FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Thursday 04th April, 2024) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. If the function $f(x)=\begin{cases}\dfrac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}} & , \ x\neq 0\\ a\log_e 2\log_e 3 & , \ x=0\end{cases}$

is continuous at x = 0, then the value of a^2 is equal to

(1) 968 (2) 1152

(3) 746 (4) 1250

Ans. (2)

Sol. $\lim\limits_{x\to 0} f(x)=a\ell n2\ell n3$

$$\lim_{n\to 0}\frac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}}=\lim_{x\to 0}\frac{\left(8^x-1\right)\left(9^x-1\right)}{\sqrt{2}-\sqrt{1+\cos x}}$$

$$\lim_{n\to 0}\left(\frac{8^x-1}{x}\right)\left(\frac{9^x-1}{x}\right)\left(\frac{x^2}{1-\cos x}\right)\left(\sqrt{2}+\sqrt{1+\cos x}\right)$$

$\therefore\ \ell n8\times\ell n9\times 2\times 2\sqrt{2}=24\sqrt{2}\ell n2\ell n3$

$\therefore\ a = 24\sqrt{2}$, $a^2 = 576 \times 2 = 1152$

2. If $\lambda > 0$, let θ be the angle between the vectors $\vec{a}=\hat{i}+\lambda\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$. If the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are mutually perpendicular, then the value of $(14\cos\theta)^2$ is equal to

(1) 25 (2) 20

(3) 50 (4) 40

Ans. (1)

Sol. $\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)=0,\ \lambda>0$

$|\vec{a}|^2-|\vec{b}|^2=0 \rightarrow 1+\lambda^2+9=9+1+4$

$\therefore\ \lambda=2,\ \cos\theta=\dfrac{\vec{a}-\vec{b}}{|\vec{a}|.|\vec{b}|}=\dfrac{3-\lambda-6}{\sqrt{14}.\sqrt{14}}$

$14\cos\theta = 3 - 8 = -5$

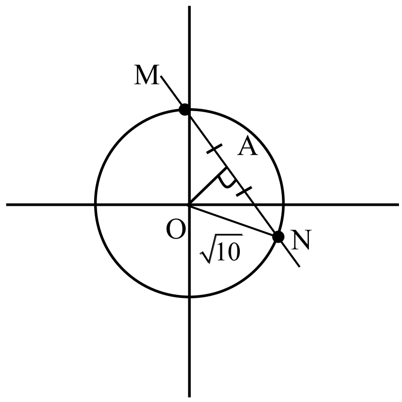
$\therefore\ (14\cos\theta)^2 = 25$

3. Let C be a circle with radius $\sqrt{10}$ units and centre at the origin. Let the line x + y = 2 intersects the circle C at the points P and Q. Let MN be a chord of C of length 2 unit and slope –1. Then, a distance (in units) between the chord PQ and the chord MN is

(1) $2-\sqrt{3}$ (2) $3-\sqrt{2}$

(3) $\sqrt{2}-1$ (4) $\sqrt{2}+1$

Ans. (2)



$C : x^2 + y^2 = 10$

$AN = \dfrac{MN}{2}=1$

$\therefore$ In $\Delta OAN \rightarrow (ON)^2 = (OA)^2 + (AN)^2$

$10 = (OA)^2 + 1 \rightarrow OA = 3$

Perpendicular distance of center from $PQ=\dfrac{|0+0-2|}{\sqrt{2}}=\sqrt{2}$

Perpendicular distance between MN and $PQ = OA+\sqrt{2}$ or $\left|OA-\sqrt{2}\right|$

$=3+\sqrt{2}$ or $3-\sqrt{2}$

4. Let a relation R on $\mathbb{N}\times\mathbb{N}$ be defined as :

(x_1,y_1) $R(x_2,y_2)$ if and only if $x_1 \le x_2$ or $y_1 \le y_2$

Consider the two statements :

(I) R is reflexive but not symmetric.

(II) R is transitive

Then which one of the following is true ?

(1) Only (II) is correct.

(2) Only (I) is correct.

(3) Both (I) and (II) are correct.

(4) Neither (I) nor (II) is correct.

Ans. (2)

Sol. All $((x_1y_1), (x_1,y_1))$ are in R where

$x_1, y_1 \in N$ $\therefore$ R is reflexive

$((1,1), (2,3)) \in R$ but $((2,3), (1,1)) \notin R$

$\therefore$ R is not symmetric

$((2,4), (3,3)) \in R$ and $((3,3), (1,3)) \in R$ but $((2,4), (1,3)) \notin R$

$\therefore$ R is not transitive

5. Let three real numbers a,b,c be in arithmetic progression and a + 1, b, c + 3 be in geometric progression. If a > 10 and the arithmetic mean of a,b and c is 8, then the cube of the geometric mean of a,b and c is

(1) 120 (2) 312

(3) 316 (4) 128

Ans. (1)

Sol. $2b = a + c$, $b^2 = (a + 1)(c + 3)$,

$$\frac{a+b+c}{3}=8 \rightarrow b=8, a+c=16$$

$64 = (a + 1)(19 - a) = 19 + 18a - a^2$

$a^2 - 18a - 45 = 0 \rightarrow (a - 15)(a + 3) = 0, (a > 10)$

$a = 15, c = 1, b = 8$

$((abc)^{1/3})^3 = abc = 120$

6. Let $A=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ and $B = I + adj(A) + (adj\ A)^2 + \ldots + (adj\ A)^{10}$. Then, the sum of all the elements of the matrix B is :

(1) –110 (2) 22

(3) –88 (4) –124

Ans. (3)

Sol. $Adj(A)=\begin{bmatrix}1 & -2\\ 0 & 1\end{bmatrix}$

$(AdjA)^2=\begin{bmatrix}1 & -4\\ 0 & 1\end{bmatrix}$

|

|

$(AdjA)^{10}=\begin{bmatrix}1 & -20\\ 0 & 1\end{bmatrix}$

$$B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}1 & -2\\ 0 & 1\end{bmatrix}+\begin{bmatrix}1 & -4\\ 0 & 1\end{bmatrix}+\ldots+\begin{bmatrix}1 & -20\\ 0 & 1\end{bmatrix}$$

$B=\begin{bmatrix}11 & -110\\ 0 & 11\end{bmatrix} \Rightarrow$ sum of elements of B

$= -88$

7. The value of $\dfrac{1\times2^2+2\times3^2+\ldots+100\times(101)^2}{1^2\times2+2^2\times3+\ldots+100^2\times101}$ is

(1) $\frac{306}{305}$ (2) $\frac{305}{301}$

(3) $\frac{32}{31}$ (4) $\frac{31}{30}$

Ans. (2)

Sol. $$\frac{1\times2^2+2\times3^2+\ldots+100\times(101)^2}{1^2\times2+2^2\times3+\ldots+100^2\times101}=\frac{\sum_{r=1}^{100}r(r+1)^2}{\sum_{r=1}^{100}r^2(r+1)}$$

$$=\frac{\sum_{r=1}^{100}(r^3+2r^2+r)}{\sum_{r=1}^{100}(r^3+r^2)}=\frac{\left(\frac{n(n+1)}{2}\right)^2+\frac{2.n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}}{\left(\frac{n(n+1)}{2}\right)^2+\frac{n(n+1)(2n+1)}{6}}$$

$$=\frac{\frac{n(n+1)}{2}\left[\frac{n(n+1)}{2}+\frac{2}{3}.(2n+1)+1\right]}{\frac{n(n+1)}{2}\left[\frac{n(n+1)}{2}+\frac{(2n+1)}{3}\right]};\text{Put } n = 100$$

$$=\frac{\frac{100(101)}{2}+\frac{2}{3}(201)+1}{\frac{100\times101}{2}+\frac{201}{3}}=\frac{5185}{5117}=\frac{305}{301}$$

8. Let $f(x)=\int_0^x \left(t+\sin\left(1-e^t\right)\right)dt, x\in\mathbb{R}$.

Then $\lim\limits_{x\to 0}\frac{f(x)}{x^3}$ is equal to

(1) $\frac{1}{6}$ (2) $-\frac{1}{6}$

(3) $-\frac{2}{3}$ (4) $\frac{2}{3}$

Ans. (2)

Sol. $\lim\limits_{x\to 0}\frac{f(x)}{x^3}$

Using L Hopital Rule.

$$\lim_{x\to 0}\frac{f'(x)}{3x^2}=\lim_{x\to 0}\frac{x+\sin\left(1-e^x\right)}{3x^2} \text{ (Again L Hopital)}$$

Using L.H. Rule

$$=\lim_{x\to 0}\frac{-\left[\sin\left(1-e^x\right)\left(-e^x\right).e^x+\cos\left(1-e^x\right).e^x\right]}{6}$$

$$=-\frac{1}{6}$$

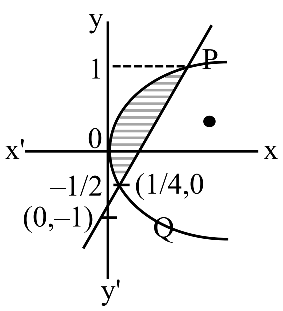
9. The area (in sq. units) of the region described by $\{(x,y) : y^2 \le 2x, \text{ and } y \ge 4x-1\}$ is

(1) $\frac{11}{32}$ (2) $\frac{8}{9}$

(3) $\frac{11}{12}$ (4) $\frac{9}{32}$

Ans. (4)

Sol.



