FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Saturday 06th April, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

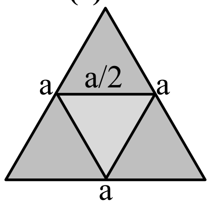
1. Let ABC be an equilateral triangle. A new triangle is formed by joining the middle points of all sides of the triangle ABC and the same process is repeated infinitely many times. If P is the sum of perimeters and Q is be the sum of areas of all the triangles formed in this process, then:

(1) $P^2 = 36\sqrt{3}Q$ (2) $P^2 = 6\sqrt{3}Q$

(3) $P = 36\sqrt{3}Q^2$ (4) $P^2 = 72\sqrt{3}Q$

Ans. (1)

Sol.



Area of first $\Delta = \dfrac{\sqrt{3}a^2}{4}$

Area of second $\Delta = \dfrac{\sqrt{3}a^2}{4}\dfrac{a^2}{4} = \dfrac{\sqrt{3}a^2}{16}$

Area of third $\Delta = \dfrac{\sqrt{3}a^2}{64}$

sum of area $= \dfrac{\sqrt{3}a^2}{4}\left(1 + \dfrac{1}{4} + \dfrac{1}{16}....\right)$

$Q = \dfrac{\sqrt{3}a^2}{4}\dfrac{1}{\frac{3}{4}} = \dfrac{a^2}{\sqrt{3}}$

perimeter of $1^{st}\ \Delta = 3a$

perimeter of $2^{nd}\ \Delta = \dfrac{3a}{2}$

perimeter of $3^{rd}\ \Delta = \dfrac{3a}{4}$

$P = 3a\left(1 + \dfrac{1}{2} + \dfrac{1}{4} + ...\right)$

$P = 3a.2 = 6a$

$a = \dfrac{P}{6}$

$Q = \dfrac{1}{\sqrt{3}} \cdot \dfrac{P^2}{36}$

$P^2 = 36\sqrt{3}Q$

2. Let A = {1, 2, 3, 4, 5}. Let R be a relation on A defined by xRy if and only if $4x \le 5y$. Let m be the number of elements in R and n be the minimum number of elements from A × A that are required to be added to R to make it a symmetric relation. Then m + n is equal to:

(1) 24 (2) 23

(3) 25 (4) 26

Ans. (3)

Sol. Given : $4x \le 5y$

then

$$R = \{(1,1),(1,2),(1,3),(1,4),(1,5),(2,2),(2,3),(2,4)\\ (2,5),(3,3),(3,4),(3,5),(4,4),(4,5),(5,4),(5,5)\}$$

i.e. 16 elements.

i.e. m = 16

Now to make R a symmetric relation add

$\{(2,1)(3,2)(4,3)(3,1)(4,2)(5,3)(4,1)(5,2)(5,1)\}$

i.e. n = 9

So m + n = 25

3. If three letters can be posted to any one of the 5 different addresses, then the probability that the three letters are posted to exactly two addresses is:

(1) $\dfrac{12}{25}$ (2) $\dfrac{18}{25}$

(3) $\dfrac{4}{25}$ (4) $\dfrac{6}{25}$

Ans. (1)

Sol. Total method $= 5^3$

faverable $= {}^5C_2\left(2^3 - 2\right) = 60$

probability $= \dfrac{60}{125} = \dfrac{12}{25}$

4. Suppose the solution of the differential equation $\frac{dy}{dx}=\frac{(2+\alpha)x-\beta y+2}{\beta x-2\alpha y-(\beta\gamma-4\alpha)}$ represents a circle passing through origin. Then the radius of this circle is :

(1) $\sqrt{17}$ (2) $\frac{1}{2}$

(3) $\frac{\sqrt{17}}{2}$ (4) 2

Ans. (3)

Sol. $\frac{dy}{dx}=\frac{(2+\alpha)x-\beta y+2}{\beta x-y(2\alpha+\beta)+4\alpha}$

$\beta xdy-(2\alpha+\beta)ydy+4\alpha dy=(2+\alpha)xdx-\beta ydx+2dx$

$\beta(xdy+ydx)-(2\alpha+\beta)ydy+4\alpha dy=(2+\alpha)xdx+2dx$

$\beta xy-\frac{(2\alpha+\beta)y^2}{2}+4\alpha y=\frac{(2+\alpha)x^2}{2}$

$\Rightarrow$ $\beta=0$ for this to be circle

$(2+\alpha)\frac{x^2}{2}+\alpha y^2+2x-4\alpha y=0$

coeff. of $x^2=y^2$ $2+a=2a$

$\Rightarrow$ $\boxed{\alpha=2}$

i.e. $2x^2+2y^2+2x-8y=0$

$x^2+y^2+x-4y=0$

$rd=\sqrt{\frac{1}{4}+4}=\frac{\sqrt{17}}{2}$

5. If the locus of the point, whose distances from the point (2, 1) and (1, 3) are in the ratio 5 : 4, is $ax^2+by^2+cxy+dx+ey+170=0$, then the value of $a^2+2b+3c+4d+e$ is equal to:

(1) 5 (2) –27

(3) 37 (4) 437

Ans. (3)

Sol. let P(x, y)

$\frac{(x-2)^2+(y-1)^2}{(x-1)^2+(y-3)^2}=\frac{25}{16}$

$9x^2+9y^2+14x-118y+170=0$

$a^2+2b+3c+4d+e$

$=81+18+0+56-118$

$=155-118$

$=37$

6. $\lim_{n\to\infty}\frac{(1^2-1)(n-1)+(2^2-2)(n-2)+.....+((n-1)^2-(n-1))\cdot 1}{(1^3+2^3+.....+n^3)-(1^2+2^2+.....+n^2)}$

is equal to:

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$

(3) $\frac{3}{4}$ (4) $\frac{1}{2}$

Ans. (2)

Sol. $\lim_{n\to\infty}\frac{\sum_{r=1}^{n-1}(r^2-r)(n-r)}{\sum_{r=1}^{n}r^3-\sum_{r=1}^{n}r^2}$

$\lim_{n\to\infty}\frac{\sum_{r=1}^{n-1}\left(-r^3+r^2(n+1)-nr\right)}{\left(\frac{n(n+1)}{2}\right)^2-\frac{n(n+1)(2n+1)}{6}}$

$\lim_{n\to\infty}\frac{\left(\frac{((n-1)n)}{2}\right)^2+\frac{(n+1)(n-1)n(2n-1)}{6}-\frac{n^2(n-1)}{2}}{\frac{n(n+1)}{2}\left(\frac{n(n+1)}{2}-\frac{2n+1}{3}\right)}$

$\lim_{n\to\infty}\frac{\frac{n(n-1)}{2}\left(\frac{-n(n-1)}{2}+\frac{(n+1)(2n-1)}{3}-n\right)}{\frac{n(n+1)}{2}\frac{3n^2+3n-4n-2}{6}}$

$\lim_{n\to\infty}\frac{(n-1)\left(-3n^2+3n+2(2n^2+n-1)-6\right)}{(n+1)(3n^2-n-2)}$

$\lim_{n\to\infty}\frac{(n-1)(n^2+5n-8)}{(n+1)(3n^2-n-2)}=\frac{1}{3}$

7. Let $0\le r\le n$. If ${}^{n+1}C_{r+1}:{}^{n}C_r:{}^{n-1}C_{r-1}=55:35:21$, then $2n+5r$ is equal to:

(1) 60 (2) 62

(3) 50 (4) 55

Ans. (3)

Ans. $\frac{{}^{n+1}C_r}{{}^{n}C_r}=\frac{55}{35}$

$\frac{(n+1)!}{(r+1)!(n-r)}!\frac{r!(n-r)!}{n!}=\frac{11}{7}$

$\frac{(n+1)}{r+1}=\frac{11}{7}$

$7n = 4 + 11r$

$$\frac{{}^{n}C_r}{{}^{n-1}C_{r-1}} = \frac{35}{21}$$

$$\frac{n!}{r!(n-r)!} = \frac{(r-1)!(n-r)!}{(n-1)!} = \frac{5}{3}$$

$$\frac{n}{r} = \frac{5}{3}$$

$3n = 5r$

By solving $r = 6$ $\quad n = 10$

$2n + 5r = 50$

8. A software company sets up m number of computer systems to finish an assignment in 17 days. If 4 computer systems crashed on the start of the second day, 4 more computer systems crashed on the start of the third day and so on, then it took 8 more days to finish the assignment. The value of m is equal to :

(1) 125 (2) 150

(3) 180 (4) 160

Ans. (2)

Sol. $17m = m + (m - 4) + (m - 4 \times 2)...+...(m - 4 \times 24)$

$17\,m = 25m - 4\,(1 + 2\,...24)$

$$8m = \frac{4\cdot 24\cdot 25}{2} = 150$$

9. If z_1, z_2 are two distinct complex number such that $\left|\frac{z_1 - 2z_2}{\frac{1}{2} - z_1\bar{z}_2}\right| = 2$, then

(1) either z_1 lies on a circle of radius 1 or z_2 lies on a circle of radius $\frac{1}{2}$

(2) either z_1 lies on a circle of radius $\frac{1}{2}$ or z_2 lies on a circle of radius 1.

(3) z_1 lies on a circle of radius $\frac{1}{2}$ and z_2 lies on a circle of radius 1.

(4) both z_1 and z_2 lie on the same circle.

