TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Wednesday 09th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM

MATHEMATICS

1. Let f be a differentiable function from R to R such that $\left|f(x)-f(y)\right| \le 2|x-y|^{\frac{3}{2}}$, for all x, y ε R. If $f(0) = 1$ then $\int_0^1 f^2(x)dx$ is equal to

(1) 0 (2) $\frac{1}{2}$ (3) 2 (4) 1

Ans. (4)

Sol. $|f(x) - f(y)| \le 2|x - y|^{3/2}$

divide both sides by $|x - y|$

$$\left|\frac{f(x)-f(y)}{x-y}\right| \le 2.|x-y|^{1/2}$$

apply limit $x \to y$

$|f'(y)| \le 0 \Rightarrow f'(y) = 0 \Rightarrow f(y) = c \Rightarrow f(x) = 1$

$$\int_0^1 1.dx = 1$$

2. If $\int_0^{\frac{\pi}{3}} \frac{\tan\theta}{\sqrt{2k\sec\theta}} d\theta = 1-\frac{1}{\sqrt{2}}, (k > 0)$, then the value of k is :

(1) 2 (2) $\frac{1}{2}$ (3) 4 (4) 1

Ans. (1)

Sol. $\frac{1}{\sqrt{2k}}\int_0^{\pi/3}\frac{\tan\theta}{\sqrt{\sec\theta}}d\theta = \frac{1}{\sqrt{2k}}\int_0^{\pi/3}\frac{\sin\theta}{\sqrt{\cos\theta}}d\theta$

$$= -\frac{1}{\sqrt{2k}}2\sqrt{\cos\theta}\Big|_0^{\pi/3} = -\frac{\sqrt{2}}{\sqrt{k}}\left(\frac{1}{\sqrt{2}}-1\right)$$

given it is $1-\frac{1}{\sqrt{2}} \Rightarrow k = 2$

3. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is

(1) 12 (2) 15 (3) 10 (4) 14

Ans. (2)

Sol. $(1 - t^6)^3 (1 - t)^{-3}$

$(1 - t^{18} - 3t^6 + 3t^{12}) (1 - t)^{-3}$

$\Rightarrow$ cofficient of t^4 in $(1 - t)^{-3}$ is

${}^{3+4-1}C_4 = {}^6C_2 = 15$

4. For each xεR, let [x] be the greatest integer less than or equal to x. Then

$\lim\limits_{x\to 0^-}\frac{x([x]+|x|)\sin[x]}{|x|}$ is equal to

(1) – sin1 (2) 0 (3) 1 (4) sin1

Ans. (1)

Sol. $\lim\limits_{x\to 0^-}\frac{x([x]+|x|)\sin[x]}{|x|}$

$x \to 0^-$

$[x] = -1 \Rightarrow \lim\limits_{x\to 0^-}\frac{x(-x-1)\sin(-1)}{-x} = -\sin 1$

$|x| = -x$

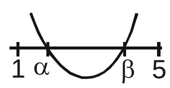
5. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1,5], then m lies in the interval:

(1) (4,5) (2) (3,4) (3) (5,6) (4) (–5,–4)

Ans. (Bonus/1)

Sol. $x^2 - mx + 4 = 0$

$\alpha,\beta \in [1,5]$



(1) $D > 0 \Rightarrow m^2 - 16 > 0$

$\Rightarrow m \in (-\infty,-4) \cup (4,\infty)$

(2) $f(1) \ge 0 \Rightarrow 5 - m \ge 0 \Rightarrow m \in (-\infty,5]$

(3) $f(5) \ge 0 \Rightarrow 29 - 5m \ge 0 \Rightarrow m \in \left(-\infty,\frac{29}{5}\right]$

(4) $1 < \frac{-b}{2a} < 5 \Rightarrow 1 < \frac{m}{2} < 5 \Rightarrow m \in (2,10)$

$\Rightarrow m \in (4,5)$

No option correct : Bonus

* If we consider $\alpha,\beta \in (1,5)$ then option (1) is correct.

