Assignment 4

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Download all python codes from

```
https://github.com/tanmaygoyal258/EE3900−Linear
−Systems−and−Signal−processing/blob/main/
Assignment4/code.py
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Download all latex codes from

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https://github.com/tanmaygoyal258/EE3900−Linear
−Systems−and−Signal−processing/blob/main/
Assignment4/main.tex
```

1 Problem

(Linear_Forms/Q.2.15) Find the equation of the line equidistant from parallel lines

$$\begin{pmatrix} 9 & 6 \end{pmatrix} \mathbf{x} = 7 \tag{1.0.1}$$

$$\begin{pmatrix} 3 & 2 \end{pmatrix} \mathbf{x} = -6 \tag{1.0.2}$$

2 Solution

In general, we can obtain the following lemma:

Lemma 2.1. *Given the two following parallel lines:*

$$a\mathbf{n}^T\mathbf{x} - c_1 = 0 \tag{2.0.1}$$

$$b\mathbf{n}^T\mathbf{x} - c_2 = 0 \tag{2.0.2}$$

The line equidistant from both parallel lines would be given by:

$$\mathbf{n}^T\mathbf{x} - \frac{\frac{c_1}{a} + \frac{c_2}{b}}{2} = 0 \tag{2.0.3}$$

Proof. The distance between a point $\mathbf{A}$ and a line $L = \mathbf{n}^T\mathbf{x} - c$ is given by:

$$\|\mathbf{P} - \mathbf{A}\| = \frac{\left|\mathbf{n}^T\mathbf{A} - c\right|}{\|\mathbf{n}\|} \tag{2.0.4}$$

where $\mathbf{P}$ is the foot of perpendicular from $\mathbf{A}$ onto L.

Consider a point $\mathbf{x}$ equidistant from both parallel lines, then:

$$\frac{\left|a\mathbf{n}^T\mathbf{x} - c_1\right|}{\|a\mathbf{n}\|} = \frac{\left|b\mathbf{n}^T\mathbf{x} - c_2\right|}{\|b\mathbf{n}\|} \tag{2.0.5}$$

$$\frac{\left|a\mathbf{n}^T\mathbf{x} - c_1\right|}{|a|} = \frac{\left|b\mathbf{n}^T\mathbf{x} - c_2\right|}{|b|} \tag{2.0.6}$$

$$\left|ab\mathbf{n}^T\mathbf{x} - bc_1\right| = \left|ab\mathbf{n}^T\mathbf{x} - ac_2\right| \tag{2.0.7}$$

$$2ab\mathbf{n}^T\mathbf{x} - bc_1 - ac_2 = 0 \tag{2.0.8}$$

$$\mathbf{n}^T\mathbf{x} - \frac{\frac{c_1}{a} + \frac{c_2}{b}}{2} = 0 \tag{2.0.9}$$

□

The two given parallel lines can be written as:

$$\begin{pmatrix} 9 & 6 \end{pmatrix} \mathbf{x} - 7 = 0 \tag{2.0.10}$$

$$\begin{pmatrix} 3 & 2 \end{pmatrix} \mathbf{x} + 6 = 0 \tag{2.0.11}$$

Since $\mathbf{x}$ is equidistant from both lines, we can write:

$$\frac{\left|\begin{pmatrix} 9 & 6 \end{pmatrix} \mathbf{x} - 7\right|}{\left\|\begin{pmatrix} 9 \\ 6 \end{pmatrix}\right\|} = \frac{\left|\begin{pmatrix} 3 & 2 \end{pmatrix} \mathbf{x} + 6\right|}{\left\|\begin{pmatrix} 3 \\ 2 \end{pmatrix}\right\|} \tag{2.0.12}$$

$$\frac{\left|\begin{pmatrix} 9 & 6 \end{pmatrix} \mathbf{x} - 7\right|}{3} = \left|\begin{pmatrix} 3 & 2 \end{pmatrix} \mathbf{x} + 6\right| \tag{2.0.13}$$

$$\left|\begin{pmatrix} 9 & 6 \end{pmatrix} \mathbf{x} - 7\right| = \left|\begin{pmatrix} 9 & 6 \end{pmatrix} \mathbf{x} + 18\right| \tag{2.0.14}$$

On further simplifcation, we get:

$$\begin{pmatrix} 18 & 12 \end{pmatrix} \mathbf{x} = -11 \tag{2.0.15}$$

$$\begin{pmatrix} 3 & 2 \end{pmatrix} \mathbf{x} = \frac{-11}{6} \tag{2.0.16}$$

Thus, we can say that the moving path of the point $\mathbf{x}$, and hence, the equidistant line is given by

$$\begin{pmatrix} 3 & 2 \end{pmatrix} \mathbf{x} = \frac{-11}{6} \tag{2.0.17}$$

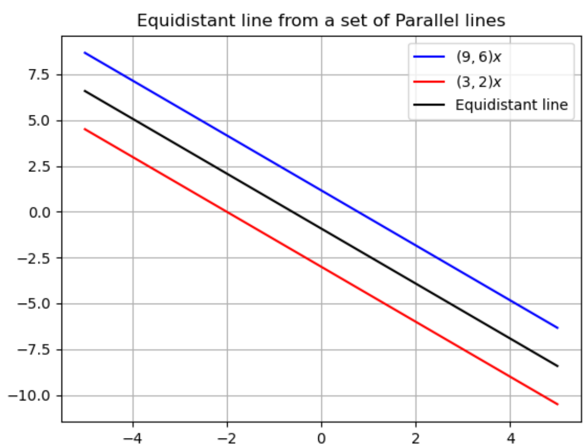

Fig. 0: The equidistant line