TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On SATURDAY 12th JANUARY., 2019) TIME : 09 : 30 AM To 12 : 30 PM

MATHEMATICS

1. For $x > 1$, if $(2x)^{2y} = 4e^{2x-2y}$, then $\left(1+\log_e 2x\right)^2 \frac{dy}{dx}$ is equal to :

(1) $\log_e 2x$

(2) $\frac{x\log_e 2x + \log_e 2}{x}$

(3) $x\log_e 2x$

(4) $\frac{x\log_e 2x - \log_e 2}{x}$

Ans. **(4)**

Sol. $(2x)^{2y} = 4e^{2x-2y}$

$2y\ell n2x = \ell n4 + 2x - 2y$

$$y = \frac{x+\ell n2}{1+\ell n2x}$$

$$y' = \frac{(1+\ell n2x) - (x+\ell n2)\frac{1}{x}}{(1+\ell n2x)^2}$$

$$y'(1+\ell n2x)^2 = \left[\frac{x\ell n2x - \ell n2}{x}\right]$$

2. The sum of the distinct real values of μ, for which the vectors, $\mu\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+\mu\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+\mu\hat{k}$ are co-planer, is :

(1) 2 (2) 0 (3) –1 (4) 1

Ans. **(3)**

Sol. $$\begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0$$

$\mu(\mu^2 - 1) - 1(\mu-1) + 1(1-\mu) = 0$

$\mu^3 - \mu - \mu + 1 + 1\ \mu = 0$

$\mu^3 - 3\mu + 2 = 0$

$\mu^3 - 1 - 3(\mu-1) = 0$

$\mu = 1,\ \mu^2 + \mu - 2 = 0$

$\mu = 1,\ \mu = -2$

sum of distinct solutions = –1

3. Let S be the set of all points in $(-\pi,\pi)$ at which the function, $f(x) = \min\{\sin x, \cos x\}$ is not differentiable. Then S is a subset of which of the following?

(1) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right\}$

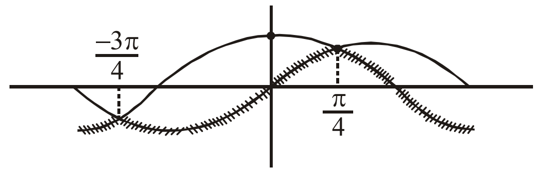
(2) $\left\{-\frac{3\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}$

(3) $\left\{-\frac{\pi}{2}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\right\}$

(4) $\left\{-\frac{\pi}{4}, 0, \frac{\pi}{4}\right\}$

Ans. **(1)**

Sol.



4. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now from an A.P. Then the sum of the original three terms of the given G.P. is

(1) 36 (2) 24

(3) 32 (4) 28

Ans. **(4)**

Sol. Let terms are $\frac{a}{r}, a, ar \rightarrow$ G.P

$\therefore\ a^3 = 512 \Rightarrow a = 8$

$\frac{8}{r} + 4, 12, 8r \rightarrow$ A.P.

$24 = \frac{8}{r} + 4 + 8r$

$r = 2,\ r = \frac{1}{2}$

$r = 2\ (4, 8, 16)$

$r = \frac{1}{2}\ (16, 8, 4)$

Sum = 28

5. The integral $\int \cos(\log_e x)dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{x}{2}[\sin(\log_e x) - \cos(\log_e x)] + C$

(2) $\frac{x}{2}[\cos(\log_e x) + \sin(\log_e x)] + C$

(3) $x[\cos(\log_e x) + \sin(\log_e x)] + C$

(4) $x[\cos(\log_e x) - \sin(\log_e x)] + C$

Ans. (2)

