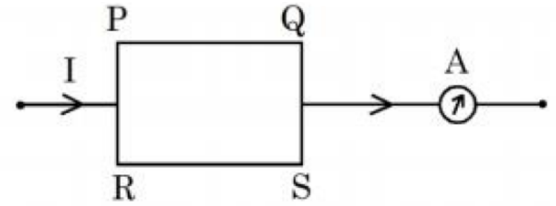
| JEE–MAIN EXAMINATION – JANUARY, 2023 | |
|---|---|
| (Held On Thursday 30th January, 2023) | TIME : 3 : 00 PM to 6 : 00 PM |

# Physics

## SECTION - A

**1.** A current carrying rectangular loop $PQRS$ is made of uniform wire. The length $PR = QS = 5$ cm and $= RS = 100$ cm. If ammeter current reading changes from $I$ to $2I$, the ratio of magnetic forces per unit length on the wire $PQ$ due to wire $RS$ in the two cases respectively $\left(f_{PQ}^{I} : f_{PQ}^{2I}\right)$ is:




(1) 1: 2 (2) 1: 3 (3) 1: 4 (4) 1: 5

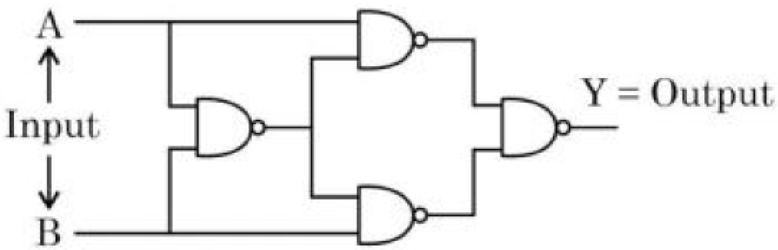
**Sol.** **(3)**

$F \propto I_1 I_2$

$\frac{F_1}{F_{2I}} = \frac{1}{4}$

Ans. (3)

**2.** The output Y for the inputs A and B of circuit is given by




Truth table of the shown circuit is:

(1)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

(2)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(3)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(4)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

**Sol.** **(3)**

**3.** Given below are two statements: one is labelled as Assertion **A** and the other is labelled as Reason R

Assertion A: Efficiency of a reversible heat engine will be highest at $-273°C$ temperature of cold reservoir.

Reason R: The efficiency of Carnot's engine depends not only on temperature of cold reservoir but it depends on the temperature of hot reservoir too and is given as $\eta = \left(1 - \frac{T_2}{T_1}\right)$.

In the light of the above statements, choose the correct answer from the options given below

(1) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(2) Both **A** and **R** are true and **R** is the correct explanation of **A**

(3) A is false but **R** is true

(4) A is true but **R** is false

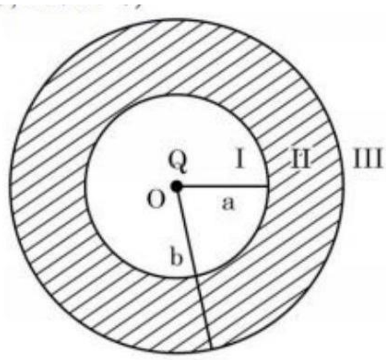
**Sol. (2)**

$$\eta = 1 - \frac{T_L}{T_H} = \frac{T_H - T_L}{T_H}$$

Efficiently of Carnot's engine will be highest at – 273º = 0K

Ans. (2)

**4.** As shown in the figure, a point charge $Q$ is placed at the centre of conducting spherical shell of inner radius $a$ and outer radius $b$. The electric field due to charge Q in three different regions I, II and III is given by: (I: $r < a$, II: $a < r < b$, III: $r > b$)



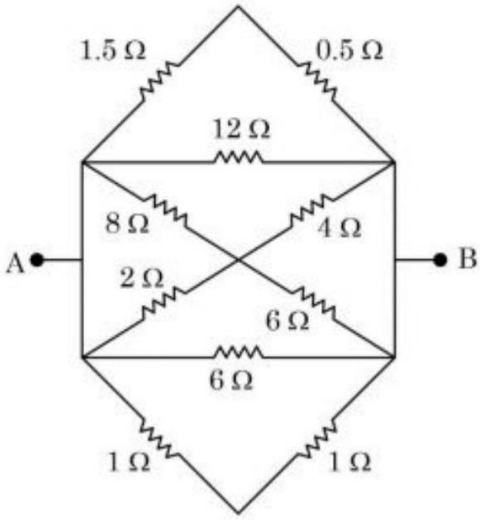

(1) $E_I = 0, E_{II} = 0, E_{III} = 0$
(2) $E_I = 0, E_{II} = 0, E_{III} \neq 0$
(3) $E_I \neq 0, E_{II} = 0, E_{III} \neq 0$
(4) $E_I \neq 0, E_{II} = 0, E_{III} = 0$

**Sol.** **Sol. (3)**

Electric field inside material of conductor is zero

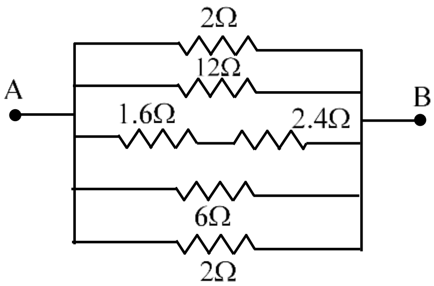
Ans. (3)

**5.** The equivalent resistance between $A$ and $B$ is




(1) $\frac{1}{3}\Omega$ (2) $\frac{1}{2}\Omega$ (3) $\frac{3}{2}\Omega$ (4) $\frac{2}{3}\Omega$

**Sol.** **(4)**




$$\frac{1}{R_{eq}}=\frac{1}{2}+\frac{1}{12}+\frac{1}{4}+\frac{1}{6}+\frac{1}{2}$$

$$=\frac{18}{12}=\frac{3}{2}$$

$$R_{eq}=\frac{2}{3}\ \Omega$$

Ans. (4)

**6.** A vehicle travels 4 km with speed of 3 km/h and another 4 km with sped of 5 km/h, then its average speed is

(1) 3.50 km/h (2) 4.25 km/h (3) 4.00 km/h (4) 3.75 km/h

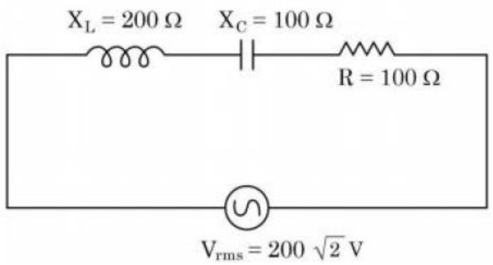
**Sol.** **(4)**

$$\frac{2}{V_{av}}=\frac{1}{3}+\frac{1}{5}=\frac{8}{15}$$

$$V_{av}=\frac{15}{4}=3.75\text{ km hr}^{-1}$$

Ans. (4)

**7.** In the given circuit, rms value of current ($I_{rms}$) through the resistor $R$ is:




(1) $2\sqrt{2}$ A (2) 2 A (3) 20 A (4) $\frac{1}{2}$ A

**Sol.** **(2)**

$$Z = \sqrt{R^2 + (X_L - X_e)^2}$$

$$Z = \sqrt{(100)^2 + (200-100)^2}$$

$$Z = 100\sqrt{2}\ \Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{200\sqrt{2}}{100\sqrt{2}} = 2\text{ A}$$

**8.** A point source of 100 W emits light with 5% efficiency. At a distance of 5 m from the source, the intensity produced by the electric field component is:

(1) $\frac{1}{2\pi}\frac{W}{m^2}$ (2) $\frac{1}{20\pi}\frac{W}{m^2}$ (3) $\frac{1}{10\pi}\frac{W}{m^2}$ (4) $\frac{1}{40\pi}\frac{W}{m^2}$

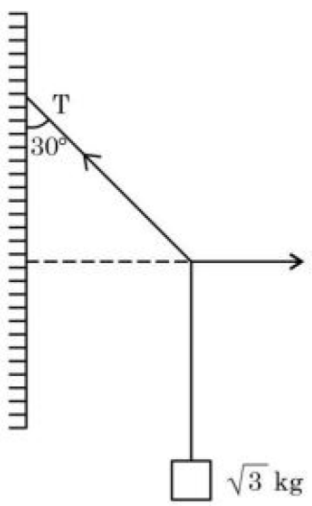
**Sol.** **(4)**

$$I_{EF} = \frac{1}{2} \times \frac{5}{4\pi(5)^2}$$

$$= \frac{1}{40\pi}\text{ w/m}^2$$

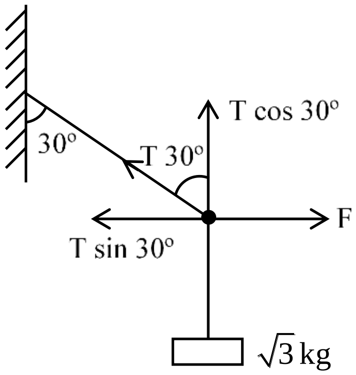
Ans: (4)

**9.** A block of $\sqrt{3}$ kg is attached to a string whose other end is attached to the wall. An unknown force F is applied so that the string makes an angle of 30° with the wall. The tension T is: (Given $g = 10\text{ ms}^{-2}$ )




(1) 20 N (2) 10 N (3) 15 N (4) 25 N

**Sol.** **(1)**




F = Tsin 30°

$\sqrt{3}g$ = T cos 30°

$\tan 30° = \frac{F}{\sqrt{3}}g$

$\frac{1}{\sqrt{3}} = \frac{F}{\sqrt{3}g}$

F = 10 N

$T = \frac{F}{\sin 30°} = 10 \times 2$

T = 10 × 2 = 20 N

Ans: (1)

**10.** Match List I with List II:

| List I | List II |
|---|---|
| A. Attenuation | I. Combination of a receiver and transmitter. |
| B. Transducer | II. process of retrieval of information from the carrier wave at receiver |
| C. Demodulation | III. converts one form of energy into another |
| D. Repeater | IV. Loss of strength of a signal while propogating through a medium. |

Choose the correct answer from the options given below:

(1) A-IV, B-III, C-I, D-II (2) A-I, B-II, C-III, D-IV

(3) A-IV, B-III, C-II, D-I (4) A-II, B-III, C-IV, D-I

**Sol.** **(3)**

Theory

**11.** An electron accelerated through a potential difference $V_1$ has a de-Broglie wavelength of $\lambda$. When the potential is changed to $V_2$, its de-Broglie wavelength increases by 50%. The value of $\left(\frac{V_1}{V_2}\right)$ is equal to

(1) 3 (2) $\frac{3}{2}$ (3) 4 (4) $\frac{9}{4}$

**Sol.** **(4)**

$$KE = \frac{P^2}{2m}$$

$$P = \frac{h}{\lambda}$$

$$eV_1 = \frac{\left(\frac{h}{\lambda}\right)^2}{2m}$$

$$eV_2 = \frac{\left(\frac{h}{1.5\lambda}\right)^2}{2m}$$

$$\frac{V_1}{V_2} = (1.5)^2 = \frac{9}{4}$$

Ans: (4)

**12.** A flask contains hydrogen and oxygen in the ratio of 2:1 by mass at temperature 27°C. The ratio of average kinetic energy per molecule of hydrogen and oxygen respectively is:

