

Assignment 1

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Download all python codes from

<https://github.com/tanmaygoyal258/EE3900-Linear-Systems-and-Signal-processing/blob/main/Assignment1/code.py>

Download all latex codes from

<https://github.com/tanmaygoyal258/EE3900-Linear-Systems-and-Signal-processing/blob/main/Assignment1/main.tex>

1 PROBLEM

(Ramsey/1.1 Points/Q.9) Prove that the points $\mathbf{A} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ are the vertices of a parallelogram. Find $\mathbf{E}, \mathbf{F}, \mathbf{G}, \mathbf{H}$, the midpoints of AB, BC, CD and AD respectively. Show that EG and FH bisect each other.

2 SOLUTION

Two lines can be said to be parallel, if their directional vectors are in the same ratio.

The directional vector of \mathbf{AB} is:

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} -1 - 3 \\ 0 - 1 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \quad (2.0.1)$$

The directional vector of \mathbf{BC} is:

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 3 - 2 \\ 1 - 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.2)$$

The directional vector of \mathbf{CD} is:

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix} 2 - (-2) \\ 2 - 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \quad (2.0.3)$$

The directional vector of \mathbf{AD} is:

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} -1 - (-2) \\ 0 - 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.4)$$

The directional vector of \mathbf{AC} is:

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} -1 - 2 \\ 0 - 2 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix} \quad (2.0.5)$$

The directional vector of \mathbf{BD} is:

$$\mathbf{B} - \mathbf{D} = \begin{pmatrix} 3 - (-2) \\ 1 - 1 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \end{pmatrix} \quad (2.0.6)$$

From (2.0.1) and (2.0.3), we can see \mathbf{AB} and \mathbf{CD} are parallel to one another. Similarly, from (2.0.2) and (2.0.4), we can see \mathbf{BC} and \mathbf{AD} are parallel to one another.

Since the two pairs of opposite lines are parallel to one another, we can say that the set of vertices represent a **parallelogram**.

We know that if the mid-point of two vectors \mathbf{X} and \mathbf{Y} is given by \mathbf{Z} , then:

$$\mathbf{Z} = \frac{\mathbf{X} + \mathbf{Y}}{2} \quad (2.0.7)$$

Thus, using (2.0.7),

\mathbf{E} is the midpoint of \mathbf{AB} , given by:

$$\mathbf{E} = \frac{\mathbf{A} + \mathbf{B}}{2} = \frac{1}{2} \begin{pmatrix} -1 + 3 \\ 0 + 1 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \quad (2.0.8)$$

\mathbf{F} is the midpoint of \mathbf{BC} , given by:

$$\mathbf{F} = \frac{\mathbf{B} + \mathbf{C}}{2} = \frac{1}{2} \begin{pmatrix} 3 + 2 \\ 1 + 2 \end{pmatrix} = \begin{pmatrix} \frac{5}{2} \\ \frac{3}{2} \end{pmatrix} \quad (2.0.9)$$

\mathbf{G} is the midpoint of \mathbf{CD} , given by:

$$\mathbf{G} = \frac{\mathbf{C} + \mathbf{D}}{2} = \frac{1}{2} \begin{pmatrix} 2 + (-2) \\ 2 + 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix} \quad (2.0.10)$$

\mathbf{H} is the midpoint of \mathbf{AD} , given by:

$$\mathbf{H} = \frac{\mathbf{A} + \mathbf{D}}{2} = \frac{1}{2} \begin{pmatrix} -1 - 2 \\ 0 + 1 \end{pmatrix} = \begin{pmatrix} -\frac{3}{2} \\ \frac{1}{2} \end{pmatrix} \quad (2.0.11)$$

Let \mathbf{P} and \mathbf{Q} be the midpoints of \mathbf{EG} and \mathbf{FH} . EG and FH would bisect one another if $\mathbf{P} = \mathbf{Q}$

$$\mathbf{P} = \frac{\mathbf{E} + \mathbf{G}}{2} = \frac{1}{2} \begin{pmatrix} 1 + 0 \\ \frac{1}{2} + \frac{3}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 2 \end{pmatrix} \quad (2.0.12)$$

$$\mathbf{Q} = \frac{\mathbf{F} + \mathbf{H}}{2} = \frac{1}{2} \begin{pmatrix} \frac{5}{2} + -\frac{3}{2} \\ \frac{3}{2} + \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.13)$$

Since $\mathbf{P} = \mathbf{Q}$, EG and FH bisect one another.

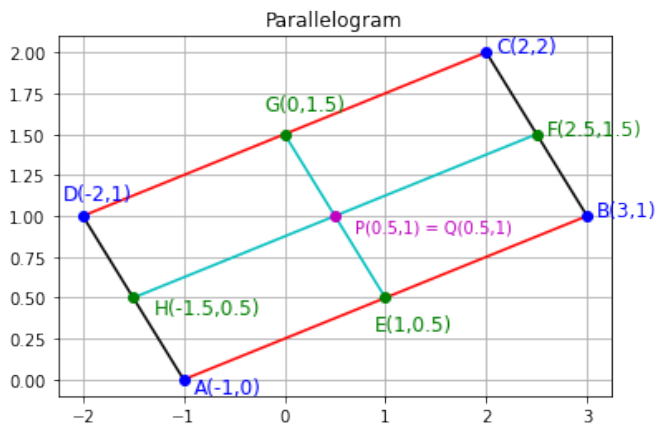


Fig. 0: The parallel lines are represented in red and black, and the initial vectors are represented in blue. The midpoints are represented in green, while the midpoints of **EG** and **FH** is shown in Magenta