Sol. $I = \int \cos(\ell n\, x)dx$

$I = \cos(\ln x).x + \int \sin(\ell n\, x)\, dx$

$\cos(\ell n\, x)x + [\sin(\ell n\, x).x - \int \cos(\ell n\, x)dx]$

$I = \frac{x}{2}[\sin(\ell n\, x) + \cos(\ell n\, x)] + C$

6. Let $S_k = \frac{1+2+3+....+k}{k}$. If

$S_1^2 + S_2^2 + ... + S_{10}^2 = \frac{5}{12}A$, then A is equal to :

(1) 303 (2) 283 (3) 156 (4) 301

Ans. (1)

Sol. $S_K = \frac{K+1}{2}$

$\Sigma S_k^2 = \frac{5}{12}A$

$\sum_{K=1}^{10}\left(\frac{K+1}{2}\right)^2 = \frac{2^2+3^2+--+11^2}{4} = \frac{5}{12}A$

$\frac{11\times 12\times 23}{6} - 1 = \frac{5}{3}A$

$505 = \frac{5}{3}A$, A = 303

7. Let S = {1,2,3, ..., 100}. The number of non-empty subsets A of S such that the product of elements in A is even is :-

(1) $2^{50}(2^{50}-1)$ (2) $2^{100}-1$

(3) $2^{50}-1$ (4) $2^{50}+1$

Ans. (1)

Sol. S = {1,2,3------100}

= Total non empty subsets-subsets with product of element is odd

$= 2^{100}-1-1[(2^{50}-1)]$

$= 2^{100} - 2^{50}$

$= 2^{50}(2^{50}-1)$

8. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observation is :

(1) 50 (2) 51 (3) 30 (4) 31

Ans. (4)

Sol. $\sum_{i=1}^{50}(x_i - 30) = 50$

$\Sigma x_i = 50\times 30 = 50$

$\Sigma x_i = 50 + 50 + 30$

Mean = $\bar{x} = \frac{\Sigma x_i}{n} = \frac{50\times 30 + 50}{50} = 30 + 1 = 31$

9. If a variable line, $3x+4y-\lambda=0$ is such that the two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2+y^2-18x-2y+78 = 0$ are on its opposite sides, then the set of all values of λ is the interval :-

(1) [12, 21] (2) (2, 17)

(3) (23, 31) (4) [13, 23]

Ans. (1)

Sol. Centre of circles are opposite side of line

$(3 + 4 - \lambda)(27 + 4 - \lambda) < 0$

$(\lambda - 7)(\lambda - 31) < 0$

$\lambda \in (7, 31)$

distance from S_1

$\left|\frac{3+4-\lambda}{5}\right| \geq 1 \Rightarrow \lambda \in (-\infty, 2] \cup [(12, \infty]$

distance from S_2

$$\left|\frac{27+4-\lambda}{5}\right| \geq 2 \Rightarrow \lambda \in (-\infty, 21] \cup [41, \infty)$$

so $\lambda \in [12, 21]$

10. A ratio of the 5th term from the beginning to the 5th term from the end in the binomial expansion of $\left(2^{\frac{1}{3}} + \frac{1}{2(3)^{\frac{1}{3}}}\right)^{10}$ is :

(1) $1 : 4(16)^{\frac{1}{3}}$ (2) $1 : 2(6)^{\frac{1}{3}}$

(3) $2(36)^{\frac{1}{3}} : 1$ (4) $4(36)^{\frac{1}{3}} : 1$

Ans. (4)

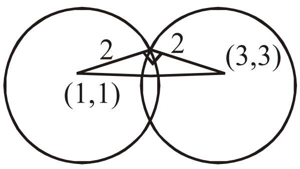
Sol. $$\frac{T_5}{T_5^1} = \frac{{}^{10}C_4 (2^{1/3})^{10-4}\left(\frac{1}{2(3)^{1/3}}\right)^4}{{}^{10}C_4\left(\frac{1}{2(3^{1/3})}\right)^{10-4}(2^{1/3})^4} = 4.(36)^{1/3}$$

11. let C_1 and C_2 be the centres of the circles $x^2+y^2-2x-2y-2 = 0$ and $x^2+y^2-6x-6y+14 = 0$ respectively. If P and Q are the points of intersection of these circles, then the area (in sq. units) of the quadrilateral PC_1QC_2 is :

(1) 8 (2) 6 (3) 9 (4) 4

Ans. (4)

Sol.