6. If

$$A=\begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t-e^{-t}\sin t & -e^{-t}\sin t+e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$$

Then A is-

(1) Invertible only if $t=\frac{\pi}{2}$

(2) not invertible for any $t \varepsilon R$

(3) invertible for all $t \varepsilon R$

(4) invertible only if $t=\pi$

Ans. (3)

Sol. $|A| = e^{-t}\begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t-\sin t & -\sin t+\cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$

$= e^{-t}[5\cos^2 t + 5\sin^2 t]\ \forall\ t \in R$

$= 5e^{-t} \neq 0\ \forall\ t \in R$

7. The area of the region

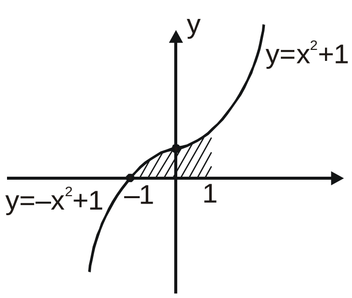
$A = \left[(x,y) : 0 \le y \le x|x| + 1 \text{ and } -1 \le x \le 1\right]$

in sq. units, is :

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) 2 (4) $\frac{4}{3}$

Ans. (3)

Sol. The graph is a follows



$$\int_{-1}^{0}\left(-x^2+1\right)dx+\int_{0}^{1}\left(x^2+1\right)dx=2$$

8. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6iz_0^{81} - 3iz_0^{93}$, then arg z is equal to:

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) 0 (4) $\frac{\pi}{6}$

Ans. (1)

Sol. $z_0 = \omega$ or ω^2 (where ω is a non-real cube root of unity)

$z = 3 + 6i(\omega)^{81} - 3i(\omega)^{93}$

$z = 3 + 3i$

$\Rightarrow \arg z = \frac{\pi}{4}$

9. Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to:

(1) $\sqrt{22}$ (2) 4 (3) $\sqrt{32}$ (4) 6

Ans. (4)

Sol. Projection of $\vec{b}$ on $\vec{a} = \frac{\vec{a}.\vec{b}}{|\vec{a}|} = |\vec{a}|$

$\Rightarrow b_1 + b_2 = 2$ ...(1)

and $(\vec{a}+\vec{b}) \perp \vec{c} \Rightarrow (\vec{a}+\vec{b}).\vec{c} = 0$

$\Rightarrow 5b_1 + b_2 = -10$ ...(2)

from (1) and (2) $\Rightarrow b_1 = -3$ and $b_2 = 5$

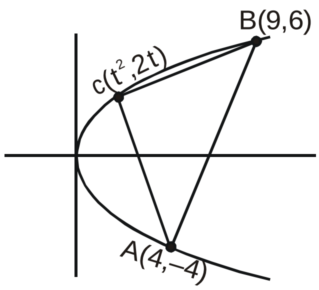
then $|\vec{b}| = \sqrt{b_1^2 + b_2^2 + 2} = 6$

10. Let A(4,–4) and B(9,6) be points on the parabola, $y^2 + 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB, is:

(1) $31\frac{3}{4}$ (2) 32 (3) $30\frac{1}{2}$ (4) $31\frac{1}{4}$

Ans. (4)

Sol.



Area = 5|t^2 – t – 6| = $5\left|\left(t-\frac{1}{2}\right)^2-\frac{25}{4}\right|$

is maximum if $t=\frac{1}{2}$

11. The logical statement

$\left[\sim\left(\sim p\vee q\right)\vee\left(p\wedge r\right)\wedge\left(\sim q\wedge r\right)\right]$ is equivalent to:

(1) $(p \wedge r) \wedge \sim q$ (2) $(\sim p \wedge \sim q) \wedge r$

(3) $\sim p \vee r$ (4) $(p \wedge \sim q) \vee r$

Ans. (1)

Sol. $s\left[\sim\left(\sim p\vee q\right)\wedge\left(p\wedge r\right)\right]\cap\left(\sim q\wedge r\right)$

$\equiv\left[\left(p\wedge\sim q\right)\vee\left(p\wedge r\right)\right]\wedge\left(\sim q\wedge r\right)$

$\equiv\left[p\wedge\left(\sim q\vee r\right)\right]\wedge\left(\sim q\wedge r\right)$

$\equiv p\wedge\left(\sim q\wedge r\right)$

$\equiv\left(p\wedge r\right)\sim q$

12. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is :

(1) $\frac{26}{49}$ (2) $\frac{32}{49}$ (3) $\frac{27}{49}$ (4) $\frac{21}{49}$

Ans. (2)

