MATHS IMPORTANT QUESTIONS.

Q.1] Solve $(\tan y + x)dx + (x\sec^2 y - 3y)dy = 0$

Sol^n → $M = \tan y + x$, $N = x\sec^2 y - 3y$

$$\frac{\partial M}{\partial y} = \sec^2 y \quad \text{———} \quad (1)$$

$$\frac{\partial N}{\partial x} = \sec^2 y \quad \text{———} \quad (2)$$

From (1) & (2) ; The given equation is exact

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int_{\text{'y' constant}} M\,dx = \int (\tan y + x)dx$$

~~Term~~

$$= x\tan y + \frac{x^2}{2}$$

$$\int_{\text{Terms free from } x} N\,dy = -\int 3y\,dy$$

$$= -\frac{3y^2}{2}$$

∴ The Solution is;

$$x\tan y + \frac{x^2}{2} - \frac{3y^2}{2} = C$$

Q2] Solve, $\frac{dy}{dx} + \frac{4x}{x^2+1} y = \frac{1}{(x^2+1)^3}$

Sol^n → It is in the form of,

$$\frac{dy}{dx} + Py = Q$$

$\therefore P = \frac{4x}{(x^2+1)}$ & $Q = \frac{1}{(x^2+1)^3}$

$\therefore \int P.dx = \int \frac{4x}{(x^2+1)} dx = 2\int \frac{2x}{x^2+1} dx$

As $\int \frac{f'(x)}{f(x)} = \log(f(x))$

$\therefore = 2\log(x^2+1) = \log(x^2+1)^2$

$\therefore$ I.F. is,

I.F. $= e^{\int P.dx} = e^{\log(x^2+1)^2} = (x^2+1)^2$

$\therefore y.(x^2+1)^2 = \int \left[(x^2+1)^2 . \frac{1}{(x^2+1)^3}\right] dx + c$

$y(x^2+1)^2 = \int \frac{1}{(x^2+1)} dx + c$

$y(x^2+1)^2 = \tan x + c$ ---- $\left[\int \frac{1}{1+x^2} = \tan x\right]$

Q3] Solve, $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$

Sol^n → $D^3y - 6D^2y + 11Dy - 6y = 0$

$(D^3 - 6D^2 + 11D - 6)y = 0$

For Complimentarry function,

A.E. is

$m^3 - 6m^2 + 11m - 6 = 0$

$(m-1)(m^2-5m+6) = 0$

1	1	-6	11	-6
		1	-5	6
	1	-5	6	$\boxed{0}$

$(m-1)(m-3)(m-2) = 0$

$m = 1, 3, 2$

$\therefore$ C.F. is,

$y_c = c_1e^{x} + c_2e^{3x} + c_3e^{2x}$

24] Solve $\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + \frac{8dy}{dx} - 4y = 0$

Sol^n → $D^3y - 5D^2y + 8Dy - 4y = 0$

$(D^3 - 5D^2 + 8D - 4)y = 0$

For Complimentary function,

A.E. is,

$m^3 - 5m^2 + 8m - 4 = 0$

$(m-1)(m^2 - 4m + 4) = 0$

$$\begin{array}{c|cccc} 1 & 1 & -5 & 8 & 4 \\ & & 1 & -4 & 4 \\ \hline & 1 & -4 & 4 & \boxed{0} \end{array}$$

$(m-1)(m-2)(m-2) = 0$

$m = 1, 2, 2$

$\therefore$ C.F. is,

$y_c = c_1e^x + (c_2 + c_3x)e^{2x}$

25] Solve, $\frac{dy}{dx} + \left(\frac{1-2x}{x^2}\right) y = 1$

Sol^n → It is in the form of,

$$\frac{dy}{dx} + Py = Q$$

$$\int P\,dx = \int \left(\frac{1-2x}{x^2}\right) dx$$

$$= \int \frac{1}{x^2}\,dx - 2\int \frac{1}{x}\,dx$$

$$= \frac{-1}{x} - 2\log x$$

$$= \frac{-1}{x} + \log\left(\frac{1}{x^2}\right)$$

$$\therefore I.F. = e^{\int P.dx} = e^{-1/x + \log(1/x^2)} = e^{-1/x} \cdot e^{\log(1/x^2)}$$

