PHYSICS

1. The kinetic energy of a particle is 10000 joule with the mass 2 kg. Find the momentum for the particle?

(1) 200 kg m/s (2) 400 kg m/s (3) 800 kg m/s (4) 600 kg m/s

Ans. (1)

Sol. $P = \sqrt{2m(K \cdot E)}$

$P = \sqrt{2 \times 2 \times 10000} = 200$ kg m/s

2. A Particle is projected vertically upward reaches 136 m height. What will be the maximum range for the particle projected with same speed ?

(1) 272 m (2) 280 m (3) 290 m (4) 300 m

Ans. (1)

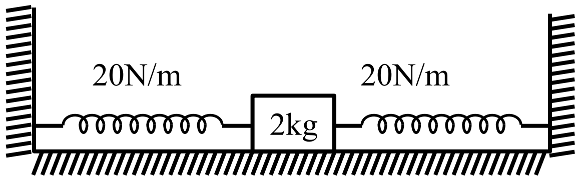
Sol. $\frac{U^2}{2g} = H_{max} = 136$ m

for maximum ranges $R = \frac{U^2}{g}$

$R_{max} = 2 \times H_{max}$

$R_{max} = 272$ m

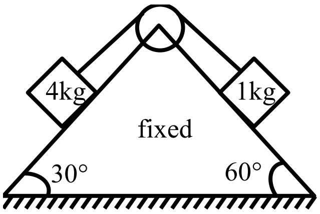
3. Given system is performing SHM with time period $T = \frac{\pi}{\sqrt{x}}$. Find x (all surfaces are smooth)?



Ans. (5)

Sol. $T = 2\pi\sqrt{\frac{2}{40}} = \frac{\pi}{\sqrt{5}}$ $\therefore x = 5$

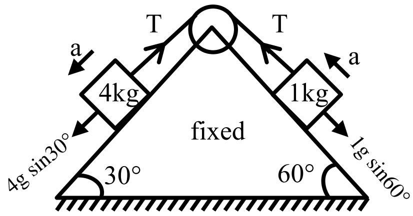
4. Find tension in string if all surfaces are smooth and string is massless.



(1) $4(\sqrt{3}+1)$N (2) $4(\sqrt{3}-1)$N (3) $(4\sqrt{3}+1)$N (4) $(4\sqrt{3}-1)$N

Ans. **(1)**

Sol.



$$a = \frac{4g\sin 30° - 1g\sin 60°}{5} = \frac{20-5\sqrt{3}}{5} = (4-\sqrt{3})\text{m/s}^2$$

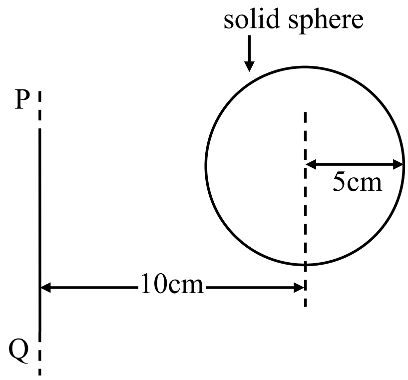
$4g\sin30° - T = 4a$

$T = 20 - 4(4-\sqrt{3})$

$= 20-16+4\sqrt{3}$

$= 4+4\sqrt{3} = 4(\sqrt{3}+1)$N

5. Radius of gyration of solid sphere about axis PQ is $\sqrt{x}\frac{R}{5}$ where R is radius of sphere. Find the value of x ?



Ans. **110**

Sol. $I_{com} = \frac{2}{5}MR^2$

||axis theorem

I_{PQ} = Icom + m(2R)^2 = $\frac{2}{5}MR^2 + 4MR^2 = \frac{22}{5}MR^2$

$I_{PQ} = MK^2$

$$\frac{25}{5}MR^2 = MK^2 \Rightarrow K = \sqrt{\frac{25}{5}} \times R = \sqrt{110}\frac{R}{5}$$

6. If equation of wave is given by y = 0.05 sin (2x – 4t). Find velocity of wave?

(1) 1 (2) 2 (3) 4 (4) 05

Ans. **(2)**

Sol. $V = \frac{\text{coefficient of t}}{\text{coefficient of x}}$

$= \frac{4}{2}$

= 2 m/sec

7. In a hydrogen atom first line wavelength of paschen series is λ = 720 nm. Find out second line wavelength of same series?

(1) 70.31 nm (2) 90 nm (3) 150 nm (4) 200 nm

Ans. **(1)**

Sol. $\frac{1}{\lambda} \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

1^{st} wavelenth $\frac{1}{\lambda_1} \propto \left(\frac{1}{3^2} - \frac{1}{4^2}\right)$

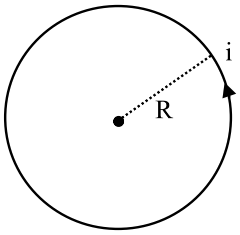
2^{nd} wavelenth $\frac{1}{\lambda_1} \propto \left(\frac{1}{3^2} - \frac{1}{5^2}\right)$

Taking ratio

$\frac{\lambda_2}{\lambda_1} = \frac{25}{256}$

$\lambda_2 = \frac{720 \times 25}{256} \approx 70.31$ nm

8. Figure shows current carrying coil of radius R. Find $\dfrac{B_{centre}}{B_{axis} \text{at } r = R}$.



(1) $4\sqrt{2}$ (2) $2\sqrt{2}$ (3) $3\sqrt{2}$ (4) $\sqrt{2}$

Ans. **(2)**

Sol. $B_C = \dfrac{\mu_0 i}{2R}$ … (1)

$$B_{r=R} = \frac{\mu_0 i R^2}{2(R^2 + R^2)^{3/2}} = \frac{\mu_0 i}{4\sqrt{2}R} \quad \text{… (2)}$$

$$\frac{B_C}{B_{r=R}} = \frac{\mu_0 i 4\sqrt{2}R}{2R\mu_0 i} = 2\sqrt{2}$$

9. Two charges q_1 & q_2 are placed in a di-electric medium 'K' at a separation d and resultant force on any charge is F_0. If both are placed in air, then what should be the separation between them so that they experience same force?

(1) $r = Kd$ (2) $r = d$ (3) $r = d\sqrt{K}$ (4) $r = K^{3/2}d$

Ans. **(3)**

Sol. **Case-I : F =** $\dfrac{Kq_1q_2}{\varepsilon_r d^2}$ **....(1)**

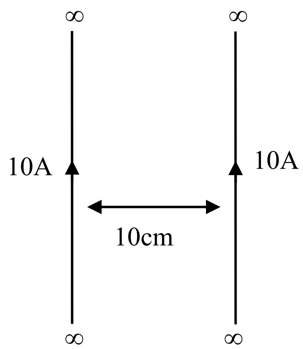
Case-II: F = $\dfrac{Kq_1q_2}{(d')^2}$ **....(2)**

Equating equation (1) & (2)

d' = $\sqrt{\varepsilon_r}\,d$

$= \sqrt{K}d$

10. If a magnetic force on 10 cm portion of one wire is F_1. Now distance is halved and current gets doubled, then force on same portion is xF_1. Find x.



