Risk_Models.py

By - Tanmay Nanda

Covariance:

$S = \frac{1}{T}X^TX$

Here:

- S is the sample covariance matrix.
- X is the matrix of returns.
- T is the number of observations.

This expression calculates the sample covariance matrix by taking the transpose of the matrix of returns (X→ X^T) and multiplying it by the original matrix of returns (X), and then dividing by the number of observations (T).

Semivariance:

Semivariance is similar to variance, but it only considers observations that are below the mean. Semivariance is a useful tool in portfolio or asset analysis because it provides a measure for downside risk.

Downside Risk:

Some investments have a finite amount of downside risk, while others have infinite risk. The purchase of a stock, for example, has a finite amount of downside risk bounded by zero. The investor can lose their entire investment, but not more. A short position in a stock, however, as accomplished through a short sale, entails unlimited downside risk since the price of the security could continue rising indefinitely.

$$Semivariance = \frac{1}{n}\sum_{i=1}^{n} \max\left(\text{Average} - r_i, 0\right)^2]$$

Here:

- (n) is the number of observations.
- (r_i) represents individual observations.
- $(Average)$ is the average or benchmark return.

Exponentially Weighted Covariance:

The formula provided describes the calculation of the exponentially weighted covariance matrix (ExpCov) for each pair of assets (i) and (j). Let me break down the components of the formula and explain each part:

$$ExpCov_{ij} = EWMA((R_i - Ri)(R_j - Rj), span)$$

1. **Returns Calculation:**
   - (R_i) and (R_j) are the returns of assets (i) and (j) respectively.
   - (R_i) and (R_j) are the respective means (averages) of the returns of assets (i) and (j).
2. **Exponentially Weighted Moving Average (EWMA):**
   - The term $((R_i - R_i) \cdot (R_j - R_j))$ represents the product of the deviations of the returns from their means.
   - EWMA is then applied to this product with a parameter called (span).
   - The EWMA function is a type of weighted moving average that gives more weight to recent observations and less weight to older observations. The parameter $(span)$ determines the decay factor of the weights.
3. **Final Calculation:**
   - The resulting value, denoted as $(ExpCov_{ij})$, is the exponentially weighted covariance between assets (i) and (j).

Here, RiRi and RjRj are the returns of assets ii and jj, R¯iRi and R¯jRj are their respective means, and span is the span parameter used in the exponentially weighted moving average (EWMA) calculation.

This type of covariance calculation is useful for capturing changes in the covariance structure over time, with more recent observations influencing the result more than older ones.

▼ **Span Parameter**

The "span" parameter in the exponentially weighted moving average (EWMA) formula determines the decay factor of the weights, influencing how much weight is given to recent observations. The calculation of the span depends on the context and the specific requirements of your analysis. In finance and time-series analysis, the span is often chosen based on the desired half-life or smoothing factor.

Here's a general guideline:

1. **Half-life (hl):**
   - The half-life is the time it takes for the weight to reduce to half its initial value.
   - The relationship between the span and half-life is given by the formula: $(hl = \frac{span}{\ln(2)})$
   - You can choose the half-life based on the rate at which you want old observations to lose their influence.
2. **Smoothing Factor ((\alpha)):**
   - The smoothing factor (α) is another parameter related to the span and is defined as: ($\alpha = \frac{2}{span+1}$)
   - Smaller values of (α) give more weight to recent observations.
3. **Direct Choice:**

- Alternatively, you can directly choose a specific value for the span based on your preferences or the characteristics of your data. Common values for span range from 10 to 1000.

Here's how you can calculate span using the half-life or smoothing factor:

- For Half-life: ($span = hl \times \ln(2)$)
- For Smoothing Factor: ($span = \frac{2}{\alpha} - 1$)

Choose the approach that aligns with your analytical needs and the characteristics of your time series data. It's often a matter of experimentation and adjusting the span based on the behaviour that we want to capture in the EWMA calculation.

▼ **EWMA**

To calculate the Exponentially Weighted Moving Average (EWMA), you can follow these steps:

1. **Choose a Span Parameter:**
   - As discussed earlier, the span parameter determines the decay factor of the weights and influences how much weight is given to recent observations. You can calculate the span based on the desired half-life or smoothing factor.
2. **Calculate the Exponential Weights:**
   - For each observation (t), calculate the exponential weight using the formula: $(w_t = (1 - \alpha)^t)$, where (α) is the smoothing factor $((0 < \alpha \leq 1))$.
3. **Calculate the EWMA:**
   - Multiply each observation by its corresponding exponential weight and sum the results to obtain the EWMA. The formula is: $EWMA = \sum_{t=0}^{T-1} w_t \cdot x_t)$, where (x_t) is the value of the time series at time (t).

Here's a step-by-step breakdown:

- Choose a value for the smoothing factor ((\alpha)) or the span.
- Calculate the exponential weights using the formula (w_t = (1 - \alpha)^t).
- Multiply each observation by its corresponding weight.
- Sum the results to obtain the EWMA.

In summary, the EWMA at time (t) is a weighted average of the historical observations, with more recent observations receiving higher weights. The choice of the smoothing factor or span depends on the specific requirements and the characteristics of the data. Adjusting these parameters allows you to control the level of smoothing and responsiveness of the EWMA to changes in the time series.