26th Feb. 2021 | Shift - 2
PHYSICS

SECTION – A

1. A tuning fork A of unknown frequency produces 5beats/s with a fork of known frequency 340 HZ. When fork A filed, the beat frequency decreases to 2beats/s. What is the frequency of fork A?

(1) 342 Hz

(2) 335 Hz

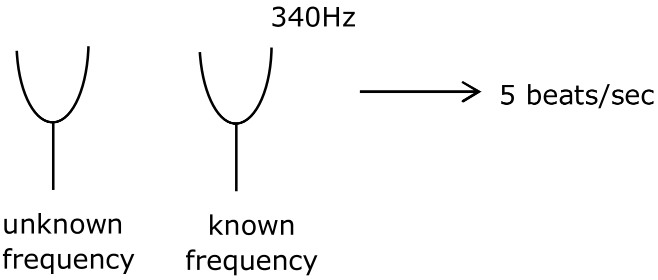
(3) 338 Hz

(4) 345 Hz

Sol. (2)

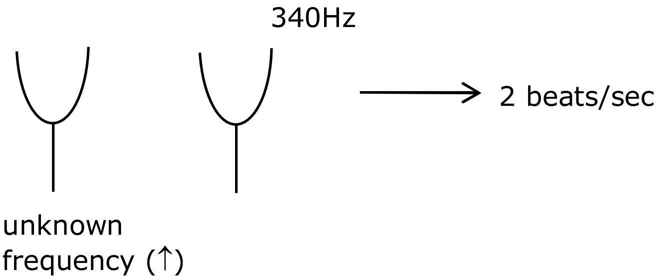
Given

BeforeFiled:



So answer should be 335 Hz or 345 Hz.

After Filed :



$340 \to 335 \xrightarrow{\text{Filed}} 336, 337, 338\ldots\ldots\ldots$

$340 \to 345 \xrightarrow{\text{Filed}} 346, 347, 348\ldots\ldots\ldots$

After filed beat/sec decreases only in case of 335 Hz.

2. The trajectory a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants and x & y are respectively the horizontal and vertical distance of the projectile from the point of projection. The angle of projection θ and the maximum height attained H are respectively given by:

(1) $\tan^{-1}\alpha, \dfrac{\alpha^2}{4\beta}$

(2) $\tan^{-1}\beta, \dfrac{\alpha^2}{2\beta}$

(3) $\tan^{-1}\left(\dfrac{\beta}{\alpha}\right), \dfrac{\alpha^2}{\beta}$

(4) $\tan^{-1}\alpha, \dfrac{4\alpha^2}{\beta}$

Sol. **(1)**

Given :

$y = \alpha x - \beta x^2$ ....(1)

for maximum height, we should find out maximum value of y from equation (1)

so, for maximum value of y

$\dfrac{dy}{dx} = 0 \Rightarrow \alpha - 2\beta x = 0$

$x = \dfrac{\alpha}{2\beta}$ ....(2)

Now, put value of x from equation (2) in quation (1)

$y = \alpha\left(\dfrac{\alpha}{2\beta}\right) - \beta\left(\dfrac{\alpha^2}{4\beta^2}\right)$

$\Rightarrow \left(\dfrac{\alpha^2}{2\beta}\right) - \left(\dfrac{\alpha^2}{4\beta}\right) \Rightarrow \dfrac{\alpha^2}{4\beta}$

So, $H_{max} = \dfrac{\alpha^2}{4\beta}$ ....(3)

As we know maximum height $H_{max} = \dfrac{u^2 \sin^2\theta}{2g}$ ...(4)

from (3) and (4) $u^2 = \left(\dfrac{\alpha^2}{4\beta}\right)\left(\dfrac{2g}{\sin^2\theta}\right)$

and range (R) = 2x = $\dfrac{u^2 \times 2\sin\theta\cos\theta}{g}$

$2\left(\dfrac{\alpha}{2\beta}\right) = \dfrac{\left(\dfrac{\alpha^2}{4\beta}\right)\left(\dfrac{2g}{\sin^2\theta}\right) \times 2\sin\theta\cos\theta}{g}$

$\tan\theta = \alpha \Rightarrow \theta = \tan^{-1}(\alpha)$

3. A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and the moment of inertia about it is I. A weight mg is attached to the cord at the end. The weight falls from rest. After falling through a distance 'h', the square of angular velocity of wheel will be:

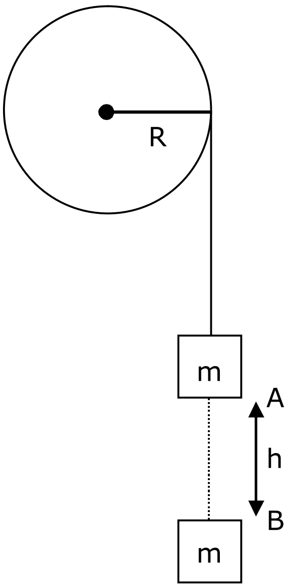
(1) $\frac{2gh}{I+mr^2}$

(2) 2gh

(3) $\frac{2mgh}{I+2mr^2}$

(4) $\frac{2mgh}{I+mr^2}$

Sol. **(4)**



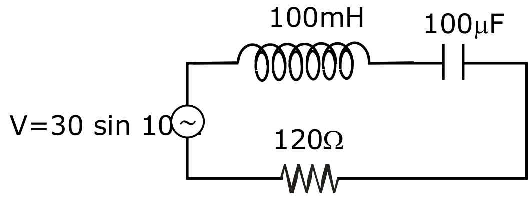
using energy conservation between A and B point

$mgh = \frac{1}{2}\,m\,(wR)^2 + \frac{1}{2}\,I\omega^2$

$2mgh = (MR^2 + I)\,\omega^2$

$\omega^2 = \frac{2mgh}{I+MR^2}$

4. Find the peak current and resonant frequency of the following circuit (as shown in figure)



(1) 0.2 A and 100 Hz
(2) 2 A and 50 Hz
(3) 2 A and 100 Hz
(4) 0.2 A and 50 Hz

Sol. **(4)**

Peak current in series LCR CKT

$$i = \frac{v_0}{z} \Rightarrow \frac{30}{\sqrt{(x_L - x_C)^2 + R^2}}$$

$$i = \frac{30}{\sqrt{(10-100)^2 + (120)^2}}$$

$$i \Rightarrow \frac{30}{150} \Rightarrow \frac{1}{5} \Rightarrow 0.2 Amp.$$

$$\therefore X_L = \omega \times L$$

$\Rightarrow$ (100) (100 × 10^{-3})$\Rightarrow$ 10

$$X_L = \frac{1}{\omega \times c} \Rightarrow \frac{1}{100 \times 100 \times 10^{-6}}$$

$$\Rightarrow \frac{10^6}{10^4} \Rightarrow 100$$

Resonance frequency $\omega = \frac{1}{\sqrt{LC}}$

$$\omega = \frac{1}{\sqrt{100 \times 10^{-3} \times 100 \times 10^{-6}}} \Rightarrow \frac{1}{\sqrt{10^{-5}}}$$

$\therefore \omega = 2\pi F$

$$F = \frac{1}{2\pi} \times \frac{1}{\sqrt{10^{-5}}}$$

$$\Rightarrow \frac{1}{2\pi}\sqrt{10^5}$$

$$\Rightarrow \frac{100}{2\pi}\sqrt{10}$$

$\Rightarrow$ 50Hz

5. The incident ray, reflected ray and the outward drawn normal are denoted by the unit vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively. Then choose the correct relation for these vectors.

