08/04/2019
Evening

# Answers & Solutions

*for*

# JEE (MAIN)-2019 (Online) Phase-2

## (Physics, Chemistry and Mathematics)

**Time : 3 hrs.** **M.M. : 360**

**Important Instructions :**

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. *¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question.

1. A cell of internal resistance r drives current through an external resistance R. The power delivered by the cell to the external resistance will be maximum when :

(1) R = 1000 r (2) R = r

(3) R = 2r (4) R = 0.001 r

**Answer (2)**

**Sol.** For maximum power in external resistance, Internal resistance = External resistance

$\Rightarrow$ R = r

2. A body of mass $m_1$ moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass $m_2$ moving with a velocity $v_2\hat{i}$. After collision, $m_1$ and $m_2$ move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively.

If $m_2 = 0.5\, m_1$ and $v_3 = 0.5\, v_1$, then $v_1$ is :

(1) $v_4 - \frac{v_2}{4}$ (2) $v_4 - v_2$

(3) $v_4 + v_2$ (4) $v_4 - \frac{v_2}{2}$

**Answer (2)**

**Sol.** $m_1v_1 + m_2v_2 = m_1v_3 + m_2v_4$

$m_1v_1 + 0.5m_1v_2 = 0.5m_1v_1 + 0.5m_1v_4$

$v_1 = v_4 - v_2$

3. In a line of sight radio communication, a distance of about 50 km is kept between the transmitting and receiving antennas. If the height of the receiving antenna is 70 m, then the minimum height of the transmitting antenna should be :

(Radius of the Earth = $6.4 \times 10^6$ m).

(1) 20 m (2) 51 m

(3) 32 m (4) 40 m

**Answer (3)**

**Sol.** $\sqrt{2\times 70\times R_E} + \sqrt{2\times h_R\times R_E} = 50\times 10^3$

Putting $R_E = 6.4 \times 10^6$ m and solving we get

$h_R = 32$ m

4. The electric field in a region is given by $\vec{E} = (Ax + B)\hat{i}$, where E is in $NC^{-1}$ and x is in metres. The values of constants are A = 20 SI unit and B = 10 SI unit. If the potential at x = 1 is $V_1$ and that at x = – 5 is $V_2$, then $V_1 - V_2$ is :

(1) 180 V (2) –520 V

(3) 320 V (4) –48 V

**Answer (1)**

**Sol.** $dV = -\vec{E}\cdot\vec{dr} = -(Ax+B)dx$

$$\int_{V_2}^{V_1} dV = \int_{-5}^{1} -(Ax+B)dx$$

$$V_1 - V_2 = \left(-A\frac{x^2}{2} - Bx\right)_{-5}^{1}$$

$$= \left(-\frac{A}{2} - B\right) + \left(\frac{A}{2}25 + B(-5)\right)$$

= 12A – 6B = 240 – 60 = 180 V

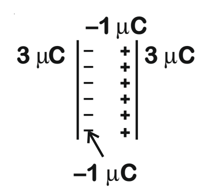
5. A parallel plate capacitor has 1 μF capacitance. One of its two plates is given +2 μC charge and the other plate, +4 μC charge. The potential difference developed across the capacitor is :

(1) 5 V (2) 1 V

(3) 3 V (4) 2 V

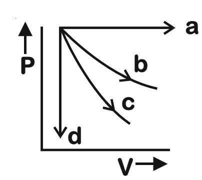
**Answer (2)**

**Sol.** $V = \frac{q}{C} = \frac{1\mu C}{1\mu F}$




= 1 V

6. The given diagram shows four processes i.e., isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by :



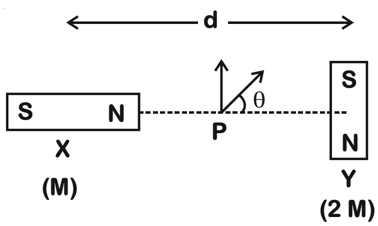

(1) a d c b (2) a d b c

(3) d a b c (4) d a c b

**Answer (3)**

**Sol.** Between the isothermal and the adiabatic processes, P-V graph for adiabatic is steeper.

7. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing through their mid-point P, at angle $\theta$ = 45° with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant? (d is much larger than the dimensions of the dipole)




(1) 0

(2) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(\frac{d}{2}\right)^3}\times qv$

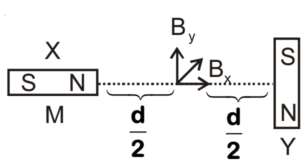
(3) $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{\left(\frac{d}{2}\right)^3}\times qv$

(4) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(\frac{d}{2}\right)^3}\times qv$

**Answer (1)**

**Sol.** $\vec{F}_m = q\left(\vec{V}\times\vec{B}\right)$

$\vec{B} = \vec{B}_x + \vec{B}_y$




Since $M_y = 2M_x$

$\Rightarrow \quad |\vec{B}_x| = |\vec{B}_y|$

$\vec{B}_{net}$ is parallel to $\vec{V}$

$\Rightarrow \quad \vec{F} = 0$

8. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to:

(1) 100 s

(2) 10 s

(3) 50 s

(4) 20 s

**Answer (4)**

**Sol.** Time for 10 oscillations = $\frac{10}{5} = 2$ s

$A = A_0\, e^{-kt}$

$\frac{1}{2} = e^{-2k} \quad \Rightarrow \quad \ln 2 = 2k$

$10^{-3} = e^{-kt} \quad \Rightarrow \quad 3\ln 10 = kt$

$t = \frac{3\ln 10}{k} = \frac{3\ln 10}{\ln 2}\times 2$

$= 6\times\frac{2.3}{0.69} \approx 20$ s

9. If Surface tension (S), Moment of Inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be:

(1) $S^{1/2}I^{3/2}h^{-1}$

(2) $S^{3/2}I^{1/2}h^{0}$

(3) $S^{1/2}I^{1/2}h^{-1}$

(4) $S^{1/2}I^{1/2}h^{0}$

**Answer (4)**

**Sol.** $[p] = MLT^{-1} = [I^x\, h^y\, S^z]$

$= M^xL^{2x}\,(ML^2T^{-1})^y\,(MT^{-2})^z$

$= M^{x+y+z}\,L^{2x+2y}\,T^{-y-2z}$

$x + y + z = 1$

$2(x + y) = 1 \quad \Rightarrow \quad x + y = \frac{1}{2} \quad \Rightarrow \quad z = \frac{1}{2}$

$y + 2z = 1 \quad \Rightarrow \quad y = 0 \quad \Rightarrow \quad x = \frac{1}{2}$

$[p] = \sqrt{IS}$

10. A circuit connected to an ac source of emf $e = e_0\sin(100t)$ with t in seconds, gives a phase difference of $\frac{\pi}{4}$ between the emf e and current i. Which of the following circuits will exhibit this?

(1) RL circuit with R = 1 kΩ and L = 10 mH

(2) RL circuit with R = 1 kΩ and L = 1 mH

(3) RC circuit with R = 1 kΩ and C = 10 μF

(4) RC circuit with R = 1 kΩ and C = 1 μF

**Answer (3)**

**Sol.** As $\phi = \frac{\pi}{4}$, $x_c = R$

11. The magnetic field of an electromagnetic wave is given by:

$$\vec{B} = 1.6\times10^{-6}\cos(2\times10^7 z + 6\times10^{15} t)(2\hat{i}+\hat{j})\frac{Wb}{m^2}$$

The associated electric field will be:

(1) $\vec{E} = 4.8\times10^2\cos\left(2\times10^7 z + 6\times10^{15} t\right)(-\hat{i}+2\hat{j})\frac{V}{m}$

(2) $\vec{E} = 4.8\times10^2\cos\left(2\times10^7 z - 6\times10^{15} t\right)(2\hat{i}+\hat{j})\frac{V}{m}$

(3) $\vec{E} = 4.8\times10^2\cos\left(2\times10^7 z - 6\times10^{15} t\right)(-2\hat{j}+\hat{i})\frac{V}{m}$

(4) $\vec{E} = 4.8\times10^2\cos\left(2\times10^7 z + 6\times10^{15} t\right)(\hat{i}-2\hat{j})\frac{V}{m}$

**Answer (1)**

**Sol.** Amplitude of electric field, $E = B_0C$

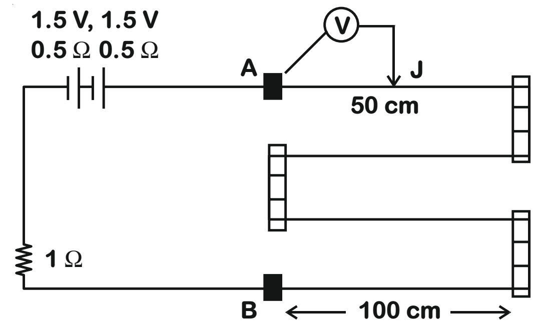
$$= 1.6\times10^{-6}\times\sqrt{5}\times3\times10^8$$

$$= 4.8\times10^2\sqrt{5}\ V/m$$

Also $\vec{E}\times\vec{B}$ is along $-\hat{k}$ (the direction of propagation)

$$\Rightarrow\ \vec{E} = 4.8\times10^2\cos(2\times10^7 z + 6\times10^{15} t)(-\hat{i}+2\hat{j})\frac{V}{m}$$

12. In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is r = 0.01 Ω/cm. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be:




(1) 0.75 V
(2) 0.50 V
(3) 0.20 V
(4) 0.25 V

**Answer (4)**

**Sol.** Resistance of potentiometer wire, $R_p = 400 \times 0.01 = 4\ \Omega$

$$\Rightarrow\ I = \frac{3}{6} = 0.5\ A$$

$\Rightarrow$ Reading of voltmeter $= I\,R_{AJ}$

$= 0.5 \times 50 \times 0.01$

$= 0.25\ V$

13. A nucleus A, with a finite de-Broglie wavelength $\lambda_A$, undergoes spontaneous fission into two nuclei B and C of equal mass. B flies in the same direction as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de-Broglie wavelength $\lambda_B$ and $\lambda_C$ of B and C are respectively:

(1) $2\lambda_A, \lambda_A$
(2) $\lambda_A, \frac{\lambda_A}{2}$
(3) $\frac{\lambda_A}{2}, \lambda_A$
(4) $\lambda_A, 2\lambda_A$

**Answer (3)**

**Sol.** $\lambda_A = \frac{h}{mV_A}$

Conservation of linear momentum

$$\Rightarrow\ mV_A = \frac{m}{2}V - \frac{m}{2}\cdot\frac{V}{2} = \frac{mV}{4}$$

$$\Rightarrow\ \lambda_A = \frac{4h}{mV}$$

$$\therefore\ V_A = \frac{V}{4}$$

$$\lambda_B = \frac{h}{\frac{m}{2}V} = \frac{2h}{mV} = \frac{\lambda_A}{2}$$

$$\lambda_c = \frac{h}{\frac{m}{2}\cdot\frac{V}{2}} = \frac{4h}{mV} = \lambda_A$$

14. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon.

