Gate Assignment 3

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Download all python codes from

https://github.com/tanmaygoyal258/EE3900−Linear−Systems−and−Signal−processing/blob/main/GateAssignment3/code.py

Download all latex codes from

https://github.com/tanmaygoyal258/EE3900−Linear−Systems−and−Signal−processing/blob/main/GateAssignment3/main.tex

1 Problem

(EC-2005/Q.21) Let

$$x(n) = \left(\frac{1}{2}\right)^n u(n) \tag{1.0.1}$$

$$y(n) = x^2(n) \tag{1.0.2}$$

and $Y(e^{j\omega})$ be the Fourier Transform of $y(n)$. Then, $Y(e^{j0})$ is:

1) $\frac{1}{4}$
2) 2
3) 4
4) $\frac{4}{3}$

2 Solution

Since the Fourier Transform is represented as $Y(e^{j\omega})$, we consider all the signals to be Discrete Time Signals, and the Fourier Transform to be a Discrete Fourier Transform.
Now,

$$x[n] = \left(\frac{1}{2}\right)^n u[n] \tag{2.0.1}$$

where

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & otherwise \end{cases} \tag{2.0.2}$$

and

$$y[n] = x^2[n] = \left(\frac{1}{4}\right)^n u^2[n] \tag{2.0.3}$$

where

$$u^2[n] = \begin{cases} 1^2 & n \geq 0 \\ 0^2 & otherwise \end{cases} = \begin{cases} 1 & n \geq 0 \\ 0 & otherwise \end{cases} = u[n] \tag{2.0.4}$$

Thus,

$$y[n] = \left(\frac{1}{4}\right)^n u[n] = \begin{cases} \left(\frac{1}{4}\right)^n & n \geq 0 \\ 0 & otherwise \end{cases} \tag{2.0.5}$$

The formula for Z transform is given by:

$$Y(z) = \sum_{n=-\infty}^{\infty} y[n] z^{-n} \tag{2.0.6}$$

Thus,

$$Y(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n u[n] z^{-n} \tag{2.0.7}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z^{-n} \tag{2.0.8}$$

$$= \sum_{n=0}^{\infty} \left(\frac{z^{-1}}{4}\right)^n \tag{2.0.9}$$

$$= \frac{1}{1 - \frac{z^{-1}}{4}}, ROC = \left|\frac{z^{-1}}{4}\right| < 1 \tag{2.0.10}$$

$$= \frac{4}{4 - z^{-1}}, ROC = |z| > \frac{1}{4} \tag{2.0.11}$$

using the formula for the infinite sum of a Geometric Progression. Since $z = e^{j0} = 1$ satisfies the ROC condition, substituting $z = e^{j0}$, we get:

$$Y(e^{j0}) = \frac{4}{4 - e^{-j0}} = \frac{4}{3} \tag{2.0.12}$$

Thus, the correct answer is **4)** $\frac{4}{3}$

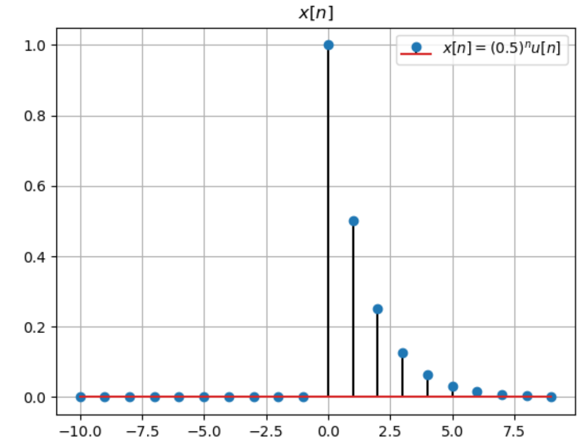

Fig. 4: $x[n] = \left(\frac{1}{2}\right)^n u[n]$

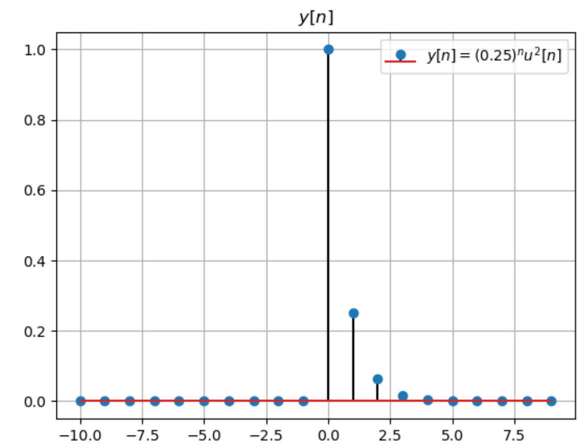

Fig. 4: $x[n] = \left(\frac{1}{4}\right)^n u^2[n]$

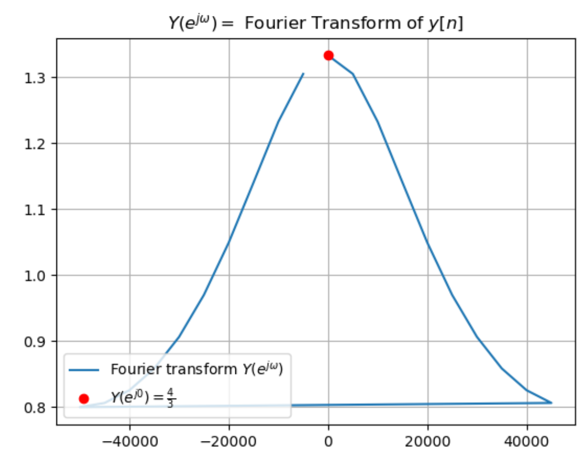

Fig. 4: $Y(e^{j\omega})$