Area = $2 \times \frac{1}{2}.4 = 2$

12. In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to :

(1) $\frac{150}{6^5}$ (2) $\frac{175}{6^5}$ (3) $\frac{200}{6^5}$ (4) $\frac{225}{6^5}$

Ans. (2)

Sol. $\_\ \_\ \_\ \underline{4}\ \underline{4}$

$$\frac{1}{6^2}\left(\frac{5^3}{6^3} + \frac{2C_1.5^2}{6^3}\right) = \frac{175}{6^5}$$

13. If the straight line, $2x-3y+17 = 0$ is perpendicular to the line passing through the points (7, 17) and (15, β), then β equals :-

(1) -5 (2) $-\frac{35}{3}$

(3) $\frac{35}{3}$ (4) 5

Ans. (4)

Sol. $\frac{17-\beta}{-8} \times \frac{2}{3} = -1$

$\beta = 5$

14. Let f and g be continuous functions on [0, a] such that $f(x) = f(a-x)$ and $g(x)+g(a-x)=4$, then $\int_0^a f(x)g(x)dx$ is equal to :-

(1) $4\int_0^a f(x)dx$ (2) $2\int_0^a f(x)dx$

(3) $-3\int_0^a f(x)dx$ (4) $\int_0^a f(x)dx$

Ans. (2)

Sol. $I = \int_0^a f(x)g(x)dx$

$I = \int_0^a f(a-x)g(a-x)dx$

$I = \int_0^a f(x)(4-g(x)dx$

$I = 4\int_0^a f(x)dx - I$

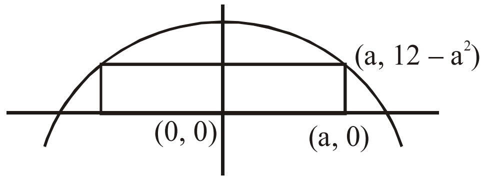
$\Rightarrow I = 2\int_0^a f(x)dx$

15. The maximum area (in sq. units) of a rectangle having its base on the x-axis and its other two vertices on the parabola, $y = 12-x^2$ such that the rectangle lies inside the parabola, is :-

(1) $20\sqrt{2}$ (2) $18\sqrt{3}$ (3) 32 (4) 36

Ans. (3)

Sol. $f(a) = 2a(12 - a)^2$



$f'(a) = 2(12 - 3a^2)$

maximum at $a = 2$

maximum area $= f(2) = 32$

16. The Boolean expression

$((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ is equivalent to:

(1) $p \wedge (\sim q)$ (2) $p \vee (\sim q)$

(3) $(\sim p) \wedge (\sim q)$ (4) $p \wedge q$

Ans. (3)

17. $\lim\limits_{x \to \pi/4} \dfrac{\cot^3 x - \tan x}{\cos\left(x + \pi/4\right)}$ is :

(1) 4 (2) $8\sqrt{2}$ (3) 8 (4) $4\sqrt{2}$

Ans. (3)

Sol. $\lim\limits_{x \to \pi/4} \dfrac{\cot^3 x - \tan x}{\cos\left(x + \dfrac{\pi}{4}\right)}$

$\lim\limits_{x \to \pi/4} \dfrac{(1 - \tan^4 x)}{\cos(x + \pi/4)}$

$2\lim\limits_{x \to \pi/4} \dfrac{(1 - \tan^2 x)}{\cos(x + \pi/4)}$

$R\lim\limits_{x \to \pi/4} \dfrac{\cos^2 x - \sin^2 x}{\dfrac{\cos x - \sin x}{\sqrt{2}}} \dfrac{1}{\cos^2 x}$

