Gate Assignment 1

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Download all python codes from

https://github.com/tanmaygoyal258/EE3900−Linear−Systems−and−Signal−processing/blob/main/GateAssignment1/code.py

Download all latex codes from

https://github.com/tanmaygoyal258/EE3900−Linear−Systems−and−Signal−processing/blob/main/GateAssignment1/main.tex

1 Problem

(EC 2017- Q.7) The input $x(t)$ and output $y(t)$ of a continous time signal are related as:

$$y(t) = \int_{t-T}^{t} x(u)\,du \tag{1.0.1}$$

The system is:

1) Linear and Time-variant
2) Linear and Time-invariant
3) Non-Linear and Time-variant
4) Non-Linear and Time-invariant

2 Solution

A necessary and sufficient condition for the linearity of a system is to check if it follows the Principle of Superposition, i.e Law of Additivity and Law of Homogeneity.

Law of Additivity:

Let the two input signals be $x_1(t)$ and $x_2(t)$, and their corresponding output signals be $y_1(t)$ and $y_2(t)$, then:

$$y_1(t) = \int_{t-T}^{t} x_1(u)\,du \tag{2.0.1}$$

$$y_2(t) = \int_{t-T}^{t} x_2(u)\,du \tag{2.0.2}$$

$$y_1(t) + y_2(t) = \int_{t-T}^{t} [x_1(u) + x_2(u)]\,du \tag{2.0.3}$$

Now, consider the input signal of $x_1(t) + x_2(t)$, then the corresponding output signal is given by $y'(t)$:

$$y'(t) = \int_{t-T}^{t} [x_1(u) + x_2(u)]\,du \tag{2.0.4}$$

Clearly, from (2.0.3) and (2.0.4):

$$y'(t) = y_1(t) + y_2(t) \tag{2.0.5}$$

Thus, the Law of Additivity holds.

Law of Homogeneity:

Consider an input signal $kx(t)$, where k is any constant. Let the corresponding output be given by $y'(t)$, then:

$$y'(t) = \int_{t-T}^{t} kx(u)\,du \tag{2.0.6}$$

$$= k\int_{t-T}^{t} x(u)\,du \tag{2.0.7}$$

$$= ky(t) \tag{2.0.8}$$

Clearly, from (2.0.8),

$$y'(t) = ky(t) \tag{2.0.9}$$

Thus, the Law of Homogeneity holds.

Since both the Laws hold, the system satisfies the Principle of Superposition, and is thus, a **linear system**.

To check for time-invariance, we would introduce a delay of t_0 in the output and input signals.
Delay in output signal:

$$y(t - t_0) = \int_{t-t_0-T}^{t-t_0} x(u)\,du \tag{2.0.10}$$

Now, we consider an input signal with a delay of t_0, given by $x(t - t_0)$, and let the corresponding output signal be given by $y'(t)$, then:

$$y'(t) = \int_{t-T}^{t} x(u - t_0)\,du \tag{2.0.11}$$

Substituting $a = u - t_0$:

$$y'(t) = \int_{t-t_0-T}^{t-t_0} x(a)\,da \tag{2.0.12}$$

Clearly, from (2.0.10) and (2.0.12):

$$y'(t) = y(t - t_0) \tag{2.0.13}$$

Thus, the system is **time-invariant**.
The correct option is **2) Linear and Time-invariant**