$$= e^{-1/x} \cdot \frac{1}{x^2}$$

$$\therefore y \cdot e^{-1/x} \cdot \frac{1}{x^2} = \int e^{-1/x} \cdot \frac{1}{x^2} \cdot (1)\,dx + C$$

put $\frac{-1}{x} = t$

$$\frac{1}{x^2}\,dx = dt$$

$$\therefore y\,e^{-1/x} \cdot \frac{1}{x^2} = \int e^{t} \cdot dt + C$$

$$y \cdot e^{-1/x} \cdot \frac{1}{x^2} = e^{t} + c$$

$$y \cdot e^{-1/x} \cdot \frac{1}{x^2} = e^{-1/x} + c$$

$$y = x^2 + ce^{1/x} \cdot x^2$$

Q6] Solve $(x - 2e^{y})dy + (y + x\sin x)dx = 0$

Sol^n →

$M = y + x\sin x$, $N = x - 2e^{y}$

$$\frac{\partial M}{\partial y} = 1 \quad \text{———} \quad (1)$$

$$\frac{\partial N}{\partial x} = 1 \quad \text{———} \quad (2)$$

From (1) & (2);

$$\frac{\partial M}{\partial y} = \frac{\partial M}{\partial x}$$

The given equation is exact.

$$\therefore \int_{\text{'y' constant}} M\,dx = \int (y + x\sin x)dy$$

$$= xy + x(-\cos x) + \sin x$$

$$\int_{\substack{\text{Terms}\\ \text{free from 'x'}}} N\,dy = \int -2e^y\,dy = -2e^y$$

$\therefore$ The solution is,

$$xy - x\cos x + \sin x - 2e^y = c$$

Q7] Find the particular integral of $(D^2 - 4D + 4)y = e^x$

Sol$^n \rightarrow$ $(D^2 - 4D + 4)y = e^x$

For Complimentary Function;

A.E. is;

$$m^2 - 4m + 4 = 0$$

$$(m-2)(m-2) = 0$$

$$m = 2, 2$$

$\therefore$ C.F. is;

$$y_c = (c_1 + c_2 x)e^{2x}$$

For P.I.;

$$y_p = \frac{1}{D^2 - 4D + 4} e^x \quad \text{------} \quad [f(a) \neq 0]$$

$$y_p = \frac{1}{((1)^2 - 4(1) + 4)} e^x$$

$$y_p = e^x$$

∴ The complete solution is;

$y = y_c + y_p$

$y = (c_1 + c_2 x) e^{2x} + e^{x}$

Q8] Find the P.I. of $(D^2 - 5D + 9)y = \sin 3x$

Sol^n →

For Complimentary Function;

A.E. is,

$m^2 - 5m + 9 = 0$

$$m = \frac{-(-5) \pm \sqrt{25 - 4(9)}}{2(1)}$$

$$m = \frac{5 \pm \sqrt{-11}}{2} = \frac{5 \pm \sqrt{11}\, i}{2}$$

∴ C.F. is;

$$y_c = e^{5/2x}\left[c_1 \cos\left(\frac{\sqrt{11}}{2}\right)x + c_2 \sin\left(\frac{\sqrt{11}}{2}\right)y\right]$$

For P.I.

$$y_p = \frac{1}{D^2 - 5D + 9} \sin 3x$$

$$y_p = \frac{1}{(-9) - 5D + 9} \sin 3x \quad ---- \left[f(a^2) \neq 0\right]$$

$$y_p = \frac{1}{-5D} \sin 3x$$

$$y_p = \frac{-1}{5} \int \sin 3x \, dx$$

$$y_p = -\frac{1}{5} \left[\frac{-\cos 3x}{3} \right]$$

$$y_p = \frac{1}{15} \cos 3x$$

$\therefore$ The complete solution is;

$$y = y_c + y_p$$

$$y = e^{5/2\, x} \left[c_1 \cos\left(\frac{\sqrt{11}}{2}\right) x + c_2 \sin\left(\frac{\sqrt{11}}{2}\right) y \right] + \frac{1}{15} \cos 3x$$

Q1) Find the approximate value of x when $x = 0.1$ given that $\frac{dy}{dx} = x$ when $y_0 = 0$ at $x = 0$ by R.K. 4th method.