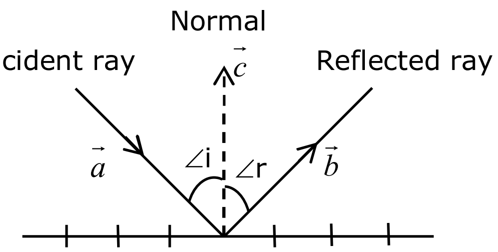
(1) $\vec{b} = 2\vec{a} + \vec{c}$

(2) $\vec{b} = \vec{a} - \vec{c}$

(3) $\vec{b} = \vec{a} + 2\vec{c}$

(4) $\vec{b} = \vec{a} - 2\left(\vec{a} \cdot \vec{c}\right)\vec{c}$

Sol. **(4)**



We see from the diagram that because of the law of reflection, the component of the unit vector $\vec{a}$ along $\vec{b}$ changes sign on reflection while the component parallel to the mirror remain unchanges.

$\vec{a} = \overrightarrow{a_{11}} + \overrightarrow{a_{\perp}}$

and $\overrightarrow{a_{\perp}} = \vec{c}\left(\vec{a}.\vec{c}\right)$

we see that the reflected unit vector is

$\vec{b} = \overrightarrow{a_{11}} - \overrightarrow{a_{\perp}} \Rightarrow \vec{a} - 2(\vec{a} \cdot \vec{c})\vec{c}$

6. A radioactive sample is undergoing α decay. At any time t_1, its activity is A and another time t_2, the activity is $\frac{A}{5}$. What is the average life time for the sample?

(1) $\frac{t_2 - t_1}{\ln 5}$

(2) $\frac{\ln(t_2 + t_1)}{2}$

(3) $\frac{t_1 - t_2}{\ln 5}$

(4) $\frac{\ln 5}{t_2 - t_1}$

Sol. **(1)**

For activity of radioactivesample

$A = A_0 e^{-\alpha t_1}$ ....(1)

$\frac{A}{5}$ $A_0 e^{-\alpha t_2}$ ....(2)

From (1)/(2)

$5 = e^{-\lambda(t_1 - t_2)}$

$\ln(5) = (t_2 - t_1)\lambda \Rightarrow \lambda = \frac{\ln(5)}{t_2 - t_1}$

avg. life $= \frac{1}{\lambda} \Rightarrow \frac{t_2 - t_1}{\ln(5)}$

7. A particle executes S.H.M., the graph of velocity as a function of displacement is:

(1) a circle

(2) a parabola

(3) an ellipse

(4) a helix

Sol. **(3)**

For a body performing SHM, relation between velocity and displacement

$v = \omega\sqrt{A^2 - x^2}$

now, square both side

$v^2 = w^2 (A^2 - x^2)$

$\Rightarrow v^2 = w^2A^2 - \omega^2x^2$

$v^2 + \omega^2x^2 = \omega^2A^2$

divide whole equation by ω^2A^2

$\frac{v^2}{\omega^2A^2} + \frac{\omega^2x^2}{\omega^2A^2} = \frac{\omega^2x^2}{\omega^2A^2}$

$\frac{v^2}{(\omega A)^2} + \frac{x^2}{(A)^2} = 1$

above equation is similar as standard equation of ellipes, so graph between velocity and displacement will be ellipes.

8. A scooter accelerates from rest for time t_1 at constant rate a_1 and then retards at constant rate a_2 for time t_2 and comes to rest. The correct value of $\frac{t_1}{t_2}$ will be:

(1) $\frac{a_1 + a_2}{a_2}$

(2) $\frac{a_2}{a_1}$

(3) $\frac{a_1 + a_2}{a_1}$

(4) $\frac{a_1}{a_2}$

Sol. **(2)**

From given information:

For 1[st] interval

$$a_1 = \frac{v_0}{t_1}$$

$v_0 = a_1 t_1$ .....(1)

For 2[nd] interval

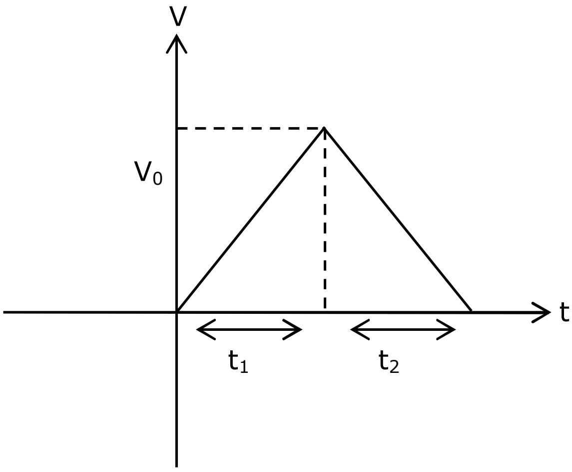
$$a_2 = \frac{v_0}{t_2}$$

$v_0 = a_2 t_2$ .....(2)

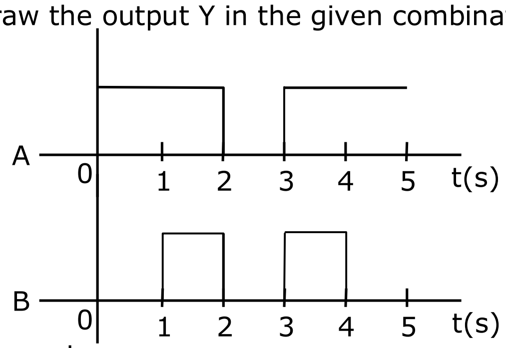
from (1) & (2)

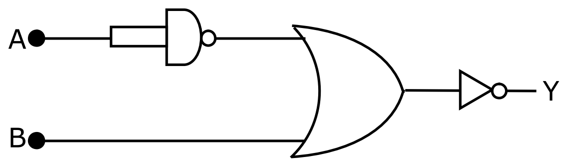
$a_1 t_1 = a_2 t2$

$$\frac{t_1}{t_2} = \frac{a_2}{a_1}$$

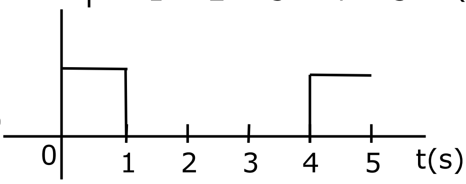


9. Draw the output Y in the given combination of gates.

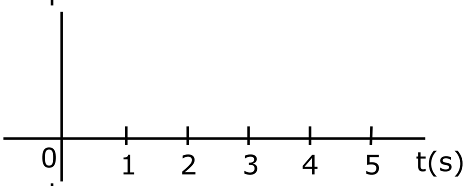




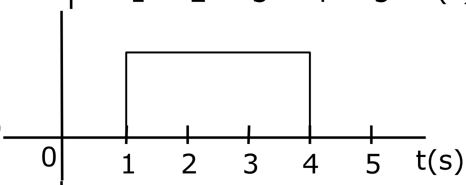
(1)



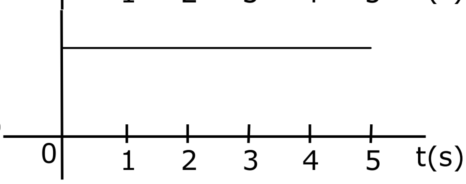
(2)



