Assignment 1

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Download all python codes from

https://github.com/tanmaygoyal258/EE3900−Linear−Systems−and−Signal−processing/blob/main/Assignment1/code.py

Download all latex codes from

https://github.com/tanmaygoyal258/EE3900−Linear−Systems−and−Signal−processing/blob/main/Assignment1/main.tex

1 Problem

Prove that the points $\mathbf{A} = \begin{pmatrix}-1\\0\end{pmatrix}$, $\mathbf{B} = \begin{pmatrix}3\\1\end{pmatrix}$, $\mathbf{C} = \begin{pmatrix}2\\2\end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix}-2\\1\end{pmatrix}$ are the vertices of a parallelogram. Find $\mathbf{E}, \mathbf{F}, \mathbf{G}, \mathbf{H}$, the midpoints of AB, BC, CD and AD respectively. Show that EG and FH bisect each other.

2 Solution

Two lines can be said to be parallel, if their directional derivatives are the same.
The directional derivative of **AB** is:

$$\begin{pmatrix}-1-3\\0-1\end{pmatrix} = \begin{pmatrix}-4\\-1\end{pmatrix} \tag{2.0.1}$$

The directional derivative of **BC** is:

$$\begin{pmatrix}3-2\\1-2\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix} \tag{2.0.2}$$

The directional derivative of **CD** is:

$$\begin{pmatrix}2-(-2)\\2-1\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix} \tag{2.0.3}$$

The directional derivative of **AD** is:

$$\begin{pmatrix}-1-(-2)\\0-1\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix} \tag{2.0.4}$$

The directional derivative of **AC** is:

$$\begin{pmatrix}-1-2\\0-2\end{pmatrix} = \begin{pmatrix}-3\\-2\end{pmatrix} \tag{2.0.5}$$

The directional derivative of **BD** is:

$$\begin{pmatrix}3-(-2)\\1-1\end{pmatrix} = \begin{pmatrix}-5\\0\end{pmatrix} \tag{2.0.6}$$

From (2.0.1) and (2.0.3), we can see **AB** and **CD** are parallel to one another. Similarly, from (2.0.2) and (2.0.4), we can see **BC** and **AD** are parallel to one another.
Since the two pairs of opposite lines are parallel to one another, we can say that the set of vertices represent a **parallelogram**.

We know that if the mid-point of two vectors $\mathbf{X}$ and $\mathbf{Y}$ is given by $\mathbf{Z}$, then:

$$\mathbf{Z} = \frac{\mathbf{X}+\mathbf{Y}}{2} \tag{2.0.7}$$

Thus, using (2.0.7),
$\mathbf{E}$ is the midpoint of **AB**, given by:

$$\mathbf{E} = \frac{1}{2}\begin{pmatrix}-1+3\\0+1\end{pmatrix} = \begin{pmatrix}1\\\frac{1}{2}\end{pmatrix} \tag{2.0.8}$$

$\mathbf{F}$ is the midpoint of **BC**, given by:

$$\mathbf{F} = \frac{1}{2}\begin{pmatrix}3+2\\1+2\end{pmatrix} = \begin{pmatrix}\frac{5}{2}\\\frac{3}{2}\end{pmatrix} \tag{2.0.9}$$

$\mathbf{G}$ is the midpoint of **CD**, given by:

$$\mathbf{G} = \frac{1}{2}\begin{pmatrix}2+(-2)\\2+1\end{pmatrix} = \begin{pmatrix}0\\\frac{3}{2}\end{pmatrix} \tag{2.0.10}$$

$\mathbf{H}$ is the midpoint of **AD**, given by:

$$\mathbf{H} = \frac{1}{2}\begin{pmatrix}-1-2\\0+1\end{pmatrix} = \begin{pmatrix}\frac{-3}{2}\\\frac{1}{2}\end{pmatrix} \tag{2.0.11}$$

Let $\mathbf{P}$ and $\mathbf{Q}$ be the midpoints of **EG** and **FH**. EG and FH would bisect one another if $\mathbf{P} = \mathbf{Q}$

$$\mathbf{P} = \frac{1}{2}\begin{pmatrix}1+0\\\frac{1}{2}+\frac{3}{2}\end{pmatrix} = \begin{pmatrix}\frac{1}{2}\\1\end{pmatrix} \tag{2.0.12}$$

$$\mathbf{Q} = \frac{1}{2}\begin{pmatrix}\frac{5}{2}+\frac{-3}{2}\\\frac{3}{2}+\frac{1}{2}\end{pmatrix} = \begin{pmatrix}\frac{1}{2}\\1\end{pmatrix} \tag{2.0.13}$$

Since $\mathbf{P} = \mathbf{Q}$, EG and FH bisects one another.

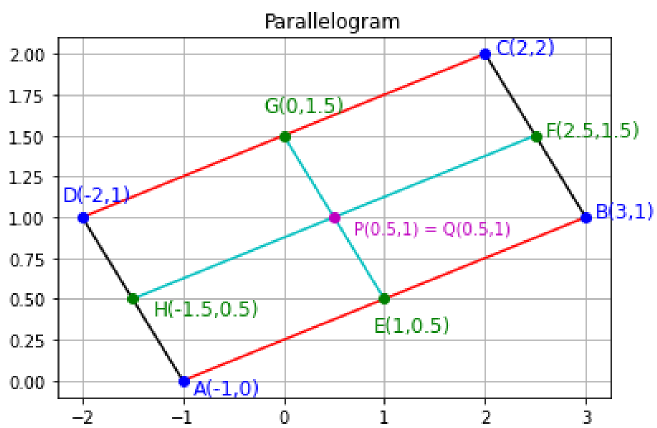

Fig. 0: The parallel lines are represented in red and black, and the initial vectors are represented in blue. The midpoints are represented in green, while the midpoints of **EG** and **FH** is shown in Magenta