

# FINAL JEE-MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16<sup>th</sup> March, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

## PHYSICS

## TEST PAPER WITH ANSWER & SOLUTION

### SECTION-A

1. One main scale division of a vernier callipers is 'a' cm and n<sup>th</sup> division of the vernier scale coincide with (n – 1)<sup>th</sup> division of the main scale. The least count of the callipers in mm is :

- (1)  $\frac{10na}{(n-1)}$                       (2)  $\frac{10a}{(n-1)}$   
 (3)  $\left(\frac{n-1}{10n}\right)a$                       (4)  $\frac{10a}{n}$

**Official Ans. by NTA (4)**

**Sol.**  $(n - 1)a = n(a')$

$$a' = \frac{(n-1)a}{n}$$

$$\therefore \text{L.C.} = 1 \text{ MSD} - 1 \text{ VSD}$$

$$= (a - a') \text{ cm}$$

$$= a - \frac{(n-1)a}{n}$$

$$= \frac{na - na + a}{n} = \frac{a}{n} \text{ cm}$$

$$= \left(\frac{10a}{n}\right) \text{ mm}$$

2. For changing the capacitance of a given parallel plate capacitor, a dielectric material of dielectric constant K is used, which has the same area as the plates of the capacitor. The thickness of the

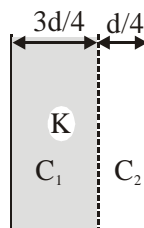
dielectric slab is  $\frac{3}{4}d$ , where 'd' is the separation

between the plates of parallel plate capacitor. The new capacitance (C') in terms of original capacitance (C<sub>0</sub>) is given by the following relation :

- (1)  $C' = \frac{3+K}{4K}C_0$                       (2)  $C' = \frac{4+K}{3}C_0$   
 (3)  $C' = \frac{4K}{K+3}C_0$                       (4)  $C' = \frac{4}{3+K}C_0$

**Official Ans. by NTA (3)**

**Sol.**



$$C_0 = \frac{\epsilon_0 A}{d}$$

$C' = C_1$  and  $C_2$  in series.

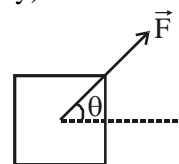
$$\text{i.e. } \frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C'} = \frac{(3d/4)}{\epsilon_0 KA} + \frac{d/4}{\epsilon_0 A}$$

$$\frac{1}{C'} = \frac{d}{4\epsilon_0 A} \left( \frac{3+K}{K} \right)$$

$$C' = \frac{4KC_0}{(3+K)}$$

3. A block of mass m slides along a floor while a force of magnitude F is applied to it at an angle  $\theta$  as shown in figure. The coefficient of kinetic friction is  $\mu_K$ . Then, the block's acceleration 'a' is given by : (g is acceleration due to gravity)



(1)  $-\frac{F}{m}\cos\theta - \mu_K \left( g - \frac{F}{m}\sin\theta \right)$

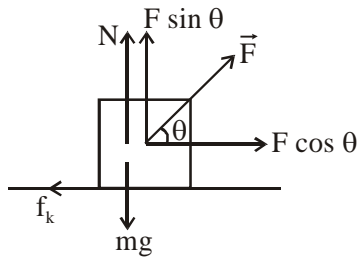
(2)  $\frac{F}{m}\cos\theta - \mu_K \left( g - \frac{F}{m}\sin\theta \right)$

(3)  $\frac{F}{m}\cos\theta - \mu_K \left( g + \frac{F}{m}\sin\theta \right)$

(4)  $\frac{F}{m}\cos\theta + \mu_K \left( g - \frac{F}{m}\sin\theta \right)$

**Official Ans. by NTA (2)**

Sol.



$$N = mg - F \sin \theta$$

$$F \cos \theta - \mu_k N = ma$$

$$F \cos \theta - \mu_k (mg - F \sin \theta) = ma$$

$$a = \frac{F}{m} \cos \theta - \mu_k \left( g - \frac{F}{m} \sin \theta \right)$$

4. The pressure acting on a submarine is  $3 \times 10^5$  Pa at a certain depth. If the depth is doubled, the percentage increase in the pressure acting on the submarine would be : (Assume that atmospheric pressure is  $1 \times 10^5$  Pa density of water is  $10^3 \text{ kg m}^{-3}$ ,  $g = 10 \text{ ms}^{-2}$ )

(1)  $\frac{200}{3}\%$  (2)  $\frac{200}{5}\%$

(3)  $\frac{5}{200}\%$  (4)  $\frac{3}{200}\%$

**Official Ans. by NTA (1)**

Sol.  $P_1 = \rho g d + P_0 = 3 \times 10^5 \text{ Pa}$

$$\therefore \rho g d = 2 \times 10^5 \text{ Pa}$$

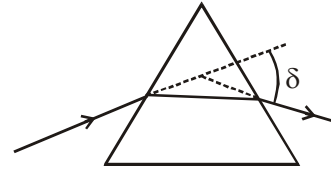
$$P_2 = 2\rho g d + P_0$$

$$= 4 \times 10^5 + 10^5 = 5 \times 10^5 \text{ Pa}$$

$$\% \text{increase} = \frac{P_2 - P_1}{P_1} \times 100$$

$$= \frac{5 \times 10^5 - 3 \times 10^5}{3 \times 10^5} \times 100 = \frac{200}{3}\%$$

5. The angle of deviation through a prism is minimum when



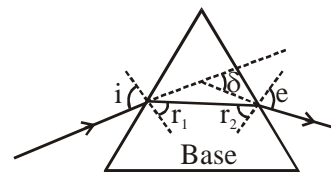
- (A) Incident ray and emergent ray are symmetric to the prism  
(B) The refracted ray inside the prism becomes parallel to its base  
(C) Angle of incidence is equal to that of the angle of emergence  
(D) When angle of emergence is double the angle of incidence

Choose the correct answer from the options given below :

- (1) Statements (A), (B) and (C) are true  
(2) Only statement (D) is true  
(3) Only statements (A) and (B) are true  
(4) Statements (B) and (C) are true

**Official Ans. by NTA (1)**

- Sol. Deviation is minimum in a prism when :  
 $i = e$ ,  $r_1 = r_2$  and ray (2) is parallel to base of prism.



6. A plane electromagnetic wave of frequency 500 MHz is travelling in vacuum along y-direction. At a particular point in space and time,  $\vec{B} = 8.0 \times 10^{-8} \hat{z} \text{ T}$ . The value of electric field at this point is :  
(speed of light =  $3 \times 10^8 \text{ ms}^{-1}$ )  
 $\hat{x}, \hat{y}, \hat{z}$  are unit vectors along x, y and z direction.

- (1)  $-24 \hat{x} \text{ V/m}$  (2)  $2.6 \hat{x} \text{ V/m}$   
(3)  $24 \hat{x} \text{ V/m}$  (4)  $-2.6 \hat{y} \text{ V/m}$

**Official Ans. by NTA (1)**

**Sol.**  $f = 5 \times 10^8 \text{ Hz}$

EM wave is travelling towards  $+\hat{j}$

$$\vec{B} = 8.0 \times 10^{-8} \hat{z} \text{ T}$$

$$\vec{E} = \vec{B} \times \vec{C} = (8 \times 10^{-8} \hat{z}) \times (3 \times 10^8 \hat{y})$$

$$= -24 \hat{x} \text{ V/m}$$

7. The maximum and minimum distances of a comet from the Sun are  $1.6 \times 10^{12} \text{ m}$  and  $8.0 \times 10^{10} \text{ m}$  respectively. If the speed of the comet at the nearest point is  $6 \times 10^4 \text{ ms}^{-1}$ , the speed at the farthest point is :

(1)  $1.5 \times 10^3 \text{ m/s}$  (2)  $6.0 \times 10^3 \text{ m/s}$

(3)  $3.0 \times 10^3 \text{ m/s}$  (4)  $4.5 \times 10^3 \text{ m/s}$

**Official Ans. by NTA (3)**

**Sol.** By angular momentum conservation :

$$mv_1 r_1 = mv_2 r_2$$

$$v_1 = \frac{48 \times 10^{14}}{1.6 \times 10^{12}} = 3000 \text{ m/sec}$$

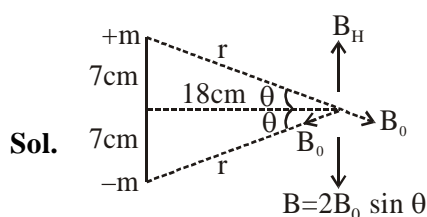
$$= 3 \times 10^3 \text{ m/sec.}$$

8. A bar magnet of length 14 cm is placed in the magnetic meridian with its north pole pointing towards the geographic north pole. A neutral point is obtained at a distance of 18 cm from the center of the magnet. If  $B_H = 0.4 \text{ G}$ , the magnetic moment of the magnet is ( $1 \text{ G} = 10^{-4} \text{ T}$ )

(1)  $2.880 \times 10^3 \text{ J T}^{-1}$  (2)  $2.880 \times 10^2 \text{ J T}^{-1}$

(3)  $2.880 \text{ J T}^{-1}$  (4)  $28.80 \text{ J T}^{-1}$

**Official Ans. by NTA (3)**



$$\text{i.e. } \frac{2\mu_0 m}{4\pi r^2} \times \frac{7}{r} = 0.4 \times 10^{-4}$$

$$\Rightarrow 2 \times 10^{-7} \times \frac{m \times 7}{(7^2 + 18^2)^{3/2}} \times 10^4$$

$$= 0.4 \times 10^{-4}$$

$$m = \frac{4 \times 10^{-2} \times (373)^{3/2}}{14}$$

$$M = m \times 14 \text{ cm} = m \times \frac{14}{100}$$

$$= \frac{0.04 \times (373)^{3/2}}{14} \times \frac{14}{100}$$

$$= 4 \times 10^{-4} \times 7203.82 = 2.88 \text{ J/T}$$

9. The volume  $V$  of an enclosure contains a mixture of three gases, 16 g of oxygen, 28 g of nitrogen and 44 g of carbon dioxide at absolute temperature  $T$ . Consider  $R$  as universal gas constant. The pressure of the mixture of gases is :

(1)  $\frac{88RT}{V}$

(2)  $\frac{3RT}{V}$

(3)  $\frac{5RT}{2V}$

(4)  $\frac{4RT}{V}$

**Official Ans. by NTA (3)**

**Sol.**  $PV = (n_1 + n_2 + n_3)RT$

$$P \times V = \left[ \frac{16}{32} + \frac{28}{28} + \frac{44}{44} \right] RT$$

$$PV = \left[ \frac{1}{2} + 1 + 1 \right] RT$$

$$P = \frac{5RT}{2V}$$

10. In thermodynamics, heat and work are :

(1) Path functions

(2) Intensive thermodynamic state variables

(3) Extensive thermodynamic state variables

(4) Point functions

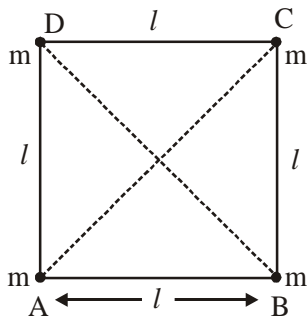
**Official Ans. by NTA (1)**

**Sol.** Heat and work are treated as path functions in thermodynamics.

$$\Delta Q = \Delta U + \Delta W$$

Since work done by gas depends on type of process i.e. path and  $\Delta U$  depends just on initial and final states, so  $\Delta Q$  i.e. heat, also has to depend on process is path.

11. Four equal masses,  $m$  each are placed at the corners of a square of length ( $l$ ) as shown in the figure. The moment of inertia of the system about an axis passing through A and parallel to DB would be :

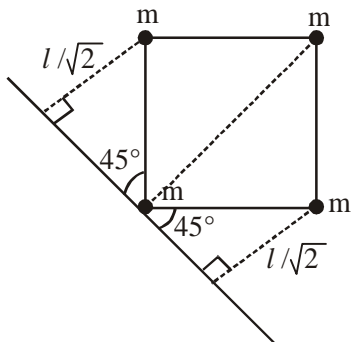


- (1)  $ml^2$                       (2)  $2 ml^2$   
 (3)  $3 ml^2$                       (4)  $\sqrt{3} ml^2$

**Official Ans. by NTA (3)**

**Sol.** Moment of inertia of point mass

$$= \text{mass} \times (\text{Perpendicular distance from axis})^2$$



Moment of Inertia

$$= m(0)^2 + m(l\sqrt{2})^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2$$

$$= 3 ml^2$$

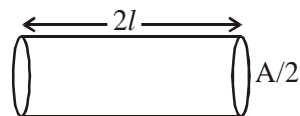
12. A conducting wire of length ' $l$ ', area of cross-section  $A$  and electric resistivity  $\rho$  is connected between the terminals of a battery. A potential difference  $V$  is developed between its ends, causing an electric current.