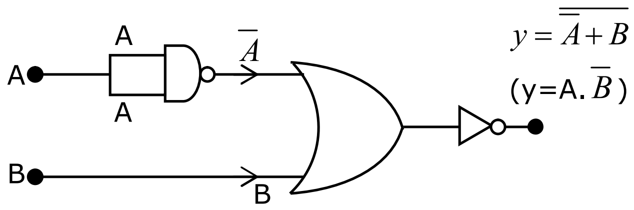
(3)



(4)



Sol. **(1)**



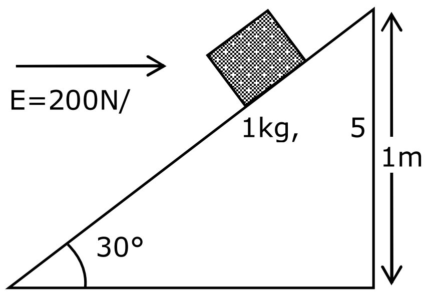
Find output expression $y = A \cdot \overline{B}$

Inputs

A	B	$y = A \cdot \overline{B}$
1	0	1
1	1	0
0	0	0
1	1	0
1	0	1

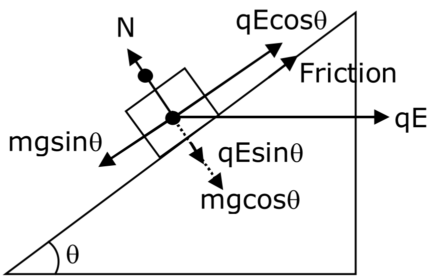
10. An inclined plane making an angle of 30º with horizontal is placed in a uniform horizontal electric field $200\frac{N}{C}$ as shown in the figure. A body of mass 1 kg and charge 5mC is allowed to slide down from rest at a height of 1m. If the coefficient of frication is 0.2, find the time taken by the body to reach the bottom.

$\left[g = 9.8m/s^2, \sin 30^\circ = \frac{1}{2}; \cos 30^\circ = \frac{\sqrt{3}}{2}\right]$



(1) 2.3 s
(2) 0.46 s
(3) 1.3 s
(4) 0.92 s

Sol. **(3)**



$F = mg\ \sin\theta - (\mu N + q\ E\ \cos\theta)$

$F = mg\ \sin\theta - \mu(mg\ \cos\theta + qE\ \sin\theta) - qE\ \cos\theta$

$F = 1 \times 10 \times \sin 30 - 0.2\ (1 \times 10 \times \cos 30^{o} + 200 \times 5 \times 10^{-3} \sin 30^{o})$

$- 200 \times 5 \times 10^{-3} \cos 30^{o}$

$F = 2.3\ N$

$a = \frac{F}{m} \Rightarrow \frac{2.3}{1} \Rightarrow 2.3 m / \sec^2$

$t = \sqrt{\frac{25}{9}} \Rightarrow \sqrt{\frac{2 \times 2}{2.3}} \Rightarrow 1.3 \sec$

11. If 'C' and 'V' represent capacity and voltage respectively then what are the dimensions of λ where $C/V = \lambda$?

(1) $\left[M^{-2}L^{-4}I^{3}T^{7}\right]$

(2) $\left[M^{-2}L^{-3}I^{2}T^{6}\right]$

(3) $\left[M^{-1}L^{-3}I^{-2}T^{-7}\right]$

(4) $\left[M^{-3}L^{-4}I^{3}T^{7}\right]$

Sol. **(1)**

$\therefore\ v = \frac{w}{q}$ and $c = \frac{q}{v}$

dimension of $\frac{c}{v}$

$\Rightarrow \frac{q}{v^2}$

$\Rightarrow \frac{q}{w^2} \times q^2 \Rightarrow \frac{q^3}{w^2}$

$\Rightarrow \frac{I^3T^3}{M^2L^4T^{-4}} \Rightarrow \left[M^{-2}L^{-4}T^{7}I^{3}\right]$

12. Given below are two statements: One is labeled as Assertion A and the other is labeled as Reason R.

Assertion A : For a simple microscope, the angular size of the object equals the angular size of the image.

Reason R : Magnification is achieved as the small object can be kept much closer to the eye than 25 cm and hence it subtends a large angle.

In the light of the above statements, choose the most appropriate answer from the options given below:

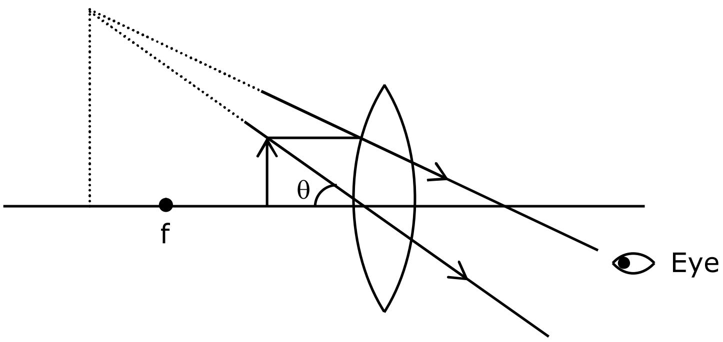
(1) Both A and R are true but R is NOT the correct explanationof A

(2) Both A and R are true and R is the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

Sol. **(2)**

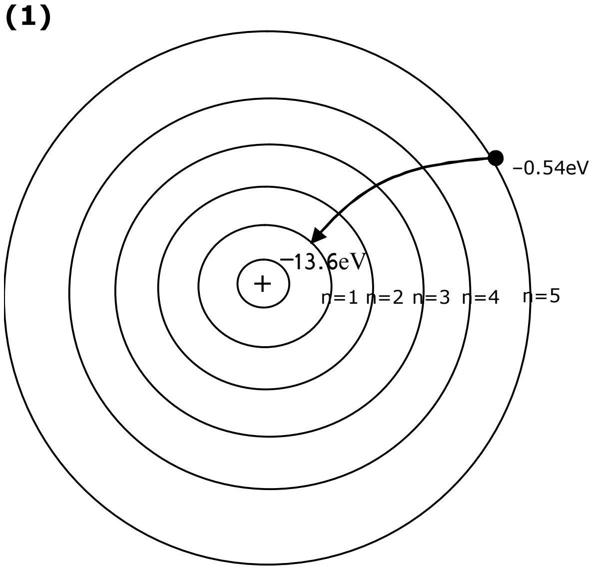


Both obtain same angle, since image can be at a distance greater than 25 cm, object can be moved closer to eye.