(1) $\frac{E}{64}$ (2) $\frac{E}{4}$

(3) $\frac{E}{16}$ (4) $\frac{E}{32}$

**Answer (3)**

**Sol.** $E = \frac{GM_E m}{R_E}$

$E' = \frac{GM_m m}{R_M}$

$\rho R_E^3 = 64\rho R_M^3$

$\Rightarrow R_E = 4\ R_M$

$$\frac{E'}{E} = \frac{M_M}{M_E} \cdot \frac{R_E}{R_M} = \frac{1}{64} .4 = \frac{1}{16}$$

$\Rightarrow E' = \frac{E}{16}$

15. In a simple pendulum experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained is 55.0 cm. The percentage error in the determination of g is close to:

(1) 6.8%

(2) 0.2%

(3) 3.5%

(4) 0.7%

**Answer (1)**

**Sol.** $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow g = 4\pi^2 \frac{l}{T^2}$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T} = \left(\frac{0.1}{55} + \frac{2\times 1}{30}\right)\times 100$$

$\approx 6.8\%$

16. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency $\omega$ is:

(1) $\sqrt{\frac{2qE}{md}}$

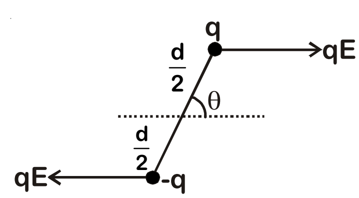
(2) $2\sqrt{\frac{qE}{md}}$

(3) $\sqrt{\frac{qE}{2md}}$

(4) $\sqrt{\frac{qE}{md}}$

**Answer (1)**

**Sol.**

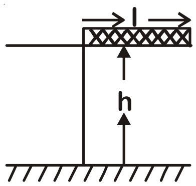



$$-Eqd\theta = I\frac{d^2\theta}{dt^2} = 2\frac{md^2}{4} \cdot \frac{d^2\theta}{dt^2}$$

$\Rightarrow \frac{d^2\theta}{dt^2} - \frac{2Eq\theta}{md}$

$\Rightarrow \omega = \sqrt{\frac{2Eq}{md}}$

17. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5 m. When released, it slips off the table in a very short time $\tau = 0.01$ s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to:




(1) 0.3

(2) 0.02

(3) 0.28

(4) 0.5

**Answer (4)**

**Sol.** Initial angular acceleration before it slips off

$$mg\frac{l}{2} = I\alpha$$

$$\alpha = \frac{3g}{2l}$$

$\therefore$ Angular speed acquire by the box in time $\tau = 0.01$ s

$$\omega = \alpha t = \frac{3g}{2l} \times 0.01 = \frac{3 \times 10 \times 0.01}{2 \times 0.3} = \frac{1}{2} \text{ rad/sec}$$

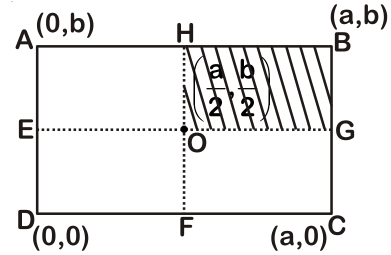
$\therefore$ The angle by which it would rotate when hits the ground

$\theta = \omega t'$

[Assuming $\omega$ = constant and $t'$ = time of fall = $\sqrt{\frac{2H}{g}} = 1 \text{ sec}$

$$\therefore \quad \theta = \frac{1}{2} \text{ radians}$$

18. A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be:




(1) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$

(2) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

(3) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$

(4) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

**Answer (2)**

**Sol.** X- coordinate of CM of remaining sheet

$$X_{cm} = \frac{MX - mx}{M - m}$$

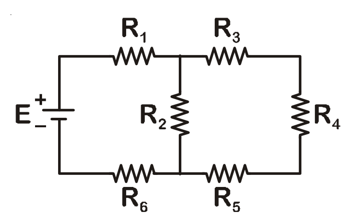
$$= \frac{(4m) \times \left(\frac{a}{2}\right) - M\left(\frac{3a}{4}\right)}{4m - m} = \frac{5a}{12}$$

Similarly $y_{cm} = \frac{5b}{12}$

$$\therefore \quad CM\left(\frac{5a}{12}, \frac{5b}{12}\right)$$

19. In the figure shown, what is the current (in Ampere) drawn from the battery? You are given :

$R_1 = 15\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = 20\ \Omega$, $R_4 = 5\ \Omega$, $R_5 = 25\ \Omega$, $R_6 = 30\ \Omega$, E = 15 V




(1) $\frac{13}{24}$

(2) $\frac{9}{32}$

(3) $\frac{20}{3}$

(4) $\frac{7}{18}$

**Answer (2)**

**Sol.** Equivalent resistance of the given circuit

$$R_{eq} = 45 + \frac{10 \times 50}{10+50} = \left(45 + \frac{50}{6}\right)\Omega = \frac{160}{3}\Omega$$

$$\therefore \quad I = \frac{15}{\frac{160}{3}} = \frac{9}{32}A$$

20. Let $|\overrightarrow{A_1}| = 3, |\overrightarrow{A_2}| = 5$ and $|\overrightarrow{A_1} + \overrightarrow{A_2}| = 5.$ The value of $(2\overrightarrow{A_1} + 3\overrightarrow{A_2}) \cdot (3\overrightarrow{A_1} - 2\overrightarrow{A_2})$ is :

(1) – 106.5 (2) – 118.5

(3) – 99.5 (4) – 112.5

**Answer (2)**

**Sol.** $(2\overrightarrow{A_1} + 3\overrightarrow{A_2}) \cdot (3\overrightarrow{A_1} - 2\overrightarrow{A_2}) = 6|\overrightarrow{A_1}|^2 + 5\overrightarrow{A_1} \cdot \overrightarrow{A_2} - 6|\overrightarrow{A_2}|^2$

$$= (6 \times 9) + 5\overrightarrow{A_1} \cdot \overrightarrow{A_2} - (6 \times 25) \quad ...(i)$$

As $25 = 9 + 25 + 2\overrightarrow{A_1} \cdot \overrightarrow{A_2}$

$\Rightarrow \quad 2\overrightarrow{A_1} \cdot \overrightarrow{A_2} = -9 \quad ...(ii)$

From (i) and (ii),

$$\Rightarrow \quad (2\overrightarrow{A_1} + 3\overrightarrow{A_2}) \cdot (3\overrightarrow{A_1} - 2\overrightarrow{A_2}) = 54 - 22.5 - 150$$

$$= -118.5$$

21. Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star.

(1) $457.5 \times 10^{-9}$ radian

(2) $305 \times 10^{-9}$ radian

(3) $152.5 \times 10^{-9}$ radian

(4) $610 \times 10^{-9}$ radian

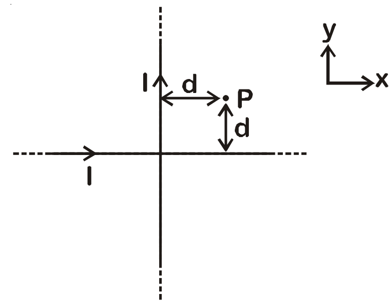
**Answer (2)**

**Sol.** $\theta = \frac{1.22\lambda}{D}$

$$\theta = \frac{1.22 \times 500 \times 10^{-9}}{200 \times 10^{-2}} = \frac{1.22 \times 500 \times 10^{-9}}{2}$$

$\theta = 305 \times 10^{-9}$ radian

22. Two very long, straight, and insulated wires are kept at 90° angle from each other in xy-plane as shown in the figure.




These wires carry currents of equal magnitude I, whose directions are shown in the figure. The net magnetic field at point P will be

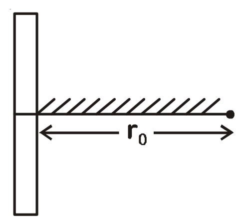
(1) Zero (2) $\frac{+\mu_0 I}{\pi d}(\hat{z})$

(3) $-\frac{\mu_0 I}{2\pi d}(\hat{x} + \hat{y})$ (4) $\frac{\mu_0 I}{2\pi d}(\hat{x} + \hat{y})$

**Answer (1)**

**Sol.** $\vec{B} = \overrightarrow{B_1} + \overrightarrow{B_2} = \frac{\mu_0 1}{2\pi d}\left[\hat{k} - \hat{k}\right] = 0$

23. A positive point charge is released from rest at a distance $r_0$ from a positive line charge with uniform density. The speed ($\nu$) of the point charge, as a function of instantaneous distance r from line charge, is proportional to :




(1) $\nu \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$

(2) $\nu \propto e^{+r/r_0}$

(3) $\nu \propto \ln\left(\frac{r}{r_0}\right)$

(4) $\nu \propto \left(\frac{r}{r_0}\right)$

**Answer (1)**

**Sol.** $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

$$\int_{V_G}^{V_P} dV = \int -\frac{\lambda}{2\pi\varepsilon_0 r} dr$$

$$\Rightarrow \quad V_P - V_G = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{r}{r_0}$$

$$\frac{1}{2}mv^2 = q(V_P - V_G)$$

$$\Rightarrow \quad v \propto \left[\ln\left(\frac{r}{r_0}\right)\right]^{\frac{1}{2}}$$

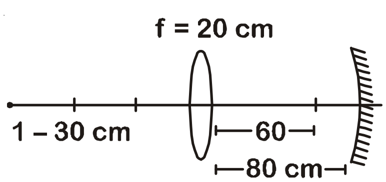
24. A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be :

(1) 30 cm (2) 25 cm

(3) 20 cm (4) 10 cm

**Answer (4)**

**Sol.**




$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore \quad \frac{1}{v} + \frac{1}{30} = \frac{1}{20}$$

$$\frac{1}{v} = \frac{30-20}{20\times 30} = \frac{10}{20\times 30}$$

v = 60 cm

So clearly radius of curvature of mirror is 20 cm. Now if the object is placed within focal plane i.e. 10 cm then image formed by mirror is virtual.

