Gate Assignment 1

Tanmay Goyal - AI20BTECH11021

Download all python codes from

```
https://github.com/tanmaygoyal258/EE3900−Linear
    −Systems−and−Signal−processing/blob/main/
    GateAssignment1/code.py
```

Download all latex codes from

```
https://github.com/tanmaygoyal258/EE3900−Linear
    −Systems−and−Signal−processing/blob/main/
    GateAssignment1/main.tex
```

1 Problem

(EC 2017- Q.7) The input $x(t)$ and output $y(t)$ of a continous time signal are related as:

$$y(t) = \int_{t-T}^{t} x(u)\, du \tag{1.0.1}$$

The system is:
1) Linear and Time-variant
2) Linear and Time-invariant
3) Non-Linear and Time-variant
4) Non-Linear and Time-invariant

2 Solution

Definition 1. *We say that a system is **linear** if and only if it follows the Principle of Superposition, i.e Law of Additivity and Law of Homogeneity.*

Definition 2. *A system is said to be **time invariant** if the output signal does not depend on the absolute time, i.e a time delay on the input signal directly equates to the delay in the output signal.*

Lemma 2.1. *The system relating the input signal $x(t)$ and output signal $y(t)$, given by*

$$y(t) = \int_{t-T}^{t} x(u)\, du \tag{2.0.1}$$

is linear and time invariant in nature.

Proof. From (1), we can say the system is linear if it follows both the laws of Additivity and Homogeneity.

Law of Additivity:

Let the two input signals be $x_1(t)$ and $x_2(t)$, and their corresponding output signals be $y_1(t)$ and $y_2(t)$, then:

$$y_1(t) = \int_{t-T}^{t} x_1(u)\, du \tag{2.0.2}$$

$$y_2(t) = \int_{t-T}^{t} x_2(u)\, du \tag{2.0.3}$$

$$y_1(t) + y_2(t) = \int_{t-T}^{t} [x_1(u) + x_2(u)]\, du \tag{2.0.4}$$

Now, consider the input signal of $x_1(t) + x_2(t)$, then the corresponding output signal is given by $y'(t)$:

$$y'(t) = \int_{t-T}^{t} [x_1(u) + x_2(u)]\, du \tag{2.0.5}$$

Clearly, from (2.0.4) and (2.0.5):

$$y'(t) = y_1(t) + y_2(t) \tag{2.0.6}$$

Thus, the Law of Additivity holds.

Law of Homogeneity:

Consider an input signal $kx(t)$, where k is any constant. Let the corresponding output be given by $y'(t)$, then:

$$y'(t) = \int_{t-T}^{t} kx(u)\, du \tag{2.0.7}$$

$$= k\int_{t-T}^{t} x(u)\, du \tag{2.0.8}$$

$$= ky(t) \tag{2.0.9}$$

Clearly, from (2.0.9),

$$y'(t) = ky(t) \tag{2.0.10}$$

Thus, the Law of Homogeneity holds.

Since both the Laws hold, the system satisfies the Principle of Superposition, and is thus, a **linear system**.

From (2) , to check for time-invariance, we would introduce a delay of t_0 in the output and input

signals.
Delay in output signal:

$$y(t - t_0) = \int_{t-t_0-T}^{t-t_0} x(u)\, du \tag{2.0.11}$$

Now, we consider an input signal with a delay of t_0, given by $x(t - t_0)$, and let the corresponding output signal be given by $y'(t)$, then:

$$y'(t) = \int_{t-T}^{t} x(u - t_0)\, du \tag{2.0.12}$$

Substituting $a = u - t_0$:

$$y'(t) = \int_{t-t_0-T}^{t-t_0} x(a)\, da \tag{2.0.13}$$

Clearly, from (2.0.11) and (2.0.13):

$$y'(t) = y(t - t_0) \tag{2.0.14}$$

Thus, the system is **time-invariant**.
The correct option is **2) Linear and Time-invariant**

