Assignment 3

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Download all python codes from

https://github.com/tanmaygoyal258/AI1103---Probability/tree/main/Assignment3/Codes

and latex-tikz codes from

https://github.com/tanmaygoyal258/AI1103---Probability/blob/main/Assignment3/main.tex

1 Problem

Let X be a random variable with a probability density function

$$f(x) = \begin{cases} 0.2 & |x| \leq 1 \\ 0.1 & 1 \leq |x| \leq 4 \\ 0 & otherwise \end{cases} \tag{1.0.1}$$

Find $\Pr(0.5 < X \leq 5)$

2 Solution

We know, if X is a continuous random variable, and its p.d.f is given by $f(x)$, then we define the c.d.f $F(x)$ as:

$$F(x) = \Pr(X \leq x) \tag{2.0.1}$$

and is given by:

$$F(x) = \int_{-\infty}^{x} f(x)\,dx \tag{2.0.2}$$

$f(x)$ is a valid p.d.f because:

1) The area under the curve of the p.d.f is 1, i.e:

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \tag{2.0.3}$$

2) $f(x) \geq 0$ for all $x \in \mathbb{R}$

Since $f(x)$ is a valid p.d.f, from (2.0.2), we get the following c.d.f:

$$F(x) = \begin{cases} 0 & x \leq -4 \\ 0.1(x+4) & -4 \leq x \leq -1 \\ 0.3 + 0.2(x+1) & -1 \leq x \leq 1 \\ 0.7 + 0.1(x-1) & 1 \leq x \leq 4 \\ 1 & 4 \leq x \end{cases} \tag{2.0.4}$$

Thus, from (2.0.1) and (2.0.4), we get:

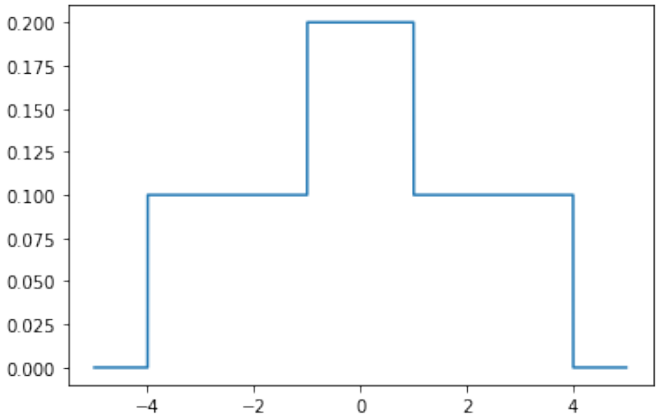
$$\Pr(X \leq 0.5) = 0.6 \tag{2.0.5}$$

$$\Pr(X \leq 5) = 1 \tag{2.0.6}$$

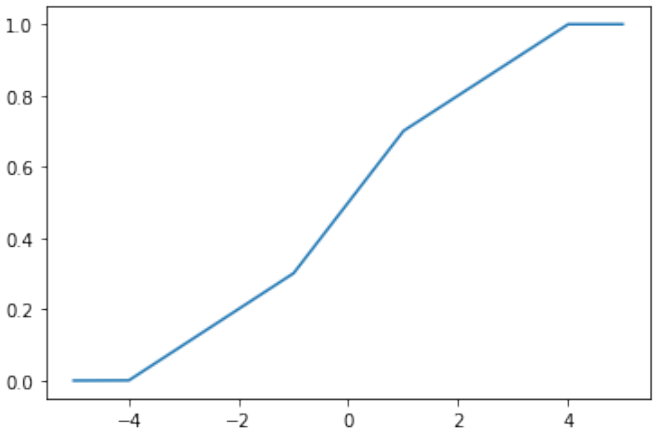
Thus,

$$\Pr(0.5 \leq X \leq 5) = \Pr(X \leq 5) - \Pr(X \leq 0.5) = 0.4 \tag{2.0.7}$$

The p.d.f is shown below:



The c.d.f is shown below:


1.0
0.8
0.6
0.4
0.2
0.0
−4
−2
0
2
4