If the length of the wire of the same material is doubled and the area of cross-section is halved, the resultant current would be :

- (1)  $\frac{1}{4} \frac{VA}{\rho l}$                       (2)  $\frac{3}{4} \frac{VA}{\rho l}$   
 (3)  $\frac{1}{4} \frac{\rho l}{VA}$                       (4)  $4 \frac{VA}{\rho l}$

**Official Ans. by NTA (1)**

**Sol.** As per the question



$$\text{Resistance} = \frac{\rho(2l)}{(A/2)} = \frac{4\rho l}{A}$$

$$\Rightarrow \text{Current} = \frac{V}{R} = \frac{VA}{4\rho l}$$

13. Time period of a simple pendulum is  $T$  inside a lift when the lift is stationary. If the lift moves upwards with an acceleration  $g/2$ , the time period of pendulum will be :

- (1)  $\sqrt{3} T$   
 (2)  $\frac{T}{\sqrt{3}}$   
 (3)  $\sqrt{\frac{3}{2}} T$   
 (4)  $\sqrt{\frac{2}{3}} T$

**Official Ans. by NTA (4)**

**Sol.** When lift is stationary

$$T = 2\pi\sqrt{\frac{L}{g}}$$

When lift is moving upwards

$\Rightarrow$  Pseudo force acts downwards

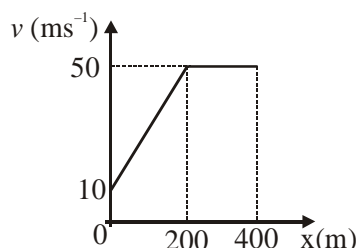
$$\Rightarrow g_{\text{eff}} = g + \frac{g}{2} = \frac{3g}{2}$$

$\Rightarrow$  New time period

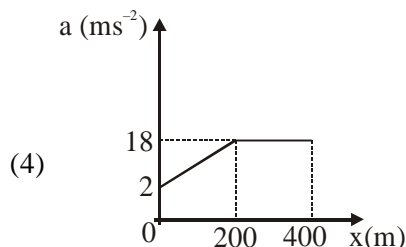
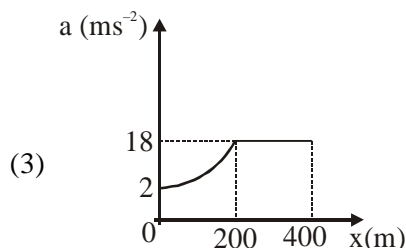
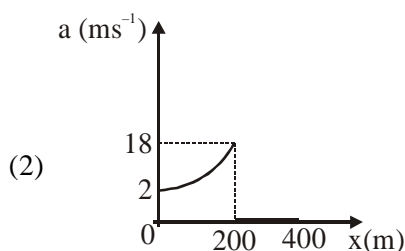
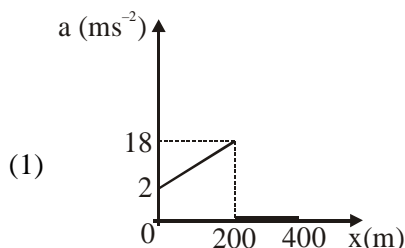
$$T' = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{2L}{3g}}$$

$$T' = \sqrt{\frac{2}{3}} T$$

- 14.** The velocity-displacement graph describing the motion of a bicycle is shown in the figure.



The acceleration-displacement graph of the bicycle's motion is best described by :



**Official Ans. by NTA (1)**

**Official Ans. by ALLEN (1 or Bonus)**

**Sol.** For  $0 \leq x \leq 200$

$$v = mx + C$$

$$v = \frac{1}{5}x + 10$$

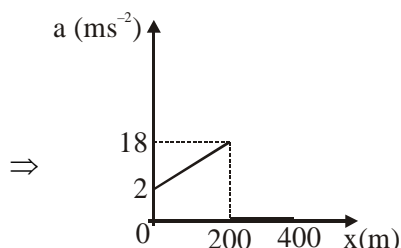
$$a = \frac{v dv}{dx} = \left(\frac{x}{5} + 10\right) \left(\frac{1}{5}\right)$$

$$a = \frac{x}{25} + 2 \Rightarrow \text{Straight line till } x = 200$$

for  $x > 200$

$$v = \text{constant}$$

$$\Rightarrow a = 0$$



Hence most appropriate option will be (1), otherwise it would be BONUS.

- 15.** A 25 m long antenna is mounted on an antenna tower. The height of the antenna tower is 75 m. The wavelength (in meter) of the signal transmitted by this antenna would be :

- (1) 300                      (2) 400  
(3) 200                      (4) 100

**Official Ans. by NTA (4)**

**Sol.** Length of Antenna =  $25\text{m} = \frac{\lambda}{4}$

$$\Rightarrow \lambda = 100\text{m}$$

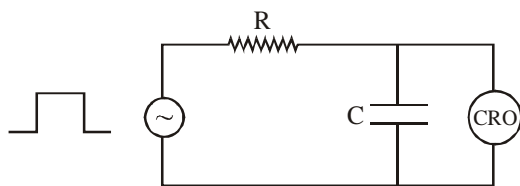
16. For an electromagnetic wave travelling in free space, the relation between average energy densities due to electric ( $U_e$ ) and magnetic ( $U_m$ ) fields is :


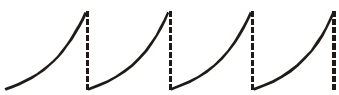

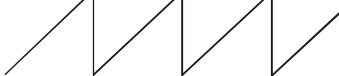
- (1)  $U_e = U_m$  (2)  $U_e > U_m$   
(3)  $U_e < U_m$  (4)  $U_e \neq U_m$

**Official Ans. by NTA (1)**

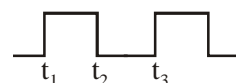
**Sol.** In EMW, Average energy density due to electric ( $U_e$ ) and magnetic ( $U_m$ ) fields is same.

17. An RC circuit as shown in the figure is driven by a AC source generating a square wave. The output wave pattern monitored by CRO would look close to :



- (1)   
(2)   
(3)   
(4) 

**Official Ans. by NTA (3)**

**Sol.** 

For  $t_1 - t_2$  Charging graph  
 $t_2 - t_3$  Discharging graph

18. The stopping potential in the context of photoelectric effect depends on the following property of incident electromagnetic radiation :

- (1) Phase (2) Intensity  
(3) Amplitude (4) Frequency

**Official Ans. by NTA (4)**

**Sol.** Stopping potential changes linearly with frequency of incident radiation.

19. A block of 200 g mass moves with a uniform speed in a horizontal circular groove, with vertical side walls of radius 20 cm. If the block takes 40 s to complete one round, the normal force by the side walls of the groove is :

- (1) 0.0314 N  
(2)  $9.859 \times 10^{-2}$  N  
(3)  $6.28 \times 10^{-3}$  N  
(4)  $9.859 \times 10^{-4}$  N

**Official Ans. by NTA (4)**

**Sol.**  $N = m\omega^2 R$

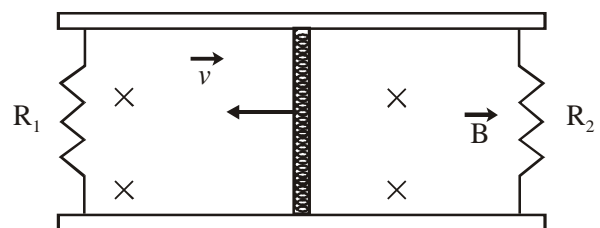
$$N = m \left[ \frac{4\pi^2}{T^2} \right] R \quad \dots(1)$$

Given  $m = 0.2$  kg,  $T = 40$  S,  $R = 0.2$  m

Put values in equation (1)

$$N = 9.859 \times 10^{-4} \text{ N}$$

20. A conducting bar of length  $L$  is free to slide on two parallel conducting rails as shown in the figure



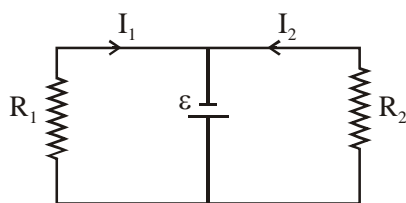
Two resistors  $R_1$  and  $R_2$  are connected across the ends of the rails. There is a uniform magnetic field  $\vec{B}$  pointing into the page. An external agent pulls the bar to the left at a constant speed  $v$ .

The correct statement about the directions of induced currents  $I_1$  and  $I_2$  flowing through  $R_1$  and  $R_2$  respectively is :

- (1) Both  $I_1$  and  $I_2$  are in anticlockwise direction  
(2) Both  $I_1$  and  $I_2$  are in clockwise direction  
(3)  $I_1$  is in clockwise direction and  $I_2$  is in anticlockwise direction  
(4)  $I_1$  is in anticlockwise direction and  $I_2$  is in clockwise direction

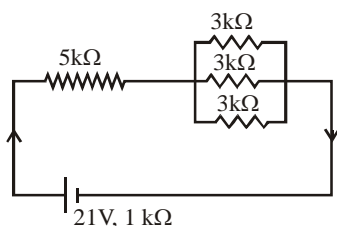
**Official Ans. by NTA (3)**

Sol.



### SECTION-B

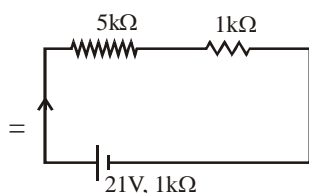
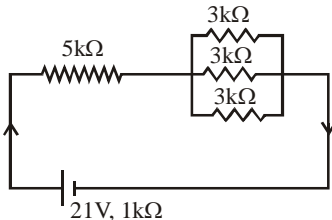
1. In the figure given, the electric current flowing through the  $5\text{ k}\Omega$  resistor is 'x' mA.



The value of x to the nearest integer is \_\_\_\_\_.

**Official Ans. by NTA (3)**

Sol.



$$I = \frac{21}{5+1+1} = 3\text{ mA}$$

2. A fringe width of 6 mm was produced for two slits separated by 1 mm apart. The screen is placed 10 m away. The wavelength of light used is 'x' nm. The value of 'x' to the nearest integer is \_\_\_\_\_.

**Official Ans. by NTA (600)**

Sol.  $\beta = \frac{\lambda D}{d}$

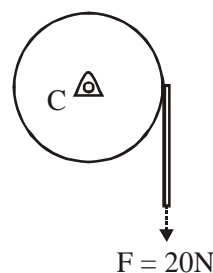
$$\lambda = \frac{\beta d}{D}$$

$$\lambda = \frac{6 \times 10^{-3} \times 10^{-3}}{10}$$

$$\lambda = 6 \times 10^{-7}\text{ m} = 600 \times 10^{-9}\text{ m}$$

$$\lambda = 600\text{ nm}$$

3. Consider a 20 kg uniform circular disk of radius 0.2 m. It is pin supported at its center and is at rest initially. The disk is acted upon by a constant force  $F = 20\text{ N}$  through a massless string wrapped around its periphery as shown in the figure.



Suppose the disk makes n number of revolutions to attain an angular speed of  $50\text{ rad s}^{-1}$ . The value of n, to the nearest integer, is \_\_\_\_\_. [Given : In one complete revolution, the disk rotates by 6.28 rad]

**Official Ans. by NTA (20)**

Sol.  $\alpha = \frac{\tau}{I} = \frac{F.R.}{mR^2/2} = \frac{2F}{mR}$

$$\alpha = \frac{2 \times 200}{20 \times (0.2)} = 10\text{ rad/s}^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$(50)^2 = 0^2 + 2(10)\Delta\theta \Rightarrow \Delta\theta = \frac{2500}{20}$$

$$\Delta\theta = 125\text{ rad}$$

$$\text{No. of revolution} = \frac{125}{2\pi} \approx 20\text{ revolution}$$

4. The first three spectral lines of H-atom in the Balmer series are given  $\lambda_1, \lambda_2, \lambda_3$  considering the Bohr atomic model, the wave lengths of first

and third spectral lines  $\left(\frac{\lambda_1}{\lambda_3}\right)$  are related by a factor of approximately 'x'  $\times 10^{-1}$ . The value of x, to the nearest integer, is \_\_\_\_\_.

