Quiz 2

Tanmay Goyal - AI20BTECH11021

Download all latex codes from

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https://github.com/tanmaygoyal258/EE3900-Linear-Systems-and-Signal-processing/blob/main/Quiz2/main.tex
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https://github.com/tanmaygoyal258/EE3900-Linear-Systems-and-Signal-processing/blob/main/Quiz2/code.py
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1 Problem

(Oppenheim/3.24(a)) Sketch the following sequence and determine their z-transform, including their region of convergence:

$$a[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k] \tag{1.0.1}$$

2 Solution

Lemma 2.1.

$$x(t) \times \delta(t-t_1) = x(t_1) \times \delta(t-t_1) \tag{2.0.1}$$

Proof.

$$\delta(t-t_1) = \begin{cases} 0 & t \neq t_1 \\ \infty & t = t_1 \end{cases} \tag{2.0.2}$$

$$\implies x(t) \times \delta(t-t_1) = \begin{cases} 0 & t \neq t_1 \\ x(t_1) \times \infty & t = t_1 \end{cases} \tag{2.0.3}$$

$$= x(t_1) \times \begin{cases} 0 & t \neq t_1 \\ \infty & t = t_1 \end{cases} \tag{2.0.4}$$

$$= x(t_1) \times \delta(t-t_1) \tag{2.0.5}$$

□

The Z-transform of a signal $x[n]$ is given by:

$$x[n] \overset{\mathcal{Z}}{\rightleftharpoons} X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \tag{2.0.6}$$

Thus, the z-transform of $a[n]$ is given by:

$$A(z) = \sum_{n=-\infty}^{\infty} a[n] z^{-n} \tag{2.0.7}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta[n-4k] z^{-n} \tag{2.0.8}$$

$$= \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta[n-4k] z^{-n} \tag{2.0.9}$$

$$= \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta[n-4k] z^{-4k} \tag{2.0.10}$$

using (2.0.1). Thus, we get:

$$A(z) = \sum_{k=-\infty}^{\infty} z^{-4k} \sum_{n=-\infty}^{\infty} \delta[n-4k] \tag{2.0.11}$$

$$= \sum_{k=-\infty}^{\infty} z^{-4k} \tag{2.0.12}$$

using the fact that:

$$\sum_{n=-\infty}^{\infty} \delta[n-m] = 1 \tag{2.0.13}$$

When $k > 0$, the term z^{-4k} becomes undefined for $z = 0$, and similarly when $k < 0$, the term z^{-4k} is undefined when $z = \pm\infty$. Thus, the region of convergence is:

$$ROC = z \in \mathbb{R} \setminus \{0, \pm\infty\} \tag{2.0.14}$$

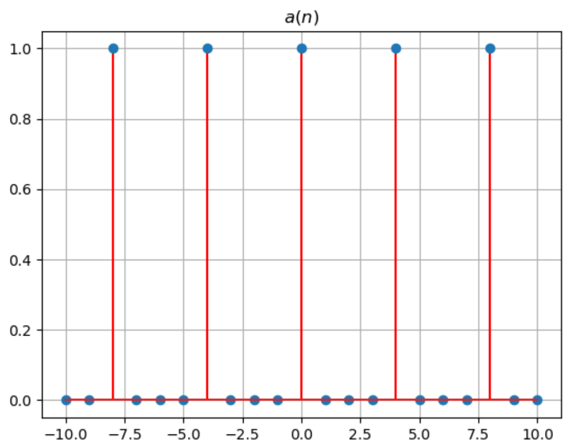

Fig. 0: $a[n]$

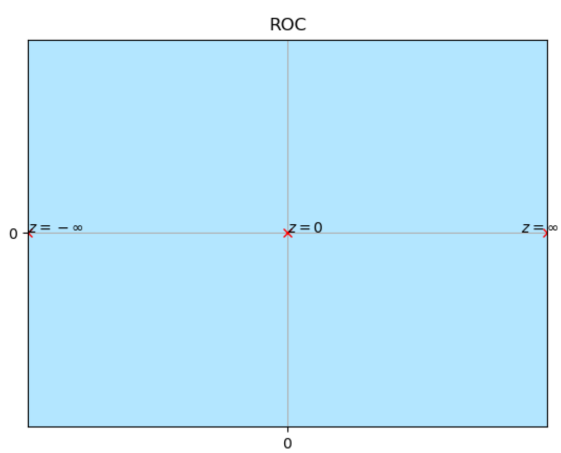

Fig. 0: ROC