JEE-Main-22-07-2021-Shift-2 (Memory Based)

PHYSICS

Question: Find the ratio of de Broglie wavelength of an electron and a proton having same Kinetic Energy. Given that the mass of electron is m_e and proton is m_p.

Options:

(a) $\dfrac{m_p}{m_e}$

(b) $\dfrac{m_e}{m_p}$

(c) $\sqrt{\dfrac{m_p}{m_e}}$

(d) $\sqrt{\dfrac{m_e}{m_p}}$

Answer: (c)

Solution:

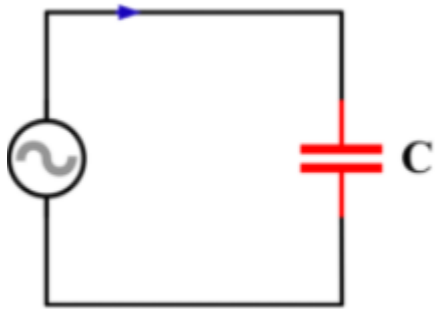de Broglie wavelength $\lambda = \dfrac{h}{\sqrt{2mk}}$

for electron, $\lambda_e = \dfrac{h}{\sqrt{2m_e k}}$ ...(i)

for proton, $\lambda_p = \dfrac{h}{\sqrt{2m_p k}}$ ...(ii)

from equation (i) and (ii)

$\dfrac{\lambda_e}{\lambda_p} = \sqrt{\dfrac{m_p}{m_e}}$

Question: If an AC source is connected to a capacitor in series. Then which of the graphs relating current and emf is correct?
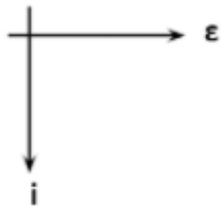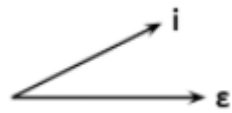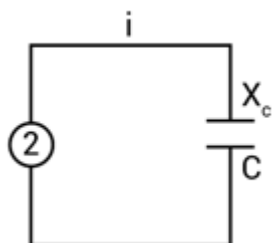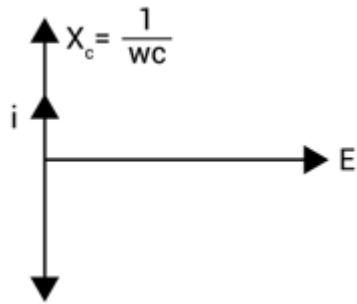
Options:

(a)

(b)

(c)

(d)

Answer: (a)

Solution:

$$i = \frac{\varepsilon}{x_C}\, \hat{j}$$

$$i = \frac{\varepsilon_0 \sin \omega t}{\frac{1}{\omega C}} \hat{j}$$

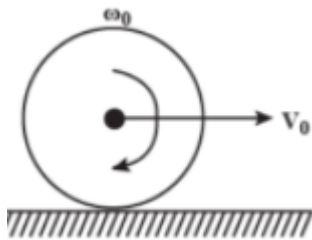$$i = \varepsilon_0 \omega C \sin \omega t \hat{j}$$

$$i = \varepsilon \omega C \hat{j}$$

option A is correct.

Question: A wheel is undergoing pure rolling on a horizontal surface as shown. The point on rim of wheel at a same horizontal level as center has velocity $\sqrt{x}V_0$. The x is
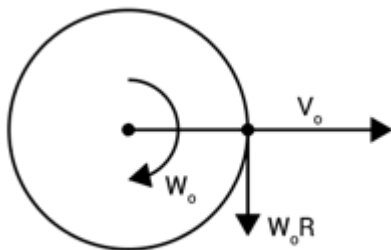

Options:
(a) 1
(b) 2
(c) 3
(d) 4
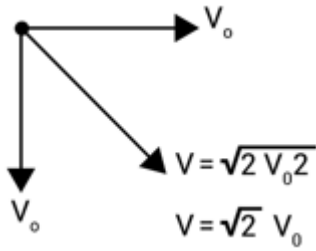**Answer:** (b)
Solution:

In pure rolling
$$V_0 = \omega_0 R$$

Therefore x =2.

Question: If a particle is performing SHM such that $X = A\sin(\omega t) + B\cos(\omega t)$. Then the equation of SHM can also be represented as $x = C\sin[\omega t + \phi]$. The C & ϕ are

Options:

(a) $\sqrt{A^2 + B^2}, \tan^{-1}\left[\dfrac{B}{A}\right]$

(b) $\sqrt{A^2 + B^2}, \tan^{-1}\left[\dfrac{A}{B}\right]$

(c) $A + B, \tan^{-1}\left[\dfrac{B}{A}\right]$

(d) None

Answer: (a)

Solution:

$X = A\sin\omega t + B\cos(\omega t)$

$X = \sqrt{A^2 + B^2}\left\{\dfrac{A}{\sqrt{A^2 + B^2}}\sin\omega t + \dfrac{B}{\sqrt{A^2 + B^2}}\cos(\omega t)\right\}$

$X = \sqrt{A^2 + B^2}\left\{\cos\phi\sin\omega t + \sin\phi\cos\omega t\right\}$

$$\begin{bmatrix} \sin\phi = \dfrac{B}{\sqrt{A^2 + B^2}} \\ \cos\phi = \dfrac{A}{\sqrt{A^2 + B^2}} \\ \tan\phi = \dfrac{B}{A} \\ \phi = \tan^{-1}\left(\dfrac{B}{A}\right) \end{bmatrix}$$

$X = \sqrt{A^2 + B^2}\sin(\omega t + \phi)$

Therefore, $C = \sqrt{A^2 + B^2}, \phi\tan^{-1}\left(\dfrac{B}{A}\right)$

Question: The intensity of sunlight at a place on earth is $0.92 wm^{-2}$. Find the amplitude of magnetic field at that place.

Options:

(a) $4.6 \times 10^{-8} T$

(b) $5.6 \times 10^{-8} T$

(c) $6.7 \times 10^{-8} T$

(d) $8.77 \times 10^{-8} T$

Answer: (d)

Solution:

$I = 0.92 w/m^2$

$I = \dfrac{1}{2} C \dfrac{B_0^2}{u_0}$

$0.92 = \dfrac{1}{2} \times \dfrac{3 \times 10^8 \times B_0^2}{1.26 \times 10^{-6}}$

$B_0 = \sqrt{\dfrac{0.92 \times 2 \times 1.26 \times 10^{-6}}{3 \times 10^8}} T$

$B_0 = 0.877 \times 10^{-7} T$

$B_0 = 8.77 \times 10^{-8} T$

Question: A long line charge of linear charge density $4 \times 10^{-5} c/m$ is placed along z-axis. An electric dipole of length 2 mm is placed along x-axis. The negative end of dipole is at 10 mm and positive end of dipole is at 12 mm. The force experienced by dipole is 4 N. Find charge of dipole

Options:

(a) $0.5 \mu C$

(b) $0.33 \mu C$

(c) $5 \mu C$

(d) $8 \mu C$

Answer: (b)

Solution:
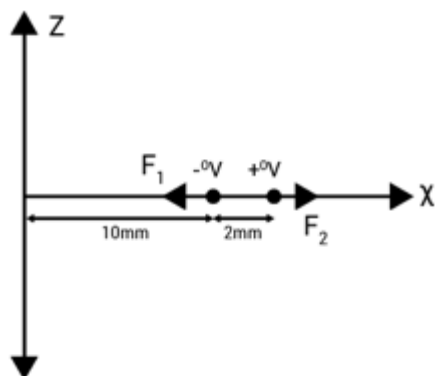
$\lambda = 4 \times 10^{-5} c/m$