Ans. (1)

Sol. $$\frac{z_1 - 2z_2}{\frac{1}{2} - z_1\bar{z}_2} \times \frac{\bar{z}_1 - 2\bar{z}_2}{\frac{1}{2} - \bar{z}_1 z_2} = 4$$

$$|z_1|^2\, 2z_1\bar{z}_2 - 2\bar{z}_1 z_2 + 4|z_2|^2$$

$$= 4\left(\frac{1}{4} - \frac{\bar{z}_1 z_2}{2} - \frac{z_1\bar{z}_2}{2} + |z_1|^2|z_2|^2\right)$$

$$z_1\bar{z}_1 + 2z_2\cdot 2\bar{z}_2 - z_1\bar{z}_1 2z_2\, 2\bar{z}_2 - 1 = 0$$

$$(z_1\bar{z}_1 - 1)(1 - 2z_2\cdot 2\bar{z}_2) = 0$$

$$\left(|z_1|^2 - 1\right)\left(|2z_2|^2 - 1\right) = 0$$

10. If the function $f(x) = \left(\frac{1}{x}\right)^{2x}$; $x > 0$ attains the maximum value at $x = \frac{1}{e}$ then :

(1) $e^{\pi} < \pi^{e}$ (2) $e^{2\pi} < (2\pi)^{e}$

(3) $e^{\pi} > \pi^{e}$ (4) $(2e)^{\pi} > \pi^{(2e)}$

Ans. (3)

Sol. Let $y = \left(\frac{1}{x}\right)^{2x}$

$$\ell ny = 2x\ell n\left(\frac{1}{x}\right)$$

$$\ell ny = -2x\ell nx$$

$$\frac{1}{y}\frac{dy}{dx} = -2(1 + \ell nx)$$

for $x > \frac{1}{e}$ f^{n} is decreasing

so, $e < \pi$

$$\left(\frac{1}{e}\right)^{2e} > \left(\frac{1}{\pi}\right)^{2\pi}$$

$e^{\pi} > \pi^{e}$

11. Let $\vec{a} = 6\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a is vector such that $|\vec{c}| \geq 6$, $\vec{a}.\vec{c} = 6|\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a}\times\vec{b}$ and $\vec{c}$ is 60°, then $|(\vec{a}\times\vec{b})\times\vec{c}|$ is equal to:

(1) $\frac{9}{2}(6 - \sqrt{6})$ (2) $\frac{3}{2}\sqrt{3}$

(3) $\frac{3}{2}\sqrt{6}$ (4) $\frac{9}{2}(6 + \sqrt{6})$

Ans. (4)

Sol. $\left|\left(\vec{a}\times\vec{b}\times\vec{c}\right)\right| = |\vec{a}\times\vec{b}||\vec{c}|\frac{\sqrt{3}}{2}$

$|\vec{c}-\vec{a}| = 2\sqrt{2}$

$|c|^2 + |a|^2 - 2\vec{c}\cdot\vec{a} = 8$

$|z|^2 + 38 - 12|z| = 8$

$|z|^2 - 12|z| + 30 = 0$

$|z| = \dfrac{12 \pm \sqrt{144-120}}{2}$

$= \dfrac{12 \pm 2\sqrt{6}}{2}$

$|z| = 6 + \sqrt{6}$

$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\ell} & \hat{j} & \hat{k} \\ 6 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix}$

$\hat{\ell} - \hat{j} + 5\hat{k}$

$|\vec{a}\times\vec{b}| = \sqrt{27}$

$|(\vec{a}\times b)\times z| = \sqrt{27}\left(6+\sqrt{6}\right)\frac{\sqrt{3}}{2}$

$\frac{9}{2}\left(6+\sqrt{6}\right)$

12. If all the words with or without meaning made using all the letters of the word "NAGPUR" are arranged as in a dictionary, then the word at 315th position in this arrangement is :

(1) NRAGUP (2) NRAGPU

(3) NRAPGU (4) NRAPUG

Ans. (3)

Sol. NAGPUR

A → 5! = 120	
G ® 5! = 120	240
NA ® 4! = 24	264
NG ® 4! = 24	288
NP ® 4! = 24	312
NRAGPU = 1	313
NRAGUP	314
NRAPGU	315

13. Suppose for a differentiable function h, h(0) = 0, h(1) = 1 and h'(0) = h'(1) = 2. If $g(x) = h(e^x)\, e^{h(x)}$, then g'(0) is equal to:

(1) 5 (2) 3

(3) 8 (4) 4

Ans. (4)

Sol. $g(x) = h\left(e^x\right)\cdot e^{h(x)}$

$g'(x) = h(e^x)\cdot e^{h(x)}\cdot h'(x) + e^{h(x)}h'\left(e^x\right)\cdot e^x$

$g'(0) = h(1)e^{h(0)}h'(0) + e^{h(0)}h'(1)$

= 2 + 2 = 4

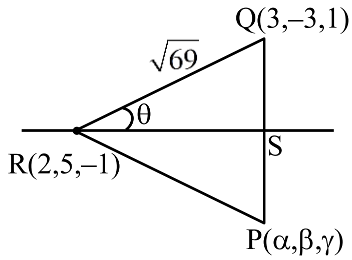
14. Let P (α, β, γ) be the image of the point Q(3, –3, 1) in the line $\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$ and R be the point (2, 5, –1). If the area of the triangle PQR is λ and $\lambda^2 = 14K$, then K is equal to:

(1) 36 (2) 72

(3) 18 (4) 81

Ans. (4)

Sol.



$RQ = \sqrt{1+64+4} = \sqrt{69}$

$\overrightarrow{RQ} = \hat{\ell} - 8\hat{j} + 2\hat{k}$

$\overrightarrow{RS} = \hat{\ell} + \hat{j} - \hat{k}$

$\cos\theta = \dfrac{\overrightarrow{RQ}\cdot\overrightarrow{RS}}{|\overrightarrow{RQ}||\overrightarrow{RS}|} = \left|\dfrac{1-8-2}{\sqrt{69}\sqrt{3}}\right| = \dfrac{9}{3\sqrt{23}}$

$\cos\theta = \dfrac{3}{\sqrt{23}} = \dfrac{RS}{RQ} = \dfrac{RS}{\sqrt{69}}$

$RS = 3\sqrt{3}$

$\sin\theta = \dfrac{\sqrt{14}}{\sqrt{23}} = \dfrac{QS}{\sqrt{69}}$

$QS = \sqrt{42}$

area = $\dfrac{1}{2}\cdot 2QS\cdot RS = \sqrt{42}\cdot 3\sqrt{3}$

$\lambda = 9\sqrt{14}$

$\lambda^2 = 81.14 = 14k$

k = 81

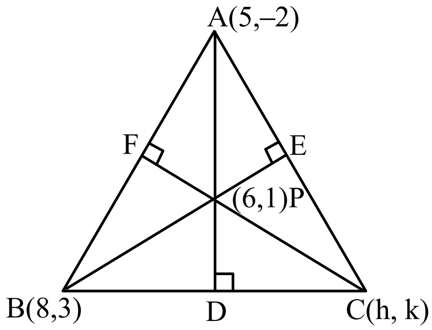
15. If P(6, 1) be the orthocentre of the triangle whose vertices are A(5, –2), B(8, 3) and C(h, k), then the point C lies on the circle.

(1) $x^2 + y^2 - 65 = 0$ (2) $x^2 + y^2 - 74 = 0$

(3) $x^2 + y^2 - 61 = 0$ (4) $x^2 + y^2 - 52 = 0$

Ans. (1)

Sol.



Slope of AD = 3

Slope of $BC = -\frac{1}{3}$

equation of BC = 3y + x – 17 = 0

slope of BE = 1

Slope of AC = –1

equation of AC is x + y – 3 = 0

point C is (–4, 7)

16. Let $f(x) = \frac{1}{7-\sin 5x}$ be a function defined on R. Then the range of the function f(x) is equal to:

(1) $\left[\frac{1}{8}, \frac{1}{5}\right]$ (2) $\left[\frac{1}{7}, \frac{1}{6}\right]$

(3) $\left[\frac{1}{7}, \frac{1}{5}\right]$ (4) $\left[\frac{1}{8}, \frac{1}{6}\right]$

Ans. (4)

Sol. $\sin 5x \in [-1,1]$

$-\sin 5x \in [-1, 1]$

$7 - \sin 5x \in [6, 8]$

$$\frac{1}{7-\sin 5x} \in \left[\frac{1}{8}, \frac{1}{6}\right]$$

17. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \left(\left(\vec{a} \times \left(\hat{i} + \hat{j}\right)\right) \times \hat{i}\right) \times \hat{i}$.

Then the square of the projection of $\vec{a}$ on $\vec{b}$ is :

(1) $\frac{1}{5}$ (2) 2

(3) $\frac{1}{3}$ (4) $\frac{2}{3}$

Ans. (2)

Sol. $\vec{a} \times \left(\hat{i} + \hat{j}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix}$

$= \hat{i} - \hat{j} + \hat{k}$

$\left(\vec{a} \times \left(\hat{i} \times \hat{j}\right)\right) \times \hat{i} = \hat{k} + \hat{j}$

$\left(\left(\vec{a} \times \left(\hat{i} \times \hat{j}\right)\right) \times i\right) \times \hat{i} = \hat{j} - \hat{k}$

projection of $\vec{a}$ on $\hat{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$= \frac{1+1}{\sqrt{2}} = \sqrt{2}$

18. If the area of the region

$\left\{(x, y) : \frac{a}{x^2} \le y \le \frac{1}{x}, 1 \le x \le 2, 0 < a < 1\right\}$ is

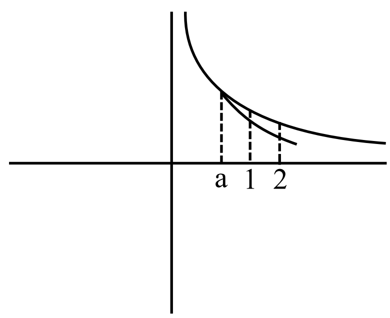
$(\log_e 2) - \frac{1}{7}$ then the value of 7a – 3 is equal to:

(1) 2 (2) 0

(3) –1 (4) 1

Ans. (3)

Sol.



area $\int_1^2\left(\frac{1}{x}-\frac{a}{x^2}\right)dx$

$\left[\ell nx+\frac{a}{x}\right]_1^2$

$\ell n2+\frac{a}{2}-a=\log_e 2-\frac{1}{7}$

$\frac{-a}{2}=-\frac{1}{7}$

$a=\frac{2}{7}$

$7a = 2$

$7a - 3 = -1$

19. If $\int\frac{1}{a^2\sin^2 x+b^2\cos^2 x}dx=\frac{1}{12}\tan^{-1}(3\tan x)+$ constant, then the maximum value of asinx + bcosx, is :

(1) $\sqrt{40}$ (2) $\sqrt{39}$

(3) $\sqrt{42}$ (4) $\sqrt{41}$

Ans. (1)