25. The ratio of mass densities of nuclei $^{40}Ca$ and $^{16}O$ is close to :

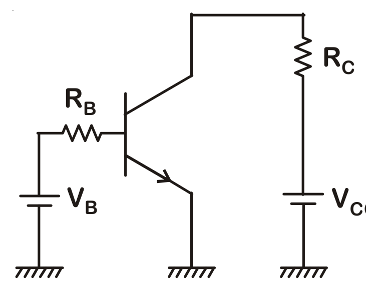
(1) 0.1 (2) 1

(3) 2 (4) 5

**Answer (2)**

**Sol.** Densities of nucleus happens to be constant, irrespective of mass number.

26. A common emitter amplifier circuit, built using an npn transistor, is shown in the figure. Its dc current gain is 250, $R_C$ = 1 kΩ and $V_{CC}$ = 10 V. What is the minimum base current for $V_{CE}$ to reach saturation?




(1) 10 μA (2) 100 μA

(3) 7 μA (4) 40 μA

**Answer (4)**

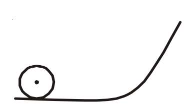
**Sol.** For saturation, $V_{CC} - i_c \times R_c = 0$

$$\Rightarrow \quad i_c = \frac{V_{CC}}{R_C} = \frac{10}{10^3} = 10^{-2} \text{ A}$$

$$\because \quad \beta = \frac{I_C}{I_B} = 250$$

$$\therefore \quad I_B = \frac{I_C}{250} = \frac{10^{-2}}{250} = 40\ \mu\text{A}$$

27. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights $h_{sph}$ and $h_{cyl}$ on the incline. The ratio $\dfrac{h_{sph}}{h_{cyl}}$ is given by



(1) $\dfrac{4}{5}$ (2) $\dfrac{2}{\sqrt{5}}$

(3) 1 (4) $\dfrac{14}{15}$

**Sol.** $mgh_{sph} = \frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{2}{5}mR^2\cdot\left(\frac{v}{R}\right)^2$

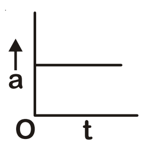
$$= \frac{7}{10}mv^2 \quad \text{...(i)}$$

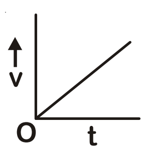
$$mgh_{cylinder} = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\left(\frac{v}{R}\right)^2 = \frac{3}{4}mv^2$$

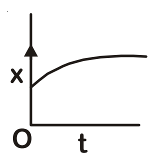
$$\Rightarrow \frac{h_{sph}}{h_{cylinder}} = \frac{7\times 4}{10\times 3} = \frac{14}{15}$$

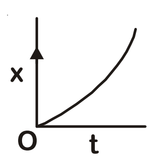
28. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represent the motion qualitatively. (a = acceleration, v = velocity,

x = displacement, t = time )

(A) 


(B) 


(C) 


(D) 


(1) (A) (2) (A), (B), (C)

(3) (A), (B), (D) (4) (B), (C)

**Answer (3)**

**Sol.** a = Constant

v = at

$x = \frac{1}{2}at^2$ [Particle starts from the origin]

A, B and D are correct graphs.

29. Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to

(1) 1.3 mm (2) 1.9 mm

(3) 1.5 mm (4) 1.7 mm

**Answer (4)**

**Sol.** $\Delta L = \frac{FL}{YA}$

$$\Rightarrow \frac{L_A}{Y_A r_A^2} = \frac{L_B}{Y_B r_B^2}$$

$$\Rightarrow r_A^2 = \sqrt{\frac{L_A}{L_B}\cdot\frac{Y_B}{Y_A}}\cdot r_B$$

$$= \sqrt{\frac{2\times 2\times 4}{3\times 7}}\times 2\text{ mm}$$

$$= \frac{4}{4.58}\times 2 = 1.7\text{ mm}$$

30. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to

[Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J/K

Avogadro Number $N_A = 6.02 \times 10^{26}$ /kg

Radius of Earth : $6.4 \times 10^6$ m

Gravitational acceleration on Earth = 10 $ms^{-2}$]

(1) $10^4$ K

(2) 650 K

(3) 800 K

(4) $3 \times 10^5$ K

**Answer (1)**

**Sol.** $V_{rms} = \sqrt{\frac{3RT}{M}} = 11.2\times 10^3$ m/s

$$\Rightarrow T = \frac{M}{3R}\times(11.2\times 10^3)^2$$

$$= \frac{2\times 10^{-3}}{3\times 8.3}\times 125.44\times 10^6 \simeq 10^4\text{ K}$$

# PART–B : CHEMISTRY

1. 0.27 g of a long chain fatty acid was dissolved in 100 cm$^3$ of hexane. 10 mL of this solution was added dropwise to the surface of water in a round watch glass. Hexane evaporates and a monolayer is formed. The distance from edge to centre of the watch glass is 10 cm. What is the height of the monolayer?

   [Density of fatty acid = 0.9 g cm$^{-3}$; $\pi$ = 3]

   (1) $10^{-8}$ m (2) $10^{-4}$ m

   (3) $10^{-2}$ m (4) $10^{-6}$ m

**Answer (4)**

**Sol.** 0.27 gm in 100 ml of hexane

$\therefore$ in 10 ml of aqueous solution only 0.027 gm acid is present

volume of 0.027 g acid $= \dfrac{0.027}{0.9}$ ml

$\therefore \quad \pi r^2 h = \dfrac{0.027}{0.9}$ (given r = 10 cm, $\pi$ = 3)

$\therefore$ h = $10^{-4}$ cm

= $10^{-6}$ m

2. The Mond process is used for the :

   (1) purification of Zr and Ti

   (2) extraction of Mo

   (3) purification of Ni

   (4) extraction of Zn

**Answer (3)**

**Sol.** Nickel is purified by Mond's process

$$\underset{\text{(impure)}}{Ni} + 4CO \xrightarrow{330-350\ k} Ni(CO)_4$$

$$Ni(CO)_4 \xrightarrow{450-470\ k} \underset{\text{(pure)}}{Ni} + 4CO$$

3. The correct statement about $ICl_5$ and $ICl_4^-$ is :

   (1) $ICl_5$ is square pyramidal and $ICl_4^-$ is tetrahedral.

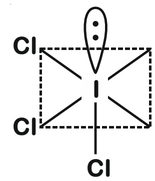
   (2) both are isostructural.

   (3) $ICl_5$ is square pyramidal and $ICl_4^-$ is square planar.

   (4) $ICl_5$ is trigonal bipyramidal and $ICl_4^-$ is tetrahedral.

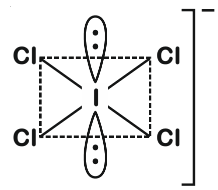
**Answer (3)**

**Sol.** $ICl_5$ is $sp^3d^2$ hybridised (5 bond pairs, 1 lone pair)




square pyramidal

$ICl_4^-$ is $sp^3d^2$ hybridised (4 bond pairs, 2 lone pairs)




square planar

4. For the following reactions, equilibrium constants are given :

$$S(s) + O_2(g) \rightleftharpoons SO_2(g);\ K_1 = 10^{52}$$

$$2S(s) + 3O_2(g) \rightleftharpoons 2SO_3(g);\ K_2 = 10^{129}$$

The equilibrium constant for the reaction,

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g) \text{ is :}$$

   (1) $10^{154}$

   (2) $10^{25}$

   (3) $10^{77}$

   (4) $10^{181}$

**Answer (2)**

**Sol.** eq$^n$ 1 $S + O_2 \rightleftharpoons SO_2$ $K_1 = 10^{52}$

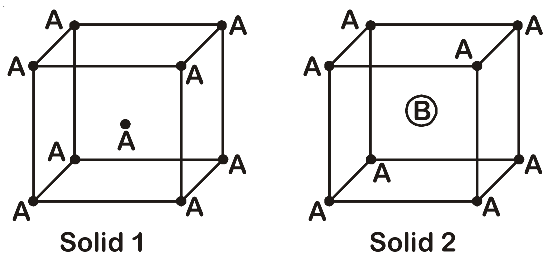
eq$^n$ 2 $2S + 3O_2 \rightleftharpoons 2SO_3$ $K_2 = 10^{129}$

eq$^n$ 3 $2SO_2 + O_2 \rightleftharpoons 2SO_3$

eq$^n$ 3 = eq$^n$ 2 – 2 (eq$^n$ 1)

$$= \frac{10^{129}}{(10^{52})^2} = 10^{25}$$

5. Consider the bcc unit cells of the solids 1 and 2 with the position of atoms as shown below. The radius of atom B is twice that of atom A. The unit cell edge length is 50% more in solid 2 than in 1. What is the approximate packing efficiency in solid 2?




(1) 45%　　(2) 65%

(3) 75%　　(4) 90%

**Answer (4)**

**Sol.** Volume occupied by atoms in solid 2

$$=\frac{4}{3}\pi r^3+\frac{4}{3}\pi(2r)^3=12\,\pi r^3$$

relationship between edge length (a) and radius of atom (r)

$$=6r=\sqrt{3}a \Rightarrow a=\frac{6r}{\sqrt{3}}$$

packing efficiency $=\dfrac{12\pi r^3}{\left(\dfrac{6r}{\sqrt{3}}\right)^3}\times 100$ = 90%

6. The compound that inhibits the growth of tumors is

(1) cis-$[Pt(Cl)_2(NH_3)_2]$

(2) trans-$[Pt(Cl)_2(NH_3)_2]$

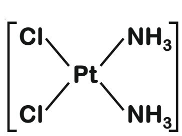
(3) cis-$[Pd(Cl)_2(NH_3)_2]$

(4) trans-$[Pd(Cl)_2(NH_3)_2]$

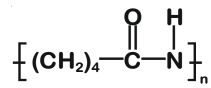
**Answer (1)**

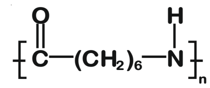
**Sol.** Cis-platin is used as an anti-cancer drug.