Ans. **8**

Sol. $F_1 = \frac{\mu_0 i^2}{2\pi r} \times l$

$F_1 \propto \frac{i^2}{r}$

$\frac{F_1}{F_2} = \frac{i_1^2 / r_1}{i_2^2 / r_2} = \frac{1}{8}$

$F_2 = 8F_1$

$\therefore \quad x = 8$

11. A circular loop of radius $\frac{10}{\sqrt{\pi}}$cm is placed in a uniform time varying magnetic field with field being perpendicular to the plane of the loop. If the field decreases from 0.5 T to zero in 0.5 sec, then induced emf in the loop at 0.25 sec. is :

(1) 1 mV (2) 10 mV (3) 5 mV (4) 100 mV

Ans. **(2)**

Sol. $|\varepsilon| = A.\frac{dB}{dt} = \pi \times \left(\frac{100}{\pi} \times 10^{-4}\right) \times \frac{0.5}{0.5} = 0.01 \text{ Volt}$

12. **Statement-1 :** When light is incident from air to water then Brewster's angle is θ_B then if light is incident from water to air then Brewster's angle is $\frac{\pi}{2}-\theta_B$.

Statement-2 : When light goes from air to any medium of refractive index is μ, then Brewster's angle (θ_B) is given by $\theta_B = \tan^{-1}(\mu)$.

(1) both statement-1 and Statement-2 is true

(2) statement-1 is true and statement-2 is false

(3) statement-1 is false and statement-2 is true

(4) both statement-1 and statement-2 are false

Ans. **(1)**

Sol. $r + r' = 90°$

$r' = 90° - r$

but $r = i$

$r' = 90° - i$

Now if light is incident from water to air then angle of incidence is $\frac{\pi}{2}-i$.

13. A cylinder has inner radius 2 mm and outer radius 4 mm. The resistivity of its material is $2.4 \times 10^{-5}\ \Omega$ m and its length is 3.14 m given. Find out its resistance between two ends?

Ans. **2**

Sol. $R = \rho\frac{\ell}{A}$

$$R = \frac{2.4\times10^{-5}\times3.14}{\pi[16-4]\times10^{-6}}$$

$R = 2\ \Omega$

14. Weight of an object on Earth is 18 N. Find out its weight (in N) at height 3200 km from the earth surface?

Ans. **8**

Sol. $R_e = 6400\text{Km}$

$$\text{height} = 3200\text{ Km} = \left(\frac{R_e}{2}\right)$$

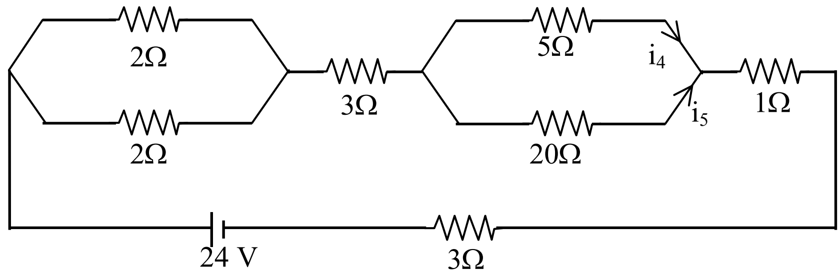
$$W_{earth} = 18\text{ N} = m\,\frac{GM_e}{R_e^{\,2}}$$

$$W' = m\,\frac{GM_e}{\left(R_e + \frac{R_e}{2}\right)^2}$$

$$W' = m\frac{GM}{R_e^2}\left(\frac{4}{9}\right) = 18 \times \frac{4}{9} = 8\text{N}$$

$$\boxed{W' = 8\text{N}}$$

15. Find the value of currents i_4 and i_5



(1) $\frac{2}{5}, \frac{8}{5}$ (2) $\frac{8}{5}, \frac{2}{5}$ (3) $\frac{3}{5}, \frac{6}{5}$ (4) $\frac{1}{5}, \frac{4}{5}$

Ans. **(2)**

Sol. $R_{eq} = \frac{2\times 2}{2+2} + 3 + \frac{5\times 20}{5+20} + 1 + 3$

$R_{eq} = 1 + 3 + 4 + 1 + 3 = 12\Omega$

$$i_{circuit} = \frac{24}{R_{eq}} = \frac{24}{12} = 2\text{A}$$

$$i_4 = i_{circuit}\,\frac{(20)}{20+5} = 2\times\frac{20}{25} = \frac{8}{5}\text{A}$$

$$i_5 = i_{circuit}\,\frac{(5)}{20+5} = \frac{2\times 5}{25} = \frac{2}{5}\text{A}$$

Ans. $\left(\frac{8}{5}, \frac{2}{5}\right)$

16. **Statement-1 :** In photodiode, the intensity of light is measured while reverse biasing the photodiode.

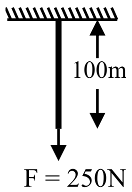
Statement-2 : Forward bias current is more than reverse bias current in PN junction.

(1) TF (2) TT (3) FF (4) FT

Ans. **(2)**

17. A force of 250 N is applied on a wire as shown

[Young Modulus = 10^{10} N/m^2, Area = 6.25×10^{-4} m^2]. Find extension (in cm) is spring ?



Ans. **0.4**

Sol. F = Kx

$$250 = \frac{\gamma A}{\ell} x$$

$$250 = \frac{10^{10} \times 6.25 \times 10^{-4} x}{100}$$

$x = 4 \times 10^{-3}$m

x = 0.4 cm

18. Match the column.

	Column-I		**Column-II**
(a)	h (Planck's constant)	(P)	$[M^1L^1T^{-1}]$
(b)	P (momentum)	(Q)	$[M^1L^2T^{-3}]$
(c)	V (stopping potential)	(R)	$[M^1L^2T^{-2}]$
(d)	ϕ (work function)	(S)	$[M^1L^2T^{-3}A^{-1}]$

Choose the correct option

(1) (a) $\to$ Q, (b) $\to$ P, (c) $\to$ S, (d) $\to$R

(2) (a) $\to$ P, (b) $\to$ Q, (c) $\to$ R, (d) $\to$ S

(3) (a) $\to$ R, (b) $\to$ P, (c) $\to$ S, (d) $\to$ Q

(4) (a) $\to$ S, (b) $\to$ P, (c) $\to$ Q, (d) $\to$ R

Ans. **(1)**

Sol. h(Planck's constant)

(a) $E = h\nu$

$$\frac{[ML^2T^{-2}]}{[T^{-1}]} = h = [M^1L^2T^{-1}] = h$$

(b) P(momentum)

$P = mv = [m][LT^{-1}] = [MLT^{-1}]$

(c) V_s (stopping potential)

$$V_s = Ed = \frac{Fd}{q} = \frac{[M^1L^1T^{-2}][L]}{[AT]} = [M^1L^2T^{-3}A^{-1}]$$

(d) Work function (ϕ)

ϕ = Energy

$\phi = [M^1L^2T^{-2}]$

19. An Electromagnetic wave propagation vector $\vec{K}$ and electric field $\vec{E}$. If ω is the angular frequency then the value of the magnetic field is?