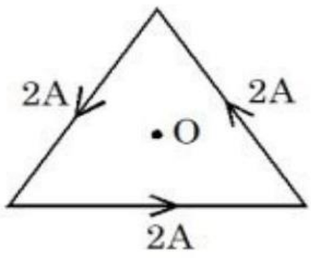
(1) 2 : 1 (2) 1 : 1 (3) 1 : 4 (4) 4 : 1

**Sol.** **(2)**

Average kinetic energy per molecule = $\frac{5}{2}KT$

Ratio = $\frac{1}{1}$

**13.** As shown in the figure, a current of 2 A flowing in an equilateral triangle of side $4\sqrt{3}$ cm. The magnetic field at the centroid O of the triangle is




(Neglect the effect of earth's magnetic field)

(1) $1.4\sqrt{3} \times 10^{-5}$ T (2) $4\sqrt{3} \times 10^{-4}$ T (3) $3\sqrt{3} \times 10^{-5}$ T (4) $\sqrt{3} \times 10^{-4}$ T

**Sol.** **(3)**

$d \tan 60° = 2\sqrt{3}$

d = 2 cm

$$B = 3\left(\frac{\mu_0 I}{2\pi d}\right)\sin 60°$$

$$B = \frac{3 \times 2 \times 10^{-7} \times 2}{2 \times 10^{-2}} \times \frac{\sqrt{3}}{2}$$

$$B = 3\sqrt{3} \times 10^{-5} T$$

14. An object is allowed to fall from a height $R$ above the earth, where $R$ is the radius of earth. Its velocity when it strikes the earth's surface, ignoring air resistance, will be

(1) $\sqrt{2gR}$ (2) $\sqrt{\frac{gR}{2}}$ (3) $2\sqrt{gR}$ (4) $\sqrt{gR}$

**Sol.** **(4)**

Use work energy theorem

$\Delta KE = w_g$

$$\frac{1}{2}mv^2 - 0 = -[u_f - u_i]$$

$$\frac{1}{2}mv^2 = -\left[-\frac{GMm}{R} - \left(-\frac{GMm}{2R}\right)\right]$$

$$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{2R}$$

$$= \frac{GMm}{R}\left(\frac{2-1}{2}\right)$$

$$\frac{1}{2}mv^2 = \frac{GMm}{2R}$$

$$V = \sqrt{\frac{GM}{R}}$$

$V = \sqrt{gR}$ $(GM = gR^2)$

15. Match List I with List II:

| List I | | List II |
|---|---|---|
| A. | Torque | I. $kg\,m^{-1}\,s^{-2}$ |
| B. | Energy density | II. $kg\,ms^{-1}$ |
| C. | Pressure gradient | III. $kg\,m^{-2}\,s^{-2}$ |
| D. | Impulse | IV. $kg\,m^2\,s^{-2}$ |

Choose the correct answer from the options given below:

(1) A – IV, B – I, C – III, D – II (2) A – IV, B – III, C – I, D – II

(3) A – IV, B – I, C – II, D – III (4) A – I, B – IV, C – III, D – II

**Sol.** **(1)**

Torque = N – m

$$= kg\ \frac{m}{sec^2}\,m$$

$$= \frac{kg\ m^2}{sec^2}$$

$$\text{Energy Density} = \frac{N-m}{m^3} = \frac{N}{m^2}$$

$$= kg\ \frac{m}{sec^2} \times \frac{1}{m^2}$$

$$\text{Pressure gradient} = \frac{\text{Pressure}}{\text{length}} = \frac{F}{A-\text{length}}$$

$= kg\ m^{-2}\ sec^{-2}$

Impulse = $\Delta P$ = $kg\ m - s^{-1}$

**16.** Given below are two statements: one is labelled as Assertion **A** and the other is labelled as Reason **R**

Assertion A: The nuclear density of nuclides $^{10}_{5}\text{B}$, $^{6}_{3}\text{Li}$, $^{56}_{26}\text{Fe}$, $^{20}_{10}\text{Ne}$ and $^{209}_{83}\text{Bi}$ can be arranged as $\rho^{N}_{\text{Bi}} > \rho^{N}_{\text{Fe}} > \rho^{N}_{\text{Ne}} > \rho^{N}_{\text{B}} > \rho^{N}_{\text{Li}}$

Reason R: The radius $R$ of nucleus is related to its mass number $A$ as $R = R_0 A^{1/3}$, where $R_0$ is a constant.

In the light of the above statements, choose the correct answer from the options given below

(1) A is false but **R** is true

(2) A is true but **R** is false

(3) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

**Sol.** **(1)**

Nuclear density is independent of A

Ans: (1)

**17.** A force is applied to a steel wire 'A', rigidly clamped at one end. As a result elongation in the wire is 0.2 mm. If same force is applied to another steel wire 'B' of double the length and a diameter 2.4 times that of the wire 'A', the elongation in the wire ' B ' will be (wires having uniform circular cross sections)

(1) $6.06 \times 10^{-2}$ mm (2) $2.77 \times 10^{-2}$ mm

(3) $3.0 \times 10^{-2}$ mm (4) $6.9 \times 10^{-2}$ mm

**Sol.** **(4)**

$$Y = \frac{F\ell}{A\Delta\ell}$$

$$F = \frac{YA\Delta\ell}{\ell}$$

$$\left(\frac{A\Delta\ell}{\ell}\right)_1 = \left(\frac{A\Delta\ell}{\ell}\right)_2$$

$$\frac{\Delta\ell_2}{\Delta\ell_1} = \frac{A_1}{A_2} \times \frac{\ell_2}{\ell_1}$$

$$\frac{(\Delta\ell)_2}{0.2} = \frac{1}{2.4 \times 2.4} \times \frac{2}{1}$$

$$(\Delta\ell)_2 = 6.9 \times 10^{-2} \text{ mm}$$

Ans: (4)

**18.** A thin prism, $P_1$ with an angle $6^{th}$ and made of glass of refractive index 1.54 is combined with another prism $P_2$ made from glass of refractive index 1.72 to produce dispersion without average deviation. The angle of prism $P_2$ is

(1) 1.3° (2) 6° (3) 4.5° (4) 7.8°

**Sol.** **(3)**

$\delta_1 = \delta_2$ [For no deviation]

$6(1.54 - 1) = A(1.72 - 1)$

$$A = \frac{18}{4} = 4.5^\circ$$

Ans: (3)

**19.** A machine gun of mass 10 kg fires 20 g bullets at the rate of 180 bullets per minute with a speed of $100 \text{ m s}^{-1}$ each. The recoil velocity of the gun is

(1) 1.5 m/s (2) 0.6 m/s (3) 2.5 m/s (4) 0.02 m/s

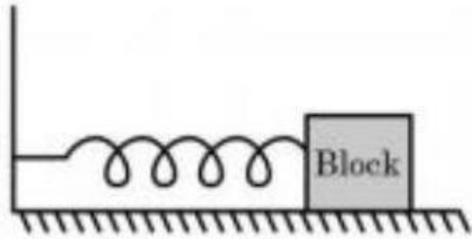
**Sol.** **(2)**

$$20 \times 10^{-3} \times \frac{180}{60} \times 100 = 10\text{ V}$$

$V = 0.6 \text{ ms}^{-1}$

Ans: (2)

**20.** For a simple harmonic motion in a mass spring system shown, the surface is frictionless. When the mass of the block is 1 kg, the angular frequency is $\omega_1$. When the mass block is 2 kg the angular frequency is $\omega_2$. The ratio $\omega_2/\omega_1$ is




(1) $1/\sqrt{2}$ (2) $\sqrt{2}$ (3) 2 (4) 1/2

**Sol.** **(1)**

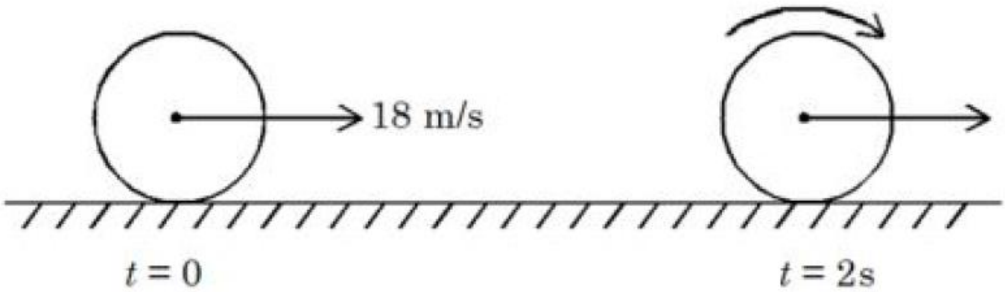
$$\omega = \sqrt{\frac{k}{m}}$$

$$\frac{\omega_2}{\omega_1} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{2}}$$

Ans: (1)

## SECTION - B

**21.** A uniform disc of mass 0.5 kg and radius $r$ is projected with velocity 18 m/s at t = 0 s on a rough horizontal surface. It starts off with a purely sliding motion at t = 0 s. After 2 s it acquires a purely rolling motion (see figure). The total kinetic energy of the disc after 2 s will be ________ J (given, coefficient of friction is 0.3 and $g = 10 \text{ m/s}^2$ ).



**Sol.** **(54)**

$a = -\mu_k g = -3$

$v = u + at$

$v = 18 - 3 \times 2 = 12\ ms^{-1}$

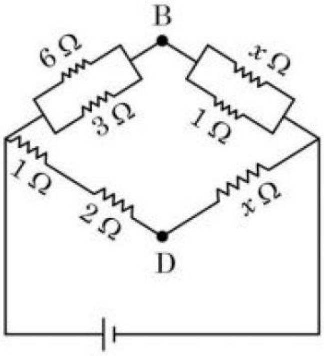
$$KE = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\left(\frac{v}{r}\right)^2$$

$$KE = \frac{3}{4}mv^2$$

$KE = 3 \times 18 = 54\ J$

Ans: (54)

**22.** If the potential difference between B and D is zero, the value of $x$ is $\frac{1}{n}\Omega$. The value of $n$ is ________.




**Sol.** **(2)**

$$\frac{2}{3} = \frac{\frac{x}{x+1}}{x}$$

$$\frac{2}{3} = \frac{1}{x+1}$$

$$x = 0.5 = \frac{1}{2}$$

$n = 2$

Ans: (2)

**23.** A stone tied to 180 cm long string at its end is making 28 revolutions in horizontal circle in every minute. The magnitude of acceleration of stone is $\frac{1936}{x}ms^{-2}$. The value of x ________.

(Take $\pi = \frac{22}{7}$)

**Sol.** **(125)**

$a = \omega^2 r$

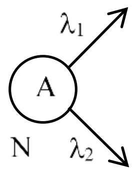
$$a = \left(\frac{28 \times 2\pi}{60}\right)^2 \times 1.8$$

$$a = \frac{1936 \times 1.8}{225} = \frac{1936}{125} ms^{-2}$$

$x = 125$

**24.** A radioactive nucleus decays by two different process. The half life of the first process is 5 minutes and that of the second process is 30 s. The effective half-life of the nucleus is calculated to be $\frac{\alpha}{11}$ s. The value of $\alpha$ is ________.