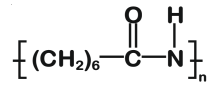
Cis $[PtCl_2(NH_3)_2]$

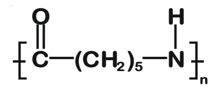


7. The structure of Nylon-6 is

(1) 

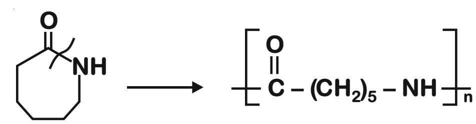
(2) 

(3) 

(4) 

**Answer (4)**

**Sol.** Nylon-6 is produced from caprolactam



Nylon-6

8. Which one of the following alkenes when treated with HCl yields majorly an anti Markovnikov product ?

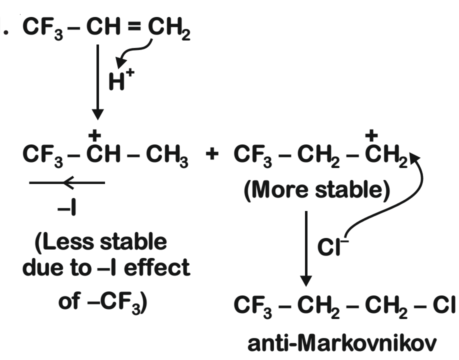
(1) $F_3C–CH=CH_2$

(2) $CH_3O–CH=CH_2$

(3) $H_2N–CH=CH_2$

(4) $Cl–CH=CH_2$

**Answer (1)**

**Sol.** 


9. The strength of 11.2 volume solution of $H_2O_2$ is [Given that molar mass of H = 1 g mol$^{-1}$ and O = 16 g mol$^{-1}$]

(1) 13.6%

(2) 1.7%

(3) 3.4%

(4) 34%

**Answer (3)**

**Sol.** 11.2 V of $H_2O_2$

$$H_2O_2 \longrightarrow H_2O + \frac{1}{2}O_2$$

11.2 L of $O_2$ at STP = 0.5 mol

It means 1 L of given $H_2O_2$ solution consist 1 mole of $H_2O_2$ (i.e., 34 g)

$$\text{strength} = \frac{34}{1000} \times 100 = 3.4\%$$

10. The covalent alkaline earth metal halide (X = Cl, Br, I) is

(1) $BeX_2$

(2) $SrX_2$

(3) $CaX_2$

(4) $MgX_2$

**Answer (1)**

**Sol.** According to Fajan's rule, graeter the polarising power of cation greater would be the covalent character.

Since $Be^{2+}$ has maximum polarising power among given cation, therefore, $BeX_2$ would be most covalent among given alkaline with metal halides.

11. Polysubstitution is a major drawback in :

(1) Reimer Tiemann reaction

(2) Acetylation of aniline

(3) Friedel Craft's acylation

(4) Friedel Craft's alkylation

**Answer (4)**

**Sol.** Polysubstitution is a major drawback in Friedel Craft's alkylation

12. The IUPAC symbol for the element with atomic number 119 would be :

(1) une (2) uun

(3) uue (4) unh

**Answer (3)**

**Sol.** Symbol for 1 is u

and for 9 is e

$\therefore$ IUPAC symbol for 119 is uue

13. 5 moles of an ideal gas at 100 K are allowed to undergo reversible compression till its temperature becomes 200 K. If $C_V$ = 28 J $K^{-1}$ $mol^{-1}$, calculate $\Delta U$ and $\Delta pV$ for this process. (R = 8.0 J $K^{-1}$ $mol^{-1}$)

(1) $\Delta U$ = 14 kJ; $\Delta(pV)$ = 18 kJ

(2) $\Delta U$ = 2.8 kJ; $\Delta(pV)$ = 0.8 kJ

(3) $\Delta U$ = 14 J; $\Delta(pV)$ = 0.8 J

(4) $\Delta U$ = 14 kJ; $\Delta(pV)$ = 4 kJ

**Answer (4)**

**Sol.** $\Delta U = n\ c_{vm}\,\Delta T = 5 \times 28 \times 100$

= 14 kJ

$$\Delta(PV) = nR(T_2 - T_1)$$

= 5 × 8 × 100

= 4 kJ

14. The calculated spin-only magnetic moments (BM) of the anionic and cationic species of $[Fe(H_2O)_6]_2$ and $[Fe(CN)_6]$, respectively, are:

(1) 2.84 and 5.92 (2) 4.9 and 0

(3) 0 and 5.92 (4) 0 and 4.9

**Answer (4)**

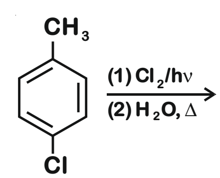
**Sol.** $[Fe(H_2O)_6]_2$ $[Fe(CN)_6]$

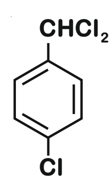
$[Fe(H_2O)_6]^{2+}$ $[Fe(CN)_6]^{4-}$

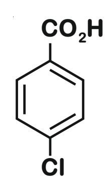
4 unpaired electrons; no unpaired electron

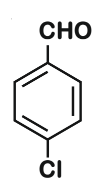
$\mu$ = 4.9; $\mu$ = 0

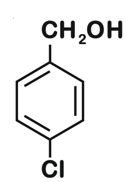
15. The major product of the following reaction is:



(1) 

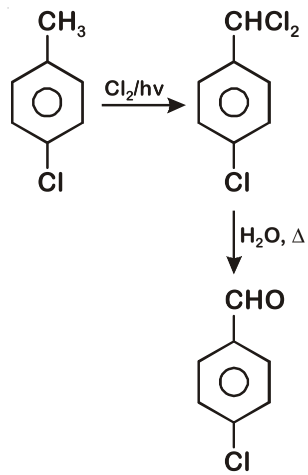
(2) 

(3) 

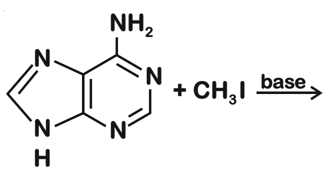
(4) 

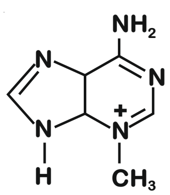
**Answer (3)**

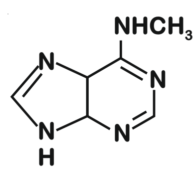
**Sol.**

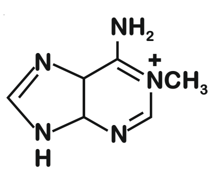


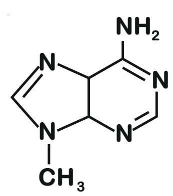
16. The major product in the following reaction is:



(1) 

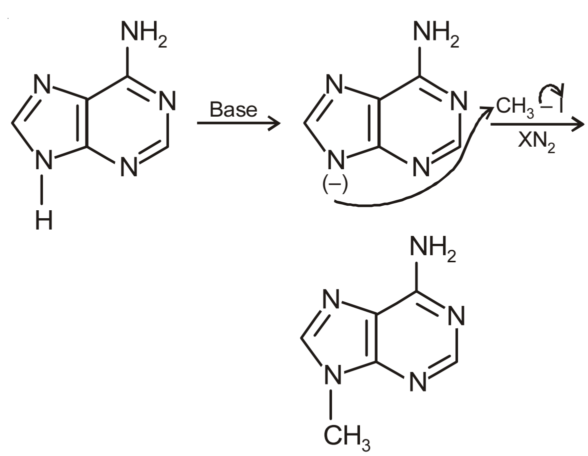
(2) 

(3) 

(4) 

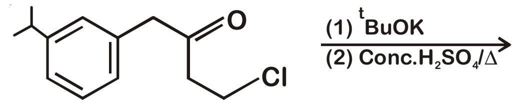
**Answer (4)**

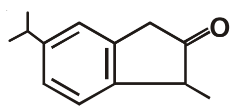
**Sol.**

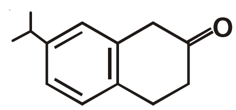


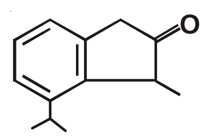
In the given compound, H-atom attached to secondary N-atom is more acidic. The base removes the more acidic H-atom and the conjugate base of the given compound attacks at $CH_3$ group to give the final product shown above.

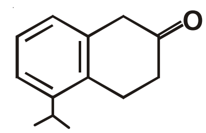
17. The major product of the following reaction is :



(1) 

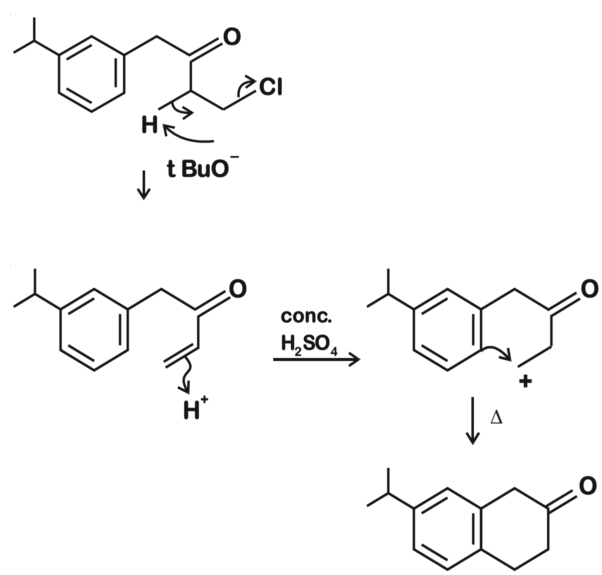
(2) 

(3) 

(4) 

**Answer (2)**

**Sol.**



18. The ion that has $sp^3d^2$ hybridization for the central atom, is :

(1) $[ICl_2]^-$ (2) $[IF_6]^-$

(3) $[BrF_2]^-$ (4) $[ICl_4]^-$

**Answer (4)**

**Sol.**

| Species | Hybridisation |
|---|---|
| $ICl_2^-$ | $sp^3d$ |
| $ICl_4^-$ | $sp^3d^2$ |
| $BrF_2^-$ | $sp^3d$ |
| $IF_6^-$ | $sp^3d^3$ |

19. Calculate the standard cell potential (in V) of the cell in which following reaction takes place :

$Fe^{2+}(aq) + Ag^{+}(aq) \rightarrow Fe^{3+}(aq) + Ag(s)$

Given that

$E^{o}_{Ag^{+}/Ag} = xV$

$E^{o}_{Fe^{2+}/Fe} = yV$

$E^{o}_{Fe^{3+}/Fe} = zV$

(1) x – y  (2) x + y – z

(3) x + 2y – 3z  (4) x – z

**Answer (3)**

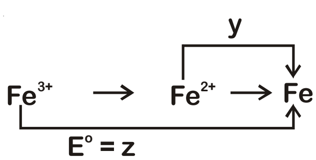
**Sol.**

$$Ag^{+}_{(aq)} + Fe^{+2}_{(aq)} \longrightarrow Fe^{3+}_{(aq)} + Ag_{(s)}$$

$$E^{o}_{cell} = E^{o}_{Ag^{+}/Ag} - E^{o}_{Fe^{3+}/Fe^{2+}}$$

To calculate

$E^{o}_{Fe^{3+}/Fe^{2+}}$




$E^{o}_{Fe^{3+}/Fe^{2+}} = 3z - 2y$

$E^{o}_{Ag^{+}/Ag} = x$

$E^{o}_{cell} = x - 3z + 2y$

20. Which of the following compounds will show the maximum 'enol' content?

(1) $CH_3COCH_2COOC_2H_5$

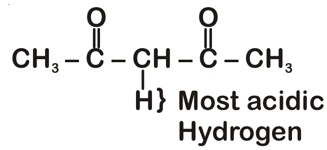
(2) $CH_3COCH_3$

(3) $CH_3COCH_2COCH_3$

(4) $CH_3COCH_2CONH_2$

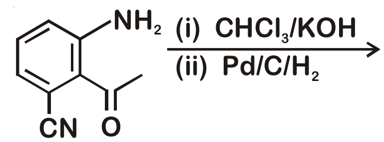
**Answer (3)**

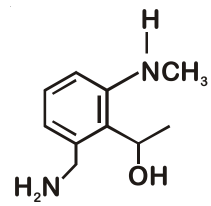
**Sol.** enol content $\alpha$ acidity of active methelene hydrogens.