Solⁿ → $\frac{dy}{dx} = x$, $h = 0.1$ | $x_0 = 0$, $y_0 = 0$

$K_1 = hf(x_0, y_0) = 0.1(0) = 0$

$K_2 = hf(x_0 + h/2, y_0 + K_1/2) = 0.1\left[0 + \frac{0.1}{2}, 0 + \frac{0.0}{2}\right]$

$K_2 = 0.1(0.05, 0) = 0.1 \times 0.05$

$K_2 = 0.005$

$K_3 = hf(x_0 + h/2, y_0 + K_2/2) = (0.05, 0.0025) 0.1$

$K_3 = 0.005$

$K_4 = hf(x_0 + h, y_0 + K_3) = 0.1(0.1, 0.005)$

$K_4 = 0.1(0.1) = 0.01$

$\therefore K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$K = 0.005$

$\therefore y = y_0 + k = 0 + 0.005$

$y = 0.005$

249] Using Taylor's series method to solve $\frac{dy}{dx} = y$ at $x_0 = 0$, $y_0 = 0.2$ at $x = 0.2$

Sol^n → $x_0 = 0$, $y_0 = 0.2$, $h = 0.2$

$y' = y \qquad y_0' = y_0 = 0.2$

$y'' = y' \qquad y_0'' = y_0' = 0.2$

$y''' = y'' \qquad y_0''' = y_0'' = 0.2$

$y^{IV} = y''' \qquad y_0^{IV} = y_0''' = 0.2$

∴ The Taylor's series is,

$$y = y_0 + x y_0' + \frac{x^2}{2!} y_0'' + \frac{x^3}{3!} y_0''' + \frac{x^4}{4!} y^{IV}$$

$$y = 0.2 + (0.2) \times 0.2 + \frac{(0.2)^2}{2} \times (0.2) + \frac{(0.2)^3}{3 \times 2} \times 0.2 + \frac{(0.2)^4}{4 \times 3 \times 2} \times 0.2$$

$$y = 0.24428$$

Q11] (a) $\frac{d^2y}{dx^2} = x^2 + 1$

Solⁿ → $D^2 y = x^2 + 1$

For P.I.

$$y_p = \frac{1}{D^2}(x^2+1)$$

$$y_p = \frac{1}{D}\left[\frac{1}{D}(x^2+1)\right]$$

$$= \frac{1}{D}\left\{\frac{x^3}{3} + x\right\}$$

$$= \frac{x^4}{4 \times 3} + \frac{x^2}{2}$$

$$= \frac{x^4}{12} + \frac{x^2}{2}$$

∴ The Total solution, is;

$$\therefore y = \frac{x^4}{12} + \frac{x^2}{2}$$

(b) $\frac{d^2y}{dx^2} = \cos 4x$

Solⁿ → $D^2 y = \cos 4x$

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For P.I $= y_p = \frac{1}{D^2} \cos 4x$

$\therefore = \frac{1}{D} \left\{ \frac{1}{D} (\cos 4x) \right\}$

$= \frac{1}{D} \left\{ \frac{\sin 4x}{4} \right\}$

$= \frac{1}{4} \int \sin 4x \, dx$

$= \frac{1}{4} \left[-\frac{\cos 4x}{4} \right]$

$= \frac{-1}{16} \cos 4x$

$\therefore$ The complete solution is;

$y = \frac{-1}{16} \cos 4x$

Q12) Solve, $(D^2-6D+9)y = \frac{e^{3x}}{x^2}$ by variation parameter method.

Sol^n →

$$(D^2-6D+9)y = \frac{e^{3x}}{x^2} \quad \text{——— (1)}$$

For C.F.

A.E. is $m^2-6m+9=0$

$(m-3)(m-3)=0$

$m = 3, 3$

∴ The C.F. is;

