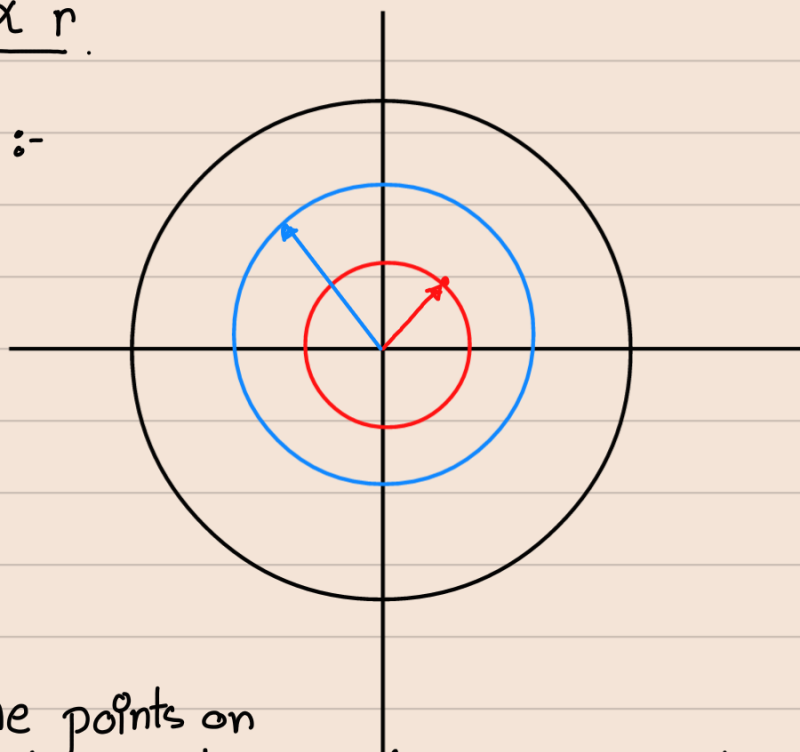


To get uniformly distributed points in a unit circle, we must select radial coordinate  $r$  such that,

$$P(r) \propto r.$$

Reason:-



All the points on the red circle have equal probability, and all the points on blue circle have equal probability. But,

$$P(\text{red circle}) = \frac{1}{2\pi r_{\text{red}}}$$

$$P(\text{blue circle}) = \frac{1}{2\pi r_{\text{blue}}}$$

Clearly, all the points don't get selected with equal probability.

Hence,  $P(r) \propto r$ .

Let  $P(r) = \alpha r$

$$\int_0^1 P(r) dr = 1 \Rightarrow \alpha = 2$$

$= 1$  ( $\because$  uniform dist)

$$P(r) = 2r$$

PDF  $q(r) = 2r$

$$q(r) = \overbrace{P(g^{-1}(r))} \cdot \left| \frac{d}{dr} g^{-1}(r) \right|$$

$$2r = \frac{d}{dr} g^{-1}(r)$$

$$\Rightarrow g^{-1}(r) = r^2$$

$$\therefore \boxed{g(r) = \sqrt{r}}$$

We randomly pick radial coordinate, and transform it using  $g(\cdot)$  to get uniform distribution.