Maximum enol content

21. The major product obtained in the following reaction is:



(1)


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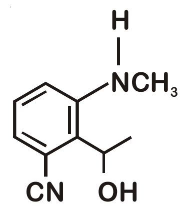

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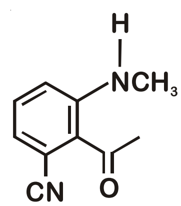
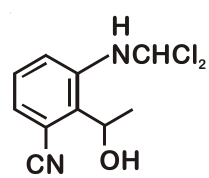
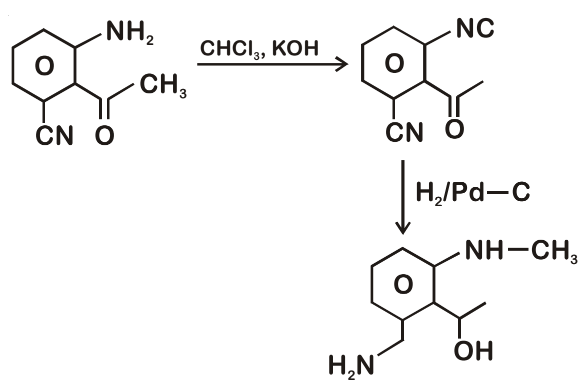
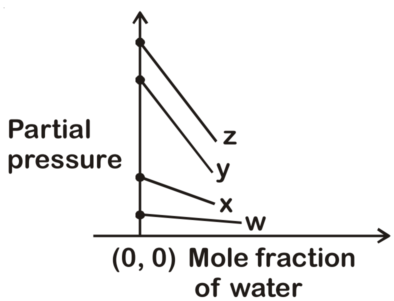


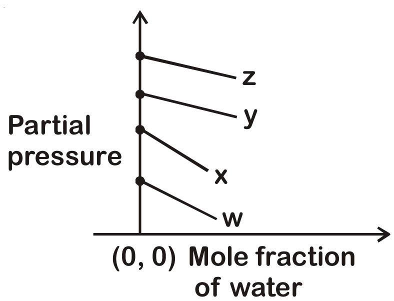
(4)


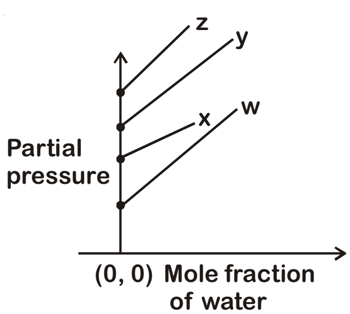
**Answer (1)**

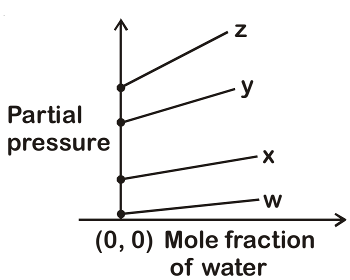
**Sol.**


22. For the solution of the gases w, x, y and z in water at 298 K, the Henrys law constants ($K_H$) are 0.5, 2, 35 and 40 kbar, respectively. The correct plot for the given data is :

(1) 


(2) 


(3) 


(4) 


**Answer (1)**

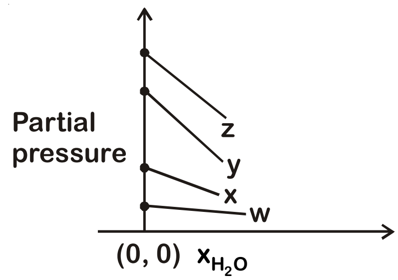
**Sol.** According to Henry's law

$P = K_H \cdot X_{gas}$

$\because \quad x_{gas} = 1 - x_{H_2O}$

$\therefore \quad P = K_H - K_H \cdot X_{H_2O}$

$y = C + mx$




| gas | $K_H$ |
|---|---|
| w | 0.5 |
| x | 2 |
| y | 35 |
| z | 50 |

23. If p is the momentum of the fastest electron ejected from a metal surface after the irradiation of light having wavelength $\lambda$, then for 1.5 p momentum of the photoelectron, the wavelength of the light should be :

(Assume kinetic energy of ejected photoelectron to be very high in comparison to work function):

(1) $\frac{3}{4}\lambda$ (2) $\frac{4}{9}\lambda$

(3) $\frac{2}{3}\lambda$ (4) $\frac{1}{2}\lambda$

**Answer (2)**

**Sol.** In photoelectric effect,

$$\frac{hc}{\lambda} = w + \text{KE of electron}$$

It is given that KE of ejected electron is very high in comparison to w.

$$\frac{hc}{\lambda} = KE \Rightarrow \frac{hc}{\lambda} = \frac{P^2}{2m}$$

New wavelength

$$\frac{hc}{\lambda_1} = \frac{(1.5P)^2}{2m} \Rightarrow \lambda' = \frac{4}{9}\lambda$$

24. Fructose and glucose can be distinguished by:

(1) Fehling's test (2) Seliwanoff's test

(3) Barfoed's test (4) Benedict's test

**Answer (2)**

**Sol.** Seliwanoff's test is used to distinguish aldose and ketose

25. Among the following molecules/ions,

$C_2^{2-}$, $N_2^{2-}$, $O_2^{2-}$, $O_2$

Which one is diamagnetic and has the shortest bond length?

(1) $O_2$ (2) $O_2^{2-}$

(3) $N_2^{2-}$ (4) $C_2^{2-}$

**Answer (4)**

**Sol.** Bond length $\propto \dfrac{1}{\text{bond order}}$

and diamagnetic species has no unpaired electron in their molecular orbitals.

| | Bond order | Magnetic character |
|---|---|---|
| $C_2^{2-}$ | 3 | diamagnetic |
| $N_2^{2-}$ | 2 | paramagnetic |
| $O_2^{2-}$ | 1 | diamagnetic |
| $O_2$ | 2 | paramagnetic |

$\therefore$ $C_2^{2-}$ has least bond length and is diamagnetic

26. For a reaction scheme $A \xrightarrow{k_1} B \xrightarrow{k_2} C$, if the rate of formation of B is set to be zero then the concentration of B is given by :

(1) $k_1k_2[A]$ (2) $(k_1 - k_2)$ [A]

(3) $\left(\dfrac{k_1}{k_2}\right)[A]$ (4) $(k_1 + k_2)$ [A]

**Answer (3)**

**Sol.** $A \xrightarrow{k_1} B \xrightarrow{k_2} C$

$$\frac{d[B]}{dt} = k_1[A] - k_2[B] = 0$$

$$[B] = \frac{k_1[A]}{k_2}$$

27. The maximum prescribed concentration of copper in drinking water is :

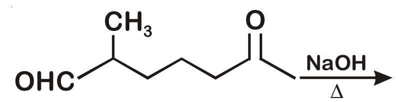
(1) 3 ppm (2) 0.05 ppm

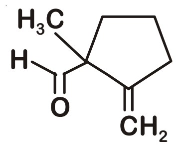
(3) 0.5 ppm (4) 5 ppm

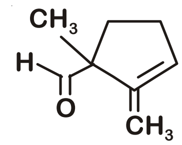
**Answer (1)**

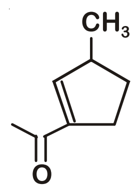
**Sol.** Maximum prescribed concentration of Cu in drinking water is 3 ppm.

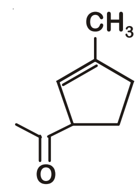
28. The major product obtained in the following reaction is



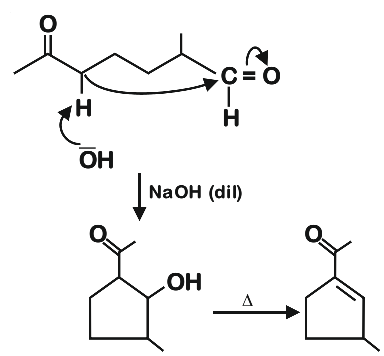

(1) 


(2) 


(3) 


(4) 


**Answer (3)**

**Sol.** 


29. The percentage composition of carbon by mole in methane is

(1) 80% (2) 75%

(3) 20% (4) 25%

**Answer (3)**

**Sol.** In $CH_4$

one atom of carbon among 5 atoms (1C + 4H atoms)

$$\therefore \quad \text{Mole \% of C} = \frac{1}{5} \times 100$$

$$= 20\%$$

30. The statement that is INCORRECT about the interstitial compounds is

(1) They are chemically reactive.

(2) They are very hard.

(3) They have high melting points.

(4) They have metallic conductivity.