Electric field due to infinitely long charged wire $\varepsilon = \dfrac{\lambda}{2\pi \in_0 x}$

$F_{net} = F_1 - F_2$

$$4 = \frac{q\lambda}{2\pi\varepsilon_0 \times 10 \times 10^{-3}} - \frac{q\lambda}{2\pi\varepsilon_0 \times 12 \times 10^{-3}}$$

$$4 = q \times \left(\frac{2q}{4\pi\varepsilon_0 10^{-2}} - \frac{2\lambda}{4\pi\varepsilon_0 \times 12 \times 10^{-3}} \right)$$

$$4 = q \times \left(\frac{2 \times 4 \times 10^{-5} \times 2 \times 10^9}{10^{-2}} - \frac{2 \times 4 \times 10^{-5} \times 9 \times 10^3}{12 \times 10^{-3}} \right)$$

$$4 = q \times 12 \times 10^{-6}$$

$$q = \frac{1}{3} \times 10^{-6} C$$

$$q = 0.33 \mu C$$

Question: The angle of Dip in plane is δ' and true Dip in magnetic meridian is δ. Then

Options:

(a) $\delta' > \delta$

(b) $\delta' < \delta$

(c) $\delta' = \delta$

(d) None

Answer: (a)

Solution:

δ' is apparent dip, let α be the angle made by dip circle with magnetic meridian.

Now rotating dip circle by $90°$ from this position. It will now make angle $90 - \alpha$ with the magnetic meridian. In this case apparent dip is δ''

Now we have relation

$\cot^2 \delta' + \cot^2 \delta'' = \cot^2 \delta$ ...(i)

Where, δ is true dip in magnetic meridian

From eq (i)

$\cot^2 \delta' < \cot^2 \delta$

or $\cot \delta' < \cot \delta$

$\Rightarrow \delta' > \delta$

Question: A semi-ring has mass m and radius R as shown. The COM of semi ring lies on Y axis at a distance of $\frac{xR}{\pi}$. Find (x).

(0,0)

Options:

(a) 1

(b) 2

(c) 3

(d) 4
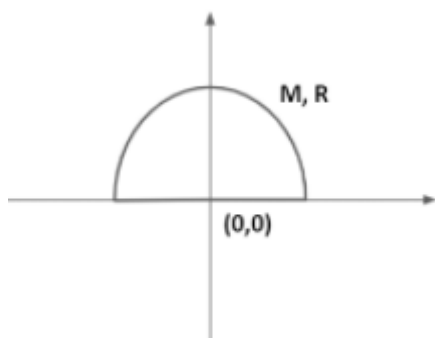
Answer: (b)

Solution:

The center of mass of the uniform semicircular ring is at $\dfrac{2R}{\pi}$ hence the value of x is 2.

Question: If a ring and a solid cylinder of same mass and radius are released from the top of an inclined plane. If the time taken by ring to some down is t_1 and time taken by cylinder is t_2 while both perform pure rolling, then choose the correct option.

Options:

(a) $t_1 > t_2$

(b) $t_1 < t_2$

(c) $t_1 = t_2$

(d) Can't be related

Answer: (a)

Solution:

$$t = \sqrt{\frac{2h}{g \sin^2 \theta}\left(1 + \frac{k^2}{R^2}\right)}$$

$$t \propto \left(1 + \frac{k^2}{R^2}\right)^{1/2}$$
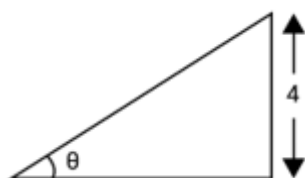
For ring

$$\frac{k^2}{R^2} = 1$$

For solid cylinder

$$\frac{k^2}{R^2} = \frac{1}{2}$$

$$t_1 \propto (1+1)^{\frac{1}{2}}$$

$$t_2 \propto \left(1 + \frac{1}{2}\right)^{\frac{1}{2}}$$

$$\frac{t_1}{t_2} = \sqrt{\frac{2}{\frac{3}{2}}} = \sqrt{\frac{4}{3}} \Rightarrow t_1 > t_2$$

Question: Identify the correct statement/statements

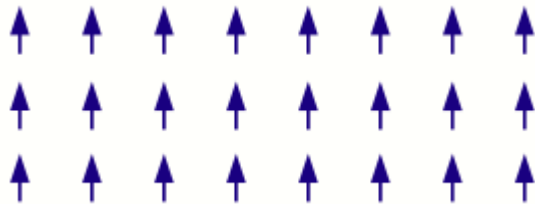Statement -1: Ferromagnetic converts to Paramagnetic on Heating.

Statement-2: Domains of Ferromagnetic material increase on Heating.

Options:

(a) 1

(b) 2

(c) Both 1 and 2

(d) Can't say

Answer: (a)

Solution:

Ferromagnetic material: All the molecular magnetic dipoles are pointed in the same direction When the ferromagnetic material heated , all magnetic dipoles are disturbed and get disoriented. Due to which the net magnetic dipole moment becomes very weak, so they behave as paramagnetic material.

Question: Find average kinetic energy of a monatomic gas molecule, assuming ideal behaviour.

(Given T is the temperature of the gas and k is Boltzmann constant)

Options:

(a) $\frac{1}{2}kT$

(b) $\frac{3}{2}kT$

(c) $\frac{5}{2}kT$

(d) kT

Answer: (b)

Solution:

The average kinetic energy of a molecule is given by

$$\frac{1}{2}f\,kT$$

Where f is degree of freedom

For monoatomic gas f = 3 so average kinetic energy is

$$\frac{3}{2}kT$$

Question: P is travelling along the vector $\vec{A} = \hat{i} + \hat{j}$, Q is moving along the vector $\vec{B} = \hat{j} + \hat{k}$, R is moving along the vector $\vec{C} = -\hat{i} + \hat{j}$. All three particles collide at a point and after collision, P moves along $\vec{A} \times \vec{B}$ and Q moves along $\vec{B} \times \vec{C}$, then the angle between the direction of P and Q after collision-

Options:

(a) $\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

(b) $\cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

(c) $\sin^{-1}\left(\dfrac{1}{3}\right)$

(d) $\cos^{-1}\left(\dfrac{1}{3}\right)$

Answer: (d)

Solution:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$$

$$\vec{\alpha}_1 = \vec{A} \times \vec{B} = i(1-0) - j(1-0) + \hat{k}(1-0)$$

$$= -i - j + k$$

$$\vec{\alpha}_1 = \vec{A} \times \vec{B} = i(1-0) - j(1-0) + \hat{k}(1-0)$$

$$= -i - j + k$$

$$\vec{\alpha}_2 = \vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ -1 & 1 & 0 \end{vmatrix}$$

$$= \hat{i}(-1) - \hat{j}(0+1) + \hat{k}(+1)$$

$$= -i - j + \hat{k}$$

Angle between $\vec{\alpha}_1$ & $\vec{\alpha}_2$

$$\vec{\alpha}_1 \cdot \vec{\alpha}_2 = (i - j + \hat{k})(-i - j + \hat{k})$$

$$= -1 + 1 + 1$$

$$\vec{\alpha}_1 \cdot \vec{\alpha}_2 = 1$$

$$|\alpha_1| = \sqrt{3}$$

$$|\alpha_2| = \sqrt{3}$$

$$\vec{\alpha}_1 \cdot \vec{\alpha}_2 = \alpha_1 \alpha_2 \cos\theta$$

$$\alpha_1 \alpha_2 \cos\theta = \vec{\alpha}_1 \cdot \vec{\alpha}_2$$

$$\left(\sqrt{3}\right)\left(\sqrt{3}\right)\cos\theta = 1$$

$$\cos\theta = \frac{1}{3}$$

$$\theta = \cos^{-1}\left(\frac{1}{3}\right)$$

Question: A TV transmitter has range of 1500 km. What is the approximate height of the TV transmission tower?