**Sol.** **(300)**




$$\frac{dN}{dt} = -(\lambda_1 + \lambda_2)N$$

$\lambda_{eq} = \lambda_1 + \lambda_2$

$$\frac{1}{t_{1/2}} = \frac{1}{300} + \frac{1}{30} = \frac{11}{300}$$

$$t_{1/2} = \left(\frac{300}{11}\right) \text{ sec}$$

Ans: (300)

**25.** A faulty thermometer reads 5°C in melting ice and 95°C in stream. The correct temperature on absolute scale will be ________ K when the faulty thermometer reads 41°C.

**Sol.** **(313)**

Ans: $\frac{41^\circ - 5^\circ}{95^\circ - 5^\circ} = \frac{R-0}{100-0}$

R = 40°C

R = 313 K

**26.** In an ac generator, a rectangular coil of 100 turns each having area $14 \times 10^{-2}\ m^2$ is rotated at 360rev/min about an axis perpendicular to a uniform magnetic field of magnitude 3.0 T. The maximum value of the emf produced will be ________ $V$.

$\left(\text{Take } \pi = \frac{22}{7}\right)$

**Sol.** **(1584)**

$E_{max} = NAB\omega$

$= 100 \times 14 \times 10^{-2} \times 3 \times \frac{360 \times 2\pi}{60}$

= 1584 V

Ans: (1584)

**27.** A body of mass 2 kg is initially at rest. It starts moving unidirectionally under the influence of a source of constant power P. Its displacement in 4 s is $\frac{1}{3}\alpha^2\sqrt{Pm}$. The value of $\alpha$ will be ________.

**Sol.** **(4)**

$$\frac{1}{2}mv^2 = pt$$

$$V = \sqrt{\frac{2pt}{m}}$$

$$\frac{dx}{dt} = \sqrt{\frac{2pt}{m}}$$

$$\int dx = \sqrt{\frac{2p}{m}}\int \sqrt{t}dt$$

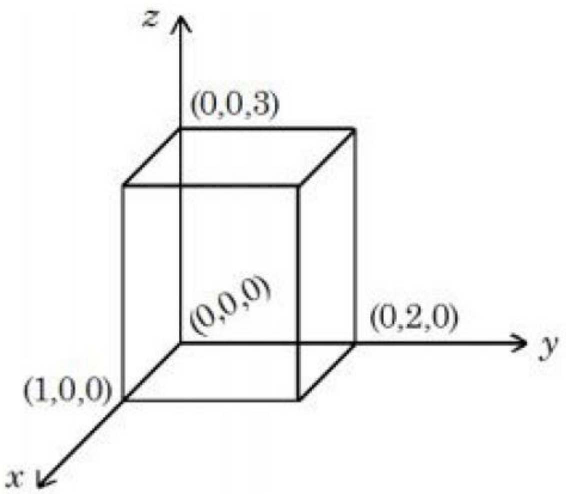
$$x = \sqrt{\frac{2p}{m}}\left[t^{3/2}\right]_0^4$$

$$x = \frac{1}{3}\times 16\sqrt{p}$$

$\propto = 4$

Ans: (4)

**28.** As shown in figure, a cuboid lies in a region with electric field $= 2x^2\hat{\imath} - 4y\hat{\jmath} + 6\hat{k}$ N/C. The magnitude of charge within the cuboid is $n \in_0 C$.
The value of $n$ is ________ (if dimension of cuboid is $1 \times 2 \times 3$ m$^3$ ).




**Sol.** **(12)**

$\phi_{net} = -8 \times 3 + 2 \times 6$

$= -12$

$$\phi_{net} = \frac{q_{inside}}{\in_0}$$

$q_{inside} = -12 \in_0$

Ans: (12)

**29.** In a Young's double slit experiment, the intensities at two points, for the path differences $\frac{\lambda}{4}$ and $\frac{\lambda}{3}$ ($\lambda$ being the wavelength of light used) are $I_1$ and $I_2$ respectively. If $I_0$ denotes the intensity produced by each one of the individual slits, then $\frac{I_1+I_2}{I_0}=$ ________.

**Sol.** **(3)**

$I = 4I_0 \cos^2 \frac{\phi}{2}$

$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$

$I_1 = 4I_0 \cos^2 \frac{\pi}{4} = 2I_0$

$I_2 = 4I_0 \cos^2 \frac{2\pi}{3} = I_0$

$\Rightarrow \frac{I_1 + I_2}{I_0} = 3$

Ans: (3)

**30.** The velocity of a particle executing SHM varies with displacement $(x)$ as $4v^2 = 50 - x^2$. The time period of oscillations is $\frac{x}{7}s$. The value of $x$ is ________.

$\left(\text{Take } \pi = \frac{22}{7}\right)$

**Sol.** **(88)**

$4v^2 = 50 - x^2$

$V = \frac{1}{2}\sqrt{50 - x^2}$

$\omega = \frac{1}{2}$

$T = \frac{2\pi}{\omega} = 4\pi = \frac{88}{7}$

$x = 88$

Ans: (88)

## Chemistry

**SECTION - A**

**31.** The $Cl - Co - Cl$ bond angle values in a fac- $[Co(NH_3)_3Cl_3]$ complex is/are:

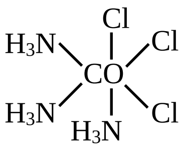
(1) $90^\circ$

(2) $90^\circ \& 120^\circ$

(3) $180^\circ$

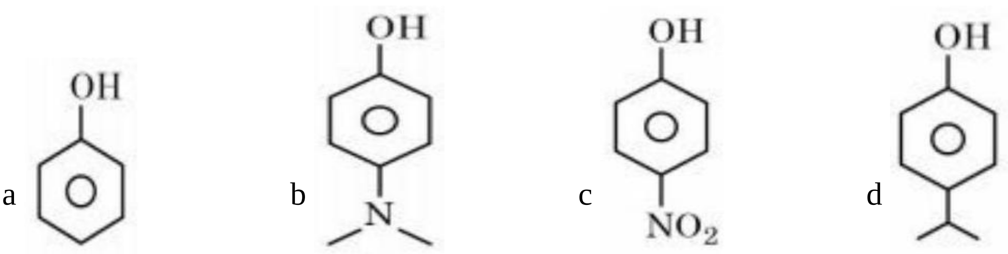
(4) $90^\circ \& 180^\circ$

**Sol.** 1




$(90^\circ)$

**32.** The correct order of $pK_a$ values for the following compounds is:




(1) c > a > d > b

(2) b > a > d > c

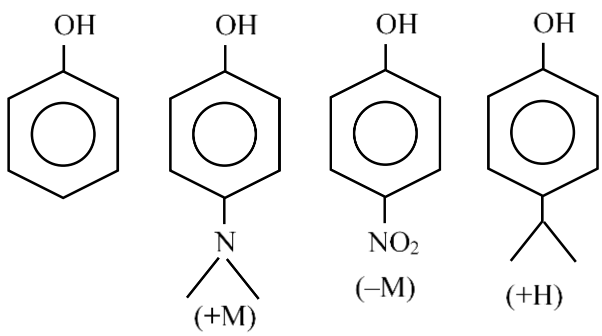
(3) $b > d > a > c$

(4) $a > b > c > d$

**Sol.** **3**

Acidic strength $\propto$ (–M, –H, – I )

$\propto \dfrac{1}{(+M, +H, +I)}$




PKa $\propto \dfrac{1}{\text{Acidic}}$ strength

A order of acidic strength: c > a > d > b

Order of PKa : c < a < d < b

**33.** Given below are two statements:

Statement I : During Electrolytic refining, the pure metal is made to act as anode and its impure metallic form is used as cathode.

Statement II : During the Hall-Heroult electrolysis process, purified $Al_2O_3$ is mixed with $Na_3AlF_6$ to lower the melting point of the mixture.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is correct but Statement II is incorrect

(2) Both Statement I and Statement II are incorrect

(3) Both Statement I and Statement II are correct

(4) Statement I is incorrect but Statement II is correct

**Sol.** **4**

Mixture of $CaF_2$ & $Na_3AlF_6$ decreasing the M.P. of $Al_2O_3$.

In electrolytic definning, pure metal is always deposited at the cathode

**34.** Match List I with List II:

| List I (Mixture) | | List II (Separation Technique) | |
|---|---|---|---|
| A. | $CHCl_3 + C_6H_5NH_2$ | I. | Steam distillation |
| B. | $C_6H_{14} + C_5H_{12}$ | II. | Differential extraction |
| C. | $C_6H_5NH_2 + H_2O$ | III. | Distillation |
| D. | Organic compound in $H_2O$ | IV. | Fractional distillation |

(1) A-IV, B-I, C-III, D-II

(2) A-III, B-IV, C-I, D-II

(3) A-III, B-I, C-IV, D-II

(4) A-II, B-I, C-III, D-IV

**Sol.** **2**

A. $CHCl_3 + C_6H_5NH_2 \rightarrow$ Distillation (III)

B. $C_6H_{14} + C_5H_{12} \rightarrow$ fractional distillation (IV)

C. $C_6H_5NH_2 \rightarrow H_2O \rightarrow$ Steam distillation (I)

D. Organic compound in $H_2O \rightarrow$ Differential extraction (II)

**35.** 1 L, 0.02M solution of $[Co(NH_3)_5SO_4]Br$ is mixed with 1 L, 0.02M solution of $[Co(NH_3)_5Br]SO_4$. The resulting solution is divided into two equal parts (X) and treated with excess of $AgNO_3$ solution and $BaCl_2$ solution respectively as shown below:

1 L solution (X) + $AgNO_3$ solution (excess) $\rightarrow$ Y

1 L Solution (X)+$BaCl_2$ solution (excess) $\rightarrow$ Z

The number of moles of Y and Z respectively are

(1) 0.02, 0.01 (2) 0.01, 0.01

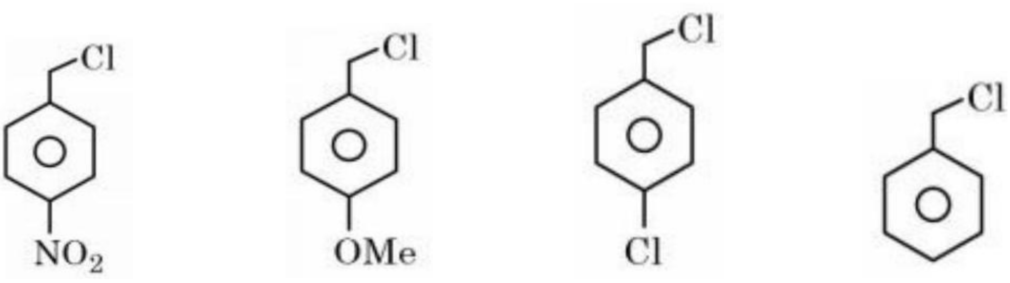
(3) 0.01, 0.02 (4) 0.02, 0.02

**Sol.** **2**

$$\left[\underset{0.01\,mol}{Co(NH_3)_5SO_4}\right]Br + \underset{excess}{AgNO_3} \rightarrow \underset{0.01\,mol}{AgBr}\downarrow$$

$$\left[\underset{0.02\,mol}{Co(NH_3)_5Br}\right]SO_4 + \underset{excess}{BaCl_2} \rightarrow \underset{0.01\,mol}{BaSO_4}\downarrow$$

**36.** Decreasing order towards SN 1 reaction for the following compounds is:




a b c d

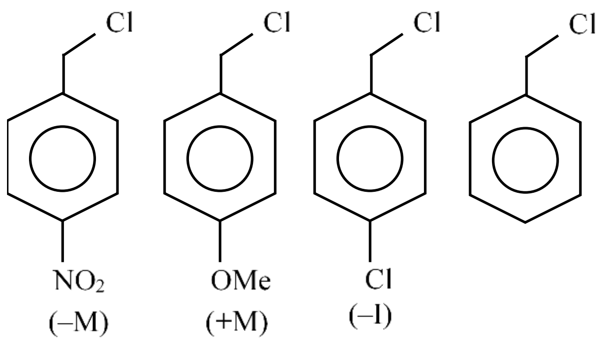
(1) a > c > d > b

(2) b > d > c > a

(3) $a > b > c > d$

(4) d > b > c > a

**Sol.** **2**




b > d > c > a

**37.** Which of the following reaction is correct?

(1) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 2\,N_2O_4 + O_2$

(2) $2LiNO_3 \xrightarrow{\Delta} 2NaNO_2 + O_2$

(3) $2LiNO_3 \longrightarrow 2Li + 2NO_2 + O_2$

(4) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$

**Sol.** **4**

$4\ LiNO_3 \xrightarrow{\Delta} 2\ Li_2O + 4\ NO_2 + O_2$

**38.** Boric acid is solid, whereas $BF_3$ is gas at room emperature because of

(1) Strong van der Waal's interaction in Boric acid

(2) Strong covalent bond in $BF_3$

(3) Strong ionic bond in Boric acid

(4) Strong hydrogen bond in Boric acid

**Sol.** **4**

Due to strong hydrogen bonding present in boric acid, boric acid present in solid form.