**Answer (1)**

**Sol.** Interstitial compounds are inert.

# PART–C : MATHEMATICS

1. The vector equation of plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 which is perpendicular to the plane x – y + z = 0 is :

(1) $\vec{r}\cdot(\hat{i}-\hat{k})+2=0$   (2) $\vec{r}\cdot(\hat{i}-\hat{k})-2=0$

(3) $\vec{r}\times(\hat{i}-\hat{k})+2=0$   (4) $\vec{r}\times(\hat{i}+\hat{k})+2=0$

**Answer (1)**

**Sol.** Equation of the plane passing through the line of intersection of x + y + z = 1 and 2x + 3y + 4z = 5 is

$(2x + 3y + 4z - 5) + \lambda\,(x + y + z - 1) = 0$

$(2 + \lambda)\,x + (3 + \lambda)\,y + (4 + \lambda)\,z + (-5 - \lambda) = 0$ ...(i)

(i) is perpendicular to x – y + z = 0

$\Rightarrow$ $(2 + \lambda)(1) + (3 + \lambda)(-1) + (4 + \lambda)(1) = 0$

$2 + \lambda - 3 - \lambda + 4 + \lambda = 0$

$\lambda = -3$

$\Rightarrow$ Equation of required plane is

$-x + z - 2 = 0$

$\Rightarrow$ $x - z + 2 = 0$

$\Rightarrow$ $\vec{r}\cdot(\hat{i}-\hat{k})+2=0$

2. Which one of the following statements is not a tautology?

(1) $(p \wedge q) \rightarrow (\sim p) \vee q$

(2) $(p \wedge q) \rightarrow p$

(3) $(p \vee q) \rightarrow (p \vee (\sim q))$

(4) $p \rightarrow (p \vee q)$

**Answer (3)**

**Sol.** By help of truth table :

| p | q | ~q | pv~q | ~p | p∧~q | pvq | p→pvq | p∧q | (p∧q)→p | ~pvq |
|---|---|---|---|---|---|---|---|---|---|---|
| T | T | F | T | F | F | T | T | T | T | T |
| T | F | T | T | F | T | T | T | F | T | F |
| F | T | F | F | T | F | T | T | F | T | T |
| F | F | T | T | T | F | F | T | F | T | T |

| $(p\wedge q)\rightarrow(\sim p)vq$ | $(pvq)\rightarrow(pv(\sim q))$ |
|---|---|
| T | T |
| T | T |
| T | F |
| T | T |

3. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is :

(1) 360   (2) 306

(3) 288   (4) 310

**Answer (4)**

**Sol.** 0, 1, 2, 3, 4, 5

| 5 | | | | = 6 × 6 × 6 = 216
6 6 6

| 4 | 5 | | | = 6 × 6 = 36
6 6

| 4 | 4 | | | = 6 × 6 = 36
6 6

| 4 | 3 | | | = 3 × 6 = 18
(5/4/3) 3 6

| 4 | 3 | 2 | | = 4
4

$\Rightarrow$ Required numbers = 216 + 36 + 36 + 18 + 4 = 310

4. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is :

(1) 5   (2) 6

(3) 8   (4) 10

**Answer (1)**

**Sol.** $\because$ Focus is $(0, 5\sqrt{3})$   $\Rightarrow |b| > |a|$

Let b > a > 0

foci (0, ± be)

$a^2 = b^2 - b^2e^2$   $\Rightarrow b^2e^2 = b^2 - a^2$

$be = \sqrt{b^2 - a^2}$

$\Rightarrow\ b^2 - a^2 = 75 \quad ...(i)$

$2b - 2a = 10$

$\Rightarrow\ b - a = 5 \quad ...(ii)$

From (i) and (ii)

$b + a = 15 \quad ...(iii)$

$\Rightarrow\ b = 10, a = 5$

Length of L·R· = $\frac{2a^2}{b} = \frac{50}{10} = 5$

5. A student scores the following marks in five tests : 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is :

(1) $\frac{100}{\sqrt{3}}$ (2) $\frac{10}{\sqrt{3}}$

(3) $\frac{100}{3}$ (4) $\frac{10}{3}$

**Answer (2)**

**Sol.** $\bar{x} = \frac{41+45+54+57+43+x}{6} = 48$

$x + 240 = 288$

$x = 48$

$$\sigma^2 = \frac{1}{6}\begin{bmatrix}(48-41)^2+(48-45)^2+(48-54)^2 \\ +(48-57)^2+(48-43)^2+(48-48)^2\end{bmatrix}$$

$$= \frac{1}{6}(49+9+36+81+25)$$

$$= \frac{200}{6} = \frac{100}{3}$$

$$\sigma = \frac{10}{\sqrt{3}}$$

6. The tangent and the normal lines at the point $(\sqrt{3}, 1)$ to the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The area of this triangle (in square units) is :

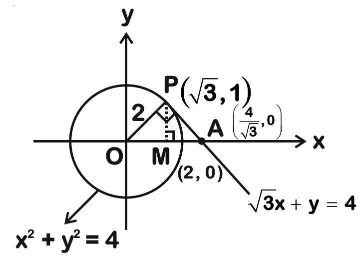
(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{4}{\sqrt{3}}$

(3) $\frac{1}{3}$ (4) $\frac{1}{\sqrt{3}}$

**Answer (1)**

**Sol.** Equation of tangent to circle at point $(\sqrt{3}, 1)$ is

$\sqrt{3}x + y = 4$




$\therefore$ Coordinate of $A = \left(\frac{4}{\sqrt{3}}, 0\right)$

Area = $\frac{1}{2} \times OA \times PM$

$= \frac{1}{2} \times \frac{4}{\sqrt{3}} \times 1 = \frac{2}{\sqrt{3}}$ Square units

7. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10} x}}} + x^{\frac{1}{12}}\right)^6$ is equal to 200, and $x > 1$, then the value of $x$ is :

(1) 10 (2) $10^3$

(3) 100 (4) $10^4$

**Answer (1)**

**Sol.** $T_4 = {}^6C_3\left(\sqrt{x^{\left(\frac{1}{1+\log_{10} x}\right)}}\right)^3 \left(x^{\frac{1}{12}}\right)^3 = 200$

$\Rightarrow\ 20x^{\frac{3}{2(1+\log_{10} x)}} \cdot x^{\frac{1}{4}} = 200$

$x^{\frac{1}{4}+\frac{3}{2(1+\log_{10} x)}} = 10$

Taking $\log_{10}$ on both sides and put $\log_{10} x = t$

$\left(\frac{1}{4} + \frac{3}{2(1+t)}\right)t = 1$

$\left(\frac{(1+t)+6}{4(1+t)}\right) \times t = 1 \quad \Rightarrow \quad t^2 + 7t = 4 + 4t$

$t^2 + 3t - 4 = 0 \quad \Rightarrow \quad t^2 + 4t - t - 4 = 0$

$\Rightarrow\ t(t + 4) - 1(t + 4) = 0$

$\Rightarrow\ t = 1$ or $t = -4$

$\log_{10} x = 1$

$\Rightarrow\ x = 10$ or if $\log_{10} x = -4$

$\Rightarrow\ x = 10^{-4}$

**Note:** There seems a printing error in this question in the original question paper.

$^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant, passes through the point :

(1) $\left(\frac{3}{4}, \frac{7}{4}\right)$ (2) $\left(-\frac{1}{3}, \frac{4}{3}\right)$

(3) $\left(\frac{1}{4}, \frac{3}{4}\right)$ (4) $\left(-\frac{1}{4}, \frac{1}{2}\right)$

**Answer (1)**

**Sol.** Intersection point of

$x^2 + y^2 = 5, \quad y^2 = 4x$

$\Rightarrow x^2 + 4x - 5 = 0$

$\Rightarrow x^2 + 5x - x - 5 = 0$

$\Rightarrow x(x + 5) - 1(x + 5) = 0$

$\therefore x = 1, -5$

Intersection point in 1st quadrant be (1, 2)

equation of tangent to $y^2 = 4x$ at (1, 2) is

$y \times 2 = 2(x + 1)$

$\Rightarrow y = x + 1$

$\Rightarrow x - y + 1 = 0 \quad \ldots(i)$

$\left(\frac{3}{4}, \frac{7}{4}\right)$ lies on (i)

9. If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct?

(1) d, e, f are in A.P. (2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

(3) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P. (4) d, e, f are in G.P.

**Answer (3)**

**Sol.** Since a, b, c are in G.P.

$\Rightarrow b^2 = ac$

Given, $ax^2 + 2bx + c = 0$

$\Rightarrow ax^2 + 2\sqrt{ac}\,x + c = 0$

$\Rightarrow \left(\sqrt{a}\,x + \sqrt{c}\right)^2 = 0$

$\Rightarrow x = -\sqrt{\frac{c}{a}}$

$\because$ $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have common root

$\Rightarrow x = -\sqrt{\frac{c}{a}}$ must satisfy $dx^2 + 2ex + f = 0$

$\Rightarrow d \cdot \frac{c}{a} + 2e\left(-\sqrt{\frac{c}{a}}\right) + f = 0$

$\frac{d}{a} - \frac{2e}{\sqrt{ac}} + \frac{f}{c} = 0$

$\Rightarrow \frac{d}{a} - \frac{2e}{b} + \frac{f}{c} = 0$

$\Rightarrow \frac{2e}{b} = \frac{d}{a} + \frac{f}{c}$

$\Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

10. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2} \left(i = \sqrt{-1}\right)$, then $\left(1 + iz + z^5 + iz^8\right)^9$ is equal to :

(1) 0 (2) $(-1 + 2i)^9$

(3) $-1$ (4) 1

**Answer (3)**

**Sol.** $z = \frac{\sqrt{3}}{2} + \frac{i}{2} = -i\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -i\omega$

where $\omega$ is not real cube root of unity

$\Rightarrow (1 + iz + z^5 + iz^8)^9 = (1 + \omega - i\omega^2 + i\omega^2)^9$

$= (1 + \omega)^9$

$= (-\omega^2)^9$

$= -\omega^{18}$

$= -1$

11. The number of integral values of m for which the equation $(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ has no real root is :

(1) Infinitely many (2) 3

(3) 2 (4) 1

**Answer (1)**

**Sol.** $(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$

equation has no real solution

$\Rightarrow D < 0$

$4(1 + 3m)^2 < 4(1 + m^2)(1 + 8m)$

$1 + 9m^2 + 6m < 1 + 8m + m^2 + 8m^3$

$8m^3 - 8m^2 + 2m > 0$

$2m(4m^2 - 4m + 1) > 0$

$2m(2m - 1)^2 > 0$

$m > 0,\ m \neq \frac{1}{2}$

$\Rightarrow$ number of integral values of m are infinitely many.