Sol. E_1 : Event of drawing a Red ball and placing a green ball in the bag

E_2 : Event of drawing a green ball and placing a red ball in the bag

E : Event of drawing a red ball in second draw

$$P(E)=P(E_1)\times P\left(\frac{E}{E_1}\right)+P(E_2)\times P\left(\frac{E}{E_2}\right)$$

$$=\frac{5}{7}\times\frac{4}{7}+\frac{2}{7}\times\frac{6}{7}=\frac{32}{49}$$

13. If $0\le x<\frac{\pi}{2}$, then the number of values of x for which sin x-sin2x+sin3x = 0, is

(1) 2 (2) 1

(3) 3 (4) 4

Ans. (1)

Sol. sinx – sin2x + sin3x = 0

$\Rightarrow$ (sinx + sin3x) – sin2x = 0

$\Rightarrow$ 2sinx. cosx – sin2x = 0

$\Rightarrow$ sin2x(2 cosx – 1) = 0

$\Rightarrow$ sin2x = 0 or $\cos x=\frac{1}{2}$

$\Rightarrow x=0,\frac{\pi}{3}$

14. The equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines

$\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is:

(1) x + 2y – 2z = 0 (2) x – 2y + z = 0

(3) 5x + 2y – 4z = 0 (4) 3x + 2y – 3z = 0

Ans. (2)

Sol. Vector along the normal to the plane containing the lines

$\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$

is $\left(8\hat{i}-\hat{j}-10\hat{k}\right)$

vector perpendicular to the vectors $2\hat{i}+3\hat{j}+4\hat{k}$ and $8\hat{i}-\hat{j}-10\hat{k}$ is $26\hat{i}-52\hat{j}+26\hat{k}$

so, required plane is

$26x - 52y + 26z = 0$

$x - 2y + z = 0$

15. Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at (1,1), then the equation of its third side is :

(1) $122y - 26x - 1675 = 0$

(2) $26x + 61y + 1675 = 0$

(3) $122y + 26x + 1675 = 0$

(4) $26x - 122y - 1675 = 0$

Ans. (4)

Sol. Equation of AB is

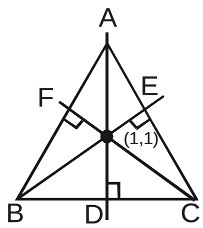
$3x - 2y + 6 = 0$

equation of AC is

$4x + 5y - 20 = 0$

Equation of BE is

$2x + 3y - 5 = 0$



Equation of CF is $5x - 4y - 1 = 0$

$\Rightarrow$ Equation of BC is $26x - 122y = 1675$

16. If $x = 3 \tan t$ and $y = 3 \sec t$, then the value of $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{4}$, is:

(1) $\frac{3}{2\sqrt{2}}$ (2) $\frac{1}{3\sqrt{2}}$ (3) $\frac{1}{6}$ (4) $\frac{1}{6\sqrt{2}}$

Ans. (4)

Sol. $\frac{dx}{dt}=3\sec^2 t$

$\frac{dy}{dt}=3\sec t\tan t$

$\frac{dy}{dx}=\frac{\tan t}{\sec t}=\sin t$

$\frac{d^2y}{dx^2}=\cos t\frac{dt}{dx}$

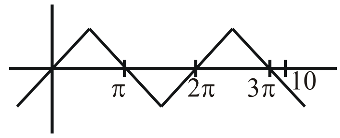
$=\frac{\cos t}{3\sec^2 t}=\frac{\cos^3 t}{3}=\frac{1}{3.2\sqrt{2}}=\frac{1}{6\sqrt{2}}$

17. If $x = \sin^{-1}(\sin 10)$ and $y=\cos^{-1}(\cos 10)$, then $y-x$ is equal to:

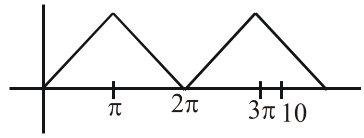
(1) π (2) 7π (3) 0 (4) 10

Ans. (1)

Sol.



