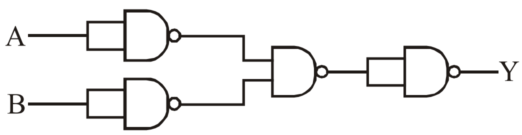
FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16th March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

SECTION-A

1. The following logic gate is equivalent to :



(1) NOR Gate (2) OR Gate
(3) AND Gate (4) NAND Gate

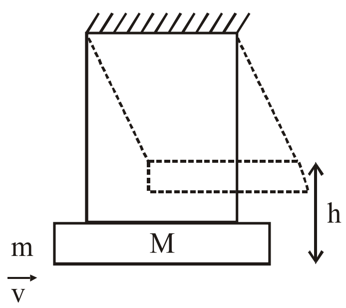
Official Ans. by NTA (1)

Sol. Truth table for the given logic gate :

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

The truth table is similar to that of a NOR gate.

2. A large block of wood of mass M = 5.99 kg is hanging from two long massless cords. A bullet of mass m = 10g is fired into the block and gets embedded in it. The (block + bullet) then swing upwards, their centre of mass rising a vertical distance h = 9.8 cm before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is : (Take g = 9.8 ms^{-2})



(1) 841.4 m/s (2) 811.4 m/s
(3) 831.4 m/s (4) 821.4 m/s

Official Ans. by NTA (3)

Sol. From energy conservation,

$$\begin{bmatrix}\text{after bullet gets embedded till the} \\ \text{system comes momentarily at rest}\end{bmatrix}$$

$$(M + m)g\,h = \frac{1}{2}(M+m)v_1^2$$

[v_1 is velocity after collision]

$$\therefore v_1 = \sqrt{2gh}$$

Applying momentum conservation, (just before and just after collision)

$$mv = (M + m)v_1$$

$$v = \left(\frac{M+m}{m}\right)v_1 = \frac{6}{10\times10^{-3}}\times\sqrt{2\times9.8\times9.8\times10^{-2}}$$

$$\approx 831.55 \text{ m / s}$$

3. A charge Q is moving $\vec{dl}$ distance in the magnetic field $\vec{B}$. Find the value of work done by $\vec{B}$.

(1) 1 (2) Infinite
(3) Zero (4) –1

Official Ans. by NTA (3)

Sol. Since force on a point charge by magnetic field is always perpendicular to $\vec{v}\left[\vec{F} = q\vec{V}\times\vec{B}\right]$

$\therefore$ Work by magnetic force on the point charge is zero.

4. What will be the nature of flow of water from a circular tap, when its flow rate increased from 0.18 L/min to 0.48 L/min ? The radius of the tap and viscosity of water are 0.5 cm and 10^{-3} Pa s, respectively.

(Density of water : 10^3 kg/m^3)

(1) Unsteady to steady flow
(2) Remains steady flow
(3) Remains turbulent flow
(4) Steady flow to unsteady flow

Official Ans. by NTA (4)

Sol. The nature of flow is determined by Reynolds Number.

$$R_e = \frac{\rho v D}{\eta}$$

$$\begin{bmatrix} \rho \rightarrow \text{density of fluid} \ ; & \eta \rightarrow \text{coefficient of} \\ v \rightarrow \text{velocity of flow} & \text{viscosity} \\ D \rightarrow \text{Diameter of pipe} & \end{bmatrix}$$

From NCERT

If $R_e < 1000$ $\rightarrow$ flow is steady

$1000 < R_e < 2000$ $\rightarrow$ flow becomes unsteady

$R_e > 2000$ $\rightarrow$ flow is turbulent

$$R_{e\,\text{initial}} = 10^3 \times \frac{0.18 \times 10^{-3}}{\pi \times (0.5 \times 10^{-2})^2 \times 60} \times \frac{1 \times 10^{-2}}{10^{-3}}$$

$= 382.16$

$$R_{e\,\text{final}} = 10^3 \times \frac{0.48 \times 10^{-3}}{\pi \times (0.5 \times 10^{-2})^2 \times 60} \times \frac{1 \times 10^{-2}}{10^{-3}}$$

$= 1019.09$

5. A mosquito is moving with a velocity $\vec{v} = 0.5t^2\,\hat{i} + 3t\,\hat{j} + 9\hat{k}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2s ?

(1) $\tan^{-1}\left(\frac{2}{3}\right)$ from x-axis

(2) $\tan^{-1}\left(\frac{2}{3}\right)$ from y-axis

(3) $\tan^{-1}\left(\frac{5}{2}\right)$ from y-axis

(4) $\tan^{-1}\left(\frac{5}{2}\right)$ from x-axis

Official Ans. by NTA (2)

Official Ans. by ALLEN (Bonus)

Sol. Given :

$\vec{v} = 0.5t^2\,\hat{i} + 3t\,\hat{j} + 9\hat{k}$

$\vec{v}_{\text{at } t=2} = 2\,\hat{i} + 6\,\hat{j} + 9\hat{k}$

$\therefore$ Angle made by direction of motion of mosquito will be,

$\cos^{-1}\frac{2}{11}$ (from x-axis) $= \tan^{-1}\frac{\sqrt{117}}{2}$

$\cos^{-1}\frac{6}{11}$ (from y-axis) $= \tan^{-1}\frac{\sqrt{85}}{6}$

$\cos^{-1}\frac{9}{11}$ (from z-axis) $= \tan^{-1}\frac{\sqrt{40}}{9}$

None of the option is matching.

Hence this question should be bonus.

6. Find out the surface charge density at the intersection of point x = 3 m plane and x-axis, in the region of uniform line charge of 8 nC/m lying along the z-axis in free space.

(1) 0.424 nC m^{-2} (2) 47.88 C/m

(3) 0.07 nC m^{-2} (4) 4.0 nC m^{-2}

Official Ans. by NTA (1)

Sol. $\frac{2K\lambda}{r} = \frac{\sigma}{\varepsilon_0}$ (x = 3m)

$\sigma = 0.424 \times 10^{-9} \frac{C}{m^2}$

7. The de-Broglie wavelength associated with an electron and a proton were calculated by accelerating them through same potential of 100 V. What should nearly be the ratio of their wavelengths ? (m_P = 1.00727 u, m_e = 0.00055u)

(1) 1860 : 1 (2) $(1860)^2$: 1

(3) 41.4 : 1 (4) 43 : 1

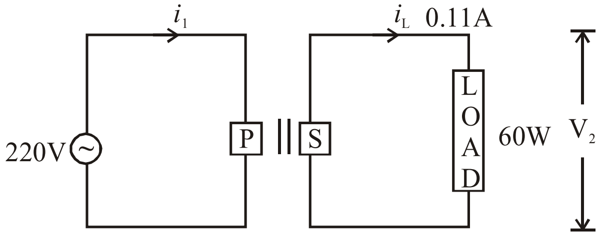
Official Ans. by NTA (4)

Sol. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$

$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1}}$

$\frac{\lambda_e}{\lambda_P} = \sqrt{\frac{m_P}{m_e}} = \sqrt{1831.4} = 42.79$

8. For the given circuit, comment on the type of transformer used :



(1) Auxilliary transformer
(2) Auto transformer
(3) Step-up transformer
(4) Step down transformer

Official Ans. by NTA (3)

Sol. $V_S = \frac{P}{i} = \frac{60}{0.11} = 545.45$

$V_P = 220$

$V_S > V_P$

$\Rightarrow$ Step up transformer

9. The half-life of Au^{198} is 2.7 days. The activity of 1.50 mg of Au^{198} if its atomic weight is 198 g mol^{-1} is, ($N_A = 6 \times 10^{23}$/mol)

(1) 240 Ci (2) 357 Ci
(3) 535 Ci (4) 252 Ci

Official Ans. by NTA (2)

Sol. $A = \lambda N$ $\left(t_{1/2} = \frac{\ln 2}{\lambda}\right)$

$N = nN_A$

$N = \left(\frac{1.5\times10^{-3}}{198}\right)N_A$

$A = \left(\frac{\ln 2}{t_{1/2}}\right)N$

1 Curie = 3.7×10^{10} Bq

A = 365 Bq

10. Calculate the value of mean free path (λ) for oxygen molecules at temperature 27°C and pressure 1.01×10^5 Pa. Assume the molecular diameter 0.3 nm and the gas is ideal. ($k = 1.38 \times 10^{-23}$ JK^{-1})

(1) 58 nm (2) 32 nm
(3) 86 nm (4) 102 nm

Official Ans. by NTA (4)

Sol. $\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A P}$

λ =102 nm

11. The refractive index of a converging lens is 1.4. What will be the focal length of this lens if it is placed in a medium of same refractive index ? (Assume the radii of curvature of the faces of lens are R_1 and R_2 respectively)

(1) 1 (2) Infinite
(3) $\frac{R_1R_2}{R_1 - R_2}$ (4) Zero

Official Ans. by NTA (2)

Sol. $\frac{1}{F} = \left[\frac{\mu_L}{\mu_S} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

If $\mu_L = \mu_S \Rightarrow \frac{1}{F} = 0 \Rightarrow F = \infty$

12. In order to determine the Young's Modulus of a wire of radius 0.2 cm (measured using a scale of least count = 0.001 cm) and length 1m (measured using a scale of least count = 1 mm), a weight of mass 1kg (measured using a scale of least count = 1g) was hanged to get the elongation of 0.5 cm (measured using a scale of least count 0.001 cm). What will be the fractional error in the value of Young's Modulus determined by this experiment ?

(1) 0.14%
(2) 0.9%
(3) 9%
(4) 1.4%

Official Ans. by NTA (4)

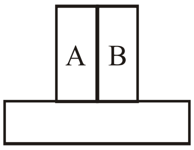
Sol. $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{FL}{Al} = \frac{mg.L}{\pi R^2.\ell}$

$$\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta L}{L} + 2.\frac{\Delta R}{R} + \frac{\Delta \ell}{\ell}$$

$$\frac{\Delta Y}{Y} \times 100 = 100\left[\frac{1}{1000} + \frac{1}{1000} + 2\left(\frac{0.001}{0.2}\right) + \frac{0.001}{0.5}\right]$$

$$= \frac{1}{10} + \frac{1}{10} + 1 + \frac{1}{5} = \frac{14}{10} = 1.4\%$$

13. A bimetallic strip consists of metals A and B. It is mounted rigidly as shown. The metal A has higher coefficient of expansion compared to that of metal B. When the bimetallic strip is placed in a cold both, it will :



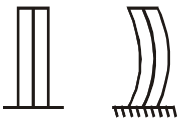
(1) Bend towards the right
(2) Not bend but shrink
(3) Neither bend nor shrink
(4) Bend towards the left

Official Ans. by NTA (4)

Sol. $\alpha_A > \alpha_B$

Length of both strips will decrease

$\Delta L_A > \Delta L_B$



14. A resistor develops 500 J of thermal energy in 20s when a current of 1.5 A is passed through it. If the current is increased from 1.5 A to 3A, what will be the energy developed in 20 s.

(1) 1500 J (2) 1000 J
(3) 500 J (4) 2000 J

Official Ans. by NTA (4)

Sol. $500 = (1.5)^2 \times R \times 20$

$E = (3)^2 \times R \times 20$

$E = 2000$ J

15. **Statement I :** A cyclist is moving on an unbanked road with a speed of $7\ \text{kmh}^{-1}$ and takes a sharp circular turn along a path of radius of 2m without reducing the speed. The static friction coefficient is 0.2. The cyclist will not slip and pass the curve ($g = 9.8\ \text{m/s}^2$)

Statement II : If the road is banked at an angle of 45°, cyclist can cross the curve of 2m radius with the speed of $18.5\ \text{kmh}^{-1}$ without slipping.