13. The recoil speed of a hydrogen atom after it emits a photon in going from n = 5 state to n = 1 state will be:

(1) 4.17 m/s (2) 4.34 m/s (3) 219 m/s (4) 3.25 m/s

Sol. **(1)**

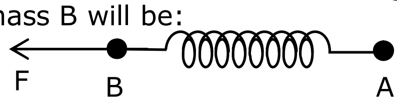


momentum (P) = $\frac{\Delta E}{C} \Rightarrow \frac{(13.6-0.54)eV}{3\times10^8}$

mv = $\frac{(13.06)\times1.6\times10^{-19}}{3\times10^8}$

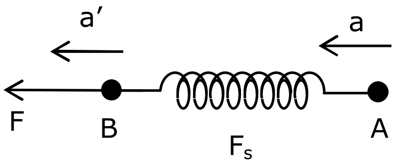
v = $\frac{(13.06)\times1.6\times10^{-19}}{3\times10^8\times1.67\times10^{-27}} \Rightarrow 4.17$ m/sec

14. Two masses A and B, each of mass M are fixed together by a massless springs. A force acts on the mass B as shown in figure. If the mass A starts moving away from mass B with acceleration 'a', than the acceleration of mass B will be:



(1) $\frac{F+Ma}{M}$ (2) $\frac{F-Ma}{M}$ (3) $\frac{Ma-F}{M}$ (4) $\frac{MF}{F+Ma}$

Sol. **(2)**



$F - F_s = Ma'$

$a' = \frac{F}{M}$ −9

$\frac{F-ma}{M}$

15. A wire of 1Ω has a length of 1 m. It is stretched till its length increases by 25%. The percentage change in a resistance to the nearest integer is:

(1) 25%

(2) 12.5%

(3) 76%

(4) 56%

Sol. **(4)**

For stretched or compressed wire

$R \propto l^2$

$\frac{R_1}{R_2} = \frac{l_1^2}{l_2^2}$

$\Rightarrow \frac{R}{R_2} = \frac{l^2}{(1.25l)^2}$

$\Rightarrow R_2 = 1.5625$ R

% increase → 56.235%

16. Given below are two statements :

Statement (1) :- A second's pendulum has a time period of 1 second.

Statement (2) :- It takes precisely one second to move between the two extreme positions.

In the light of the above statements, choose the correct answer from the options give below.

(1) Both Statement I and Statement II are false

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II is true

Sol. **(3)**

As we know time period of second's penduklum is 2 sec, so statement (1) is incorrect.

Time taken between two extreme points in second's pendulum is 1 sec.

Above statement is correct because time taken by particle performing SHM between two extreme position is T/2.

Here, T = 2 sec.

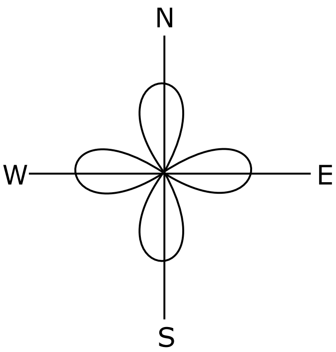
So, time = 2/2 = 1 sec

17. An aeroplane, with its wings spread 10 m, is flying at a speed of 180 km/h in a horizontal direction. The total intensity of earth's field at that part is 2.5×10^{-4} Wb/m^2 and the angle of dip is 60°. The emf induced between the tips of the plane wings will be ________.

(1) 88.37 mV (2) 62.50 mV

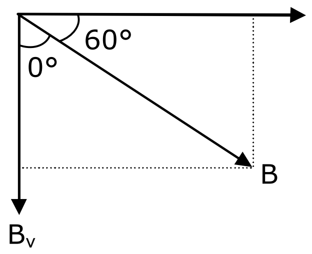
(3) 54.125 mV (4) 108.25 mV

Sol. **(4)**



$$\sum = B \perp v\ell$$

$$\sin 60° = \frac{B_v}{B}$$



$$\frac{\sqrt{3}}{2} = \frac{B_v}{B}$$

$$Bv = \frac{\sqrt{3}}{2}B$$

$$E = \frac{\sqrt{3}}{2}B\ell v$$

$$= \frac{\sqrt{3}}{2} \times 2.5 \times 10^{-4} \times 10 \times 180 \times \frac{5}{18}$$

$$= \frac{\sqrt{3}}{2} \times 2.5 \times 5 \times 10^{-2} \quad = 10.825 \times 10^{-2} \qquad = 108.25 \text{ mV}$$

18. The length of metallic wire is l_1 when tension in it is T_1. It is l_2 when the tension is T_2. The original length of the wire will be :

(1) $\frac{l_1 + l_2}{2}$

(2) $\frac{T_1 l_1 - T_2 l_2}{T_2 - T_1}$

(3) $\frac{T_2 l_1 + T_1 l_2}{T_1 + T_2}$

(4) $\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$

Sol. (4)

From young's modulus relation $\left(y = \frac{\frac{F}{A}}{\left(\frac{\Delta l}{l}\right)} \right)$

we can write for 1st case

$$\frac{T_1}{A} = \frac{y(\ell_1 - \ell)}{\ell}$$

we can write for 2^{nd} case

$$\frac{T_2}{A} = \frac{y(\ell_2 - \ell)}{\ell}$$

$$\frac{T_1}{T_2} = \frac{\ell_1 - \ell}{\ell_2 - \ell}$$

$$T_1\ell_2 - T_1\ell = T_2\ell_1 - T_2\ell$$

$$\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1} = \ell$$

19. The internal energy (U), pressure (P) and volume (V) of an ideal gas are related as U = 3PV + 4. The gas is :

(1) polyatomic only

(2) monoatomic only

(3) either monoatomic or diatomic

(4) diatomic only.

Sol. (1)

U = 3 PV + 4

$\frac{f}{2}$ PV = 3PV + 4 $\qquad \therefore u = \frac{f}{2} nRT$

$f = 6 + \frac{8}{PV}$ $\qquad \therefore Pv = nRT$

f>6 $\therefore$ Polyatomic gas.

20. Given below are two statements :

Statement – I : An electric dipole is placed at the centre of a hollow sphere. The flux of electric field through the sphere is zero but the electric field is not zero anywhere in the sphere.

Statement – II : If R is the radius of a solid metallic sphere and Q be the total charge on it. The electric field at any point on the spherical surface of radius r (< R) is zero but theelectric flux passing through this closed spherical surface of radius r is not zero.

In the light of the above statements. Choose the correct answerfrom the option given below :

Option :

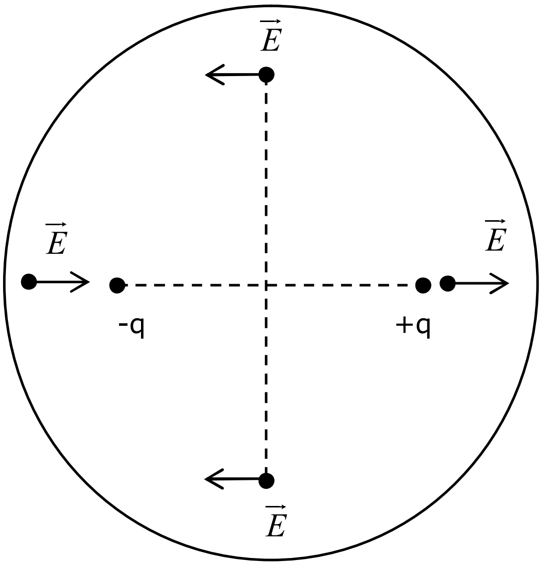
(1) Statement I is true but Statement II is false

(2) Statement I is false but Statement II is true

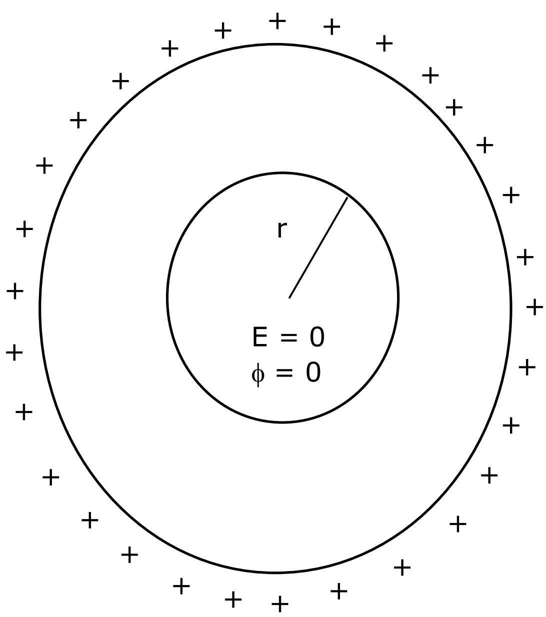
(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Sol. (1)



Statement – 1 $\rightarrow$ Correct



Statement – 2 $\rightarrow$ Incorrect

SECTION – B

1. If thehighest frequency modulating a carrier is 5 kHz, then the number of AM broadcast stations accommodated in a 90 kHz bandwidth are ________________________.