$x = \sin^{-1}(\sin 10) = 3\pi - 10$



$y = \cos^{-1}(\cos 10) = 4\pi - 10$

$y - x = \pi$

18. If the lines $x = ay+b$, $z = cy + d$ and $x=a'z + b'$, $y = c'z + d'$ are perpendicular, then:

(1) $cc' + a + a' = 0$

(2) $aa' + c + c' = 0$

(3) $ab' + bc' + 1 = 0$

(4) $bb' + cc' + 1 = 0$

Ans. (2)

Sol. Line $x = ay + b$, $z = cy + d \Rightarrow \frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$

Line $x = a'z + b'$, $y = c'z + d'$

$\Rightarrow \frac{x-b'}{a'}=\frac{y-d'}{c'}=\frac{z}{1}$

Given both the lines are perpendicular

$\Rightarrow aa' + c' + c = 0$

19. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2-11x+\alpha = 0$ are rational numbers is :

(1) 2 (2) 5 (3) 3 (4) 4

Ans. (3)

Sol. $6x^2 - 11x + \alpha = 0$

given roots are rational

$\Rightarrow$ D must be perfect square

$\Rightarrow 121 - 24\alpha = \lambda^2$

$\Rightarrow$ maximum value of α is 5

$\alpha = 1 \Rightarrow \lambda \notin I$

$\alpha = 2 \Rightarrow \lambda \notin I$

$\alpha = 3 \Rightarrow \lambda \in I$ $\Rightarrow$ 3 integral values

$\alpha = 4 \Rightarrow \lambda \in I$

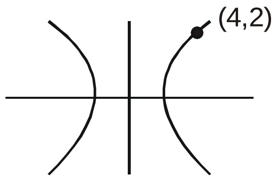
$\alpha = 5 \Rightarrow \lambda \in I$

20. A hyperbola has its centre at the origin, passes through the point (4,2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is :

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{3}{2}$ (3) $\sqrt{3}$ (4) 2

Ans. (1)

Sol.



$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

2a = 4 a = 2

$$\frac{x^2}{4}-\frac{y^2}{b^2}=1$$

Passes through (4,2)

$$4-\frac{4}{b^2}=1 \Rightarrow b^2=\frac{4}{3} \Rightarrow e=\frac{2}{\sqrt{3}}$$

21. Let A={xεR:x is not a positive integer}

Define a function f :A→R as $f(x)=\frac{2x}{x-1}$ then f is

(1) injective but not surjective

(2) not injective

(3) surjective but not injective

(4) neither injective nor surjective

Ans. (1)

Sol. $f(x)=2\left(1+\frac{1}{x-1}\right)$

$$f'(x)=-\frac{2}{(x-1)^2}$$

⇒ f is one-one but not onto

22. If $f(x)=\int\frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx,(x\geq 0)$ and $f(0)=0$, then the value of $f(1)$ is :

(1) $-\frac{1}{2}$ (2) $\frac{1}{2}$ (3) $-\frac{1}{4}$ (4) $\frac{1}{4}$

Ans. (4)

Sol. $\int\frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx$

$$=\int\frac{5x^{-6}+7x^{-8}}{\left(\frac{1}{x^7}+\frac{1}{x^5}+2\right)^2}dx=\frac{1}{2+\frac{1}{x^5}+\frac{1}{x^7}}+C$$

As $f(0)=0$, $f(x)=\frac{x^7}{2x^7+x^2+1}$

$$f(1)=\frac{1}{4}$$

23. If the circles $x^2+y^2-16x-20y+164=r^2$ and $(x-4)^2+(y-7)^2=36$ intersect at two distinct points, then:

(1) 0 < r < 1 (2) 1 < r < 11

(3) r > 11 (4) r = 11

Ans. (2)

Sol. $x^2+y^2-16x-20y+164=r^2$

A(8,10), $R_1=r$

$(x-4)^2+(y-7)^2=36$

B(4,7), $R_2=6$

$|R_1-R_2|<AB<R_1+R_2$

⇒ 1 < r < 11

24. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50sq. units, then the number of elements in the set S is:

(1) 9 (2) 18 (3) 32 (4) 36

Ans. (4)

Sol. Let A(α,0) and B(0,β)

be the vectors of the given triangle AOB

⇒ |αβ| = 100

⇒ Number of triangles

= 4 × (number of divisors of 100)