$y_c = (c_1+c_2x)e^{3x}$

$y_c = c_1e^{3x} + c_2e^{3x}.x$

∴ $u = e^{3x}$, $v = e^{3x}.x$

$u' = 3e^{3x}$, $v' = [3e^{3x}.x + e^{3x}]$

$v' = e^{3x}[3x+1]$

Let,

$y_p = Au + Bv \quad \text{——— (2)}$

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Now, $W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = \begin{vmatrix} e^{3x} & e^{3x}.x \\ 3e^{3x} & e^{3x}[3x+1] \end{vmatrix}$

$$= e^{3x}.e^{3x} \begin{vmatrix} 1 & x \\ 3 & 3x+1 \end{vmatrix}$$

$$= e^{6x}[\cancel{3x}+1-\cancel{3x}]$$

$$\underline{\underline{W = e^{6x}}}$$

$$A = \int \frac{-VX}{W} = -\int \frac{e^{3x}.\cancel{x}.e^{3x}}{x^2.e^{6x}}\,dx$$

$$= -\int \frac{\cancel{e^{6x}}.}{x\,\cancel{e^{6x}}}\,dx$$

$$= -\log x = \log(1/x)$$

$$B = \int \frac{uX}{W} = \int \frac{e^{3x}.e^{3x}}{x^2 e^{6x}}\,dx$$

$$= \int \frac{1}{x^2}\,dx$$

$$= \frac{-1}{x}$$

put value of A & B in equation ②;

$$y_p = \log(1/x).e^{3x} + (-1/\cancel{x})e^{3x}.\cancel{x}$$

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$y_p = e^{3x}[\log(1/x) - 1]$

$\therefore$ The complete solution is;

$y = y_c + y_p$

$y = (c_1 + c_2 x)e^{3x} + e^{3x}[\log(1/x) - 1]$

Q13) Solve $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$

Sol^n $\rightarrow$ $\frac{dy}{dx} = \frac{e^x}{e^y}(e^x - e^y)$

$e^y \frac{dy}{dx} = e^{2x} - e^{xy}$

$e^y \frac{dy}{dx} + e^{xy} = e^{2x}$

$e^y \frac{dy}{dx} + e^x . e^y = e^{2x}$

put $e^y = v$

$e^y \frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dv}{dx} + ve^x = e^{2x}$

I.F. is $e^{\int P.dx} = e^{\int e^x dx} = e^{e^x}$

$\therefore e^{e^x} . v = \int e^{2x} . e^{e^x} . dx + c$

put $e^x = t$

$e^x dx = dt$

$\therefore e^{e^x} . v = \int e^t . t . dt + c$

$e^{e^x} . v = e^t . t - e^t (1) + c$

$e^{e^x} . v = e^t (t - 1) + c$

$e^{e^x} . e^y = e^{e^x} (e^x - 1) + c$

$e^y = e^x - 1 + c . e^{-e^x}$

Q14] Solve $(x^3 y^3 + xy) dy = dx$

Sol^n → $\frac{dx}{dy} = x^3 y^3 + xy$

$\frac{dx}{dy} - xy = x^3 y^3$

Divide by x^3;

$$\frac{1}{x^3}\frac{dx}{dy} - \frac{y}{x^2} = x^3y^3$$

put $+\frac{1}{x^2} = v$

$$-\frac{2}{x^3}\frac{dx}{dy} = \frac{dv}{dy}$$

$$\therefore -\frac{1}{2}\frac{dv}{dy} + vy = y^3$$

$$\frac{dv}{dy} - 2vy = -2y^3$$

I.F. is $e^{-2\int y \cdot dy} = e^{-2y^2/2} = e^{-y^2}$

$$e^{-y^2} \cdot v = -2\int e^{-y^2} \cdot y^3\, dy + c$$

$$= -2\int e^{-y^2} \cdot y^2 \cdot y \cdot dy + c$$

put $y^2 = t$

$$2y\, dy = dt$$

$$= -\int e^{-t} \cdot t\, dt + c$$

$$e^{-y^2}.V = -\left[(-e^{-t}).t - (e^{-t})(1)\right] + c$$

$$e^{-y^2}.V = e^{-t}(t+1) + c$$

$$e^{-y^2}.\frac{1}{x^2} = e^{-y^2}(y^2+1) + c$$

Q15] Solve $\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^2$ —— (1)