**39.** Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason **R**.

Assertion A: Antihistamines do not affect the secretion of acid in stomach.

Reason : Antiallergic and antacid drugs work on different receptors.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is false but R is true

(2) Both A and R are true but R is not the correct explanation of A

(3) Both A and R are true and R is the correct explanation of A

(4) A is true but R is false

**Sol.** **3**

**40.** Formulae for Nessler's reagent is:

(1) $HgI_2$

(2) $K_2HgI_4$

(3) $KHgI_3$

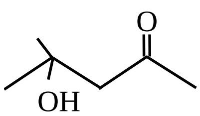
(4) $KHg_2I_2$

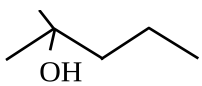
**Sol.** **2**

Nessler's reagent

$K_2HgI_4 + KOH$

**41.** Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: 

can be easily reduced using Zn-Hg/HCl to 


Reason R: $Zn - Hg/HCl$ is used to reduce carbonyl group to $-CH_2-$ group.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is true but R is false

(2) Both A and R are true and R is the correct explanation of A

(3) A is false but R is true

(4) Both A and R are true but R is not the correct explanation of A

**Sol.** **2**

**42.** Maximum number of electrons that can be accommodated in shell with n=4

(1) 16 (2) 32 (C) 72 (D) 50

**Sol.** **2**

Max $e^-$ that can be accommodated in shell = $2n^2$

(n=4)

$2(4)^2=32$

**43.** The wave function (Ψ) of 2 s is given by

$\Psi_{2s} = \frac{1}{2\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{1/2}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$

At $r = r_0$, radial node is formed. Thus, $r_0$ in terms of $a_0$

(1) $r_0 = 4a_0$

(2) $r_0 = \frac{a_0}{2}$

(3) $r_0 = a_0$

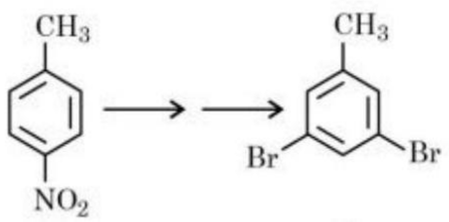
(4) $r_0 = 2a_0$

**Sol.** **4**

At node $\psi_{2s} = 0$

$2 - \frac{r_0}{a_0} = 0$

$r_0 = 2a_0$

**44.** 


(X) (Y)

In the above conversion of compound (X) to product (Y), the sequence of reagents to be used will be:

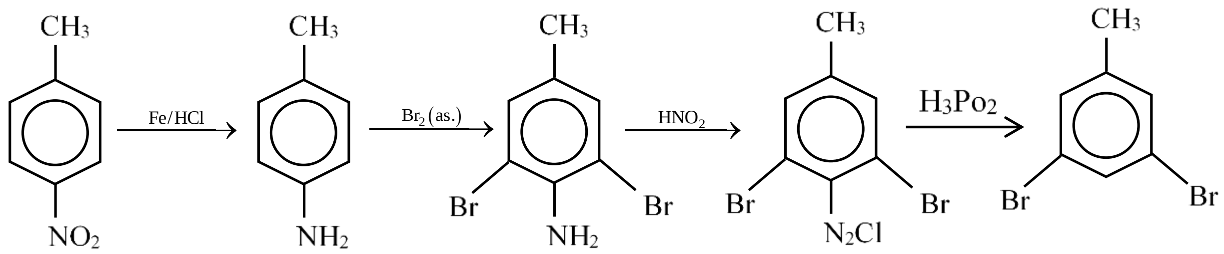
(1) (i) $Br_2(aq)$ (ii) $LiAIH_4$ (iii) $H_3O^+$

(2) (i) $Br_2, Fe$ (ii) $Fe, H^+$ (iii) $LiAIH_4$

(3) (i) $Fe, H^+$ (ii) $Br_2$ (aq) (iii) $HNO_2$ (iv) $H_3PO_2$

(4) (i) $Fe, H^+$ (ii) $Br_2$ (aq) (iii) $HNO_2$ (iv) CuBr

**Sol.** **3**



**45.** Match List I with List II:

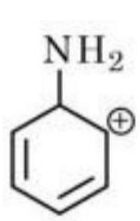
| List I (Complexes) | List II (Hybridisation) |
|---|---|
| A. $[Ni(CO)_4]$ | I. $sp^3$ |
| B. $[Cu(NH_3)_4]^{2+}$ | II. $dsp^2$ |
| C. $[Fe(NH_3)_6]^{2+}$ | III. $sp^3d^2$ |
| D. $[Fe(H_2O)_6]^{2+}$ | IV. $d^2sp^3$ |

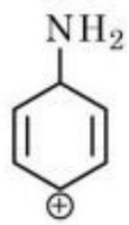
(1) A-I, B-II, C-IV, D-III
(2) A-II, B-I, C-III, D-IV
(3) A-II, B-I, C-IV, D-III
(4) A-I, B-II, C-III, D-IV

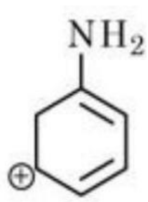
**Sol.** **1**

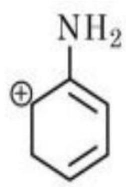
| Complex | Hyloridisation |
|---|---|
| (A) $Ni(CO)_4$ | $sp^3$ |
| (B) $[Cu(NH_3)_4]^{+2}$ | $dsp^2$ |
| (C) $[Fe(NH_3)_6]^{+2}$ | $d^2sp^3$ |
| (D) $[Fe(H_2O)_6]^{+2}$ | $sp^3d^2$ |

**46.** The most stable carbocation for the following is:










a b c d

(1) a (2) c (3) d (4) b

**Sol.** **3**

**47.** Chlorides of which metal are soluble in organic solvents:
(1) K (2) Be (3) Mg (4) Ca

**Sol.** **2**

Due to smaller size, $Be^{+2}$. will show more polarising power, hence, Be will have maximum covalent character & most soluble in organic solvent.

**48.** $KMnO_4$ oxidises $I^-$ in acidic and neutral/faintly alkaline solution, respectively, to

(1) $IO_3^-$ & $IO_3^-$

(2) $I_2$ & $IO_3^-$

(3) $I_2$ & $I_2$

(4) $IO_3^-$ & $I_2$

**Sol.** **2**

$2\ KMnO_4 + 10I^- + 16H^+ \rightarrow 2Mn^{+2} + 8H_2O + 5I_2$

neutral/faintly alkaline sol$^n$.

$2\ MnO_4^- + I^- + H_2O \rightarrow 2\ MnO_2 + 2OH^- + IO_3^-$

**49.** Bond dissociation energy of "E-H" bond of the "$H_2E$ " hydrides of group 16 elements (given below), follows order.

A. O

B. S

C. Se

D. Te

Choose the correct from the options given below:

(1) $B > A > C > D$

(2) $A > B > D > C$

(3) $A > B > C > D$

(4) $D > C > B > A$

**Sol.** **3**

$H_2O > H_2S > H_2Se > H_2Te$

**50**. The water quality of a pond was analysed and its BOD was found to be 4. The pond has

(1) Highly polluted water

(2) Slightly polluted water

(3) Water has high amount of fluoride compounds

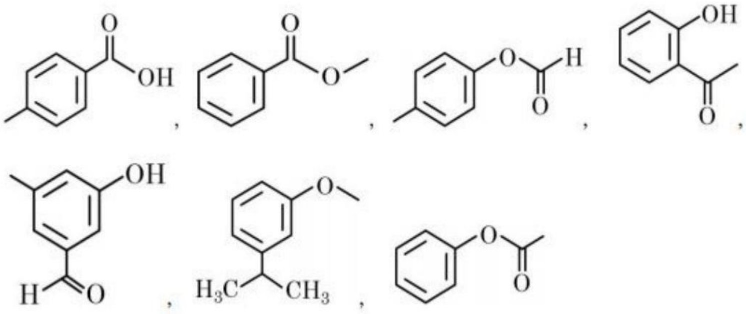
(4) Very clean water

**Sol.** **4**

Clean water have BOD value less than 5 ppm while highly polluted water have. BOD value of 17 ppm or more.

# SECTION B

**51.** Number of compounds from the following which will not dissolve in cold $NaHCO_3$ and NaOH solutions but will dissolve in hot NaOH solution is



**Sol.** **3**

**52.** 1 mole of ideal gas is allowed to expand reversibly and adiabatically from a temperature of 27°C. The work done is 3 kJ $mol^{-1}$. The final temperature of the gas is_______ K (Nearest integer). Given $C_V$ = 20 J $mol^{-1}$ $K^{-1}$

**Sol.** **150**

q = 0

$\Delta U = W = nC_v\Delta T$

$= 1 \times 20 \times [T_2 - 300] = -3000$

$= T_2 - 300 = -150$

$= T_2 = 150$ K

**53.** A short peptide on complete hydrolysis produces 3 moles of glycine (G), two moles of leucine (L) and two moles of valine (V) per mole of peptide. The number of peptide linkages in it are

**Sol.** **6**

**54.** Lead storage battery contains 38% by weight solution of $H_2SO_4$. The van't Hoff factor is 2.67 at this concentration. The temperature in Kelvin at which the solution in the battery will freeze is __ (Nearest integer). Given $K_f = 1.8$ K kg $mol^{-1}$

**Sol.** **243**

$\Delta T_f = i \cdot k_f \cdot m$

$m = \frac{38}{98} \times \frac{1000}{62}$

$\Delta T_f = 2.67 \times 1.8 \times \frac{38}{98} \times \frac{1000}{62}$

$\Delta T_f = 30.05$

F.P. = 273 – 30 = 243 K

**55.** The strength of 50 volume solution of hydrogen peroxide is__________ $g/\mathrm{L}$ (Nearest integer).

Given:

Molar mass of $H_2O_2$ is 34 g $mol^{-1}$

Molar volume of gas at STP $= 22.7$ L.