Shaded area $=\int_{-\frac{1}{2}}^{1}\left(x_{Right}-x_{Left}\right)dy$

$$\left|\begin{array}{l} y^2=2x \\ y=4x-1 \quad\quad \text{Solve} \\ y=1, y=-\frac{1}{2}\end{array}\right.$$

Shaded area $=\int_{-\frac{1}{2}}^{1}\left(\frac{y+1}{4}-\frac{y^2}{2}\right)dy$

$$=\left(\frac{1}{4}\left(\frac{y^2}{2}+y\right)-\frac{y^3}{6}\right)_{-\frac{1}{2}}^{1}=\frac{9}{32}$$

10. The area (in sq. units) of the region $S=\left\{z\in\mathbb{C}; |z-1|\le 2; (z+\bar{z})+i(z-\bar{z})\le 2, \text{Im}(z)\ge 0\right\}$ is

(1) $\frac{7\pi}{3}$ (2) $\frac{3\pi}{2}$

(3) $\frac{17\pi}{8}$ (4) $\frac{7\pi}{4}$

Ans. (2)

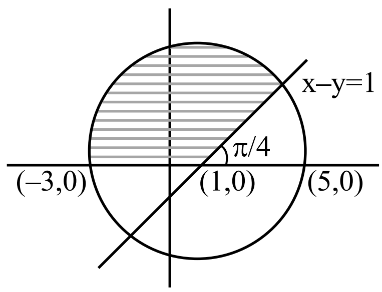
Sol. Put z = x + iy

$|z-1|\le 2 \Rightarrow (x-1)^2+y^2\le 4 \quad \ldots(1)$

$(z+\bar{z})+i(z-\bar{z})\le 2 \Rightarrow 2x+i(2iy)\le 2$

$\Rightarrow x-y\le 1 \quad \ldots(2)$

$\text{Im}(z)\ge 0 \Rightarrow y\ge 0 \quad \ldots(3)$



Required area

= Area of semi-circle – area of sector A

$\frac{1}{2}\pi(2)^2-\frac{\pi}{2}$

$=\frac{3\pi}{2}$

11. If the value of the integral $\int_{-1}^{1} \frac{\cos\alpha x}{1+3^x}dx$ is $\frac{2}{\pi}$.

Then, a value of α is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Ans. (2)

Sol. Let $I = \int_{-1}^{+1} \frac{\cos\alpha x}{1+3^x}dx$ …(I)

$$I = \int_{-1}^{+1} \frac{\cos\alpha x}{1+3^{-x}}dx$$

$$\left(\text{using} \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right) \quad \text{…(II)}$$

Add (1) and (II)

$$2I = \int_{-1}^{+1} \cos(\alpha x)dx = 2\int_0^1 \cos(\alpha x)dx$$

$$I = \frac{\sin\alpha}{\alpha} = \frac{2}{\pi}(\text{given})$$

$$\therefore \ \alpha = \frac{\pi}{2}$$

12. Let $f(x) = 3\sqrt{x-2} + \sqrt{4-x}$ be a real valued function. If α and β are respectively the minimum and the maximum values of f, then $\alpha^2 + 2\beta^2$ is equal to

(1) 44 (2) 42

(3) 24 (4) 38

Ans. (2)

Sol. $f(x) = 3\sqrt{x-2} + \sqrt{4-x}$

$x-2 \geq 0 \ \& \ 4-x \geq 0$

$\therefore x \in [2, 4]$

Let $x = 2\sin^2\theta + 4\cos^2\theta$

$\therefore f(x) = 3\sqrt{2}|\cos\theta| + \sqrt{2}|\sin\theta|$

$\therefore \sqrt{2} \leq 3\sqrt{2}|\cos\theta| + \sqrt{2}|\sin\theta| \leq \sqrt{9\times2+2}$

$\sqrt{2} \leq 3\sqrt{2}|\cos\theta| + \sqrt{2}|\sin\theta| \leq \sqrt{20}$

$\therefore \alpha = \sqrt{2} \quad \beta = \sqrt{20}$

$\alpha^2 + 2\beta^2 = 2 + 40 = 42$

13. If the coefficients of x^4, x^5 and x^6 in the expansion of $(1 + x)^n$ are in the arithmetic progression, then the maximum value of n is :

(1) 14 (2) 21

(3) 28 (4) 7

Ans. (1)

Sol. Coeff. of $x^4 = {}^nC_4$

Coeff. of $x^5 = {}^nC_5$

Coeff. of $x^6 = {}^nC_6$

${}^nC_4, {}^nC_5, {}^nC_6$ …. AP

$2.{}^nC_5 = {}^nC_4 + {}^nC_6$

$$2 = \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5} \quad \left\{\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right\}$$

$$2 = \frac{5}{n-4} + \frac{n-5}{6}$$

$12(n-4) = 30 + n^2 - 9n + 20$

$n^2 - 21n + 98 = 0$

$(n-14)(n-7) = 0$

$n_{max} = 14 \qquad n_{min} = 7$

14. Consider a hyperbola H having centre at the origin and foci and the x-axis. Let C_1 be the circle touching the hyperbola H and having the centre at the origin. Let C_2 be the circle touching the hyperbola H at its vertex and having the centre at one of its foci. If areas (in sq. units) of C_1 and C_2 are 36π and 4π, respectively, then the length (in units) of latus rectum of H is

(1) $\frac{28}{3}$ (2) $\frac{14}{3}$

(3) $\frac{10}{3}$ (4) $\frac{11}{3}$

Ans. (1)

Sol. Let H : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ $(b^2 = a^2(e^2 - 1))$

$\therefore$ eq^n of $C_1 = x^2 + y^2 = a^2$

Ar. = 36π

$\pi a^2 = 36\pi$

a = 6

Now radius of C_2 can be a(e – 1) or a(e + 1)

for r = a(e – 1)	for r = a(e + 1)
Ar. = 4π	$\pi r^2 = 4\pi$
$\pi a^2(e-1)^2 = 4\pi$	$a^2(e+1)^2 = 4$
$36\pi(e-1)^2 = 4\pi$	$36(e+1)^2 = 4$
$e-1=\frac{1}{3}$	$e+1=\frac{1}{3}$
$e=\frac{4}{3}$	$-\frac{2}{3}$
	Not possible

$\therefore b^2 = 36\left(\frac{16}{9}-1\right) = 28$

$\therefore LR = \frac{2b^2}{a}=\frac{2\times 28}{6}=\frac{28}{3}$

15. If the mean of the following probability distribution of a random variable X;

X	0	2	4	6	8
P(X)	a	2a	a + b	2b	3b

is $\frac{46}{9}$, then the variance of the distribution is

(1) $\frac{581}{81}$ (2) $\frac{566}{81}$

(3) $\frac{173}{27}$ (4) $\frac{151}{27}$

Ans. (2)

Sol. $\sum P_i = 1$

a + 2a + a + b + 2b + 3b = 1

4a + 6b = 1 …. (I)

E(x) = mean = $\frac{46}{9}$

$\sum P_i X_i = \frac{46}{9} \Rightarrow 4a + 4a + 4b + 12b + 24b = \frac{46}{9}$

$8a + 40b = \frac{46}{9}$

$4a + 20b = \frac{23}{9}$ …. (II)

Subtract (I) from (II) we get

$b = \frac{1}{9}$ & $a = \frac{1}{12}$

Variance = $E(x_i^2) - E(x_i)^2$

$E(x_i^2) = 0^2 \times 9^2 + 2^2 \times 2a + 4^2(a + b) + 6^2(2b) + 8^2(3b)$

= 24a + 280b

Put $a = \frac{1}{12}$ $b = \frac{1}{9}$

$E(x_i^2) = 2 + \frac{280}{9} = \frac{298}{9}$

$\therefore \sigma^2 = E(x_i^2) - E(x_i)^2$

$=\frac{298}{9}-\left(\frac{46}{9}\right)^2$

$\sigma^2 = \frac{298}{9}-\frac{2116}{81}$

$=\frac{566}{81}$

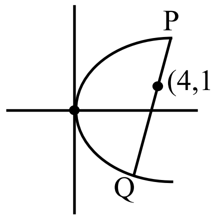
16. Let PQ be a chord of the parabola $y^2 = 12x$ and the midpoint of PQ be at (4,1). Then, which of the following point lies on the line passing through the points P and Q ?

(1) (3,–3) (2) $\left(\frac{3}{2},-16\right)$

(3) (2,–9) (4) $\left(\frac{1}{2},-20\right)$

Ans. (4)

Sol.



$T = S_1$

y – 6(x + 4)

= 1 – 48

6x – y = 23

Option 4 $\left(\frac{1}{2},-20\right)$ will satisfy

17. Given the inverse trigonometric function assumes principal values only. Let x, y be any two real numbers in [–1,1] such that

$\cos^{-1}x - \sin^{-1}y = \alpha, \frac{-\pi}{2} \le \alpha \le \pi$.

Then, the minimum value of $x^2 + y^2 + 2xy \sin\alpha$ is

(1) –1 (2) 0

(3) $\frac{-1}{2}$ (4) $\frac{1}{2}$

Ans. (2)

Sol. $\cos^{-1}x - \left(\frac{\pi}{2} - \cos^{-1}y\right) = \alpha$

$\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2} + \alpha$

$\alpha \in \left[-\frac{\pi}{2}, \pi\right], \frac{\pi}{2} + \alpha \in \left[0, \frac{3\pi}{2}\right]$

$\cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right) = \frac{\pi}{2} + \alpha$

$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -\sin\alpha$

$(xy + \sin\alpha) = (1 - x^2)(1 - y^2)$

$x^2y^2 + 2xy\sin a + \sin^2 a = 1 - x^2 - y^2 + x^2y^2$

$x^2 + y^2 + 2xy \sin\alpha = 1 - \sin^2\alpha$

$x^2 + y^2 + 2xy\sin\alpha = \cos^2\alpha$

Min. value of $\cos^2\alpha = 0$

At $\boxed{\alpha = \frac{\pi}{2}}$

Option (2) is correct

18. Let y = y(x) be the solution of the differential equation

$(x^2 + 4)^2dy + (2x^3y + 8xy - 2)dx = 0$. If y(0) = 0, then y(2) is equal to

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{16}$

(3) 2π (4) $\frac{\pi}{32}$

Ans. (4)

Sol. $\frac{dy}{dx} + y\left(\frac{2x^3 + 8x}{(x^2+4)^2}\right) = \frac{2}{(x^2+4)^2}$

$\frac{dy}{dx} + y\left(\frac{2x}{x^2+4}\right) = \frac{2}{(x^2+4)^2}$

$IF = e^{\int \frac{2x}{x^2+4} dx}$

$IF = x^2 + 4$

$y \times (x^2 + 4) = \int \frac{2}{(x^2+4)^2} \times (x^2+4)$

$y(x^2 + 4) = 2\int \frac{dx}{x^2 + 2^2}$

$y(x^2+4) = \frac{2}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

$0 = 0 + c = c = 0$

$y(x^2 + 4) = \tan^{-1}\left(\frac{x}{2}\right)$

y at x = 2

$y(4 + 4) = \tan^{-1}(1)$

$\boxed{y(2) = \frac{\pi}{32}}$

Option (4) is correct

19. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = x\hat{i} + 2\hat{j} + 3\hat{k}, x \in \mathbb{R}$. If $\vec{d}$ is the unit vector in the direction of $\vec{b} + \vec{c}$ such that $\vec{a}.\vec{d} = 1$, then $(\vec{a} \times \vec{b}).\vec{c}$ is equal to

(1) 9 (2) 6

(3) 3 (4) 11

Ans. (4)

Sol. $\vec{d} = \lambda\left(\vec{b}+\vec{c}\right)$

$\vec{a}.\vec{d} = \lambda\left(\vec{b}.\vec{a}+\vec{c}.\vec{a}\right)$

$1 = \lambda(1 + x + 5)$

$1 = \lambda(x + 6)$ ….(1)

$\left|\vec{d}\right| = 1$ $\boxed{\frac{1}{\lambda} = x + 6}$

$\left|\lambda\left(\vec{b}+\vec{c}\right)\right| = 1$

$\left|\lambda\left((x+2)\hat{i}+6\hat{j}-2\hat{k}\right)\right| = 1$

$\lambda^2((x + 2)^2 + 6^2 + 2^2) = 1$

$x^2 + 4x + 4 + 36 + 4 = (x + 6)^2$

$x^2 + 4x + 44 = x^2 + 12x + 36$

$8x = 8, x = 1$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -5 \\ x & 2 & 3 \end{vmatrix} = \left(\vec{a}\times\vec{b}\right).\vec{c}$$

$$\begin{vmatrix} 0 & 0 & 1 \\ -2 & 9 & -4 \\ x-2 & -1 & 3 \end{vmatrix} = 2 - 9(x - 2)$$

$= 20 - 9x$

at $x = 1$

$20 - 9 = 11$

Option 4 is correct

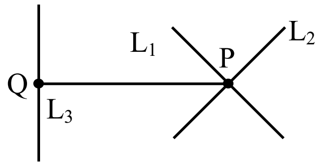
20. Let P the point of intersection of the lines $\frac{x-2}{1}=\frac{y-4}{5}=\frac{z-2}{1}$ and $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-3}{2}$.