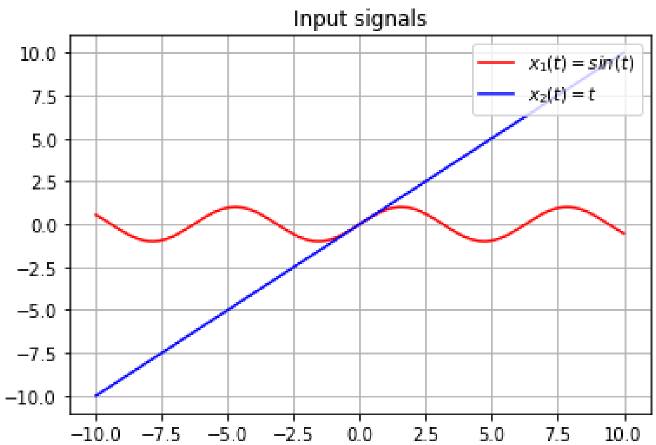

Fig. 4: $x_1(t) = \sin t$ and $x_2(t) = t$

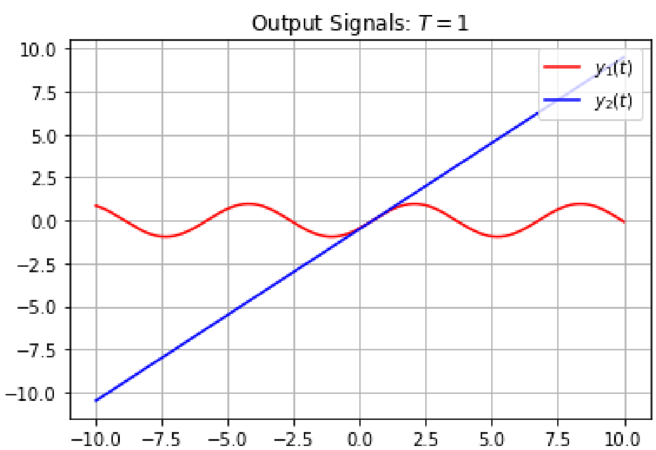

Fig. 4: $y_1(t)$ and $y_2(t)$

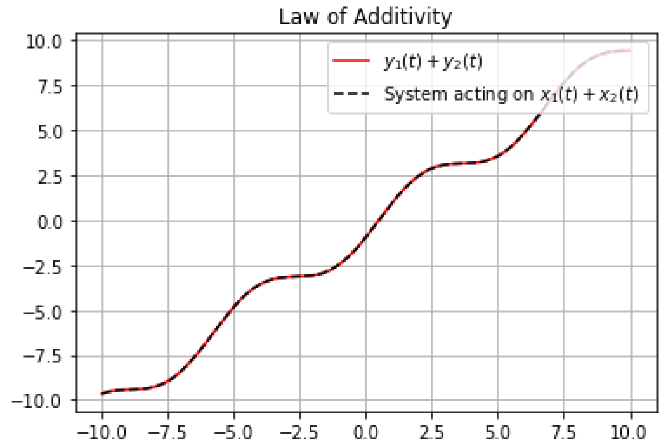

Fig. 4: Law of Additivity

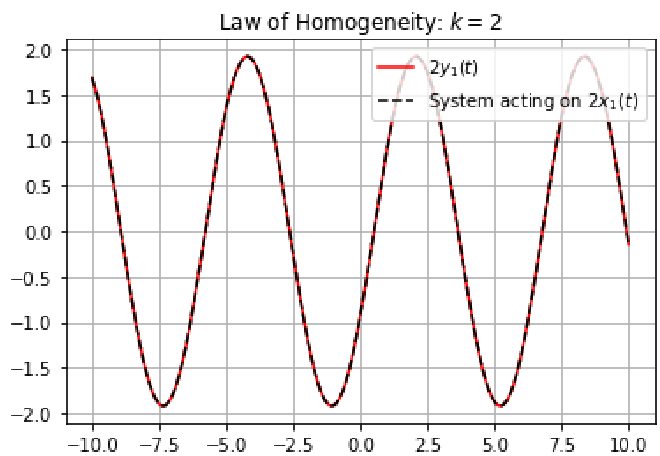

Fig. 4: Law of Homogeneity

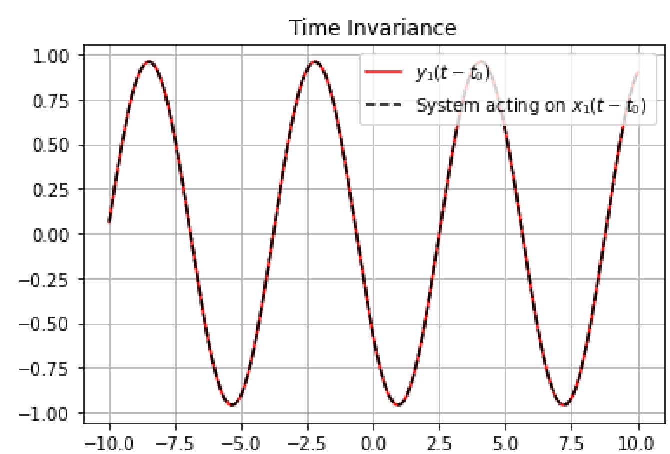

Fig. 4: Time invariance