(1) $\omega(\vec{K} \times \vec{E})$ (2) $\frac{I}{\omega}(\vec{K} \times \vec{E})$ (3) $\vec{K} \times \vec{E}$ (4) $\vec{E} \times \vec{K}$

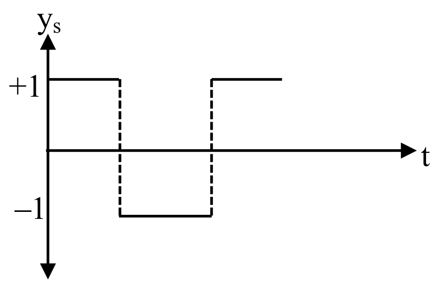
Ans. **(2)**

Sol. $C = \frac{E}{B}$ and $C = \frac{\omega}{K}$

$\frac{\omega}{K} = \frac{E}{B} \Rightarrow B = \frac{EK}{\omega}$

and $(\vec{K} \times \vec{E})$ is direction of propagation of $\vec{B}$.

20. A signal of square shape is superimposed with a carrier wave $y_c = 2\sin(\omega_c t - kx)$, then modulation index of amplitude modulated wave is



(1) 1 : 2 (2) 1 : 4 (3) 4 : 1 (4) 2 : 1

Ans. **(1)**

Sol. $\mu = \frac{A_m}{A_c} = \frac{1}{2}$

21. **Statement 1 :** If temperature of a gas is increased from – 73ºC to 527°C then its rms velocity becomes double.

Statement 2 : Product of pressure and volume is equal to translational kinetic energy of an ideal gas.

(1) Statement 1 is true, statement-II is true

(2) Statement 1 is false, statement-II is true

(3) Statement 1 is true, statement-II is false

(4) Statement 1 is false, statement-II is false

Ans. **(3)**

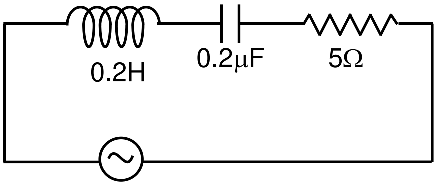
Sol. Statement-1 $V_{rms} = \sqrt{\frac{3RT}{M_0}}$

$$\frac{V_{rms_1}}{V_{rms_2}} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{200}{800}} = \frac{1}{2}$$

$$2V_{rms_1} = V_{rms_2}$$

Statement-2 $K.\varepsilon_T = \frac{3}{2}PV$

22. Calculate the ratio of quality factor and band width for the following circuit.



Ans. **8**

Sol. For an RLC circuit

band with = $\frac{R}{L} = \frac{5}{0.2}$Hz

for an RLC circuit factor $\frac{\sqrt{L}}{R\sqrt{C}} = \frac{\sqrt{0.2}}{5\times\sqrt{0.2}\times10^{-6}} = 200$

$\frac{Q}{B\ \text{width}} = \frac{200}{25} = \frac{8}{1}$

23. A radioactive substance $^{218}_{84}X$ undergoes following decay:

$$^{218}_{84}X \xrightarrow{\alpha-\text{Decay}} A \xrightarrow{\beta^{-}-\text{Decay}} B \xrightarrow{\alpha-\text{Decay}} C \xrightarrow{\beta^{+}-\text{Decay}} D \xrightarrow{\gamma-\text{Decay}} Y$$

Then product y is :

(1) $^{210}_{84}Y$ (2) $^{210}_{80}Y$ (3) $^{208}_{84}Y$ (4) $^{210}_{82}Y$

Ans. **(2)**

Sol. By mass conservation : 218 – 4 × 2 = 210

By Charge conservation : 84 – 2 × 2 + (–1) + 1 × 1 = 80

24. 1 gm liquid is converted into vapour under 3 × 10^5 Pa. 10% of heat is used to expand volume by 1600 cm^3. What is the increase in internal energy:-

(1) 4800 (2) 4320 (3) 4300 (4) 400

Ans. **(2)**

Sol. 10% of heat is used in expansion

Rest 90% will increase internal energy

$Q \times \frac{10}{100} = P.\Delta V = 3 \times 10^5 \times 1600 \times 10^{-6}$

$0.1Q = 48 \times 10 = 480$

$Q = 4800$ J

$\Delta U = 0.9\ Q = 0.9 \times 4800 = \boxed{4320J}$

25. Choose the correct option based on the following statements

(a) Photoelectric effect is explained by wave theory

(b) Stopping potential may depend on work function

(c) If intensity of light increases then photoelectric current also increases

(d) If intensity of light increases then maximum kinetic energy of photoelectrons increases.

(1) (a, d) (2) (a, c) (3) c (2) (b, c, d)

Ans **(3)**

Basic Theory

26. If $A = 3\hat{i} - 2\hat{j} + b\hat{k}$ and $\dot{B} = a\hat{i} + \frac{7}{2}\hat{j} + 2\hat{k}$ and A & $\dot{B}$ are perpendicular to each other, also $2a - 3b = -4$. If $\frac{a}{b} = \frac{x}{2}$. The value of x is ?

Ans. **(1)**

Sol. $\dot{A}.\dot{B} = 0$

$3a - 7 + 2b = 0$

$3a + 2b = 7$

$\Rightarrow a = 1\ \&\ b = 2$

CHEMISTRY

1. $[Co(NH_3)_5Cl]Cl_2$ primary and secondary valency will be :

[Coordination compound]

Sol. 3, 6

2. $CoCl_4^{2-}$ electronic configuration $\rightarrow e^m t_2^n$.

Calculate m + n. n = number of unpaired electrons

Ans. **7** **[Coordination compound]**

Sol. $3d^7$ WFL

$EC \rightarrow e^4 t_2^3$

m = 4, no. of unpaired electron = 3

3. Graph of X-ray frequency $(\nu)^n$ v/s atomic number (Z) is linear. Find the value of n.

(1) $\frac{1}{2}$ (2) 1 (3) $-\frac{1}{2}$ (4) –1

Ans. **(1)** **[Periodic properties]**

Sol. $\nu^n \propto Z$

$\sqrt{\nu} = a(Z-b)$

$\therefore \; n = \frac{1}{2}$

4. The wavelength of first line of paschen series is 720 nm, then calculate wavelength of second line of paschen series ?