Sol^n →

put $\log z = y$

$$\frac{1}{z}\frac{dz}{dx} = \frac{dy}{dx}$$

$$\frac{dz}{dx} = z\frac{dy}{dx}$$

$$\cancel{z}\frac{dy}{dx} + \frac{\cancel{z}}{x}y = \frac{\cancel{z}}{x^2}y^2$$

$$\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$$

Divide by y^2

$$y^{-2}\frac{dy}{dx} + \frac{1}{xy} = \frac{1}{x^2}$$

put $y^{-1} = v$

$$-\frac{1}{y^2}\frac{dy}{dx} = \frac{dv}{dx}$$

$$\therefore -\frac{dv}{dx} + \frac{v}{x} = \frac{1}{x^2}$$

$$\frac{dv}{dx} - \frac{v}{x} = -\frac{1}{x^2} \quad \text{———} \quad (2)$$

$$\therefore \text{ I.F. is } e^{\int P.dx} = e^{-\int 1/x \, dx} = e^{-\log x} = e^{\log(1/x)}$$

$$e^{\int P.dx} = \frac{1}{x} \;//$$

$$\therefore \frac{1}{x}.v = -\int \frac{1}{x}.\frac{1}{x^2}\,dx + c$$

$$\frac{1}{x}.v = -\left[\frac{-1}{2x^2}\right] + c$$

$$\frac{1}{x}.\frac{1}{y} = \frac{1}{2x^2} + c$$

$$\frac{1}{xy} = \frac{1}{2x^2} + c$$

$$\frac{1}{x\log z} = \frac{1}{2x^2} + c$$

$$\frac{1}{\log z} = \frac{1}{2x} + Cx$$

Q16] Solve $4x^2y\frac{dy}{dx} = 3x(3y^2+2) + (3y^2+2)^3$

Sol^n →

put $3y^2+2 = v$

$$6y\frac{dy}{dx} = \frac{dv}{dx}$$

$$\therefore \frac{\cancel{4}^{2}x^2y}{3\cancel{6}y}\frac{dv}{dx} - 3xV = \cancel{(3y^2+2)^3}\, V^3$$

$$\frac{2x^2}{3}\frac{dv}{dx} - 3xV = V^3$$

$$x^2\frac{dv}{dx} - \frac{9}{2}xV = \frac{3V^3}{2}$$

$$\frac{dv}{dx} - \frac{9}{2}\frac{V}{x} = \frac{3V^3}{2x^2}$$

Divide by V^3;

$$V^{-3}\frac{dv}{dx} - \frac{9}{2}\frac{1}{V^2x} = \frac{3}{2}\times\frac{1}{x^2}$$

put $\frac{1}{v^2} = t$

$$\frac{-2}{v^3}\frac{dv}{dx} = \frac{dt}{dx}$$

$$-\frac{dt}{dx} - \frac{9}{\cancel{2}}\frac{t}{x} \times \cancel{2} = \frac{3}{\cancel{2}} \times \frac{1}{x^2} \times \cancel{2}$$

$$\frac{dt}{dx} + \frac{9}{\cancel{1}}\frac{t}{\cancel{x}} = \frac{-3}{\cancel{1}x^2}$$

$$\therefore \text{I.F. is } e^{\int P.dx} = e^{\frac{9}{\cancel{1}}\int \frac{1}{x}dx} = e^{\log x^9} = x^9$$

$$x^9.t = \frac{-3}{\cancel{1}}\int \frac{x^9}{x^2}\,dx + c$$

$$x^9.t = \frac{-3}{\cancel{1}}\left[\frac{x^8}{8}\right] + c$$

$$x^9.t = \frac{-3x^8}{8} + c$$

$$x^9.\frac{1}{v^2} = \frac{-3x^8}{8} + c$$

$$\frac{x^9}{(3y^2+2)^2} = \frac{-3}{8}x^8 + c$$

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Q17) Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = \sin(\log x)$ —— (1)

Sol^n → put $z = \log x$ / $x = e^z$

$$x^2\frac{d^2y}{dx^2} = D(D-1)y$$

$$x\frac{dy}{dx} = Dy$$

$$D(D-1)y - 3Dy + 5y = \sin z \quad \text{—— (2)}$$

$$[D^2 - D - 3D + 5]y = \sin z$$

For C.F.