**Sol. 150**

Molarity = $\frac{\text{Volume Strength}}{11.35}$

Strength (g/lit) = Molarity × mol. Wt

$= \frac{50}{11.35} \times 34 ==$ 150 gm/lit

**56.** The electrode potential of the following half cell at 298 K

$X|X^{2+}(0.001M) \parallel Y^{2+}(0.01M)|Y$ is__________ $\times 10^{-2}$ V (Nearest integer).

Given: $E^0_{X^{2+}|X} = -2.36$ V

$E^0_{Y^{+2}|Y}$

$E^0Y^{2+1Y} = +0.36$ V

$\frac{2.303RT}{F} = 0.06$ V

**Sol. 275**

$x + y^{+2} \rightarrow y + x^{+2}$

$E^o$ Cell = $E^o_{Cathode} - E^o_{Anode}$

$E^o$ Cell = 0.36 – (–2.36) = 2.72 V

$E_{Cell} = 2.72 - \frac{0.06}{2} \log \frac{x^{+2}}{y^{+2}}$

$E_{Cell} = 2.72 - \frac{0.06}{2} \log \frac{0.001}{0.01}$

= 2.72 + 0.03 = 2.75 V

= $275 \times 10^{-2}$ V

**57.** An organic compound undergoes first order decomposition. If the time taken for the 60% decomposition is 540 s, then the time required for 90% decomposition will be is______ s. (Nearest integer).

Given: $\ln 10 = 2.3; \log 2 = 0.3$

**Sol. 1350**

$K = \frac{2.303}{540}\log\frac{100}{40}$

$K = \frac{2.303}{540}\times 0.4$

$t_{90} = \frac{2.303\times 540}{2.303\times 0.4}\log\frac{100}{10}$

$t_{90} = 1350$

**58.** Consider the following equation:

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g), \Delta H = -190$ kJ

The number of factors which will increase the yield of $SO_3$ at equilibrium from the following is

A. Increasing temperature

B. Increasing pressure

C. Adding more $SO_2$

D. Adding more $O_2$

E. Addition of catalyst

**Sol.** **3**

The yield of $SO_3$ at equilibrium will be due to:

B. Increasing pressure

C. Adding more $SO_2$

D. Adding more $O_2$

**59.** Iron oxide FeO, crystallises in a cubic lattice with a unit cell edge length of 5.0Å. If density of the FeO in the crystal is 4.0 g $cm^{-3}$, then the number of FeO units present per unit cell is_______ (Nearest integer)

Given: Molar mass of Fe and O is 56 and 16 g $mol^{-1}$ respectively. $N_A = 6.0 \times 10^{23}$ $mol^{-1}$

**Sol.** **4**

$d = \frac{z\times M}{N_0 \times a^3}$

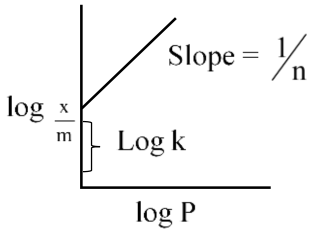
$4 = \frac{z\times 72}{6\times 10^{23}\times 125\times 10^{-24}}$

$Z = 4.166 \cong 4$

**60.** The graph of $\log \frac{x}{m}$ vs $\log p$ for an adsorption process is a straight line inclined at an angle of $45°$ with intercept equal to $0.6020$. The mass of gas adsorbed per unit mass of adsorbent at the pressure of $0.4$ atm is________ $\times 10^{-1}$ (Nearest integer)

Given: $\log 2 = 0.3010$

**Sol.** **16**




Slope = tan 45° = 1

logK = 0.6020 = log 4

K = 4

$\frac{x}{m} = KP^{1/n}$

$\frac{x}{m} = 4(0.4)1 = 16 \times 10^{-1}$

# Mathematics

## SECTION - A

**61.** A vector $\vec{v}$ in the first octant is inclined to the x-axis at $60°$, to the y-axis at 45 and to the z-axis at an acute angle. If a plane passing through the points $(\sqrt{2}, -1, 1)$ and $(a, b, c)$, is normal to $\vec{v}$, then

(1) $\sqrt{2}a + b + c = 1$ (2) $a + \sqrt{2}b + c = 1$

(3) $a + b + \sqrt{2}c = 1$ (4) $\sqrt{2}a - b + c = 1$

**Sol.** **2**

$\cos\alpha = \cos 60$ $\cos\beta = \cos 45$

$\ell = \frac{1}{2}$ $m = \frac{1}{\sqrt{2}}$

$l^2 + m^2 + n^2 = 1$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + n^2 = 1$ $n^2 = 1 - \frac{3}{4} = \frac{1}{4}$

$\boxed{n = \frac{1}{2}}$

Direction of $\vec{v}$ is $\frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{2}\hat{k}$

Equation of plane through ( $\sqrt{2}$ , –1, 1) & Normla to $\vec{v}$ is

$\frac{1}{2}(x - \sqrt{2}) + \frac{1}{\sqrt{2}}(y+1) + \frac{1}{2}(z-1) = 0$

It passes through (a,b,c)

$\left(a - \sqrt{2}\right) + \sqrt{2}(b+1) + (c-1) = 0$

$\Rightarrow a + \sqrt{2}b + c = \sqrt{2} - \sqrt{2} + 1$

$\Rightarrow \boxed{a + \sqrt{2}b + c = 1}$

**62.** Let $a, b, c > 1, a^3, b^3$ and $c^3$ be in A.P., and $\log_a b, \log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is $-444$, then abc is equal to:

(1) $\frac{125}{8}$ (2) 216 (3) 343 (4) $\frac{343}{8}$

**Sol.** **2**

If $\log_a b, \log_c a, \log_b c \rightarrow$ G.P.

$(\log_c a)^2 = \log_a b \times \log_b c$

$(\log_c a)^2 = \log_a c$

$\Rightarrow (\log_c a)^2 = \frac{1}{\log_c a}$

$\Rightarrow (\log_c a)^3 = 1$

$\Rightarrow \log_c a = 1$

$\boxed{a = c}$

If $a^3 b^3 c^3 \to$ A.P

$2b^3 = a^3 + c^3$

If a = c

$\Rightarrow \boxed{a = b = c}$

For AP

$A = \frac{a+4a+a}{3}$ $\qquad D = \frac{a-8a+a}{10}$

$A = 2a$ $\qquad D = \frac{-3a}{5}$

$S_{20} = \frac{20}{2}\left[2\times 2a + (20-1)\left(\frac{-3a}{5}\right)\right]$

$= 10\left[4a - \frac{57a}{5}\right]$

$= 10\left[-\frac{37a}{5}\right] = -444$

$\Rightarrow a = \frac{444\times 5}{37\times 10}$

$\boxed{a = 6}$

$\Rightarrow \boxed{abc = 6\times 6\times 6 = 216}$

**63.** Let $a_1 = 1, a_2, a_3, a_4$, ..... be consecutive natural numbers.

Then $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + \cdots. + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$ is equal to

(1) $\cot^{-1}(2022) - \frac{\pi}{4}$

(2) $\frac{\pi}{4} - \cot^{-1}(2022)$

(3) $\tan^{-1}(2022) - \frac{\pi}{4}$

(4) $\frac{\pi}{4} - \tan^{-1}(2022)$

**Sol.** **3**

$a_1 = 1, a_2, a_3, .....a_n$ be consecative natural numbers.

$\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + ..... + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$

$\Rightarrow \quad T_K = \tan^{-1}\left(\frac{1}{1+K(K+1)}\right)$

$= \tan^{-1}\left(\frac{K+1-K}{1+K(K+1)}\right)$

$= \tan^{-1}(K+1) - \tan^{-1} K$

$T_1 = \tan^{-1} 2 - \tan^{-1} 1$

$T_2 = \tan^{-1} 3 - \tan^{-1} 2$

$T_3 = \tan^{-1} 4 - \tan^{-1} 3$

$\vdots$

$\underline{T_{2021} = \tan^{-1}(2022) - \tan^{-1}(2021)}$

On adding

$\Sigma T_n = \tan^{-1}(2022) - \tan^{-1}(1)$

$\boxed{\sum_{n=1}^{2021} T_n = \tan^{-1}(2022) - \frac{\pi}{4}}$

**64.** Let $\lambda \in \mathbb{R}, \vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}, \vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$

If $((\vec{a}+\vec{b}) \times (\vec{a} \times \vec{b})) \times (\vec{a} - \vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$, then $|\lambda(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b})|^2$ is equal to

(1) 132 (2)136 (3) 140 (4) 144

**Sol.** **3**

$\left((\vec{a}+\vec{b})\times(\vec{a}\times\vec{b})\right)\times\left(\vec{a}-\vec{b}\right)- = 8\hat{i} - 40\hat{j} - 24\hat{k}$

$\Rightarrow \left(\vec{a}\times\left(\vec{a}\times\vec{b}\right) + \vec{b}\times\left(\vec{a}\times\vec{b}\right)\right)\times(\vec{a}-\vec{b})$

$\Rightarrow \left(\left(\vec{a}\cdot\vec{b}\right)\vec{a} - \left(\vec{a}\cdot\vec{a}\right)\vec{b} + (\vec{b}\cdot\vec{b})\vec{a} - (\vec{b}\cdot\vec{a})\vec{b}\right)\times(\vec{a}-\vec{b})$

$\Rightarrow 0 - (\vec{a}\cdot\vec{b})(\vec{a}\times\vec{b}) - a^2(\vec{b}\times\vec{a}) + 0 - b^2(\vec{a}\times\vec{b}) - (\vec{a}\cdot\vec{b})\vec{b}\times\vec{a} = 8\hat{i} - 40\hat{j} - 24\hat{k}$

$\Rightarrow \left(a^2 - b^2\right)(\vec{a}\times\vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$

$\left(\left(\lambda^2+4+9\right) - \left(1+\lambda^2+4\right)\right)\left(\vec{a}\times\vec{b}\right)$

$8\left(\vec{a}\times\vec{b}\right) = 8(\hat{i} - 5\hat{j} - 3\hat{k})$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \lambda & 2 & -3 \\ 1 & -\lambda & 2 \end{vmatrix} = \hat{i} - 5\hat{j} - 3\hat{k}$$

$\hat{i}(4-3\lambda) - \hat{j}(2\lambda+3) + \hat{k}(-\lambda^2-2) = \hat{i} - 5\hat{j} - 3\hat{k}$

$\Rightarrow \quad 4 - 3\lambda = 1 \qquad 2\lambda + 3 = 5 \qquad -\lambda^2 - 2 = -3$

$3\lambda = 3 \qquad \lambda^2 = 1$

$\boxed{\lambda = 1} \qquad \boxed{\lambda = 1}$

$\left|\lambda(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right| = \left|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right|^2$

$\Rightarrow \quad \left|-\vec{a}\times\vec{b} + \vec{b}\times\vec{a}\right|^2 = \left|2(\vec{a}\times\vec{b})\right|^2 = 4(1+25+9) = 140$

**65.** Let q be the maximum integral value of p in $[0,10]$ for which the roots of the equation $x^2 - px + \frac{5}{4}p = 0$ are rational. Then the area of the region $\{(x,y): 0 \le y \le (x-q)^2, 0 \le x \le q\}$ is

(1) 243 (2) 164 (3) $\frac{125}{3}$ (4) 25

**Sol.** **1**

$$x^2 - px + \frac{5}{4}p = 0$$

Roots are rational

D = A perfect square

$$p^2 - 4(1)\frac{5}{4}p$$

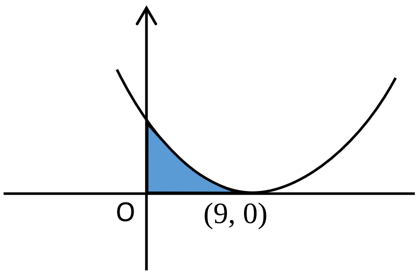
$p^2 - 5p$ = A perfect square

for p = 0, p = 5, p = 9 the D is a perfect square

$\therefore$ maximum integral of p is 9.

$$\boxed{q = 9}$$

$$\{(x, y); 0 \le y \le (x-9)^2, 0 \le x \le 9\}$$




Area = $\int_0^9 (x-9)^2 dx$

$$\Rightarrow \left.\frac{(x-9)^3}{3}\right]_0^3$$

$$\Rightarrow 0 - \frac{(0-9)^3}{3}$$

$$\Rightarrow \frac{9 \times 9 \times 9}{3}$$

= 243

**66.** Let f, g and h be the real valued functions defined on $\mathbb{R}$ as

$$f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 1, & x = 0 \end{cases}, g(x) = \begin{cases} \frac{\sin(x+1)}{(x+1)}, & x \neq -1 \\ 1, & x = -1 \end{cases}$$

and $h(x) = 2[x] - f(x)$, where [x] is the greatest integer $\le$ x.