Options:

(a) 175 km

(b) 200 km

(c) 150 km

(d) 125 km

Answer: (a)

Solution:

Range d = 1500 km, Radius of earth R_e=6400 km

$$d = \sqrt{2hR_e}$$

$$h = \frac{d^2}{2R_e} = \frac{\left(1500\times10^3\right)^2}{2\times6400\times10^3} \approx 175\,km$$

Question: A ray of light is incident on a prism at an angle of incidence 45° and ray is undergoing minimum deviation such that the angle of deviation is 30°. Find the angle of prism

Answer: -60°

Solution:

When set at minimum deviation position, $A - 2i - d$

$$i - 45^\circ, d - 30^\circ$$

$$A - 2\times45 - 30 - 60^\circ$$

Question: In 5 min, a body changes temperature from $75^\circ C$ to $65^\circ C$ at room temperature $25^\circ C$. What temperature will it have after another 5 min?

Options:

(a) $60^\circ C$

(b) $57^\circ C$

(c) $55^\circ C$

(d) $53^\circ C$.

Answer: (b)

Solution:

$T_1 = 75^\circ C, T_2 = 65^\circ C, t = 5min, T_0 = 25^\circ C, T = ?$

$$\frac{T_1 - T_2}{t} = k\left(\frac{(T_1 + T_2)}{2} - T_0\right) \Rightarrow \frac{10}{t} = 45k \quad \dots \textbf{(i)}$$

$$\frac{T_2 - T}{t} = k\left(\frac{(T + T_2)}{2} - T_0\right) \Rightarrow \frac{65 - T}{t} = k\left(\frac{(T + 65)}{2} - 25\right) \dots \text{(ii)}$$

From (i) and (ii) $\dfrac{10}{65 - T} = \dfrac{45}{\left(\dfrac{(T + 65)}{2} - 25\right)}$

on solving for $T, T = 57^\circ C$

Question: A section of a railway track is clamped between two nails. In summers it gets heated up and as a result, some elastic energy is stored in it. If $\Delta T = 10^\circ C, \propto = 10^{-6\circ} C^{-1} = $ coefficient of linear expansion, $Y = 10^{11} Nm^{-2} = $ Young's modulus, $A = 0.01m^2 = $ Area of cross section. Find the energy stored per unit length in the railway track.

Options:

(a) $5 \times 10^{-2} Jm^{-1}$

(b) $3 \times 10^{-2} Jm^{-1}$

(c) $2 \times 10^{-2} Jm^{-1}$

(d) $10^{-2} Jm^{-1}$

Answer: (a)

Solution:

$\alpha = 10^{-6\circ} C^{-1}, \Delta T = 10^\circ C, Y = 10^{11} N/m^2, A = 0.01m^2 = 10^{-2} m^2$

$\ell = \alpha L \Delta T \Rightarrow \dfrac{\ell}{L} = \alpha \Delta T$ (strain)

$= \left(10 \times 10^{-6}\right)$

$E_{\text{per unit length}} = \dfrac{1}{2} \times Y \times (\text{ strain})^2 \times \text{ area}$

$= \dfrac{1}{2} \times 10^{11} \times \left(10 \times 10^{-6}\right)^2 \times 10^{-2}$

$= 5 \times 10^{-2} J/m$

Question: Mass number of a nucleus is 184. It undergoes alpha decay. If Q value is 5.5 MeV, find kinetic energy of the emitted alpha particle.

Options:

(a) **0.4 MeV**

(b) **5 MeV**

(c) **5.4 MeV**

(d) **0.1 MeV**

Answer: (c)

Solution:

$M \rightarrow X + \alpha$

$E = \dfrac{p^2}{2m}$

Let Total energy = T

$T = E_\alpha + E_X$

$M = M_\alpha + M_X$

$M_X = 180$

Conservation of momentum

$p_\alpha = p_X$

$M E_\alpha = M_x E_X$

$E_\alpha = \dfrac{M_x}{M} T$

$E_\alpha = \dfrac{184}{180} \times 5.5 = 5.4 MeV$

Question: A pendulum has time period of small oscillations as T. What will be the new time period by length of pendulum is reduced to $\dfrac{1}{16}^{th}$ of its initial value?

Options:

(a) $\dfrac{T}{2}$

(b) $\dfrac{T}{4}$

(c) $\dfrac{T}{8}$

(d) $\dfrac{T}{16}$

Answer: (b)

Solution:

$T = 2\pi \sqrt{\dfrac{L}{g}}$

$T' = 2\pi \sqrt{\dfrac{L'}{g}}$

$L' = \dfrac{L}{16}$

$\therefore T' = 2\pi \sqrt{\dfrac{L}{16g}} = \dfrac{1}{4}\left(2\pi \sqrt{\dfrac{L}{g}} \right)$

$$T' = \frac{T}{4}$$

Question: Find projection of $\vec{A}$ on $\vec{B}$ given $\vec{A} = \hat{i} + \hat{j}$ & $\vec{B} = \hat{i} + \hat{j} + \hat{k}$

Options:

(a) $\dfrac{\sqrt{3}}{2}$

(b) $\dfrac{2}{\sqrt{3}}$

(c) $\dfrac{3}{2}$

(d) 2

Answer: (b)

Solution:

We know that

Projection of $\vec{a}$ on $\vec{b}$ **is**

$$= \frac{\vec{a} \cdot \vec{b}}{\left|\vec{b}\right|}$$

Given:

$$\vec{A} = \hat{i} + \hat{j}$$

$$\vec{B} = \hat{i} + \hat{j} + \hat{k}$$

$\therefore$ Projection $= \dfrac{(\hat{i} + \hat{j}) \cdot (\hat{i} + \hat{j} + \hat{k})}{\sqrt{1^2 + 1^2 + 1^2}}$

$$= \frac{1+1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

Question: A bullet of mass 4 gm is shot from a gun of mass 4 kg . Bullet has speed $50 ms^{-1}$. Find the impulse of bullet and velocity of recoil of gun?

Options:

(a) $200 kgm/s, 5m/s$

(b) $0.2 kgm/s, 5m/s$

(c) $0.2 kgm/s, 5cm/s$

(d) $200 kgm/s, 5cm/s$

Answer: (b)

Solution:

|Impulse|=|Change in momentum|

Mass of bullet $= 4 gm = 4 \times 10^{-3} Kg$

Here initial velocity of bullet = 0

Here final velocity of bullet = 50 m/s

$$\therefore \left| \Delta \vec{P} \right| = m \left| \left(\vec{V}_f - \vec{V}_i \right) \right|$$

$$= 4 \times 10^{-3} (50 - 0)$$

$$= 2 \times 10^{-1} kgm/s$$

$$= 0.2 kgm/s$$

By conservation of linear momentum

$$0 = \left(4 \times 10^{-3} \right) (50) - 4(V)$$

Here V=recoil velocity of gun

$$\therefore V = 0.05 m/s = 5 cm/s$$

Question: An 80 kg object brought down by 80 cm at constant speed. Find work done by external agent.