Then, the shortest distance of P from the line $4x = 2y = z$ is

(1) $\frac{5\sqrt{14}}{7}$ (2) $\frac{\sqrt{14}}{7}$

(3) $\frac{3\sqrt{14}}{7}$ (4) $\frac{6\sqrt{14}}{7}$

Ans. (3)



$L_1 \equiv \frac{x-2}{1}=\frac{y-4}{5}=\frac{z-2}{1}=\lambda$

$P\left(\lambda+2, 5\lambda+4, \lambda+2\right)$

$L_2 \equiv \frac{x-3}{2}=\frac{y-2}{3}=\frac{z-3}{2}$

$P\left(2\mu+3, 3\mu+2, 2\mu+3\right)$

$\lambda + 2 = 2\mu + 3$ $3\mu + 2 = 5\lambda + 4$

$\lambda = 2\mu + 1$ $3\mu = 5\lambda + 2$

$3\mu = 5(2\mu + 1) + 2$

$3\mu = 10\mu + 7$

$\mu = -1$ $\lambda = -1$

Both satisfies (P)

$P(1,-1,1)$

$L_3 \equiv \frac{x}{1/4}=\frac{y}{1/2}=\frac{z}{1}$

$L_3 = \frac{x}{1}=\frac{y}{2}=\frac{z}{4}=k$

Coordinates of $Q(k,2k,4k)$

DR's of PQ = $<k-1, 2k + 1, 4k - 1>$

$PQ \perp$ to L_3

$(k - 1) + 2(2k + 1) + 4(4k - 1) = 0$

$k - 1 + 4k + 2 + 16k - 4 = 0$

$k = \frac{1}{7}$

$Q\left(\frac{1}{7},\frac{2}{7},\frac{4}{7}\right)$

$$PQ = \sqrt{\left(1-\frac{1}{7}\right)^2+\left(-1-\frac{2}{7}\right)^2+\left(1-\frac{4}{7}\right)^2}$$

$$= \sqrt{\frac{36}{49}+\frac{81}{49}+\frac{9}{49}} = \frac{\sqrt{126}}{7}$$

$PQ = \frac{3\sqrt{14}}{7}$

Option-3 will satisfy

SECTION-B

21. Let S = {$\sin^2 2\theta$: $(\sin^4\theta + \cos^4\theta)x^2 + (\sin 2\theta)x + (\sin^6\theta + \cos^6\theta) = 0$ has real roots}. If α and β be the smallest and largest elements of the set S, respectively, then $3((\alpha - 2)^2 + (\beta - 1)^2)$ equals…..

Ans. (4)

Sol. $D = (\sin 2\theta)^2 - 4\left(1 - \frac{\sin^2 2\theta}{2}\right)\left(1 - \frac{3}{4}\sin^2 2\theta\right)$

$= (\sin 2\theta)^2 - 4\left(1 - \frac{5}{4}\sin^2 2\theta + \frac{3}{8}\sin^4 2\theta\right)$

$D = -\frac{3}{2}\sin^4 2\theta + 6\sin^2 2\theta - 4 > 0$

$3\sin^4 2\theta - 12\sin^2 2\theta + 8 < 0$

$\sin^2 2\theta = \frac{12 \pm \sqrt{12^2 - 12.8}}{6} = \frac{12 \pm 4\sqrt{3}}{6} = \frac{6 \pm 2\sqrt{3}}{3}$

$\sin^2 2\theta = 2 \pm \frac{2}{\sqrt{3}}$, but $\sin^2 2\theta \in [0, 1]$

$\therefore\ \alpha = 2 - \frac{2}{\sqrt{3}}, \beta = 1 \rightarrow (\alpha - 2)^2 = \frac{4}{3}, (\beta - 1)^2 = 0$

$\boxed{3(\alpha - 2)^2 + (\beta - 1)^2 = 4}$

22. If $\int \operatorname{cosec}^5 x\,dx = \alpha \cot x \operatorname{cosec} x\left(\operatorname{cosec}^2 x + \frac{3}{2}\right) + \beta \log_e\left|\tan\frac{x}{2}\right| + C$

where $\alpha, \beta \in \mathbb{R}$ and C is constant of integration, then the value of $8(\alpha + \beta)$ equals …..

Ans. (1)

Sol. $\int \operatorname{cosec}^3 x \cdot \operatorname{cosec}^2 x\,dx = I$

By applying integration by parts

$I = -\cot x \operatorname{cosec}^3 x + \int \cot x\left(-3\operatorname{cosec}^2 x \cot x \operatorname{cosec} x\right)dx$

$I = -\cot x \operatorname{cosec}^3 x - 3\int \operatorname{cosec}^3 x\left(\operatorname{cosec}^2 x - 1\right)dx$

$I = -\cot x \operatorname{cosec}^3 x - 3I + 3\int \operatorname{cosec}^3 x\,dx$

let

$I_1 = \int \operatorname{cosec}^3 x\,dx = -\operatorname{cosec} x \cot x - \int \cot^2 x \operatorname{cosec} x\,dx$

$I_1 = -\operatorname{cosec} x \cot x - \int\left(\operatorname{cosec}^2 x - 1\right)\operatorname{cosec} x\,dx$

$2I_1 = -\operatorname{cosec} x \cot x + \ell n\left|\tan\frac{x}{2}\right|$

$I_1 = -\frac{1}{2}\operatorname{cosec} x \cot x + \frac{1}{2}\ell n\left|\tan\frac{x}{2}\right|$

$4I = -\cot x \operatorname{cosec}^3 x - \frac{3}{2}\operatorname{cosec} x \cot x + \frac{3}{2}\ell n\left|\tan\frac{x}{2}\right| + 4c$

$I = -\frac{1}{4}\operatorname{cosec} x \cot x\left(\operatorname{cosec}^2 x + \frac{3}{2}\right) + \frac{3}{8}\ell n\left|\tan\frac{x}{2}\right| + c$

$\therefore\ \alpha = \frac{-1}{4}, \beta = \frac{3}{8} \rightarrow \boxed{8(\alpha + \beta) = 1}$

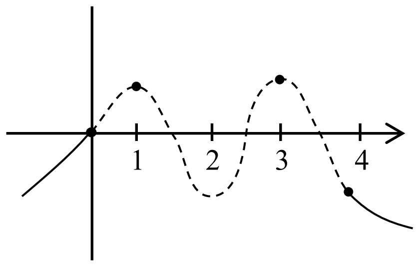
23. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a thrice differentiable function such that f(0) = 0, f(1) = 1, f(2) = –1, f(3) = 2 and f(4) = –2. Then, the minimum number of zeros of (3f' f'' + ff''') (x) is …..

Ans. (5)

Sol. $(3f'f'' + ff''')(x) = \left(\left(ff'' + (f')^2\right)(x)\right)'$

$\left(ff'' + (f')^2\right)(x) = ((ff')(x))'$

$\therefore\ (3f'f'' + f''')(x) = (f(x) \cdot f'(x))''$



min. roots of f(x) $\rightarrow$ 4

$\therefore$ min. roots of f '(x) $\rightarrow$ 3

$\therefore$ min. roots of $(f(x) \cdot f'(x)) \rightarrow 7$

$\boxed{\therefore\ \text{min. roots of } (f(x) \cdot f'(x))'' \rightarrow 5}$

24. Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by

$f(x) = \frac{2x}{\sqrt{1 + 9x^2}}$. If the composition of

$f, \underbrace{(f \circ f \circ f \circ ... \circ f)}_{10\text{ times}}(x) = \frac{2^{10}x}{\sqrt{1 + 9\alpha x^2}}$, then the value of $\sqrt{3\alpha + 1}$ is equal to …..

Ans. (1024)

Sol. $f(f(x)) = \frac{2f(x)}{\sqrt{1+9f^2(x)}} = \frac{4x}{\sqrt{1+9x^2+9.2^2x^2}}$

$f(f(f(x))) = \frac{2^3x/\sqrt{1+9x^2}}{\sqrt{1+9(1+2^2)\frac{2^2x^2}{1+9x^2}}} = \frac{2^3x}{\sqrt{1+9x^2(1+2^2+2^4)}}$

$\therefore$ By observation

$\alpha = 1+2^2+2^4+....+2^{18} = 1\left(\frac{(2^2)^{10}-1}{2^2-1}\right) = \frac{2^{20}-1}{3}$

$3\alpha+1 = 2^{20} \rightarrow \sqrt{3\alpha+1} = 2^{10} = \boxed{1024}$

25. Let A be a 2 × 2 symmetric matrix such that $A\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\7\end{bmatrix}$ and the determinant of A be 1.

If $A^{-1} = \alpha A + \beta I$, where I is an identity matrix of order 2 × 2, then $\alpha + \beta$ equals …..

Ans. (5)

Sol. Let $A = \begin{bmatrix}a & b\\b & d\end{bmatrix}$

$\begin{bmatrix}a & b\\b & d\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\7\end{bmatrix}, ad-b^2=1$

$a + b = 3, b + d = 7, (3 - b)(7 - b) - b^2 = 1$

$21 - 10b = 1 \rightarrow b = 2, a = 1, d = 5$

$A = \begin{bmatrix}1 & 2\\2 & 5\end{bmatrix}, A^{-1} = \begin{bmatrix}5 & -2\\-2 & 1\end{bmatrix}$

$A^{-1} = \alpha A + \beta I$

$\begin{bmatrix}5 & -2\\-2 & 1\end{bmatrix} = \begin{bmatrix}\alpha+\beta & 2\alpha\\2\alpha & 5\alpha+\beta\end{bmatrix}$

$\alpha = -1, \beta = 6 \rightarrow \boxed{\alpha+\beta=5}$

26. There are 4 men and 5 women in Group A, and 5 men and 4 women in Group B. If 4 persons are selected from each group, then the number of ways of selecting 4 men and 4 women is …..