Sol. $\int\frac{\sec^2 x dx}{a^2\tan^2 x+b^2}$

let tanx = t

$\sec^2 dx = dt$

$\int\frac{dt}{a^2t^2+b^2}$

$\frac{1}{a^2}\int\frac{dt}{t^2+\left(\frac{b}{a}\right)^2}$

$\frac{1}{a^2}\frac{1}{\frac{b}{a}}\tan^{-1}\left(\frac{t}{b}a\right)+c$

$\frac{1}{ab}\tan^{-1}\left(\frac{\alpha}{b}\tan x\right)+c$

on comparing $\frac{a}{b}=3$

ab = 12

a = 6, b = 2

maximum value of

6 sinx + 2cosx is $\sqrt{40}$

20. If A is a square matrix of order 3 such that det(A) = 3 and

det(adj(–4 adj(–3 adj(3 adj($(2A)^{-1}$))))) = 2^m3^n,

then m +| 2n is equal to:

(1) 3 (2) 2

(3) 4 (4) 6

Ans. (3)

Sol. $|A|=3$

$\left|\text{adj}(-4\text{adj}(-3\text{adj }(3\text{adj}\left((2A)^{-1}\right))))\right|$

$\left|-4\text{adj}\left(-3\text{adj}(3\text{adj}(2A)^{-1}\right)\right|^2$

$4^6\left|\text{adj}\left(-3\text{adj}\left(3\text{adj}(2A)^{-1}\right)\right)\right|^2$

$2^{12}\cdot3^{12}\left|3\text{adj}(2A)^{-1}\right|^8$

$2^{12}\cdot3^{12}\cdot3^{24}\left|\text{adj}(2A)^{-1}\right|^8$

$2^{12}\cdot3^{36}\left|(2A)^{-1}\right|^{16}$

$2^{12}\cdot3^{36}\frac{1}{|2A|^{16}}$

$2^{12}\cdot3^{36}\frac{1}{2^{48}|A|^{16}}$

$2^{12}\cdot3^{36}\frac{1}{2^{48}\cdot3^{16}}$

$\frac{3^{20}}{2^{36}}=2^{-36}\cdot3^{20}$

m = – 36 n = 20

m + 2n = 4

SECTION-B

21. Let [t] denote the greatest integer less than or equal to t. Let f: $[0, \infty) \to R$ be a function defined by $f(x)=\left[\frac{x}{2}+3\right]-\left[\sqrt{x}\right]$. Let S be the set of all points in the interval [0, 8] at which f is not continuous. Then $\sum_{a\in S} a$ is equal to ________.

Ans. (17)

Sol. $\left[\frac{x}{2}+3\right]$ is discontinuous at x = 2,4,6,8

$\sqrt{x}$ is discontinuous at x = 1,4

F(x) is discontinuous at x = 1,2,6,8

$\sum a = 1+2+6+8=17$

22. The length of the latus rectum and directrices of a hyperbola with eccentricity e are 9 and $x = \pm\frac{4}{\sqrt{3}}$, respectively. Let the line $y - \sqrt{3}x + \sqrt{3} = 0$ touch this hyperbola at (x_0, y_0). If m is the product of the focal distances of the point (x_0, y_0), then $4e^2 + m$ is equal to _______.

NTA Ans. (61)

Ans. (Bonus)

Sol. Given $\frac{2b^2}{a} = 9$ and $\frac{a}{e} = \pm\frac{4}{\sqrt{3}}$

equation of tangent $y - \sqrt{3}x + \sqrt{3} = 0$

by equation of tangent

Let slope = S = $\sqrt{3}$

Constant = $-\sqrt{3}$

By condition of tangency

$\Rightarrow 6 = 6a^2 - 9a$

$\Rightarrow a = 2, b^2 = 9$

Equation of Hyperbola is

$\frac{x^2}{4}-\frac{y^2}{9}=1$ and for tangent

Point of contact is $(4, 3\sqrt{3}) = (x_0, y_0)$

Now $e = \sqrt{1+\frac{9}{4}} = \frac{\sqrt{13}}{2}$

Again product of focal distances

$m = (x_0e + a)(x_0e - a)$

$m + 4e^2 = 20e^2 - a^2$

$= 20 \times \frac{13}{4} - 4 = 61$

(There is a printing mistake in the equation of directrix $x = \pm\frac{4}{\sqrt{3}}$.

Corrected equation is $x = \pm\frac{4}{\sqrt{13}}$ for directrix, as eccentricity must be greater than one, so question must be bonus)

23. If $S(x) = (1 + x) + 2(1 + x)^2 + 3(1 + x)^3 + \ldots + 60(1 + x)^{60}$, $x \neq 0$, and $(60)^2 S(60) = a(b)^b + b$, where a, b $\in$ N, then (a + b) equal to _______

Ans. (3660)

Sol.

$S(x)=(1+x) + 2(1+x)^2 + 3(1+x)^3+..+ 60(1 + x)^{60}$

$(1+x)S = \quad (1+x)^2 + \ldots\ldots \quad 59(1+x)^{60} + 60(1+x)^{61}$

$$-xS = \frac{(1+x)\left((1+x)^{60}-1\right)}{x} - 60(1+x)^{61}$$

Put x = 60

$$-60S = \frac{61\left((61)^{60}-1\right)}{60} - 60(61)^{61}$$

on solving 3660

24. Let [t] denote the largest integer less than or equal to t. If

$$\int_0^3\left([x^2]+\left[\frac{x^2}{2}\right]\right)dx = a+b\sqrt{2}-\sqrt{3}-\sqrt{5}+c\sqrt{6}-\sqrt{7},$$

where a, b, c $\in$ z, then a + b + c is equal to ______

Ans. (23)

Sol. $\int_0^3\left[x^2\right]dx + \int_0^3\left[\frac{x^2}{2}\right]dx$

$$= \int_0^1 0\,dx + \int_1^{12} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx$$

$$+\int_{\sqrt{3}}^{2} 3\,dx + \int_{2}^{\sqrt{5}} 4\,dx + \int_{\sqrt{5}}^{\sqrt{6}} 5\,dx$$

$$+\int_{\sqrt{6}}^{\sqrt{7}} 6\,dx + \int_{\sqrt{7}}^{\sqrt{8}} 7\,dx + \int_{\sqrt{8}}^{3} 8\,dx$$

$$+\int_{0}^{\sqrt{2}} 0\,dx + \int_{\sqrt{2}}^{2} 1\,dx$$

$$+\int_{2}^{\sqrt{6}} 2\,dx + \int_{\sqrt{6}}^{\sqrt{8}} 3\,dx + \int_{\sqrt{8}}^{3} 4dx = 31 - 6\sqrt{2} - \sqrt{3} - \sqrt{5}$$

$$-2\sqrt{6} - \sqrt{7}$$

$a = 31 \quad b = -6 \quad c = -2$

$a + b + c = 31 - 6 - 2 = 23$

25. From a lot of 12 items containing 3 defectives, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Let items in the sample be drawn one by one without replacement. If variance of X is $\frac{m}{n}$, where gcd(m, n) = 1, then n – m is equal to _______.

Ans. (71)

Sol. $a = 1 - \frac{{}^{3}C_5}{{}^{12}C_5}$

$b = 3.\frac{{}^{9}C_4}{{}^{12}C_5}$

$c = 3.\frac{{}^{9}C_3}{{}^{12}C_5}$

$d = 1.\frac{{}^{9}C_2}{{}^{12}C_5}$

$u = 0.a + 1.b + 2.c + 3.d = 1.25$

$\sigma^2 = 0.a + 1.b + 4.c + 9d - u^2$

$\sigma^2 = \frac{105}{176}$

Ans. 176 – 105 = 71

26. In a triangle ABC, BC = 7, AC = 8, AB = $\alpha \in N$ and $\cos A = \frac{2}{3}$. If $49\cos(3C) + 42 = \frac{m}{n}$, where gcd(m, n) = 1, then m + n is equal to _______

Ans. (39)

26. In a triangle ABC, BC = 7, AC = 8, AB = $\alpha \in N$ and $\cos A = \frac{2}{3}$. If $49\cos(3C) + 42 = \frac{m}{n}$, where gcd(m, n) = 1, then m + n is equal to _______

Ans. (39)

Sol. $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\frac{2}{3} = \frac{8^2 + c^2 - 7^2}{2 \times 8 \times c}$

$C = 9$

$\cos C = \frac{7^2 + 8^2 - 9^2}{2 \times 7 \times 8} = \frac{2}{7}$

$49 \cos 3C + 42$

$49(4 \cos^3 C - 3 \cos C) + 42$

$49\left(4\left(\frac{2}{7}\right)^3 - 3\left(\frac{2}{7}\right)\right) + 42$

$= \frac{32}{7}$

$m + n = 32 + 7 = 39$

27. If the shortest distance between the lines $\frac{x-\lambda}{3} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $\frac{x+2}{-3} = \frac{y+5}{2} = \frac{z-4}{4}$ is $\frac{44}{\sqrt{30}}$, then the largest possible value of $|\lambda|$ is equal to _______.

Ans. (43)

Sol. $\bar{a}_1 = \lambda\hat{i} + 2\hat{j} + \hat{k}$

$\bar{a}_2 = -2\hat{i} - 5\hat{j} + 4\hat{k}$

$\vec{p} - = 3\hat{i} - \hat{j} + \hat{k}$

$\vec{q} - = -3\hat{i} + 2\hat{j} + 4\hat{k}$

$(\lambda + 2)\hat{i} + 7\hat{j} - 3\hat{k} = \bar{a}_1 - \bar{a}_2$

$\vec{p} \times \vec{q} - = -6\hat{i} - 15\hat{j} + 3\hat{k}$

$$\frac{44}{\sqrt{30}}=\frac{|-6\lambda-12-105-9|}{\sqrt{(-6)^2+(-15)^2+3^2}}$$

$$\frac{44}{\sqrt{30}}=\frac{|6\lambda+126|}{3\sqrt{30}}$$

$$132=|6\lambda+126|$$

$\lambda = 1, \lambda = -43$

$|\lambda| = 43$

28. Let α, β be roots of $x^2+\sqrt{2}x-8=0$.

If $U_n=\alpha^n+\beta^n$, then $\frac{U_{10}+\sqrt{12}U_9}{2U_8}$

is equal to _______.

