There are multiple steps to a Monte-Carlo simulation.

(1) Random Variable Generation

- These variables represent the uncertain elements of the Model.

- The choice of probability distribution depends on the nature of the data.

Common distributions are Gaussian, log-normal, uniform & binomial distribution

For our case, and for most financial models, the gaussian/normal distribution is used.

$$\begin{bmatrix}\text{Normal} \\ \text{Distribution}\end{bmatrix} \equiv X \sim N(\mu, \sigma^2)$$

here $\Rightarrow$ μ = mean

σ^2 = variance

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Now, random samples from this distribution can be generated from algorithms like the Box-Muller transform.

1) Box Muller Transform

The Box-Muller transform takes two independent random numbers from a uniform distribution and transforms them into two independent normally distributed random variables. This is particularly useful because computers are typically good at generating random numbers in a uniform distribution, which can then be transformed into a normal distribution.

for 2 independent random variables

$\equiv$

U_1 & U_2

{uniformly distributed in interval $(0,1)$}

the transform converts these 2 independent standard normally distributed random variables Z_0 & Z_1

$$\Rightarrow Z_0 = \sqrt{-2\ln(U_1)}\,\cos(2\pi U_2)$$

$$\Rightarrow Z_1 = \sqrt{-2\ln(U_1)}\,\sin(2\pi U_2)$$

WHY IT WORKS

because it effectively maps the uniform distribution onto a circular distribution & then projects points from this circle onto the x & y axes, which results in a normal distribution due to the properties of sine & cosine functions in a circle.

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(2) Repeated Sampling

The monte carlo simulation relies heavily on the law of large nos.

LAW OF LARGE NOS ⇒

Avg of the results obtained from a large no. of trials is much more likely to be closer to the expected value, and tends closer with increasing no. of results.

Iterations: Each iteration uses a new set of random variables, with accuracy increasing with no. of iterations.

Now in each run, a specific outcome is calculated on the basis of the model being simulated.

3 Aggregation & Analysis

After running the simulations, results are aggregated to make a statistical distribution.

Mean $\Rightarrow \bar{X} = \frac{1}{N}\sum_{i=1}^{N} O_i$

S.d $\Rightarrow \sigma = \sqrt{\frac{\sum_{i=1}^{N}(O_i-\bar{X})^2}{N-1}}$

④ Estimating Probabilities

Probability Estimation ⇒

e.g.: to estimate the probability of a stock's return falling below a certain threshold, we carry out ⇒

$$\frac{\text{no. of times event occurs}}{\text{total no. of iterations}}$$

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⑤ Application in Portfolio Optimization

- **Future Value Estimation** ⇒

$$\text{future stock value} = \text{current stock value} \times (1+r)$$

{portfolio analysis}

return of portfolio $= \sum_{i=1}^{n} W_i R_i$

where → W_i = weights

R_i = assets returns

Other calculations for the portfolio can be carried out on the basis of the result obtained above