

Assignment 5

Tanmay Goyal - AI20BTECH11021

Download all python codes from

<https://github.com/tanmaygoyal258/EE3900-Linear-Systems-and-Signal-processing/blob/main/Assignment5/code.py>

Download all latex codes from

<https://github.com/tanmaygoyal258/EE3900-Linear-Systems-and-Signal-processing/blob/main/Assignment5/main.tex>

The roots can be verified using the python code. As we can see from the graph, $\sqrt{5}x^2 + x + \sqrt{5} = 0$ does not intersect the x-axis anywhere, and hence, has no real roots.

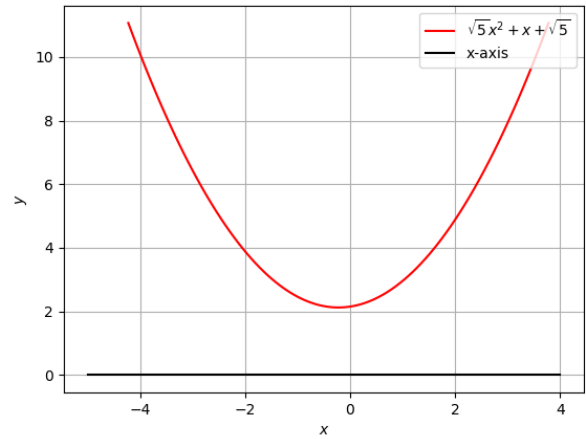


Fig. 0: Graph of $\sqrt{5}x^2 + x + \sqrt{5} = 0$

1 PROBLEM

(Quadratic Forms/Q.2.23) Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$

2 SOLUTION

Let $y = \sqrt{5}x^2 + x + \sqrt{5} = 0$.

Then, y can be represented in the vector form as:

$$y = \mathbf{x}^T \begin{pmatrix} \sqrt{5} & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} \frac{1}{2} & 0 \end{pmatrix} \mathbf{x} + \sqrt{5} \quad (2.0.1)$$

where

$$\mathbf{x} = \begin{pmatrix} x \\ 0 \end{pmatrix} \quad (2.0.2)$$

Substituting $y = 0$, we get:

$$\mathbf{x}^T \begin{pmatrix} \sqrt{5} & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} \frac{1}{2} & 0 \end{pmatrix} \mathbf{x} + \sqrt{5} = 0 \quad (2.0.3)$$

$$\sqrt{5}x^2 + x + \sqrt{5} = 0 \quad (2.0.4)$$

$$x^2 + \frac{1}{\sqrt{5}}x + 1 = 0 \quad (2.0.5)$$

$$x^2 + \frac{1}{\sqrt{5}}x + \left(\frac{1}{2\sqrt{5}}\right)^2 - \left(\frac{1}{2\sqrt{5}}\right)^2 + 1 = 0 \quad (2.0.6)$$

$$\left(x + \frac{1}{2\sqrt{5}}\right)^2 + 1 - \frac{1}{20} = 0 \quad (2.0.7)$$

$$\left(x + \frac{1}{2\sqrt{5}}\right)^2 + \frac{19}{20} = 0 \quad (2.0.8)$$

Clearly, for no value of x would we get the value of this expression to be 0, and hence, this equation has no real roots.