In the light of the above statements, choose the correct answer from the options given below.

(1) Statement I is incorrect and statement II is correct

(2) Statement I is correct and statement II is incorrect

(3) Both statement I and statement II are false

(4) Both statement I and statement II are true

Official Ans. by NTA (4)

Sol. **Statement I :**

$$v_{max} = \sqrt{\mu R g} = \sqrt{(0.2) \times 2 \times 9.8}$$

$v_{max} = 1.97$ m/s

7 km/h = 1.944 m/s

Speed is lower than v_{max}, hence it can take safe turn.

Statement II

$$v_{max} = \sqrt{Rg\left[\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right]}$$

$$= \sqrt{2 \times 9.8\left[\frac{1+0.2}{1-0.2}\right]} = 5.42\ \text{m/s}$$

18.5 km/h = 5.14 m/s

Speed is lower than v_{max}, hence it can take safe turn.

16. Two identical antennas mounted on identical towers are separated from each other by a distance of 45 km. What should nearly be the minimum height of receiving antenna to receive the signals in line of sight ? (Assume radius of earth is 6400 km)

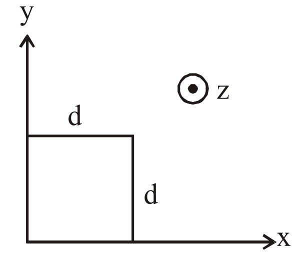
(1) 19.77 m (2) 39.55 m
(3) 79.1 m (4) 158.2 m

Official Ans. by NTA (2)

Sol. $D = 2\sqrt{2Rh}$

$$h = \frac{D^2}{8R} = \frac{45^2}{8 \times 6400}\text{km} \cong 39.55\,\text{m}$$

17. The magnetic field in a region is given by $\vec{B} = B_0\left(\frac{x}{a}\right)\hat{k}$. A square loop of side d is placed with its edges along the x and y axes. The loop is moved with a constant velocity $\vec{v} = v_0\hat{i}$. The emf induced in the loop is :



(1) $\frac{B_0 v_0^2 d}{2a}$ (2) $\frac{B_0 v_0 d}{2a}$

(3) $\frac{B_0 v_0 d^2}{a}$ (4) $\frac{B_0 v_0 d^2}{2a}$

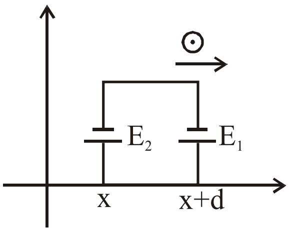
Official Ans. by NTA (3)

Sol. $E_1 = \frac{B_0(x+d)}{a}v_0 d$

$E_2 = \frac{B_0(x)}{a}v_0 d$

$E_{net} = E_1 - E_2$

$E_{net} = \frac{B_0 v_0 d^2}{a}$



18. Amplitude of a mass-spring system, which is executing simple harmonic motion decreases with time. If mass = 500g, Decay constant = 20 g/s then how much time is required for the amplitude of the system to drop to half of its initial value ? (ln 2 = 0.693)

(1) 34.65 s (2) 17.32 s

(3) 0.034 s (4) 15.01 s

Official Ans. by NTA (1)

Sol. $A = A_0 e^{-\gamma t} = A_0 e^{-\frac{bt}{2m}}$

$\frac{A_0}{2} = A_0 e^{-\frac{bt}{2m}}$

$\frac{bt}{2m} = \ln 2$

$t = \frac{2m}{b}\ln 2 = \frac{2 \times 500 \times 0.693}{20}$

t = 34.65 second.

19. Calculate the time interval between 33% decay and 67% decay if half-life of a substance is 20 minutes.

(1) 60 minutes (2) 20 minutes

(3) 40 minutes (4) 13 minutes

Official Ans. by NTA (2)

Sol. $N_1 = N_0 e^{-\lambda t_1}$

$\frac{N_1}{N_0} = e^{-\lambda t_1}$

$0.67 = e^{-\lambda t_1}$

$\ln(0.67) = -\lambda t_1$

$N_2 = N_0 e^{-\lambda t_2}$

$\frac{N_2}{N_0} = e^{-\lambda t_2}$

$0.33 = e^{-\lambda t_2}$

$\ln(0.33) = -\lambda t_2$

$\ln(0.67) - \ln(0.33) = \lambda t_1 - \lambda t_2$

$\lambda(t_1 - t_2) = \ln\left(\frac{0.67}{0.33}\right)$

$\lambda(t_1 - t_2) \cong \ln 2$

$t_1 - t_2 \simeq \frac{\ln 2}{\lambda} = t_{1/2}$

Half life = $t_{1/2}$ = 20 minutes.

20. Red light differs from blue light as they have :

(1) Different frequencies and different wavelengths

(2) Different frequencies and same wavelengths

(3) Same frequencies and same wavelengths

(4) Same frequencies and different wavelengths

Official Ans. by NTA (1)

Sol. Red light and blue light have different wavelength and different frequency.

SECTION-B

1. The energy dissipated by a resistor is 10 mJ in 1s when an electric current of 2 mA flows through it. The resistance is _______ Ω. (Round off to the Nearest Integer)

Official Ans. by NTA (2500)

Sol. **Ans. (2500)**

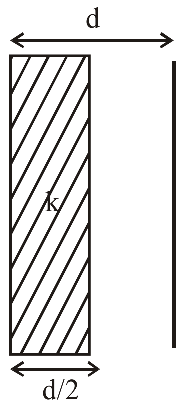
$Q = i^2 RT$

$R = \frac{Q}{i^2 t} = \frac{10 \times 10^{-3}}{4 \times 10^{-6} \times 1} = 2500\ \Omega$

2. In a parallel plate capacitor set up, the plate area of capacitor is 2 m^2 and the plates are separated by 1m. If the space between the plates are filled with a dielectric material of thickness 0.5 m and area $2m^2$ (see fig.) the capacitance of the set-up will be ______ ε_0.

(Dielectric constant of the material = 3.2)

(Round off to the Nearest Integer)



Official Ans. by NTA (3)

Sol. Ans. (3)

$$C = \frac{\varepsilon_0 A}{\frac{d}{2K}+\frac{d}{2}} = \frac{2\varepsilon_0 A}{\frac{d}{K}+d}$$

$$= \frac{2\times 2\varepsilon_0}{\frac{1}{3.2}+1} = \frac{4\times 3.2}{4.2}\varepsilon_0$$

$$= 3.04\ \varepsilon_0$$

3. A force $\vec{F} = 4\hat{i}+3\hat{j}+4\hat{k}$ is applied on an intersection point of x = 2 plane and x-axis. The magnitude of torque of this force about a point (2, 3, 4) is ________. (Round off to the Nearest Integer)

Official Ans. by NTA (20)

Sol. Ans. (20)

$$\vec{\tau} = \vec{r}\times\vec{F}$$

$$\vec{r} = (2\hat{i}) - (2\hat{i}+3\hat{j}+4\hat{k}) = -3\hat{j}-4\hat{k}$$

$$\& \ \vec{F} = 4\hat{i}+3\hat{j}+4\hat{k}$$

$$\vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -4 \\ 4 & 3 & 4 \end{vmatrix}$$

$$= \hat{i}(-12+12) - \hat{j}(0+16) + \hat{k}(0+12)$$

$$= -16\hat{i}+12\hat{k}$$

$$\therefore \ |\vec{\tau}| = \sqrt{16^2+12^2} = 20$$

4. If one wants to remove all the mass of the earth to infinity in order to break it up completely. The amount of energy that needs to be supplied will be $\frac{x}{5}\frac{GM^2}{R}$ where x is ____ (Round off to the Nearest Integer)

(M is the mass of earth, R is the radius of earth, G is the gravitational constant)

Official Ans. by NTA (3)

Sol. Ans. (3)

Energy given = $U_f - U_i$

$$= 0 - \left(-\frac{3}{5}\frac{GM^2}{R}\right)$$

$$= \frac{3}{5}\frac{GM^2}{R}$$

$$x = 3$$

5. A deviation of 2° is produced in the yellow ray when prism of crown and flint glass are achromatically combined. Taking dispersive powers of crown and flint glass are 0.02 and 0.03 respectively and refractive index for yellow light for these glasses are 1.5 and 1.6 respectively. The refracting angles for crown glass prism will be ________° (in degree)

(Round off to the Nearest Integer)

Official Ans. by NTA (12)

Sol. **Ans. (12)**

$\omega_1 = 0.02$; $\mu_1 = 1.5$; $\omega_2 = 0.03$; $\mu_2 = 1.6$

Achromatic combination

$\therefore\ \theta_{net} = 0$

$\theta_1 - \theta_2 = 0$

$\theta_1 = \theta_2$

$\omega_1\delta_1 = \omega_2\delta_2$

& $\delta_{net} = \delta_1 - \delta_2 = 2°$

$$\delta_1 - \frac{\omega_1\delta_1}{\omega_2} = 2°$$

$$\delta_1\left(1 - \frac{\omega_1}{\omega_2}\right) = 2°$$

$$\delta_1\left(1 - \frac{2}{3}\right) = 2°$$

$\delta_1 = 6°$

$\delta_1 = (\mu_1 - 1)\ A_1$

$6° = (1.5 - 1)\ A_1$

$A_1 = 12°$

6. A body of mass 2kg moves under a force of $(2\hat{i} + 3\hat{j} + 5\hat{k})N$. It starts from rest and was at the origin initially. After 4s, its new coordinates are (8, b, 20). The value of b is __________. (Round off to the Nearest Integer)

Official Ans. by NTA (12)

Sol. **Ans. (12)**

$$\vec{a} = \frac{\vec{F}}{m} = \frac{2\hat{i} + 3\hat{j} + 5\hat{k}}{2}$$

$$= \hat{i} + 1.5\hat{j} + 2.5\hat{k}$$

$$\vec{\tau} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$= 0 + \frac{1}{2}(\hat{i} + 1.5\hat{j} + 2.5\hat{k})\ (16)$$

$$= 8\hat{i} + 12\hat{j} + 20\hat{k}$$

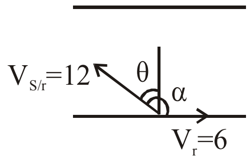
b = 12

7. A swimmer can swim with velocity of 12 km/h in still water. Water flowing in a river has velocity 6 km/h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is _________°. (Round off to the Nearest Integer) (find the angle in degree)

Official Ans. by NTA (120)

Sol. **Ans. (12)**

$12\sin\theta = v_r$



$$\sin\theta = \frac{1}{2}$$

$\theta = 30°$

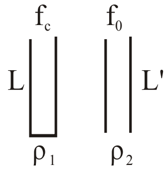
$\therefore\ \alpha = 120°$

8. A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open pipe is $\frac{x}{3}L\sqrt{\frac{\rho_1}{\rho_2}}$ where x is ________. (Round off to the Nearest Integer)

Official Ans. by NTA (4)

Sol. **Ans. (4)**

$f_c = f_0$



$$\frac{3V_C}{4L} = \frac{2V_0}{2L'}$$

$$\frac{3V_C}{4L} = \frac{V_0}{L'}$$

$$L' = \frac{4L}{3}\frac{V_0}{V_C} = \frac{4L}{3}\sqrt{\frac{B.\rho_1}{\rho_2.B}}$$ (B is bulk modulus)