Sol. **(9)**

$$\text{No. of station} = \frac{\text{Band width}}{2\times \text{Highest Band width}}$$

$$\Rightarrow \frac{90}{2\times 5}$$

$\Rightarrow 9$

2. 1 mole of rigid diatomic gas performs a work of $\frac{Q}{5}$ when heat Q is supplied to it. The molar heat capacity of the gas during this transformation is $\frac{xR}{8}$. The value of x is __________.

Sol. **(25)**

From thermodynamics law:

$\Delta Q = \Delta U + \Delta W$ .....(1)

$Q = nC_v\Delta T + \frac{Q}{5}$

$Q - \frac{Q}{5} = 1 \times \frac{5}{2}R \times \Delta T$

$Q = \frac{25}{8}R\,\Delta T$ ......(2) $\therefore Q = n\,c\,\Delta T$

$C = \frac{25}{8}R$ given $C = \frac{xR}{8}$

$x = 25$

3. A particle excutes S.H.M with amplitude 'a' and time period T. The displacement of the particle when its speed is half of maximum speed is $\frac{\sqrt{x}\,a}{2}$. The value of x is _______________

Sol. **(3)**

Fora particle excutes S.H.M

$V = \omega\sqrt{a^2 - x^2}$

Given $V = \frac{V_{max}}{2} \Rightarrow \frac{A\omega}{2}$

$$\frac{A^2\omega^2}{4} = \omega^2 a^2 - \omega^2 x^2$$

$$x = \frac{\sqrt{3}}{2}a$$

4. Two stream of photons, possessing energies equal to twice and ten times the work function of metal are incident on the metal surface successively. The value of ratio of maximum velocities of the photoelectrons emitted in the two respective cases is x : y. The value of x is ________.

Sol. (1)

Forphotoelectric effectk.E_{max} = E – ϕ

$E_1 = 2\phi$, $k_1 = \phi$

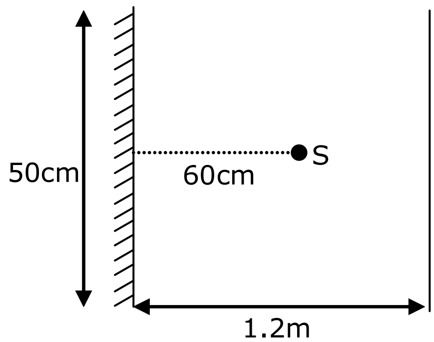
$E_2 = 10\phi$, $k_2 = 9\phi$

$\therefore V \propto \sqrt{k}$ $\left(k = \frac{1}{2}mv^2\right)$

$\frac{v_1}{v_2} = \sqrt{\frac{1}{9}} \Rightarrow \frac{1}{3} = \frac{x}{y}$

x = 1

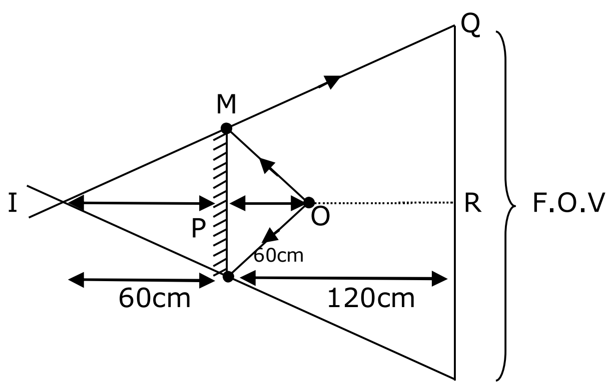
5. A point source of light S, placed at a distance 60 cm infront of the centre of plane mirror of width 50 cm, hangs vertically on a wall. A man walks infront of the mirror along a line parallel to the mirror at a distance 1.2 m from it (see in the figure). The distance between the extreme points where he can see the image of the light source in the mirror is ______________cm



Sol. (150)

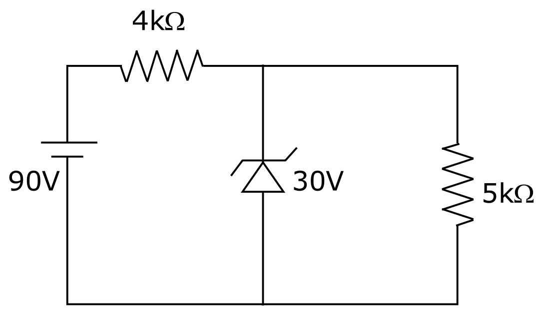
from similar triangle IMP and IQR

$\frac{QR}{25} = \frac{180}{60} \Rightarrow QR = 7$

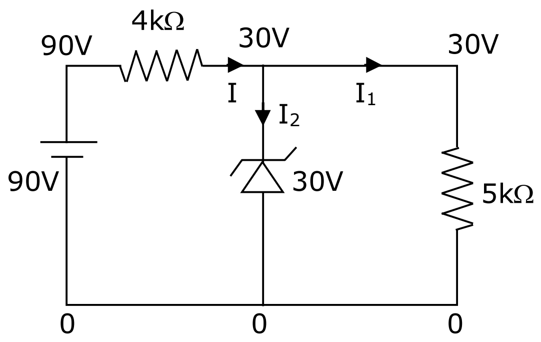


F.O.V. = 2 × 75 ⇒ 150 cm

6. The zener diode has a V_Z= 30 V. The current passing through the diode for the following ciruit is ________________mA.



Sol. (9)

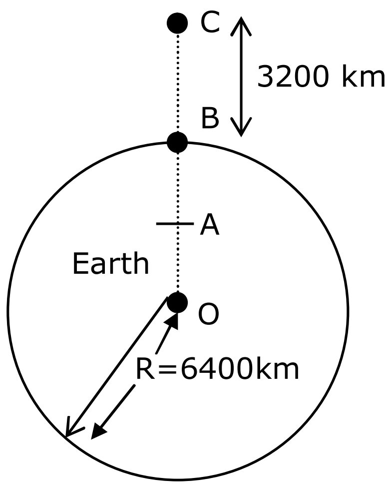


$$I = \frac{90-30}{4} = 15\text{mA}$$

$$I_I = \frac{30}{5K\Omega} = 6\text{mA}$$

$$I_2 = 15\text{mA} - 6\text{mA} = 9\text{mA}$$

7. In the reported figure of earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of OA : AB will be x : y. The value of x is ________________.