A.E. is $m^2 - 4m + 5 = 0$

$$m = \frac{4 \pm \sqrt{16 - 5 \times 4 \times 1}}{2(1)}$$

$$m = 2 \pm i$$

$\therefore$ C.F. is,

$$y_c = e^{2\cancel{x}}[C_1\cos \cancel{x} + C_2 \sin \cancel{x}]$$

$\therefore$ For P.I.;

$$y_p = \frac{1}{D^2 - 4D + 5}\sin z$$

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$$y_p = \frac{1}{(-1)-4D+5} \sin z$$

$$y_p = \frac{1}{-4D+4} \sin z$$

$$y_p = \frac{-1}{4} \frac{1}{(D-1)} \sin z$$

$$y_p = \frac{-1}{4}\left[\frac{(D+4)}{(D^2-4^2)} \sin z\right]$$

$$y_p = \frac{-1}{4}\left[\frac{(D+4)\sin z}{-1-16}\right]$$

$$y_p = \frac{+1}{4\times 17}\left[D \sin z + 4 \sin z\right]$$

$$y_p = \frac{1}{68}\left[\cos z + 4 \sin z\right]$$

∴ The complete solution is;

$$y = y_p + y_c$$

$$y = \frac{1}{68}\left[\cos z + 4\sin z\right] + e^{2z}\left[c_1 \cos z + c_2 \sin z\right]$$

$$y = 1/68\left[\cos \log x + 4 \sin \log x\right] + e^{2z}\left[c_1 \cos \log x + c_2 \sin \log x\right]$$

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218] Solve $(D^2-1)y = e^{-x}\sin(e^{-x}) + \cos(e^{-x})$ by variation parameter method.

For C.F.

A.E. is $m^2-1=0$

$m = \pm 1$

$\therefore$ $y_c = \cancel{c_1\cos x + c_2\sin x}$

$y_c = c_1 e^{-x} + c_2 e^{x}$

$u = e^{-x}$, $v = e^{x}$

$u' = -e^{-x}$, $v' = e^{x}$

$$W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = \begin{vmatrix} e^{-x} & e^{x} \\ -e^{-x} & e^{x} \end{vmatrix}$$

$$= e^{-x}\cdot e^{x}\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}$$

$$W = e^{0}(1+1) = 2$$

Let,

$y_p = Au + Bv$ ——— (I)

$$A = \int \frac{-vX}{W} = \frac{-1}{2}\int e^{x}.\left[e^{-x}\sin(e^{-x}) + \cos(e^{-x})\right]dx$$

$$A = \frac{-1}{2} e^{x}\cos(e^{-x})$$

$$\left[e^{x}\left[f(x) + f'(x)\right] = e^{x} f(x) \right]$$

$$B = \int \frac{uX}{W} = \frac{-1}{2}\int e^{-x}\left[e^{-x}\sin(e^{-x}) + \cos(e^{-x})\right] dx$$

$\cancel{B =}$ put $e^{-x} = t$

$-e^{-x}\,dx = dt$

$$B = \frac{-1}{2}\int (t\sin t + \cos t)\,dt$$

$$B = \frac{-1}{2}\left[(-\cos t).t - (-\sin t).(1) + \sin t\right]dt$$

$$B = \frac{-1}{2}\left[-t\cos t + 2\sin t\right]$$

$$B = \frac{1}{2} t.\cos t - \sin t$$

$$B = \frac{1}{2} e^{-x}\cos e^{-x} - \sin e^{-x}$$

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Put value of A & B in equation ①;

$$y_p = e^{-x}\left[\frac{-1}{2}e^{x}\cos(e^{-x})\right] + e^{x}\left[\frac{1}{2}e^{-x}\cos e^{-x} - \sin e^{-x}\right]$$

$$y_p = \frac{-1}{2}\cos e^{-x} + \frac{1}{2}\cos e^{-x} + (-e^{x}\sin e^{-x})$$

$$y_p = -e^{x}\sin(e^{-x})$$

∴ The complete solution is,

$$y = c_1 e^{-x} + c_2 e^{-x} - e^{x}\sin(e^{-x})$$

Q19] Solve $(5+2x)^2\frac{d^2y}{dx^2} - 6(5+2x)\frac{dy}{dx} + 8y = 6x$ ——①

Solⁿ→ putting $5+2x = V$ | $2x = V-5$

$$2 = \frac{dV}{dx}$$

The equation becomes,

$$V^2\, 4\frac{d^2y}{dx^2} - 6.2V.\frac{dy}{dx} + 8y = 3(V-5)$$

Dividing equation by 4;

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$$v^2 \frac{d^2y}{dx^2} - 3v\frac{dy}{dx} + 2y = \frac{3}{4}(v-5) \quad \text{——— (2)}$$

put $\log v = z$, $v = e^z$.