Ans. **492.1875 nm** **[Atomic Structure]**

Sol. Paschen series first line : $4 \rightarrow 3$

$$\frac{1}{\lambda_1} = R_H Z^2 \left(\frac{1}{9} - \frac{1}{16}\right)$$

$$\frac{1}{720} = R_H Z^2 \left(\frac{7}{16\times 9}\right) \text{ ... (i)}$$

Paschen series second line : $5 \rightarrow 3$

$$\frac{1}{\lambda_2} = R_H Z^2 \left(\frac{1}{9} - \frac{1}{25}\right)$$

$$\frac{1}{\lambda_2} = R_H Z^2 \left(\frac{16}{9\times 25}\right) \text{ ... (ii)}$$

eq. (i) / (ii)

$$\frac{\lambda_2}{720} = \left(\frac{7}{16\times 9}\right) \times \left(\frac{9\times 25}{16}\right)$$

$\lambda_2 = 492.1875$ nm

5. Find correct order of covalent character : **[Chemical bonding]**

(A) KF < KI

(B) CuCl > NaCl

(C) LiF > KF

(1) A & B only (2) A & C only (3) A, B & C (4) B & C only

Ans. (3)

6. Freezing point of solution is less than that of pure solvent, which of the following statements are correct ?

(A) Vapour pressure of solution is less than that of pure solvent

(B) Vapour pressure of solution is greater than that of pure solvent

(C) Only solvent molecules will freeze

(D) Only solute molecules will freeze

(1) A & B only (2) A & C only (3) C & D only (D) A, B & D only

Ans. (2) **[Solution & colligative properties]**

7. For which of the following aqueous ion, spin only magnetic moment is 3.87 BM ?

(1) T_i^{2+} (2) V^{2+} (3) Cr^{2+} (4) Mn^{2+}

Ans. (2) **[d & f-block]**

Sol. $V^{2+} = 3d^3\ 4s^0$

8. Correct order of strength of H-bond in the following :

(A) Liquid water (B) Ice (C) Impure water

(1) A > B > C (2) A < B < C (3) B > A > C (4) A = B > C

Ans. (3) **[Hydrogen]**

9. How many reactions are nonspontaneous at 300 K. For independent reaction ΔH & ΔS values are given

(1) $\Delta H = -25$ kJ/mole, $\Delta S = -80$ J/mole (2) $\Delta H = +25$ kJ/mole, $\Delta S = -50$ J/mole

(3) $\Delta H = 22$ kJ/mole, $\Delta S = +50$ J/mole (4) $\Delta H = -22$ kJ/mole, $\Delta S = 80$ J/mole

Ans. (2) **[Thermodynamics-2 (2nd law & 3rd law)]**

Sol. (1) Spontaneous

(2) Non-Spontaneous

(3) Non spontaneous

(4) Spontaneous

10. Buffer solution of pH = 5 prepared by mixing 25 ml, 0.2M CH_3COONa and 25ml, 0.02M CH_3COOH, if Ka of $CH_3COOH = x \times 10^{-5}$ find x.

Ans. (10) **[Ionic Equilibrium (Elementary)]**

Sol. $pH = pKa + \log \frac{0.1}{0.01}$

$5 = pKa + \log 10$

$pKa = 4$

$Ka = 10^{-4}$

$= 10 \times 10^{-5}$

$= 10$

11.

	Column-I	**Column-II**
	(A) Zone refining	(P) pig iron
	(B) Electrolysis	(Q) Al
	(C) Reverberatory furnace	(R) Si
	(D) Blast furnace	(S) Cu

	A	B	C	D
(1)	R	Q	S	P
(2)	Q	P	S	R
(3)	P	S	Q	R
(4)	S	P	R	Q

Ans. (1) **[Metallurgy]**

12. How many statements are correct regarding Arrhenius equation ? $\left(K = Ae^{-E_a/RT}\right)$

(I) Slope of graph between ln K v/s $\frac{1}{T}$ is $-\frac{E_a}{R}$

(II) On increasing E_a, rate constant decreases

(III) On increasing temperature, temperature coefficient decreases

(IV) On increasing activation energy fraction of molecules crossing energy barrier increases

Ans. (3) **[Chemical Kinetics]**

Sol. (I), (II) & (III) are correct.

13. 5g of NaOH is mixed with 450 ml of de-ionized water to form stock solution. What volume of this stock solution is used to prepare 500ml, 0.1M solution.

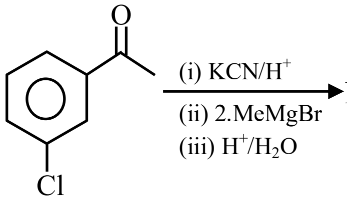
Ans. 180 ml **[Mole Concept]**

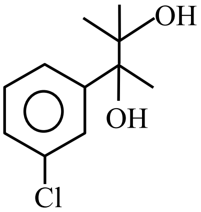
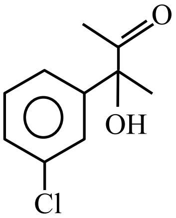
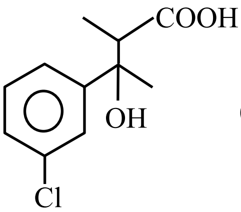
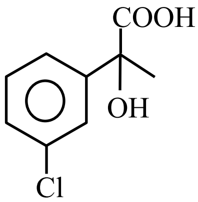
Sol. $M_1V_1 = M_2V_2$

$\frac{5}{40} \times \frac{1000}{450} \times V = 0.1 \times 500$

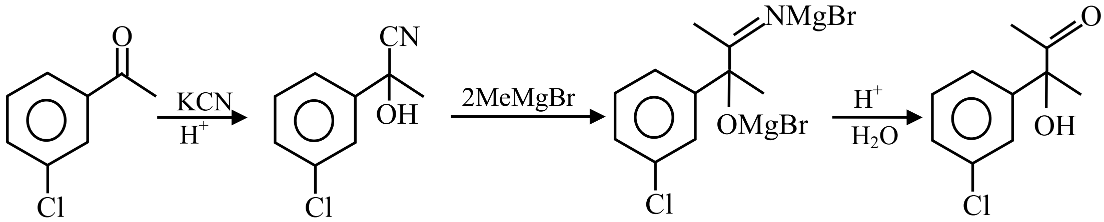
$V = \frac{0.1 \times 500 \times 40 \times 450}{5 \times 1000}$

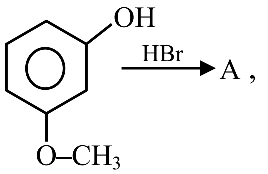
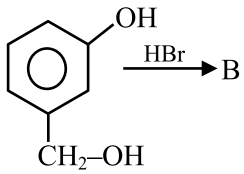
$V = 180$ ml

14.  Final product is : **[Carbonyl compounds]**

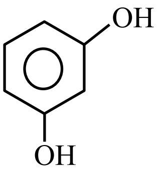
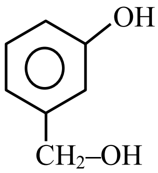
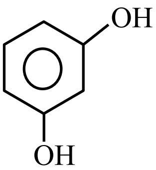
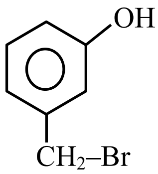
(1)  (2)  (3)  (4) 

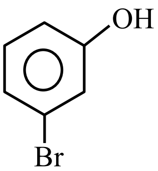
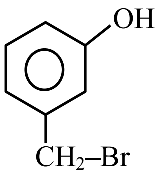
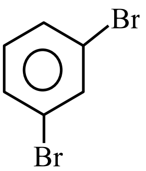
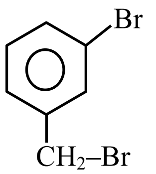
Ans. (2)

Sol. 

