B\right)} \tag{2.0.1}$$

Then, from (1.0.1) :

$$\frac{\Pr\left(N_1 = i, N = a\right)}{\Pr\left(N = a\right)} = \frac{1}{a} \tag{2.0.2}$$

$$\Pr\left(N_1 = i, N = a\right) = \frac{\Pr\left(N = a\right)}{a} = \frac{1}{a}\frac{e^{-\lambda}\lambda^a}{a!} \tag{2.0.3}$$

using the fact that, for a Poisson Distribution:

$$\Pr\left(N = a\right) = \frac{e^{-\lambda}\lambda^a}{a!} \tag{2.0.4}$$

where λ is the parameter.
Thus, from (2.0.4), we obtain:

$$\Pr\left(N_2 = a - i, N = a\right) = \frac{1}{a}\frac{e^{-\lambda}\lambda^a}{a!} \tag{2.0.5}$$

To obtain p.d.f of N_2, we sum over all possible values of N:

$$\Pr\left(N_2 = a - i\right) = \sum_{a=0}^{\infty}\frac{1}{a}\frac{e^{-\lambda}\lambda^a}{a!} \tag{2.0.6}$$

Thus,

$$\begin{aligned}
E(N_2) &= \sum_{a=0}^{\infty}\sum_{i=1}^{a}(a-i)\times\frac{1}{a}\frac{e^{-\lambda}\lambda^a}{a!} && (2.0.7)\\
&= \sum_{a=0}^{\infty}\frac{e^{-\lambda}\lambda^a}{a!}\sum_{i=1}^{a}\frac{a-i}{a} && (2.0.8)\\
&= \sum_{a=0}^{\infty}\frac{e^{-\lambda}\lambda^a}{a!}\left(a - \frac{(a+1)}{2}\right) && (2.0.9)\\
&= \sum_{a=0}^{\infty}\frac{e^{-\lambda}\lambda^a}{a!}\frac{a-1}{2} && (2.0.10)\\
&= \frac{e^{-\lambda}}{2}\left[\sum_{a=0}^{\infty}\frac{a\lambda^a}{a!} - \sum_{a=0}^{\infty}\frac{\lambda^a}{a!}\right] && (2.0.11)\\
&= \frac{e^{-\lambda}}{2}\left[\lambda\sum_{a=1}^{\infty}\frac{\lambda^{a-1}}{(a-1)!} - \sum_{a=0}^{\infty}\frac{\lambda^a}{a!}\right] && (2.0.12)\\
&= \frac{e^{-\lambda}}{2}\left[\lambda e^{\lambda} - e^{\lambda}\right] && (2.0.13)\\
&= \frac{\lambda - 1}{2} && (2.0.14)
\end{aligned}$$