**Official Ans. by NTA (15)**

**Sol.** For 1<sup>st</sup> line

$$\frac{1}{\lambda_1} = RZ^2 \left( \frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$\frac{1}{\lambda_1} = RZ^2 \frac{5}{36} \quad \dots(i)$$

For 3<sup>rd</sup> line

$$\frac{1}{\lambda_3} = RZ^2 \left( \frac{1}{2^2} - \frac{1}{5^2} \right)$$

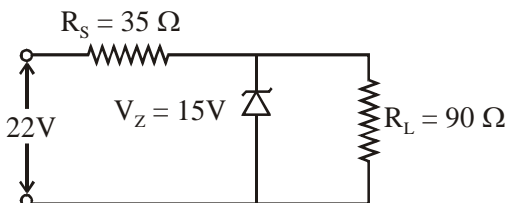
$$\frac{1}{\lambda_3} = RZ^2 \frac{21}{100} \quad \dots(ii)$$

(ii) + (i)

$$\frac{\lambda_1}{\lambda_3} = \frac{21}{100} \times \frac{36}{5} = 1.512 = 15.12 \times 10^{-1}$$

$x \approx 15$

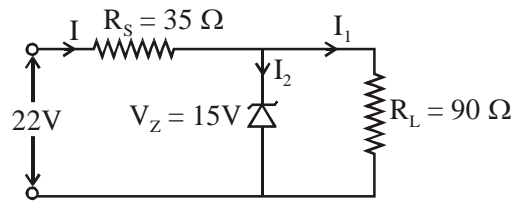
5. The value of power dissipated across the zener diode ( $V_z = 15$  V) connected in the circuit as shown in the figure is  $x \times 10^{-1}$  watt.



The value of x, to the nearest integer, is \_\_\_\_\_.

**Official Ans. by NTA (5)**

**Sol.**



$$\text{Voltage across } R_s = 22 - 15 = 7\text{V}$$

$$\text{Current through } R_s = I = \frac{7}{35} = \frac{1}{5} \text{ A}$$

$$\text{Current through } 90\Omega = I_2 = \frac{15}{90} = \frac{1}{6} \text{ A}$$

$$\text{Current through zener} = \frac{1}{5} - \frac{1}{6} = \frac{1}{30} \text{ A}$$

Power through zener diode

$$P = VI$$

$$P = 15 \times \frac{1}{30} = 0.5 \text{ watt}$$

$$P = 5 \times 10^{-1} \text{ watt}$$

6. A sinusoidal voltage of peak value 250 V is applied to a series LCR circuit, in which  $R = 8\Omega$ ,  $L = 24$  mH and  $C = 60\mu\text{F}$ . The value of power dissipated at resonant condition is 'x' kW. The value of x to the nearest integer is \_\_\_\_\_.

**Official Ans. by NTA (4)**

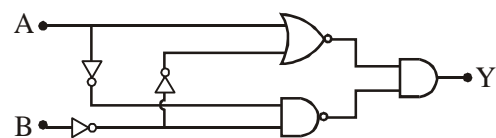
**Sol.** At resonance power (P)

$$P = \frac{(V_{\text{rms}})^2}{R}$$

$$P = \frac{(250/\sqrt{2})^2}{8} = 3906.25 \text{ W}$$

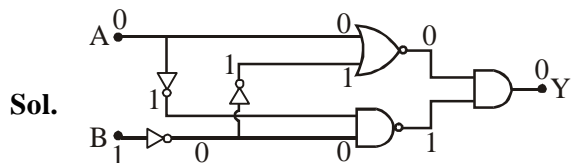
$$\approx 4 \text{ kW}$$

7. In the logic circuit shown in the figure, if input A and B are 0 to 1 respectively, the output at Y would be 'x'. The value of x is \_\_\_\_\_.



**Official Ans. by NTA (0)**





8. The resistance  $R = \frac{V}{I}$ , where  $V = (50 \pm 2)V$  and  $I = (20 \pm 0.2)A$ . The percentage error in  $R$  is 'x' %. The value of 'x' to the nearest integer is \_\_\_\_\_.

**Official Ans. by NTA (5)**

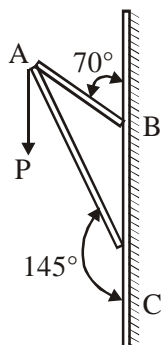
**Sol.**  $\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$

$$\% \text{ error in } R = \frac{2}{50} \times 100 + \frac{0.2}{20} \times 100$$

$$\% \text{ error in } R = 4 + 1$$

$$\% \text{ error in } R = 5\%$$

9. Consider a frame that is made up of two thin massless rods AB and AC as shown in the figure. A vertical force  $\vec{P}$  of magnitude 100 N is applied at point A of the frame.

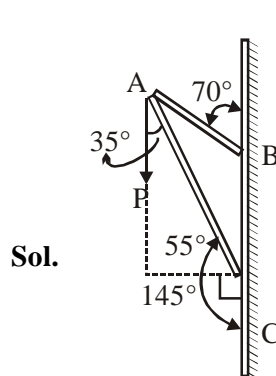


Suppose the force is  $\vec{P}$  resolved parallel to the arms AB and AC of the frame. The magnitude of the resolved component along the arm AC is xN. The value of x, to the nearest integer, is \_\_\_\_\_.

[Given :  $\sin(35^\circ) = 0.573$ ,  $\cos(35^\circ) = 0.819$

$\sin(110^\circ) = 0.939$ ,  $\cos(110^\circ) = -0.342$  ]

**Official Ans. by NTA (82)**



Component along AC

$$= 100 \cos 35^\circ N$$

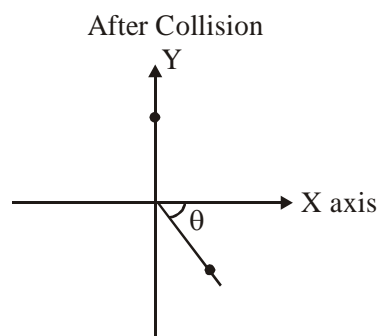
$$= 100 \times 0.819 N$$

$$= 81.9 N$$

$$\approx 82 N$$

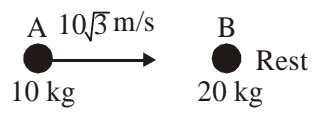
10. A ball of mass 10 kg moving with a velocity  $10\sqrt{3} \text{ ms}^{-1}$  along X-axis, hits another ball of mass 20 kg which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the pieces starts moving along Y-axis at a speed of 10 m/s. The second piece starts moving at a speed of 20 m/s at an angle  $\theta$  (degree) with respect to the X-axis.

The configuration of pieces after collision is shown in the figure. The value of  $\theta$  to the nearest integer is \_\_\_\_\_.

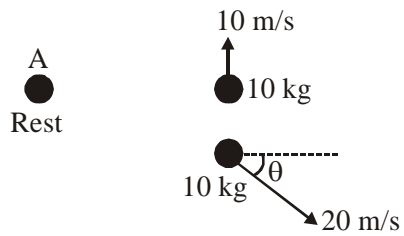


**Official Ans. by NTA (30)**

**Sol. Before Collision**



**After Collision**



From conservation of momentum along x axis;

$$\vec{P}_i = \vec{P}_f$$

$$10 \times 10\sqrt{3} = 200 \cos \theta$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\theta = 30^\circ$$

**FINAL JEE–MAIN EXAMINATION – MARCH, 2021**  
**(Held On Tuesday 16<sup>th</sup> March, 2021) TIME : 9 : 00 AM to 12 : 00 NOON**

**CHEMISTRY**

**SECTION-A**

1. Given below are two statement : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : Size of  $\text{Bk}^{3+}$  ion is less than  $\text{Np}^{3+}$  ion.

Reason R : The above is a consequence of the lanthanoid contraction.

In the light of the above statements, choose the correct answer from the options given below :

- (1) A is false but R is true
- (2) Both A and R are true but R is not the correct explanation of A
- (3) Both A and R are true and R is the correct explanation of A
- (4) A is true but R is false

**Official Ans. by NTA (3)**

**Official Ans. by ALLEN (4)**

- Sol.** Size of  ${}_{97}\text{Bk}^{3+}$  ion is less than that of  ${}_{93}\text{Np}^{3+}$  due to actinoid contraction.

As we know that in a period from left to right ionic radius decreases and in actinide series it is due to actinoid contraction.

2. Which among the following pairs of Vitamins is stored in our body relatively for longer duration ?

- (1) Thiamine and Vitamin A
- (2) Vitamin A and Vitamin D
- (3) Thiamine and Ascorbic acid
- (4) Ascorbic acid and Vitamin D

**Official Ans. by NTA (2)**

- Sol.** Vitamin-A & Vitamin-D

3. Given below are two statements :

Statement I : Both  $\text{CaCl}_2 \cdot 6\text{H}_2\text{O}$  and  $\text{MgCl}_2 \cdot 8\text{H}_2\text{O}$  undergo dehydration on heating.

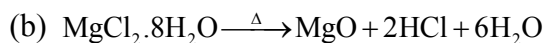
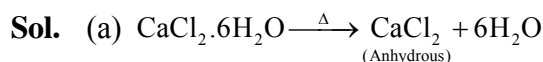
Statement II :  $\text{BeO}$  is amphoteric whereas the oxides of other elements in the same group are acidic.

In the light of the above statements, choose the correct answer from the options given below :

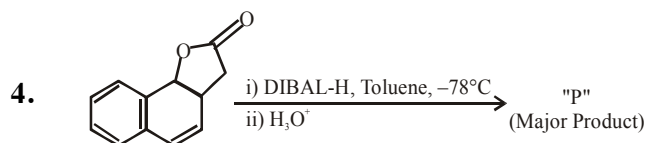
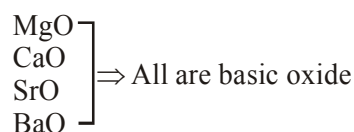
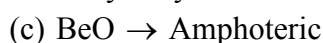
- (1) Statement I is false but statement II is true
- (2) Both statement I and statement II are false
- (3) Both statement I and statement II are true
- (4) Statement I is true but statement II is false

**TEST PAPER WITH ANSWER & SOLUTION**

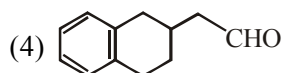
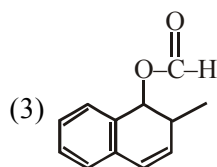
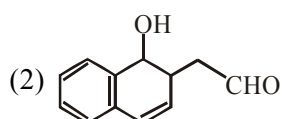
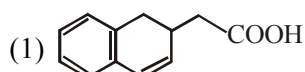
**Official Ans. by NTA (2)**



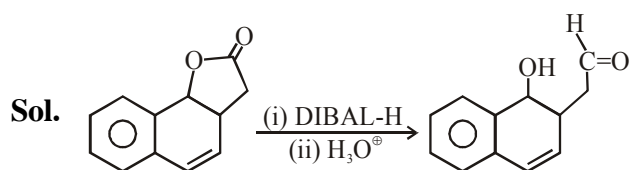
The dehydration of hydrated chloride of calcium can be achieved. The corresponding hydrated chloride of magnesium on heating suffer hydrolysis.



The product "P" in the above reaction is :



**Official Ans. by NTA (2)**



DIBAL can not reduce double bond  
 It can reduce cyclic ester.

5. Match List-I with List-II :

List-I	List-II
Industrial process	Application
(a) Haber's process	(i) $\text{HNO}_3$ synthesis
(b) Ostwald's process	(ii) Aluminium extraction
(c) Contact process	(iii) $\text{NH}_3$ synthesis
(d) Hall-Heroult process	(iv) $\text{H}_2\text{SO}_4$ synthesis

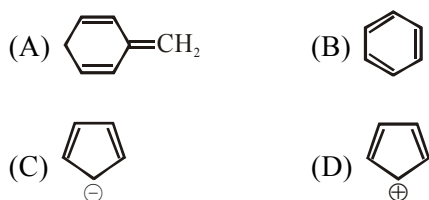
Choose the correct answer from the options given below :

- (1) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)  
 (2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)  
 (3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)  
 (4) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

**Official Ans. by NTA (3)**

- Sol.** (a) Haber's process is used for  $\text{NH}_3$  synthesis.  
 (b) Ostwald's process is used for  $\text{HNO}_3$  synthesis.  
 (c) Contact process is used for  $\text{H}_2\text{SO}_4$  synthesis.  
 (d) In Hall-Heroult process, electrolytic reduction of impure alumina can be done. (Aluminium extraction)

6. Among the following, the aromatic compounds are :

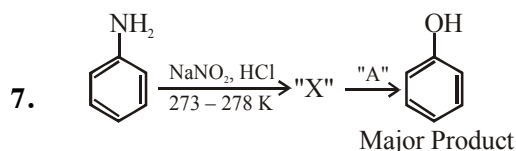


Choose the correct answer from the following options :

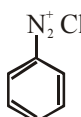
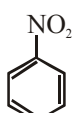
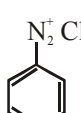
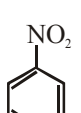
- (1) (A) and (B) only  
 (2) (B) and (C) only  
 (3) (B), (C) and (D) only  
 (4) (A), (B) and (C) only

**Official Ans. by NTA (2)**

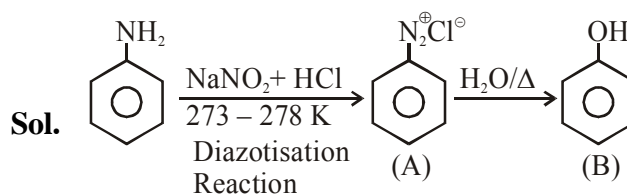
- Sol.** (A) Non-Aromatic (B) Aromatic  
 (C) Aromatic (D) Anti-Aromatic



In the above chemical reaction, intermediate "X" and reagent/condition "A" are :

- (1) X-  ; A-  $\text{H}_2\text{O}/\text{NaOH}$   
 (2) X-  ; A-  $\text{H}_2\text{O}/\Delta$   
 (3) X-  ; A-  $\text{H}_2\text{O}/\Delta$   
 (4) X-  ; A-  $\text{H}_2\text{O}/\text{NaOH}$

**Official Ans. by NTA (3)**



8. Given below are two statements :

Statement I : The  $E^\circ$  value of  $\text{Ce}^{4+} / \text{Ce}^{3+}$  is + 1.74 V.