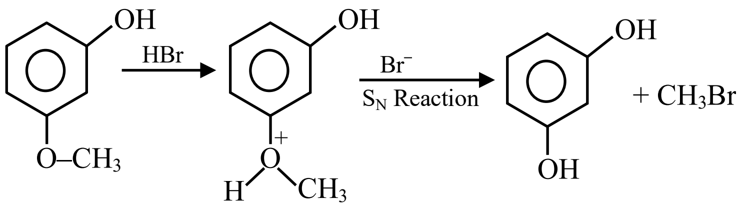
15.  ,  **[Alcohols, Phenols & Ethers]**

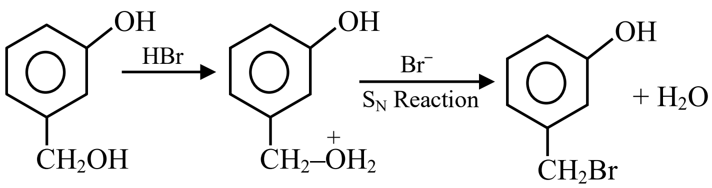
Products A and B are respectively

(1)  &  (2)  & 

(3)  &  (4)  & 

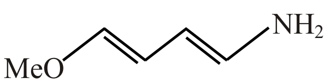
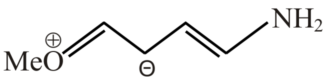
Ans. **(2)**

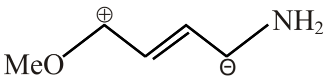
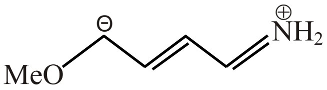
Sol. 



16. Which of the following is correct stability order of the given resonance structures?

[GOC-1]

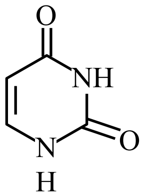
(a)  (b) 

(c)  (d) 

(1) a > b > c > d (2) b > a > d > c (3) a > b > d > c (4) a > d > b > c

Ans. (3)

17. Mass percentage of nitrogen in uracil is **[Biomolecules]**

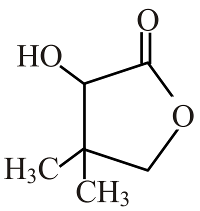
 (Uracil)

Ans. **25**

Sol. Molecular formula of uracil is $C_4H_4N_2O_2$

Molecular mass of uracil is 112

$$\% \text{ of N in uracil} = \frac{28}{112} \times 100 = 25$$

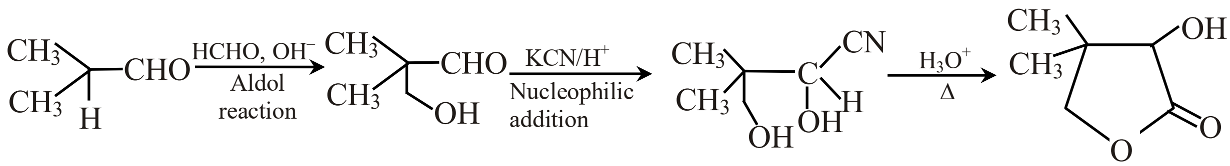
18. Compound (X) $\xrightarrow[\text{(2) KCN/H}^+ \text{ (3) H}_3\text{O}^+ \text{ (4) }\Delta]{\text{(1) HCHO/OH}^-}$  **[Carbonyl compounds]**

X will be :

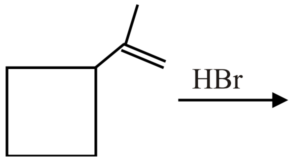
(1) $CH_3–CHO$ (2) $(CH_3)_2CH–CHO$

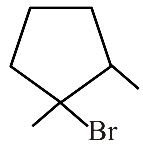
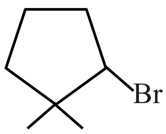
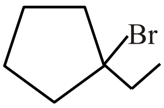
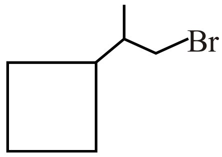
(3) $CH_3–\underset{}{\overset{H}{\overset{|}{C}}}H–CHO$ (4) $CH_3–\underset{\underset{O}{\|}}{C}–CH_3$

Ans. **(2)**

Sol. 

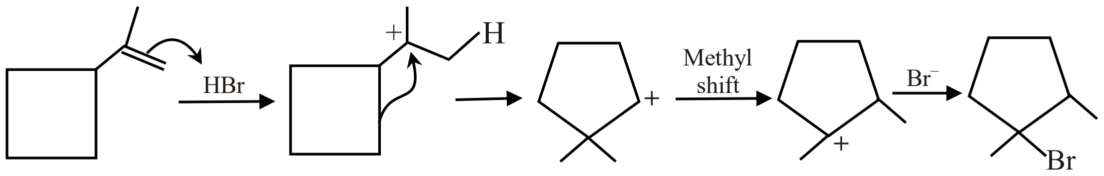
19. Major product of the given reaction will be : **[Hydrocarbons]**



(1)  (2)  (3)  (4) 

Ans. (1)

Sol.



Major product

20. Which of the following statements is correct? **[Haloalkanes & Haloarenes]**

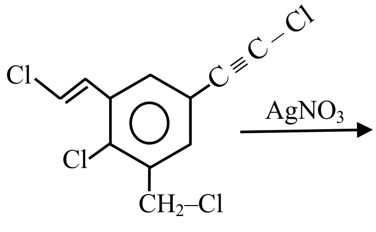
(1) All radicals are known as freons. (2) Freons cause skin cancer.

(3) Freons are chlorofluoro carbon. (4) Freons are used in sunscreen lotion.

Ans. (3)

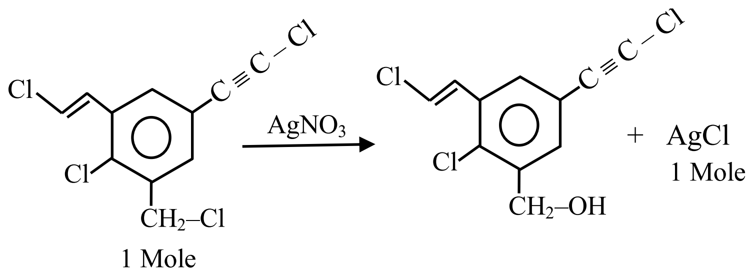
Sol. Freons are chlorofluoro carbon. Other given statements are wrong.

21. How many moles of AgCl are formed in the given reaction? **[Haloalkanes & Haloarenes]**



Ans. (1)

Sol.



Only chlorine atom attached with sp^3 hybrid carbon (haloalkane) in given molecule reacts with $AgNO_3$ and produces white ppt of AgCl, so only one mole AgCl is formed.

22. **Statement-I** **[Chemistry in every day life]**

Noradrenaline is one of the neurotransmitter.

Statement-II

Low level of noradrenaline is not cause of depression in humans.

(1) Both Statement-I and Statement-II are correct.

(2) Both Statement-I and Statement-II are incorrect.

(3) Statement-I is correct but Statement-II is incorrect.

(4) Statement-I is incorrect but Statement-II is correct.