Ans. (5626)

Sol.

From Group A	From Group B	Ways of selection
4M	4W	${}^4C_4\,{}^4C_4 = 1$
3M 1W	1M 3W	${}^4C_3\,{}^5C_1\,{}^5C_1\,{}^4C_3 = 400$
2M 2W	2M 2W	${}^4C_2\,{}^5C_2\,{}^5C_2\,{}^4C_2 = 3600$
1M 3W	3M 1W	${}^4C_1\,{}^5C_3\,{}^5C_3\,{}^4C_1 = 1600$
4W	4M	${}^5C_4\,{}^5C_4 = 25$
Total		5626

Ans. 5626

27. In a tournament, a team plays 10 matches with probabilities of winning and losing each match as $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Let x be the number of matches that the team wins, and y be the number of matches that team loses. If the probability $P(|x-y| \le 2)$ is p, then 3^9p equals……

Ans. (8288)

Sol. $P(W) = \frac{1}{3}$ $\qquad$ $P(L) = \frac{2}{3}$

x = number of matches that team wins

y = number of matches that team loses

$|x-y| \le 2$ and x + y = 10

$|x-y| = 0, 1, 2 \quad x, y \in N$

Case-I : $|x-y| = 0 \Rightarrow x = y$

$\because\ x+y=10 \Rightarrow x=5=y$

$P(|x-y|=0) = {}^{10}C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^5$

Case-II : $|x-y| = 1 \Rightarrow x-y = \pm 1$

$x = y + 1$	$x = y - 1$
$\because\ x+y=10$	$\because\ x+y=10$
$2y = 9$	$2y = 11$
Not possible	Not possible

Case-III : $|x-y|=2 \Rightarrow x-y=\pm 2$

$x-y=2$ OR $x-y=-2$

$\because\ x+y=10$ $\quad\because\ x+y=10$

$x=6, y=4$ $\quad x=4, y=6$

$$P(|x-y|=2) = {}^{10}C_6\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^4 + {}^{10}C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^6$$

$$p = {}^{10}C_5\frac{2^5}{3^{10}} + {}^{10}C_6\frac{2^4}{3^{10}} + {}^{10}C_4\frac{2^6}{3^{10}}$$

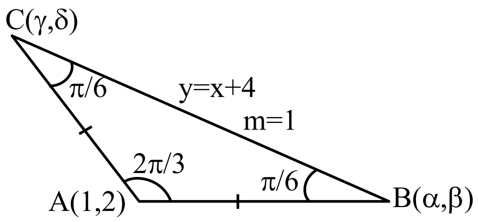
$$3^9 p = \frac{1}{3}\left({}^{10}C_5 2^5 + {}^{10}C_6 2^4 + {}^{10}C_4 2^6\right)$$

= 8288

28. Consider a triangle ABC having the vertices A(1,2), B(α,β) and C(γ,δ) and angles $\angle ABC = \frac{\pi}{6}$ and $\angle BAC = \frac{2\pi}{3}$. If the points B and C lie on the line y = x + 4, then $\alpha^2 + \gamma^2$ is equal to .....

Ans. (14)

Sol.



Equation of line passes through point A(1, 2) which makes angle $\frac{\pi}{6}$ from y = x + 4 is

$$y-2 = \frac{1\pm\tan\frac{\pi}{6}}{1\mp\tan\frac{\pi}{6}}(x-1)$$

$$y-2 = \frac{\sqrt{3}\pm 1}{\sqrt{3}\mp 1}(x-1)$$

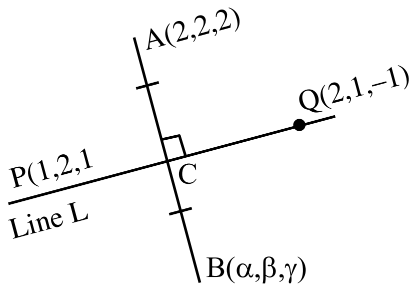
⊕	⊖
$y-2=(2+\sqrt{3})(x-1)$	$y-2=(2-\sqrt{3})(x-1)$
solve with $y=x+4$	solve with $y=x+4$
$x+2=(2+\sqrt{3})x-2-\sqrt{3}$	$x+2=(2-\sqrt{3})x-2+\sqrt{3}$
$x=\frac{4+\sqrt{3}}{1+\sqrt{3}}$	$x=\frac{4-\sqrt{3}}{1-\sqrt{3}}$

$$\alpha^2+\gamma^2 = \left(\frac{4+\sqrt{3}}{1+\sqrt{3}}\right)^2 + \left(\frac{4-\sqrt{3}}{1-\sqrt{3}}\right)^2$$

$$\alpha^2+\gamma^2 = 14$$

29. Consider a line L passing through the points P(1,2,1) and Q(2,1,–1). If the mirror image of the point A(2,2,2) in the line L is (α,β,γ), then α + β + 6γ is equal to .....

Ans. (6)



DR's of Line L ≡ –1 : 1 : 2

DR's of AB ≡ α – 2 : β – 2 : γ – 2

$AB \perp_{ar} L \Rightarrow 2-\alpha+\beta-2+2\gamma-4=0$

$2\gamma+\beta-\alpha=4$ ...(1)

Let C is mid-point of AB

$$C\left(\frac{\alpha+2}{2},\frac{\beta+2}{2},\frac{\gamma+2}{2}\right)$$

DR's of PC = $\frac{\alpha}{2}:\frac{\beta-2}{2}:\frac{\gamma}{2}$

line L | | PC $\Rightarrow \frac{-\alpha}{2}=\frac{\beta-2}{2}=\frac{\gamma}{4}=K(\text{let})$

α = – 2K

β = 2K + 2

γ = 4K

use in (1) $\Rightarrow K=\frac{1}{6}$

value of α + β + 6γ = 24K + 2 = 6

30. Let $y = y(x)$ be the solution of the differential equation $(x + y + 2)^2\, dx = dy$, $y(0) = -2$. Let the maximum and minimum values of the function $y = y(x)$ in $\left[0, \frac{\pi}{3}\right]$ be α and β, respectively. If $(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta\sqrt{3}, \gamma, \delta \in \mathbb{Z}$, then $\gamma + \delta$ equals …..

Ans. (31)

Sol. $\frac{dy}{dx} = (x + y + 2)^2$ ...(1), $y(0) = -2$

Let $x + y + 2 = v$

$1 + \frac{dy}{dx} = \frac{dv}{dx}$

from (1) $\frac{dv}{dx} = 1 + v^2$

$\int \frac{dv}{1 + v^2} = \int dx$

$\tan^{-1}(v) = x + C$

$\tan^{-1}(x + y + 2) = x + C$

at $x = 0$ $y = -2 \Rightarrow C = 0$

$\Rightarrow \tan^{-1}(x + y + 2) = x$

$y = \tan x - x - 2$

$f(x) = \tan x - x - 2, x \in \left[0, \frac{\pi}{3}\right]$

$f'(x) = \sec^2 x - 1 > 0 \Rightarrow f(x) \uparrow$

$f_{min} = f(0) = -2 = \beta$

$f_{max} = f\left(\frac{\pi}{3}\right) = \sqrt{3} - \frac{\pi}{3} - 2 = \alpha$

now $(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta\sqrt{3}$

$\Rightarrow (3\alpha + \pi)^2 + \beta^2 = (3\sqrt{3} - 6)^2 + 4$

$\gamma + \delta\sqrt{3} = 67 - 36\sqrt{3}$

$\Rightarrow \gamma = 67$ and $\delta = -36 \Rightarrow \gamma + \delta = 31$

SECTION-A

31. The translational degrees of freedom (f_t) and rotational degrees of freedom (f_r) of CH_4 molecule are :

(1) $f_t = 2$ and $f_r = 2$ (2) $f_t = 3$ and $f_r = 3$

(3) $f_t = 3$ and $f_r = 2$ (4) $f_t = 2$ and $f_r = 3$

Ans. **(2)**

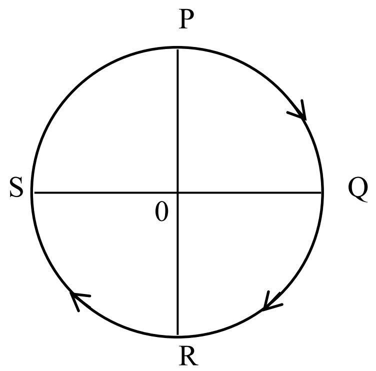
Sol. Since CH_4 is polyatomic Non-Linear

D.O.F of CH_4

T. DOF = 3

R DOF = 3

32. A cyclist starts from the point P of a circular ground of radius 2 km and travels along its circumference to the point S. The displacement of a cyclist is :

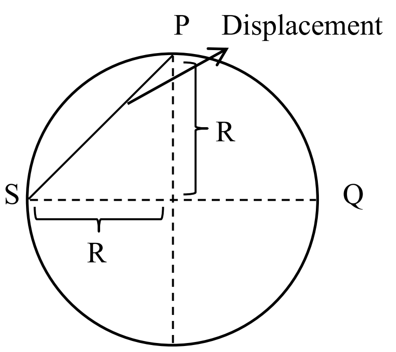


(1) 6 km (2) $\sqrt{8}$ km

(3) 4 km (4) 8 km

Ans. **(2)**

Sol.



$\therefore$ Displacement = $R\sqrt{2} = 2\sqrt{2} = \sqrt{8}$ km

33. The magnetic moment of a bar magnet is 0.5 Am^2. It is suspended in a uniform magnetic field of 8×10^{-2} T. The work done in rotating it from its most stable to most unstable position is :

(1) 16×10^{-2} J (2) 8×10^{-2} J

(3) 4×10^{-2} J (4) Zero

Ans. **(2)**

Sol. At stable equilibrium

$U = -mB \cos 0° = -mB$

At unstable equilibrium

$U = -mB \cos 180° = + mB$

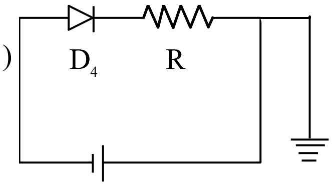
$W = \Delta U$

W.D. = 2 mB

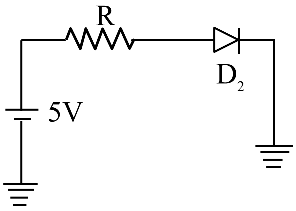
$= 2\,(0.5)\, 8 \times 10^{-2} = 8 \times 10^{-2}$ J

34. Which of the diode circuit shows correct biasing used for the measurement of dynamic resistance of p-n junction diode :

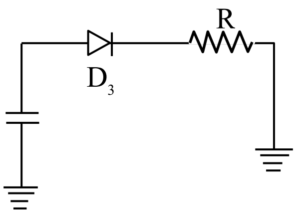
(1)



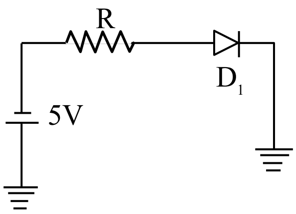
(2)



(3)



(4)



Ans. **(2)**

Sol. Diode should be in forward biased to calculate dynamic resistance

Hence correct answer would be 2.