Statement II : Ce is more stable in  $\text{Ce}^{4+}$  state than  $\text{Ce}^{3+}$  state.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (1) Both statement I and statement II are correct  
 (2) Statement I is incorrect but statement II is correct  
 (3) Both statement I and statement II are incorrect  
 (4) Statement I is correct but statement II is incorrect

**Official Ans. by NTA (4)**

**Sol.** The  $E^\circ$  value for  $Ce^{4+}/Ce^{3+}$  is +1.74 V because the most stable oxidation state of lanthanide series elements is +3.  
It means  $Ce^{3+}$  is more stable than  $Ce^{4+}$ .

9. The functions of antihistamine are :

- (1) Antiallergic and Analgesic
- (2) Antacid and antiallergic
- (3) Analgesic and antacid
- (4) Antiallergic and antidepressant

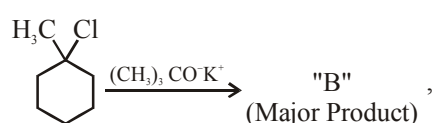
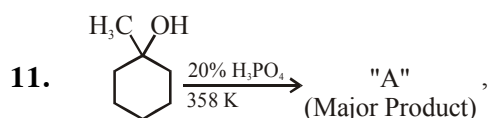
**Official Ans. by NTA (2)**

10. Which of the following is Lindlar catalyst ?

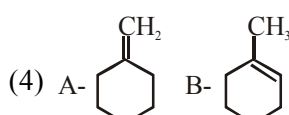
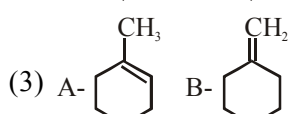
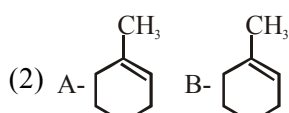
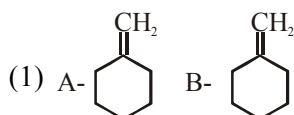
- (1) Zinc chloride and HCl
- (2) Cold dilute solution of  $KMnO_4$
- (3) Sodium and Liquid  $NH_3$
- (4) Partially deactivated palladised charcoal

**Official Ans. by NTA (4)**

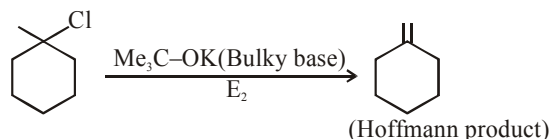
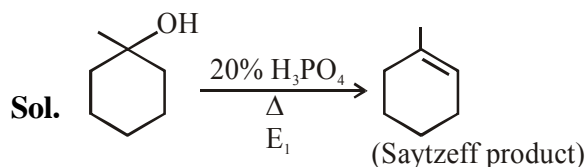
**Sol.** Partially deactivated palladised charcoal ( $H_2/pd/CaCO_3$ ) is lindlar catalyst.



The product "A" and "B" formed in above reactions are :



**Official Ans. by NTA (3)**



12. Given below are two statements :

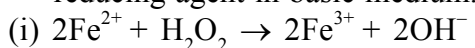
Statement I :  $H_2O_2$  can act as both oxidising and reducing agent in basic medium.

Statement II : In the hydrogen economy, the energy is transmitted in the form of dihydrogen. In the light of the above statements, choose the correct answer from the options given below :

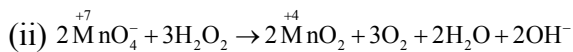
- (1) Both statement I and statement II are false
- (2) Both statement I and statement II are true
- (3) Statement I is true but statement II is false
- (4) Statement I is false but statement II is true

**Official Ans. by NTA (2)**

**Sol.** (a)  $H_2O_2$  can acts as both oxidising and reducing agent in basic medium.



In this reaction,  $H_2O_2$  acts as oxidising agent.



In this reaction,  $H_2O_2$  acts as reducing agent.

(b) The basic principle of hydrogen economy is the transportation and storage of energy in the form of liquids or gaseous dihydrogen.

Advantage of hydrogen economy is that energy is transmitted in the form of dihydrogen and not as electric power

13. The type of pollution that gets increased during the day time and in the presence of  $O_3$  is :

- (1) Reducing smog
- (2) Oxidising smog
- (3) Global warming
- (4) Acid rain

**Official Ans. by NTA (2)**

**Sol.** In presence of ozone( $O_3$ ), oxidising smog gets increased during the day time because automobiles and factories produce main components of the photochemical smog (oxidising smog) results from the action of sunlight on unsaturated hydrocarbon and nitrogen oxide.

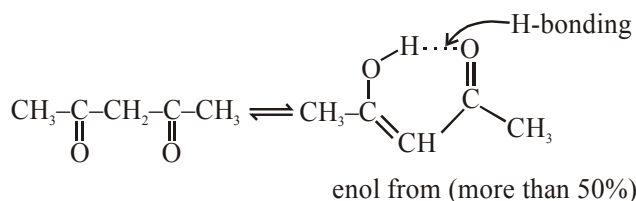
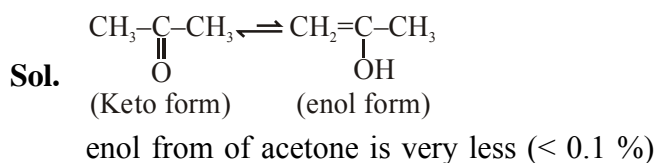
Ozone is strong oxidising agent and can react with the unburnt hydrocarbons in the polluted air to produce chemicals.

14. Assertion A : Enol form of acetone [ $\text{CH}_3\text{COCH}_3$ ] exists in  $< 0.1\%$  quantity. However, the enol form of acetyl acetone [ $\text{CH}_3\text{COCH}_2\text{OCCH}_3$ ] exists in approximately 15% quantity.

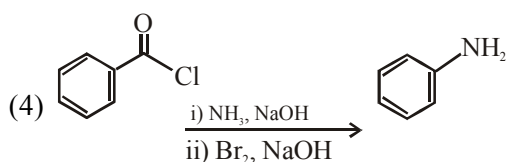
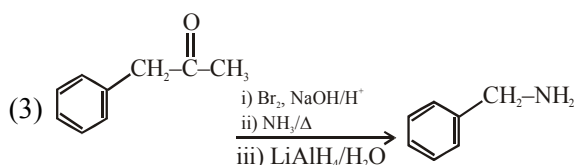
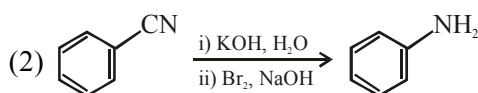
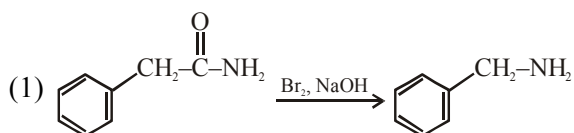
Reason R : enol form of acetyl acetone is stabilized by intramolecular hydrogen bonding, which is not possible in enol form of acetone. Choose the correct statement :

- (1) A is false but R is true
- (2) Both A and R are true and R is the correct explanation of A
- (3) Both A and R are true but R is not the correct explanation of A
- (4) A is true but R is false

**Official Ans. by NTA (2)**

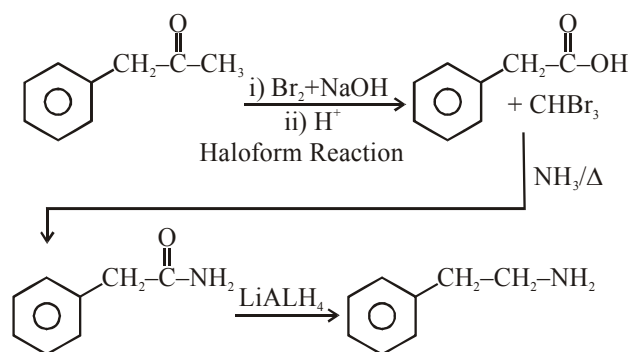


15. Which of the following reaction DOES NOT involve Hoffmann Bromamide degradation ?



**Official Ans. by NTA (3)**

**Sol.**



$\Rightarrow$  This reaction does not involve haffmann bromanide degradation.

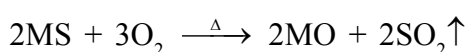
$\Rightarrow$  Rest all options involve haffmann bromamide degradation during the reaction of  $\text{Br}_2 + \text{NaOH}$  with amide.

16. The process that involves the removal of sulphur from the ores is :

- (1) Smelting
- (2) Roasting
- (3) Leaching
- (4) Refining

**Official Ans. by NTA (2)**

**Sol.** In roasting process, metal sulphide (MS) ore are converted into metal oxide and sulphur is remove in the form of  $\text{SO}_2$  gas.



17. Match List-I with List-II :

**List-I**

**List-II**

Name of oxo acid      Oxidation state of 'P'

- |                           |          |
|---------------------------|----------|
| (a) Hypophosphorous acid  | (i) +5   |
| (b) Orthophosphoric acid  | (ii) +4  |
| (c) Hypophosphoric acid   | (iii) +3 |
| (d) Orthophosphorous acid | (iv) +2  |
|                           | (v) +1   |

Choose the correct answer from the options given below :

- (1) (a)-(v), (b)-(i), (c)-(ii), (d)-(iii)
- (2) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)
- (3) (a)-(iv), (b)-(v), (c)-(ii), (d)-(iii)
- (4) (a)-(v), (b)-(iv), (c)-(ii), (d)-(iii)

**Official Ans. by NTA (1)**

**Sol.** (a) Hypophosphorus acid :  $\text{H}_3\text{PO}_2$

$$(+1) 3 + x + (-2)2 = 0$$

$$x = +1$$

(b) Orthophosphoric acid :  $\text{H}_3\text{PO}_4$

$$(+1) 3 + x + (-2)4 = 0$$

$$x = +5$$

(c) Hypophosphoric acid :  $\text{H}_4\text{P}_2\text{O}_6$

$$(+1) 4 + 2x + (-2)6 = 0$$

$$x = +4$$

(d) Orthophosphorous acid :  $\text{H}_3\text{PO}_3$

$$(+1)3 + x + (-2)3 = 0$$

$$x = +3$$

**18.** Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R :

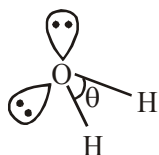
**Assertion A :** The H–O–H bond angle in water molecule is  $104.5^\circ$ .

**Reason R :** The lone pair – lone pair repulsion of electrons is higher than the bond pair - bond pair repulsion.

- (1) A is false but R is true
- (2) Both A and R are true, but R is not the correct correct explanation of A
- (3) A is true but R is false
- (4) Both A and R are true, and R is the correct explanation of A

**Official Ans. by NTA (4)**

**Sol.**  $\text{H}_2\text{O}$



$$\theta = 104.5^\circ$$

the hybridisation of oxygen in water molecule is  $\text{sp}^3$ .

So electron geometry of water molecule is tetrahedral and the bond angle should be  $109^\circ 28'$  but as we know that lone pair-lone pair repulsion of electrons is higher than the bond pair-bond pair repulsion because lone pair is occupied more space around central atom than that of bond pair.

**19.** In chromatography technique, the purification of compound is independent of :

- (1) Mobility or flow of solvent system
- (2) Solubility of the compound
- (3) Length of the column or TLC Plate
- (4) Physical state of the pure compound

**Official Ans. by NTA (4)**

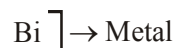
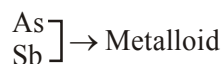
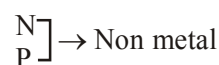
**Sol.** In chromatography technique, the purification of a compound is independent of the physical state of the pure compound.

**20.** A group 15 element, which is a metal and forms a hydride with strongest reducing power among group 15 hydrides. The element is :

- (1) Sb (2) P (3) As (4) Bi

**Official Ans. by NTA (4)**

**Sol.** In group 15



Hydrides of group 15 elements are



In  $\text{NH}_3$ , hydrogen atom gets partial positive charge due to less electronegativity.