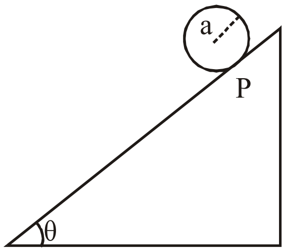
$$= \frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$$

x = 4

9. A solid disc of radius 'a' and mass 'm' rolls down without slipping on an inclined plane making an angle θ with the horizontal. The acceleration of the disc will be $\frac{2}{b} g \sin\theta$ where b is ______. (Round off to the Nearest Integer)

(g = acceleration due to gravity)

(θ = angle as shown in figure)



Official Ans. by NTA (3)

Sol. Ans. (3)

$$a = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3} g\sin\theta$$

b = 3

10. For an ideal heat engine, the temperature of the source is 127°C. In order to have 60% efficiency the temperature of the sink should be _______°C. (Round off to the Nearest Integer)

Official Ans. by NTA (113)

Official Ans. by ALLEN (–113)

Sol. Ans. (–113)

$$n = 0.60 = 1 = \frac{T_L}{T_H}$$

$$\frac{T_L}{T_H} = 0.4 \Rightarrow T_L = 0.4 \times 400$$

$$= 160 \text{ K}$$

$$= -113°\text{C}$$

SECTION-A

1. The green house gas/es is (are) :

(A) Carbon dioxide

(B) Oxygen

(C) Water vapour

(D) Methane

Choose the most appropriate answer from the options given below :

(1) (A) and (C) only

(2) (A) only

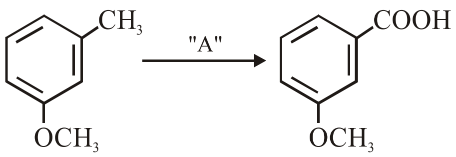
(3) (A), (C) and (D) only

(4) (A) and (B) only

Official Ans. by NTA (3)

Sol. The green house gases are CO_2, $H_2O_{(vapour)}$ & CH_4.

2.



In the above reaction, the reagent "A" is :

(1) $NaBH_4$, H_3O^+

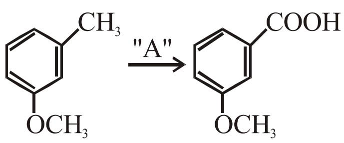
(2) $LiAlH_4$

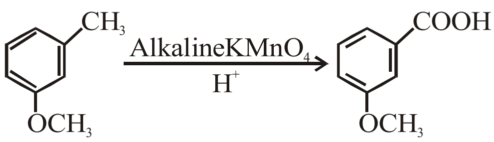
(3) Alkaline $KMnO_4$, H^+

(4) HCl, Zn-Hg

Official Ans. by NTA (3)

Sol.





;

3. Which of the following reduction reaction CANNOT be carried out with coke ?

(1) $Al_2O_3 \rightarrow Al$

(2) $ZnO \rightarrow Zn$

(3) $Fe_2O_3 \rightarrow Fe$

(4) $Cu_2O \rightarrow Cu$

Official Ans. by NTA (1)

Sol. Reduction of $Al_2O_3 \rightarrow Al$ is carried out by electrolytic reduction of its fused salts.

ZnO, Fe_2O_3 & Cu_2O can be reduce by carbon.

4. Identify the elements X and Y using the ionisation energy values given below :

	Ionization energy (kJ/mol) 1st	2nd
X	495	4563
Y	731	1450

(1) X = Na ; Y = Mg

(2) X = Mg ; Y = F

(3) X = Mg ; Y = Na

(4) X = F ; Y = Mg

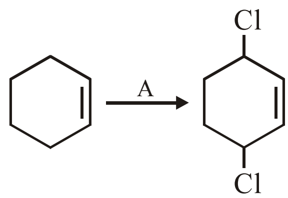
Official Ans. by NTA (1)

Sol. $Na \rightarrow [Ne]\ 3s^1$ IE_1 is very low but IE_2 is very high due to stable noble gas configuration of Na^+.

$Mg \rightarrow [Ne]\ 3s^2$ IE_1 & $IE_2 \rightarrow$ Low

IE_3 is very high.

5.



Identify the reagent(s) 'A' and condition(s) for the reaction :

(1) A = HCl; Anhydrous $AlCl_3$

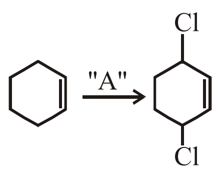
(2) A = HCl, $ZnCl_2$

(3) A = Cl_2; UV light

(4) A = Cl_2; dark, Anhydrous $AlCl_3$

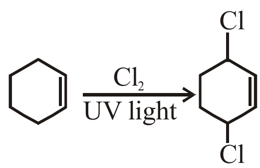
Official Ans. by NTA (3)

Sol.



For substitution at allylic position in the given compound, the reagent used is Cl_2/uv light.

The reaction is free radical halogenation.



6. The secondary structure of protein is stabilised by:
(1) Peptide bond
(2) glycosidic bond
(3) Hydrogen bonding
(4) van der Waals forces
Official Ans. by NTA (3)

Sol. The secondary structure of protein includes two type :
(a) α-Helix (b) β-pleated sheet
In α-Helix structure, the poly peptide chain is coil around due to presence of Intramolecular H-Bonding.

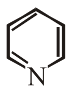
7. Fex_2 and Fey_3 are known when x and y are :
(1) x = F, Cl, Br, I and y = F, Cl, Br
(2) x = F, Cl, Br and y = F, Cl, Br, I
(3) x = Cl, Br, I and y = F, Cl, Br, I
(4) x = F, Cl, Br, I and y = F, Cl, Br, I
Official Ans. by NTA (1)

Sol. $\underset{\text{(Unstable)}}{2FeI_3} \longrightarrow \underset{\text{(Stable)}}{2FeI_2} + I_2$
Due to strong reducing nature of I^-
$2Fe^{3+} + 2I^- \longrightarrow 2Fe^{2+} + I_2$
remaining halides of Fe^{2+} & Fe^{3+} are stable.

8. Which of the following polymer is used in the manufacture of wood laminates ?
(1) *cis*-poly isoprene
(2) Melamine formaldehyde resin
(3) Urea formaldehyde resin
(4) Phenol and formaldehyde resin
Official Ans. by NTA (3)

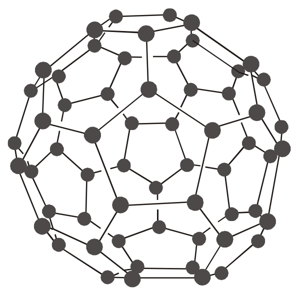
Sol. Urea –HCHO resin is used in manufacture of wood laminates.

9. **Statement I :** Sodium hydride can be used as an oxidising agent.
Statement II : The lone pair of electrons on nitrogen in pyridine makes it basic.
Choose the CORRECT answer from the options given below :
(1) Both statement I and statement II are false
(2) Statement I is true but statement II is false
(3) Statement I is false but statement II is true
(4) Both statement I and statement II are true
Official Ans. by NTA (3)

Sol. (1) NaH (sodium Hydride) is used as a reducing reagent.
(2)  In pyridine, due to free electron on N atom, it is basic in nature.
Hence statement I is false & II is true.

10. The INCORRECT statement regarding the structure of C_{60} is :
(1) The six-membered rings are fused to both six and five-membered rings.
(2) Each carbon atom forms three sigma bonds.
(3) The five-membered rings are fused only to six-membered rings.
(4) It contains 12 six-membered rings and 24 five-membered rings.
Official Ans. by NTA (4)

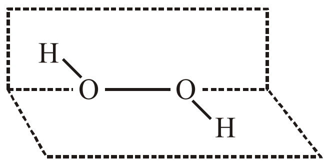
Sol. Structure of C_{60}



It contain 20 hexagons 20 and 12 pentagons 12 so option 4 is incorrect.

11. The correct statements about H_2O_2 are :
(A) used in the treatment of effluents.
(B) used as both oxidising and reducing agents.
(C) the two hydroxyl groups lie in the same plane.
(D) miscible with water.
Choose the correct answer from the options given below :
(1) (A), (B), (C) and (D)
(2) (A), (B) and (D) only
(3) (B), (C) and (D) only
(4) (A), (C) and (D) only
Official Ans. by NTA (2)

Sol.



Structure of H_2O_2
(Open book type) $\rightarrow$ Non planar
H_2O_2 is used in the treatment of effluents.

$H_2O_2 \xrightarrow{R.A.} O_2$
$H_2O_2 \xrightarrow{O.A.} H_2O$
act as both O.A & R.A.

H_2O_2 is miscible in water due to hydrogen bonding.

12. Ammonolysis of Alkyl halides followed by the treatment with NaOH solution can be used to prepare primary, secondary and tertiary amines. The purpose of NaOH in the reaction is :

(1) to remove basic impurities

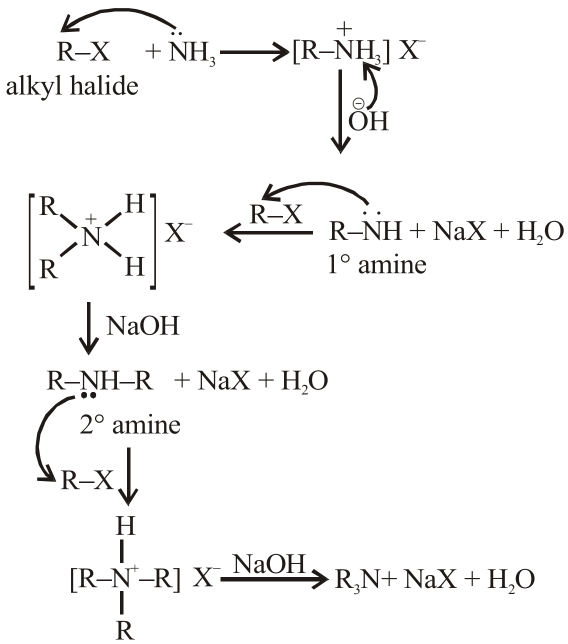
(2) to activate NH_3 used in the reaction

(3) to remove acidic impurities

(4) to increase the reactivity of alkyl halide

Official Ans. by NTA (3)

Sol.

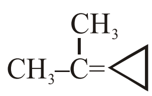


So the purpose of NaOH in the above reactions in to remove acidic impurities.

13. An unsaturated hydrocarbon X on ozonolysis gives A. Compound A when warmed with ammonical silver nitrate forms a bright silver mirror along the sides of the test tube. The unsaturated hydrocarbon X is :

(1) $CH_3-\underset{CH_3}{\underset{|}{C}}=\underset{CH_3}{\underset{|}{C}}-CH_3$

(2)



(3) $HC\equiv C-CH_2-CH_3$

(4) $CH_3-C\equiv C-CH_3$

Official Ans. by NTA (3)

Sol. $\underset{\text{Unsaturated Hydrocarbon}}{(X)} \xrightarrow{\text{Ozonolysis}} (A) \xrightarrow[\text{(Tollen's regent)}]{\text{Ammonical } AgNO_3} \underset{\text{silver mirror}}{Ag\downarrow}$

As (A) compound given positive tollen's test hence it may consist –CHO (aldehyde group). or it can be HCOOH

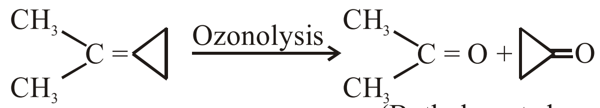
So for the given option :

(3) $CH_3-CH_2-C\equiv CH \xrightarrow{\text{Ozonolysis}} CH_3CH_2COOH + \underset{\substack{(A)\\ \text{+ve tollen's test}}}{HCOOH}$

and for other compounds (options):

(1) $(CH_3)_2C=C(CH_3)_2 \xrightarrow{\text{Ozonolysis}} (CH_3)_2C=O$ (Does not show tollen's Test)

(2)



(Both do not show tollen's test)

(3) $CH_3-C\equiv C-CH_3 \xrightarrow{\text{Ozonolysis}} 2CH_3-COOH$ (Does not show tollen's test)

14. Which of the following is least basic ?

(1) $(CH_3CO)\ddot{N}HC_2H_5$

(2) $(C_2H_5)_3\ddot{N}$

(3) $(CH_3CO)_2\ddot{N}H$

(4) $(C_2H_5)_2\ddot{N}H$

Official Ans. by NTA (3)

Sol. For the given compounds :

(1) $CH_3-\overset{O}{\overset{||}{C}}-\ddot{N}H-C_2H_5$; L.P. on Nitrogen is delocalised.