$4\sqrt{2}\lim\limits_{x \to \pi/4} (\cos x + \sin x) = 8$

18. Considering only the principal values of inverse functions, the set $A = \left\{x \geq 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \dfrac{\pi}{4}\right\}$

(1) is an empty set

(2) Contains more than two elements

(3) Contains two elements

(4) is a singleton

Ans. (4)

Sol. $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$

$\Rightarrow \dfrac{5x}{1 - 6x^2} = 1$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$x = -1$ or $x = \dfrac{1}{6}$

$x = \dfrac{1}{6} \quad \because x > 0$

19. An ordered pair(α, β) for which the system of linear equations

$(1+\alpha)x + \beta y + z = 2$

$\alpha x + (1+\beta)y + z = 3$

$\alpha x + \beta y + 2z = 2$ has a unique solution is

(1) (1,–3) (2) (–3,1)

(3) (2, 4) (4) (–4, 2)

Ans. (3)

Sol. For unique solution

$$\Delta \neq 0 \Rightarrow \begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

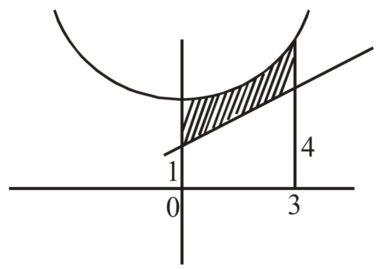
$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0 \Rightarrow \alpha + \beta \neq -2$$

20. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is :

(1) $\dfrac{15}{4}$ (2) $\dfrac{15}{2}$ (3) $\dfrac{21}{2}$ (4) $\dfrac{17}{4}$

Ans. (2)

Sol.



Req. area $= \int_0^3 (x^2 + 2)dx - \dfrac{1}{2}.5.3 = 9 + 6 - \dfrac{15}{2} = \dfrac{15}{2}$

21. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2+m(m-4)x+2 = 0$, then the least value of m for which $\lambda+\frac{1}{\lambda}=1$, is :

(1) $2-\sqrt{3}$ (2) $4-3\sqrt{2}$

(3) $-2+\sqrt{2}$ (4) $4-2\sqrt{3}$

Ans. (2)

Sol. $3m^2x^2 + m(m - 4)x + 2 = 0$

$\lambda+\frac{1}{\lambda}=1$, $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=1$, $\alpha^2 + \beta^2 = \alpha\beta$

$(\alpha + \beta)^2 = 3\alpha\beta$

$\left(-\frac{m(m-4)}{3m^2}\right)^2=\frac{3(2)}{3m^2}$, $\frac{(m-4)^2}{9m^2}=\frac{6}{3m}$

$(m - 4)^2 = 18$, $m = 4\pm\sqrt{18},\ 4\pm3\sqrt{2}$

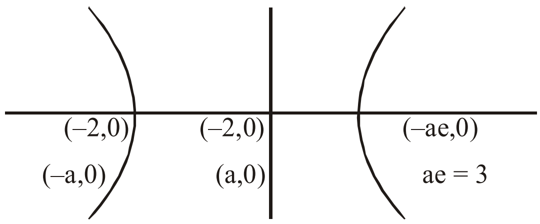
22. If the vertices of a hyperbola be at (–2, 0) and (2, 0) and one of its foci be at (–3, 0), then which one of the following points does not lie on this hyperbola?

(1) $\left(4,\sqrt{15}\right)$ (2) $\left(-6,2\sqrt{10}\right)$

(3) $\left(6,5\sqrt{2}\right)$ (4) $\left(2\sqrt{6},5\right)$

Ans. (3)

Sol.