But in  $\text{BiH}_3$ , hydrogen atom gets partial negative charge because hydrogen is more electronegative than bismuth.

i.e.  $\text{BiH}_3$  is a strong reducing agent than others because we know that  $\text{H}^-$  is a strong reducing agent.

### SECTION-B

**1.** For the reaction  $\text{A(g)} \rightleftharpoons \text{B(g)}$  at 495 K,  $\Delta_r G^\circ = -9.478 \text{ kJ mol}^{-1}$ .

If we start the reaction in a closed container at 495 K with 22 millimoles of A, the amount of B is the equilibrium mixture is \_\_\_\_\_ millimoles. (Round off to the Nearest Integer).

$$[R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}; \ln 10 = 2.303]$$

**Official Ans. by NTA (20)**

**Sol.**  $\Delta G^\circ = -RT \ln K_{eq}$

Given  $\Delta G^\circ = -9.478 \text{ KJ/mole}$

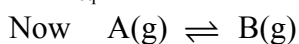
$T = 495\text{K} \quad R = 8.314 \text{ J mol}^{-1}$

So  $-9.478 \times 10^3 = -495 \times 8.314 \times \ln K_{eq}$

$\ln K_{eq} = 2.303$

$= \ln 10$

So  $K_{eq} = 10$



$t = 0 \quad 22 \quad 0$

$t = t \quad 22-x \quad x$

$K_{eq} = \frac{[B]}{[C]} = \frac{x}{22-x} = 10$

or  $x = 20$

So millmoles of B = 20

2. Complete combustion of 750 g of an organic compound provides 420 g of  $\text{CO}_2$  and 210 g of  $\text{H}_2\text{O}$ . The percentage composition of carbon and hydrogen in organic compound is 15.3 and \_\_\_\_\_ respectively. (Round off to the Nearest Integer)

**Official Ans. by NTA (3)**

**Sol.** 44 gm  $\text{CO}_2$  have 12 gm carbon

So, 420 gm  $\text{CO}_2 \Rightarrow \frac{12}{44} \times 420$

$\Rightarrow \frac{1260}{11} \text{ gm carbon}$

$\Rightarrow 114.545 \text{ gram carbon}$

So, % of carbon =  $\frac{114.545}{750} \times 100$

$\approx 15.3\%$

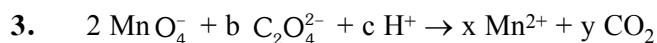
18 gm  $\text{H}_2\text{O} \Rightarrow 2 \text{ gm H}_2$

210 gm  $\Rightarrow \frac{2}{18} \times 210$

$= 23.33 \text{ gm H}_2$

So, %  $\text{H}_2 \Rightarrow \frac{23.33}{750} \times 100 = 3.11\%$

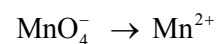
$\approx 3\%$



If the above equation is balanced with integer coefficients, the value of c is \_\_\_\_\_. (Round off to the Nearest Integer).

**Official Ans. by NTA (16)**

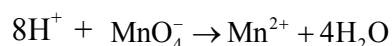
**Sol.** Writing the half reaction  
oxidation half reaction



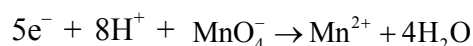
balancing oxygen



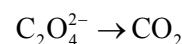
balancing Hydrogen



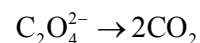
balancing charge



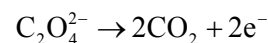
Reduction half



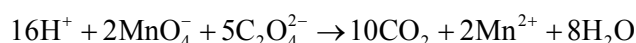
Balancing carbon



Balancing charge



Net equation

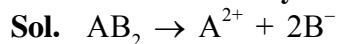


So  $c = 16$

4.  $\text{AB}_2$  is 10% dissociated in water to  $\text{A}^{2+}$  and  $\text{B}^-$ . The boiling point of a 10.0 molal aqueous solution of  $\text{AB}_2$  is \_\_\_\_\_ $^\circ\text{C}$ . (Round off to the Nearest Integer).

[Given : Molal elevation constant of water  $K_b = 0.5 \text{ K kg mol}^{-1}$  boiling point of pure water =  $100^\circ\text{C}$ ]

**Official Ans. by NTA (106)**



$t = 0 \quad a \quad 0 \quad 0$

$t = t \quad a - a\alpha \quad a\alpha \quad 2a\alpha$

$n_T = a - a\alpha + a\alpha + 2a\alpha$

$= a(1 + 2\alpha)$

so  $i = 1 + 2\alpha$

Now  $\Delta T_b = i \times m \times K_b$

$\Delta T_b = (1 + 2\alpha) \times m \times K_b$

$\alpha = 0.1 \quad m = 10 \quad K_b = 0.5$

$\Delta T_b = 1.2 \times 10 \times 0.5$

$= 6$

So boiling point = 106

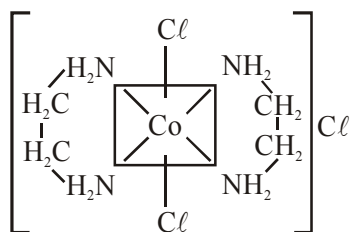
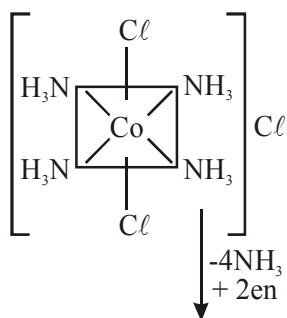


5. The equivalents of ethylene diamine required to replace the neutral ligands from the coordination sphere of the trans-complex of  $\text{CoCl}_3 \cdot 4\text{NH}_3$  is \_\_\_\_\_. (Round off to the Nearest Integer).

**Official Ans. by NTA (2)**

**Sol.** trans -  $\text{CoCl}_3 \cdot 4\text{NH}_3$

or



As we know that ethylene diamine is a bidentate ligand and ammonia is a monodentate ligand.

It means overall two ethylene diamine is required to replace all neutral ligands (four ammonia) from the coordination sphere of this complex.

6. A 6.50 molal solution of  $\text{KOH}$  (aq.) has a density of  $1.89 \text{ g cm}^{-3}$ . The molarity of the solution is \_\_\_\_\_  $\text{mol dm}^{-3}$ . (Round off to the Nearest Integer).

[Atomic masses: K :39.0 u; O :16.0 u; H :1.0 u]

**Official Ans. by NTA (9)**

**Sol.** 6.5 molal  $\text{KOH}$  = 1000gm solvent has 6.5 moles  $\text{KOH}$

so wt of solute =  $6.5 \times 56$

= 364 gm

wt of solution =  $1000 + 364 = 1364$

$$\text{Volume of solution} = \frac{1364}{1.89} \text{ ml}$$

$$\begin{aligned} \text{Molarity} &= \frac{\text{mole of solute}}{V_{\text{solution in Litre}}} \\ &= \frac{6.5 \times 1.89 \times 1000}{1364} \\ &= 9.00 \end{aligned}$$

7. When light of wavelength 248 nm falls on a metal of threshold energy 3.0 eV, the de-Broglie wavelength of emitted electrons is \_\_\_\_\_ Å. (Round off to the Nearest Integer).

[Use :  $\sqrt{3} = 1.73$ ,  $h = 6.63 \times 10^{-34} \text{ Js}$

$m_e = 9.1 \times 10^{-31} \text{ kg}$  ;  $c = 3.0 \times 10^8 \text{ ms}^{-1}$  ;

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ ]

**Official Ans. by NTA (9)**

**Sol.** Energy incident =  $\frac{hc}{\lambda}$

$$= \frac{6.63 \times 10^{-34} \times 3.0 \times 10^8}{248 \times 10^{-9} \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= \frac{6.63 \times 3 \times 100}{248 \times 1.6}$$

$$= 0.05 \text{ eV} \times 100 = 5 \text{ eV}$$

Now using

$$E = \phi + \text{K.E.}$$

$$5 = 3 + \text{K.E.}$$

$$\text{K.E.} = 2 \text{ eV} = 3.2 \times 10^{-19} \text{ J}$$

$$\text{for de Broglie wavelength } \lambda = \frac{h}{mv}$$

$$\text{K.E.} = \frac{1}{2}mv^2$$

$$\text{so } v = \sqrt{\frac{2\text{K.E.}}{m}}$$

$$\text{hence } \lambda = \frac{h}{\sqrt{2\text{K.E.} \times m}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 3.2 \times 10^{-19} \times 9.1 \times 10^{-31}}}$$

$$= \frac{6.63}{7.6} \times \frac{10^{-34}}{10^{-25}} = \frac{66.3 \times 10^{-10} \text{ m}}{7.6}$$

$$= 8.72 \times 10^{-10} \text{ m}$$

$$\approx 9 \times 10^{-10} \text{ m}$$

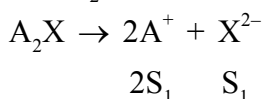
$$= 9 \text{ Å}$$

8. Two salts  $A_2X$  and  $MX$  have the same value of solubility product of  $4.0 \times 10^{-12}$ . The ratio of their molar solubilities i.e.  $\frac{S(A_2X)}{S(MX)} = \underline{\hspace{2cm}}$ .

(Round off to the Nearest Integer).

**Official Ans. by NTA (50)**

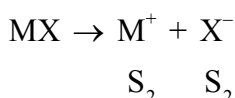
**Sol.** For  $A_2X$



$$K_{sp} = 4S_1^3 = 4 \times 10^{-12}$$

$$S_1 = 10^{-4}$$

for  $MX$



$$K_{sp} = S_2^2 = 4 \times 10^{-12}$$

$$S_2 = 2 \times 10^{-6}$$

$$\text{so } \frac{S_{A_2X}}{S_{MX}} = \frac{10^{-4}}{2 \times 10^{-6}} = 50$$

9. A certain element crystallises in a bcc lattice of unit cell edge length  $27 \text{ \AA}$ . If the same element under the same conditions crystallises in the fcc lattice, the edge length of the unit cell in  $\text{\AA}$  will be  $\underline{\hspace{2cm}}$ . (Round off to the Nearest Integer).

[Assume each lattice point has a single atom]

[Assume  $\sqrt{3} = 1.73$ ,  $\sqrt{2} = 1.41$ ]

**Official Ans. by NTA (33)**

**Sol.** For BCC  $\sqrt{3} a = 4r$

$$\text{so } r = \frac{\sqrt{3}}{4} \times 27$$

for FCC  $a = 2\sqrt{2} r$

$$= 2 \times \sqrt{2} \times \frac{\sqrt{3}}{4} \times 27$$

$$= \frac{\sqrt{3}}{\sqrt{2}} \times 27$$

$$= 33$$

10. The decomposition of formic acid on gold surface follows first order kinetics. If the rate constant at  $300 \text{ K}$  is  $1.0 \times 10^{-3} \text{ s}^{-1}$  and the activation energy  $E_a = 11.488 \text{ kJ mol}^{-1}$ , the rate constant at  $200 \text{ K}$  is  $\underline{\hspace{2cm}} \times 10^{-5} \text{ s}^{-1}$ . (Round off to the Nearest Integer).

