

Assignment 4

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Download all python codes from

<https://github.com/tanmaygoyal258/EE3900-Linear-Systems-and-Signal-processing/blob/main/Assignment4/code.py>

Download all latex codes from

<https://github.com/tanmaygoyal258/EE3900-Linear-Systems-and-Signal-processing/blob/main/Assignment4/main.tex>

Consider a point \mathbf{x} equidistant from both parallel lines, then:

$$\frac{|a\mathbf{n}^T \mathbf{x} - c_1|}{\|a\mathbf{n}\|} = \frac{|b\mathbf{n}^T \mathbf{x} - c_2|}{\|b\mathbf{n}\|} \quad (2.0.5)$$

$$\frac{|a\mathbf{n}^T \mathbf{x} - c_1|}{|a|} = \frac{|b\mathbf{n}^T \mathbf{x} - c_2|}{|b|} \quad (2.0.6)$$

$$|ab\mathbf{n}^T \mathbf{x} - bc_1| = |ab\mathbf{n}^T \mathbf{x} - ac_2| \quad (2.0.7)$$

$$2ab\mathbf{n}^T \mathbf{x} - bc_1 - ac_2 = 0 \quad (2.0.8)$$

$$\mathbf{n}^T \mathbf{x} - \frac{\frac{c_1}{a} + \frac{c_2}{b}}{2} = 0 \quad (2.0.9)$$

□

1 PROBLEM

(Linear_Forms/Q.2.15) Find the equation of the line equidistant from parallel lines

$$(9 \ 6)\mathbf{x} = 7 \quad (1.0.1)$$

$$(3 \ 2)\mathbf{x} = -6 \quad (1.0.2)$$

2 SOLUTION

In general, we can obtain the following lemma:

Lemma 2.1. Given the two following parallel lines:

$$a\mathbf{n}^T \mathbf{x} - c_1 = 0 \quad (2.0.1)$$

$$b\mathbf{n}^T \mathbf{x} - c_2 = 0 \quad (2.0.2)$$

The line equidistant from both parallel lines would be given by:

$$\mathbf{n}^T \mathbf{x} - \frac{\frac{c_1}{a} + \frac{c_2}{b}}{2} = 0 \quad (2.0.3)$$

Proof. The distance between a point \mathbf{A} and a line $L = \mathbf{n}^T \mathbf{x} - c$ is given by:

$$\|\mathbf{P} - \mathbf{A}\| = \frac{|\mathbf{n}^T \mathbf{A} - c|}{\|\mathbf{n}\|} \quad (2.0.4)$$

where \mathbf{P} is the foot of perpendicular from \mathbf{A} onto L .

The two given parallel lines can be written as:

$$(9 \ 6)\mathbf{x} - 7 = 0 \quad (2.0.10)$$

$$(3 \ 2)\mathbf{x} + 6 = 0 \quad (2.0.11)$$

Since \mathbf{x} is equidistant from both lines, we can write:

$$\frac{|(9 \ 6)\mathbf{x} - 7|}{\left\| \begin{pmatrix} 9 \\ 6 \end{pmatrix} \right\|} = \frac{|(3 \ 2)\mathbf{x} + 6|}{\left\| \begin{pmatrix} 3 \\ 2 \end{pmatrix} \right\|} \quad (2.0.12)$$

$$\frac{|(9 \ 6)\mathbf{x} - 7|}{3} = |(3 \ 2)\mathbf{x} + 6| \quad (2.0.13)$$

$$|(9 \ 6)\mathbf{x} - 7| = |(9 \ 6)\mathbf{x} + 18| \quad (2.0.14)$$

On further simplification, we get:

$$(18 \ 12)\mathbf{x} = -11 \quad (2.0.15)$$

$$(3 \ 2)\mathbf{x} = \frac{-11}{6} \quad (2.0.16)$$

Thus, we can say that the moving path of the point \mathbf{x} , and hence, the equidistant line is given by

$$(3 \ 2)\mathbf{x} = \frac{-11}{6} \quad (2.0.17)$$

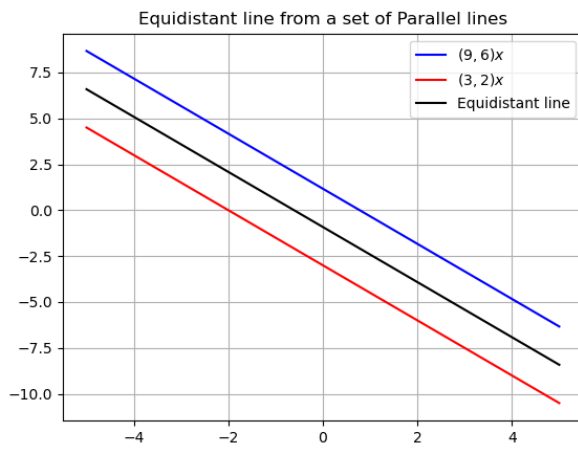


Fig. 0: The equidistant line