$\square$

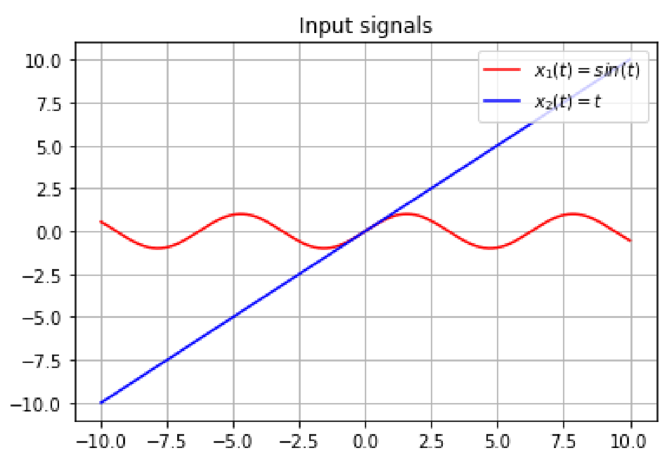

Fig. 4: $x_1(t) = \sin t$ and $x_2(t) = t$

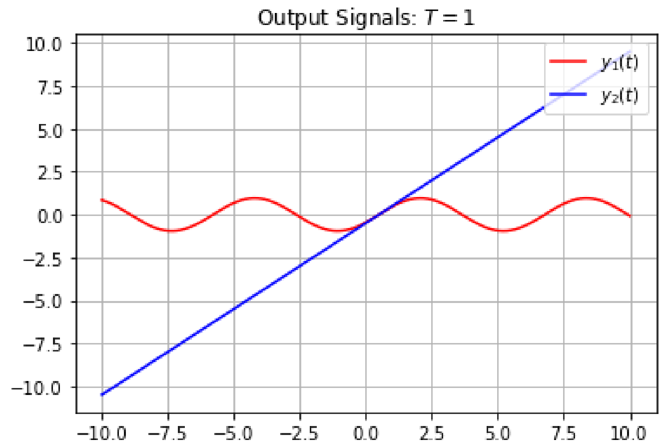

Fig. 4: $y_1(t)$ and $y_2(t)$

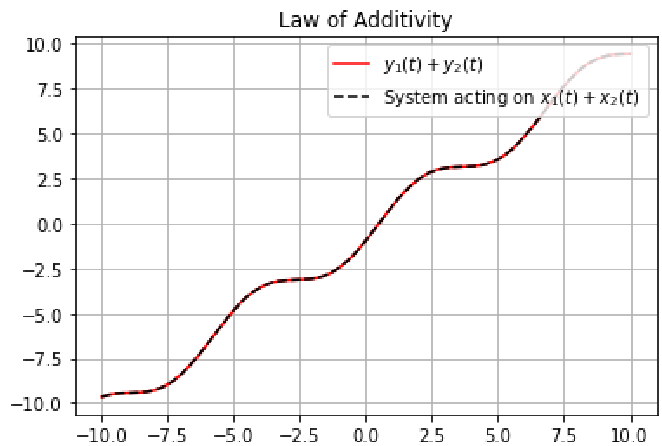

Fig. 4: Law of Additivity

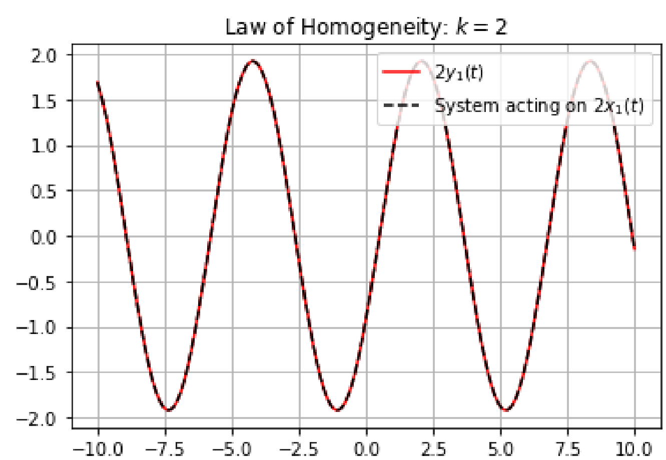

Fig. 4: Law of Homogeneity

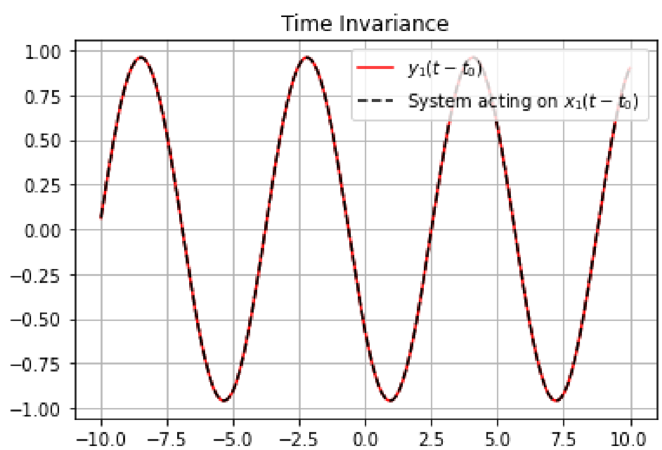

Fig. 4: Time invariance