(2) $CH_3CH_2-\underset{CH_2CH_3}{\underset{|}{\ddot{N}}}-CH_2CH_3$; L.P. on Nitrogen is localised.

(3) $CH_3-\underset{O}{\underset{||}{C}}-\ddot{N}H-\underset{O}{\underset{||}{C}}-CH_3$; L.P. on Nitrogen is delocalised due to conjugation with both $-\underset{O}{\underset{||}{C}}-$

(Hence least basic)

(4) $CH_3-CH_2-\ddot{N}H-CH_2-CH_3$; L.P. on Nitrogen is localised.

15. The characteristics of elements X, Y and Z with atomic numbers, respectively, 33, 53 and 83 are :

(1) X and Y are metalloids and Z is a metal.

(2) X is a metalloid, Y is a non-metal and Z is a metal.

(3) X, Y and Z are metals.

(4) X and Z are non-metals and Y is a metalloid

Official Ans. by NTA (2)

Sol. $X = {}_{33}As \rightarrow$ Metalloid

$Y = {}_{53}I \rightarrow$ Nonmetal

$Z = {}_{83}Bi \rightarrow$ Metal

16. Match List-I with List-II

List-I Test/Reagents/Observation(s)	List-II Species detected
(a) Lassaigne's Test	(i) Carbon
(b) Cu(II) oxide	(ii) Sulphur
(c) Silver nitrate	(iii) N, S, P, and halogen
(d) The sodium fusion extract gives black precipitate with acetic acid and lead acetate	(iv) Halogen Specifically

The correct match is :

(1) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

(2) (a)-(i), (b)-(iv), (c)-(iii), (d)-(ii)

(3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(4) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

Official Ans. by NTA (3)

Sol. Match list :-

(a) Lassaigne's Test	(iii) N, S, P and Halogen
(b) Cu(II) Oxide	(i) Carbon
(c) $AgNO_3$	(iv) Halogen specifically.
(d) Sodium fusion extract given black precipitate with acetic acid and lead acetate ($CH_3COOH/(CH_3COO)_2Pb$)	(ii) Sulphur

Option-(a)-(iii) ; (b)-(i) ; (c)-(iv) ; (d)-(ii)

17. The INCORRECT statements below regarding colloidal solutions is :

(1) A colloidal solution shows colligative properties.

(2) An ordinary filter paper can stop the flow of colloidal particles.

(3) The flocculating power of Al^{3+} is more than that of Na^{+}.

(4) A colloidal solution shows Brownian motion of colloidal particles.

Official Ans. by NTA (2)

Sol. * Colloidel solution exhibits colligative properties

* An ordinary filter can not stop the flow of colloidal particles.

* Flocculating power increases with increase the opposite charge of electrolyte.

* Colloidal particles show brownian motion.

18. Arrange the following metal complex/ compounds in the increasing order of spin only magnetic moment. Presume all the three, high spin system.

(Atomic numbers Ce = 58, Gd = 64 and Eu = 63.)

(a) $(NH_4)_2[Ce(NO_3)_6]$ (b) $Gd(NO_3)_3$ and

(c) $Eu(NO_3)_3$

Answer is :

(1) (b) < (a) < (c)

(2) (c) < (a) < (b)

(3) (a) < (b) < (c)

(4) (a) < (c) < (b)

Official Ans. by NTA (4)

Sol. (a) ${}_{58}Ce \rightarrow [Xe]4f^2\ 5d^0\ 6s^2$

In complex $Ce^{4+} \rightarrow [Xe]\ 4f^0\ 5d^0\ 6s^0$

there is no unpaired electron so $\mu_m = 0$

(b) ${}_{64}Gd^{3+} \rightarrow [Xe]4f^7\ 5d^0\ 6s^0$

contain seven unpaired electrons so,

$\mu_m = \sqrt{7(7+2)} = \sqrt{63}$ B.M.

(c) ${}_{63}Eu^{3+} \rightarrow [{}_{54}Xe]4f^6\ 5d^0\ 6s^0$

contain six unpaired electron

so, $\mu_m = \sqrt{6(6+2)} = \sqrt{48}$ B.M.

Hence, order of spin only magnetic movement

$\boxed{b > c > a}$

19. The exact volumes of 1 M NaOH solution required to neutralise 50 mL of 1 M H_3PO_3 solution and 100 mL of 2 M H_3PO_2 solution, respectively, are :

(1) 100 mL and 100 mL

(2) 100 mL and 50 mL

(3) 100 mL and 200 mL

(4) 50 mL and 50 mL

Official Ans. by NTA (3)

Sol. $H_3PO_3 + 2NaOH \rightarrow Na_2HPO_3 + 2H_2O$

50 ml   1M

1M   V = ?

$$\Rightarrow \frac{n_{NaoH}}{n_{H_3PO_3}} = \frac{2}{1}$$

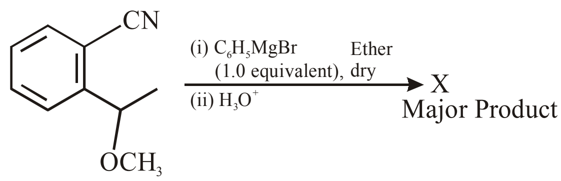
$$\Rightarrow \frac{1\times V}{50\times 1} = \frac{2}{1} \Rightarrow \boxed{V_{NaOH} = 100\,ml}$$

$H_3PO_2 + 2NaOH \rightarrow NaH_2PO_3 + H_2O$

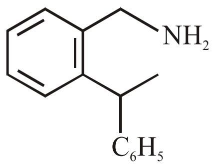
100 ml   1M

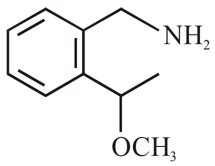
2M   V = ?

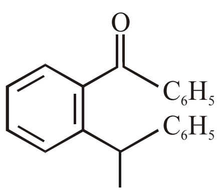
$$\Rightarrow \frac{n_{NaoH}}{n_{H_3PO_3}} = \frac{1}{1} \quad \Rightarrow \frac{1\times V}{2\times 100} = \frac{1}{1} \Rightarrow \boxed{V_{NaOH} = 200\,ml}$$

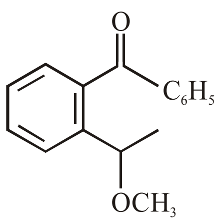
20. 

The structure of X is :

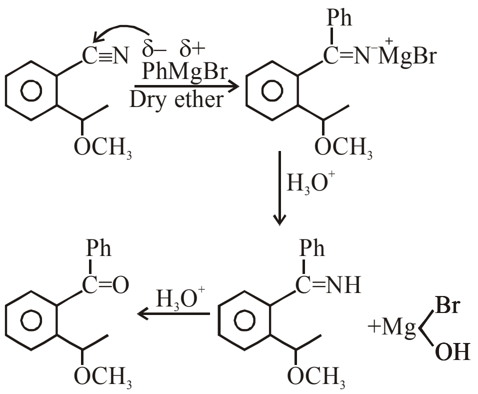
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (4)

Sol. 

SECTION-B

1. Ga (atomic mass 70 u) crystallizes in a hexagonal close packed structure. The total number of voids in 0.581 g of Ga is _______ $\times 10^{21}$. (Round off to the Nearest Integer).

Official Ans. by NTA (15)

Sol. HCP structure : Per atom, there will be one octahedral void (OV) and two tetrahedral voids (TV).

Therefore total three voids per atom are present in HCP structure.

$\rightarrow$ therefore total no of atoms of Ga will be-

$$= \frac{\text{Mass}}{\text{Molar Mass}} \times N_A = \frac{0.581g}{70g/mol} \times 6.023\times 10^{23}$$

$\rightarrow$ Now, total Number of voids = 3 × total no. of atoms

$$= 3\times \frac{0.581}{70} \times 6.023\times 10^{23} = 14.99 \times 10^{21}$$

$$\simeq 15\times 10^{21}$$

2. A 5.0 m mol dm^{-3} aqueous solution of KCl has a conductance of 0.55 mS when measured in a cell constant 1.3 cm^{-1}. The molar conductivity of this solution is _______ $mSm^2\ mol^{-1}$. (Round off to the Nearest Integer)

Official Ans. by NTA (143)

Official Ans. by ALLEN (14)

Sol. Given conc^n of KCl = $\frac{m.mol}{L}$

: Conductance (G) = 0.55 mS

: Cell constant $\left(\frac{\ell}{A}\right) = 1.3\ cm^{-1}$

To Calculate : Molar conductivity (λ_m) of sol.

$\rightarrow$ Since $\boxed{\lambda_m = \frac{1}{1000} \times \frac{k}{m}}$ ......(1)

$\rightarrow$ Molarity = $5\times 10^{-3}\ \frac{mol}{L}$

$$\rightarrow \text{Conductivity} = G\times\left(\frac{\ell}{A}\right) = 0.55\ mS \times \frac{1.3}{\frac{1}{100}} m^{-1}$$

$= 55 \times 1.3 \quad mSm^{-1}$

$$\text{eq}^n\ (1) \quad \lambda_m = \frac{1}{1000} \times \frac{55\times 1.3}{\left(\frac{5}{1000}\right)} \frac{mSm^2}{mol}$$

$$\Rightarrow \lambda_m = 14.3 \frac{mSm^2}{mol}$$

3. A and B decompose via first order kinetics with half-lives 54.0 min and 18.0 min respectively. Starting from an equimolar non reactive mixture of A and B, the time taken for the concentration of A to become 16 times that of B is ______ min. (Round off to the Nearest Integer).

Official Ans. by NTA (108)

Sol. Given $\underset{A}{t_2} = 54$ min $\quad\quad \underset{B}{T_{1/2}} = 18$ min

t = 0 'x' M $\quad\quad$ t = 0 'x' M

$\Rightarrow$ To calculate : $[A_t] = 16 \times [B_t]$ ....(1) time = ?

$\Rightarrow$ For I order kinetic : $\boxed{[A_t] = \frac{A_0}{(2)^n}}$

n $\rightarrow$ no of Half lives

$\Rightarrow$ Now from the relation (1)

$[A_t] = 16 \times [B_t]$

$\Rightarrow \frac{x}{(2)^{n_1}} = \frac{x}{(2)^{n_2}} \times 16 \Rightarrow (2)^{n_2} = (2)^{n_1} \times (2)^4$

$\Rightarrow n_2 = n_1 + 4 \Rightarrow \frac{t}{(t_{1/2})_2} = \frac{t}{(t_{1/2})_1} + 4$

$\Rightarrow t\left(\frac{1}{18} - \frac{1}{54}\right) = 4 \Rightarrow t = \frac{4 \times 18 \times 54}{36}$

$\Rightarrow \boxed{t = 108\,\text{min}}$

4. In Duma's method of estimation of nitrogen, 0.1840 g of an organic compound gave 30 mL of nitrogen collected at 287 K and 758 mm of Hg pressure. The percentage composition of nitrogen in the compound is ______. (Round off to the Nearest Integer).