Options:

(a) **Zero**

(b) $+640J$

(c) $-640J$

(d) 645

Answer: (c)

Solution:

$m = 80 \ kg$

$\Delta h = -80cm = -0.8m$ [height decreased] .

$g = 10m/s^2$

Work done=Change in potential energy

[K.E. doesn't change]

$$W = \Delta U$$

$$= mg(\Delta h)$$

$$= 80(10)(-0.8)$$

$$= -640J$$

Question: Q. A body is projected from surface of earth with escape velocity. Find the time taken by body to attain height h is (M = mass of earth, R = Radius of earth, G = Gravitational constant)

Options:

(a) $\dfrac{2 \left[(R+h)^{\frac{3}{2}} - R^{\frac{3}{2}} \right]}{3\sqrt{2GM}}$

(b) $\dfrac{\left[(R+h)^{\frac{3}{2}} - R^{\frac{3}{2}} \right]}{3\sqrt{GM}}$

(c) $\dfrac{\left[(R+h)^{\frac{1}{2}} - R^{\frac{1}{2}}\right]}{3\sqrt{2GM}}$

(d) None

Answer: (a)

Solution:

Total energy of body will remain zero

T.E=0

At a distance 'h' from surface of earth

M=mass of earth

M=mass of body

R=Radius of earth

$\dfrac{1}{2}mv^2 - G\dfrac{Mm}{h} = 0$

$v = \sqrt{\dfrac{2GM}{h}}$

$\dfrac{dh}{dt} = \sqrt{\dfrac{2GM}{h}}$

$\Rightarrow \sqrt{h}\,dh = \sqrt{2GM}\,dt$

$\Rightarrow \int_{R}^{(R+h)} \sqrt{h}\,dh = \sqrt{2GM}\int_{0}^{t} dt$

$\dfrac{2}{3}\left[h^{3/2}\right]_{R}^{R+h} = \sqrt{2GM}\,t$

$\Rightarrow \dfrac{\dfrac{2}{3}\left[(R+h)^{3/2} - R^{3/2}\right] = t}{\sqrt{2GM}}$

$\Rightarrow t = \dfrac{2}{3}\dfrac{\left[(R+h)^{3/2} - R^{3/2}\right]}{3\sqrt{2GM}}$

Question: A cell of emf E is attached in a circuit having 5 ohms and 2 ohms resistor one by one. PD across 5 ohms comes out to be 1.25 V and across 2 ohms as 1 volt. If $E = \dfrac{x}{10}$, then find x .

Answer: (a)

Solution:

Let the internal resistance of cell be 'r'. Then when we connect 5Ω in the circuit

$i_1 = \dfrac{E}{5+r}$

And P.D across 5Ω ohm will be

$$V_5 = 5i_1 = 5\left(\frac{E}{5+r}\right) = 1.25...(1)$$

When we connect 2Ω

$$i_2 = \frac{E}{2+r}$$

& P.D across 2Ω is

$$V_2 = 2i_2 = 2\left(\frac{E}{2+r}\right) = 1...(2) \text{ Given}$$

1) can be written as

$$\frac{5E}{5+r} = 1.25 = \frac{5}{4}$$

$$\Rightarrow \frac{E}{5+r} = \frac{1}{4}$$

$$\Rightarrow 4E = 5+r..(3)$$

2) can be written as

$$\frac{2E}{2+r} = 1$$

$$\Rightarrow 2E = 2+r...(4)$$

Solving 3 and 4 we get

$$E = \frac{3}{2} = \frac{15}{10}$$

$$\therefore x = 15$$

CHEMISTRY

Question: More dissolved oxygen is found in?

Options:

(a) Boiling water

(b) Water at 4°C

(c) Water at 80°C

(d) Polluted water

Answer: (b)

Solution: As temperature increases, solubility of gas decreases.

Question: Match the following.

Compound	Hybridisation
I. SF_4	**(A)** sp^3d^2
II. IF_5	**(B)** sp^3d
III. NO_2^+	**(C)** sp^3
IV. NH_4^+	**(D)** sp
	(E) sp^3d^3

Options:

(a) I → (B); II → (A); III → (D); IV → (C)

(b) I → (C); II → (B); III → (A); IV → (D)

(c) I → (D); II → (B); III → (C); IV → (A)

(d) I → (B); II → (C); III → (A); IV → (D)

Answer: (a)

Solution:

$SF_4 – sp^3d$

$IF_5 – sp^3d^2$

$NO_2^+ – sp$

$NH_4^+ - sp^3$
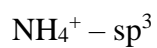
Question: Methylation of 10 g of benzene gives 9.2 g of toluene, the % yield is:

Options:

(a) 92 %

(b) 78 %

(c) 90 %

(d) 70 %

Answer: (b)

Solution:

$$\underset{78g}{C_6H_6} \xrightarrow[AlCl_3]{CH_3Cl} \underset{92g}{C_6H_5-CH_3}$$
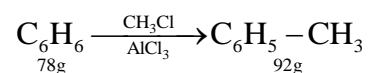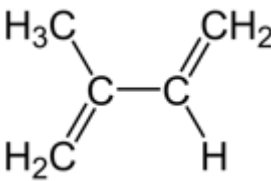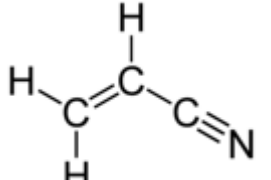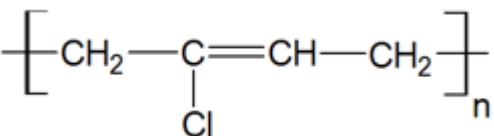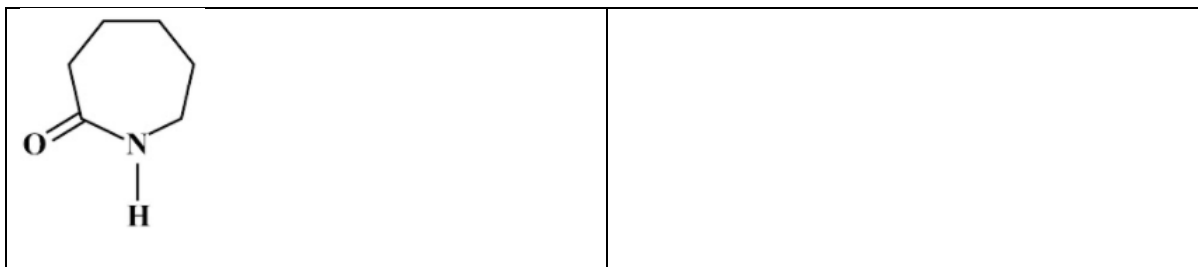
$$10 \text{ g benzene gives} = \frac{92}{78} \times 10 \text{ g}$$

$$\% \text{ yield} = \frac{9.2}{920/78} \times 100 = 78\%$$

Question: Identify the structure of isoprene, neoprene, acrylonitrile, caprolactam are