35. Arrange the following in the ascending order of wavelength :

(A) Gamma rays (λ_1) (B) x-ray (λ_2)

(C) Infrared waves (λ_3) (D) Microwaves (λ_4)

Choose the most appropriate answer from the options given below :

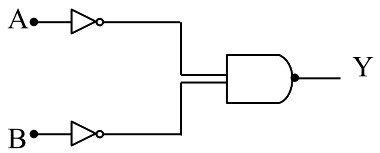
(1) $\lambda_4 < \lambda_3 < \lambda_1 < \lambda_2$ (2) $\lambda_4 < \lambda_3 < \lambda_2 < \lambda_1$

(3) $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$ (4) $\lambda_2 < \lambda_1 < \lambda_4 < \lambda_3$

Ans. **(3)**

Sol. $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$

36. Identify the logic gate given in the circuit :



(1) NAND - gate (2) OR - gate

(3) AND gate (4) NOR gate

Ans. **(2)**

Sol. $Y = \overline{\overline{A}.\overline{B}}$

By De-Morgan Law

$Y = \overline{\overline{A+B}}$

$Y = A + B$

Hence OR gate

37. The width of one of the two slits in a Young's double slit experiment is 4 times that of the other slit. The ratio of the maximum of the minimum intensity in the interference pattern is :

(1) 9 :1 (2) 16 : 1

(3) 1 : 1 (4) 4 : 1

Ans. **(1)**

Sol. Since, Intensity $\propto$ width of slit (ω)

so, $I_1 = I$, $I_2 = 4I$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = I$$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 9I$$

$$\frac{I_{max}}{I_{min}} = \frac{9I}{I} = \frac{9}{1}$$

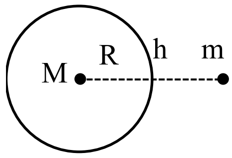
38. Correct formula for height of a satellite from earths surface is :

(1) $\left(\frac{T^2R^2g}{4\pi}\right)^{1/2} - R$ (2) $\left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R$

(3) $\left(\frac{T^2R^2}{4\pi^2 g}\right)^{1/3} - R$ (4) $\left(\frac{T^2R^2}{4\pi^2}\right)^{-1/3} + R$

Ans. **(2)**

Sol.



$$\Rightarrow \frac{GMm}{(R+h)^2} = \frac{mv^2}{(R+h)}$$

$$\Rightarrow \frac{GM}{(R+h)} = v^2 \text{ ....(1)}$$

$$\Rightarrow v = (R + h)\omega$$

$$\Rightarrow v = (R+h)\frac{2\pi}{T} \text{ ....(2)}$$

$$\Rightarrow \frac{GM}{R^2} = g$$

$$\Rightarrow GM = gR^2 \text{ ....(3)}$$

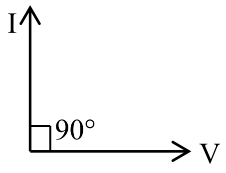
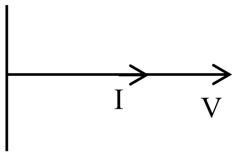
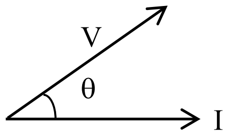
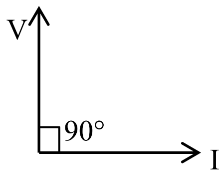
Put value from (2) & (3) in eq. (1)

$$\Rightarrow \frac{gR^2}{(R+h)} = (R+h)^2\left(\frac{2\pi}{T}\right)^2$$

$$\Rightarrow \frac{T^2R^2g}{(2\pi)^2} = (R+h)^3$$

$$\Rightarrow \left[\frac{T^2R^2g}{(2\pi)^2}\right]^{1/3} - R = h$$

39. Match List I with List II

	List–I		List–II
A.	Purely capacitive circuit	I.	
B.	Purely inductive circuit	II.	
C.	LCR series at resonance	III.	
D.	LCR series circuit	IV.	

Choose the correct answer from the options given below :

(1) A-I, B-IV, C-III, D-II

(2) A-IV, B-I, C-III, D-II

(3) A-IV, B-I, C-II, D-III

(4) A-I, B-IV, C-II, D-III

Ans. **(4)**

Sol. **A** – V lags by 90° from I hence option (I) is correct.

B – V lead by 90° from I hence option (IV) is correct

C – In LCR resonance $X_L = X_C$. Hence circuit is purely resistive so option (II) is correct

D – In LCR series V is at some angle from I hence (III) is correct

Hence option (4) is correct.

40. Given below are two statements :

Statement I : The contact angle between a solid and a liquid is a property of the material of the solid and liquid as well.

Statement II : The rise of a liquid in a capillary tube does not depend on the inner radius of the tube.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true.

(3) Statement I is true but Statement II is false.

(4) Both Statement I and Statement II are true.

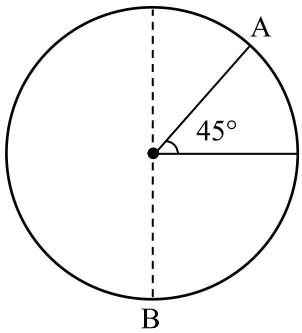
Ans. **(3)**

Sol. Statement I is correct as we know contact angle depends on cohesine and adhesive forces.

Statement II is incorrect because height of liquid is given by h = $= \dfrac{2T\cos\theta_C}{\rho g r}$ where r is radius of Tube (assuming length of capillary is sufficient)

Hence option (3) is correct.

41. A body of m kg slides from rest along the curve of vertical circle from point A to B in friction less path. The velocity of the body at B is :



(Vertical Circle)

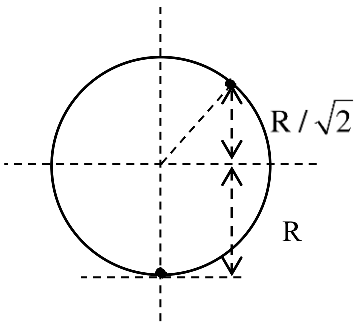
(given, R = 14 m, g = 10 m/s^2 and $\sqrt{2}$ = 1.4)

(1) 19.8 m/s (2) 21.9 m/s

(3) 16.7 m/s (4) 10.6 m/s

Ans. **(2)**

Sol.



Apply W.E.T. from A to B

$\Rightarrow W_{mg} = K_B - K_A$

$\Rightarrow mg \times \left(\frac{R}{\sqrt{2}} + R\right) = \frac{1}{2}mv_B^2 - 0 \ \{v_A = 0 \text{ rest}\}$

$\Rightarrow mgR\frac{\left(\sqrt{2}+1\right)}{\sqrt{2}} = \frac{1}{2}mv_B^2$

$\Rightarrow \sqrt{gR\frac{2\left(\sqrt{2}+1\right)}{\sqrt{2}}} = v_B$

$\Rightarrow \sqrt{\frac{10\times14\times2(2.4)}{1.4}} = v_B$

$\Rightarrow 21.9 = v_B$

Hence option (2) is correct

42. An electric bulb rated 50 W – 200 V is connected across a 100 V supply. The power dissipation of the bulb is :

(1) 12.5 W (2) 25 W

(3) 50 W (4) 100 W

Ans. (1)

Sol. Rated power & voltage gives resistance

$$R = \frac{V^2}{P} = \frac{(200)^2}{50} = \frac{40000}{50}$$

R = 800

$$P = \frac{\left(V_{applied}\right)^2}{R} = \frac{(100)^2}{800}$$

P = 12.5 watt

Hence option 1 is correct.

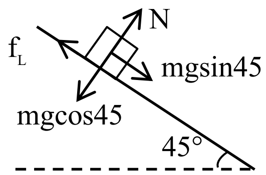
43. A 2 kg brick begins to slide over a surface which is inclined at an angle of 45° with respect to horizontal axis. The co-efficient of static friction between their surfaces is :

(1) 1 (2) $\frac{1}{\sqrt{3}}$

(3) 0.5 (4) 1.7

Ans. (1)

Sol.



mg sin 45 = f_L

mg cos 45 = N

$f_L = \mu_s N$

$\mu_s = \tan 45 = 1$

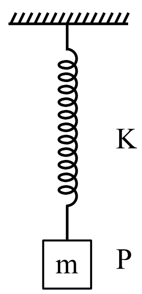
or

$\tan\theta = \mu_s$ (θ is angle of repose)

$\tan 45 = \mu_s = 1$

correct option (1)

44. In simple harmonic motion, the total mechanical energy of given system is E. If mass of oscillating particle P is doubled then the new energy of the system for same amplitude is :



(1) $\frac{E}{\sqrt{2}}$ (2) E

(3) $E\sqrt{2}$ (4) 2E

Ans. (2)

Sol. T.E. = $\frac{1}{2}kA^2$

since A is same T.E. will be same

correct option (2)

45. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Number of photons increases with increase in frequency of light.

Reason R : Maximum kinetic energy of emitted electrons increases with the frequency of incident radiation.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both **A** and **R** are correct and **R** is **NOT** the correct explanation of **A**.

(2) **A** is correct but **R** is not correct.

(3) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(4) **A** is not correct but **R** is correct.

Ans. **(4)**

Sol. Intensity of light $I = \frac{nh\nu}{A}$

Here n is no. of photons per unit time.

$n = \frac{IA}{h\nu}$ so on increasing frequency ν, n decreases taking intensity constant.

$k_{max} = h\nu - \phi$

So on increasing ν, kinetic energy increases.