Ans. (4)

Sol. $$\frac{\alpha^{10}+\beta^{10}+\sqrt{2}\left(\alpha^9+\beta^9\right)}{2\left(\alpha^8+\beta^8\right)}$$

$$\frac{\alpha^8\left(\alpha^2+\sqrt{2}\alpha\right)+\beta^8\left(\beta^2+\sqrt{2}\beta\right)}{2\left(\alpha^8+\beta^8\right)}$$

$$\frac{8\alpha^8+8\beta^8}{2\left(\alpha^8+\beta^8\right)}=4$$

29. If the system of equations

$2x + 7y + \lambda z = 3$

$3x + 2y + 5z = 4$

$x + \mu y + 32z = -1$

has infinitely many solutions, then $(\lambda - \mu)$ is equal to ________ :

Ans. (38)

Sol. $D = D_1 = D_2 = D_3 = 0$

$$D_3=\begin{vmatrix}2 & 7 & 3\\ 3 & 2 & 4\\ 1 & \mu & -1\end{vmatrix}=0\Rightarrow\mu=-39$$

$$D=\begin{vmatrix}2 & 7 & \lambda\\ 3 & 2 & 5\\ 1 & -39 & 32\end{vmatrix}=0\Rightarrow\lambda=-1$$

$$\lambda-\mu=38$$

30. If the solution y(x) of the given differential equation $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$ passes through the point $\left(\frac{\pi}{2},0\right)$, then the value of $e^{y\left(\frac{\pi}{6}\right)}$ is equal to ______.

Ans. (3)

Sol. $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$

$$\Rightarrow d\left(\left(e^y+1\right)\sin x\right)=0$$

$$\left(e^y+1\right)\sin x=C$$

It passes through $\left(\frac{\pi}{2},0\right)$

$\Rightarrow c=2$

Now, $x=\frac{\pi}{6}$

$\Rightarrow e^y=3$

PHYSICS | TEST PAPER WITH SOLUTION

SECTION-A

31. The longest wavelength associated with Paschen series is : (Given $R_H = 1.097 \times 10^7$ SI unit)

(1) 1.094×10^{-6}m (2) 2.973×10^{-6}m

(3) 3.646×10^{-6}m (4) 1.876×10^{-6}m

Ans. (4)

Sol. For longest wavelength in Paschen's series:

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For longest $n_1 = 3$

$n_2 = 4$

$$\frac{1}{\lambda} = R\left[\frac{1}{(3)^2} - \frac{1}{(4)^2}\right]$$

$$\frac{1}{\lambda} = R\left[\frac{1}{9} - \frac{1}{16}\right]$$

$$\frac{1}{\lambda} = R\left[\frac{16-9}{16\times 9}\right]$$

$$\Rightarrow \lambda = \frac{16\times 9}{7R} = \frac{16\times 9}{7\times 1.097\times 10^7}$$

$\lambda = 1.876 \times 10^{-6}$ m

32. A total of 48 J heat is given to one mole of helium kept in a cylinder. The temperature of helium increases by 2°C. The work done by the gas is : (Given, $R = 8.3$ J $K^{-1}mol^{-1}$.)

(1) 72.9 J (2) 24.9 J

(3) 48 J (4) 23.1 J

Ans. (4)

Sol. 1^{st} law of thermodynamics

$\Delta Q = \Delta U + W$

$\Rightarrow +48 = nC_v\Delta T + W$

$$\Rightarrow 48 = (1)\left(\frac{3R}{2}\right)(2) + W$$

$\Rightarrow W = 48 - 3 \times R$

$\Rightarrow W = 48 - 3 \times (8.3)$

$$\Rightarrow \boxed{W = 23.1 \text{ Joule}}$$

33. In finding out refractive index of glass slab the following observations were made through travelling microscope 50 vernier scale division = 49 MSD; 20 divisions on main scale in each cm

For mark on paper

MSR = 8.45 cm, VC = 26

For mark on paper seen through slab

MSR = 7.12 cm, VC = 41

For powder particle on the top surface of the glass slab

MSR = 4.05 cm, VC = 1

(MSR = Main Scale Reading, VC = Vernier Coincidence)

Refractive index of the glass slab is:

(1) 1.42 (2) 1.52

(3) 1.24 (4) 1.35

Ans. (1)

Sol. $$1 \text{ MSD} = \frac{1\text{cm}}{20} = 0.05 \text{ cm}$$

$$1 \text{ VSD} = \frac{49}{50} \text{ MSD} = \frac{49}{50} \times 0.05 \text{ cm} = 0.049 \text{ cm}$$

LC = 1MSD – 1VSD = 0.001 cm

For mark on paper, $L_1 = 8.45$ cm + 26 × 0.001 cm = 84.76 mm

For mark on paper through slab, L2 = 7.12 cm + 41× 0.001 cm = 71.61 mm

For powder particle on top surface, ZE = 4.05 cm + 1 × 0.001 cm = 40.51 mm

$\therefore$ actual $L_1 = 84.76 - 40.51 = 44.25$ mm

actual L2 = 71.61 – 40.51 = 31.10 mm

$$L_2 = \frac{L_1}{\mu}$$

$$\Rightarrow \mu = \frac{L_1}{L_2} = \frac{44.25}{31.10} = 1.42$$

34. In the given electromagnetic wave $E_y = 600 \sin (\omega t - kx)\ Vm^{-1}$, intensity of the associated light beam is (in W/m^2); (Given $\in_0 = 9\times10^{-12}C^2N^{-1}m^{-2}$)

(1) 486 (2) 243

(3) 729 (4) 972

Ans. **(1)**

Sol. Intensity $= \frac{1}{2}\varepsilon_0 E_0^2 c$

$$= \frac{1}{2} \times 9 \times 10^{-12} \times (600)^2 \times 3 \times 10^8$$

$$= \frac{9}{2} \times 36 \times 3 = 486 \text{ w/m}^2$$

35. Assuming the earth to be a sphere of uniform mass density, a body weighed 300 N on the surface of earth. How much it would weigh at R/4 depth under surface of earth ?

(1) 75 N (2) 375 N

(3) 300 N (4) 225 N

Ans. **(4)**

Sol. At surface: mg = 300 N

$$m = \frac{300}{g_s}$$

At Depth $\frac{R}{4}$: $g_d = g_s\left[1-\frac{d}{R}\right]$

$$g_d = g_s\left[1-\frac{R}{4R}\right]$$

$$g_d = \frac{3g_s}{4}$$

weight at depth $= m \times g_d$

$$= m \times \frac{3g_s}{4}$$

$$= \frac{3}{4}\times 300$$

$$= 225 \text{ N}$$

36. The acceptor level of a p-type semiconductor is 6eV. The maximum wavelength of light which can create a hole would be : Given hc = 1242 eV nm.

(1) 407 nm (2) 414 nm

(3) 207 nm (4) 103.5 nm

Ans. **(3)**

Sol. Energy $= \frac{hc}{\lambda}$;

$$E = \frac{1240}{\lambda(\text{nm})}\text{eV}$$

$$6 = \frac{1240}{\lambda(\text{nm})}$$

$$\lambda = \frac{1240}{6} = 207\text{nm}$$

37. A car of 800 kg is taking turn on a banked road of radius 300 m and angle of banking 30°. If coefficient of static friction is 0.2 then the maximum speed with which car can negotiate the turn safely : ($g = 10\ m/s^2$, $\sqrt{3} = 1.73$)

(1) 70.4 m/s (2) 51.4 m/s

(3) 264 m/s (4) 102.8 m/s

Ans. **(2)**

Sol. m = 800 kg

r = 300 m

$\theta = 30°$

$\mu_s = 0.2$

$$V_{max} = \sqrt{Rg\left[\frac{\tan\theta+\mu}{1-\mu\tan\theta}\right]}$$

$$= \sqrt{300\times g\times\left[\frac{\tan 30°+0.2}{1-0.2\times\tan 30°}\right]}$$

$$= \sqrt{300\times 10\times\left[\frac{0.57+0.2}{1-0.2\times 0.57}\right]}$$

$V_{max} = 51.4$ m/s

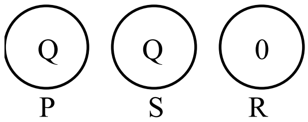
38. Two identical conducting spheres P and S with charge Q on each, repel each other with a force 16N. A third identical uncharged conducting sphere R is successively brought in contact with the two spheres. The new force of repulsion between P and S is :

(1) 4 N (2) 6 N

(3) 1 N (4) 12 N

Ans. **(2)**

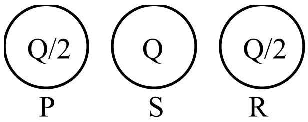
Sol.



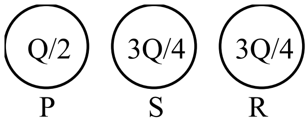
$F_{PS} \propto Q^2$

$F_{PS} = 16$ N

Now   If P & R are brought in contact then



Now If S & R are brought in contact then



New force between P & S is :

$F_{PS} \propto \frac{Q}{2} \times \frac{3Q}{4}$

$F_{PS} \propto \frac{3Q^2}{8} = \frac{3}{8} \times 16 = 6N$

39. In a coil, the current changes form –2 A to +2A in 0.2 s and induces an emf of 0.1 V. The self-inductance of the coil is :

(1) 5 mH (2) 1 mH

(3) 2.5 mH (4) 4 mH

Ans. (1)

Sol. $(\text{Emf})_{\text{induced}} = -L\frac{di}{dt}$

In magnitude form,

$|\text{Emf}_{\text{ind}}| = \left|(-)L\frac{di}{dt}\right|$

$\Rightarrow 0.1 = \frac{(L)[+2-(-2)]}{0.2}$

$\Rightarrow \boxed{L = \frac{0.1 \times 0.2}{4} = 5\text{mH}}$

40. For the thin convex lens, the radii of curvature are at 15 cm and 30 cm respectively. The focal length the lens is 20 cm. The refractive index of the material is :

(1) 1.2 (2) 1.4

(3) 1.5 (4) 1.8

Ans. (3)

Sol. $\frac{1}{f} = \left(\frac{\mu_{\text{lens}}}{\mu_{\text{air}}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\Rightarrow \frac{1}{+20} = \left(\frac{\mu}{1} - 1\right)\left(\frac{1}{+15} - \frac{1}{(-30)}\right)$

$\Rightarrow \frac{1}{20} = (\mu - 1)\left(\frac{3}{30}\right)$

$\Rightarrow \mu - 1 = \frac{1}{2}$

$\Rightarrow \boxed{\mu = 1 + \frac{1}{2} = \frac{3}{2} = 1 \cdot 5}$

41. Energy of 10 non rigid diatomic molecules at temperature T is :

(1) $\frac{7}{2}RT$ (2) 70 K_BT

(3) 35 RT (4) 35 K_BT

Ans. (4)

Sol. Degree of freedom(f) = 5 + 2(3N – 5)

f = 5 + 2(3 × 2 –1) = 7

energy of one molecule = $\frac{f}{2}K_BT$

energy of 10 molecules

$= 10\left(\frac{f}{2}K_BT\right) = 10\left(\frac{7}{2}K_BT\right) = 35\ K_BT$

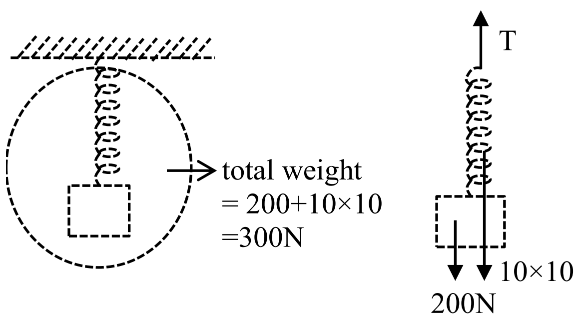
42. A body of weight 200 N is suspended form a tree branch thought a chain of mass 10 kg. The branch pulls the chain by a force equal to (if g = 10 m/s^2):

(1) 150 N (2) 300 N

(3) 200 N (4) 100 N

Ans. (2)

Sol.