Ans. **(3)**

MATHEMATICS

1. If $I = \int_0^{\pi/2} \frac{(\sin x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}}$ dx, then the value of I is

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{3\pi}{4}$ (4) $\frac{3\pi}{2}$

Ans. **(1)**

Sol. $I = \int_0^{\pi/2} \frac{(\sin x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}}\, dx$

$I = \int_0^{\pi/2} \frac{(\cos x)^{2023}}{(\cos x)^{2023} + (\sin x)^{2023}}$ .... (a + b – X property)

$2I = \int_0^{\pi/2} dx \Rightarrow I = \frac{\pi}{4}$

2. If $I = \int_0^3 \left|x^2 - 3x + 2\right| dx$, then find the value of 12I

Ans. **(22)**

Sol. $I = \int_0^1 \left(x^2 - 3x + 2\right) dx - \int_1^2 \left(x^2 - 3x + 2\right) dx$

$+\int_2^3 \left(x^2 - 3x + 2\right) dx$

$I = \left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right)_0^1 - \left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right)_1^2$

$+\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right)_2^3$

$= \left(\frac{1}{3} - \frac{3}{2} + 2\right) - \left(\frac{8}{3} - 6 + 4 - \left(\frac{1}{3} - \frac{3}{2} + 2\right)\right) + 9 - \frac{27}{2} + 6 - \left(\frac{8}{3} - 6 + 4\right)$

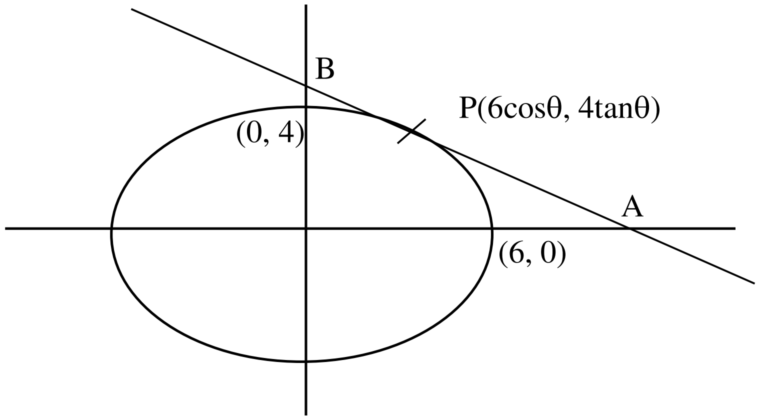
$= \frac{5}{6} - \left(\frac{2}{3} - \frac{5}{6}\right) + \frac{3}{2} - \frac{2}{3}$

$I = \frac{1}{3} + \frac{3}{2} = \frac{11}{6} \Rightarrow 12I = 22$

3. A tangent at P on the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$ is drawn. If this tangent cuts x -axis & y-axis at the points A and B respectively then find minimum possible value of AB.

Ans. **(10)**

Sol.



Let P = (6 cosθ, 4 sinθ)

Equation of tangent will be

$$\frac{x\cos\theta}{6}+\frac{y\sin\theta}{4}=1$$

$$\therefore\ AB = \sqrt{\frac{36}{\cos^2\theta}+\frac{16}{\sin^2\theta}}=\sqrt{36\left(1+\tan^2\theta\right)+16\left(1+\cot^2\theta\right)}$$

Since $\dfrac{36\tan^2\theta+16\cot^2\theta}{2}\ge\sqrt{36\tan^2\theta\cdot 16\cot^2\theta}$

$36\tan^2\theta + 16\cot^2\theta \ge 2 \times 6 \times 4$

$AB_{min} = \sqrt{52+36\tan^2\theta+16\cot^2\theta}=10$

4. If $\sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$, then the value of α is-

(1) 1011 (2) 1012 (3) 2022 (4) 2024

Ans. **(2)**

Sol. $\sum_{r=0}^{n} r^2 \cdot {}^{n}C_r$

$\sum_{r=0}^{n} (r(r-1)+r)\,{}^{n}C_r$

$\sum_{r=0}^{n} \left\{n(n-1)\,{}^{n-2}C_{r-2} + n\,{}^{n-1}C_{r-1}\right\}$

$= n(n-1)2^{n-2} + n \times 2^{n-1}$

$= 2023\ [2022\ 2^{2021} + 2^{2022}]$

$= 2023 \times 2^{2022} \times 1012$

5. There are 12 subjects in a class, out of which 5 are language subjects. A student has to choose 5 subjects in which atmost 2 are language subjects. Find no. of ways to do so.

(1) 546 (2) 540 (3) 456 (4) 567

Ans. **(1)**

Sol. ${}^{7}C_5 + {}^{7}C_4\,{}^{5}C_1 + {}^{7}C_3\,{}^{5}C_2$

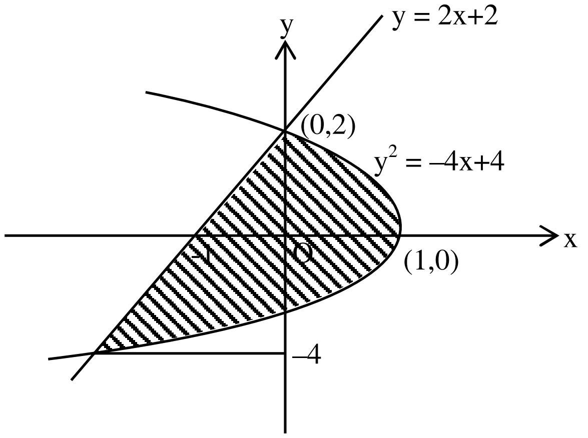
$= 21 + 175 + 350 = 546$

6. Find the area bounded by the curves $y^2 = -4x + 4$ and $y = 2x + 2$.

(1) 27 (2) 9 (3) $\frac{27}{4}$ (4) $\frac{9}{2}$

Ans. (2)

Sol. $A = \int_{-4}^{2}\left(\frac{4-y^2}{4} - \frac{2y-4}{4}\right) dy = \int_{-4}^{2}\frac{1}{4}\left(4 - y^2 - 2y + 4\right) dy = \frac{1}{4}\int_{-4}^{2}\left(-y^2 - 2y + 8\right) dy$



$= \frac{1}{4}\left[-\frac{y^3}{3} - y^2 + 8y\right]_{-4}^{2} = \frac{1}{4}\left[\left(-\frac{8}{3} - 4 + 16\right) - \left(\frac{64}{3} - 16 - 32\right)\right] = \frac{1}{4} \times \left(-\frac{72}{3} + 60\right) = 9$ sq. units

7. If $x^2 - 4x + 3 = x[x] - [x]$, where [.] represents the greatest integer function then :

(1) No. of solutions in $(-\infty, 1)$ are 1 (2) No. of solutions in $(-\infty, \infty)$ are 1

(3) No. of solutions in $(1, \infty)$ are 2 (4) No. of solutions in $(3, \infty)$ are infinite

Ans. (2)

Sol. $(x - 1)(x - 3) = (x - 1) . [x]$

$x - 3 = [x]$ or $x = 1$

Case-I : $x \in I$ Case-II : $x \notin I$

$x - 3 = x$

No solution No solution

$\therefore$ only 1 solution in R.