46. According to Bohr's theory, the moment of momentum of an electron revolving in 4th orbit of hydrogen atom is :

(1) $8\frac{h}{\pi}$ (2) $\frac{h}{\pi}$

(3) $2\frac{h}{\pi}$ (4) $\frac{h}{2\pi}$

Ans. **(3)**

Sol. Moment of momentum is $\vec{r} \times \vec{P}$

$\vec{L} = \vec{r} \times m\vec{v}$

$L = mvr = \frac{nh}{2\pi} = \frac{4h}{2\pi} = \frac{2h}{\pi}$

47. A sample of gas at temperature T is adiabatically expanded to double its volume. Adiabatic constant for the gas is γ = 3/2. The work done by the gas in the process is : (μ = 1 mole)

(1) $RT\left[\sqrt{2}-2\right]$ (2) $RT\left[1-2\sqrt{2}\right]$

(3) $RT\left[2\sqrt{2}-1\right]$ (4) $RT\left[2-\sqrt{2}\right]$

Ans. **(4)**

Sol. $W = \frac{nR\Delta T}{1-\gamma}$

$TV^{\gamma-1} = \text{constant} = T_f(2V)^{\gamma-1}$

$T_f = T\left(\frac{1}{2}\right)^{1/2} = \frac{T}{\sqrt{2}}$

$W = \frac{R\left(\frac{T}{\sqrt{2}} - T\right)}{1-\frac{3}{2}} = 2RT\frac{\left(\sqrt{2}-1\right)}{\sqrt{2}}$

$= RT\left(2-\sqrt{2}\right)$

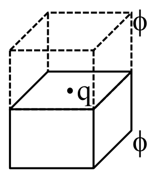
48. A charge q is placed at the center of one of the surface of a cube. The flux linked with the cube is :-

(1) $\frac{q}{4\in_0}$ (2) $\frac{q}{2\in_0}$

(3) $\frac{q}{8\in_0}$ (4) Zero

Ans. **(2)**

Sol. From



$2\phi = \frac{q}{\in_0}$

$\phi = \frac{q}{2\in_0}$

49. Applying the principle of homogeneity of dimensions, determine which one is correct.

where T is time period, G is gravitational constant, M is mass, r is radius of orbit.

(1) $T^2 = \frac{4\pi^2 r}{GM^2}$ (2) $T^2 = 4\pi^2 r^3$

(3) $T^2 = \frac{4\pi^2 r^3}{GM}$ (4) $T^2 = \frac{4\pi^2 r^2}{GM}$

Ans. (3)

Sol. According to principle of homogeneity dimension of LHS should be equal to dimensions of RHS so option (3) is correct.

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$\left[T^2\right] = \frac{\left[L^3\right]}{\left[M^{-1}L^3T^{-2}\right]\left[M\right]}$$

(Dimension of G is $\left[M^{-1}L^3T^{-2}\right]$)

$$\left[T^2\right] = \frac{\left[L^3\right]}{\left[L^3T^{-2}\right]} = \left[T^2\right]$$

50. A 90 kg body placed at 2R distance from surface of earth experiences gravitational pull of :

(R = Radius of earth, g = 10 ms^{-2})

(1) 300 N (2) 225 N

(3) 120 N (4) 100 N

Ans. (4)

Sol. Value of g = $g_s\left(1+\frac{h}{R}\right)^{-2}$

$= g_s\left(1+2\right)^{-2} = \frac{g_s}{9}$

Here g_s = gravitational acceleration at surface

Force = mg = 90 × $\frac{g_s}{9}$ = 100 N

SECTION-B

51. The displacement of a particle executing SHM is given by x = 10 sin $\left(\omega t + \frac{\pi}{3}\right)$m . The time period of motion is 3.14 s. The velocity of the particle at t = 0 is ______ m/s.

Ans. (10)

Sol. Given,

$$T = 3.14 = \frac{2\pi}{\omega}$$

ω = 2 rad/s

$$x = 10\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$v = \frac{dx}{dt} = 10\omega\cos\left(\omega t + \frac{\pi}{3}\right)$$

at t = 0

$v = 10\omega\cos\left(\frac{\pi}{3}\right) = 10\times 2\times\frac{1}{2}$ [using ω = 2 rad/s]

v = 10 m/s

52. A bus moving along a straight highway with speed of 72 km/h is brought to halt within 4s after applying the brakes. The distance travelled by the bus during this time (Assume the retardation is uniform) is _______m.

Ans. (40)

Sol. Initial velocity = u = 72 km/h = 20 m/s

v = u + at

$\Rightarrow$ 0 = 20 + a × 4

a = –5 m/s^2

$v^2 - u^2 = 2as$

$\Rightarrow 0^2 - 20^2 = 2(-5).s$

s = 40 m

53. A parallel plate capacitor of capacitance 12.5 pF is charged by a battery connected between its plates to potential difference of 12.0 V. The battery is now disconnected and a dielectric slab ($\in_r$ = 6) is inserted between the plates. The change in its potential energy after inserting the dielectric slab is _______× 10^{-12} J.

Ans. (750)

Sol. Before inserting dielectric capacitance is given $C_0 = 12.5$ pF and charge on the capacitor $Q = C_0V$

After inserting dielectric capacitance will become $\in_r C_0$.

Change in potential energy of the capacitor $= E_i - E_f$

$$= \frac{Q^2}{2C_i} - \frac{Q^2}{2C_f} = \frac{Q^2}{2C_0}\left[1 - \frac{1}{\in_r}\right]$$

$$= \frac{(C_0V)^2}{2C_0}\left[1 - \frac{1}{\in_r}\right] = \frac{1}{2}C_0V^2\left[1 - \frac{1}{\in_r}\right]$$

Using $C_0 = 12.5$ pF, $V = 12$ V, $\in_r = 6$

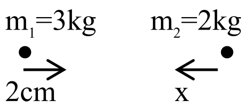
$$= \frac{1}{2}(12.5)\times 12^2\left[1 - \frac{1}{6}\right] = \frac{1}{2}(12.5)\times 12^2 \times \frac{5}{6}$$

$= 750$ pJ $= 750 \times 10^{-12}$J

54. In a system two particles of masses $m_1 = 3$kg and $m_2 = 2$kg are placed at certain distance from each other. The particle of mass m_1 is moved towards the center of mass of the system through a distance 2cm. In order to keep the center of mass of the system at the original position, the particle of mass m_2 should move towards the center of mass by the distance ____ cm.

Ans. **(3)**

Sol.



$$\Delta X_{C.O.M.} = \frac{m_1\Delta x_1 + m_2\Delta x_2}{m_1 + m_2}$$

$$\Rightarrow 0 = \frac{3\times 2 + 2(-x)}{3+2}$$

$\Rightarrow x = 3$ cm

55. The disintegration energy Q for the nuclear fission of ${}^{235}U \rightarrow {}^{140}Ce + {}^{94}Zr + n$ is ____MeV.

Given atomic masses of

${}^{235}U : 235.0439u; {}^{140}Ce; 139.9054u$,

${}^{94}Zr : 93.9063u; n : 1.0086u$,

Value of $c^2 = 931$ MeV/u.

Ans. **(208)**

Sol. ${}^{235}U \rightarrow {}^{140}Ce + {}^{94}Zr + n$

Disintegration energy

$Q = (m_R - m_P).c^2$

$m_R = 235.0439$ u

$m_P = 139.9054u + 93.9063u + 1.0086$ u

$= 234.8203u$

$Q = (235.0439u - 234.8203u)c^2$

$= 0.2236\ c^2$

$= 0.2236 \times 931$

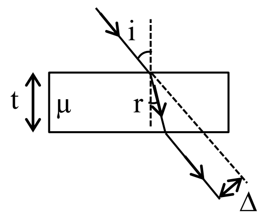
$Q = 208.1716$

56. A light ray is incident on a glass slab of thickness $4\sqrt{3}$ cm and refractive index $\sqrt{2}$. The angle of incidence is equal to the critical angle for the glass slab with air. The lateral displacement of ray after passing through glass slab is ____cm.

(Given sin 15° = 0.25)

Ans. **(2)**

Sol.



$\mathbf{i} = \theta_C$

$\Rightarrow i = \sin^{-1}\left(\frac{1}{\mu}\right)$

$\Rightarrow i = 45°$

and according to snell's law

$1 \sin 45° = \sqrt{2}\sin r$

$\Rightarrow r = 30°$

Lateral displacement $\Delta = \frac{t\sin(i-r)}{\cos r}$

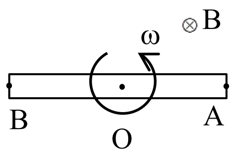
$$\Rightarrow \Delta = \frac{4\sqrt{3}\times\sin 15°}{\cos 30°}$$

$\Rightarrow \Delta = 2$cm

57. A rod of length 60 cm rotates with a uniform angular velocity 20 rad s^{-1} about its perpendicular bisector, in a uniform magnetic field 0.5 T. The direction of magnetic field is parallel to the axis of rotation. The potential difference between the two ends of the rod is ____V.

Ans. **(0)**

Sol.



$$\because V_0 - V_A = \frac{B\omega\ell^2}{2}$$

$$V_0 - V_B = \frac{B\omega\ell^2}{2}$$

$$\therefore\ V_A = V_B \therefore\ V_A - V_B = 0$$

58. Two wires A and B are made up of the same material and have the same mass. Wire A has radius of 2.0 mm and wire B has radius of 4.0 mm. The resistance of wire B is 2Ω. The resistance of wire A is _____Ω.

Ans. **(32)**

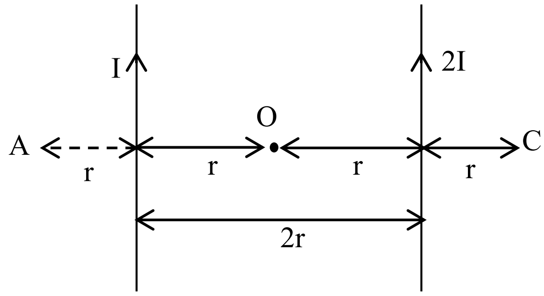
Sol. $\because R = \frac{\rho\ell}{A} = \frac{\rho V}{A^2}$

$$\therefore \frac{R_A}{R_B} = \frac{A_B^2}{A_A^2} = \frac{r_B^4}{r_A^4}$$

$$\Rightarrow \frac{R_A}{2} = \left[\frac{4\times10^{-3}}{2\times10^{-3}}\right]^4$$

$\Rightarrow R_A = 32\ \Omega.$

59. Two parallel long current carrying wire separated by a distance 2r are shown in the figure. The ratio of magnetic field at A to the magnetic field produced at C is $\frac{x}{7}$. The value of x is ___.