$$D(D-1)y - 3Dy + 2y = 3/4\,(e^z - 5)$$

$$[D^2 - D - 3D + 2]y = \frac{3}{4}(e^z - 5)$$

$$[D^2 - 4D + 2]y = \frac{3}{4}(e^z - 5)$$

For. C.F;

A.E. is $m^2 - 4m + 2 = 0$

$$m = \frac{4 \pm \sqrt{16-8}}{2(1)}$$

$$m = \frac{4 \pm 2\sqrt{2}}{2}$$

$$m = 2+\sqrt{2},\ 2-\sqrt{2}$$

$\therefore$ C.F. is;

$$y_c = C_1 e^{(2+\sqrt{2})z} + C_2 e^{(2-\sqrt{2})z}$$

For P.I.

$$y_p = \frac{1}{D^2 - 4D + 2}\left(\frac{3}{4}e^z - \frac{15}{4}\right)$$

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$$y_p = \frac{3}{4}\frac{1}{D^2-4D+2}e^z - \frac{15}{4}\frac{1}{D^2-4D+2}e^0$$

$$y_p = \frac{3}{4}\frac{1}{(-1)}e^z - \frac{15}{4}\times\frac{1}{2}$$

$$y_p = \frac{-3}{4}e^z - \frac{15}{8}$$

$\therefore$ The complete solution is,

$$y = C_1 e^{(2+\sqrt{2})z} + C_2 e^{(2+\sqrt{2})z} - \frac{3}{4}e^z - \frac{15}{8}$$

$$y = C_1 V^{(2+\sqrt{2})} + C_2 V^{(2-\sqrt{2})} - \frac{3}{4}\cancel{V} - \frac{15}{8}$$

$$y = C_1(5+2x)^{(2+\sqrt{2})} + C_2(5+2x)^{(2-\sqrt{2})} - \frac{3}{4}(5+2x) - \frac{15}{8}$$

22] Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$

Sol^n → $(D^2-2D+1)y = xe^x \sin x$

For C.F.

A.E is $m^2-2m+1=0$

$(m-1)(m-1)=0$

$m = 1, 1$

$$y_c = (c_1 + c_2 x)e^x$$

for P.I.;

$$y_p = \frac{1}{D^2 - 2D + 1} x e^x \sin x$$

$$= e^x \frac{1}{(D+1)^2 - 2(D+1) + 1} x \sin x$$

$$= e^x \frac{1}{D^2 + \cancel{1} + \cancel{2D} - \cancel{2D} - \cancel{2} + \cancel{1}} x \sin x$$

$$= e^x \frac{1}{D^2} \sin x . x$$

$$= e^x \frac{1}{D} \int x \sin x \, dx$$

$$= e^x \frac{1}{D} \left[(-\cos x) x - (-\sin x)(1) \right]$$

$$= e^x \int \left[-x\cos x + \sin x \right] dx$$

$$= e^x \left[- \left\{ x \sin x - (-\cos x)(1) \right\} - \cos x \right]$$

$$= e^x \left[- \left\{ x \sin x + \cos x \right\} - \cos x \right]$$

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$= -e^{x}[x\sin x + 2\cos x]$

$\therefore$ The complete solution is;

$y = y_c + y_p$

$y = (c_1 + c_2 x)e^{x} + \{-e^{x}(x\sin x + 2\cos x)\}$

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Q21]

Q3] * Apply Runge-Kutta Method of Fourth order to find approximate value of y at $x = 0.2$, if $\frac{dy}{dx} = x + y^2$ given $y = 1$, at $x_0 = 0$ in steps $h = 0.1$

Sol^n → (i) $x_0 = 0$, $y_0 = 1$
$x_1 = 0.1$, $h = 0.1$

$k_1 = hf(x_0, y_0) = 0.1(0 + 1^2) = 0.1 \times 1 = 0.1$

$k_2 = hf(x_0 + h/2, y_0 + k_1/2) = 0.1[0.05, 1 + 0.05]$
$k_2 = 0.1(0.05, 1.05)$
$k_2 = 0.1(0.05 + (1.05)^2) = 0.11525$