(Given :  $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$ )

**Official Ans. by NTA (10)**

**Sol.**  $K_{300} = 10^{-4}$   $K_{200} = ?$

$$E_a = 11.488 \text{ KJ/mole} \quad R = 8.314 \text{ J/mole-K}$$

$$\text{so } \ln\left(\frac{K_{300}}{K_{200}}\right) = \frac{E_a}{R} \left(\frac{1}{200} - \frac{1}{300}\right)$$

$$\ln\left(\frac{K_{300}}{K_{200}}\right) = \frac{11.488 \times 1000 \times 100}{8.314 \times 200 \times 300}$$

$$= 2.303$$

$$= \ln 10$$

$$\text{so } \frac{K_{300}}{K_{200}} = 10$$

$$K_{200} = \frac{1}{10} \times K_{300} = 10^{-4}$$

$$= 10 \times 10^{-5} \text{ sec}^{-1}$$

# FINAL JEE-MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16<sup>th</sup> March, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

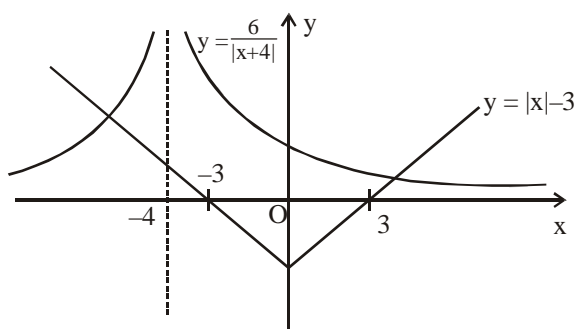
## MATHEMATICS

### SECTION-A

1. The number of elements in the set  $\{x \in \mathbb{R} : (|x| - 3)|x + 4| = 6\}$  is equal to  
 (1) 3 (2) 2 (3) 4 (4) 1

**Official Ans. by NTA (2)**

**Sol.**  $x \neq -4$   
 $(|x| - 3)(|x + 4|) = 6$   
 $\Rightarrow |x| - 3 = \frac{6}{|x + 4|}$

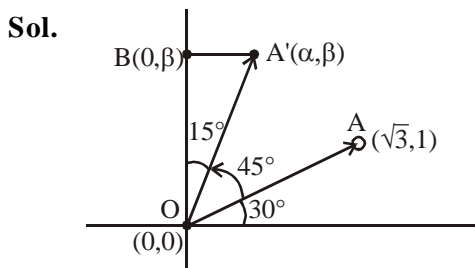


No. of solutions = 2

2. Let a vector  $\alpha\hat{i} + \beta\hat{j}$  be obtained by rotating the vector  $\sqrt{3}\hat{i} + \hat{j}$  by an angle  $45^\circ$  about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices  $(\alpha, \beta)$ ,  $(0, \beta)$  and  $(0, 0)$  is equal to

- (1)  $\frac{1}{2}$  (2) 1 (3)  $\frac{1}{\sqrt{2}}$  (4)  $2\sqrt{2}$

**Official Ans. by NTA (1)**



$$\begin{aligned} \text{Area of } \triangle(OA'B) &= \frac{1}{2} OA' \cos 15^\circ \times OA' \sin 15^\circ \\ &= \frac{1}{2} (OA')^2 \frac{\sin 30^\circ}{2} \\ &= (3+1) \times \frac{1}{8} = \frac{1}{2} \end{aligned}$$

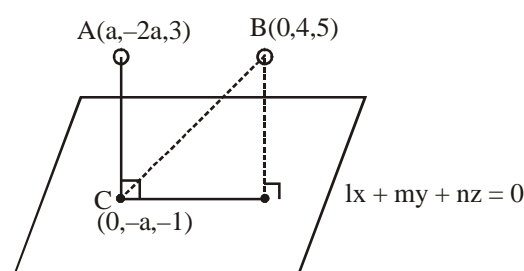
## TEST PAPER WITH SOLUTION

3. If for  $a > 0$ , the feet of perpendiculars from the points  $A(a, -2a, 3)$  and  $B(0, 4, 5)$  on the plane  $lx + my + nz = 0$  are points  $C(0, -a, -1)$  and  $D$  respectively, then the length of line segment  $CD$  is equal to :

- (1)  $\sqrt{31}$  (2)  $\sqrt{41}$   
 (3)  $\sqrt{55}$  (4)  $\sqrt{66}$

**Official Ans. by NTA (4)**

**Sol.**



$$C \text{ lies on plane} \Rightarrow -ma - n = 0 \Rightarrow \frac{m}{n} = -\frac{1}{a} \dots (1)$$

$$\vec{CA} \parallel l\hat{i} + m\hat{j} + n\hat{k}$$

$$\frac{a-0}{l} = \frac{-a}{m} = \frac{4}{n} \Rightarrow \frac{m}{n} = -\frac{a}{4} \dots (2)$$

From (1) & (2)

$$-\frac{1}{a} = \frac{-a}{4} \Rightarrow a^2 = 4 \Rightarrow a = 2 \quad (\text{since } a > 0)$$

$$\text{From (2)} \quad \frac{m}{n} = \frac{-1}{2}$$

$$\text{Let } m = -t \Rightarrow n = 2t$$

$$\frac{2}{l} = \frac{-2}{-t} \Rightarrow l = t$$

$$\text{So plane : } t(x - y + 2z) = 0$$

$$BD = \frac{6}{\sqrt{6}} = \sqrt{6} \quad C \equiv (0, -2, -1)$$

$$\begin{aligned} CD &= \sqrt{BC^2 - BD^2} \\ &= \sqrt{(0^2 + 6^2 + 6^2) - (\sqrt{6})^2} \\ &= \sqrt{66} \end{aligned}$$

4. Consider three observations a, b and c such that  $b = a + c$ . If the standard deviation of  $a + 2$ ,  $b + 2$ ,  $c + 2$  is d, then which of the following is true ?

- (1)  $b^2 = 3(a^2 + c^2) + 9d^2$   
 (2)  $b^2 = a^2 + c^2 + 3d^2$   
 (3)  $b^2 = 3(a^2 + c^2 + d^2)$   
 (4)  $b^2 = 3(a^2 + c^2) - 9d^2$

**Official Ans. by NTA (4)**

**Sol.** For a, b, c

$$\text{mean} = \frac{a+b+c}{3} (= \bar{x})$$

$$b = a + c$$

$$\Rightarrow \bar{x} = \frac{2b}{3} \quad \dots(1)$$

$$\text{S.D. } (a + 2, b + 2, c + 2) = \text{S.D. } (a, b, c) = d$$

$$\Rightarrow d^2 = \frac{a^2 + b^2 + c^2}{3} - (\bar{x})^2$$

$$\Rightarrow d^2 = \frac{a^2 + b^2 + c^2}{3} - \frac{4b^2}{9}$$

$$\Rightarrow 9d^2 = 3(a^2 + b^2 + c^2) - 4b^2$$

$$\Rightarrow b^2 = 3(a^2 + c^2) - 9d^2$$

5. If for  $x \in \left(0, \frac{\pi}{2}\right)$ ,  $\log_{10} \sin x + \log_{10} \cos x = -1$

$$\text{and } \log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1), n > 0,$$

then the value of n is equal to :

- (1) 20      (2) 12      (3) 9      (4) 16

**Official Ans. by NTA (2)**

**Sol.**  $x \in \left(0, \frac{\pi}{2}\right)$

$$\log_{10} \sin x + \log_{10} \cos x = -1$$

$$\Rightarrow \log_{10} \sin x \cdot \cos x = -1$$

$$\Rightarrow \sin x \cdot \cos x = \frac{1}{10} \quad \dots(1)$$

$$\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$$

$$\Rightarrow \sin x + \cos x = 10^{\left(\log_{10} \sqrt{n} - \frac{1}{2}\right)} = \sqrt{\frac{n}{10}}$$

by squaring

$$1 + 2\sin x \cdot \cos x = \frac{n}{10}$$

$$\Rightarrow 1 + \frac{1}{5} = \frac{n}{10} \Rightarrow n = 12$$

6. Let  $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ ,  $i = \sqrt{-1}$ . Then, the system of

linear equations  $A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$  has :

- (1) A unique solution  
 (2) Infinitely many solutions  
 (3) No solution  
 (4) Exactly two solutions

**Official Ans. by NTA (3)**

**Sol.**  $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

$$A^2 = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = 2 \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$A^4 = 2^2 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 8 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A^8 = 64 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 128 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$$\Rightarrow 128 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$$\Rightarrow 128 \begin{bmatrix} x-y \\ -x+y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$$\Rightarrow x-y = \frac{1}{16} \quad \dots(1)$$

$$\& \quad -x+y = \frac{1}{2} \quad \dots(2)$$

$\Rightarrow$  From (1) & (2) : No solution.

7. If the three normals drawn to the parabola,  $y^2 = 2x$  pass through the point (a, 0)  $a \neq 0$ , then 'a' must be greater than :

- (1)  $\frac{1}{2}$       (2)  $-\frac{1}{2}$       (3) -1      (4) 1

**Official Ans. by NTA (4)**

**Sol.** For standard parabola

For more than 3 normals (on axis)

$$x > \frac{L}{2} \quad (\text{where } L \text{ is length of L.R.})$$

For  $y^2 = 2x$

L.R. = 2

for  $(a, 0)$

$$a > \frac{\text{L.R.}}{2} \Rightarrow a > 1$$

8. Let the position vectors of two points P and Q be  $3\hat{i} - \hat{j} + 2\hat{k}$  and  $\hat{i} + 2\hat{j} - 4\hat{k}$ , respectively. Let R and S be two points such that the direction ratios of lines PR and QS are  $(4, -1, 2)$  and  $(-2, 1, -2)$ , respectively. Let lines PR and QS intersect at T. If the vector  $\overrightarrow{TA}$  is perpendicular to both  $\overrightarrow{PR}$  and  $\overrightarrow{QS}$  and the length of vector  $\overrightarrow{TA}$  is  $\sqrt{5}$  units, then the modulus of a position vector of A is :

(1)  $\sqrt{482}$  (2)  $\sqrt{171}$

(3)  $\sqrt{5}$  (4)  $\sqrt{227}$

**Official Ans. by NTA (2)**

**Sol.** P(3, -1, 2)

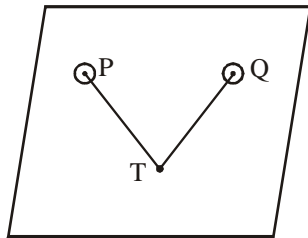
Q(1, 2, -4)

$$\overrightarrow{PR} \parallel 4\hat{i} - \hat{j} + 2\hat{k}$$

$$\overrightarrow{QS} \parallel -2\hat{i} + \hat{j} - 2\hat{k}$$

dr's of normal to the plane containing P, T & Q will be proportional to :

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 2 \\ -2 & 1 & -2 \end{vmatrix}$$



$$\therefore \frac{\ell}{0} = \frac{m}{4} = \frac{n}{2}$$

For point, T :  $\overrightarrow{PT} = \frac{x-3}{4} = \frac{y+1}{-1} = \frac{z-2}{2} = \lambda$

$$\overrightarrow{QT} = \frac{x-1}{-2} = \frac{y-1}{1} = \frac{z+4}{-2} = \mu$$

$$T : (4\lambda + 3, -\lambda - 1, 2\lambda + 2)$$

$$\cong (2\mu + 1, \mu + 2, -2\mu - 4)$$

$$4\lambda + 3 = -2\mu + 1 \Rightarrow 2\lambda + \mu = -1$$

$$\lambda + \mu = -3 \Rightarrow \lambda = 2$$

$$\& \mu = -5 \quad \lambda + \mu = -3 \Rightarrow \lambda = 2$$

$$\text{So point T : } (11, -3, 6)$$

$$\overrightarrow{OA} = (11\hat{i} - 3\hat{j} + 6\hat{k}) \pm \left( \frac{2\hat{j} + \hat{k}}{\sqrt{5}} \right) \sqrt{5}$$

$$\overrightarrow{OA} = (11\hat{i} - 3\hat{j} + 6\hat{k}) \pm (2\hat{j} + \hat{k})$$

$$\overrightarrow{OA} = 11\hat{i} - \hat{j} + 7\hat{k}$$

or

$$9\hat{i} - 5\hat{j} + 5\hat{k}$$

$$|\overrightarrow{OA}| = \sqrt{121 + 1 + 49} = \sqrt{171}$$

or

$$\sqrt{81 + 25 + 25} = \sqrt{131}$$

9. Let the functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  be defined as :

$$f(x) = \begin{cases} x+2, & x < 0 \\ x^2, & x \geq 0 \end{cases} \text{ and } g(x) = \begin{cases} x^3, & x < 1 \\ 3x-2, & x \geq 1 \end{cases}$$

Then, the number of points in  $\mathbb{R}$  where  $(f \circ g)(x)$  is NOT differentiable is equal to :

(1) 3 (2) 1

(3) 0 (4) 2

**Official Ans. by NTA (2)**

**Sol.**  $f(g(x)) = \begin{cases} g(x)+2, & g(x) < 0 \\ (g(x))^2, & g(x) \geq 0 \end{cases}$

$$= \begin{cases} x^3+2, & x < 0 \\ x^6, & x \in [0, 1) \\ (3x-2)^2, & x \in [1, \infty) \end{cases}$$

$$(f \circ g(x))' = \begin{cases} 3x^2, & x < 0 \\ 6x^5, & x \in (0, 1) \\ 2(3x-2) \times 3, & x \in (1, \infty) \end{cases}$$

At 'O'

L.H.L.  $\neq$  R.H.L. (Discontinuous)

At '1'

L.H.D. = 6 = R.H.D.

$\Rightarrow f \circ g(x)$  is differentiable for  $x \in \mathbb{R} - \{0\}$

10. Which of the following Boolean expression is a tautology ?

- (1)  $(p \wedge q) \vee (p \vee q)$   
 (2)  $(p \wedge q) \vee (p \rightarrow q)$   
 (3)  $(p \wedge q) \wedge (p \rightarrow q)$   
 (4)  $(p \wedge q) \rightarrow (p \rightarrow q)$

**Official Ans. by NTA (4)**

**Sol.**

p	q	$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

$(p \wedge q) \rightarrow (p \rightarrow q)$  is tautology

11. Let a complex number  $z$ ,  $|z| \neq 1$ ,

satisfy  $\log_{\frac{1}{\sqrt{2}}} \left( \frac{|z|+11}{(|z|-1)^2} \right) \leq 2$ . Then, the largest

value of  $|z|$  is equal to \_\_\_\_\_ .