[Given : Aqueous tension at 287 K = 14 mm of Hg]

Official Ans. by NTA (19)

Sol. In Duma's method of estimation of Nitrogen. 0.1840 gm of organic compound gave 30 mL of nitrogen which is collected at 287 K & 758 mm of Hg.

Given ;

Aqueous tension at 287 K = 14 mm of Hg.

Hence actual pressure = (758 – 14)

= 744 mm of Hg.

Volume of nitrogen at STP = $\frac{273 \times 744 \times 30}{287 \times 760}$

V = 27.935 mL

$\because$ 22400 mL of N_2 at STP weighs = 28 gm.

$\therefore$ 27.94 mL of N_2 at STP weighs =

$\left(\frac{28}{22400} \times 27.94\right)$gm

= 0.0349 gm

Hence % of Nitrogen = $\left(\frac{0.0349}{0.1840} \times 100\right)$

= 18.97 %

Rond off. Answer = 19 %

5. The number of orbitals with n = 5, m_1 = + 2 is ______. (Round off to the Nearest Integer).

Official Ans. by NTA (3)

Sol. For, n = 5

ℓ = (0, 1, 2, 3, 4)

If ℓ = 0, m = 0

ℓ = 1, m = {–1, 0, +1}

ℓ = 2, m = {–2, –1, 0, +1, **+2**}

ℓ = 3, m = {–3, –2, –1, 0, +1, **+2**, +3}

ℓ = 4, m = {–4, –3, –2, –1, 0, +1, **+2**, +3, +4}

5d, 5f and 5g subshell contain one-one orbital having m_ℓ = +2

6. At 363 K, the vapour pressure of A is 21 kPa and that of B is 18 kPa. One mole of A and 2 moles of B are mixed. Assuming that this solution is ideal, the vapour pressure of the mixture is ________ kPa. (Round of to the Nearest Integer).

Official Ans. by NTA (19)

Sol. Given $P_A^0 = 21\text{kPa} \Rightarrow P_B^0 = 18\text{kPa}$

$\rightarrow$ An Ideal solution is prepared by mixing 1 mol A and 2 mol B.

$\rightarrow X_A = \frac{1}{3}$ and $X_B = \frac{2}{3}$

$\rightarrow$ Acc to Raoult's low

$P_T = X_A P_A^0 + X_B P_B^0$

$\Rightarrow P_T = \left(\frac{1}{3} \times 21\right) + \left(\frac{2}{3} \times 18\right)$

$\Rightarrow P_T$ = 7 +12 = 19 KPa

7. Sulphurous acid (H_2SO_3) has $Ka_1 = 1.7 \times 10^{-2}$ and $Ka_2 = 6.4 \times 10^{-8}$. The pH of 0.588 M H_2SO_3 is _________. (Round off to the Nearest Integer)

Official Ans. by NTA (1)

Sol. H_2SO_3 [Dibasic acid]

c = 0.588 M

⇒ pH of solution Þ due to First dissociation only since $K_{a_1} >> Ka_2$

⇒ First dissociation of H_2SO_3

$$H_2SO_3(aq) \rightleftharpoons H^{\oplus}(aq) + HSO_3^-(aq) : ka_1 = 1.7 \times 10^{-2}$$

t = 0 C

t C–x x x

$$\Rightarrow \quad Ka_1 = \frac{1.7}{100} = \frac{[H^{\oplus}][HSO_3^-]}{[H_2SO_3]}$$

$$\Rightarrow \frac{1.7}{100} = \frac{x^2}{(0.58 - x)}$$

⇒ $1.7 \times 0.588 - 1.7x = 100\, x^2$

⇒ $100x^2 + 1.7x - 1 = 0$

$$\Rightarrow \quad [H^{\oplus}] = x = \frac{-1.7 + \sqrt{(1.7)^2 + 4 \times 100 \times 1}}{2 \times 100} = 0.09186$$

Therefore pH of sol. is : pH = – log $[H^{\oplus}]$

⇒ pH = –log (0.09186) = 1.036 ≃ 1

8. When 35 mL of 0.15 M lead nitrate solution is mixed with 20 mL of 0.12 M chromic sulphate solution, ________ $\times 10^{-5}$ moles of lead sulphate precipitate out. (Round off to the Nearest Integer).

Official Ans. by NTA (525)

Sol. $3\,Pb\,(NO_3)_2 + Cr_2\,(SO_4)_3 \rightarrow 3PbSO_4 + 2Cr(NO_3)_3$

35 ml 20 ml

0.15 M 0.12 M

= 5.25 m.mol = 2.4 m.mol 5.25 m.mol

= 5.25×10^{-3} mol

therefore moles of $PbSO_4$ formed = 5.25×10^{-3}

= 525×10^{-5}

9. At 25°C, 50 g of iron reacts with HCl to form $FeCl_2$. The evolved hydrogen gas expands against a constant pressure of 1 bar. The work done by the gas during this expansion is _______ J.

(Round off to the Nearest Integer)

[Given : R = 8.314 J mol^{-1} K^{-1}. Assume, hydrogen is an ideal gas]

[Atomic mass off Fe is 55.85 u]

Official Ans. by NTA (2218)

Sol. T = 298 K, R = 8.314 $\frac{J}{mol\,K}$

→ Chemical reaction is

$Fe + 2HCl \rightarrow FeCl_2 + H_2(g)$

50g P = 1 bar

$= \frac{50}{55.85}$ mol

$\frac{50}{55.85}$ mol

→ Work done for 1 mol gas

$= -P_{ext} \times \Delta V$

$= \Delta ng\, RT$

$= -1 \times 8.314 \times 298$ J

→ Work done for $\frac{50}{55.85}$ mol of gas

$= -1.8314 \times 298 \times \frac{50}{55.85}$ J

= –2218.059 J

≃ –2218 J

10. $[Ti(H_2O)_6]^{3+}$ absorbs light of wavelength 498 nm during a d – d transition. The octahedral splitting energy for the above complex is _____ $\times 10^{-19}$ J. (Round off to the Nearest Integer). h = 6.626×10^{-34} Js; c = 3×10^8 ms^{-1}.

Official Ans. by NTA (4)

Sol. $\lambda_{absorbed}$ = 498 nm (given)

The octahedral spilitting energy

$$\Delta_0 \text{ or } E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{498 \times 10^{-9}}$$

$= 0.0399 \times 10^{-17}$ J

$= 3.99 \times 10^{-19}$ J

$= 4.00 \times 10^{-19}$ J (round off)

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16th March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. The maximum value of

$$f(x)=\begin{vmatrix}\sin^2 x & 1+\cos^2 x & \cos 2x\\ 1+\sin^2 x & \cos^2 x & \cos 2x\\ \sin^2 x & \cos^2 x & \sin 2x\end{vmatrix}, x\in R \text{ is:}$$

(1) $\sqrt{7}$ (2) $\frac{3}{4}$ (3) $\sqrt{5}$ (4) 5

Official Ans by NTA (3)

Sol. $C_1 + C_2 \to C_1$

$$\begin{vmatrix}2 & 1+\cos^2 x & \cos 2x\\ 2 & \cos^2 x & \cos 2x\\ 1 & \cos^2 x & \sin 2x\end{vmatrix}$$

$R_1 - R_2 \to R_1$

$$\begin{vmatrix}0 & 1 & 0\\ 2 & \cos^2 x & \cos 2x\\ 1 & \cos^2 x & \sin 2x\end{vmatrix}$$

Open w.r.t. R_1

$-(2 \sin 2x - \cos 2x)$

$\cos 2x - 2 \sin 2x = f(x)$

$$f(x)\big|_{max}=\sqrt{1+4}=\sqrt{5}$$

2. Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of event A is equal to :

(1) $\frac{9}{56}$ (2) $\frac{4}{9}$ (3) $\frac{3}{7}$ (4) $\frac{11}{27}$

Official Ans by NTA (2)

Sol. Total cases :

$\underline{6}\cdot\underline{6}\cdot\underline{5}\cdot\underline{4}\cdot\underline{3}\cdot\underline{2}$

$n(s) = 6 \cdot 6!$

Favourable cases :

Number divisible by 3 ≡

Sum of digits must be divisible by 3

Case-I

1, 2, 3, 4, 5, 6

Number of ways = 6!

Case-II

0, 1, 2, 4, 5, 6

Number of ways = $5\cdot 5!$

Case-III

0, 1, 2, 3, 4, 5

Number of ways = $5\cdot 5!$

n(favourable) = $6! + 2\cdot 5\cdot 5!$

$$P=\frac{6!+2\cdot 5\cdot 5!}{6\cdot 6!}=\frac{4}{9}$$

3. Let $\alpha \in R$ be such that the function

$$f(x)=\begin{cases}\dfrac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, & x\neq 0\\ \alpha, & x=0\end{cases}$$

is continuous at x = 0, where $\{x\} = x - [x]$, $[x]$ is the greatest integer less than or equal to x. Then :

(1) $\alpha=\frac{\pi}{\sqrt{2}}$ (2) $\alpha = 0$

(3) no such α exists (4) $\alpha=\frac{\pi}{4}$

Official Ans by NTA (3)

Sol.

$$\lim_{x\to 0^+} f(x)=f(0)=\lim_{x\to 0^-}(x)$$

$$\lim_{x\to 0^+}\frac{\cos^{-1}(1-x^2)\cdot\sin^{-1}(1-x)}{x(1-x)(1+x)}$$

$$\lim_{x\to 0^+}\frac{\cos^{-1}(1-x^2)}{x\cdot 1\cdot 1}\cdot\frac{\pi}{2}$$

Let $1 - x^2 = \cos\theta$

$$\frac{\pi}{2}\lim_{x\to 0^+}\frac{\theta}{\sqrt{1-\cos\theta}}$$

$$\frac{\pi}{2}\lim_{\theta\to 0^+}\frac{\theta}{\sqrt{2}\sin\frac{\theta}{2}}=\frac{\pi}{\sqrt{2}}$$

Now, $\lim\limits_{x\to 0^-}\dfrac{\cos^{-1}\left(1-(1+x)^2\right)\sin^{-1}(-x)}{(1+x)-(1+x)^3}$

$$\lim_{x\to 0^-}\frac{\frac{\pi}{2}\left(-\sin^{-1}x\right)}{(1+x)(2+x)(-x)}$$

$$\lim_{x\to 0^-}\frac{\frac{\pi}{2}}{1\cdot 2}\cdot\frac{\sin^{-1}x}{x}=\frac{\pi}{4}$$

$\Rightarrow$ RHL $\neq$ LHL

Function can't be continuous

$\Rightarrow$ No value of α exist

4. If (x, y, z) be an arbitrary point lying on a plane P which passes through the point (42, 0, 0), (0, 42, 0) and (0, 0, 42), then the value of expression

$$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-19}{(x-11)^2(z-12)^2}$$

$$+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

(1) 0 (2) 3 (3) 39 (4) –45

Official Ans by NTA (2)

Sol. Plane passing through (42, 0, 0), (0, 42, 0), (0, 0, 42)

