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Assignment 5

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Download all python codes from

https://github.com/tanmaygoyal258/EE3900-Linear -Systems-and-Signal-processing/blob/main/ Assignment5/code.py

Download all latex codes from

https://github.com/tanmaygoyal258/EE3900-Linear -Systems-and-Signal-processing/blob/main/ Assignment5/main.tex

1 Problem

(Quadratic Forms/Q.2.23) Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$

2 Solution

Let
$$y = \sqrt{5}x^2 + x + \sqrt{5} = 0$$
.

Then, y can be represented in the vector form as:

$$y = \mathbf{x}^T \begin{pmatrix} \sqrt{5} & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} + \sqrt{5}$$
 (2.0.1)

where

$$\mathbf{x} = \begin{pmatrix} x \\ 0 \end{pmatrix} \tag{2.0.2}$$

Substituting y = 0, we get:

$$\mathbf{x}^T \begin{pmatrix} \sqrt{5} & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} + \sqrt{5} = 0 \tag{2.0.3}$$

$$\sqrt{5}x^2 + x + \sqrt{5} = 0 \tag{2.0.4}$$

$$\left(x - \left(\frac{-1}{2\sqrt{5}}\right)\right) \left(x - \left(\frac{-1}{2\sqrt{5}}\right)\right) = 0 \qquad (2.0.5)$$

$$x = \begin{pmatrix} \frac{-1}{2\sqrt{5}} \\ \frac{\sqrt{19}}{2\sqrt{5}} \end{pmatrix}, \begin{pmatrix} \frac{-1}{2\sqrt{5}} \\ \frac{-\sqrt{19}}{2\sqrt{5}} \end{pmatrix}$$
 (2.0.6)

$$x = \begin{pmatrix} -0.223 \\ 0.974 \end{pmatrix}, \begin{pmatrix} -0.223 \\ -0.974 \end{pmatrix}$$
 (2.0.7)

The roots can be verified using the python code. As we can see from the graph, $\sqrt{5}x^2 + x + \sqrt{5} = 0$ does not intersect the x-axis anywhere, and hence has no real roots.

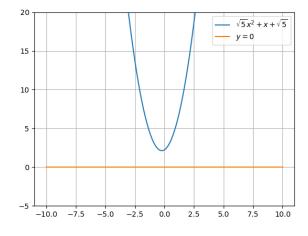


Fig. 0: Graph of $\sqrt{5}x^2 + x + \sqrt{5} = 0$