Column I	Column-II
I. H₃C, CH₂ structure	**(A)** isoprene
II. acrylonitrile structure	**(B)** acrylonitrile
III. $\left[CH_2-C=CH-CH_2 \right]_n$ with Cl	**(C)** neoprene
IV.	**(D)** Caprolactam

Options:

(a) I → (B); II → (A); III → (D); IV → (C)

(b) I → (C); II → (B); III → (A); IV → (D)

(c) I → (D); II → (B); III → (C); IV → (A)

(d) I → (A); II → (B); III → (C); IV → (D)

Answer: (d)

Solution:

⇒ isoprene

⇒ acrylonitrile

⇒ neoprene

⇒ Caprolactam
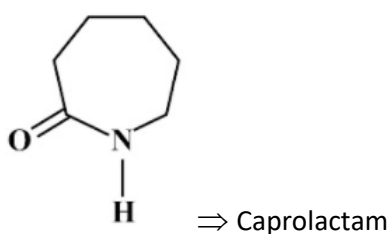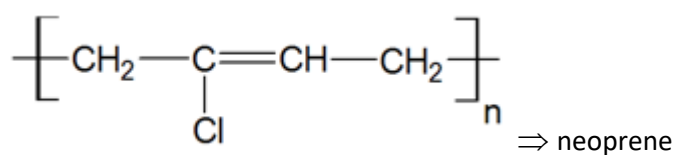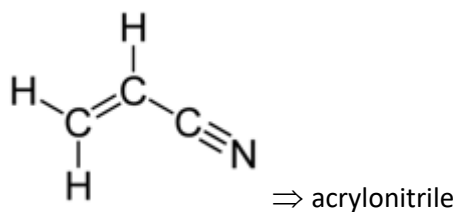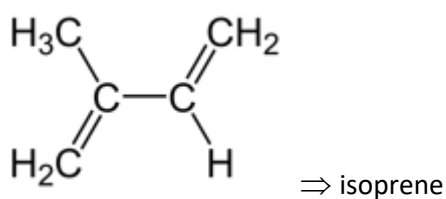
Question: Isotope of hydrogen which emits low energy **β**-particle with half-life value greater than 12 years is:
Options:

(a) Protium

(b) Tritium

(c) Deutrium

(d) None of the above

Answer: (c)

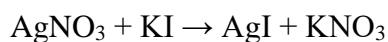**Solution:** Half-life of tritium is 12.33 yrs

Question: When $AgNO_3$ is added to KI solution, the solution produced is:
Options:

(a) KI/NO_3^-

(b) AgI/Ag^+

(c) AgI/NO_3^-

(d) $AgNO_3/NO_3^-$

Answer: (b)

Solution: When $AgNO_3$ is added to KI solution, Ag+ is an excess so we get accumulation of Ag^+ ions outside AgI, giving positively charged sol

$AgNO_3 + KI \rightarrow AgI + KNO_3$

Question: Which statement is not true for DI Mendeleev?

Options:

(a) He wrote the book Principles of chemistry

(b) He invented accurate barometer

(c) He proposed periodic table when structure of atoms were unknown

(d) Element with atomic number 101 is named after his name

Answer: (b)

Solution: Other options are true, only (b) is wrong.

Question: Which of the following 0.06 M solution has lowest freezing point?
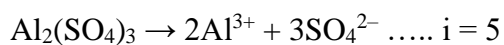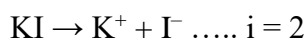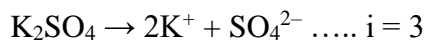
Options:

(a) K_2SO_4

(b) KI

(c) $C_6H_{12}O_6$

(d) $Al_2(SO_4)_3$

Answer: (d)

Solution:

$K_2SO_4 \rightarrow 2K^+ + SO_4^{2-}$ ..... i = 3

$KI \rightarrow K^+ + I^-$ ..... i = 2

$Al_2(SO_4)_3 \rightarrow 2Al^{3+} + 3SO_4^{2-}$ ..... i = 5

$C_6H_{12}O_6 \rightarrow$ No dissociation .... i = 1

More the value of i, greater will be depression in freezing point

Question: Number of acyclic structural isomers for pentene are :
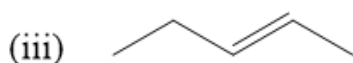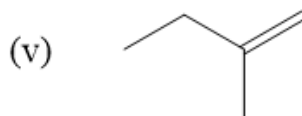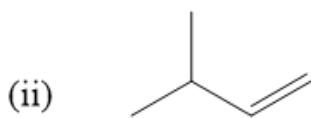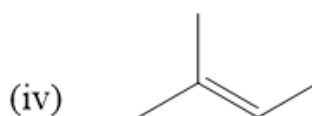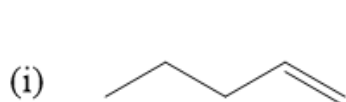
Options:

(a) 2

(b) 3

(c) 4

(d) 5

Answer: (d)

Solution:

Question: Thiamine and pyridoxine are respectively:

(a) Vitamin B_1 and Vitamin B_6

(b) Vitamin B_2 and Vitamin B_{12}

(c) Vitamin C and Vitamin A

(d) Vitamin C_1 and Vitamin D

Answer: (a)

Solution: Thiamine: Vitamin B_1

Pyridoxine: Vitamin B_6

Question: Which of the following is most reducing agent in group 15 elements?

Options:

(a) AsH_3

(b) SbH_3

(c) BiH_3

(d) PH_3

Answer: (c)

Solution: E-H bond energy decreases down the group.

So, best reducing agent in group 15 is BiH_3

Question: Total number of unpaired electron $[Co(NH_3)_6]Cl_2$ and $[Co(NH_3)_6]Cl_3$

Options:

(a) 3, 0

(b) 1, 2

(c) 1, 0

(d) 2, 1

Answer: (c)

Solution:

$[Co(NH_3)_6]Cl_2$ contains 1 unpaired electron while $[Co(NH_3)_6]Cl_3$ contains 0 unpaired electron

Question:
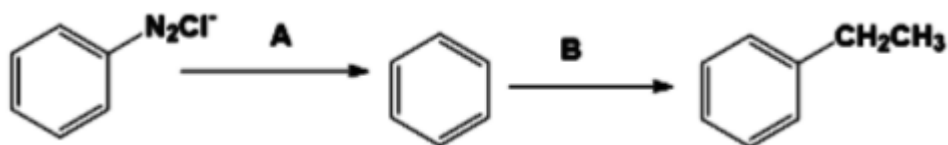
A and B respectively are

Options:

(a) A → H_3PO_2/H_2O; B → Anh. $AlCl_3$, C_2H_5Cl

(b) A → $KMnO_4/H_2O$; B → H_2O

(c) A → HCl; B → C_2H_5OH

(d) Zn(Hg), HCl; B → Anh. $AlCl_3$

Answer: (a)

Solution:
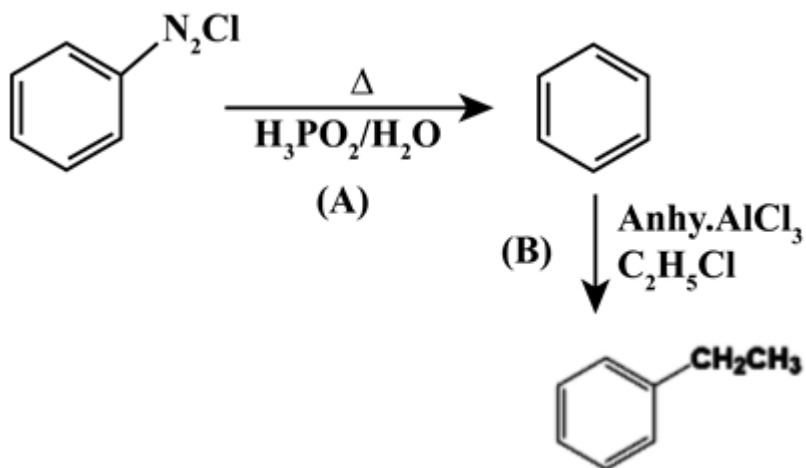
Question: Which of the following pair is paramagnetic as well as coloured?