Then the value of $\lim_{x\to 1} g(h(x-1))$ is

(1) $-1$ (2) 0 (3) $\sin(1)$ (4) 1

**Sol.** **4**

LHL

$$\lim_{\delta \to 0} g(h(-\delta)) \quad \delta > 0$$

$$\lim_{\delta \to 0} g(-2+1)$$

$\Rightarrow g(-1) = 1$

RHL

$\lim\limits_{\delta \to 0} g(h(\delta))$

$\lim\limits_{\delta \to 0} g(2\times 0\text{-}1)$

$\lim\limits_{\delta \to 0} g(-1)$

$\lim\limits_{x \to 1} g(h(x-1) = 1$

**67.** Let S be the set of all values of $a_1$ for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \ldots . a_{100}$ is 25 . Then S is

(1) N (2) $\phi$ (3) {99} (4) {9}

**Sol.** **1**

Let $a_1 = n$ $a_2 = n+1$ $a_3 = n+2$ ......

$$\bar{x} = \frac{n+(n+1)+(n+2)+\ldots\ldots n+99}{100}$$

$$= \frac{100n + \frac{100\times 99}{2}}{100} = n + \frac{99}{2}$$

Mean deviation about the mean

$$\frac{1}{100}\sum |x_i - \bar{x}|$$

$$\Rightarrow \frac{1}{100}\left(\frac{99}{2}+\frac{97}{2}+\frac{95}{2}+\ldots\ldots+\frac{97}{2}+\frac{99}{2}\right)$$

$$\Rightarrow \frac{2}{100}\left(\frac{99}{2}+\frac{97}{2}+\frac{95}{2}+\ldots\ldots 50\,\text{terms}\right)$$

$$\Rightarrow \frac{2}{100}\times\frac{1}{2}\times(50)^2 = \frac{50\times 50}{100} = 25$$

It is 25 irrespective of the value of

$\therefore n \in N$

$\Rightarrow \boxed{S = N}$

**68.** For $\alpha, \beta \in \mathbb{R}$, suppose the system of linear equations

$x - y + z = 5$

$2x + 2y + \alpha z = 8$

$3x - y + 4z = \beta$

has infinitely many solutions. Then $\alpha$ and $\beta$ are the roots of

(1) $x^2 + 14x + 24 = 0$ (2) $x^2 + 18x + 56 = 0$

(3) $x^2 - 18x + 56 = 0$ (4) $x^2 - 10x + 16 = 0$

**Sol.** **3**

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{vmatrix} = 0$$

$1(8+\alpha)+1(8-3\alpha)+1(-2-6)=0$

$\Rightarrow\ 8+\alpha+8-3\alpha-8=0$

$-2\alpha=-8$

$\boxed{\alpha=4}$

$D_1 = 0$

$$\Rightarrow \begin{vmatrix} 5 & -1 & 1 \\ 8 & 2 & 4 \\ \beta & -1 & 4 \end{vmatrix} = 0$$

$5(8+4)+1(32-4\beta)+1(-8-2\beta)=0$

$60+32-4\beta-8-2\beta=0$

$\Rightarrow\ -6\beta=-84$

$\boxed{\beta=14}$

Equation having roots as $\alpha \,\&\, \beta$

$\boxed{x^2-18x+56=0}$

**69.** Let $\vec{a}$ and $\vec{b}$ be two vectors, Let $|\vec{a}|=1, |\vec{b}|=4$ and $\vec{a}\cdot\vec{b}=2$. If $\vec{c}=(2\vec{a}\times\vec{b})-3\vec{b}$, then the value of $\vec{b}\cdot\vec{c}$ is

(1) $-24$ (2) $-84$ (3) $-48$ (4) $-60$

**Sol.** **3**

$\vec{b}.\vec{c}=(2\vec{a}\times\vec{b}).\vec{b}-3\vec{b}.\vec{b}$

$=\ 0-3b^2$

$=-3\times 16=-48$

$\boxed{\vec{b}.\vec{c}=-48}$

**70.** If the functions $f(x)=\frac{x^3}{3}+2bx+\frac{ax^2}{2}$ and $g(x)=\frac{x^3}{3}+ax+bx^2, a\neq 2b$ have a common extreme point, then $a+2b+7$ is equal to:

(1) $\frac{3}{2}$ (2) 3 (3) 4 (4) 6

**Sol.** **4**

$f'(x)=x^2+2b+ax=0$

$g'(x)=x^2+a+2bx=0$

x = 1 is the common root

1 + 2b + a = 0

2b + a + 1 + 6 = 6

$\boxed{2b+a+7=6}$

**71.** If P is a $3 \times 3$ real matrix such that $P^T = aP + (a-1)I$, where $a > 1$, then

(1) $|\text{AdjP}| = \frac{1}{2}$ (2) $|\text{AdjP}| = 1$

(3) P is a singular matrix (4) $|\text{AdjP}| > 1$

**Sol.** **2**

$$\left(P^T\right)^T = aP^T(a-1)I$$

$$P = a(aP + (a-1)I) + (a-1)I$$

$$= a^2P + (a^2 - a)I + (a-1)I$$

$$= a^2P + (a^2 - a + a - 1)I$$

$$P = a^2P + (a^2-1)I \quad \Rightarrow \quad P = (1-a^2) = (a^2-1)I$$

$$\boxed{P = -I}$$

$$|\text{AdjP}| = |P|^{3-1} = (-1)^2$$

$$= 1$$

**72.** The number of ways of selecting two numbers a and b, $a \in \{2,4,6,\ldots,100\}$ and $b \in \{1,3,5,\ldots,99\}$ such that 2 is the remainder when $a + b$ is divided by 23 is

(1) 268 (2) 108 (3) 54 (4) 186

**Sol.** **2**

| a + b = 25, | | a + b = 71 | | a + b = 117 | | a + b = 163 | |
|---|---|---|---|---|---|---|---|
| a | b | a | b | a | b | a | b |
| 2 | 23 | 2 | 69 | 18 | 99 | 64 | 99 |
| 4 | 21 | 4 | 67 | 20 | 97 | 66 | 97 |
| ⋮ | | ⋮ | | ⋮ | | ⋮ | |
| 24 | 1 | 70 | 1 | 100 | 17 | 100 | 63 |
| 12 cases | | 35 cases | | 42 cases | | 19 cases | |

Total ways = 12 + 35 + 42 +19

= 108

**73.** $\lim\limits_{n\to\infty} \frac{3}{n}\left\{4 + \left(2+\frac{1}{n}\right)^2 + \left(2+\frac{2}{n}\right)^2 + \cdots + \left(3-\frac{1}{n}\right)^2\right\}$ is equal to

(1) 12 (2) $\frac{19}{3}$ (3) 0 (4) 19

**Sol.** **4**

$$\lim_{x\to\infty} \frac{3}{n}\left\{\underset{\substack{\uparrow \\ (2+0/n)^2}}{4}\ \left(2+\frac{1}{n}\right)^2 + \left(2+\frac{2}{n}\right)^2 + \ldots. + \left(3-\frac{1}{n}\right)^2\right\}$$

$$\Rightarrow \sum_{r=0}^{n-1} \frac{3}{n}\left(2+\frac{r}{n}\right)^2$$

$$\Rightarrow \int_0^1 3(2+x)^2\,dx$$

$$\Rightarrow \left[(2+x)^3\right]_0^1$$

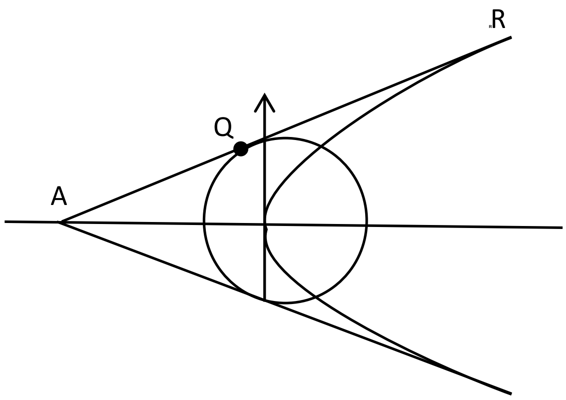
$$\Rightarrow (2+1)^3-(2+0)^3$$

$$\Rightarrow 27-8=19$$

**74.** Let A be a point on the x-axis. Common tangents are drawn from A to the curves $x^2+y^2=8$ and $y^2=16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to

(1) 81 (2) 72 (3) 76 (4) 64

**Sol.** **2**




$y^2=16x$ circle

Tangent $x^2+y^2=8$

$y=mx+\frac{4}{m}$ Tagent

$y=mx\pm 2\sqrt{2}\sqrt{1+m^2}$

$\frac{4}{m}=\pm 2\sqrt{2}\sqrt{1+m^2}$

$\frac{16}{m^2}=8+8m^2$

$8m^4+8m^2=16$

$m^4+m^2=2$

$m^2=1,-2$

Let $m=1$

$\underline{m>1}$

$\therefore y=x+4$

Point of tangecy at parabola

$Q\left(\frac{4}{m^2},\frac{8}{m}\right)$

$Q(4,8)$

Point of tangency at circle

$eq^n$ at tangent at $R(x_1, y_1)$

is $\boxed{T=0}$

$xx_1 + yy_1 = 8$

Comparison with

$x - y + 4 = 0$

$\frac{x_1}{1} = \frac{y_1}{-1} = -\frac{8}{4}$

$x_1 = -2 \qquad y_1 = 2$

$R(-2,2)$

Now $QR^2 = \sqrt{(4+2)^2 + (8-2)^2}$

$\boxed{QR^2 = 72}$

**75.** If a plane passes through the points $(-1, k, 0), (2, k, -1), (1,1,2)$ and is parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$, then the value of $\frac{k^2+1}{(k-1)(k-2)}$ is