$ae = 3$, $e=\frac{3}{2}$, $b^2=4\left(\frac{9}{4}-1\right)$, $b^2 = 5$

$\frac{x^2}{4}-\frac{y^2}{5}=1$

23. If $\frac{z-\alpha}{z+\alpha}(\alpha\in R)$ is a purely imaginary number and $|z| = 2$, then a value of α is :

(1) 1 (2) 2 (3) $\sqrt{2}$ (4) $\frac{1}{2}$

Ans. (2)

Sol. $\frac{z-\alpha}{z+\alpha}+\frac{\bar{z}-\alpha}{\bar{z}+\alpha}=0$

$z\bar{z}+z\alpha-\alpha\bar{z}-\alpha^2+z\bar{z}-z\alpha+\bar{z}\alpha-\alpha^2=0$

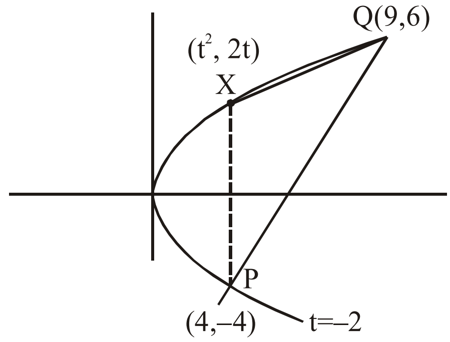
$|z|^2 = \alpha^2$, $a = \pm2$

24. Let P(4, –4) and Q(9, 6) be two points on the parabola, $y^2= 4x$ and let X be any point on the arc POQ of this parabola, where O is the vertex of this parabola, such that the area of ΔPXQ is maximum. Then this maximum area (in sq. units) is :

(1) $\frac{125}{4}$ (2) $\frac{125}{2}$ (3) $\frac{625}{4}$ (4) $\frac{75}{2}$

Ans. (1)

Sol.



$y^2 = 4x$

$2yy' = 4$

$y'=\frac{1}{t}=2$, $t=\frac{1}{2}$

$$\text{Area} = \frac{1}{2}\begin{vmatrix}\frac{1}{4} & 1 & 1\\ 9 & 6 & 1\\ 4 & -4 & 1\end{vmatrix}=\frac{125}{4}$$

25. the perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is:

(1) $11/\sqrt{6}$ (2) $6\sqrt{11}$ (3) 11 (4) $11\sqrt{6}$

Ans. (1)

Sol. $\begin{vmatrix} i & j & k\\ 3 & 5 & 7\\ 1 & 4 & 7\end{vmatrix}$

$\hat{i}(35-28)-\hat{j}(21.7)+\hat{k}(12-5)$

$7\hat{i}-14\hat{j}+7\hat{k}$

$\hat{i}-2\hat{j}+\hat{k}$

$1(x + 2) - 2(y - 2) + 1(z+15) = 0$

$x - 2y + z + 11 = 0$

$\frac{11}{\sqrt{4+1+1}}=\frac{11}{\sqrt{6}}$

26. The maximum value of $3\cos\theta+5\sin\left(\theta-\frac{\pi}{6}\right)$ for any real value of θ is :

(1) $\sqrt{19}$ (2) $\frac{\sqrt{79}}{2}$ (3) $\sqrt{31}$ (4) $\sqrt{34}$

Ans. **(1)**

Sol. $y=3\cos\theta+5\left(\sin\theta\frac{\sqrt{3}}{2}-\cos\theta\frac{1}{2}\right)$

$\frac{5\sqrt{3}}{2}\sin\theta+\frac{1}{2}\cos\theta$

$y_{max}=\sqrt{\frac{75}{4}+\frac{1}{4}}=\sqrt{19}$

27. A tetrahedron has vertices P(1, 2, 1), Q(2, 1, 3), R(–1,1,2) and O(0, 0, 0). The angle between the faces OPQ and PQR is :

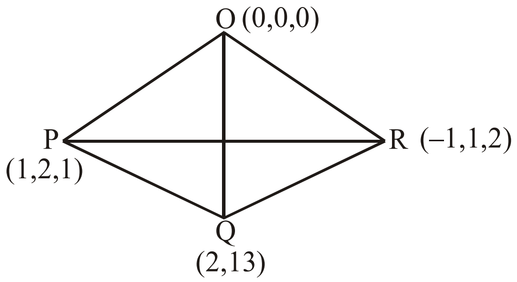
(1) $\cos^{-1}\left(\frac{9}{35}\right)$ (2) $\cos^{-1}\left(\frac{19}{35}\right)$