Options:

(a) Mn^{7+}, Mn^{2+}

(b) Mn^{2+}, Cu^{2+}

(c) Mn^{7+}, Cu^{2+}

(d) Mn^{7+}, Cu^{2+}

Answer: (b)

Solution: Mn^{2+} and Cu^{2+} both are paramagnetic as well as coloured

Mn^{2+} – 2 unpaired electrons and light pink colour

Cu^{2+} – 1 unpaired electrons and blue colour

Question: Which of the following about B$_2$H$_6$ is correct?

Options:

(a) Diborare is obtained by NaBH$_4$ + I$_2$

(b) It is a planar molecule

(c) Boron are sp^2 hybridised

(d) Contains one 3c-3e bond

Answer: (a)

Solution: It is non-planar molecule in which B atoms are sp^3 hybridised. Molecule contains two 3c-2e bonds

Question: Which compound does not give Friedel craft reaction?

Options:

(a) Benzene

(b) Aniline

(c) Toluene

(d) Ethyl benzene

Answer: (b)

Solution: Lone pair of $\overset{..}{N}H_2$ combines with AlCl$_3$ due to which further reaction is not possible
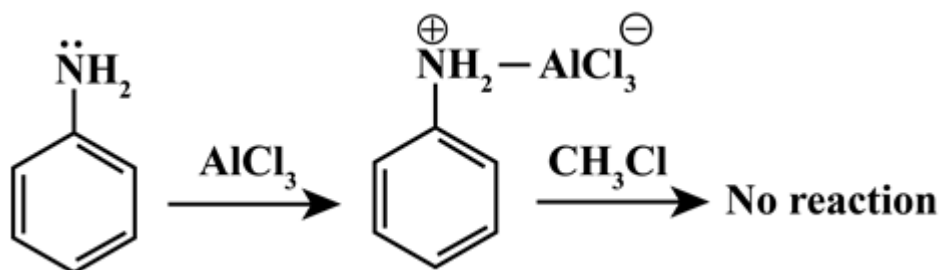
Question: Which of the following

Options:

(a) 3-ethye hex-3-ene

(b) 3-methyl hex-2-ene

(c) hex-1-ene

(d) None of these

Answer: (c)

Solution:

GI Possible

GI Possible

GI not possible
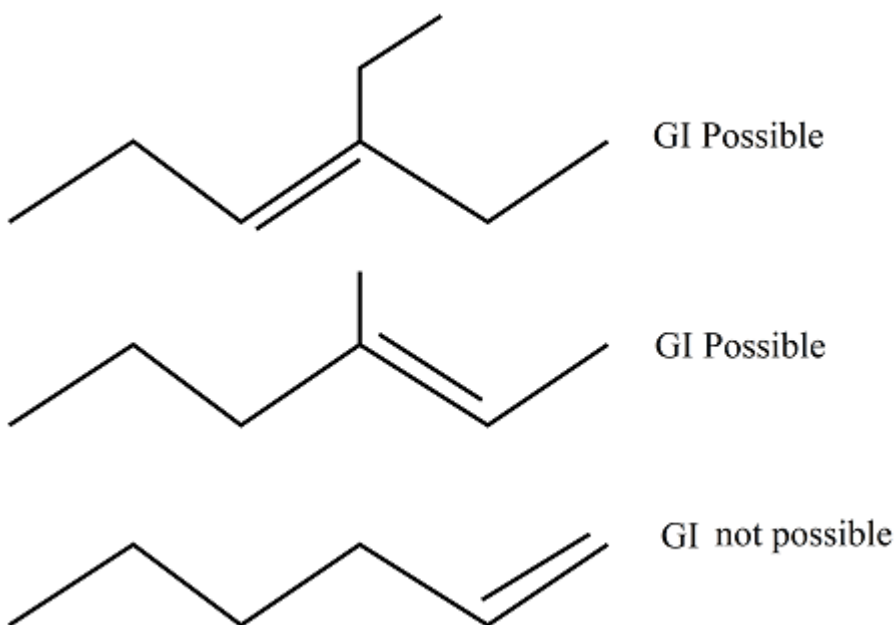
Question: If concentration of glucose ($C_6H_{12}O_6$) in blood is 0.72 g/L. Its molarity is:

Answer: 0.004

Solution:

$$\text{Molarity} = \frac{w}{M \times V}$$

M of glucose = 180 g/mol

$$\text{Molarity} = \frac{0.72}{180} = 0.004\,M$$

Question: Number of electrons in p-orbitals of Vanadium

Answer: 12.00

Solution: $_{23}V = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^2, 3d^3$

Total p electrons = 12

Question: For $N_2O_4 \rightarrow 2NO_2$ $K_P = 476$ find K_C at 288 K

Answer: 2.00

$KP = K_C(RT)^{\Delta ng}$

$47.6 = K_C[0.0821 \times 288]$

$K_C = \dfrac{47.6}{23.65} = 2$

JEE-Main-22-07-2021-Shift-2 (Memory Based)

MATHEMATICS

Question: The number of solution of $\sin^7 x + \cos^7 x = 1$, $x \in (0, 4\pi]$

Options:
(a) 7
(b) 11
(c) 9
(d) 5

Answer: (d)

Solution:

$\sin^7 x + \cos^7 x = 1$

If $\sin x \neq 0$ and $\cos x \neq 0$

$\sin^7 x < \sin^2 x$ and $\cos^7 x < \cos^2 x$

Adding both we get

$\sin^7 x + \cos^7 x < \sin^2 x + \cos^2 x$

$\Rightarrow \sin^7 x + \cos^7 x < 1$

Hence $\sin^7 x + \cos^7 x = 1$ is only possible

When $\sin x = 1$ and $\cos x = 0$

or $\cos x = 1$ and $\sin x = 0$

$\Rightarrow x = 0, \dfrac{\pi}{2}, \dfrac{5\pi}{2}, 2\pi, 4\pi$

Question: Let s_n denote sum of first n-terms of ap, $s_{10} = s_{30}$, $s_5 = 140$ then $s_{20} - s_6$

Options:
(a) 1872
(b) 1842
(c) 1852
(d) 1862

Answer: (d)

Solution:

$s_{10} = 5[2a + 9d] = 530 \Rightarrow 2a + 9d = 106$

$$s_5 = \frac{5}{2}[2a + 4d] = 140 \Rightarrow 2a + 4d = 56$$

$$\Rightarrow d = 10 \text{ and } a = 8$$

$$\therefore s_{20} - s_6 = 10[2a + 19d] - 3[2a + 5d]$$

$$= 14a + 175d = 1862$$

Question: If the domain $f(x) = \dfrac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\dfrac{2x-1}{2}\right)}}$ is the integral (α, β) then $\alpha + \beta$ is