Ans. **(5)**

Sol. $B_A = \frac{\mu_0 i}{2\pi r} + \frac{\mu_0(2i)}{2\pi(3r)} = \frac{5\mu_0 i}{6\pi r}$

$$B_C = \frac{\mu_0(2i)}{2\pi r} + \frac{\mu_0 i}{2\pi(3r)} = \frac{7\mu_0 i}{6\pi r}$$

$$\therefore \frac{B_A}{B_C} = \frac{5}{7}$$

$\therefore x = 5$

60. Mercury is filled in a tube of radius 2 cm up to a height of 30 cm. The force exerted by mercury on the bottom of the tube is ___N.
(Given, atmospheric pressure = 10^5 Nm^{-2}, density of mercury = 1.36×10^4 kg m^{-3}, g = 10 ms^{-2}, $\pi = \frac{22}{7}$)

Ans. **(177)**

Sol. $F = P_0A + \rho_m ghA$

$$= \mathbf{10^5} \times \frac{22}{7}\times\left(2\times10^{-2}\right)^2$$

$$+1.36\times10^4\times10\times\left(30\times10^{-2}\right)\left(\frac{22}{7}\times\left(2\times10^{-2}\right)^2\right)$$

F = 51.29 + 125.71 = 177 N

SECTION-A

61. The equilibrium constant for the reaction

$$SO_3(g) \rightleftharpoons SO_2(g) + \frac{1}{2}O_2(g)$$

is $K_C = 4.9 \times 10^{-2}$. The value of K_C for the reaction given below is

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g) \text{ is}$$

(1) 4.9 (2) 41.6

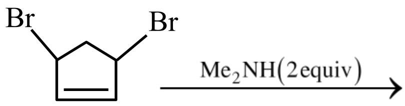
(3) 49 (4) 416

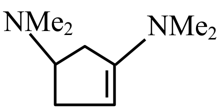
Ans. (4)

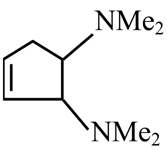
Sol. $K'_C = \left(\frac{1}{K_C}\right)^2 = \left(\frac{1}{4.9 \times 10^{-2}}\right)^2$

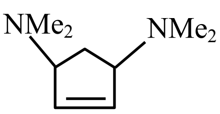
$K'_C = 416.49$

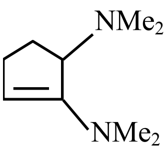
62. Find out the major product formed from the following reaction. [Me: $-CH_3$]



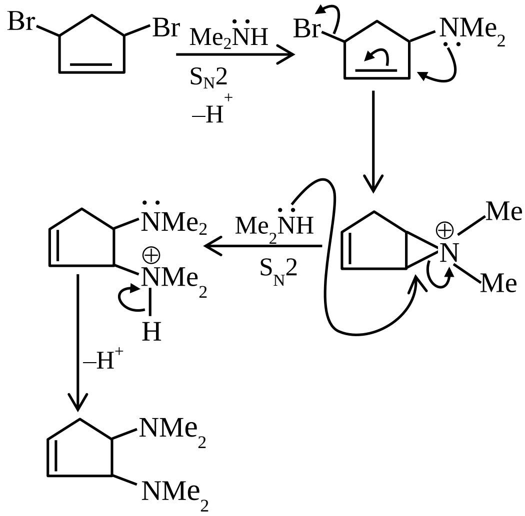
(1) 

(2) 

(3) 

(4) 

Ans. (2)

Sol. 

The above mechanism valid for both cis and trans isomers. So the products are same for both cis and trans isomers.

63. When MnO_2 and H_2SO_4 is added to a salt (A), the greenish yellow gas liberated as salt (A) is :

(1) NaBr (2) CaI_2

(3) KNO_3 (4) NH_4Cl

Ans. (4)

Sol. $2NH_4Cl + MnO_2 + 2H_2SO_4 \xrightarrow{\Delta} MnSO_4$

$+(NH_4)_2SO_4 + 2H_2O + \underset{\text{greenish yellow solution}}{Cl_2} \uparrow$

64. The correct statement/s about Hydrogen bonding is/are :

A. Hydrogen bonding exists when H is covalently bonded to the highly electro negative atom.

B. Intermolecular H bonding is present in o-nitro phenol

C. Intramolecular H bonding is present in HF.

D. The magnitude of H bonding depends on the physical state of the compound.

E. H-bonding has powerful effect on the structure and properties of compounds.

Choose the **correct** answer from the options given below :

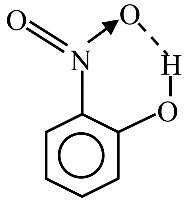
(1) A only (2) A, D, E only

(3) A, B, D only (4) A, B, C only

Ans. (2)

Sol. (A) Generally hydrogen bonding exists when H is covalently bonded to the highly electronegative atom like F, O, N.

(B) Intramolecular H bonding is present in

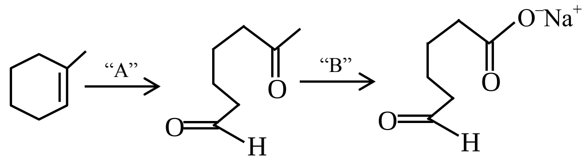


(C) Intermolecular Hydrogen bonding is present in HF

(D) The magnitude has Hydrogen bonding in solid state is greater than liquid state.

(E) Hydrogen bonding has powerfull effect on the structure & properties of compound like melting point, boiling point, density etc.

65.



In the above chemical reaction sequence "A'' and "B" respectively are :

(1) O_3, Zn/H_2O and $NaOH_{(alc.)}$ / I_2

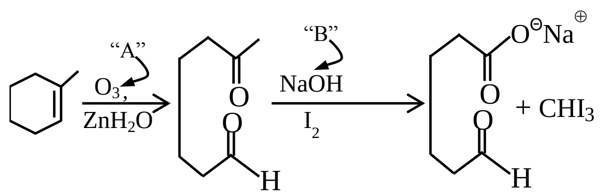
(2) H_2O, H^+ and $NaOH_{(alc.)}$ / I_2

(3) H_2O, H^+ and $KMnO_4$

(4) O_3, Zn/H_2O and $KMnO_4$

Ans. **(1)**

Sol.



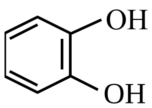
66. Common name of Benzene-1, 2-diol is

(1) quinol (2) resorcinol

(3) catechol (4) o-cresol

Ans. **(3)**

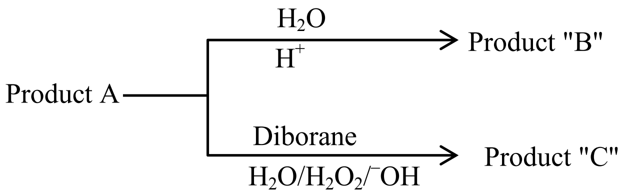
Sol.



IUPAC name : Benzene-1,2-diol

Common name : catechol

67. $CH_3-CH_2-CH_2-Br+NaOH \xrightarrow{C_2H_5OH} \text{Product 'A'}$



Consider the above reactions, identify product B and product C.

(1) B = C = 2-Propanol

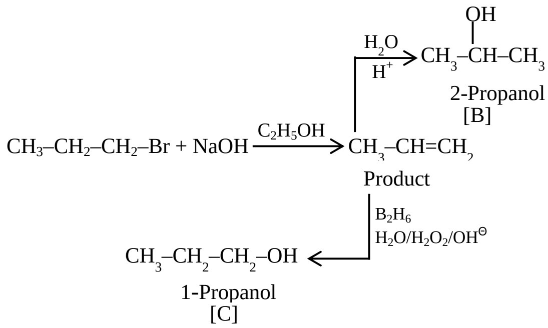
(2) B = 2-Propanol C = 1-Propanol

(3) B = 1-Propanol C = 2-Propanol

(4) B = C = 1-Propanol

Ans. **(2)**

Sol.



68. The adsorbent used in adsorption chromatography is/are

A. silica gel B. alumina

C. quick lime D. magnesia

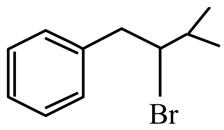
Choose the **most appropriate** answer from the options given below :

(1) B only (2) C and D only

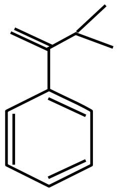
(3) A and B only (4) A only

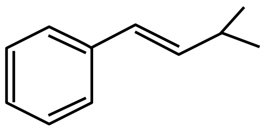
Ans. **(3)**

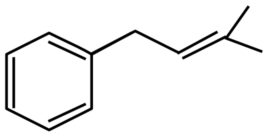
Sol. The most common polar and acidic support used is adsorption chromatography is silica. The surface silanol groups on their supported to adsorb polar compound and work particularly well for basic substances. Alumina is the example of polar and basic adsorbent that is used in adsorption chromatography.

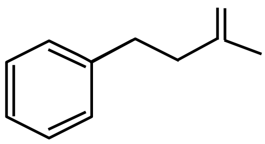
69.  $\xrightarrow[\Delta]{KOH(alc)}$ major product "P"

Product P is

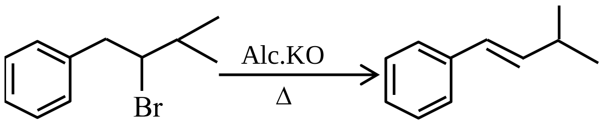
(1) 

(2) 

(3) 

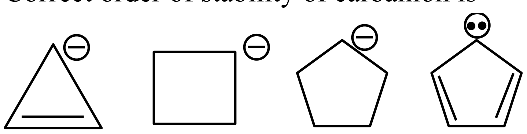
(4) 

Ans. **(2)**

Sol. 

(Major)

70. Correct order of stability of carbanion is



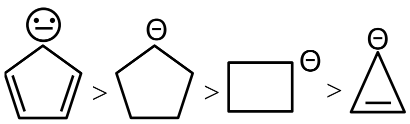
a b c d

(1) c > b > d > a (2) a > b > c > d

(3) d > a > c > b (4) d > c > b > a

Ans. **(4)**

Sol. As we know compound (d) is aromatic and the compound (a) is anti-aromatic. Hence compound (d) is most stable and compound (a) is least stable among these in compound (b) and (c) carbon atom of that positive charge is sp^3 hybridised they on the basis of angle strain theory compound (c) is more stable than compound (b).



71. The correct order of the first ionization enthalpy is

(1) Al > Ga > Tl (2) Ga > Al > B

(3) B > Al > Ga (4) Tl > Ga > Al

Ans. **(4)**

Sol. (i) due to lanthanide contraction $T\ell$ has more I.E. as compared to Ga and $A\ell$

(ii) due to scandide contraction Ga has more I.E. as compared to $A\ell$

72. If an iron (III) complex with the formula $\left[Fe(NH_3)_x(CN)_y\right]^-$ has no electron in its e_g orbital, then the value of x + y is

(1) 5 (2) 6

(3) 3 (4) 4

Ans. **(2)**

Sol. Complex is $[\overset{III}{Fe}(NH_3)_2(CN)_4]^{\ominus}$

x = 2

y = 4

so x + y = 6

73. Fuel cell, using hydrogen and oxygen as fuels,

A. has been used in spaceship

B. has as efficiency of 40% to produce electricity

C. uses aluminium as catalysts

D. is eco-friendly

E. is actually a type of Galvanic cell only

(1) A,B,C only (2) A,B,D only

(3) A,B,D,E only (4) A,D,E only

Ans. **(4)**

Sol. Fuel cell is used in spaceship and it is type of galvanic cell.