Chain block system is in equilibrium so

T = 200 + 100 = 300 N.

43. When UV light of wavelength 300 nm is incident on the metal surface having work function 2.13 eV, electron emission takes place. The stopping potential is : (Given hc = 1240 eV nm)

(1) 4 V (2) 4.1 V (3) 2 V (4) 1.5 V

Ans. (3)

Sol. $\frac{hc}{\lambda} - \phi = e.V_s$

$\Rightarrow \frac{1240}{300}\,eV - 2.13\ eV = eVs$

$\Rightarrow 4.13\ eV - 2.13\ eV = eVs.$

$\Rightarrow$ So, $\boxed{V_s = 2volt}$

44. The number of electrons flowing per second in the filament of a 110 W bulb operating at 220 V is : (Given e = 1.6×10^{-19} C)

(1) 31.25×10^{17} (2) 6.25×10^{18}

(3) 6.25×10^{17} (4) 1.25×10^{19}

Ans. (1)

Sol. Power (P) = V.I

$\Rightarrow 110 = (220)\ (I)$

$\Rightarrow I = 0.5\ A$

Now, $I = \frac{n \cdot e}{t}$

$\Rightarrow 0.5 = \left(\frac{n}{t}\right)(1.6 \times 10^{-19})$

$\Rightarrow \frac{n}{t} = \frac{0.5}{1.6 \times 10^{-19}}$

$\Rightarrow \boxed{\frac{n}{t} = 31.25 \times 10^{17}}$

45. When kinetic energy of a body becomes 36 times of its original value, the percentage increase in the momentum of the body will be :

(1) 500% (2) 600%

(3) 6% (4) 60%

Ans. (1)

Sol. Kinetic energy (K) = $\frac{P^2}{2m}$

$\Rightarrow P = \sqrt{2mK}$

If $K_f = 36\ K_i$

So, $P_f = 6\ P_i$

% increase in momentum = $\frac{P_f - P_i}{P_i} \times 100\%$

$= \frac{6P_i - P_i}{P_i} \times 100\%$

$= 500\%$

46. Pressure inside a soap bubble is greater than the pressure outside by an amount :

(given : R = Radius of bubble, S = Surface tension of bubble)

(1) $\frac{4S}{R}$ (2) $\frac{4R}{S}$

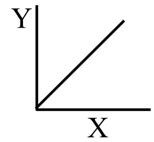
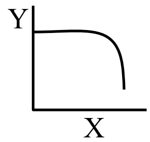
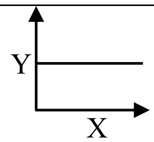
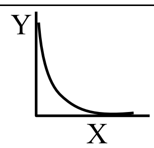
(3) $\frac{S}{R}$ (4) $\frac{2S}{R}$

Ans. (1)

Sol. There are two liquid-air surfaces in bubble so

$\Delta P = 2\left(\frac{2S}{R}\right) = \frac{4S}{R}$

47. Match List-I with List-II

List-I (Y vs X)		List-II (Shape of Graph)	
(A)	Y = magnetic susceptibility X = magnetising field	(I)	
(B)	Y = magnetic field X = distance from centre of a current carrying wire for x < a (where a=radius of wire)	(II)	
(C)	Y = magnetic field X = distance from centre of a current carrying wire for x > a (where a = radius of wire)	(III)	
(D)	Y= magnetic field inside solenoid X = distance from center	(IV)	

Choose the correct answer from the options given below :

(1) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

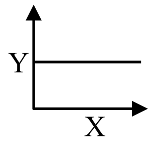
(2) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

(3) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

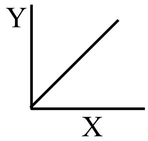
Ans. (4)

Sol. (A) Graph between Magnetic susceptibility and magnetising field is :



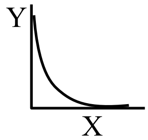
(B) magnetic field due to a current carrying wire for x < a :

$$B = \frac{\mu_0 \, i \, r}{2\pi a^2}$$

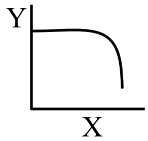


(C) magnetic field due to a current carrying wire for x > a :

$$B = \frac{\mu_0 \, i}{2\pi a}$$



(D) magnetic field inside solenoid varies as:



48. In a vernier calliper, when both jaws touch each other, zero of the vernier scale shifts towards left and its 4^{th} division coincides exactly with a certain division on main scale. If 50 vernier scale divisions equal to 49 main scale divisions and zero error in the instrument is 0.04 mm then how many main scale divisions are there in 1 cm ?

(1) 40 (2) 5
(3) 20 (4) 10

NTA Ans. (3)

Sol. 4^{th} division coincides with 3^{rd} division then

0.004 cm = 4VSD – 3MSD

49MSD = 50 VSD

$$1\text{MSD} = \frac{1}{N} \text{ cm}$$

$$0.004 = 4\left\{\frac{49}{50}\text{MSD}\right\} - 3\text{MSD}$$

$$0.004 = \left(\frac{196}{50} - 3\right)\left(\frac{1}{N}\right)$$

$$N = \frac{46}{50} \times \frac{1000}{4} = \frac{46 \times 1000}{200} = 230$$

49. Given below are two statements :

Statement (I) : Dimensions of specific heat is $[L^2T^{-2}K^{-1}]$

Statement (II) : Dimensions of gas constant is $[M\,L^2T^{-1}K^{-1}]$

(1) Statement (I) is incorrect but statement (II) is correct

(2) Both statement (I) and statement (II) are incorrect

(3) Statement (I) is correct but statement (II) is incorrect

(4) Both statement (I) and statement (II) are correct

Ans. (3)

Sol. $\Delta Q = mS\Delta T$

$$\mathbf{s} = \frac{\Delta Q}{m\Delta T}$$

$$[\mathbf{s}] = \left[\frac{ML^2T^{-2}}{MK}\right]$$

$$[\mathbf{s}] = [L^2\,T^{-2}\,K^{-1}]$$

Statement-(I) is correct

$$PV = nRT \quad \Rightarrow R = \frac{PV}{nT}$$

$$[R] = \frac{[ML^{-1}T^{-2}][L^3]}{[\text{mol}][K]}$$

$$[R] = [ML^2T^{-2}\,\text{mol}^{-1}K^{-1}]$$

Statement-II is incorrect

50. A body projected vertically upwards with a certain speed from the top of a tower reaches the ground in t_1. If it is projected vertically downwards from the same point with the same speed, it reaches the ground in t_2. Time required to reach the ground, if it is dropped from the top of the tower, is :

(1) $\sqrt{t_1 t_2}$ (2) $\sqrt{t_1 - t_2}$

(3) $\sqrt{\frac{t_1}{t_2}}$ (4) $\sqrt{t_1 + t_2}$

Ans. (1)

Sol. $t_1 = \frac{u+\sqrt{u^2+2gh}}{g}$

$t_2 = \frac{-u+\sqrt{u^2+2gh}}{g}$

$t = \frac{\sqrt{2gh}}{g}$

$t_1 t_2 = \frac{(u^2+2gh)-u^2}{g^2} = \frac{2gh}{g^2} = t^2$

$\Rightarrow t = \sqrt{t_1 t_2}$

SECTION-B

51. In Franck-Hertz experiment, the first dip in the current-voltage graph for hydrogen is observed at 10.2 V. The wavelength of light emitted by hydrogen atom when excited to the first excitation level is ______ nm.

(Given hc = 1245 eV nm, e = 1.6 × 10^{-19}C).

Ans. (122)

Sol. $10.2 \text{ eV} = \frac{hc}{\lambda}$

$\lambda = \frac{1245 \text{ eV} - \text{nm}}{10.2\text{eV}} = 122.06 \text{ nm}$

52. For a given series LCR circuit it is found that maximum current is drawn when value of variable capacitance is 2.5 nF. If resistance of 200Ω and 100 mH inductor is being used in the given circuit. The frequency of ac source is _____ × 10^3 Hz. (given $\pi^2 = 10$)

Ans. (10)

Sol. for maximum current, circuit must be in resonance.

$f_0 = \frac{1}{2\pi\sqrt{L \times C}}$

$f_0 = \frac{1}{2\pi\sqrt{100\times10^{-3}\times2.5\times10^{-9}}}$

$= \frac{1}{2\pi\sqrt{25\times10^{-11}}}$

$= \frac{1}{2\pi\times5}\times10^5\times\sqrt{10}\text{ Hz}$

$= \frac{100}{10}\times10^3\text{ Hz}$

$f_0 = 10 \times 10^3$ Hz

53. A particle moves in a straight line so that its displacement x at any time t is given by $x^2=1+t^2$. Its acceleration at any time t is x^{-n} where n = ______.

Ans. (3)

Sol. $x^2 = 1 + t^2$

$2x\frac{dx}{dt} = 2t$

$xv = t$

$x\frac{dv}{dt} + v\frac{dx}{dt} = 1$

$x.a+v^2 = 1$

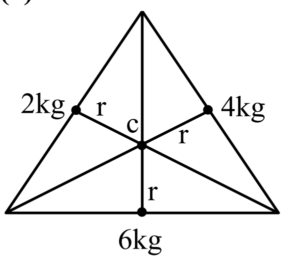
$a = \frac{1-v^2}{x} = \frac{1-t^2/x^2}{x}$

$a = \frac{1}{x^3} = x^{-3}$

54. Three balls of masses 2kg, 4kg and 6kg respectively are arranged at centre of the edges of an equilateral triangle of side 2 m. The moment of inertia of the system about an axis through the centroid and perpendicular to the plane of triangle, will be ______ kg m^2.