= 4 × 9 = 36

25. The sum of the follwing series

$$1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2+....+5^2)}{11}+....$$ up to 15 terms, is:

(1) 7820 (2) 7830 (3) 7520 (4) 7510

Ans. (1)

Sol. $$T_n = \frac{(3+(n-1)\times 3)(1^2+2^2+....+n^2)}{(2n+1)}$$

$$T_n = \frac{3.\frac{n(n+1)(2n+1)}{6}}{2n+1} = \frac{n^2(n+1)}{2}$$

$$S_{15} = \frac{1}{2}\sum_{n=1}^{15}(n^3+n^2) = \frac{1}{2}\left[\left(\frac{15(15+1)}{2}\right)^2 + \frac{15\times 16\times 31}{6}\right]$$

$= 7820$

26. Let a, b and c be the 7th, 11th and 13th terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to:

(1) $\frac{1}{2}$ (2) 4

(3) 2 (4) $\frac{7}{13}$

Ans. (2)

Sol. a = A + 6d

b = A + 10d

c = A + 12d

a,b,c are in G.P.

$\Rightarrow (A + 10d)^2 = (A + 6d)(a + 12d)$

$$\Rightarrow \frac{A}{d} = -14$$

$$\frac{a}{c} = \frac{A+6d}{A+12d} = \frac{6+\frac{A}{d}}{12+\frac{A}{d}} = \frac{6-14}{12-14} = 4$$

27. If the system of linear equations

$x-4y+7z = g$

$3y - 5z = h$

$-2x + 5y - 9z = k$

is consistent, then :

(1) g + h + k = 0

(2) 2g + h + k = 0

(3) g + h + 2k = 0

(4) g + 2h + k = 0

Ans. (2)

Sol. $P_1 \equiv x - 4y + 7z - g = 0$

$P_2 \equiv 3x - 5y - h = 0$

$P_3 \equiv -2x + 5y - 9z - k = 0$

Here $\Delta = 0$

$2P_1 + P_2 + P_3 = 0$ when $2g + h + k = 0$

28. Let $f:[0,1]\rightarrow R$ be such that $f(xy) = f(x).f(y)$ for all $x,y,\varepsilon[0,1]$, and $f(0)\neq 0$. If $y = y(x)$ satisfies the differential equation, $\frac{dy}{dx} = f(x)$ with $y(0) = 1$, then $y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)$ is equal to

(1) 4 (2) 3 (3) 5 (4) 2

Ans. (2)

Sol. $f(xy) = f(x).\ f(y)$

$f(0) = 1$ as $f(0) \neq 0$

$\Rightarrow f(x) = 1$

$$\frac{dy}{dx} = f(x) = 1$$

$\Rightarrow y = x + c$

At, x = 0, y = 1 $\Rightarrow$ c = 1

y = x + 1

$$\Rightarrow y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right) = \frac{1}{4}+1+\frac{3}{4}+1 = 3$$

29. A data consists of n observations:

$x_1,\ x_2,\ .....,\ x_n$. If $\sum_{i=1}^{n}(x_i+1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i-1)^2 = 5n$, then the standard deviation of this data is :

(1) 5 (2) $\sqrt{5}$ (3) $\sqrt{7}$ (4) 2

Ans. (2)

Sol. $\sum\left(x_i+1\right)^2=9n$ ...(1)

$\sum\left(x_i-1\right)^2=5n$ ...(2)

(1) + (2) $\Rightarrow$ $\sum\left(x_1^2+1\right)=7n$

$\Rightarrow \dfrac{\sum x_i^2}{n}=6$

(1) - (2) $\Rightarrow$ $4\Sigma x_i=4n$

$\Rightarrow$ $\Sigma x_i=n$

$\Rightarrow$ $\dfrac{\Sigma x_i}{n}=1$

$\Rightarrow$ variance = 6 – 1 = 5

$\Rightarrow$ Standard diviation $=\sqrt{5}$

30. The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repitition of digits allowed) is equal to :

(1) 250 (2) 374 (3) 372 (4) 375

Ans. (2)

Sol.

a_1	a_2	a_3

Number of numbers = $5^3 - 1$

a_4	a_1	a_2	a_3

2 ways for a_4

Number of numbers = 2×5^3

Required number = $5^3 + 2 \times 5^3 - 1$

= 374