Sol. **(4)**

$$\frac{GM}{\left(\frac{3R}{2}\right)^2} = \frac{GMr}{R^3}$$

OA = $\frac{4R}{9}$ =r

AB = R - $\frac{4R}{9} = \frac{5R}{9}$

OA : AB

$\frac{4R}{9} : \frac{5R}{9} \Rightarrow$ 4:5 = x:y

(x=4)

8. 27 similar drops of mercury are maintained at 10 V each. All these spherical drops combine into a single big drop. The potential energy of the bigger drop is ________________ times that of a smaller drop.

Sol. **(243)**

For self energy of sphere (conducting)

$$U = \frac{kq^2}{2r}$$

For small drop $\rightarrow U_i = \frac{kq^2}{2r}$ ....(1)

After combine small drops volume remains same as bigger drop

$$\therefore \quad \frac{4}{3}\pi r^3 \times n = \frac{4}{3}\pi R^3$$

R = $(n)^{\frac{1}{3}}$r ....(2)

For large drop$\rightarrow U_f = \frac{k(nq)^2}{2 \times 3R}$ ....(3)

From equation (1), (2), (3)

$$\frac{U_f}{U_i} = (n)^{5/3}$$

$\Rightarrow (27)^{5/3}$

$\Rightarrow$ 243

9. The volume V of a given mass of monatomic gas changes with temperature T according to the relation $V = KT^{\frac{2}{3}}$. The work done when temperature changes by 90 K will be xR. The value of x is ________________.
[R =universal gas constant]

Sol. **(60)**
Given: $V = k\,T^{2/3}$
$V^{3/2} = (k)^{3/2}\,T$
$TV^{-3/2}$ = const. ......(1)
and $TV^{\gamma-1}$ = const. ......(2)
From (1) & (2)

$$-\frac{3}{2} = \gamma - 1$$

$$\gamma = -\frac{1}{2}$$

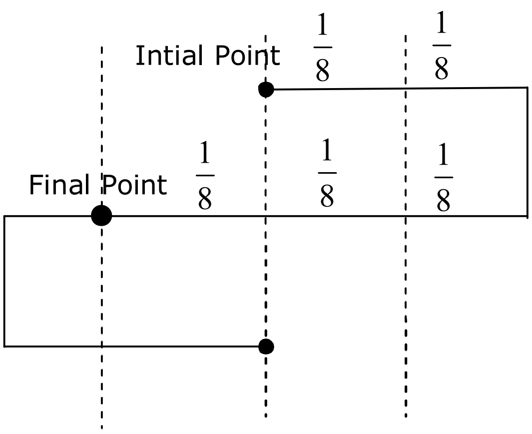
Work done (w) = $\dfrac{nR\Delta T}{\gamma - 1}$

W = $\dfrac{1 \times R \times 90}{-\frac{1}{2} - 1}$ |W| = 60R x = 60

10. Time period of a simple pendulum is T. The time taken to complete $\frac{5}{8}$ oscillations starting from mean position is $\frac{\alpha}{\beta}$T. The value of α is ___________.

Sol. **(7)**
For given $\left(\frac{5}{8}\right)$ oscillation, we can write it as $\rightarrow \left(\frac{1}{2} + \frac{1}{8}\right)$

And we know for half oscillations time $\rightarrow \frac{T}{2}$



For final point $\rightarrow \pi + \frac{\pi}{6}$ $\Rightarrow \frac{7\pi}{6}$

Time $\rightarrow \frac{7T}{12} \rightarrow$ given $\rightarrow \frac{\alpha}{\beta}T$ α = 7p

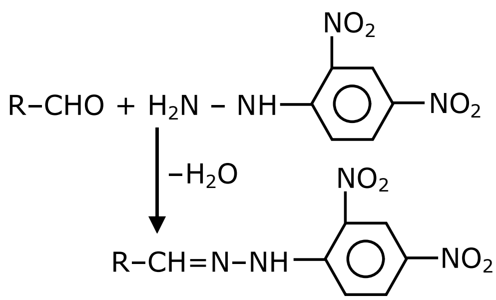
26th Feb. 2021 | Shift - 2
CHEMISTRY

Section - A

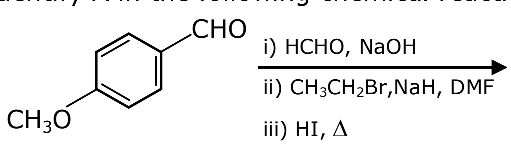
1. 2,4-DNP test can be used to identify:
(1) aldehyde
(2) halogens
(3) ether
(4) amine

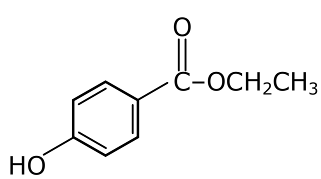
Ans. **(1)**

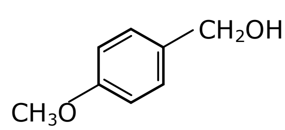
Sol.

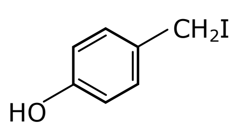


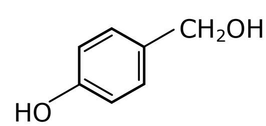
2. Identify A in the following chemical reaction.



(1) 

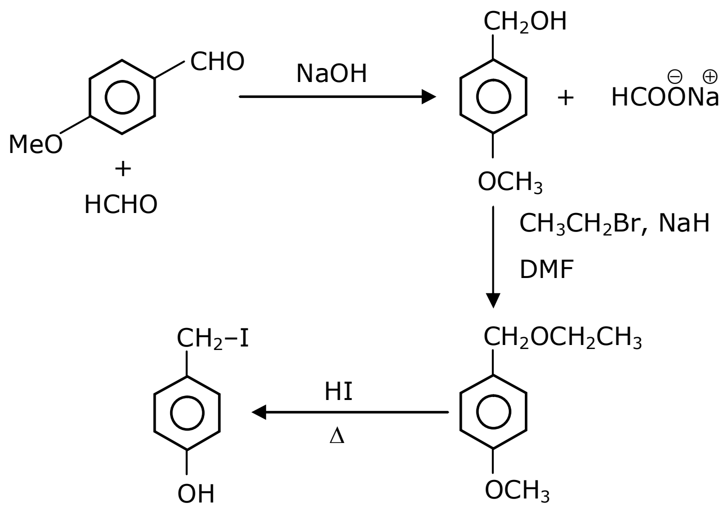
(2) 

(3) 

(4) 

Ans. **(3)**

Sol.



3. The nature of charge on resulting colloidal particles when $FeCl_3$ is added to excess of hot water is:

(1) positive

(2) neutral

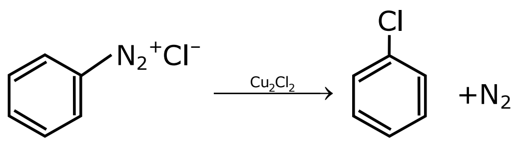
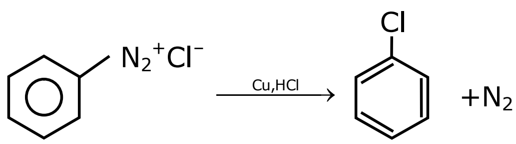
(3) sometimes positive and sometimes negative

(4) negative

Ans. **(1)**

Sol. If $FeCl_3$ is added to excess of hot water, a positively charged sol of hydrated ferric oxide is formed due to adsorption of Fe^{3+} ions.