12. Let the numbers 2, b, c be in an A.P. and $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If det(A) $\in$ [2, 16], then c lies in the interval :

(1) [2, 3] (2) $(2 + 2^{3/4}, 4)$

(3) $[3, 2 + 2^{3/4}]$ (4) [4, 6]

**Answer (4)**

**Sol.** $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{vmatrix}$

$c_2 \to c_2 - c_1,\ c_3 \to c_3 - c_1$

$$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 2 & b-2 & c-2 \\ 4 & (b-2)(b+2) & (c-2)(c+2) \end{vmatrix}$$

$$= (b-2)(c-2)(c-b)$$

2, b, c are in A.P. $\Rightarrow (b - 2) = (c - b) = d,\ c - 2 = 2d$

$\Rightarrow |A| = d.2d.d = 2d^3$

$\because |A| \in [2,16] \Rightarrow 1 \le d^3 \le 8 \Rightarrow 1 \le d \le 2$

$4 \le 2d + 2 \le 6 \ \Rightarrow 4 \le c \le 6$

13. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is :

(1) 5 (2) 2

(3) 4 (4) 3

**Answer (3)**

**Sol.** $p = P(H) = \frac{1}{2},\ q = 1 - p = \frac{1}{2}$

$P(x \ge 1) \ge \frac{9}{10}$

$1 - P(x = 0) \ge \frac{9}{10}$

$1 - {}^nC_0\left(\frac{1}{2}\right)^n \ge \frac{9}{10}$

$\frac{1}{2^n} \le 1 - \frac{9}{10} \Rightarrow \frac{1}{2^n} \le \frac{1}{10}$

$2^n \ge 10 \quad \Rightarrow n \ge 4$

$\Rightarrow n_{min} = 4$

14. Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x. Then $|\vec{a} \times \vec{b}| = r$ is possible if :

(1) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$

(2) $\sqrt{\frac{3}{2}} < r \le 3\sqrt{\frac{3}{2}}$

(3) $0 < r \le \sqrt{\frac{3}{2}}$

(4) $r \ge 5\sqrt{\frac{3}{2}}$

**Answer (4)**

**Sol.** $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix}$

$= (2 + x)\hat{i} + (x - 3)\hat{j} - 5\hat{k}$

$|\vec{a} \times \vec{b}| = r = \sqrt{(2+x)^2 + (x-3)^2 + (-5)^2}$

$\Rightarrow r = \sqrt{4 + x^2 + 4x + x^2 + 9 - 6x + 25}$

$= \sqrt{2x^2 - 2x + 38} = \sqrt{2\left(x^2 - x + \frac{1}{4}\right) + 38 - \frac{1}{2}}$

$= \sqrt{2\left(x - \frac{1}{2}\right)^2 + \frac{75}{2}}$

$\Rightarrow r \ge \sqrt{\frac{75}{2}} \Rightarrow r \ge 5\sqrt{\frac{3}{2}}$

15. The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to :

(1) $2-\frac{3}{2^{17}}$

(2) $1-\frac{11}{2^{20}}$

(3) $2-\frac{21}{2^{20}}$

(4) $2-\frac{11}{2^{19}}$

**Answer (4)**

**Sol.** $S=\sum_{k=1}^{20} k.\frac{1}{2^k}$

$$S=\frac{1}{2}+2.\frac{1}{2^2}+3.\frac{1}{2^3}+....+20.\frac{1}{2^{20}}$$

$$\frac{1}{2}S=\frac{1}{2^2}+2.\frac{1}{2^3}+....+19\frac{1}{2^{20}}+20\frac{1}{2^{21}}$$

On subtracting

$$\frac{S}{2}=\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+....+\frac{1}{2^{20}}\right)-20\frac{1}{2^{21}}$$

$$=\frac{\frac{1}{2}\left(1-\frac{1}{2^{20}}\right)}{1-\frac{1}{2}}-20.\frac{1}{2^{21}}=1-\frac{1}{2^{20}}-10.\frac{1}{2^{20}}$$

$$\frac{S}{2}=1-11.\frac{1}{2^{20}}\Rightarrow S=2-11.\frac{1}{2^{19}}=2-\frac{11}{2^{19}}$$

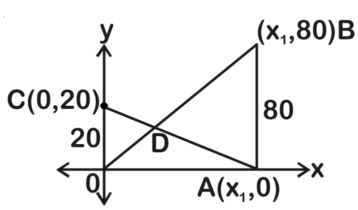
16. Two vertical poles of heights, 20 m and 80 m stand apart on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is :

(1) 16 (2) 18

(3) 15 (4) 12

**Answer (1)**

**Sol.**




equation of line OB and AC are respectively

$$y=\frac{80}{x_1}x \qquad ...(i)$$

$$\frac{x}{x_1}+\frac{y}{20}=1 \qquad ...(ii)$$

For intersection point, from equations (i) and (ii)

$$\frac{y}{80}+\frac{y}{20}=1$$

$\Rightarrow$ y + 4y = 80

$\Rightarrow$ y = 16 m

$\Rightarrow$ Height of intersection point is 16 m

17. Let f : [–1, 3] $\rightarrow$ R be defined as

$$f(x)=\begin{cases}|x|+[x], & -1\le x<1\\ x+|x|, & 1\le x<2\\ x+[x], & 2\le x\le 3,\end{cases}$$

where [t] denotes the greatest integer less than or equal to t. Then, f is discontinuous at :

(1) Only one point

(2) Only two points

(3) Only three points

(4) Four or more points

**Answer (3)**

**Sol.** $f(x)=\begin{cases}|x|+[x], & -1\le x<1\\ x+|x|, & 1\le x<2\\ x+[x], & 2\le x\le 3\end{cases}$

$$=\begin{cases}-x-1, & -1\le x<0\\ x, & 0\le x<1\\ 2x, & 1\le x<2\\ x+2, & 2\le x<3\\ 6, & x=3\end{cases}$$

$\Rightarrow$ $f(-1) = 0$, $f(-1^+) = 0$

$f(0^-) = -1$, $f(0) = 0$, $f(0^+) = 0$

$f(1^-) = 1$, $f(1) = 2$, $f(1^+) = 2$

$f(2^-) = 4$, $f(2) = 4$, $f(2^+) = 4$

$f(3^-) = 5$, $f(3) = 6$

f(x) is discontinuous at x = {0, 1, 3}

18. Given that the slope of the tangent to a curve $y = y(x)$ at any point $(x,y)$ is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is

(1) $x \log_e|y| = -2(x-1)$

(2) $x \log_e|y| = x - 1$

(3) $x \log_e|y| = 2(x-1)$

(4) $x^2 \log_e|y| = -2(x-1)$

**Answer (3)**

**Sol.** $\frac{dy}{dx} = \frac{2y}{x^2} \Rightarrow \int \frac{dy}{y} = 2\int \frac{dx}{x^2}$

$\Rightarrow \ln|y| = -\frac{2}{x} + C$ ...(i)

(i) passes through (1, 1)

$\Rightarrow C = 2$

$\Rightarrow \ln|y| = -\frac{2}{x} + 2$

$x\ln|y| = -2 + 2x$

$x\ln|y| = -2(1-x) = 2(x-1)$

19. Suppose that the points $(h, k)$, $(1, 2)$ and $(-3, 4)$ lie on the line $L_1$. If a line $L_2$ passing through the points $(h, k)$ and $(4, 3)$ is perpendicular to $L_1$, then $\frac{k}{h}$ equals

(1) 3 (2) $-\frac{1}{7}$

(3) 0 (4) $\frac{1}{3}$

**Answer (4)**

**Sol.** (h, k), (1, 2) and (–3, 4) and collinear

$$\begin{vmatrix} h & k & 1 \\ 1 & 2 & 1 \\ -3 & 4 & 1 \end{vmatrix} = 0 \Rightarrow -2h - 4k + 10 = 0$$

$\Rightarrow h + 2k = 5$ ...(i)

$m_{L_1} = \frac{4-2}{-3-1} = \frac{2}{-4} = -\frac{1}{2} \Rightarrow m_{L_2} = 2$

$\Rightarrow m_{L_2} = \frac{k-3}{h-4} = 2 \Rightarrow k - 3 = 2h - 8$

$2h - k = 5$ ...(ii)

from (i) and (ii)

$h = 3, k = 1 \Rightarrow \frac{k}{h} = \frac{1}{3}$

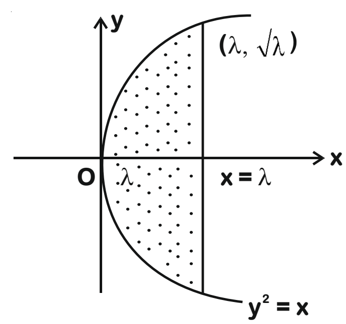
20. Let $S(\alpha) = \{(x, y) : y^2 \le x, 0 \le x \le \alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a $\lambda$, $0 < \lambda < 4$, $A(\lambda) : A(4) = 2 : 5$, then $\lambda$ equals

(1) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (2) $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$

(3) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (4) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

**Answer (4)**

**Sol.**




$A(\lambda) = 2 \times \frac{2}{3}(\lambda \times \sqrt{\lambda}) = \frac{4}{3}\lambda^{3/2}$

$\Rightarrow \frac{A(\lambda)}{A(4)} = \frac{2}{5} \Rightarrow \frac{\lambda^{3/2}}{8} = \frac{2}{5}$

$\lambda = \left(\frac{16}{5}\right)^{2/3} = 4.\left(\frac{4}{25}\right)^{1/3}$

21. If a point $R(4, y, z)$ lies on the line segment joining the points $P(2, -3, 4)$ and $Q(8, 0, 10)$, then the distance of R from the origin is

(1) $\sqrt{53}$ (2) $2\sqrt{21}$

(3) 6 (4) $2\sqrt{14}$

**Answer (4)**

**Sol.** P, Q, R are collinear

$\Rightarrow \overrightarrow{PR} = \lambda\overrightarrow{PQ}$

$2\hat{i} + (y+3)\hat{j} + (z-4)\hat{k} = \lambda[6\hat{i} + 3\hat{j} + 6\hat{k}]$

$\Rightarrow 6\lambda = 2, y + 3 = 3\lambda, z - 4 = 6\lambda$

$\Rightarrow \lambda = \frac{1}{3}, y = -2, z = 6$

$\Rightarrow$ point R (4, –2, 6)

$\Rightarrow OR = \sqrt{(4)^2 + (-2)^2 + (6)^2} = \sqrt{16 + 4 + 36}$

$= \sqrt{56} = 2\sqrt{14}$

22. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is

(1) 4 : 5 : 6

(2) 3 : 4 : 5

(3) 5 : 9 : 13

(4) 5 : 6 : 7

**Answer (1)**

**Sol.** Let a > b > c

$\because$ A = 2C

$\Rightarrow$ A + B + C = $\pi$

$\Rightarrow$ B = $\pi$ – 3C ...(i)

$\because$ a + c = 2b

$\Rightarrow$ sinA + sinC = 2sinB ...(ii)

$\Rightarrow$ sinA = sin(2C), sinB = sin3C

$\Rightarrow$ From (ii), sin2C + sinC = 2sin3C

(2cosC + 1)sinC = 2sinC (3 – 4 $\sin^2$C)