Options:

(a) $\dfrac{1}{2}$

(b) $\dfrac{3}{2}$

(c) 1

(d) 2

Answer: (b)

Solution:

$$f(x) = \frac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\dfrac{2x-1}{2}\right)}}$$

$$0 < \sin^{-1}\left(\frac{2x-1}{2}\right) \le \frac{\pi}{2}$$

$$0 < \frac{2x-1}{2} \le 1$$

$$0 < 2x - 1 \le 2$$

$$1 < 2x \le 3$$

$$\frac{1}{2} < x \le \frac{3}{2} \qquad ....(1)$$

$$x^2 - x + 1 = x^2 - x + \frac{1}{4} - \frac{1}{4} + 1 = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} \ge \frac{3}{4}$$

Also, $x^2 - x + 1 = x^2 - x + \dfrac{1}{4} - \dfrac{1}{4} + 1 = \left(x - \dfrac{1}{2}\right)^2 + \dfrac{3}{4}$

$x^2 - x + 1 \leq 1$

$x^2 - x \leq 0$

$x(x-1) \leq 0$

$x \in [0,1] \qquad ....(2)$

From (1) and (2)

$x \in \left(\dfrac{1}{2}, 1 \right]$

$\alpha = \dfrac{1}{2}, \beta = 1$

$\alpha + \beta = \dfrac{1}{2} + 1 = \dfrac{3}{2}$

Question: $11^n > 10^n + 9^n$ number of integers satisfy the relation in $\{1, 2, 3, ..., 100\}$

Answer: 96
Solution:

$f(x) = \left(\dfrac{10}{11} \right)^x + \left(\dfrac{9}{11} \right)^x$

$f'(x) = \left(\dfrac{10}{11} \right)^x \ln^{\frac{10}{11}} + \left(\dfrac{9}{11} \right) \ln^{\frac{8}{11}} < 0$

$f(x)$ is decreasing

$f(4) > 1$ and $f(5) < 1$

Hence $x = 1, 2, 3, 4$ will not satisfy

$x = 5, 6, 7, ..., 100$ will satisfy

Number of integers = 96

Question: 0, 2 ,4, 6, 8 number of numbers > 10000 which can be formed, if repetition not allowed.
Answer: 96
Solution:

Number of numbers formed = $4 \times 4 \times 3 \times 2 \times 1 = 96$

Question: $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, number of matrices 'B' which can be formed such that AB = BA;

B can have elements {1, 2, 3, 4, 5}

Answer: 5^5

Solution:

Let $B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\because AB = BA$

$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow d = b, e = a, f = c, g = h$

So, i can be selected in 5 ways and each pair by 5, so total ways = 5^5

Question: 4 die rolled, numbers are first in 2×2 matrices. Find the probability that the matrices is non-singular & all entries are different.

Answer: $\dfrac{80}{81}$

Solution:

Let $a = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

For A to be singular, $ad = bc$

$\therefore a, d = 1, 6$ and $b, c = 3, 2 \Rightarrow 8$

$a, d = 2, 6$ and $b, c = 4, 3 \Rightarrow 8$

So, total case of singular matrix = $8 + 8 = 16$

$\therefore$ Total non-singular matrix = $6 \times 5 \times 4 \times 3 - 16$

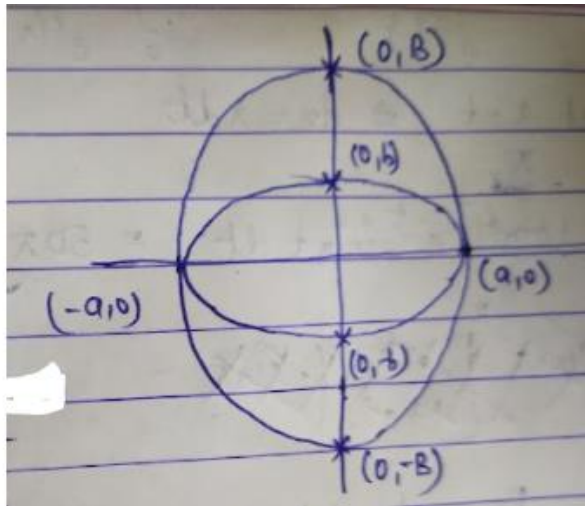$$\therefore \ \text{Probability} = \frac{344}{360} = \frac{43}{45}$$

Question: If $E_1 : \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1; \ a > b$, E_2 is an ellipse which touches E_1 at the end of major axis and end of minor axis E_1 are the foci of E_2. The eccentricity of both ellipse are equal. Find 'e'

Options:

(a)

(b)

(c)

(d)

Answer: ()

Solution:

Let $E_2 : \dfrac{x^2}{a^2} + \dfrac{y^2}{B^2} = 1$

Where $B > a$

$$\because \ e_1 = e_2 \Rightarrow 1 - \frac{b^2}{a^2} = 1 - \frac{a^2}{B^2} \Rightarrow B^2 = \frac{a^4}{b^2}$$

Also, foci of $E_2 = Be = b \Rightarrow e = \dfrac{b}{B} = \dfrac{b^2}{a^2} = 1 - e^2$

$$\therefore \ e = \frac{\sqrt{5} - 1}{2}$$

Question: If $36x^2 + 36y^2 - 108x + 120x + C = 0$ the circle does not cut/touch coordinate axis. Find the range of 'c'.

Options:

(a) $(100, 165)$

(b) $(100, 156)$

(c)

(d)

Answer: ()

Solution:

$36x^2 + 36y^2 - 108x + 120y + C = 0$

$x^2 + y^2 - \dfrac{108x}{36} + \dfrac{120y}{36} + \dfrac{C}{36} = 0$

$x^2 + y^2 - 3x + \dfrac{10}{3}y + \dfrac{C}{36} = 0$

Circle does not touch or cut axes

$\Rightarrow g^2 - C < 0$ and $f^2 - C < 0$

$\Rightarrow \left(\dfrac{3}{2}\right)^2 - \dfrac{C}{36} < 0$ and $\left(\dfrac{10}{6}\right)^2 - \dfrac{C}{36} < 0$

$\Rightarrow \dfrac{9}{4} - \dfrac{C}{36} < 0$ and $C > 36\left(\dfrac{100}{36}\right)$

$\Rightarrow C > \dfrac{36 \times 5}{4}$ and $\Rightarrow C > 100$ ....(2)

$\Rightarrow C > 81$ ....(1)

Also $g^2 + f^2 - C > 0$

$\Rightarrow \left(\dfrac{3}{2}\right)^2 + \left(\dfrac{3}{3}\right)^2 - \dfrac{C}{36} > 0$

$\Rightarrow \dfrac{C}{36} < \dfrac{9}{4} + \dfrac{25}{9}$

$\Rightarrow C < 81 + 100$

$\Rightarrow C < 181$ ....(3)

From (1), (2) and (3)

$C \in (100, 181)$

Question: If $f(1) + f(2) + f(3) = 3$

$A = \{0, 1, 2, 3, ..., 9\}$

No. of objective functions $f : A \to A$ which satisfy?