4. Match **List-I** with **List-II**

List-I	**List-II**
(a) 	(i) Wurtz reaction
(b) 	(ii) Sandmeyer reaction
(c) $2CH_3CH_2Cl + 2Na \xrightarrow{\text{Ether}} C_2H_5-C_2H_5 + 2NaCl$	(iii) Fitting reaction
(d) $2C_2H_5Cl + 2Na \xrightarrow{\text{Ether}} C_6H_5-C_6H_5 + 2NaCl$	(iv) Gatterman reaction

Choose the correct answer from the option given below:

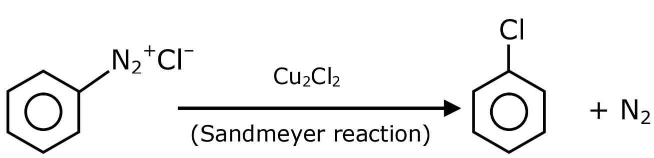
(1) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

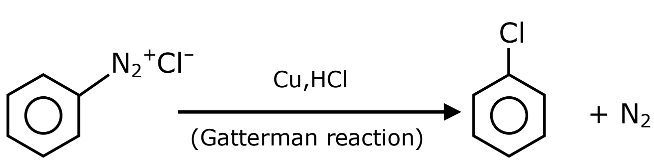
(2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(3) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

(4) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Ans. **(3)**

Sol. (a) 

(b) 

(c) $2CH_3-CH_2Cl + 2Na \xrightarrow[\text{(Wurtz reaction)}]{\text{Ether}} C_2H_5-C_2H_5+2NaCl$

(D) $2C_6H_5Cl + 2Na \xrightarrow[\text{(Fitting reaction)}]{\text{Ether}} C_6H_5-C_6H_5+2NaCl$

5. In $\overset{1}{CH_2} = \overset{2}{C} = \overset{3}{CH}–\overset{4}{CH_3}$ molecule, the hybridization of carbon 1, 2, 3 and 4 respectively are:

(1) sp^2, sp, sp^2, sp^3

(2) sp^2, sp^2, sp^2, sp^3

(3) sp^2, sp^3, sp^2, sp^3

(4) sp^3, sp, sp^3, sp^3

Ans. **(1)**

Sol. $\underset{sp^2}{CH_2} = \underset{sp}{C} = \underset{sp^2}{CH} - \underset{sp^3}{CH_3}$

6. Match List-I with List-II.

List-I	**List-II**
(a) Sucrose	(i) β-D-Galactose and β-D-Glucose
(b) Lactose	(ii) α-D-Glucose and β-D-Fructose
(c) Maltose	(iii) α-D- Glucose and α-D-Glucose

Choose the correct answer from the options given below:

(1) (a)-(iii), (b)-(ii), (c)-(i)

(2) (a)-(iii), (b)-(i), (c)-(ii)

(3) (a)-(i), (b)-(iii), (c)-(ii)

(4) (a)-(ii), (b)-(i), (c)-(iii)

Ans. **(4)**

Sol. Sucrose $\rightarrow \alpha$-D- Glucose and β-D- Fructose

Lactose $\rightarrow \beta$-D- Galactose and β-D- Glucose

Maltose $\rightarrow \alpha$-D- Glucose and α-D- Glucose

7. Which pair of oxides is acidic in nature?

(1) N_2O, BaO

(2) CaO, SiO_2

(3) B_2O_3, CaO

(4) B_2O_3, SiO_2

Ans. **(4)**

Sol. B_2O_3 and SiO_2 both are oxides of non-metal and hence are acidic in nature.

8. Calgon is used for water treatment. Which of the following statement is NOT true about calgon?

(1) Calgon contains the 2^{nd} most abundant element by weight in the earth's crust.

(2) It is also known as Graham's salt.

(3) It is polymeric compound and is water soluble.

(4) It doesnot remove Ca^{2+} ion by precipitation.

Ans. (1)

Sol. $Na_6(PO_3)_6$ or $Na_6P_6O_{18}$

Order of abundance of element in earth crust is

O > Si > Al > Fe > Ca > Na > Mg > K

So second most abundant element in earth crust is Si not Ca.

9. Ceric ammonium nitrate and $CHCl_3$/alc. KOH are used for the identification of functional groups present in _________and________respectively.

(1) alcohol, amine (2) amine, alcohol

(3) alcohol, phenol (4) amine, phenol

Ans. (1)

Sol. Alcohol give positive test with ceric ammonium nitrate and primary amines gives carbyl amine test with $CHCl_3$, KOH.

10. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: In TlI_3, isomorphous to CsI_3, the metal is present in +1 oxidation state.

Reason R: Tl metals has fourteen *f* electrons in its electronic configuration.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both A and R are correct and R is the correct explanation of A

(2) A is not correct but R is correct

(3) Both A and R are correct R is NOT the correct explanation of A

(4) A is correct but R is not correct

Ans. (3)

Sol. $T\ell I_3$ is $T\ell^+ \ I_3^-$

CsI_3 is $Cs^+ \ I_3^-$

Thallium shows $T\ell^+$ state due to inert pair effect.

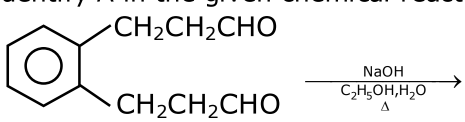
11. The correct order of electron gain enthalpy is:

(1) S > Se > Te > O
(2) O > S > Se > Te
(3) S > O > Se > Te
(4) Te > Se > S > O

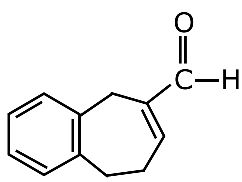
Ans. **(1)**

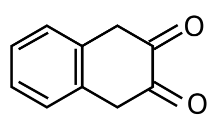
Sol. Electron gain enthalpy of O is very low due to small size.

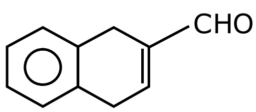
12. Identify A in the given chemical reaction.

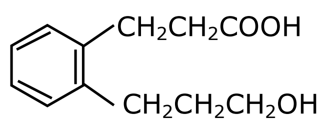


A (Major product)

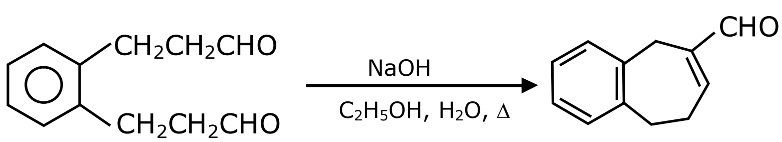
(1) 

(2) 

(3) 

(4) 

Ans. **(1)**

Sol. 

(Internal aldol condensation)

13. Match List-I with List-II

List-I	**List-II**
(a) Siderite	(i) Cu
(b) Calamine	(ii) Ca
(c) Malachite	(iii) Fe
(d) Cryolite	(iv) Al
	(v) Zn

Choose the correct answer from the options given below:

(1) (a)-(i), (b)-(ii), (c)-(v), (d)-(iii)
(2) (a)-(iii), (b)-(v), (c)-(i), (d)-(iv)
(3) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
(4) (a)-(iii), (b)-(i), (c)-(v), (d)-(ii)

Ans. **(2)**

Sol.