8. If $\begin{bmatrix} 1 & \alpha & \alpha^2 \\ c & a & b \\ 1 & 1 & 1 \end{bmatrix}$ is a singular matrix and α is a root of the equation $(a - b)x^2 + (b - c)x + (c - a) = 0$,

then the value of $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(c-a)(a-b)} + \frac{(c-a)^2}{(a-b)(b-c)}$ is

(1) 3 (2) 6 (3) 9 (4) 12

Ans. **(1)**

Sol. $\begin{bmatrix} 1 & \alpha & \alpha^2 \\ c & a & b \\ 1 & 1 & 1 \end{bmatrix} = 0$

$\Rightarrow (a - b) + (b - c)\alpha + (c - a)\alpha^2 = 0$

Also, $(a - b)\alpha^2 + (b - c)\alpha + (c - a) = 0$

$$\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(c-a)(a-b)} + \frac{(c-a)^2}{(a-b)(b-c)} = \frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$$

$$= \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$$

9. Two lines are given by $\frac{x-2}{3} = \frac{y-1}{3} = \frac{z-0}{2}$ and $\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-1}{3}$ then shortest distance between lines is-

(1) $\frac{6}{\sqrt{43}}$ (2) $\frac{11}{\sqrt{43}}$ (3) $\frac{3}{\sqrt{43}}$ (4) $\frac{5}{\sqrt{43}}$

Ans. **(2)**

Sol. s.d. $= \left| \frac{(a_1 - a_2).(\overline{p} \times \overline{q})}{(\overline{p} \times \overline{q})} \right|$

$= \left| \frac{(\hat{i} - \hat{j} - \hat{k}).(5\hat{i} - 3\hat{j} - 3\hat{k})}{|5\hat{i} - 3\hat{j} - 3\hat{k}|} \right|$

$= \frac{11}{\sqrt{43}}$

10. Let $f(x) = \begin{bmatrix} x^2 \sin\frac{1}{x} & ; & x \neq 0 \\ 0 & ; & x = 0 \end{bmatrix}$, then

(1) f is continuous and f' is discontinuous at x = 0

(2) f and f' both are continuous at x = 0

(3) f and f' both are discontinuous at x = 0

(4) f is discontinuous and f' is continuous at x = 0

Ans. **(1)**

Sol. $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} x^2 \sin\frac{1}{x} = 0 = f(0)$

$\therefore$ f(x) continuous at x = 0

$f'(x) = 2x\sin\frac{1}{x} + x^2\left(\cos\frac{1}{x}\right)\left(\frac{-1}{x^2}\right)$

$f'(x) = 2x \sin\frac{1}{x} - \cos\frac{1}{x}$

$\lim\limits_{x \to 0} f'(x)$ does not exists

$\therefore$ f'(x) is discontinuous at x = 0

11. $\lim_{t\to 0}\left(1^{\frac{1}{\sin^2 t}}+2^{\frac{1}{\sin^2 t}}+....+n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$ is equal to

(1) n^2 (2) n (3) $\frac{n(n+1)}{2}$ (4) $n^2 + n$

Ans. (1)

Sol. $n\left[\left(\frac{1}{n}\right)^{\frac{1}{\sin^2 t}}+\left(\frac{2}{n}\right)^{\frac{1}{\sin^2 t}}+....+\left(\frac{n-1}{n}\right)^{\frac{1}{\sin^2 t}}+1\right]^{\sin^2 t} = n$

12. Find the minimum distance of the point (7, –4, –3) from the plane formed by the points (2, 2, –1), (3, 4, 2) and (7, 0, 6).

(1) $\frac{\sqrt{19}}{4}$ (2) $\sqrt{19}$ (3) $\frac{\sqrt{19}}{3}$ (4) $\sqrt{\frac{19}{2}}$

Ans. (4)

Sol. $\vec{n} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 4 & -4 & 4\end{vmatrix} = <5, 2, -3>$

Plane : $5x + 2y - 3z = 17$

Distance $= \left|\frac{35-8+9-17}{\sqrt{25+4+9}}\right| = \frac{19}{\sqrt{38}} = \sqrt{\frac{19}{2}}$

13. If 'N' is decided by rolling a normal die and $\frac{'k'}{6}$ is the probability that the system of equations

$x + y + z = 0$
$Nx + y + z = 2$
$3x + (N - 3)y + z = 6$

has a unique solution, then find sum of all possible values of 'k' and 'n'

Ans. **(20)**

Sol. $D \neq 0$ for unique solution,

$\Rightarrow \begin{vmatrix}1 & 1 & 1\\ N & 1 & 1\\ 3 & N-3 & 1\end{vmatrix} \neq 0$

$\Rightarrow (N-1)(N-4) \neq 0$

$\Rightarrow N \neq 1, 4$

$\therefore$ N can be 2, 3, 5, 6

Also, required probability $= \frac{4}{6} \Rightarrow k = 4$

Hence, sum $= (2 + 3 + 5 + 6) + 4$

$= 20$

14. Numbers are formed using digits 1, 2, 3, 4, 1, 2, 3, 4 & 1 then the number of 9 digits numbers such that even digits occupy even places are-

Ans. **(60)**

Sol. 2, 2, 4, 4 occupy 2^{nd}, 4^{th} , 6^{th} and 8^{th} places

no. of numbers = $\frac{4!}{2!.2!} \cdot \frac{5!}{3!.2!} = 60$

15. A circle with centre ≡ (2, 0) and largest possible radius is inscribed in ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$.

If the circle passes through the point $(1, \alpha)$, then find value of $5\alpha^2$.

Ans. 59

Sol. $P \equiv (6 \cos \theta, 4 \sin \theta)$

$N ; \frac{36x}{6\cos\theta} - \frac{16y}{4\sin\theta} = 20$

Passes (2, 0)

$\frac{6}{\cos\theta} = 10 \Rightarrow \cos \theta = \frac{3}{5}$

$\Rightarrow P \equiv \left(\frac{18}{5}, \frac{16}{5}\right)$

$R = \sqrt{\frac{64}{25}+\frac{256}{25}} = \sqrt{\frac{320}{25}}$

$S : (x - 2)^2 + y^2 = \frac{320}{25}$

Passes $(1, \alpha)$

$\alpha^2 = \frac{64}{5} - 1 = \frac{59}{5}$

16. If $\left(1-\sqrt{3}i\right)^{200} = 2^{199}(p + iq)$ then the equation whose roots are $p + q + q^2$ and $p - q + q^2$ is

(1) $x^2 + 4x - 1 = 0$ (2) $x^2 + 4x + 1 = 0$ (3) $x^2 - 4x + 1 = 0$ (4) $x^2 + 2x + 2 = 0$

Ans. (3)

Sol. $2^{200}\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{200} = 2^{199} (p + iq)$

$(-2\omega)^{200}$

$2^{200} . \omega^2 =$

$2^{200}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right) = 2^{199} (p + iq)$

$p = -1, q = -\sqrt{3}$

so roots are $p + q + q^2 = -1 - \sqrt{3} + 3 = 2 - \sqrt{3}$

$p - q + q^2 = -1 + \sqrt{3} + 3 = 2 + \sqrt{3}$

Equation is $x^2 - 4x + 1 = 0$

17. Tangent is drawn at a point on the parabola $y^2 = 24x$, it intersects the hyperbola $xy = 2$ at points A and B such that locus of mid point of AB is a parabola whose.