Ans. (4)

Sol.



$r = \frac{1}{\sqrt{3}}$

Moment of inertia about C and perpendicular to the plane is :

$I = r^2\,[2+4+6]$

$= \frac{1}{3}\times12$

$I = 4 \text{ kg-m}^2$

55. A coil having 100 turns, area of 5 × $10^{-3}$$m^2$, carrying current of 1 mA is placed in uniform magnetic field of 0.20 T such a way that plane of coil is perpendicular to the magnetic field. The work done in turning the coil through 90° is _____ μJ.

Ans. (100)

Sol. $W = \Delta U = U_f - U_i$

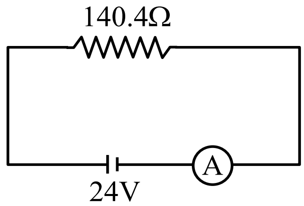
$W = (-\vec{\mu}.\vec{B})_f - (-\vec{\mu}.\vec{B})_i$

$= 0 + (\vec{\mu}.\vec{B})_i$

$= (100 \times 5 \times 10^{-3} \times 1 \times 10^{-3}) \times 0.2$ J

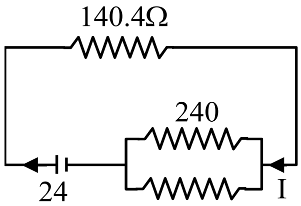
$= 1 \times 10^{-4}$ J = 100 μJ

56. In the given figure an ammeter A consists of a 240Ω coil connected in parallel to a 10 Ω shunt. The reading of the ammeter is ______ mA.



Ans. (160)

Sol.



Req

$= 140.4 + \frac{240 \times 10}{240 + 10}$

Req $= 140.4 + \frac{2400}{250}$

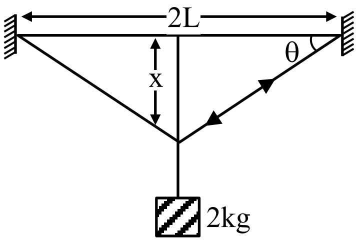
Req. = 150Ω

$\therefore$ Current in ammeter $= \frac{24}{150}$

$= 160$ mA

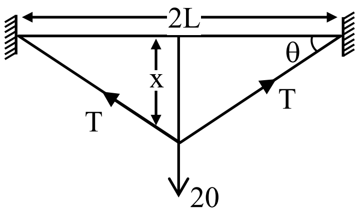
57. A wire of cross sectional area A, modulus of elasticity 2×10^{11} Nm^{-2} and length 2 m is stretched between two vertical rigid supports. When a mass of 2 kg is suspended at the middle it sags lower from its original position making angle $\theta = \frac{1}{100}$ radian on the points of support. The value of A is _____ $\times 10^{-4}$ m^2 (consider x<<L).

(given : g=10 m/s^2)



Ans. (1)

Sol.



In vertical derection

$2T \sin\theta = 20$

using small angle approximation $\sin\theta = \theta$

$\theta = \frac{1}{100}$

$\therefore \quad T = \frac{10}{\theta}$

$T = 1000N$

Change in length $\Delta L = 2\sqrt{x^2 + L^2} - 2L$

$= 2L\left[1 + \frac{x^2}{2L^2} - 1\right]$

$\Delta L = \frac{x^2}{L}$

$\therefore$ Modulus of elasticity $= \frac{\text{stress}}{\text{strain}}$

$2 \times 10^{11} = \frac{10^3}{A \times \frac{x^2}{L}} \times 2L$

$\therefore \quad A = 1 \times 10^{-4}\ m^2$

58. Two coherent monochromatic light beams of intensities I and 4I are superimposed. The difference between maximum and minimum possible intensities in the resulting beam is x I. The value of x is______.

Ans. (8)

Sol. $I_{max} = \left(\sqrt{I} + \sqrt{4I}\right)^2 = 9I$

$I_{min} = \left(\sqrt{4I} - \sqrt{I}\right)^2 = I$

$\therefore I_{max} - I_{min} = 8I$

59. Two open organ pipes of length 60 cm and 90 cm resonate at 6th and 5th harmonics respectively. The difference of frequencies for the given modes is _____ Hz.

(Velocity of sound in air = 333 m/s)

Ans. (740)

Sol. The difference in frequency in open organ pipe =

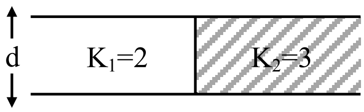
$f = \frac{nv}{2L}$

$\Delta f = \frac{6v}{2 \times 0.6} - \frac{5v}{2 \times 0.9}$

$v = 333$ m/s

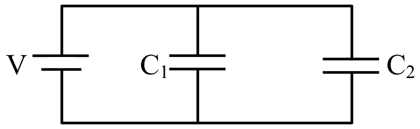
$\Delta f = 740$ Hz

60. A capacitor of 10 μF capacitance whose plates are separated by 10 mm through air and each plate has area 4 cm^2 is now filled equally with two dielectric media of $K_1 = 2$, $K_2 = 3$ respectively as shown in figure. If new force between the plates is 8 N. The supply voltage is _____ V.



NTA Ans. (80)

Sol.



$C_{eq} = C_1 + C_2$

$$C_1 = \frac{2 \in_0 A}{2 \times d} = 10\mu F$$

$$C_2 = \frac{3 \in_0 A}{2d} = 15\mu F$$

$C_{eq} = 25$ μF

Now the charge on $C_1 = 10V$ μc

$C_2 = 1.5$ V μC.

Now force between the plates $\left[F = \frac{Q^2}{2A \in_0}\right]$

$$\frac{100V^2 \times 10^{-12}}{2 \times 2 \times 10^{-4} \in_0} + \frac{225V^2 \times 10^{-12}}{2 \times 2 \times 10^{-4} \times \in_0} = 8$$

$325\ V^2 = 8 \times 4 \times 10^{-4} \times 8.85$

$$V^2 = \frac{32 \times 8.85 \times 10^{-4}}{325}$$

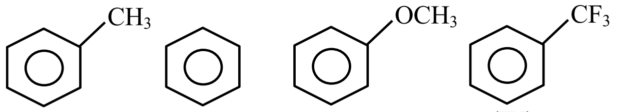
$$\therefore\ V = \sqrt{\frac{283.2 \times 10^{-4}}{325}}$$

$$\boxed{V = 0.93 \times 10^{-2}}$$

CHEMISTRY	TEST PAPER WITH SOLUTION

SECTION-A

61.



(I) (II) (III) (IV)

The **correct** arrangement for decreasing order of electrophilic substitution for above compounds

(1) (IV) > (I) > (II) > (III)

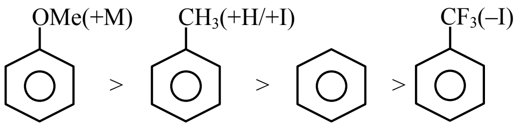
(2) (III) > (I) > (II) > (IV)

(3) (II) > (IV) > (III) > (I)

(4) (III) > (IV) > (II) > (I)

Ans. (2)

Sol.



62. Molality (m) of 3 M aqueous solution of NaCl is:

(Given : Density of solution = 1.25 g mL^{-1}, Molar mass in g mol^{-1} : Na-23, Cl-35.5)

(1) 2.90 m (2) 2.79 m

(3) 1.90 m (4) 3.85 m

Ans. (2)

Sol. 3 moles are present in 1 litre solution

$$\text{molality} = \frac{3\times1000}{1.25\times1000-[3\times58.5]} = 2.79\text{ m}$$

63. The incorrect statements regarding enzymes are:

(A) Enzymes are biocatalysts.

(B) Enzymes are non-specific and can catalyse different kinds of reactions.

(C) Most Enzymes are globular proteins.

(D) Enzyme - oxidase catalyses the hydrolysis of maltose into glucose.

Choose the correct answer from the option given below:

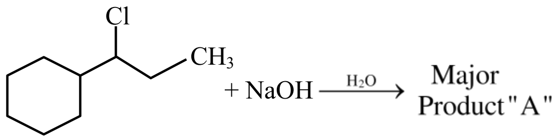
(1) (B) and (C) (2) (B), (C) and (D)

(3) (B) and (D) (4) (A), (B) and (C)

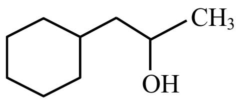
Ans. (3)

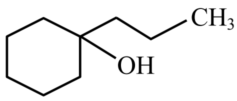
Sol. Direct NCERT Based

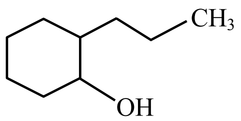
64.

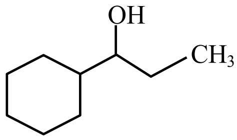


Consider the above chemical reaction. Product "A" is:

(1)


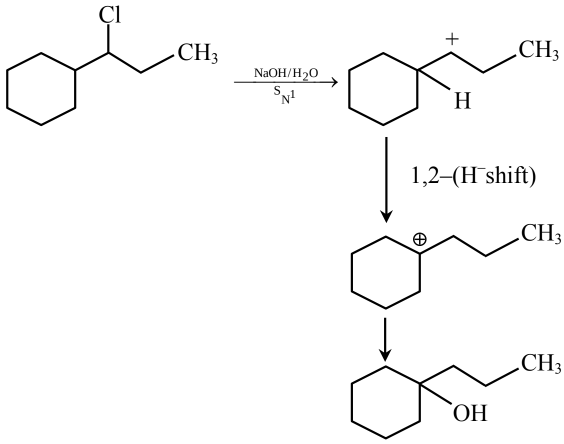
(2)


(3)


(4)


Ans. (2)

Sol.



(Major Product)

65. During the detection of acidic radical present in a salt, a student gets a pale yellow precipitate soluble with difficulty in NH_4OH solution when sodium carbonate extract was first acidified with dil. HNO_3 and then $AgNO_3$ solution was added. This indicates presence of:

(1) Br^- (2) CO_3^{2-}

(3) I^- (4) Cl^-

Ans. (1)

Sol. $Ag^+ + I^- \rightarrow AgI$ Yellow ppt.

