Assignment 5

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Download all python codes from

```
https://github.com/tanmaygoyal258/EE3900−Linear
−Systems−and−Signal−processing/blob/main/
Assignment5/code.py
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Download all latex codes from

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https://github.com/tanmaygoyal258/EE3900−Linear
−Systems−and−Signal−processing/blob/main/
Assignment5/main.tex
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1 Problem

(Quadratic Forms/Q.2.23) Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$

2 Solution

Let $y = \sqrt{5}x^2 + x + \sqrt{5} = 0$.
Then, y can be represented in the vector form as:

$$y = \mathbf{x}^T \begin{pmatrix} \sqrt{5} & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2\begin{pmatrix} \frac{1}{2} & 0 \end{pmatrix} \mathbf{x} + \sqrt{5} \tag{2.0.1}$$

where

$$\mathbf{x} = \begin{pmatrix} x \\ 0 \end{pmatrix} \tag{2.0.2}$$

Subsituting $y = 0$, we get:

$$\mathbf{x}^T \begin{pmatrix} \sqrt{5} & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2\begin{pmatrix} \frac{1}{2} & 0 \end{pmatrix} \mathbf{x} + \sqrt{5} = 0 \tag{2.0.3}$$

$$\sqrt{5}x^2 + x + \sqrt{5} = 0 \tag{2.0.4}$$

$$x^2 + \frac{1}{\sqrt{5}}x + 1 = 0 \tag{2.0.5}$$

$$x^2 + \frac{1}{\sqrt{5}}x + \left(\frac{1}{2\sqrt{5}}\right)^2 - \left(\frac{1}{2\sqrt{5}}\right)^2 + 1 = 0 \tag{2.0.6}$$

$$\left(x + \frac{1}{2\sqrt{5}}\right)^2 + 1 - \frac{1}{20} = 0 \tag{2.0.7}$$

$$\left(x + \frac{1}{2\sqrt{5}}\right)^2 + \frac{19}{20} = 0 \tag{2.0.8}$$

Clearly, for no value of x would we get the value of this expression to be 0, and hence, this equation has no real roots.

The roots can be verified using the python code. As we can see from the graph, $\sqrt{5}x^2 + x + \sqrt{5} = 0$ does not intersect the x-axis anywhere, and hence, has no real roots.

Affine Transformation
Consider $y = \sqrt{5}x^2 + x + \sqrt{5}$, which can be written in the vector form as:

$$\mathbf{x}^T \begin{pmatrix} \sqrt{5} & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2\begin{pmatrix} \frac{1}{2} & \frac{-1}{2} \end{pmatrix} \mathbf{x} + \sqrt{5} = 0 \tag{2.0.9}$$

$$\mathbf{V} = \begin{pmatrix} \sqrt{5} & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} \frac{1}{2} \\ \frac{-1}{2} \end{pmatrix}, f = \sqrt{5} \tag{2.0.10}$$

For obtaining the affine transformation, we use:

$$\mathbf{x} = \mathbf{P}\mathbf{y} + \mathbf{c} \tag{2.0.11}$$

The corresponding eigenvalues of $\mathbf{V}$ are:

$$\lambda_1 = 0, \lambda_2 = \sqrt{5} \tag{2.0.12}$$

$$\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{5} \end{pmatrix} \tag{2.0.13}$$

The corresponding eigenvectors are:

$$\mathbf{p}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \mathbf{p}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.14}$$

$$\mathbf{P} = \begin{pmatrix} \mathbf{p}_1 & \mathbf{p}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.0.15}$$

Since $|\mathbf{V}| = 0$,

$$\begin{pmatrix} \mathbf{u}^\top + \eta \mathbf{p}_1^\top \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \tag{2.0.16}$$

$$\eta = \mathbf{u}^\top \mathbf{p}_1 \tag{2.0.17}$$

$$\implies \eta = \frac{-1}{2} \tag{2.0.18}$$

$$\implies \begin{pmatrix} \frac{1}{2} & -1 \\ \sqrt{5} & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -\sqrt{5} \\ \frac{-1}{2} \\ 0 \end{pmatrix} \tag{2.0.19}$$

$$\implies \mathbf{c} = \begin{pmatrix} \frac{-1}{2\sqrt{5}} \\ \frac{19}{4\sqrt{5}} \end{pmatrix} \tag{2.0.20}$$

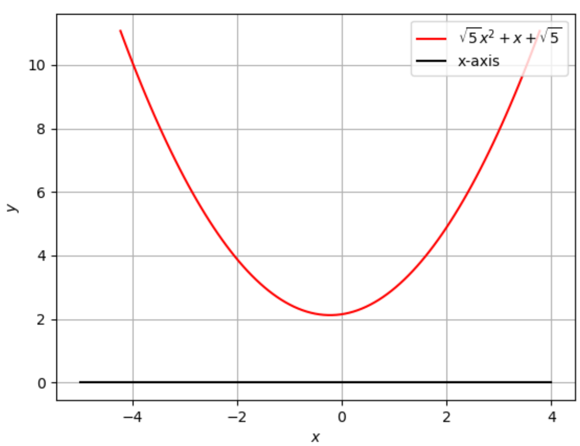

Fig. 0: Graph of $\sqrt{5}x^2 + x + \sqrt{5} = 0$