From intercept from, equation of plane is

$x + y + z = 42$

$\Rightarrow$ $(x - 11) + (y - 19) + (z - 12) = 0$

let $a = x - 11$, $b = y - 19$, $c = z - 12$

$a + b + c = 0$

Now, given expression is

$$3+\frac{a}{b^2c^2}+\frac{b}{a^2c^2}+\frac{c}{a^2b^2}-\frac{42}{14abc}$$

$$3+\frac{a^3+b^3+c^3-3abc}{a^2b^2c^2}$$

If $a + b + c = 0$

$\Rightarrow$ $a^3 + b^3 + c^3 = 3\,abc$

$\Rightarrow$ 3

5. Consider the integral

$$I=\int_0^{10}\frac{[x]\,e^{[x]}}{e^{x-1}}dx,$$

where [x] denotes the greatest integer less than or equal to x. Then the value of I is equal to:

(1) $9(e - 1)$ (2) $45(e + 1)$

(3) $45(e - 1)$ (4) $9(e + 1)$

Official Ans by NTA (3)

Sol. $I=\displaystyle\int_0^{10}[x]\cdot e^{[x]-x+1}$

$$I=\int_0^1 0dx+\int_1^2 1\cdot e^{2-x}+\int_2^3 2\cdot e^{3-x}+.....+\int_9^{10}9\cdot e^{10-x}dx$$

$$\Rightarrow\quad I=\sum_{n=0}^{9}\int_n^{n+1}n\cdot e^{n+1-x}dx$$

$$=-\sum_{n=0}^{9}n\left(e^{n+1-x}\right)_n^{n+1}$$

$$=-\sum_{n=0}^{9}n\cdot(e^0-e^1)$$

$$=(e-1)\sum_{n=0}^{9}n$$

$$=(e-1)\cdot\frac{9\cdot 10}{2}$$

$= 45(e - 1)$

6. Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then the equation of tangent to C at P(2,1) is :

(1) $x - y = 1$ (2) $2x + y = 5$
(3) $x + 3y = 5$ (4) $x + 2y = 4$

Official Ans by NTA (1)

Sol. Given $y^2 = 4x$

Mirror image on $y = x \Rightarrow$ C : $x^2 = 4y$

$$2x = 4\cdot\frac{dy}{dx} \;\Rightarrow\; \frac{dy}{dx} = \frac{x}{2}$$

$$\left.\frac{dy}{dx}\right|_{P(2,1)} = \frac{2}{2} = 1$$

Equation of tangent at (2, 1)

$\Rightarrow$ $y - 1 = 1(x - 2)$

$\Rightarrow$ $x - y = 1$

7. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + (\tan x)\,y = \sin x,\ 0 \le x \le \frac{\pi}{3}$, with $y(0) = 0$, then $y\left(\frac{\pi}{4}\right)$ equal to :

(1) $\frac{1}{4}\log_e 2$ (2) $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$

(3) $\log_e 2$ (4) $\frac{1}{2}\log_e 2$

Official Ans by NTA (2)

Sol. $\frac{dy}{dx} + (\tan x)y = \sin x\ ;\ 0 \le x \le \frac{\pi}{3}$

$$\text{I.F.} = e^{\int \tan x\,dx} = e^{\ell n \sec x} = \sec x$$

$$y \sec x = \int \tan x\,dx$$

$$y \sec x = \int \tan x dx$$

$$y \sec x = \ell n\,|\sec x| + C$$

$x = 0,\ y = 0 \quad \Rightarrow \quad \therefore\ c = 0$

$$y \sec x = \ell n\,|\sec x|$$

$$y = \cos x \cdot \ell n\,|\sec x|$$

$$y|_{x=\frac{\pi}{4}} = \left(\frac{1}{\sqrt{2}}\right)\cdot \ell n\sqrt{2}$$

$$y|_{x=\frac{\pi}{4}} = \frac{1}{2\sqrt{2}}\log_e 2$$

8. Let A = {2, 3, 4, 5, ... , 30} and '$\simeq$' be an equivalence relation on A × A, defined by (a, b) $\simeq$ (c, d), if and only if ad = bc. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to :

(1) 5 (2) 6 (3) 8 (4) 7

Official Ans by NTA (4)

Sol. A = {2, 3, 4, 5, ... , 30}

(a, b) $\simeq$ (c, d) $\Rightarrow$ ad = bc

(4, 3) $\simeq$ (c, d) $\Rightarrow$ 4d = 3c

$$\Rightarrow \frac{4}{3} = \frac{c}{d}$$

$\frac{c}{d} = \frac{4}{3}$ & c, d $\in$ {2, 3, ..., 30}

(c, d) = {(4, 3), (8, 6), (12, 9), (16, 12), (20, 15), (24, 18), (28, 21)}

No. of ordered pair = 7

9. Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, $(a < 0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is euqal to :

(1) $\sqrt{11}$ (2) $\sqrt{7}$ (3) $\sqrt{6}$ (4) $\sqrt{10}$

Official Ans by NTA (3)

Sol. $x^2 + y^2 + ax + 2ay + c = 0$

$$2\sqrt{g^2 - c} = 2\sqrt{\frac{a^2}{4} - c} = 2\sqrt{2}$$

$$\Rightarrow \frac{a^2}{4} - c = 2 \qquad ...(1)$$

$2\sqrt{f^2 - c} = 2\sqrt{a^2 - c} = 2\sqrt{5}$

$\Rightarrow$ $a^2 - c = 5$ ...(2)

(1) & (2)

$\frac{3a^2}{4} = 3 \Rightarrow a = -2 \quad (a < 0)$

$\therefore$ $c = -1$

Circle $\Rightarrow$ $x^2 + y^2 - 2x - 4y - 1 = 0$

$\Rightarrow$ $(x - 1)^2 + (y - 2)^2 = 6$

Given $x + 2y = 0 \Rightarrow m = -\frac{1}{2}$

$m_{tangent} = 2$

Equation of tangent

$\Rightarrow$ $(y - 2) = 2(x - 1) \pm \sqrt{6}\sqrt{1+4}$

$\Rightarrow 2x - y \pm \sqrt{30} = 0$

Perpendicular distance from (0, 0) = $\left|\frac{\pm\sqrt{30}}{\sqrt{4+1}}\right| = \sqrt{6}$

10. The least value of $|z|$ where z is complex number which satisfies the inequality

$$\exp\left(\frac{(|z|+3)(|z|-1)}{\left||z|+1\right|}\log_e 2\right) \ge \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|,$$

$i = \sqrt{-1}$, is equal to :

(1) 3 (2) $\sqrt{5}$ (3) 2 (4) 8

Official Ans by NTA (1)

Sol. $\exp\left(\frac{(|z|+3)(|z|-1)}{\left||z|+1\right|}\ell n\, 2\right) \ge \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|$

$\Rightarrow 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \ge \log_{\sqrt{2}}(16)$

$\Rightarrow 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \ge 2^3$

$\Rightarrow \frac{(|z|+3)(|z|-1)}{(|z|+1)} \ge 3$

$\Rightarrow$ $(|z| + 3)(|z| - 1) \ge 3(|z| + 1)$

$|z|^2 + 2|z| - 3 \ge 3|z| + 3$

$\Rightarrow$ $|z|^2 + |z| - 6 \ge 0$

$\Rightarrow$ $(|z| - 3)(|z| + 2) \ge 0 \Rightarrow |z| - 3 \ge 0$

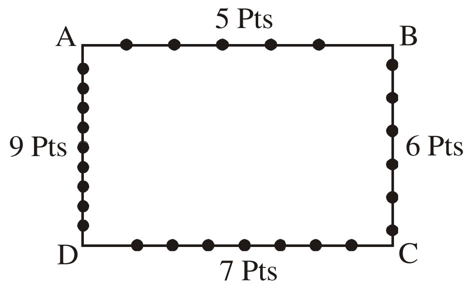
$\Rightarrow$ $|z| \ge 3$ $\Rightarrow$ $|z|_{min} = 3$

11. Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to :

(1) 795 (2) 1173 (3) 1890 (4) 717

Official Ans by NTA (4)

Sol.



α = Number of triangles

$\alpha = 5\cdot6\cdot7 + 5\cdot7\cdot9 + 5\cdot6\cdot9 + 6\cdot7\cdot9$

$= 210 + 315 + 270 + 378$

$= 1173$

β = Number of Quadrilateral

$\beta = 5\cdot6\cdot7\cdot9 = 1890$

$\beta - \alpha = 1890 - 1173 = 717$

12. If the point of intersections of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = 4b$, $b > 4$ lie on the curve $y^2 = 3x^2$, then b is equal to:

(1) 12 (2) 5 (3) 6 (4) 10

Official Ans by NTA (1)

Sol. $y^2 = 3x^2$

and $x^2 + y^2 = 4b$

Solve both we get

so $x^2 = b$

$\frac{x^2}{16} + \frac{3x^2}{b^2} = 1$

$$\frac{b}{16}+\frac{3}{b}=1$$

$b^2 - 16b + 48 = 0$

$(b - 12)(b - 4) = 0$

$b = 12,\ b > 4$

13. Given that the inverse trigonometric functions take principal values only. Then, the number of real values of x which satisfy $\sin^{-1}\left(\frac{3x}{5}\right)+\sin^{-1}\left(\frac{4x}{5}\right)=\sin^{-1}x$ is equal to:

(1) 2 (2) 1 (3) 3 (4) 0

Official Ans by NTA (3)

Sol. $\sin^{-1}\frac{3x}{5}+\sin^{-1}\frac{4x}{5}=\sin^{-1}x$

$$\sin^{-1}\left(\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}\right)=\sin^{-1}x$$

$$\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}=x$$

$x = 0,\ 3\sqrt{25-16x^2}+4\sqrt{25-9x^2}=25$

$4\sqrt{25-9x^2}=25-3\sqrt{25-16x^2}$ squaring we get

$16(25-9x^2) = 625+9(25-16x^2) - 150\sqrt{25-16x^2}$

$400 = 625 + 225 - 150\sqrt{25-16x^2}$

$\sqrt{25-16x^2} = 3 \Rightarrow 25 - 16x^2 = 9$

$\Rightarrow x^2 = 1$

Put x = 0, 1, –1 in the original equation

We see that all values satisfy the original equation.

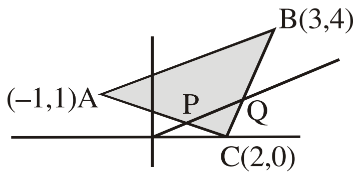
Number of solution = 3

14. Let A(–1, 1), B(3, 4) and C(2, 0) be given three points. A line y = mx, m > 0, intersects lines AC and BC at point P and Q respectively. Let A_1 and A_2 be the areas of ΔABC and ΔPQC respectively, such that $A_1 = 3A_2$, then the value of m is equal to :

(1) $\frac{4}{15}$ (2) 1 (3) 2 (4) 3

Official Ans by NTA (2)

Sol.