(1) $\frac{17}{5}$ (2) $\frac{13}{6}$ (3) $\frac{6}{13}$ (4) $\frac{5}{17}$

**Sol. 2**

$Eq^n$ of plane

$$\begin{vmatrix} x-2 & y-k & 3+1 \\ 1-2 & 1-k & 2+1 \\ -1-2 & k-k & 0+1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-k & 3+1 \\ -1 & 1-k & 3 \\ -3 & 0 & 1 \end{vmatrix} = 0$$

$(x-2)(1-k-0) - (y-k)(-1+9) + (3+1)(0+3-3k) = 0$

$\Rightarrow (1-k)x - 8y + (3-3k)z - 2 + 2k + 8k + 3 - 3k = 0$

$(1-K)x - 8y + (3-3K)z + 7K + 1 = 0$

Plane is parallel to the line L:

$\therefore (1-k)1 - 8.1 + (3-3K)(-1) = 0$

$\Rightarrow 1 - k - 8 - 3 + 3k = 0$

$2k = 10$

$\boxed{k=5}$

$\frac{k^2+1}{(k-1)(k-2)}$

$= \frac{25+1}{(5-1)(5-2)} = \frac{26}{4\times 3} = \frac{13}{6}$

**76.** The range of the function $f(x) = \sqrt{3-x} + \sqrt{2+x}$ is:

(1) $[2\sqrt{2}, \sqrt{11}]$ (2) $[\sqrt{5}, \sqrt{13}]$ (3) $[\sqrt{2}, \sqrt{7}]$ (4) $[\sqrt{5}, \sqrt{10}]$

**Sol.** **4**

$3-x \geq 0$ $\quad$ $2+x \geq 0$

$x \leq 3$ $\quad$ $x \geq -2$

$x \in [-2, 3]$

Now, $f(-2) = \sqrt{3+2} = \sqrt{5}$

$f(3) = \sqrt{2+3} = \sqrt{5}$

$f(x) = \sqrt{3-x} + \sqrt{2+x}$

$$f'(x) = \frac{1}{2\sqrt{3-x}} x - 1 + \frac{1}{2\sqrt{2+x}} = 0$$

$$\Rightarrow \frac{1}{\sqrt{3-x}} = \frac{1}{\sqrt{2+x}}$$

$\Rightarrow 3-x = 2+x$

$\Rightarrow 2x = 1$

$x = 1/2$

$$f\left(\frac{1}{2}\right) = \sqrt{3 - 1/2} + \sqrt{2 + 1/2}$$

$$= \sqrt{\frac{5}{2}} + \sqrt{\frac{5}{2}} \Rightarrow 2\sqrt{\frac{5}{2}} \Rightarrow \sqrt{10}$$

Range $= \left[\sqrt{5}, \sqrt{10}\right]$

**77.** The solution of the differential equation $\frac{dy}{dx} = -\left(\frac{x^2+3y^2}{3x^2+y^2}\right), y(1) = 0$ is

(1) $\log_e|x+y| - \frac{xy}{(x+y)^2} = 0$ (2) $\log_e|x+y| + \frac{2xy}{(x+y)^2} = 0$

(3) $\log_e|x+y| - \frac{2xy}{(x+y)^2} = 0$ (4) $\log_e|x+y| + \frac{xy}{(x+y)^2} = 0$

**Sol.** **2**

$y = Vx$

$$\frac{dy}{dx} = V + \frac{xdv}{dx}$$

$$V + \frac{xdv}{dx} = -\frac{x^2 + 3v^2x^2}{3x^2 + v^2x^2}$$

$$x\frac{dv}{dx} = -\frac{1+3v^2}{3+v^2} - v$$

$$x\frac{dv}{dx} = -\frac{1+3v^2+3v+v^3}{3+v^2}$$

$$\int \frac{3+v^2}{1+3v^2+3v+v^3} dv = -\int \frac{dx}{x}$$

$$\Rightarrow \int \frac{3+v^2}{(1+v)^3}dv = -\ln x + C$$

Let $v+1=t$

$dv = dt$

$$\int \frac{3+(t-1)^2}{t^3}dt = -\ln x + c$$

$$\Rightarrow \int \frac{t^2-2t+4}{t^3}dt$$

$$\Rightarrow \int \left(\frac{1}{t}-\frac{2}{t^2}+\frac{4}{t^3}\right)dt = -\ln x + c$$

$$\Rightarrow \ln t + \frac{2}{t} - \frac{4}{2t^2} = -\ln x + C$$

$$\Rightarrow \ln\left(\frac{y}{x}+1\right)^{-1} \frac{2}{\frac{y}{x}+1} - \frac{4}{2\left(y/x+1\right)^2} = -\ln x + C$$

$$\Rightarrow \ln\left(\frac{y+x}{x}\right) + \frac{2x}{y+x} - \frac{2x^2}{(x+y)^2} = -\ln x + c$$

$$\Rightarrow \ln|x+y| + \frac{2x}{(x+y)^2}(x+y-x) = C$$

$$\Rightarrow \boxed{\ln|x+y| + \frac{2xy}{(x+y)^2} = C}$$

**78.** The parabolas : $ax^2 + 2bx + cy = 0$ and $dx^2 + 2ex + fy = 0$ intersect on the line $y = 1$. If a, b, c, d, e, f are positive real numbers and a, b, c are in G.P., then

(1) d, e, f are in G.P.

(2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

(3) d, e, f are in A.P.

(4) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

**Sol.** **2**

at $y=1$, Both curve intersect

$$\Rightarrow \left.\begin{matrix} ax^2+2bx+c=0 \\ dx^2+2ex+f=0 \end{matrix}\right\} \text{Common Root}$$

Given a, b, c are in G.P

$$\boxed{b^2 = ac}$$

$\Rightarrow D = 4b^2 - 4ac = 0$ for the first equation

$\Rightarrow$ Both the Root are equal

$\because$ sum of the roots $= -2\frac{b}{a}$

$$\alpha + \alpha = -2\frac{b}{a}$$

$$\boxed{\alpha = -\frac{b}{a}}$$

It satisfies the second equation also

$$d\left(-\frac{b}{a}\right)^2 + 2e\left(-\frac{b}{a}\right) + f = 0$$

$$d\left(\frac{b^2}{a^2}\right) - \frac{2eb}{a} + f = 0$$

$$d\left(\frac{ac}{a^2}\right) - 2e\frac{b}{a} + f = 0$$

$$\frac{d}{a} - \frac{2eb}{ac} + \frac{f}{c} = 0$$

$$\frac{d}{a} - \frac{2eb}{b^2} + \frac{f}{c} = 0$$

$$\Rightarrow \boxed{2\frac{e}{b} = \frac{d}{a} + \frac{f}{c}} \Rightarrow \boxed{\frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in AP}}$$

**79.** Consider the following statements:

P : I have fever

Q: I will not take medicine

R : I will take rest.

The statement "If I have fever, then I will take medicine and I will take rest" is equivalent to:

(1) $((\sim P) \vee \sim Q) \wedge ((\sim P) \vee R)$

(2) $(P \vee Q) \wedge ((\sim P) \vee R)$

(3) $((\sim P) \vee \sim Q) \wedge ((\sim P) \vee \sim R)$

(4) $(P \vee \sim Q) \wedge (P \vee \sim R)$

**Sol.** **1**

$$P \to (\sim Q \wedge R)$$

$$\sim P V(\sim Q \wedge R)$$

$$\Rightarrow ((\sim P) V(\sim Q)) \wedge ((\sim P) v R)$$

**80.** $x = (8\sqrt{3} + 13)^{13}$ and $y = (7\sqrt{2} + 9)^9$. If $[t]$ denotes the greatest integer $\leq t$, then

(1) $[x]$ is odd but $[y]$ is even

(2) $[x] + [y]$ is even

(3) $[x]$ and $[y]$ are both odd

(4) $[x]$ is even but $[y]$ is odd

**Sol.** **2**

Let $x = I_1 + f_1$

$$\left(8\sqrt{3} + 13\right)^{13} = I_1 + f_1$$

$$\left(8\sqrt{3} - 13\right)^{13} = f_1{}'(\text{let})$$

On Subtaction

$\left(8\sqrt{3}+13\right)^{13}-\left(8\sqrt{b}-13\right)^{13}=I+f_1-f_1'$

$2\left[{}^{13}C_1\left(8\sqrt{3}\right)^{12}\right]13+{}^{13}C_3\left(8\sqrt{3}\right)^{10}13^3$

$+----]=I+0$

$\because I=\text{Even Number}$

$[x]=\text{Even}$

similarly,

Let $y=I_2+f_2$

$\left(7\sqrt{2}+9\right)^9=I_2+f_2$

$\left(7\sqrt{2}-9\right)^9=f_2'$

On Subtaction

$\left(7\sqrt{2}+9\right)^9-\left(7\sqrt{2}-9\right)^9=I_2+f_2-f_2^1$

$2\left[{}^9C_1\left(7\sqrt{2}\right)^8.9+{}^9C_3\left(7\sqrt{2}\right)^7 9^2---\right]=I_2+0$

$I_2=\text{Even}$

$$\boxed{\begin{aligned}\because [x]+[y]&=\text{Even}+\text{Even}\\&=\text{Even}\end{aligned}}$$

## SECTION - B

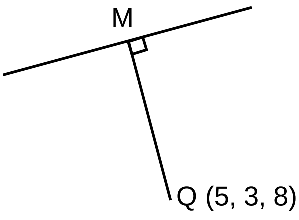
**81.** Let a line L pass through the point $P(2,3,1)$ and be parallel to the line $x+3y-2z-2=0=x-y+2z$. If the distance of L from the point $(5,3,8)$ is $\alpha$, then $3\alpha^2$ is equal to ______.

**Sol.** **158**

The Direction ratio of line

$$\begin{vmatrix} i & j & k \\ 1 & 3 & -2 \\ 1 & -1 & 2 \end{vmatrix} \begin{aligned}&=i(6-2)-j(2+2)+k(-1-3)\\&=4i-4j-4k\end{aligned}$$

Equation of line L




$$\frac{x-2}{1}=\frac{y-3}{-1}=\frac{z-1}{-1}=\lambda(\alpha d)$$

Let $M(\lambda+2, -\lambda+3, -\lambda+1)$

DR's of MQ is $<\lambda+2-5, -\lambda+3-3, -\lambda+1-8>$

$< \lambda - 3, -\lambda . -\lambda > 7 >$

$\because\ L \perp MQ$

$\Rightarrow (\lambda - 3)(1) + (-\lambda)(-1) + (-\lambda - 7)(-1) = 0$

$\Rightarrow \lambda - 3 + \lambda + \lambda + 7 = 0$

$\Rightarrow 3\lambda = -4 \Rightarrow \lambda = -\frac{4}{3}$

$\therefore\ M\left(-\frac{4}{3}+2, \frac{+4}{3}+3, \frac{4}{3}+1\right) = \left(\frac{2}{3}, \frac{13}{3}, \frac{7}{3}\right)$

$MQ = \alpha$

$\therefore\ 3\alpha^2 = 3 \times \left(\left(5-\frac{2}{3}\right)^2 + \left(3-\frac{13}{3}\right)^2 + \left(8-\frac{7}{5}\right)^2\right)$

$= 3\left(\frac{169}{9}+\frac{16}{9}+\frac{289}{9}\right) \Rightarrow \frac{474}{9} = 158$

**82.** A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is p. Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colour is q. If $p : q = m : n$, where m and n are coprime, then $m + n$ is equal to ______.