74. Choose the **Incorrect** Statement about Dalton's Atomic Theory

(1) Compounds are formed when atoms of different elements combine in any ratio

(2) All the atoms of a given element have identical properties including identical mass

(3) Matter consists of indivisible atoms

(4) Chemical reactions involve recorganization of atoms

Ans. **(1)**

Sol. In compound atoms of different elements combine in fixed ratio by mass.

75. Match List I with List II

	LIST I		LIST II
A.	α- Glucose and α-Galactose	I.	Functional isomers
B.	α- Glucose and β-Glucose	II.	Homologous
C.	α- Glucose and α-Fructose	III.	Anomers
D.	α- Glucose and α-Ribose	IV.	Epimers

Choose the **correct** answer from the options given below:

(1) A-III, B-IV, C-II, D-I

(2) A-III, B-IV, C-I, D-II

(3) A-IV, B-III, C-I, D-II

(4) A-IV, B-III, C-II, D-I

Ans. **(3)**

Sol. Based on biomolecules theory and structure of these named compounds –

(A) α-Glucose and α-Galactose (IV) Epimers.

(B) α-Glucose and β-Glucose (III) Anomers

(C) α-Glucose and α-Fructose (I) Functional isomers

(D) α-Glucose and α-Ribose (II) Homologous

76. Given below are two statements:

Statement I : The correct order of first ionization enthalpy values of Li, Na, F and Cl is Na < Li < Cl < F.

Statement II : The correct order of negative electron gain enthalpy values of Li, Na, F and Cl is Na < Li < F < Cl

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is false but Statement II is true

(4) Statement I is true but Statement II is false

Ans. **(1)**

Sol.. (i)

	Na	<	Li	<	Cl	<	F
	↓		↓		↓		↓
I.E.$_1$ in kJ/mol	496		520		1256		1681

(ii)

	Na	<	Li	<	F	<	Cl
	↓		↓		↓		↓
$\Delta_{eg}H$ in kJ/mol	–53		–60		–328		–349

77. For a strong electrolyte, a plot of molar conductivity against (concentration)$^{1/2}$ is a straight line, with a negative slope, the correct unit for the slope is

(1) S cm^2 mol$^{-3/2}$ L$^{1/2}$ (2) S cm^2 mol^{-1} L$^{1/2}$

(3) S cm^2 mol$^{-3/2}$ L (4) S cm^2 mol$^{-3/2}$ L$^{-1/2}$

Ans. **(1)**

Sol. $\Lambda_m = \Lambda_m^o - A\sqrt{C}$

Units of $A\sqrt{C}$ = S cm^2 mole^{-1}

Uits of A = S cm^2 mole$^{-3/2}$ L$^{1/2}$

78. A first row transition metal in its +2 oxidation state has a spin-only magnetic moment value of 3.86 BM. The atomic number of the metal is

(1) 25 (2) 26

(3) 22 (4) 23

Ans. **(4)**

Sol. $_{22}Ti^{+2} \Rightarrow [Ar]3d^2$

$_{23}V^{+2} \Rightarrow [Ar]3d^3$

$_{25}Mn^{+2} \Rightarrow [Ar]3d^5$

$_{26}Fe^{+2} \Rightarrow [Ar]3d^6$

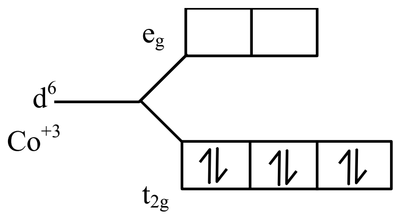
79. The number of unpaired d-electrons in $[Co(H_2O)_6]^{3+}$ is______

(1) 4 (2) 2

(3) 0 (4) 1

Ans. **(3)**

Sol. $\Rightarrow [Co(H_2O)_6]^{+3}$



No unpaired electrons

80. The number of species from the following that have pyramidal geometry around the central atom is________

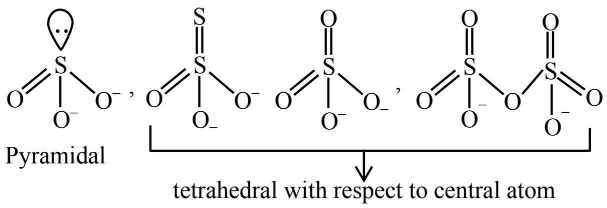
$S_2O_3^{2-}, SO_4^{2-}, SO_3^{2-}, S_2O_7^{2-}$

(1) 4 (2) 3

(3) 1 (4) 2

Ans. **(3)**

Sol.

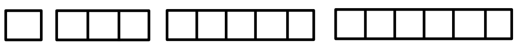


SECTION-B

81. The maximum number of orbitals which can be identified with n = 4 and $m_l = 0$ is_____

Ans. **(4)**

Sol. n = 4 ,



4s 4p 4d 4f

$m_\ell \Rightarrow$ 1 1 1 1

So answer is 4.

82. Number of compounds/species from the following with non-zero dipole moment is_____

$BeCl_2$, BCl_3, NF_3, XeF_4, CCl_4, H_2O H_2S, HBr, CO_2, H_2, HCl

Ans. **(5)**

Sol. $\underset{(\mu \neq 0)}{\text{Polar molecule}}: NF_3, H_2O, H_2S, HBr, HCl$

$\underset{(\mu = 0)}{\text{Non Polar molecule}}: BeCl_2, BCl_3, XeF_4, CCl_4, CO_2, H_2$

So answer is 5.

83. Three moles of an ideal gas are compressed isothermally from 60 L to 20 L using constant pressure of 5 atm. Heat exchange Q for the compression is – ____ Lit. atm.

Ans. **(200)**

Sol. As isothermal $\Delta U = 0$

and process is irreversible

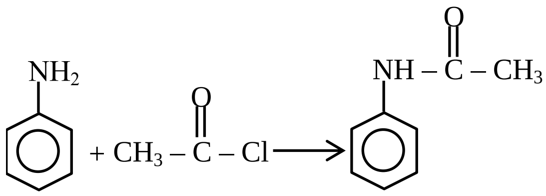
$Q = -W = -[-P_{ext}(V_2 - V_1)]$

$Q = 5(20 - 60) = -200$ atm-L

84. From 6.55 g of aniline, the maximum amount of acetanilide that can be prepared will be___$\times 10^{-1}$ g.

Ans. **(95)**

Sol.



93 g aniline form 135 gm acetanlide

so 6.55 g anilne form $\frac{135}{93} \times 6.55 = 9.5$

95×10^{-1}

85. Consider the following reaction, the rate expression of which is given below

$A + B \rightarrow C$

rate = k $[A]^{1/2}[B]^{1/2}$

The reaction is initiated by taking 1M concentration A and B each. If the rate constant (k) is 4.6×10^{-2} s^{-1}, then the time taken for A to become 0.1 M is_____sec. (nearest integer)

Ans. **(50)**

Sol. $K = \frac{2.303}{t}\log\frac{1}{0.1}$

$4.6 \times 10^{-2} = \frac{2.303}{t}$

t = 50 sec.

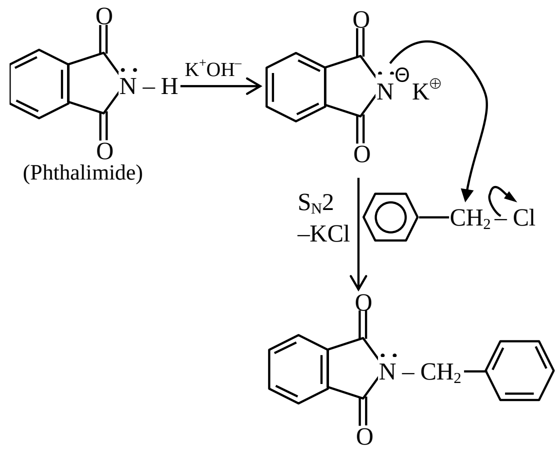
86. Phthalimide is made to undergo following sequence of reactions.

$\text{Phthalimide} \xrightarrow[\text{(ii)Benzylchloride}]{\text{(i)KOH}} \text{'P'}$

Total number of π bonds present in product 'P' is/are

Ans. **(8)**

Sol.



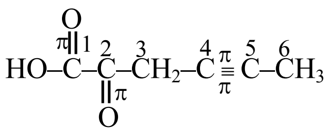
(P)

Total number of π-bonds present in product P is 8

87. The total number of 'sigma' and 'Pi' bonds in 2-oxohex-4-ynoic acid is____.

Ans. **(18)**

Sol.



2-Oxohex-4-ynoic acid

Number of σ-bonds = 14

Number of π-bonds = 4

= 18

88. A first row transition metal with highest enthalpy of atomisation, upon reaction with oxygen at high temperature forms oxides of formula M_2O_n (where n = 3,4,5). The 'spin-only' magnetic moment value of the amphoteric oxide from the above oxides is___ BM (near integer)

(Given atomic number : Sc : 21, Ti : 22, V : 23, Cr : 24, Mn : 25, Fe : 26, Co : 27, Ni : 28 ,Cu : 29, Zn : 30)

Ans. **(0)**

Sol. 'V' has highest enthalpy of atomisation (515 kJ/mol) among first row transition elements.

V_2O_5

Here 'V' is in +5 oxidation state

$V^{+5} \Rightarrow 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6$ (no unpaired electrons)

89. 2.7 Kg of each of water and acetic acid are mixed, The freezing point of the solution will be –x °C. Consider the acetic acid does not dimerise in water, nor dissociates in water x = ______(nearest integer)

[Given : Molar mass of water = 18 g mol^{-1}, acetic acid = 60 g mol^{-1}]

K_f H_2O : 1.86 K kg mol^{-1}

K_f acetic acid : 3.90 K kg mol^{-1}

freezing point : H_2O = 273 K, acetic acid = 290 K]

Ans. **(31)**

Sol. As moles of water > moles of CH_3COOH

water is solvent.

$T^\circ_F - (T_F)_S = K_F \times M$

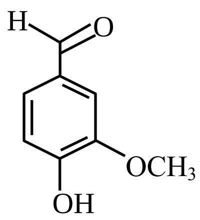
$$0 - (T_F)_S = 1.86 \times \frac{2700/60}{2700/1000}$$

$(T_F)_S = -31°C.$

90. Vanillin compound obtained from vanilla beans, has total sum of oxygen atoms and π electrons is__

Ans. **(11)**

Sol. Vanillin compound is an organic compound molecular formula $C_8H_8O_3$. It is a phenolic aldehyde. Its functional compounds include aldehyde, hydroxyl and ether. It is the primary component of the extract of the vanilla beans.



Total sum of oxygen atoms and π-electrons is 3 + 8 = 11

Total number of oxygen atoms = 3

Total number of π-bonds = 4

∴ Total number of π-electrons = 8