$P \equiv (x_1, mx_1)$

$Q \equiv (x_2, mx_2)$

$$A_1=\frac{1}{2}\begin{vmatrix}3 & 4 & 1\\ 2 & 0 & 1\\ -1 & 1 & 1\end{vmatrix}=\frac{13}{2}$$

$$A_2=\frac{1}{2}\begin{vmatrix}x_1 & mx_1 & 1\\ x_2 & mx_2 & 1\\ 2 & 0 & 1\end{vmatrix}$$

$$A_2=\frac{1}{2}\left|2(mx_1-mx_2)\right|=m\left|x_1-x_2\right|$$

$$A_1 = 3A_2 \Rightarrow \frac{13}{2} = 3m\left|x_1-x_2\right|$$

$$\Rightarrow \left|x_1-x_2\right|=\frac{16}{6m}$$

AC : x + 3y = 2

BC : y = 4x – 8

$$P : x + 3y = 2 \ \&\ y = mx \Rightarrow x_1 = \frac{2}{1+3m}$$

$$Q : y = 4x - 8 \ \&\ y = mx \Rightarrow x_2 = \frac{8}{4-m}$$

$$\left|x_1-x_2\right|=\left|\frac{2}{1+3m}-\frac{8}{4-m}\right|$$

$$= \left|\frac{-26m}{(1+3m)(4-m)}\right| = \frac{26m}{(3m+1)|m-4|}$$

$$= \frac{26m}{(3m+1)(4-m)}$$

$$|x_1 - x_2| = \frac{13}{6m}$$

$$\frac{26m}{(3m+1)(4-m)} = \frac{13}{6m}$$

$\Rightarrow$ $12m^2 = -(3m + 1)(m - 4)$

$\Rightarrow$ $12m^2 = -(3m^2 - 11m - 4)$

$\Rightarrow$ $15m^2 - 11m - 4 = 0$

$\Rightarrow$ $15m^2 - 15m + 4m - 4 = 0$

$\Rightarrow$ $(15m + 4)(m - 1) = 0$

$\Rightarrow$ $m = 1$

15. Let f be a real valued function, defined on $R - \{-1, 1\}$ and given by

$$f(x) = 3\log_e\left|\frac{x-1}{x+1}\right| - \frac{2}{x-1}.$$

Then in which of the following intervals, function f(x) is increasing?

(1) $(-\infty,-1)\cup\left(\left[\frac{1}{2},\infty\right)-\{1\}\right)$

(2) $(-\infty, \infty) - \{-1, 1\}$

(3) $\left(-1,\frac{1}{2}\right]$

(4) $\left(-\infty,\frac{1}{2}\right]-\{-1\}$

Official Ans by NTA (1)

Sol. $f(x) = 3\ell n(x-1) - 3\ell n(x+1) - \frac{2}{x-1}$

$$f'(x) = \frac{3}{x-1} - \frac{3}{x+1} + \frac{2}{(x-1)^2}$$

$$f'(x) = \frac{4(2x-1)}{(x-1)^2(x+1)}$$

$f'(x) \geq 0$

$\Rightarrow$ $x \in (-\infty,-1)\cup\left[\frac{1}{2},1\right)\cup(1,\infty)$

16. Let $f : S \to S$ where $S = (0, \infty)$ be a twice differentiable function such that $f(x + 1) = xf(x)$. If $g : S \to R$ be defined as $g(x) = \log_e f(x)$, then the value of $|g''(5) - g''(1)|$ is equal to :

(1) $\frac{205}{144}$ (2) $\frac{197}{144}$ (3) $\frac{187}{144}$ (4) 1

Official Ans by NTA (1)

Sol. $\ln f(x + 1) = \ln(xf(x))$

$\ln f(x + 1) = \ln x + \ln f(x)$

$\Rightarrow$ $g(x + 1) = \ln x + g(x)$

$\Rightarrow$ $g(x + 1) - g(x) = \ln x$

$\Rightarrow$ $g''(x + 1) - g''(x) = -\frac{1}{x^2}$

Put x = 1, 2, 3, 4

$g''(2) - g''(1) = -\frac{1}{1^2}$ ...(1)

$g''(3) - g''(2) = -\frac{1}{2^2}$ ...(2)

$g''(4) - g''(3) = -\frac{1}{3^2}$ ...(3)

$g''(5) - g''(4) = -\frac{1}{4^2}$ ...(4)

Add all the equation we get

$$g''(5) - g''(1) = -\frac{1}{1^2} - \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2}$$

$$|g''(5) - g''(1)| = \frac{205}{144}$$

17. Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx = 1$ and P(x) leaves remainder 5 when it is divided by $(x - 2)$. Then the value of $9(b + c)$ is equal to:

(1) 9 (2) 15 (3) 7 (4) 11

Official Ans by NTA (3)

Sol. $\int_0^1 (x^2+bx+c)dx = 1$

$\frac{1}{3}+\frac{b}{2}+c=1 \quad \Rightarrow \quad \frac{b}{2}+c=\frac{2}{3}$

$3b+6c=4$ ...(1)

P(2) = 5

4 + 2b + c = 5

2b + c = 1 ...(2)

From (1) & (2)

$b = \frac{2}{9}$ & $c = \frac{5}{9}$

9(b + c) = 7

18. If the foot of the perpendicular from point (4, 3, 8) on the line $L_1 : \frac{x-a}{l}=\frac{y-2}{3}=\frac{z-b}{4}$, $l \neq 0$ is (3, 5, 7), then the shortest distance between the line L_1 and line $L_2 : \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{6}}$ (3) $\sqrt{\frac{2}{3}}$ (4) $\frac{1}{\sqrt{3}}$

Official Ans by NTA (2)

Sol. (3,5,7) satisfy the line L_1

$\frac{3-a}{\ell}=\frac{5-2}{3}=\frac{7-b}{4}$

$\frac{3-a}{\ell}=1$ & $\frac{7-b}{4}=1$

$a + \ell = 3$ ...(1) & b = 3 ...(2)

$\vec{v}_1 = <4,3,8> - <3,5,7>$

$\vec{v}_1 = <1,-2,1>$

$\vec{v}_2 = <\ell,3,4>$

$\vec{v}_1.\vec{v}_2 = 0 \Rightarrow \ell-6+4=0 \Rightarrow \ell=2$

$a+\ell=3 \Rightarrow a = 1$

$L_1 : \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$

$L_2 : \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$

A = <1, 2, 3>

B = <2, 4, 5>

$\overrightarrow{AB} = <1,2,2>$

$\vec{p} = 2\hat{i}+3\hat{j}+4\hat{k}$

$\vec{q} = 3\hat{i}+4\hat{j}+5\hat{k}$

$\vec{p}\times\vec{q} = -\hat{i}+2\hat{j}-\hat{k}$

Shortest distance = $\left|\frac{\overrightarrow{AB}\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right| = \frac{1}{\sqrt{6}}$

19. Let C_1 be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx}=y^2-x^2$, $x > 0$. Let the curve C_2 be the solution of $\frac{2xy}{x^2-y^2}=\frac{dy}{dx}$. If both the curves pass through (1,1), then the area enclosed by the curves C_1 and C_2 is equal to :

(1) $\pi - 1$ (2) $\frac{\pi}{2}-1$ (3) $\pi + 1$ (4) $\frac{\pi}{4}+1$

Official Ans by NTA (2)

Sol. $\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$, $x \in (0, \infty)$

put y = vx

$x\frac{dv}{dx}+v=\frac{v^2-1}{2v}$

$\frac{2v}{v^2+1}dv = -\frac{dx}{x}$

Integrate,

$ln(v^2 + 1) = - ln x + C$

$ln\left(\frac{y^2}{x^2}+1\right) = -ln x + C$

put x = 1, y = 1, C = $ln2$

$$ln\left(\frac{y^2}{x^2}+1\right)=-lnx+ln2$$

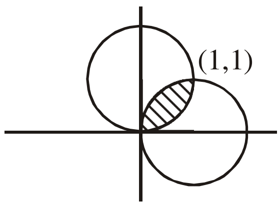
$\Rightarrow$ $x^2 + y^2 - 2x = 0$ (Curve C_1)

Similarly,

$$\frac{dy}{dx}=\frac{2xy}{x^2-y^2}$$

Put y = vx

$x^2 + y^2 - 2y = 0$



required area = $2\int_0^1\left(\sqrt{2x-x^2}-x\right)dx = \frac{\pi}{2} - 1$

20. Let $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+5\hat{k}$. If $\vec{r}\times\vec{a}=\vec{b}\times\vec{r}$, $\vec{r}.\left(\alpha\hat{i}+2\hat{j}+\hat{k}\right)=3$ and $\vec{r}.\left(2\hat{i}+5\hat{j}-\alpha\hat{k}\right)=-1$, $\alpha \in$ R, then the value of $\alpha+\left|\vec{r}\right|^2$ is equal to :

(1) 9 (2) 15 (3) 13 (4) 11

Official Ans by NTA (2)

Sol. $\vec{r}\times\vec{a}=\vec{b}\times\vec{r} \Rightarrow \vec{r}\times(\vec{a}+\vec{b})=0$

$\vec{r}=\vec{\lambda}(\vec{a}+\vec{b}) \Rightarrow \vec{r}=\vec{\lambda}(\hat{i}+2\hat{j}-3\hat{k}+2\hat{i}-3\hat{j}+5\hat{k})$

$\vec{r}=\vec{\lambda}(3\hat{i}-\hat{j}+2\hat{k})$ ...(1)

$\vec{r}\cdot(\alpha\hat{i}+2\hat{j}+\hat{k})=3$

Put $\vec{r}$ from (1) $\alpha\lambda=1$ ...(2)

$\vec{r}\cdot(2\hat{i}+5\hat{j}-\alpha\hat{k})=-1$

Put $\vec{r}$ from (1) $2\lambda\alpha-\lambda=1$ ...(3)

Solve (2) & (3)

$\alpha = 1, \lambda=1$

$\Rightarrow \vec{r}=3\hat{i}-\hat{j}+2\hat{k}$

$\left|\vec{r}\right|^2=14$ & $\alpha = 1$

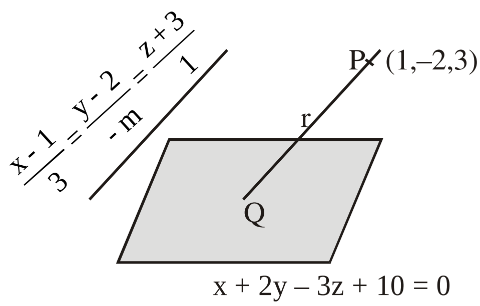
$\alpha+\left|\vec{r}\right|^2 = 15$

SECTION-B

1. If the distance of the point (1, –2, 3) from the plane x + 2y – 3z + 10 = 0 measured parallel to the line, $\frac{x-1}{3}=\frac{2-y}{m}=\frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of |m| is equal to ______.

Official Ans by NTA (2)

Sol.



$$\text{DC of line} \equiv \left(\frac{3}{\sqrt{m^2+10}},\frac{-m}{\sqrt{m^2+10}},\frac{1}{\sqrt{m^2+10}}\right)$$

$$Q\equiv\left(1+\frac{3r}{\sqrt{m^2+10}},-2+\frac{-mr}{\sqrt{m^2+10}},3+\frac{r}{\sqrt{m^2+10}}\right)$$

Q lies on x + 2y – 3z + 10 = 0

$$1+\frac{3r}{\sqrt{m^2+10}}-4-\frac{2mr}{\sqrt{m^2+10}}-9-\frac{3r}{\sqrt{m^2+10}}+10=0$$

$$\Rightarrow \frac{r}{\sqrt{m^2+10}}(3-2m-3)=2$$

$$\Rightarrow \frac{r}{\sqrt{m^2+10}}(-2m)=2$$

$r^2m^2 = m^2 + 10$

$\frac{7}{2}m^2 = m^2 + 10 \Rightarrow \frac{5}{2}m^2 = 10 \Rightarrow m^2 = 4$

$|m| = 2$

2. Consider the statistics of two sets of observations as follows :

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to __________.