$k_3 = 0.1(0.05, 1 + 0.11525/2) = 0.11685$

$k_4 = 0.1(0.1, 1 + 0.11685) = 0.13473$

$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$
$k = 0.11648$

$y_1 = y_0 + k = 1 + 0.11648 = 1.11648$

(ii) At $x = 0.2$
$x_0 = 0.1$, $y_0 = 1.11648$
$x_1 = 0.2$, $h = 0.1$

$k_1 = hf(x_0, y_0) = 0.1(0.1 + 1.11648^2)$
$k_1 = 0.13465$

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$K_2 = hf(x_0 + h/2, y_0 + K_1/2) = 0.1\left[0.1 + \frac{0.1}{2}, 1.11648 + \frac{0.13465}{2}\right]$

$K_2 = 0.1[0.15, 1.1833]$

$K_2 = 0.15514$

$K_3 = hf(x_0 + h/2, y_0 + K_2/h) = 0.1[0.15, 1.19405]$

$K_3 = 0.15757$

$K_4 = hf(x_0 + h, y_0 + K_3) = 0.1(0.2, 1.27405)$

$K_4 = 0.18232$

$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4] = 0.157706$

$y_1 = y_0 + K = 1.27354$

10/02/17

229] Assignment (E)

Q1] $\frac{dy}{dx} = \log(x+y)$, $y(1) = 2$, for $x = 1.2$ & $x = 1.4$

$h = 0.2$

$x_0 = 1$, $y_0 = 2$, $h = 0.2$, $x_1 = 1.2$

$y(1.2) = 2.2332$

Solⁿ → b) at $x = 1.4$

we take $x_0 = 1.2$, $y_0 = 2.2332$

$x_1 = 1.4$, $h = 0.2$

$y_1 = y_0 + h f(x_0, y_0) = 2.2332 + 0.2 \log(1.2 + 2.2332)$

$y_1 = 2.4799$

Now,

$y_1^{(1)} = y_0 + \frac{0.2}{2}\left[\log(1.2 + 2.2332) + \log(1.4 + 2.4799)\right]$

$y_1^{(1)} = 2.4921$

$y_1^{(2)} = 2.2332 + \frac{0.2}{2}\left[\log(1.2 + 2.2332) + \log(1.4 + 2.4921)\right]$

$y_1^{(2)} = 2.4924$

$y_{(1)}^{(3)} = 2.4924$

The second and third approximation gives same value at $x = 1.4$ is 2.4924

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224] Solve
$$(2xy^4e^y+2xy^3+y)dx+(x^2y^4e^y-x^2y^2-3x)dy=0 \quad \text{①}$$

Sol^n →
$$M = 2xe^y y^4+2xy^3+y$$
$$N = x^2y^4e^y - x^2y^2-3x$$

$$\frac{\partial M}{\partial y} = 2x\left[4y^3e^y+e^y y^4\right]+6xy^2+1$$

$$\frac{\partial M}{\partial y} = 8xy^3e^y+2xe^y y^4+6xy^2+1 \quad \text{②}$$

$$\frac{\partial N}{\partial x} = e^y y^4\left[2x\right]\cancel{e^y+e^y x^2 y^4}-2xy^2-3$$

$$\frac{\partial N}{\partial x} = 2xy^4e^y-2xy^2-3$$

$$\therefore \frac{\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)}{M} = \frac{\cancel{8}-4\cancel{(2e^y xy^3+2xy^2+1)}}{y\cancel{(2e^y xy^3+2xy^2+1)}}$$

$$= \frac{-4}{y}$$

$$\therefore \text{I.F. is I.F.} = e^{\int P.dy} = e^{-4\int 1/y\, dy}$$
$$= e^{-4\log y} = \frac{1}{y^4}$$

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Multiply I.F to eq^n (I) we get;

$$\left[2xe^y + \frac{2x}{y} + \frac{1}{y^3}\right]dx + \left[x^2e^y - \frac{x^2}{y^2} - \frac{3x}{y^4}\right]dy = 0$$

which is exact

$$\int_{\text{'y' constant}} M\,dx = \int\left(2xe^y + \frac{2x}{y} + \frac{1}{y^3}\right)dx$$

$$= x^2e^y + \frac{x^2}{y} + \frac{x}{y^3}$$

$$\int_{\text{Terms free from 'x'}} N\,dy = \int 0\,dy = 0$$

$\therefore$ The solution is;

$$x^2e^y + \frac{x^2}{y} + \frac{x}{y^3} = c$$

$$x^2y^3e^y + x^2y^2 + x = cy^3$$