Answer: $3! \times 7!$

Solution:

$f(1) + f(2) + f(3) = 3$

Only possible combination is 0 + 1+ 2

So total bijective function $= 3! \times 7!$

Question: If $P : y^2 = \alpha x;\ L : 2x + y = k$

L is a tangent to $x^2 - y^2 = 3$ and P. Find 'α'

Options:
(a) -24
(b) 24
(c) -19
(d) 19

Answer: (b)

Solution:

$2x + y = k$ is tangent to $x^2 - y^2 = 3$

$y = -2x + k$

$c^2 = a^2 m^2 - a^2$

$\Rightarrow k^2 = 3(-2)^2 - 3$

$\Rightarrow k^2 = 9$

$\Rightarrow k = \pm 3$

$y = -2x + 3$ and $y = -2x - 3$

If $y = mx + c$ is tangent to $y^2 = 4ax$ then $c = \dfrac{a}{m}$

$\Rightarrow 3 = \dfrac{\alpha}{4(-2)}$ or $-3 = \dfrac{\alpha}{4(-2)}$

$\Rightarrow \alpha = -24$ or $\alpha = 24$

Question: If 'n' is the number of solutions of $z^2 + 3\bar{z} = 0$, where $z \in C$, then find $\sum_{k=0}^{\infty} \dfrac{1}{n^k}$

Answer: $\dfrac{1}{2}$

Solution:

$z = x + iy$

$(x + iy)^2 + 3(x - iy) = 0$

$x^2 - y^2 + 2ixy + 3x - 3iy = 0$

$x^2 - y^2 + 3x + i(2xy - 3y) = 0$

$\Rightarrow x^2 - y^2 + 3x = 0$ and $y(2x - 3) = 0$

If $y = 0$ $x^2 + 3x = 0$ $\Rightarrow y = 0$ or $x = \dfrac{3}{2}$

$\Rightarrow x = 0, -3$

And if $x = \dfrac{3}{2}$

$y^2 = \left(\dfrac{3}{2}\right)^2 + 3\left(\dfrac{3}{2}\right)$

$= \dfrac{9}{4} + 3\left(\dfrac{3}{2}\right) = \dfrac{9}{4} + \dfrac{9}{2}$

$= \dfrac{27}{4}$

Hence 3 solutions

$\sum_{k=0}^{1} \dfrac{1}{3^k} = \dfrac{1}{3} + \dfrac{1}{3^2} + \dots \infty$

$= \dfrac{\dfrac{1}{3}}{1 - \dfrac{1}{3}} = \dfrac{1}{2}$

Question: ω be a cube root of unity r_1, r_2, r_3 are the numbers obtained on the dice then the probability of $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$?

Options:

(a) $\dfrac{1}{18}$

(b) $\dfrac{1}{9}$

(c) $\dfrac{2}{9}$

(d) $\dfrac{1}{3}$

Answer: (c)

Solution:

$r_1, r_2, r_3 \in \{1,2,3,4,5,6\}$

r_1, r_2, r_3 are of the form $3k, 3k+1, 3k+2$

Required Probability $= \dfrac{3! \times {}^2C_1 \times {}^2C_1 \times {}^2C_1}{6 \times 6 \times 6} = \dfrac{6 \times 8}{216} = \dfrac{2}{9}$

Question: The number of elements in the set $\{x \in R : (|x|-3)|x+4| = 6\}$ is equal to
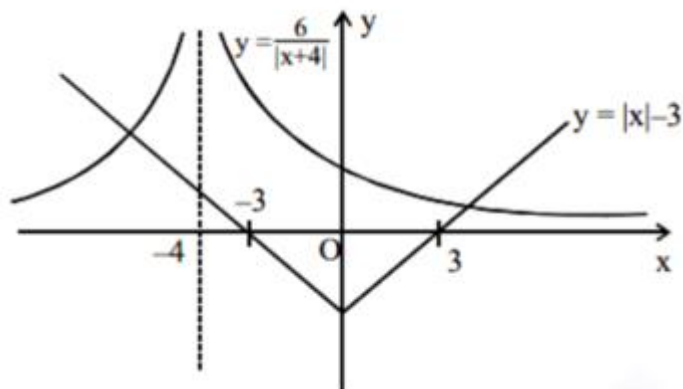
Options:

(a) 3

(b) 4

(c) 2

(d) 1

Answer: (c)

Solution:

$x \neq -4$

$(|x|-3)(|x+4|) = 6$

$\Rightarrow |x| - 3 = \dfrac{6}{|x+4|}$

Number of solutions = 2

Question: $\int \dfrac{e^x\left(2-x^2\right)}{(1-x)\sqrt{1-x^2}}\,dx$

Answer: $\dfrac{2}{9}$

Solution:

$I = \int \dfrac{e^x\left(2-x^2\right)}{1-\sqrt{1-x^2}}$

$= \int e^x\left(\dfrac{1}{(1-x)\sqrt{1-x^2}} + \dfrac{\sqrt{1+x}}{\sqrt{1-x}}\right)dx$

Let $\sqrt{\dfrac{1+x}{1-x}} = f(x)$

Then $f'(x) = \dfrac{1}{(1-x)\sqrt{1-x^2}}$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$= e^x f(x) + c$

$\Rightarrow I = e^x\sqrt{\dfrac{1+x}{1-x}} + c$

$r_1, r_2, r_3 \in \{1, 2, 3, 4, 5, 6\}$

r_1, r_2, r_3 are of the form $3k,\ 3k+1,\ 3k+2$

Required Probability $= \dfrac{3! \times {}^2C_1 \times {}^2C_1 \times {}^2C_1}{6\times6\times6} = \dfrac{6\times8}{216} = \dfrac{2}{9}$

Question: $\displaystyle\int_{0}^{100\pi} \dfrac{\sin^2 x}{e^{\left(\frac{x}{2}-\left[\frac{x}{\pi}\right]\right)}}\,dx = \dfrac{\alpha\pi^3}{1+4\pi^2}$, $\alpha \in R$ $[x]$ is greatest integer

Options:

(a) $50(e-1)$

(b) $150\left(e^{-1}-1\right)$

(c) $200\left(1-e^{-1}\right)$

(d) $100(1-e)$

Answer: (c)

Solution:

$$\int_0^{100\pi} \frac{\sin^2 x}{e^{\left\{\frac{x}{\pi}\right\}}}\,dx = 100\int_0^{\pi} \frac{\sin^2 x}{e^{\frac{x}{\pi}}}\,dx$$

Let $\dfrac{x}{\pi} = t \Rightarrow dx = \pi dt$

$$I = 100\pi \int_0^1 e^{-t} \sin^2 \pi t\,dt = 50\pi \int_0^1 \left(1 - \cos 2\pi t\right)e^{-t}\,dt$$

$$I = 50\pi \left[\left(-e^{-t}\right)^1 - \left(\frac{e^{-t}}{1+4\pi^2}\left\{-\cos 2\pi t + 2\pi \sin 2\pi t\right\}\right)_0^1 \right]$$

$$= 50\pi \left[\left(1 - \frac{1}{e}\right) - \frac{1}{\left(1+4\pi^2\right)}\left\{\frac{-1}{e} + 1\right\} \right]$$

$$= 50\pi \left(1 - \frac{1}{e}\right)\left(\frac{4\pi^2}{1+4\pi^2}\right) = \frac{200\pi^3}{1+4\pi^2}\left(1 - e^{-1}\right)$$

$$\therefore \ \alpha = 200\left(1 - e^{-1}\right)$$