

ASSIGNMENT

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Aim: Develop a program C++ or Java based on number theory such as Chinese Remainder Theorem.

Theory:

Relatively Prime Numbers:

Two integers are termed as relatively prime if the only common factor between them is 1.
i.e. Greatest Common Divisor $(m, n) = 1$

Two distinct primes are always relatively prime.

Example:

$$18 = 2 \times 3 \times 3$$

$$35 = 7 \times 5$$

so 18 and 35 are relatively prime.

Set of residues

It is a set of non negative integers less than n

$$\mathbb{Z}_n = \{0, 1, 2, \dots, (n-1)\}$$

Chinese Remainder Theorem (CRT)

Let $m_1, m_2, m_3, \dots, m_k$ be pair wise relatively prime positive integers. that is,

$$\gcd(m_i, m_j) = 1 \quad \text{for } 1 \leq i < j \leq k$$

Steps in CRT:

1. Find $M = m_1 \times m_2 \times \dots \times m_k$. this is the common modulus
2. Find $M_1 = M/m_1, M_2 = M/m_2 \dots M_k = M/m_k$
3. Find the multiplicative inverse of $M_1, M_2, M_3 \dots M_k$ using corresponding moduli

(m_1, m_2, \dots, m_k) call the inverses $M_1^{-1}, M_2^{-1}, \dots, M_k^{-1}$

4. The solution to the simultaneous equations is

$$x = (a_1 \times M_1 \times M_1^{-1} + a_2 \times M_2 \times M_2^{-1} + \dots + a_k \times M_k \times M_k^{-1}) \pmod{M}$$

Conclusion: Chinese remainder theorem for 3 numbers was implemented successfully in C++ and Java.