(3) $\cos^{-1}\left(\frac{17}{31}\right)$ (4) $\cos^{-1}\left(\frac{7}{31}\right)$

Ans. **(1)**

Sol. $\overrightarrow{OP}\times\overrightarrow{OQ}=(\hat{i}+2\hat{j}+\hat{k})\times(2\hat{i}+\hat{j}+3\hat{k})$

$5\hat{i}-\hat{j}-3\hat{k}$



$\overrightarrow{PQ}\times\overrightarrow{PR}=(\hat{i}-\hat{j}+2\hat{k})\times(-2\hat{i}-\hat{j}+\hat{k})$

$\hat{i}-5\hat{j}-3\hat{k}$

$\cos\theta=\frac{5+5+9}{\left(\sqrt{25+9+1}\right)^2}=\frac{19}{35}$

28. Lety = y(x) be the solution of the differential equation, $x\frac{dy}{dx}+y=x\log_e x,(x>1)$. If $2y(2)=\log_e 4-1$, then y(e) is equal to :-

(1) $\frac{e^2}{4}$ (2) $\frac{e}{4}$ (3) $-\frac{e}{2}$ (4) $-\frac{e^2}{2}$

Ans. **(2)**

Sol. $\frac{dy}{dx}=\frac{y}{x}=\ell n x$

$e^{\int\frac{1}{x}dx}=x$

$xy=\int x\ell nx+C$

$\ell n x\frac{x^2}{2}-\int\frac{1}{x}\cdot\frac{x^2}{2}$

$xy=\frac{x}{2}\ell n x-\frac{x^2}{4}$ +C, for $2y(2)=2\ell n2-1$

$\Rightarrow C=0$

$y=\frac{x}{2}\ell n x-\frac{x}{4}$

$y(e)=\frac{e}{4}$

29. Let $P=\begin{bmatrix}1&0&0\\3&1&0\\9&3&1\end{bmatrix}$ and $Q=[q_{ij}]$ be two 3×3 matrices such that $Q-P^5=I_3$. Then $\frac{q_{21}+q_{31}}{q_{32}}$ is equal to:

(1) 15 (2) 9 (3) 135 (4) 10

Ans. **(4)**

Sol. $P=\begin{bmatrix}1&0&0\\3&1&0\\9&3&1\end{bmatrix}$

$P^2=\begin{bmatrix}1&0&0\\3+3&1&0\\9+9+9&3+3&1\end{bmatrix}$

$P^3=\begin{bmatrix}1&0&0\\3+3+3&1&0\\6.9&3+3+3&1\end{bmatrix}$

$P^n=\begin{bmatrix}1&0&0\\3n&1&0\\\frac{n(n+1)}{2}3^2&3n&1\end{bmatrix}$

$P^5=\begin{bmatrix}1&0&0\\5.3&1&0\\15.9&5.3&1\end{bmatrix}$

$Q = P^5 + I_3$

$$Q = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$$

$$\frac{q_{21}+q_{31}}{q_{32}} = \frac{15+135}{15} = 10$$

Aliter

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$$

$P = I + X$

$$X = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$$

$$X^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{pmatrix}$$

$X^3 = 0$

$P^5 = I + 5X + 10X^2$

$Q = P^5 + I = 2I + 5X + 10X^2$

$$Q = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 15 & 0 & 0 \\ 15 & 15 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 90 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \quad Q = \begin{pmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{pmatrix}$$

30. Consider three boxes, each containing 10 balls labelled 1,2,....,10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, (i = 1, 2, 3). Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is :

(1) 82 (2) 240 (3) 164 (4) 120

Ans. (4)

Sol. No. of ways = $10C_3$ = 120