Official Ans by NTA (5)

Sol. $\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1n_2}{(n_1+n_2)}(\bar{x}_1 - \bar{x}_2)^2$

$n_1 = 10$, $n_2 = n$, $\sigma_1^2 = 2$, $\sigma_2^2 = 1$

$\bar{x}_1 = 2$, $\bar{x}_2 = 3$, $\sigma^2 = \frac{17}{9}$

$\frac{17}{9} = \frac{10\times 2 + n}{n+10} + \frac{10n}{(n+10)^2}(3-2)^2$

$\Rightarrow \frac{17}{9} = \frac{(n+20)(n+10)+10n}{(n+10)^2}$

$\Rightarrow 17n^2 + 1700 + 340\,n = 90n + 9(n^2+30n+200)$

$\Rightarrow 8n^2 - 20n - 100 = 0$

$2n^2 - 5n - 25 = 0$

$\Rightarrow (2n + 5)(n - 5) = 0 \Rightarrow n = \underset{\substack{\downarrow \\ \text{(Rejected)}}}{\frac{-5}{2}}, 5$

Hence n = 5

3. Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2 × 1 matrices with real entries such that A = XB, where $X = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$, and k ∈ R. If $a_1^2 + a_2^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$ and $(k^2+1)b_2^2 \neq -2b_1b_2$, then the value of k is _______.

Official Ans by NTA (1)

Sol. A = XB

$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$\begin{bmatrix} \sqrt{3}\,a_1 \\ \sqrt{3}\,a_2 \end{bmatrix} = \begin{bmatrix} b_1 - b_2 \\ b_1 + kb_2 \end{bmatrix}$

$b_1 - b_2 = \sqrt{3}\,a_1$ ...(1)

$b_1 + kb_2 = \sqrt{3}\,a_2$ ...(2)

Given, $a_1^2 + a_2^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$

$(1)^2 + (2)^2$

$(b_1 + b_2)^2 + (b_1 + kb_2)^2 = 3(a_1^2 + a_2^2)$

$a_1^2 + a_2^2 = \frac{2}{3}b_1^2 + \frac{(1+k^2)}{3}b_2^2 + \frac{2}{3}b_1b_2(k-1)$

Given, $a_1^2 + a_2^2 = \frac{2}{3}b_1^2 + \frac{2}{3}b_2^2$

On comparing we get

$\frac{k^2+1}{3} = \frac{2}{3} \Rightarrow k^2 + 1 = 2$

$\Rightarrow k = \pm 1$ ....(3)

& $\frac{2}{3}(k-1) = 0 \Rightarrow k = 1$ ...(4)

From both we get k = 1

4. For real numbers α, β, γ and δ, if

$$\int \frac{(x^2-1) + \tan^{-1}\left(\frac{x^2+1}{x}\right)}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$= \alpha\log_e\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)$

$+\beta\tan^{-1}\left(\frac{\gamma(x^2-1)}{x}\right)+\delta\tan^{-1}\left(\frac{x^2+1}{x}\right)+C$

where C is an arbitrary constant, then the value of $10(\alpha + \beta\gamma + \delta)$ is equal to __________.

Official Ans by NTA (6)

Sol. $\int\frac{(x^2-1)dx}{(x^4+3x^2+1)\tan^{-1}\left(x+\frac{1}{x}\right)}+\int\frac{dx}{x^4+3x^2+1}$

$$\int\frac{\left(1-\frac{1}{x^2}\right)dx}{\left(\left(x+\frac{1}{x}\right)^2+1\right)\tan^{-1}\left(x+\frac{1}{x}\right)}+\frac{1}{2}\int\frac{(x^2+1)-(x^2-1)dx}{x^4+3x^2+1}$$

Put $\tan^{-1}\left(x+\frac{1}{x}\right)=t$

$$\int\frac{dt}{t}+\frac{1}{2}\int\frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+5}-\frac{1}{2}\int\frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2+1}$$

Put $x-\frac{1}{x}=y$, $x+\frac{1}{x}=z$

$$\log_e t+\frac{1}{2}\int\frac{dy}{y^2+5}-\frac{1}{2}\int\frac{dz}{z^2+1}$$

$$= \log_e\tan^{-1}\left(x+\frac{1}{x}\right)+\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}x}\right)$$

$$-\frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right)+C$$

$\alpha = 1,\ \beta = \frac{1}{2\sqrt{5}},\ \gamma = \frac{1}{\sqrt{5}},\ \delta = \frac{-1}{2}$

or

$\alpha = 1,\ \beta = \frac{-1}{2\sqrt{5}},\ \gamma = \frac{-1}{\sqrt{5}},\ \delta = \frac{-1}{2}$

$10(\alpha + \beta\gamma + \delta) = 10\left(1+\frac{1}{10}-\frac{1}{2}\right) = 6$

5. Let $f : R \to R$ and $g : R \to R$ be defined as

$f(x)=\begin{cases} x+a, & x<0 \\ |x-1|, & x\ge 0\end{cases}$ and

$g(x)=\begin{cases} x+1, & x<0 \\ (x-1)^2+b, & x\ge 0\end{cases}$

where a, b are non-negative real numbers. If (gof)(x) is continuous for all $x \in R$, then a + b is equal to ___________.

Official Ans by NTA (1)

Sol. $g[f(x)] = \left[\begin{array}{ll} f(x)+1 & f(x)<0 \\ (f(x)-1)^2+b & f(x)\ge 0\end{array}\right.$

$$g[f(x)]=\left[\begin{array}{ll} x+a+1 & x+a<0\ \&\ x<0 \\ |x-1|+1 & |x-1|<0\ \&\ x\ge 0 \\ (x+a-1)^2+b & x+a\ge 0\ \&\ x<0 \\ (|x-1|-1)^2+b & |x-1|\ge 0\ \&\ x\ge 0\end{array}\right.$$

$$g[f(x)]=\left[\begin{array}{ll} x+a+1 & x\in(-\infty,-a)\ \&\ x\in(-\infty,0) \\ |x-1|+1 & x\in\phi \\ (x+a-1)^2+b & x\in[-a,\infty)\ \&\ x\in(-\infty,0) \\ (|x-1|-1)^2+b & x\in R\ \&\ x\in[0,\infty)\end{array}\right.$$

$$g[f(x)]=\left[\begin{array}{ll} x+a+1 & x\in(-\infty,-a) \\ (x+a-1)^2+b & x\in[-a,0) \\ (|x-1|-1)^2+b & x\in[0,\infty)\end{array}\right.$$

g(f(x)) is continuous

at $x = -a$ & at $x = 0$

$1 = b + 1$ & $(a-1)^2 + b = b$

$b = 0$ & $a = 1$

$\Rightarrow a + b = 1$

6. Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a}, \frac{1}{b}, 6$ be in A.P., where a, b > 0. Then 72(a + b) is equal to ________.

Official Ans by NTA (14)

Sol. $a^2 = \frac{b}{16} \Rightarrow \frac{1}{b} = \frac{1}{16a^2}$

$$\frac{2}{b} = \frac{1}{a} + 6$$

$$\frac{1}{8a^2} = \frac{1}{a} + 6$$

$$\frac{1}{a^2} - \frac{8}{a} - 48 = 0$$

$$\frac{1}{a} = 12, -4 \quad \Rightarrow \quad a = \frac{1}{12}, -\frac{1}{4}$$

$$a = \frac{1}{12},\ a > 0$$

$b = 16a^2 = \frac{1}{9}$

$\Rightarrow 72(a + b) = 6 + 8 = 14$

7. In ΔABC, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC, then the value of 2R + r (in cm) is equal to _______.

Official Ans by NTA (15)

Sol. $\Delta = \frac{1}{2}.5.12.\sin A = 30$

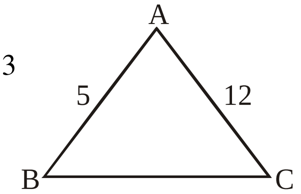
sinA = 1

A = 90° $\Rightarrow$ BC = 13

BC = 2R = 13

$r = \frac{\Delta}{S} = \frac{30}{15} = 2$

2R + r = 15



8. Let n be a positive integer. Let

$$A = \sum_{k=0}^{n} (-1)^k {}^nC_k \left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k\right]$$

If $63A = 1 - \frac{1}{2^{30}}$, then n is equal to _______.

Official Ans by NTA (6)

Sol. $A = \sum_{k=0}^{n} {}^nC_k \left[\left(-\frac{1}{2}\right)^k + \left(\frac{-3}{4}\right)^k + \left(\frac{-7}{8}\right)^k + \left(\frac{-15}{16}\right)^k + \left(\frac{-37}{32}\right)^k\right]$

$$A = \left(1-\frac{1}{2}\right)^n + \left(1-\frac{3}{4}\right)^n + \left(1-\frac{7}{8}\right)^n + \left(1-\frac{15}{16}\right)^n + \left(1-\frac{31}{32}\right)^n$$

$$A = \frac{1}{2^n} + \frac{1}{4^n} + \frac{1}{8^n} + \frac{1}{16^n} + \frac{1}{32^n}$$

$$A = \frac{1}{2^n}\left(\frac{1-\left(\frac{1}{2^n}\right)^5}{1-\frac{1}{2^n}}\right) \Rightarrow A = \frac{\left(1-\frac{1}{2^{5n}}\right)}{(2^n-1)}$$

$(2^n - 1)A = 1 - \frac{1}{2^{5n}}$, Given $63A = 1 - \frac{1}{2^{30}}$

Clearly 5n = 30

n = 6

9. Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

If $\vec{c}.\left(\hat{i} + \hat{j} + 3\hat{k}\right) = 8$ then the value of $\vec{c}.\left(\vec{a} \times \vec{b}\right)$ is equal to _________.

Official Ans by NTA (28)

Sol. $\vec{c} = \lambda(\vec{a} \times \vec{b})$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$(\vec{a} \times \vec{b}) = 3\hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{c} \cdot (\hat{i} + \hat{j} + 3\hat{k}) = \lambda(3\hat{i} - 2\hat{j} + \hat{k}) . (\hat{i} + \hat{j} + 3\hat{k})$$

$\Rightarrow \lambda(4)=8 \quad \Rightarrow \quad \lambda=2$

$$\vec{c}=2(\vec{a}\times\vec{b})$$

$$\vec{c}.(\vec{a}\times\vec{b})=2|\vec{a}\times\vec{b}|^2=28$$

10. Let

$$S_n(x)=\log_{a^{1/2}} x+\log_{a^{1/3}} x+\log_{a^{1/6}} x$$

$$+\log_{a^{1/11}} x+\log_{a^{1/18}} x+\log_{a^{1/27}} x+.....$$

up to n-terms, where a > 1. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to _______.

Official Ans by NTA (16)

Sol. $S_n(x) = (2+3+6+11+18+27+.......+\text{n-terms})\log_a x$

Let $S_1 = 2 + 3 + 6 + 11 + 18 + 27 + ... + T_n$

$S_1 = 2 + 3 + 6 + ... + T_n$

———————————————————

$T_n = 2 + 1 + 3 + 5 + ... + \text{n terms}$

$T_n = 2 + (n-1)^2$

$$S_1 = \Sigma T_n = 2n + \frac{(n-1)n(2n-1)}{6}$$

$$\Rightarrow S_n(x)=\left(2n+\frac{n(n-1)(2n-1)}{6}\right)\log_a x$$

$S_{24}(x) = 1093$ (Given)

$$\log_a x\left(48+\frac{23.24.47}{6}\right)=1093$$

$$\log_a x = \frac{1}{4} \quad .... (1)$$

$S_{12}(2x) = 265$

$S_{12}(2x) = 265$

$$\log_a(2x)\left(24+\frac{11.12.23}{6}\right)=265$$

$$\log_a 2x = \frac{1}{2} \quad .... (2)$$

(2) – (1)

$$\log_a 2x - \log_a x = \frac{1}{4}$$

$$\log_a 2 = \frac{1}{4} \Rightarrow a = 16$$