$Ag^+ + Cl^- \rightarrow AgCl$ White ppt

$Ag^+ + Br^- \rightarrow AgBr$ Pale yellow ppt

66. How can an electrochemical cell be converted into an electrolytic cell ?

(1) Applying an external opposite potential greater than E^0_{cell}

(2) Reversing the flow of ions in salt bridge.

(3) Applying an external opposite potential lower than E^0_{cell}.

(4) Exchanging the electrodes at anode and cathode.

Ans. (1)

Sol. Applied external potential should be greater than E^0_{cell} in opposite direction.

67. Arrange the following elements in the increasing order of number of unpaired electrons in it.

(A) Sc (B) Cr

(C) V (D) Ti

(E) Mn

Choose the correct answer from the options given below:

(1) (C) < (E) < (B) < (A) < (D)

(2) (B) < (C) < (D) < (E) < (A)

(3) (A) < (D) < (C) < (B) < (E)

(4) (A) < (D) < (C) < (E) < (B)

Ans. (4)

Sol. **Unpaired electron**

Sc[Ar] $4s^2\ 3d^1$	1
Cr[Ar] $4s^1\ 3d^5$	6
V[Ar] $4s^2\ 3d^3$	3
Ti : [Ar] $4s^2\ 3d^2$	2
Mn : [Ar] $4s^2\ 3d^5$	5

68. Match **List-I** with **List-II.**

List-I **Alkali Metal**	**List-II** **Emission Wavelength in nm**
(A) Li	(I) 589.2
(B) Na	(II) 455.5
(C) Rb	(III) 670.8
(D) Cs	(IV) 780.0

Choose the **correct** answer from the options given below:

(1) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

(2) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

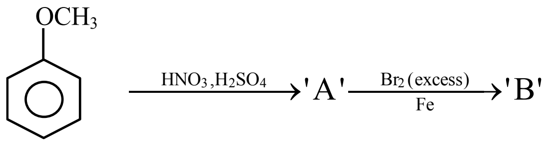
(3) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(4) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)

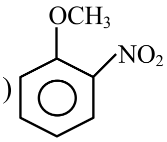
Ans. (2)

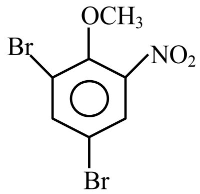
Sol. Fact Based

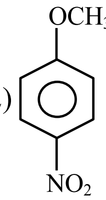
69. The major products formed:

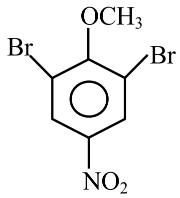


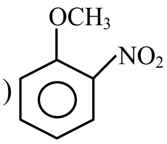
A and B respectively are:

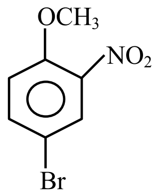
(1) 

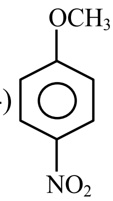
and 

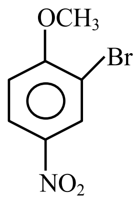
(2) 

and 

(3) 

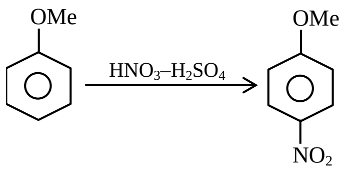
and 

(4) 

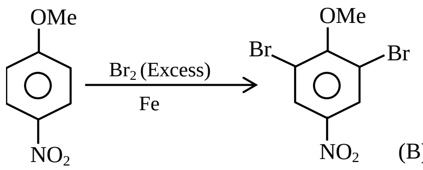
and 

Ans. (2)

Sol.



(A)



(B)

70. The incorrect statement regarding the geometrical isomers of 2-butene is:

(1) cis-2-butene and trans-2-butene are not interconvertible at room temperature.

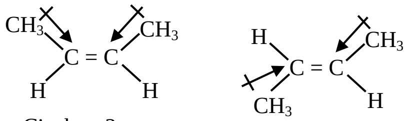
(2) cis-2-butene has less dipole moment than trans-2-butene.

(3) trans-2-butene is more stable than cis-2-butene.

(4) cis-2-butene and trans-2-butene are stereoisomers.

Ans. **(2)**

Sol.



Cis–but–2–ene (Polar) Trans–but–2–ene (Non Polar)

Cis–but–2–ene has higher Dipole moment than trans–but–2–ene.

71. Given below are two statements:

Statement I: PF_5 and BrF_5 both exhibit sp^3d hybridisation.

Statement II: Both SF_6 and $[Co(NH_3)_6]^{3+}$ exhibit sp^3d^2 hybridisation.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) **Statement I** is true but **Statement II** is false

(2) Both **Statement I** and **Statement II** are true

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is false but **Statement II** is true

Ans. **(3)**

Sol.

	Hybridisation		Hybridisation
PF_5	sp^3d	SF_6	sp^3d^2
BrF_5	sp^3d^2	$[Co(NH_3)_6]^{+3}$	d^2sp^3

Both Statement (1) and (2) are false.

72. The number of ions from the following that are expected to behave as oxidising agent is:

Sn^{4+}, Sn^{2+}, Pb^{2+}, Tl^{3+}, Pb^{4+}, Tl^{+}

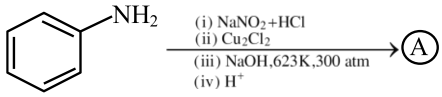
(1) 3 (2) 4

(3) 1 (4) 2

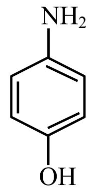
Ans. **(4)**

Sol. Due to inert pair effect; $T\ell^{+3}$ and Pb^{+4} can behave as oxidising agents.

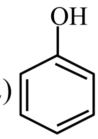
73. Identify the product (A) in the following reaction.



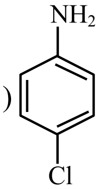
(1)



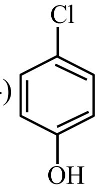
(2)



(3)

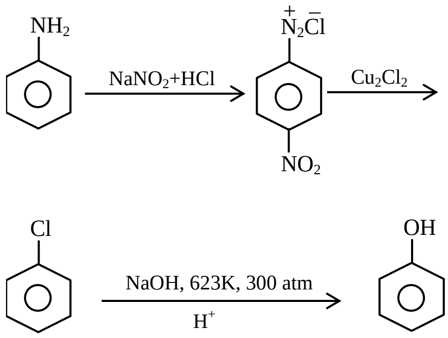


(4)



Ans. **(2)**

Sol.



(A)

74. The correct statements among the following, for a "chromatography" purification method is:

(1) Organic compounds run faster than solvent in the thin layer chromatographic plate.

(2) Non-polar compounds are retained at top and polar compounds come down in column chromatography.

(3) R_f of a polar compound is smaller than that of a non-polar compound.

(4) R_f is an integral value.

Ans. **(3)**

Sol. Non polar compounds are having higher value of R_f than polar compound.

75. Evaluate the following statements related to group 14 elements for their correctness.

(A) Covalent radius decreases down the group from C to Pb in a regular manner.

(B) Electronegativity decreases from C to Pb down the group gradually.

(C) Maximum covalence of C is 4 whereas other elements can expand their covalence due to presence of d orbitals.

(D) Heavier elements do not form pπ-pπ bonds.

(E) Carbon can exhibit negative oxidation states.

Choose the **correct** answer from the options given below:

(1) (C), (D) and (E) Only (2) (A) and (B) Only

(3) (A), (B) and (C) Only (4) (C) and (D) Only

Ans. **(1)**

Sol. (A) Down the group; radius increases

(B) EN does not decrease gradually from C to Pb.

(C) Correct.

(D) Correct.

(E) Range of oxidation state of carbon ; –4 to +4

76. Match **List-I** with the **List-II**

List-I **Reaction**	**List-II** **Type of redox reaction**
(A) $N_{2(g)} + O_{2(g)} \rightarrow 2NO_{(g)}$	(I) Decomposition
(B) $2Pb(NO_3)_{2(s)} \rightarrow 2PbO_{(s)} + 4NO_{2(g)} + O_{2(g)}$	(II) Displacement
(C) $2Na_{(s)} + 2H_2O_{(l)} \rightarrow 2NaOH_{(aq.)} + H_{2(g)}$	(III) Disproportionation
(D) $2NO_{2(g)} + 2\,^{-}OH_{(aq.)} \rightarrow NO^{-}_{2(aq.)} + NO^{-}_{3(aq.)} + H_2O_{(l)}$	(IV) Combination

Choose the **correct** answer from the options given below:

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(2) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(3) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(4) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Ans. **(4)**

Sol. A → (IV)

B → (I)

C → (II)

D → (III)

77. Consider the given reaction, identify the major product P.

$$CH_3-COOH \xrightarrow[\text{(iv) } H_2O/\bar{O}H,\Delta]{\text{(i) } LiAlH_4 \quad \text{(ii) PCC} \quad \text{(iii) } HCN/\bar{O}H} "P"$$

(1) $CH_3-CH_2-CH_2-OH$

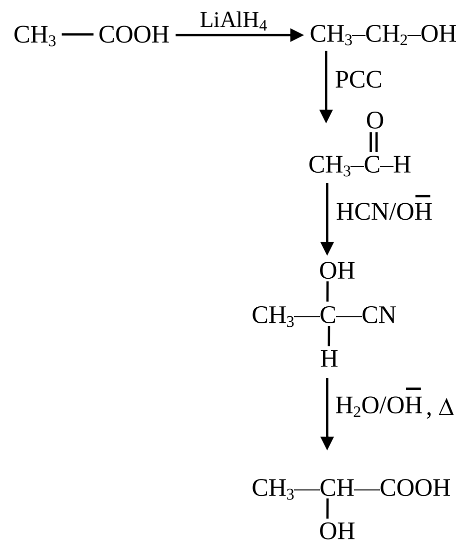
(2) $CH_3-CH_2-C(=O)-NH_2$

(3) $CH_3-C(=O)-CH_2CH_3$

(4) $CH_3-CH(OH)-COOH$

Ans. **(4)**

Sol.