$\Rightarrow$ 2cosC + 1 = 6 – 8(1 – $\cos^2$C)

$\Rightarrow$ $8\cos^2$C – 2cosC – 3 =0

$$\Rightarrow \cos C = \frac{3}{4} \text{ or } \cos C = -\frac{1}{2}$$

$\because$ C is acute angle

$$\Rightarrow \cos C = \frac{3}{4}, \sin A = 2\sin C\cos C = 2\times\frac{\sqrt{7}}{4}\times\frac{3}{4}$$

$$\Rightarrow \sin C = \frac{\sqrt{7}}{4}, \sin B = \frac{3\sqrt{7}}{4} - \frac{4\sqrt{7}}{4}\times\frac{7}{16} = \frac{5\sqrt{7}}{16}$$

$\Rightarrow$ sinA : sinB : sinC :: a : b : c is 6 : 5 : 4

23. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is

(1) $\frac{2}{3}\sqrt{3}$

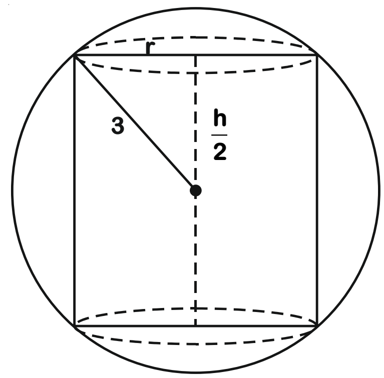
(2) $2\sqrt{3}$

(3) $\sqrt{3}$

(4) $\sqrt{6}$

**Answer (2)**

**Sol.** Let radius of base and height of cylinder be r and h respectively.




$$\therefore \quad r^2 + \frac{h^2}{4} = 9 \quad ...(i)$$

$\because$ Volume of cylinder

V = $\pi r^2 h$

$$V = \pi h\left(9 - \frac{h^2}{4}\right)$$

$$V = 9\pi h - \frac{\pi}{4}h^3$$

$$\therefore \quad \frac{dV}{dh} = 9\pi - \frac{3}{4}\pi h^2$$

For maxima/minima

$$\frac{dV}{dh} = 0$$

$$\Rightarrow \quad h = \sqrt{12}$$

$$\text{and } \frac{d^2V}{dh^2} = -\frac{3}{2}\pi h$$

$$\therefore \quad \left(\frac{d^2V}{dh^2}\right)_{h=\sqrt{12}} < 0$$

$\Rightarrow$ Volume is maximum when $h = 2\sqrt{3}$

24. If the system of linear equations

x – 2y + kz = 1

2x + y + z = 2

3x – y – kz = 3

has a solution (x, y, z), z $\neq$ 0, then (x, y) lies on the straight line whose equation is

(1) 3x – 4y – 4 = 0 (2) 3x – 4y – 1 = 0

(3) 4x – 3y – 1 = 0 (4) 4x – 3y – 4 = 0

**Answer (4)**

**Sol.** $x - 2y + kz = 1,\ 2x + y + z = 2,\ 3x - y - kz = 3$

$$\Delta = \begin{vmatrix} 1 & -2 & k \\ 2 & 1 & 1 \\ 3 & -1 & -k \end{vmatrix} = 1(-k+1) + 2(-2k-3) + k(-2-3)$$

$$= -k + 1 - 4k - 6 - 5k = -10k - 5 = -5(2k+1)$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 & k \\ 2 & 1 & 1 \\ 3 & -1 & -k \end{vmatrix} = -5(2k+1)$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & k \\ 2 & 2 & 1 \\ 3 & 3 & -k \end{vmatrix} = 0,\ \Delta_3 = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & 2 \\ 3 & -1 & 3 \end{vmatrix} = 0$$

$\because\ z \neq 0$

$\Rightarrow\ \Delta = 0$

$\Rightarrow\ k = -\frac{1}{2}$

$\therefore$ System of equation has infinite many solutions.

Let $z = \lambda \neq 0$ then $x = \frac{10 - 3\lambda}{10}$ and $y = -\frac{2\lambda}{5}$

$\therefore$ (x, y) must lie on line $4x - 3y - 4 = 0$

25. If $f(1) = 1$, $f'(1) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is :

(1) 33 (2) 12

(3) 9 (4) 15

**Answer (1)**

**Sol.** Let $g(x) = f(f(f(x))) + (f(x))^2$

On differentiating both sides w.r.t. x we get

$g'(x) = f'(f(f(x)))\ f'(f(x))\ f'(x) + 2f(x)\ f'(x)$

$g'(1) = f'(f(f(1)))\ f'(f(1))\ f'(1) + 2f(1)\ f'(1)$

$= f'(f(1))\ f'(1)\ f'(1) + 2f(1)\ f'(1)$

$= 3 \times 3 \times 3 + 2 \times 1 \times 3 = 27 + 6 = 33$

26. Let $f(x) = \int_0^x g(t)dt$, where g is a non-zero even function. If $f(x + 5) = g(x)$, then $\int_0^x f(t)dt$, equals:

(1) $\int_{x+5}^{5} g(t)dt$ (2) $2\int_{5}^{x+5} g(t)dt$

(3) $\int_{5}^{x+5} g(t)dt$ (4) $5\int_{x+5}^{5} g(t)dt$

**Answer (1)**

**Sol.** $f(x) = \int_0^x g(t)dt,$ ...(i)

$g(-x) = g(x),$ ...(ii)

$f(x + 5) = g(x)$ ...(iii)

From (i)

$f'(x) = g(x)$

Let $I = \int_0^x f(t)dt,$

Put $t = \lambda - 5$

$\Rightarrow\ I = \int_5^{x+5} f(\lambda - 5)d\lambda$

$\because\ f(x + 5) = g(x)$

$\Rightarrow\ f(-x + 5) = g(-x) = g(x)$ ...(iv)

$I = \int_5^{x+5} f(\lambda - 5)d\lambda,$

$I = \int_5^{x+5} -f(5 - \lambda)d\lambda$

($\because$ $f(0) = 0$, $g(x)$ is even $\Rightarrow$ $f(x)$ is odd)

$I = -\int_5^{x+5} g(\lambda)d\lambda = \int_{x+5}^{5} g(t)dt$ (from (iv))

27. Let $f(x) = a^x$ $(a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals:

(1) $2f_1(x)f_1(y)$ (2) $2f_1(x + y)f_1(x - y)$

(3) $2f_1(x + y)f_2(x - y)$ (4) $2f_1(x)f_2(y)$

**Answer (1)**

**Sol.** $f(x) = a^x = \left(\frac{a^x + a^{-x}}{2}\right) + \left(\frac{a^x - a^{-x}}{2}\right)$

where $f_1(x) = \frac{a^x + a^{-x}}{2}$ is even function

$f_2(x) = \frac{a^x - a^{-x}}{2}$ is odd function

$\Rightarrow\ f_1(x + y) + f_1(x - y)$

$= \left(\frac{a^{x+y} + a^{-x-y}}{2}\right) + \left(\frac{a^{x-y} + a^{-x+y}}{2}\right)$

$= \frac{1}{2}\left[a^x(a^y + a^{-y}) + a^{-x}(a^y + a^{-y})\right]$

$= \frac{(a^x + a^{-x})(a^y + a^{-y})}{2}$

$= 2f_1(x).f_1(y)$

28. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}} + C$ where C is a constant of integration, then the function f(x) is equal to:

(1) $-\frac{1}{2x^3}$ (2) $\frac{3}{x^2}$

(3) $-\frac{1}{6x^3}$ (4) $-\frac{1}{2x^2}$

**Answer (1)**

**Sol.** $I = \int \frac{dx}{x^3(1+x^6)^{2/3}} = \int \frac{dx}{x^7(1+x^{-6})^{2/3}}$

Put $1 + x^{-6} = t^3$

$\Rightarrow -6x^{-7}dx = 3t^2\,dt$

$\Rightarrow \frac{dx}{x^7} = -\frac{1}{2}t^2dt$

$\Rightarrow I = \int -\frac{1}{2}\frac{t^2dt}{t^2}$

$= -\frac{1}{2}t + C$

$= -\frac{1}{2}(1+x^{-6})^{\frac{1}{3}} + C$

$= -\frac{1}{2}\frac{(1+x^6)^{\frac{1}{3}}}{x^2} + C$

$= -\frac{1}{2x^3}x(1+x^6)^{\frac{1}{3}} + C$

$\Rightarrow f(x) = -\frac{1}{2x^3}$

29. If the eccentricity of the standard hyperbola passing through the point (4, 6) is 2, then the equation of the tangent to the hyperbola at (4, 6) is:

(1) $2x - 3y + 10 = 0$ (2) $x - 2y + 8 = 0$

(3) $3x - 2y = 0$ (4) $2x - y - 2 = 0$

**Answer (4)**

**Sol.** Let equation of hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{...(i)}$$

$\Rightarrow b^2 = a^2(e^2 - 1)$

$\because$ e = 2 $\Rightarrow$ $\boxed{b^2 = 3a^2}$ ...(ii)

(i) passes through (4, 6)

$$\Rightarrow \frac{16}{a^2} - \frac{36}{b^2} = 1 \quad \text{...(iii)}$$

From (ii) and (iii)

$a^2 = 4, b^2 = 12$

$\Rightarrow$ Equation of hyperbola is $\frac{x^2}{4} - \frac{y^2}{12} = 1$

Equation of tangent to the hyperbola at (4, 6) is

$\frac{4x}{4} - \frac{6y}{12} = 1$

$\Rightarrow x - \frac{y}{2} = 1$

$\Rightarrow 2x - y = 2$

30. Let $f : R \to R$ be a differentiable function satisfying $f'(3) + f'(2) = 0$.

Then $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to:

(1) e (2) 1

(3) $e^2$ (4) $e^{-1}$

**Answer (2)**

**Sol.** $I = \lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$

form : $1^\infty$

$\Rightarrow I = e^{I_1}$, where

$I_1 = \lim_{x\to 0}\left(\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)} - 1\right)\frac{1}{x}\right)$

$= \lim_{x\to 0}\left(\left(\frac{f(3+x)-f(3)-f(2-x)+f(2)}{1+f(2-x)-f(2)}\right)\frac{1}{x}\right)$

form : $\frac{0}{0}$

Using L.H. Rule

$I_1 = \lim_{x\to 0}\left(\frac{f'(3+x)+f'(2-x)}{1}\right).\lim_{x\to 0}\left(\frac{1}{1+f(2-x)-f(2)}\right)$

$= f'(3) + f'(2) = 0$

$\Rightarrow I = e^{I_1} = 1$