- (1) 8 (2) 7 (3) 6 (4) 5

**Official Ans. by NTA (2)**

**Sol.**  $\log_{\frac{1}{\sqrt{2}}} \left( \frac{|z|+11}{(|z|-1)^2} \right) \leq 2$

$$\frac{|z|+11}{(|z|-1)^2} \geq \frac{1}{2}$$

$$2|z| + 22 \geq (|z| - 1)^2$$

$$2|z| + 22 \geq |z|^2 + 1 - 2|z|$$

$$|z|^2 - 4|z| - 21 \leq 0$$

$$\Rightarrow |z| \leq 7$$

$\therefore$  Largest value of  $|z|$  is 7

12. If  $n$  is the number of irrational terms in the expansion of  $(3^{1/4} + 5^{1/8})^{60}$ , then  $(n-1)$  is divisible by :

- (1) 26 (2) 30 (3) 8 (4) 7

**Official Ans. by NTA (1)**

**Sol.**  $(3^{1/4} + 5^{1/8})^{60}$   
 ${}^{60}C_r (3^{1/4})^{60-r} (5^{1/8})^r$

$${}^{60}C_r (3)^{\frac{60-r}{4}} (5)^{\frac{r}{8}}$$

For rational terms.

$$\frac{r}{8} = k; \quad 0 \leq r \leq 60$$

$$0 \leq 8k \leq 60$$

$$0 \leq k \leq \frac{60}{8}$$

$$0 \leq k \leq 7.5$$

$$k = 0, 1, 2, 3, 4, 5, 6, 7$$

$\frac{60-8k}{4}$  is always divisible by 4 for all value of  $k$ .

Total rational terms = 8

Total terms = 61

irrational terms = 53

$$n - 1 = 53 - 1 = 52$$

52 is divisible by 26.

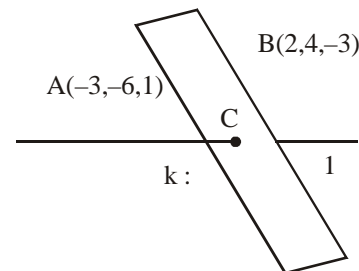
13. Let  $P$  be a plane  $lx + my + nz = 0$  containing

the line,  $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$ . If plane  $P$  divides the line segment  $AB$  joining points  $A(-3, -6, 1)$  and  $B(2, 4, -3)$  in ratio  $k : 1$  then the value of  $k$  is equal to :

- (1) 1.5 (2) 3 (3) 2 (4) 4

**Official Ans. by NTA (3)**

**Sol.**



Point C is

$$\left( \frac{2k-3}{k+1}, \frac{4k-6}{k+1}, \frac{-3k+1}{k+1} \right)$$

$$\frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3}$$

Plane  $lx + my + nz = 0$

$$l(-1) + m(2) + n(3) = 0$$

$$-l + 2m + 3n = 0 \quad \dots\dots(1)$$

It also satisfy point  $(1, -4, -2)$

$$l - 4m - 2n = 0 \quad \dots\dots(2)$$

Solving (1) and (2)

$$2m + 3n = 4m + 2n$$

$$n = 2m$$

$$l - 4m - 4m = 0$$

$$l = 8m$$

$$\frac{l}{8} = \frac{m}{1} = \frac{n}{2}$$

$$l : m : n = 8 : 1 : 2$$

Plane is  $8x + y + 2z = 0$

It will satisfy point C

$$8 \left( \frac{2k-3}{k+1} \right) + \left( \frac{4k-6}{k+1} \right) + 2 \left( \frac{-3k+1}{k+1} \right) = 0$$

$$16k - 24 + 4k - 6 - 6k + 2 = 0$$

$$14k = 28 \quad \therefore k = 2$$

14. The range of  $a \in \mathbb{R}$  for which the function

$$f(x) = (4a - 3)(x + \log_e 5) + 2(a - 7) \cot\left(\frac{x}{2}\right) \sin^2\left(\frac{x}{2}\right),$$

$x \neq 2n\pi, n \in \mathbb{N}$ , has critical points, is :

- (1)  $(-3, 1)$  (2)  $\left[-\frac{4}{3}, 2\right]$   
 (3)  $[1, \infty)$  (4)  $(-\infty, -1]$

**Official Ans. by NTA (2)**

- Sol.**  $f(x) = (4a - 3)(x + \log_e 5) + (a - 7) \sin x$

$$f(x) = (4a - 3)(1) + (a - 7) \cos x = 0$$

$$\Rightarrow \cos x = \frac{3 - 4a}{a - 7}$$

$$-1 \leq \frac{3 - 4a}{a - 7} < 1$$

$$\frac{3 - 4a}{a - 7} + 1 \geq 0$$

$$\frac{3 - 4a + a - 7}{a - 7} \geq 0$$

$$\frac{-3a - 4}{a - 7} \geq 0$$

$$\frac{3a + 4}{a - 7} \leq 0$$

$$\frac{3 - 4a}{a - 7} < 1$$

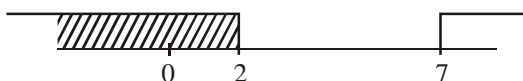
$$\frac{3 - 4a}{a - 7} - 1 < 0$$

$$\frac{3 - 4a - a + 7}{a - 7} < 0$$

$$\frac{-5a + 10}{a - 7} < 0$$

$$\frac{5a - 10}{a - 7} > 0$$

$$\frac{5(a - 2)}{a - 7} > 0$$



$$a \in \left[-\frac{4}{3}, 2\right)$$

Check end point  $\left[-\frac{4}{3}, 2\right)$

15. A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is :

- (1)  $\frac{3}{4}$  (2)  $\frac{52}{867}$   
 (3)  $\frac{39}{50}$  (4)  $\frac{22}{425}$

**Official Ans. by NTA (3)**

- Sol.**  $E_1$  : Event denotes spade is missing

$$P(E_1) = \frac{1}{4}; P(\bar{E}_1) = \frac{3}{4}$$

A : Event drawn two cards are spade

$$P(A) = \frac{\frac{1}{4} \times \left(\frac{{}^{12}C_2}{{}^{51}C_2}\right) + \frac{3}{4} \times \left(\frac{{}^{13}C_2}{{}^{51}C_2}\right) + \frac{3}{4} \times \left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)}{\frac{1}{4} \times \left(\frac{{}^{12}C_2}{{}^{51}C_2}\right) + \frac{3}{4} \times \left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)}$$

$$= \frac{39}{50}$$

16. Let  $[x]$  denote greatest integer less than or

equal to  $x$ . If for  $n \in \mathbb{N}$ ,  $(1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j$ ,

then  $\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4 \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$  is equal to :

- (1) 2 (2)  $2^{n-1}$   
 (3) 1 (4)  $n$

**Official Ans. by NTA (3)**

- Sol.**  $(1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j$

$$(1 - x + x^3)^n = a_0 + a_1 x + a_2 x^2 + \dots + a_{3n} x^{3n}$$

$$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} = \text{Sum of } a_0 + a_2 + a_4 + \dots$$

$$\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} = \text{Sum of } a_1 + a_3 + a_5 + \dots$$

put  $x = 1$

$$1 = a_0 + a_1 + a_2 + a_3 + \dots + a_{3n} \quad \dots (A)$$

Put  $x = -1$

$$1 = a_0 - a_1 + a_2 - a_3 + \dots + (-1)^{3n} a_{3n} \quad \dots (B)$$

Solving (A) and (B)

$$a_0 + a_2 + a_4 + \dots = 1$$

$$a_1 + a_3 + a_5 + \dots = 0$$

$$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4 \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} = 1$$

17. If  $y = y(x)$  is the solution of the differential equation,  $\frac{dy}{dx} + 2y \tan x = \sin x$ ,  $y\left(\frac{\pi}{3}\right) = 0$ , then the maximum value of the function  $y(x)$  over  $\mathbb{R}$  is equal to :

- (1) 8      (2)  $\frac{1}{2}$       (3)  $-\frac{15}{4}$       (4)  $\frac{1}{8}$

**Official Ans. by NTA (4)**

**Sol.**  $\frac{dy}{dx} + 2y \tan x = \sin x$

I.F. =  $e^{\int 2 \tan x dx} = e^{2 \ln \sec x}$

I.F. =  $\sec^2 x$

$y \cdot (\sec^2 x) = \int \sin x \cdot \sec^2 x dx$

$y \cdot (\sec^2 x) = \int \sec x \tan x dx$

$y \cdot (\sec^2 x) = \sec x + C$

$x = \frac{\pi}{3}; y = 0$

$\Rightarrow C = -2$

$\Rightarrow y = \frac{\sec x - 2}{\sec^2 x} = \cos x - 2 \cos^2 x$

$y = t - 2t^2 \Rightarrow \frac{dy}{dt} = 1 - 4t = 0 \Rightarrow t = \frac{1}{4}$

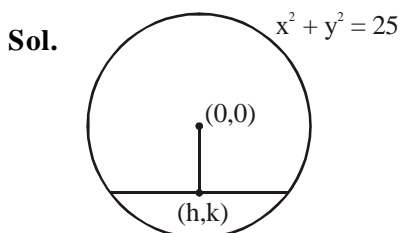
$\therefore \max = \frac{1}{4} - \frac{1}{8} = \frac{2-1}{8} = \frac{1}{8}$

18. The locus of the midpoints of the chord of the circle,  $x^2 + y^2 = 25$  which is tangent to the

hyperbola,  $\frac{x^2}{9} - \frac{y^2}{16} = 1$  is :

- (1)  $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$   
 (2)  $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$   
 (3)  $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$   
 (4)  $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

**Official Ans. by NTA (4)**



Equation of chord

$y - k = -\frac{h}{k}(x - h)$

$ky - k^2 = -hx + h^2$

$hx + ky = h^2 + k^2$

$y = -\frac{hx}{k} + \frac{h^2 + k^2}{k}$

tangent to  $\frac{x^2}{9} - \frac{y^2}{16} = 1$

$c^2 = a^2m^2 - b^2$

$\left(\frac{h^2 + k^2}{k}\right)^2 = 9\left(-\frac{h}{k}\right)^2 - 16$

$(x^2 + y^2)^2 = 9x^2 - 16y^2$

19. The number of roots of the equation,

$(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$

in the interval  $[0, \pi]$  is equal to :

- (1) 3      (2) 4      (3) 8      (4) 2

**Official Ans. by NTA (2)**

**Sol.**  $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$

$(81)^{\sin^2 x} + \frac{(81)^1}{(18)^{\sin^2 x}} = 30$

$(81)^{\sin^2 x} = t$

$t + \frac{81}{t} = 30$

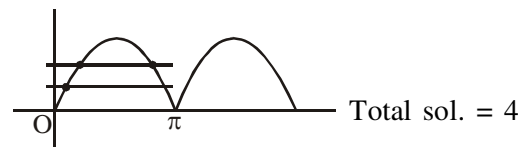
$t^2 - 30t + 81 = 0$

$(t - 3)(t - 27) = 0$

$(81)^{\sin^2 x} = 3^1$  or  $(81)^{\sin^2 x} = 3^3$

$3^{4\sin^2 x} = 3^1$  or  $3^{4\sin^2 x} = 3^3$

$\sin^2 x = \frac{1}{4}$  or  $\sin^2 x = \frac{3}{4}$



20. Let  $S_k = \sum_{r=1}^k \tan^{-1}\left(\frac{6^r}{2^{2r+1} + 3^{2r+1}}\right)$ . Then  $\lim_{k \rightarrow \infty} S_k$  is equal to :

(1)  $\tan^{-1}\left(\frac{3}{2}\right)$       (2)  $\frac{\pi}{2}$

(3)  $\cot^{-1}\left(\frac{3}{2}\right)$       (4)  $\tan^{-1}(3)$

**Official Ans. by NTA (3)**



**Sol.**  $S_k = \sum_{r=1}^k \tan^{-1} \left( \frac{6^r}{2^{2r+1} + 3^{2r+1}} \right)$

Divide by  $3^{2r}$

$$\sum_{r=1}^k \tan^{-1} \left( \frac{\left(\frac{2}{3}\right)^r}{\left(\frac{2}{3}\right)^{2r} \cdot 2 + 3} \right)$$

$$\sum_{r=1}^k \tan^{-1} \left( \frac{\left(\frac{2}{3}\right)^r}{3 \left( \left(\frac{2}{3}\right)^{2r+1} + 1 \right)} \right)$$

Let  $\left(\frac{2}{3}\right)^r = t$

$$\sum_{r=1}^k \tan^{-1} \left( \frac{\frac{t}{3}}{1 + \frac{2}{3}t^2} \right)$$

$$\sum_{r=1}^k \tan^{-1} \left( \frac{t - \frac{2t}{3}}{1 + t \cdot \frac{2t}{3}} \right)$$

$$\sum_{r=1}^k \left( \tan^{-1}(t) - \tan^{-1}\left(\frac{2t}{3}\right) \right)$$

$$\sum_{r=1}^k \left( \tan^{-1}\left(\frac{2}{3}\right)^r - \tan^{-1}\left(\frac{2}{3}\right)^{r+1} \right)$$

$$S_k = \tan^{-1}\left(\frac{2}{3}\right) - \tan^{-1}\left(\frac{2}{3}\right)^{k+1}$$

$$S_\infty = \lim_{k \rightarrow \infty} \left( \tan^{-1}\left(\frac{2}{3}\right) - \tan^{-1}\left(\frac{2}{3}\right)^{k+1} \right)$$

$$= \tan^{-1}\left(\frac{2}{3}\right) - \tan^{-1}(0)$$

$$\therefore S_\infty = \tan^{-1}\left(\frac{2}{3}\right) = \cot^{-1}\left(\frac{3}{2}\right)$$

## SECTION-B

1. Consider an arithmetic series and a geometric series having four initial terms from the set  $\{11, 8, 21, 16, 26, 32, 4\}$ . If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to \_\_\_\_\_.