**Sol.** **14**

$p = 1.\frac{1}{6}$

$q = \left({}^6C_1 . \frac{1}{6}.\frac{1}{6}.\frac{1}{6}.\frac{5}{6}\right)\frac{4!}{3!} = \frac{5}{216} \times 4 = \frac{5}{54}$

$\frac{p}{q} = \frac{1/6}{5/54} = \frac{9}{5}$

$m = 9$

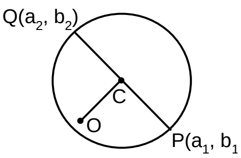
$n = 5$

$m + n = 9 + 5 = 14$

**83.** Let $P(a_1, b_1)$ and $Q(a_2, b_2)$ be two distinct points on a circle with center $C(\sqrt{2}, \sqrt{3})$. Let O be the origin and OC be perpendicular to both CP and CQ.

If the area of the triangle OCP is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to ______.

**Sol.** **24**




OC is $\perp^r$ to both CP & CQ

$\Rightarrow$ PQ is a Diameter

Area of $\Delta OCP = \dfrac{\sqrt{35}}{2}$

$\dfrac{1}{2} \times CP \times OC = \dfrac{\sqrt{35}}{2}$

$CP \times \sqrt{2+3} = \sqrt{35}$

$CP = \sqrt{7} \Rightarrow$ radius $= \sqrt{7}$

Now $OP^2 = OC^2 + PC^2$

$a_1^2 + b_1^2 = 2+3+7 = 12$

Similarly

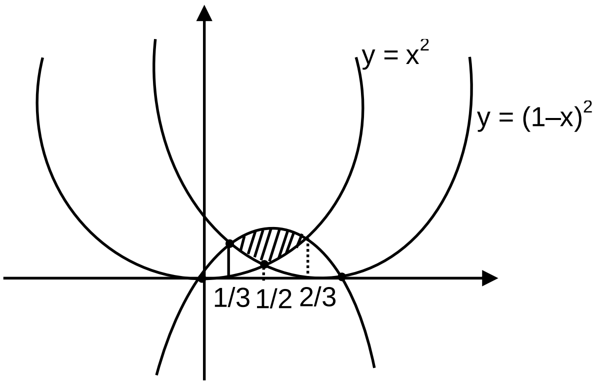
$OQ^2 = OC^2 + CQ^2$

$a_1^2 + b_1^2 = 2+3+7 = 12$

$\therefore\ a_1^2 + a_2^2 + b_1^2 + b_2^2 = 24$

**84.** Let A be the area of the region $\{(x, y): y \geq x^2, y \geq (1-x)^2, y \leq 2x(1-x)\}$. Then 540 A is equal to ______.

**Sol.** **25**




| | | |
|---|---|---|
| $x^2 = (1-x)^2$ | $x^2 = 2x - 2x^2$ | $(1-x)^2 + 2x - 2x^2$ |
| $x^3 = 1 + x^2 - 2x$ | $3x^1 = 2x$ | $1 + x^2 - 2x = 2x - 2x^2$ |
| $x = \dfrac{1}{2}$ | $x(3x - 2) = 0$ | $\Rightarrow 3x^2 - 4x + 1 = 0$ |
| | $x = 0, \dfrac{2}{3}$ | $\Rightarrow 3x^2 - 3x - x + 1 = 0$ |
| | | $\Rightarrow 3x(x-1) - 1(x-1) = 0$ |
| | | $x = 1, \dfrac{1}{3}$ |

Required Area

$$A = \int_{\frac{1}{3}}^{\frac{1}{2}} \left\{(2x-2x^2)-(1-x)^2\right\}dx + \int_{\frac{1}{2}}^{\frac{2}{3}} \left((2x-2x^2)-x^2\right)dx$$

$$\Rightarrow \left[x^2 - \frac{2x^3}{3} + \frac{(1-x)^3}{3}\right]_{\frac{1}{3}}^{\frac{1}{2}} + (x^2 - x^3)_{1/2}^{2/3}$$

$$\Rightarrow \left(\frac{1}{4} - \frac{2}{3}.\frac{1}{8} + \frac{1}{8.3}\right) - \left(\frac{1}{9} - \frac{2}{3}.\frac{1}{27} + \frac{8}{27.3}\right) + \left(\frac{4}{9} - \frac{8}{27}\right) - \left(\frac{1}{4} - \frac{1}{8}\right)$$

$$\Rightarrow -\frac{1}{24} - \frac{1}{9} - \frac{6}{3\times 27} + \frac{4}{9} - \frac{8}{27} + \frac{1}{8}$$

$$\Rightarrow -\frac{1}{24} + \frac{3}{9} - \frac{10}{27} + \frac{3}{24} = \frac{-27+216-240+81}{24\times 27} = \frac{297-267}{24\times 27} = A$$

$$540\ A = 540 \times \frac{30}{24\times 27} = 25$$

**85.** The $8^{th}$ common term of the series

$$S_1 = 3 + 7 + 11 + 15 + 19 + \cdots..$$
$$S_2 = 1 + 6 + 11 + 16 + 21 + \cdots.$$

is _______.

**Sol.** **151**

$8^{th}$ common term of the series

$S_1 = 3 + 7 + 11 + 15 + 19 + \ldots\ldots\ldots$

$S_2 = 1 + 6 + 11 + 16 + 21 + \ldots\ldots\ldots$

First common term = 11

common diff of the AP of common terms

= L.C.M of {4, 5}

= 20

$\therefore$ AP

11, 31, 51, .....

$T_8 = 11 + (8 - 1)20$

$= 11 + 140$

$T_8 = 151$

**86.** Let $A = \{1,2,3,5,8,9\}$. Then the number of possible functions $f: A \to A$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is equal to _______.

**Sol.** **1**

| LHL | RHL |
|---|---|
| $\lim\limits_{h\to 0} g(H(1-h-1))$ | $\lim\limits_{h\to 0} g(H(1+h-1))$ |
| $\lim\limits_{h\to 0} g(2(-1)-f(-h))$ | $\lim\limits_{h\to 0} g(H(h))$ |
| $\lim\limits_{h\to 0} g\left(-2-\frac{1-h}{\lvert -h\rvert}\right)$ | $\Rightarrow \lim\limits_{h\to 0} g(2(0)+(h))$ |
| $\Rightarrow g\left(-2-\frac{-1}{1}\right)$ | g (0 – 1) |
| $\Rightarrow 2(-1) = +1$ | $\Rightarrow 1$ |

$\therefore \lim\limits_{h\to 0} g(H(x-1) = 1$

**87.** If $\int \sqrt{\sec 2x - 1}dx = \alpha \log_e \left|\cos 2x + \beta + \sqrt{\cos 2x\left(1 + \cos\frac{1}{\beta}x\right)}\right| + \text{constant}$, then $\beta - \alpha$ is equal to ______.

**Sol.** **1**

$$I = \int \sqrt{\sec 2x - 1}dx$$

$$\Rightarrow \int \sqrt{\frac{1-\cos 2x}{\cos 2x}}dx$$

$$\Rightarrow \int \frac{\sqrt{2}\sin x}{\sqrt{2\cos^2 x - 1}}dx$$

Let $\sqrt{2}\cos x = t$

$-\sqrt{\alpha}\sin x \; dx = dt$

$$I = \int \frac{-dt}{\sqrt{t^2-1}} = \ln\left|t + \sqrt{t'-1}\right| + c$$

$$\Rightarrow -\ln\left|\sqrt{2}\cos x + \sqrt{2\cos^2 x - 1}\right| + c$$

$$\Rightarrow \frac{-1}{2}\ln\left|\left(\sqrt{2}\cos x + \sqrt{\cos 2x}\right)^2\right| + c$$

$$\Rightarrow \frac{-1}{2}\ln\left|2\cos^2 x + \cos 2x + 2\sqrt{2}\cos x\sqrt{\cos 2x}\right| + c$$

$$\Rightarrow \frac{-1}{2}\ln\left|1 + \cos 2x + \cos 2x + 2\sqrt{2}\sqrt{\cos 2x} \times \sqrt{\frac{1+\cos x}{2}}\right| + c$$

$$\Rightarrow \frac{-1}{2}\ln\left|2\cos 2x + 1 + 2\sqrt{\cos 2x\left(1 + \cos 2x\right)}\right| + c$$

$$\Rightarrow \frac{-1}{2}\ln\left|\cos 2x + \frac{1}{2}\sqrt{\cos 2x\left(1 + \cos 2x\right)}\right| + c$$

$$\alpha = \frac{-1}{2} \qquad \beta = \frac{1}{2}$$

$$\therefore \boxed{\beta - \alpha = \frac{1}{2} - \left(\frac{-1}{2}\right) = 1}$$

**88.** If the value of real number $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real root is $\frac{3}{\sqrt{2\beta}}$ then $\beta$ is equal to ______.

**Sol.** **13**

$$x^2 - 5ax + 1 = 0$$

$$x^2 - ax - 5 = 0$$

$$-\quad +\quad +$$

$$-4ax + 6 = 0$$

$x = \frac{6}{4a} = \frac{3}{2a}$ (common root)

$\therefore \left(\frac{3}{2a}\right)^2 - 5a\left(\frac{3}{2a}\right) + 1 = 0$

$\Rightarrow 9 - 30a^2 + 4a^2 = 0$

$\Rightarrow 26a^2 = 9$

$a^2 = \frac{9}{26} \Rightarrow a = \frac{3}{\sqrt{26}} = \frac{3}{\sqrt{2\beta}}$

$\beta = 13$

**89.** $50^{th}$ root of a number x is 12 and $50^{th}$ root of another number y is 18 . Then the remainder obtained on dividing $(x + y)$ by 25 is ______.

**Sol.** **23**

$x^{\frac{1}{50}} = 12$ $\qquad$ $y^{\frac{1}{50}} = 18$

Remainder when x + y is division by 25.

$x = 12^{50}$ $\qquad$ $y = 18^{50}$

$x + y = 12^{50} + 18^{50}$

$= 6^{50}(2^{50} + 3^{50})$

$= (5 + 1)^{50}\ ((2^2)^{25} + (3^2)^{25})$

$= (25\lambda_1 + 1)\ ((5-1)^{25} + (10-1)^{25})$

$= (25\lambda_1 + 1)\ (25(\lambda_2 + \lambda_3) - 2)$

$= (25\lambda_1 + 1)\ (25K - 2)$

$\Rightarrow 25\lambda_1 . 25K - 50\lambda_1 + 25K - 2$

$\Rightarrow 25n_1 - 2$

$\Rightarrow 25n_2 + 23$

Remainder = 23

**90.** The number of seven digits odd numbers, that can be formed using all the seven digits 1,2,2,2,3,3,5 is ______.

**Sol.** **240**

The no. of 7 digit odd Numbers that can be formed using

1, 2, 2, 2, 3, 3, 5

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | 1 |

$\frac{\lfloor 6}{\lfloor 3 \lfloor 2} = \frac{720}{12} = 60$

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | 3 |

$\frac{\lfloor 6}{\lfloor 3} = \frac{720}{6} = 120$

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | 5 |

$\frac{\lfloor 6}{\lfloor 3 \lfloor 2} = \frac{720}{12} = 60$

= 240