78. The correct IUPAC name of $[PtBr_2(PMe_3)_2]$ is:

(1) bis(trimethylphosphine)dibromoplatinum(II)

(2) bis[bromo(trimethylphosphine)]platinum(II)

(3) dibromobis(trimethylphosphine)platinum(II)

(4) dibromodi(trimethylphosphine)platinum(II)

Ans. **(3)**

Sol. Dibromo bis(trimethylphosphine) platinum (II)

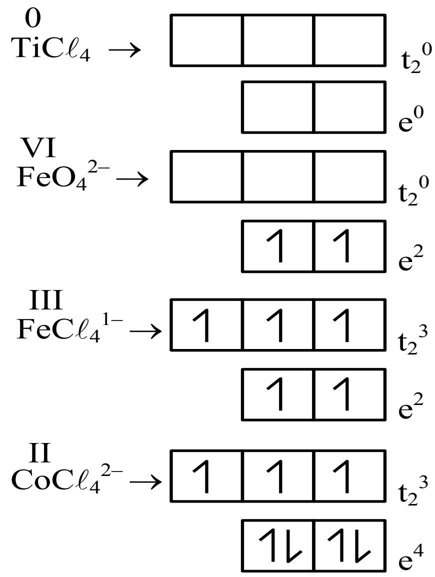
79. Match **List-I** with **List-II**

List-I	**List-II**
Tetrahedral Complex	**Electronic configuration**
(A) $TiCl_4$	(I) e^2, t_2^0
(B) $[FeO_4]^{2-}$	(II) e^4, t_2^3
(C) $[FeCl_4]^-$	(III) e^0, t_2^0
(D) $[CoCl_4]^{2-}$	(IV) e^2, t_2^3

Choose the **correct** answer from the option given below:

(1) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)
(2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(3) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(4) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Ans. (4)



Sol.

80. The ratio $\frac{K_P}{K_C}$ for the reaction:

$$CO_{(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons CO_{2(g)} \text{ is:}$$

(1) $(RT)^{1/2}$ (2) RT

(3) 1 (4) $\frac{1}{\sqrt{RT}}$

Ans. (4)

Sol. $CO(g) + \frac{1}{2}O_2(g) \rightleftharpoons CO_2(g)$

$$\Delta n_g = 1 - \left(1 + \frac{1}{2}\right) = -\frac{1}{2}$$

$$\frac{K_P}{K_C} = (RT)^{\Delta n_g} = \frac{1}{\sqrt{RT}}$$

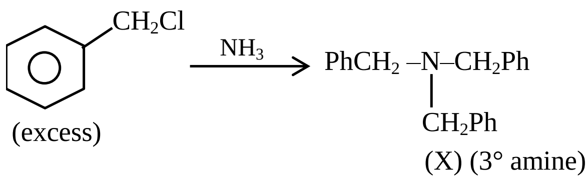
SECTION-B

81. An amine (X) is prepared by ammonolysis of benzyl chloride. On adding p-toluenesulphonyl chloride to it the solution remains clear. Molar mass of the amine (X) formed is______ g mol^{-1}.

(Given molar mass in gmol^{-1} C : 12, H : 1, O : 16, N : 14)

Ans. (287)

Sol.



Molar Mass of (X) is 287 g mol^{-1}

82. Consider the following reactions

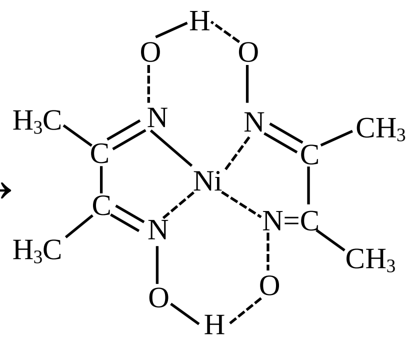
$NiS + HNO_3 + HCl \rightarrow A + NO + S + H_2O$

$$A + NH_4OH + \begin{matrix} H_3C - C = N - OH \\ | \\ H_3C - C = N - OH \end{matrix} \longrightarrow B + NH_4Cl + H_2O$$

The number of protons that do not involve in hydrogen bonding in the product B is______.

Ans. (12)

Sol. **B →**



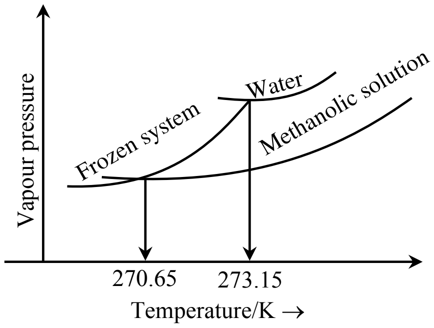
$3NiS + 2HNO_3 + 6HCl$

$$\longrightarrow 3NiCl_2 + 2NO + 3S + 4H_2O$$

$$NiCl_2 + 2NH_4OH + \begin{matrix} H_3C - C = N - OH \\ | \\ H_3C - C = N - OH \end{matrix}$$

$$\rightarrow NH_4Cl + H_2O + (B)$$

83. When 'x' × 10^{-2} mL methanol (molar mass = 32 g; density = 0.792 g/cm^3) is added to 100 mL water (density = 1 g/cm^3), the following diagram is obtained.



x =………………(nearest integer)

[Given: Molal freezing point depression constant of water at 273.15 K is 1.86 K kg mol^{-1}]

Ans. **(543)**

Sol. $\Delta T_f = 273.15 - 270.65 = 2.5$ K

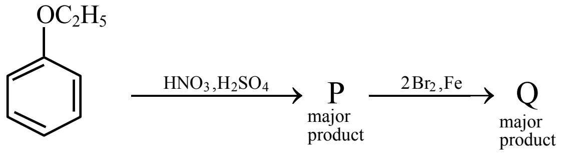
$\Delta T_f = K_f m \Rightarrow 2.5 = 1.86 \times \dfrac{n}{0.1}$

$\Rightarrow n = 0.1344$ moles

$\Rightarrow w = 0.1344 \times 32 = 4.3$ g

Volume $= \dfrac{4.3}{0.792} = 5.43$ ml $= 543 \times 10^{-2}$ ml

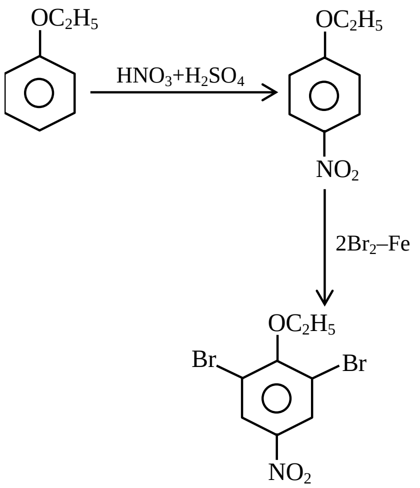
84.



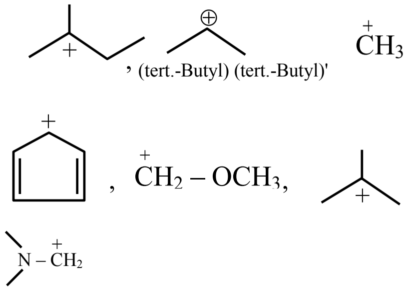
The ratio of number of oxygen atoms to bromine atoms in the product Q is_____ × 10^{-1}.

Ans. **(15)**

Sol.

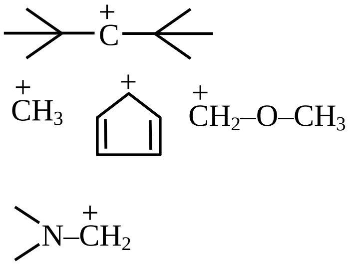


85. Number of carbocation from the following that are **not** stabilized by hyperconjugation is……….. .



Ans. **(5)**

Sol.



86. For the reaction at 298 K, 2A + B → C. ΔH = 400 kJ mol^{-1} and ΔS = 0.2 kJ mol^{-1} K^{-1}. The reaction will become spontaneous above______ K.

Ans. **(2000)**

Sol. $\Delta G = 0$

$T = \dfrac{\Delta H}{\Delta S} = \dfrac{400}{0.2} = 2000$ K

87. Total number of species from the following with central atom utilising 2p^2 hybrid orbitals for bonding is…………. .

NH_3, SO_2, SiO_2, $BeCl_2$, C_2H_2, C_2H_4, BCl_3, HCHO, C_6H_6, BF_3, $C_2H_4Cl_2$

Ans. **(6)**

Sol. Central atom utilising sp^2 hybrid orbitals

SO_2, C_2H_4, BCl_3, HCHO, C_6H_6, BF_3

88. Consider the two different first order reactions given below

$A + B \rightarrow C$ (Reaction 1)

$P \rightarrow Q$ (Reaction 2)

The ratio of the half life of Reaction 1 : Reaction 2 is 5 : 2. If t_1 and t_2 represent the time taken to complete $2/3^{rd}$ and $4/5^{th}$ of Reaction 1 and Reaction 2, respectively, then the value of the ratio $t_1 : t_2$ is________$\times 10^{-1}$ (nearest integer).

[Given: $\log_{10}(3) = 0.477$ and $\log_{10}(5) = 0.699$]

Ans. **(17)**

Sol. $\dfrac{(t_{1/2})_I}{(t_{1/2})_{II}} = \dfrac{K_2}{K_1} = \dfrac{5}{2}$

$$\therefore\ K_1t_1 = \ell n \frac{1}{1-\frac{2}{3}} = \ell n3$$

$$K_2t_2 = \ell n \frac{1}{1-\frac{4}{5}} = \ell n5$$

$$\Rightarrow \frac{K_1}{K_2} \times \frac{t_1}{t_2} = \frac{0.477}{0.699}$$

$$\Rightarrow \frac{t_1}{t_2} = \frac{0.477}{0.699} \times \frac{5}{2} = 1.7 = 17 \times 10^{-1}$$

89. For hydrogen atom, energy of an electron in first excited state is – 3.4 eV, K.E. of the same electron of hydrogen atom is x eV. Value of x is______ $\times 10^{-1}$ eV. (Nearest integer)

Ans. **(34)**

90. Among VO_2^+, MnO_4^- and $Cr_2O_7^{2-}$, the spin-only magnetic moment value of the species with least oxidising ability is……………….BM (Nearest integer).

(Given atomic member V = 23, Mn = 25, Cr = 24)

Ans. **(0)**

Sol. For 3d transition series;

Oxidising power : $V^{+5} < Cr^{+6} < Mn^{+7}$

V^{+5} : [Ar] $4s^0\ 3d^0$

Number of unpaired electron = 0

$\boxed{\mu = 0}$