(1) Directrix is $x = \frac{3}{2}$ (2) Latus rectum is 3 (3) Directrix is $x = -\frac{3}{4}$ (4) Latus rectum is $\frac{3}{2}$

Ans. **(2)**

Sol. Tangent to the parabola is $x - ty + 6t^2 = 0$ ...(i)

Equation of chord of $xy = 2$ with middle point M(h, k), is $T = S_1$

$\Rightarrow \frac{xk + yh}{2} - 2 = hk - 2$

$\Rightarrow xk + yh - 2hk = 0$ ...(ii)

Comparing equation (i) and (ii) gives

$k^2 = -3h$

or locus of M is $y^2 = -3x$

Hence length of latus rectum is 3

18. If $y = y(x)$ is solution of differential equation $x^3dy + (xy - 1)\,dx = 0$ and $y\left(\frac{1}{2}\right) = (3 - e)$, then y(1) is equal to

(1) e (2) 1 (3) $e^{\frac{1}{e}}$ (4) e^2

Ans. (2)

Sol. $\frac{dy}{dx} = \frac{1 - xy}{x^3}$

$\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}$

I.F. $= e^{\int \frac{1}{x^2} dx} = e^{-\frac{1}{x}}$

$y.\left(e^{-\frac{1}{x}}\right) = \int \frac{1}{x^3} e^{-\frac{1}{x}} dx$

Let $-\frac{1}{x} = t \Rightarrow \frac{1}{x^2}\, dx = dt$

$y\, e^{-\frac{1}{x}} = \int -te^t\, dt = -[te^t - e^t] + c$

$y\, e^{-\frac{1}{x}} = \frac{1}{x}\, e^{-\frac{1}{x}} + e^{-\frac{1}{x}} + c$

$y = \frac{1}{x} + 1 + c\,e^{\frac{1}{x}}$ where $y\left(\frac{1}{2}\right) = 3 - e$

$3 - e = 2 + 1 + ce^2$

$c = -\frac{1}{e}$

$y = \frac{1}{x} + 1 - e^{\frac{1}{x} - 1}$

$x = 1$

$y = \frac{1}{1} + 1 - 1 \Rightarrow y = 1$

19. If A and B are two square matrices of same order such that $A^2 B = A^2 + B$, then

(1) $A^2B = BA^2$ (2) $A^2 B = -BA^2$ (3) A = I or B = I (4) $A^2 = I$

Ans. **(1)**

Sol. $A^2 B = A^2 + B$

$\Rightarrow (A^2 - I)(B - I) = I$

$\Rightarrow (A^2 - I)(B - I) = (B - I)(A^2 - I)$

$\Rightarrow A^2B = BA^2$

20. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$ is equal to

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{2}$ (4) 0

Ans. **(2)**

21. Consider a G.P. with 4^{th} term 500. If S_n denotes sum of first 'n' terms of G.P. such that $S_6 > S_5 + 1$ and $S_7 < S_6 + 1$. If common ratio of G.P. is $\left(\frac{1}{m}\right)$ where $m \in N$; then find number of possible values of m.

Ans. **(15)**

Sol. $ar^3 = 500$

$S_6 > S_5 + 1 \Rightarrow T_6 > 1 \Rightarrow ar^5 > 1$

$S_7 < S_6 + 1 \Rightarrow T_7 < 1 \Rightarrow ar^6 < 1$

$\therefore r^2 > \frac{1}{500}$ and $r^3 < \frac{1}{500}$

$\Rightarrow 3\sqrt{500} < m < \sqrt{500}$

$m \in [8, 22]$

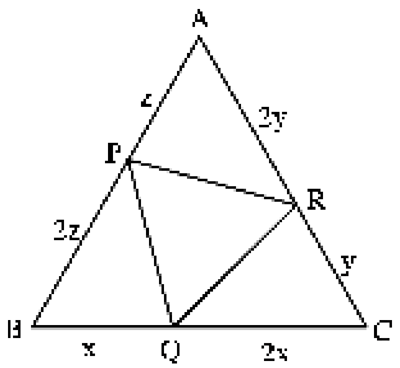
Number of values of m = 15

22. If P, Q, R lies on the sides AB, BC and CA respectively of triangle ABC dividing them in the ratio 1 : 2, then the ratio of areas of triangle ABC and triangle PQR is

(1) 2 (2) 3 (3) 4 (4) $\frac{5}{2}$

Ans. (2)

Sol. Area of $(\Delta BPQ + \Delta CQR + \Delta APR + \Delta PQR)$ = area of ΔABC



$\frac{1}{2}$ x (2z) sin B + $\frac{1}{2}$ (2x) (y) sin c + $\frac{1}{2}$ (z) (2y) sin A + $\Delta PQR = \Delta$

$\Delta ABC = \Delta = \frac{1}{2}$ (3z) (3y) (sin A) $= \frac{1}{2}$(3x) (3z) sin B $= \frac{1}{2}$ (3x) (3y) sin C

$\frac{2\Delta}{9} + \frac{2\Delta}{9} + \frac{2\Delta}{9} + \Delta PQR = \Delta$

$\Delta PQR = \Delta - \frac{6\Delta}{9} = \frac{\Delta}{3}$

Ratio of $\Delta ABC : \Delta PQR = \frac{\Delta}{\frac{\Delta}{3}} = 3$

23. If R : N $\rightarrow$ N such that aRb is gcd (a, b) = 1 & 2a $\neq$ b, then relation R is.

(1) Reflexive and transitive (2) Reflexive but not transitive

(3) Symmetric but not transitive (4) Symmetric and transitive

Ans. **(3)**

Sol. Not reflexive

because gcd(a, a) = a (which is not always equal to 1)

Symmetric

gcd(a, b) = gcd(b, a) = 1 hence symmetric

Transitive

gcd (5, 3) = 1, gcd(3,25) = 1 but gcd (5,25) = 5

hence not transitive

24. Consider the vectors $\vec{u} = \frac{\hat{i}+11\hat{j}-9\hat{k}}{2}$ and $\vec{v} = \hat{i} + \hat{j} + \hat{k}$. Consider a vector $\vec{w}$ such that

$\vec{v} \times \vec{w} = \vec{u} + \lambda \vec{v}$; $\vec{v} . \vec{w} = 2$ then find $\vec{u} . \vec{w}$. (Data may be different)

(1) 4 (2) 3 (3) 2 (4) 1

Ans. **(4)**

Sol. $\vec{v} \times (\vec{v} \times \vec{w}) = \vec{v} \times \vec{u}$

$2\vec{v} - 3\vec{w} = -10\hat{i} + 5\hat{j} + 5\hat{k}$

$\Rightarrow \vec{w} = \frac{2\hat{i}+2\hat{j}+2\hat{k}+10\hat{i}-5\hat{j}-5\hat{k}}{3}$

$\vec{w} = 4\hat{i} - \hat{j} - \hat{k}$

$\therefore \vec{u} . \vec{w} = 2 + \left(-\frac{11}{2}\right) + \frac{9}{2} = 1$