**Official Ans. by NTA (3)**

**Sol. GP :** 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192

**AP :** 11, 16, 21, 26, 31, 36

Common terms : 16, 256, 4096 only

2. Let  $f : (0, 2) \rightarrow \mathbb{R}$  be defined as

$$f(x) = \log_2 \left( 1 + \tan \left( \frac{\pi x}{4} \right) \right).$$

Then,  $\lim_{n \rightarrow \infty} \frac{2}{n} \left( f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \dots + f(1) \right)$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (1)**

**Sol.**  $E = 2 \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} f\left(\frac{r}{n}\right)$

$$E = \frac{2}{\ln 2} \int_0^1 \ln \left( 1 + \tan \frac{\pi x}{4} \right) dx \quad \dots(i)$$

replacing  $x \rightarrow 1 - x$

$$E = \frac{2}{\ln 2} \int_0^1 \ln \left( 1 + \tan \frac{\pi}{4} (1 - x) \right) dx$$

$$E = \frac{2}{\ln 2} \int_0^1 \ln \left( 1 + \tan \left( \frac{\pi}{4} - \frac{\pi}{4} x \right) \right) dx$$

$$E = \frac{2}{\ln 2} \int_0^1 \ln \left( 1 + \frac{1 + \tan \frac{\pi}{4} x}{1 + \tan \frac{\pi}{4} x} \right) dx$$

$$E = \frac{2}{\ln 2} \int_0^1 \ln \left( \frac{2}{1 + \tan \frac{\pi x}{4}} \right) dx$$

$$E = \frac{2}{\ln 2} \int_0^1 \left( \ln 2 - \ln \left( 1 + \tan \frac{\pi x}{4} \right) \right) dx \quad \dots(ii)$$

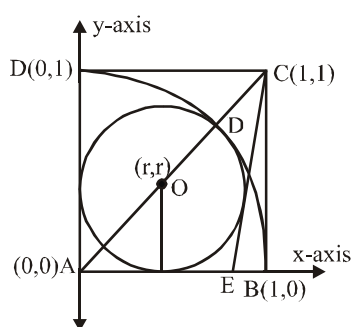
equation (i) + (ii)

$$E = 1$$

3. Let ABCD be a square of side of unit length. Let a circle  $C_1$  centered at A with unit radius is drawn. Another circle  $C_2$  which touches  $C_1$  and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle  $C_2$  meet the side AB at E. If the length of EB is  $\alpha + \sqrt{3}\beta$ , where  $\alpha, \beta$  are integers, then  $\alpha + \beta$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (1)**

**Sol.**



Here  $AO + OD = 1$  or  $(\sqrt{2} + 1)r = 1$

$$\Rightarrow r = \sqrt{2} - 1$$

equation of circle  $(x - r)^2 + (y - r)^2 = r^2$

Equation of CE

$$y - 1 = m(x - 1)$$

$$mx - y + 1 - m = 0$$

It is tangent to circle

$$\therefore \left| \frac{mr - r + 1 - m}{\sqrt{m^2 + 1}} \right| = r$$

$$\left| \frac{(m-1)r + 1 - m}{\sqrt{m^2 + 1}} \right| = r$$

$$\frac{(m-1)^2 (r-1)^2}{m^2 + 1} = r^2$$

$$\text{Put } r = \sqrt{2} - 1$$

On solving  $m = 2 - \sqrt{3}, 2 + \sqrt{3}$

Taking greater slope of CE as

$$2 + \sqrt{3}$$

$$y - 1 = (2 + \sqrt{3})(x - 1)$$

$$\text{Put } y = 0$$

$$-1 = (2 + \sqrt{3})(x - 1)$$

$$\frac{-1}{2 + \sqrt{3}} \times \left( \frac{2 - \sqrt{3}}{2 - \sqrt{3}} \right) = x - 1$$

$$x - 1 = \sqrt{3} - 1$$

$$EB = 1 - x = 1 - (\sqrt{3} - 1)$$

$$EB = 2 - \sqrt{3}$$

4. If  $\lim_{x \rightarrow 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x} = 2$ , then  $a + b + c$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (4)**

$$\text{Sol. } \lim_{x \rightarrow 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x} = 2$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{a \left( 1 + x + \frac{x^2}{2!} \dots \right) - b \left( 1 - \frac{x^2}{2!} + \dots \right) + c \left( 1 - x + \frac{x^2}{2!} \right)}{\left( \frac{x \sin x}{x} \right) x} = 2$$

$$a - b + c = 0 \quad \dots(1)$$

$$a - c = 0 \quad \dots(2)$$

$$\& \frac{a + b + c}{2} = 2$$

$$\Rightarrow \boxed{a + b + c = 4}$$

5. The total number of  $3 \times 3$  matrices A having entries from the set  $\{0, 1, 2, 3\}$  such that the sum of all the diagonal entries of  $AA^T$  is 9, is equal to \_\_\_\_\_.

**Official Ans. by NTA (766)**

$$\text{Sol. Let } A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

diagonal elements of

$$AA^T, \quad a^2 + b^2 + c^2, \quad d^2 + e^2 + f^2, \quad g^2 + h^2 + i^2$$

$$\text{Sum} = a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 9$$

$$a, b, c, d, e, f, g, h, i \in \{0, 1, 2, 3\}$$

	Case	No. of Matrices
(1)	All - 1s	$\frac{9!}{9!} = 1$
(2)	One $\rightarrow$ 3 remaining-0	$\frac{9!}{1! \times 8!} = 9$
(3)	One-2 five-1s three-0s	$\frac{9!}{1! \times 5! \times 3!} = 8 \times 63$
(4)	two - 2's one-1 six-0's	$\frac{9!}{2! \times 6!} = 63 \times 4$

$$\text{Total no. of ways} = 1 + 9 + 8 \times 63 + 63 \times 4 = \boxed{766}$$

6. Let

$$P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix} \text{ and } A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega + 1 \end{bmatrix}$$

where  $\omega = \frac{-1 + i\sqrt{3}}{2}$ , and  $I_3$  be the identity

matrix of order 3. If the determinant of the matrix  $(P^{-1}AP - I_3)^2$  is  $\alpha\omega^2$ , then the value of  $\alpha$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (36)**

**Sol.** Let  $M = (P^{-1}AP - I)^2$

$$= (P^{-1}AP)^2 - 2P^{-1}AP + I$$

$$= P^{-1}A^2P - 2P^{-1}AP + I$$

$$PM = A^2P - 2AP + P$$

$$= (A^2 - 2A.I + I^2)P$$

$$\Rightarrow \text{Det}(PM) = \text{Det}((A - I)^2 \times P)$$

$$\Rightarrow \text{Det}P \cdot \text{Det}M = \text{Det}(A - I)^2 \times \text{Det}(P)$$

$$\Rightarrow \text{Det} M = (\text{Det}(A - I))^2$$

$$\text{Now } A - I = \begin{bmatrix} 1 & 7 & \omega^2 \\ -1 & -\omega - 1 & 1 \\ 0 & -\omega & -\omega \end{bmatrix}$$

$$\text{Det}(A - I) = (\omega^2 + \omega + \omega) + 7(-\omega) + \omega^3 = -6\omega$$

$$\text{Det}((A - I)^2) = 36\omega^2$$

$$\Rightarrow \alpha = 36$$

7. If the normal to the curve

$$y(x) = \int_0^x (2t^2 - 15t + 10) dt \text{ at a point } (a, b) \text{ is}$$

parallel to the line  $x + 3y = -5$ ,  $a > 1$ , then the value of  $|a + 6b|$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (406)**

$$\text{Sol. } y(x) = \int_0^x (2t^2 - 15t + 10) dt$$

$$y'(x) \Big|_{x=a} = [2x^2 - 15x + 10]_a = 2a^2 - 15a + 10$$

$$\text{Slope of normal} = -\frac{1}{3}$$

$$\Rightarrow 2a^2 - 15a + 10 = 3 \Rightarrow a = 7$$

$$\& \quad a = \frac{1}{2} \text{ (rejected)}$$

$$b = y(7) = \int_0^7 (2t^2 - 15t + 10) dt$$

$$= \left[ \frac{2t^3}{3} - \frac{15t^2}{2} + 10t \right]_0^7$$

$$\Rightarrow 6b = 4 \times 7^3 - 45 \times 49 + 60 \times 7$$

$$|a + 6b| = 406$$

8. Let the curve  $y = y(x)$  be the solution of the

differential equation,  $\frac{dy}{dx} = 2(x+1)$ . If the

numerical value of area bounded by the curve

$y = y(x)$  and x-axis is  $\frac{4\sqrt{8}}{3}$ , then the value of  $y(1)$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (2)**

$$\text{Sol. } \frac{dy}{dx} = 2(x+1)$$

$$\Rightarrow \int dy = \int 2(x+1) dx$$

$$\Rightarrow y(x) = x^2 + 2x + C$$

$$\text{Area} = \frac{4\sqrt{8}}{3}$$

$$-1 + \sqrt{1-C}$$

$$\Rightarrow 2 \int_{-1}^{-1+\sqrt{1-C}} (-(x+1)^2 - C + 1) dx = \frac{4\sqrt{8}}{3}$$

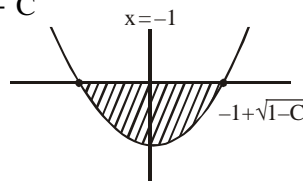
$$\Rightarrow 2 \left[ -\frac{(x+1)^3}{3} - Cx + x \right]_{-1}^{-1+\sqrt{1-C}} = \frac{4\sqrt{8}}{3}$$

$$\Rightarrow -(\sqrt{1-C})^3 + 3c - 3C\sqrt{1-C}$$

$$-3 + 3\sqrt{1-C} - 3C + 3 = 2\sqrt{8}$$

$$\Rightarrow C = -1$$

$$\Rightarrow f(x) = x^2 + 2x - 1, \quad f(1) = 2$$



9. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a continuous function such that  $f(x) + f(x+1) = 2$ , for all  $x \in \mathbb{R}$ . If

$$I_1 = \int_0^8 f(x) dx \text{ and } I_2 = \int_{-1}^3 f(x) dx, \text{ then the value}$$

of  $I_1 + 2I_2$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (16)**

- Sol.**  $f(x) + f(x+1) = 2$   
 $\Rightarrow f(x)$  is periodic with period = 2

$$I_1 = \int_0^8 f(x) dx = 4 \int_0^2 f(x) dx$$

$$= 4 \int_0^1 (f(x) + f(1+x)) dx = 8$$

$$\text{Similarly } I_2 = 2 \times 2 = 4$$

$$I_1 + 2I_2 = 16$$

10. Let  $z$  and  $w$  be two complex numbers such that

$$w = z\bar{z} - 2z + 2, \left| \frac{z+i}{z-3i} \right| = 1 \text{ and } \operatorname{Re}(w) \text{ has}$$

minimum value. Then, the minimum value of  $n \in \mathbb{N}$  for which  $w^n$  is real, is equal to \_\_\_\_\_.

**Official Ans. by NTA (4)**

**Sol.**  $w = z\bar{z} - 2z + 2$

$$\left| \frac{z+i}{z-3i} \right| = 1$$

$$\Rightarrow |z+i| = |z-3i|$$

$$\Rightarrow z = x + i, x \in \mathbb{R}$$

$$w = (x+i)(x-i) - 2(x+i) + 2$$

$$= x^2 + 1 - 2x - 2i + 2$$

$$\operatorname{Re}(w) = x^2 - 2x + 3$$

$$\text{For min } (\operatorname{Re}(w)), x = 1$$

$$\Rightarrow w = 2 - 2i = 2(1-i) = 2\sqrt{2} e^{-i\frac{\pi}{4}}$$

$$w^n = (2\sqrt{2})^n e^{-i\frac{n\pi}{4}}$$

For real & minimum value of  $n$ ,

$$n = 4$$

