Assignment 1

Tanmay Goyal - AI20BTECH11021

Download all python codes from

https://github.com/tanmaygoyal258/EE3900−Linear−Systems−and−Signal−processing/blob/main/Assignment1/code.py

Download all latex codes from

https://github.com/tanmaygoyal258/EE3900−Linear−Systems−and−Signal−processing/blob/main/Assignment1/main.tex

1 Problem

Prove that the points $\mathbf{A} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ are the vertices of a parallelogram. Find $\mathbf{E}, \mathbf{F}, \mathbf{G}, \mathbf{H}$, the midpoints of AB, BC, CD and AD respectively. Show that EG and FH bisect each other.

2 Solution

Two lines can be said to be parallel, if their directional vectors are in the same ratio.
The directional vector of **AB** is:

$$\begin{pmatrix} -1-3 \\ 0-1 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \tag{2.0.1}$$

The directional vector of **BC** is:

$$\begin{pmatrix} 3-2 \\ 1-2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2.0.2}$$

The directional vector of **CD** is:

$$\begin{pmatrix} 2-(-2) \\ 2-1 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \tag{2.0.3}$$

The directional vector of **AD** is:

$$\begin{pmatrix} -1-(-2) \\ 0-1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2.0.4}$$

The directional vector of **AC** is:

$$\begin{pmatrix} -1-2 \\ 0-2 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix} \tag{2.0.5}$$

The directional vector of **BD** is:

$$\begin{pmatrix} 3-(-2) \\ 1-1 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \end{pmatrix} \tag{2.0.6}$$

From (2.0.1) and (2.0.3), we can see **AB** and **CD** are parallel to one another. Similarly, from (2.0.2) and (2.0.4), we can see **BC** and **AD** are parallel to one another.
Since the two pairs of opposite lines are parallel to one another, we can say that the set of vertices represent a **parallelogram**.

We know that if the mid-point of two vectors **X** and **Y** is given by **Z**, then:

$$\mathbf{Z} = \frac{\mathbf{X}+\mathbf{Y}}{2} \tag{2.0.7}$$

Thus, using (2.0.7),
E is the midpoint of **AB**, given by:

$$\mathbf{E} = \frac{\mathbf{A}+\mathbf{B}}{2} = \frac{1}{2}\begin{pmatrix} -1+3 \\ 0+1 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \tag{2.0.8}$$

F is the midpoint of **BC**, given by:

$$\mathbf{F} = \frac{\mathbf{B}+\mathbf{C}}{2} = \frac{1}{2}\begin{pmatrix} 3+2 \\ 1+2 \end{pmatrix} = \begin{pmatrix} \frac{5}{2} \\ \frac{3}{2} \end{pmatrix} \tag{2.0.9}$$

G is the midpoint of **CD**, given by:

$$\mathbf{G} = \frac{\mathbf{C}+\mathbf{D}}{2} = \frac{1}{2}\begin{pmatrix} 2+(-2) \\ 2+1 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix} \tag{2.0.10}$$

H is the midpoint of **AD**, given by:

$$\mathbf{H} = \frac{\mathbf{A}+\mathbf{D}}{2} = \frac{1}{2}\begin{pmatrix} -1-2 \\ 0+1 \end{pmatrix} = \begin{pmatrix} \frac{-3}{2} \\ \frac{1}{2} \end{pmatrix} \tag{2.0.11}$$

Let **P** and **Q** be the midpoints of **EG** and **FH**. EG and FH would bisect one another if $\mathbf{P} = \mathbf{Q}$

$$\mathbf{P} = \frac{\mathbf{E}+\mathbf{G}}{2} = \frac{1}{2}\begin{pmatrix} 1+0 \\ \frac{1}{2}+\frac{3}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} \tag{2.0.12}$$

$$\mathbf{Q} = \frac{\mathbf{F}+\mathbf{H}}{2} = \frac{1}{2}\begin{pmatrix} \frac{5}{2}+\frac{-3}{2} \\ \frac{3}{2}+\frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} \tag{2.0.13}$$

Since $\mathbf{P} = \mathbf{Q}$, EG and FH bisects one another.

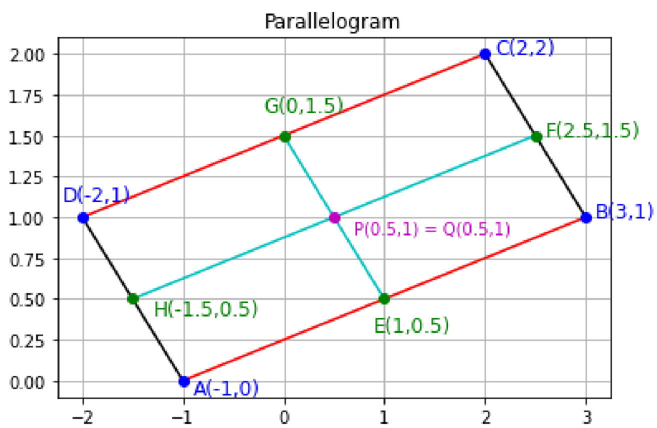

Fig. 0: The parallel lines are represented in red and black, and the initial vectors are represented in blue. The midpoints are represented in green, while the midpoints of **EG** and **FH** is shown in Magenta