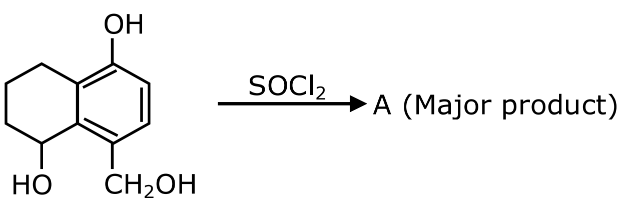
Siderite - $FeCO_3$

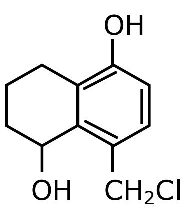
Calamine - $ZnCO_3$

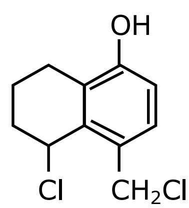
Malachite - $CuCO_3.Cu(OH)_2$

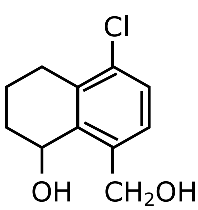
Cryolite - Na_3AlF_6

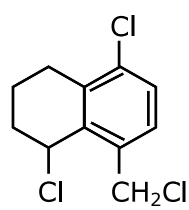
14. Identify A in the given reaction



(1) 

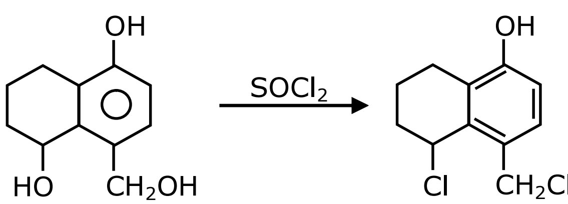
(2) 

(3) 

(4) 

Ans. **(2)**

Sol.



15. Match List-I with List-II.

List-I	**List-II**
(a) Sodium Carbonate	(i) Deacon
(b) Titanium	(ii) Caster-Kellner
(c) Chlorine	(iii) Van-Arkel
(d) Sodium hydroxide	(iv) Solvay

Choose the correct answer from the option given below:

(1) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)

(2) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

(3) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

(4) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

Ans. **(2)**

Sol. Sodium carbonate Na_2CO_3 & $NaHCO_3$

Titanium : Van arkel method

$$T_i + I_2 \xrightarrow{T1} T_i \underset{(g)}{I_4}$$

$$T_i \underset{(g)}{I_4} \xrightarrow[T_2 > T_1]{T_2} \underset{\substack{\text{Refined} \\ \text{titanium}}}{T_i} + 2\underset{(g)}{I_2}$$

Chlorine : Decon's process

$$HCl + O_2 \xrightarrow{CuCl_2} H_2O + Cl_2$$

Sodium hydroxide :- Caster-Kellner cell

16. Match List-I with List-II.

List-I (Molecule)	**List-II (Bond order)**
(a) Ne_2	(i) 1
(b) N_2	(ii) 2
(c) F_2	(iii) 0
(d) O_2	(iv) 3

Choose the correct answer from the options given below:

(1) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii) (2) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

(3) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii) (4) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

Ans. **(1)**

Sol.

Ne_2O	BO = 0
N_2	BO = 3
F_2	BO = 1
O_2	BO = 2

As per molecular orbital theory

17. Which of the following forms of hydrogen emits low energy β^- particles?

(1) Proton H^+

(2) Deuterium ${}^2_1H$

(3) Protium ${}^1_1H$

(4) Tritium ${}^3_1H$

Ans. **(4)**

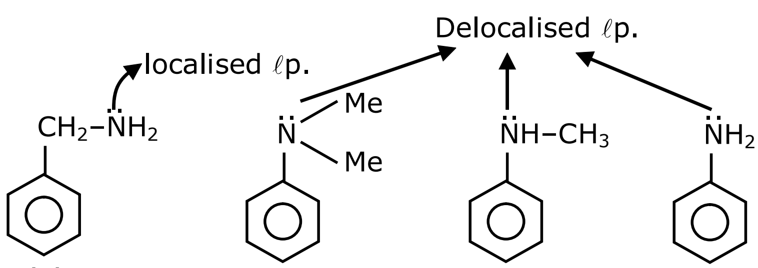
Sol. Tritium isotope of hydrogen is radioactive and emits low energy β^- particles. It is because of high n/p ratio of tritium which makes nucleus unstable.

18. A. Phenyl methanamine
B. N, N-Dimethylaniline
C. N-Methyl aniline
D. Benzenamine
Choose the correct order of basic nature of the above amines.

(1) D > C > B > A
(2) D > B > C > A
(3) A > C > B > D
(4) A > B > C > D

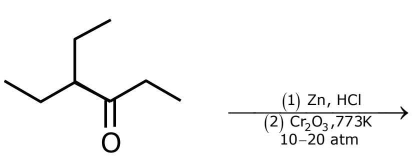
Ans. **(4)**

Sol.

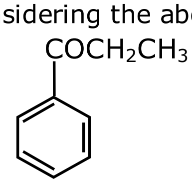


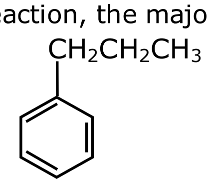
(a) (b) (c) (d)

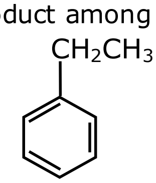
19.

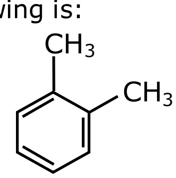


Considering the above reaction, the major product among the following is:

(1) 

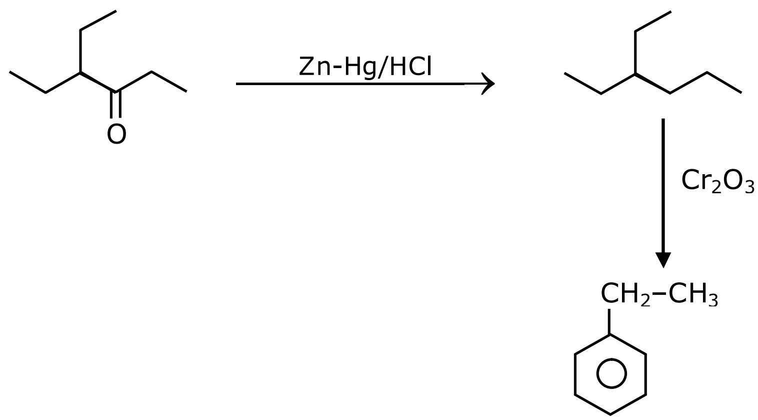
(2) 

(3) 

(4) 

Ans. **(3)**

Sol.



(Aromatisation)

20. Seliwanoff test and Xanthoproteic test are used for the identification of ___________ and ____________ respectively

(1) ketoses, proteins
(2) proteins, ketoses
(3) aldoses, ketoses
(4) ketoses, aldoses

Ans. **(1)**

Sol. Seliwanoff test and Xanthaproteic test are used for identification of 'Ketoses' and proteins respectively.

Section - B

1. The $NaNO_3$ weighed out to make 50 mL of an aqueous solution containing 70.0 mg Na^+ per mL is_________g. (Rounded off to the nearest integer)
[Given: Atomic weight in g mol^{-1}. Na: 23; N: 14; O : 16]

Ans. **13**

Sol. Na^+ = 70 mg/mL

W_{Na^+} in 50mL solution = 70 × 50mg

= 3500 mg

= 3.5 gm

Moles of Na^+ in 50 ml solution = $\frac{3.5}{23}$

Moles of $NaNO_3$ = moles of Na^+

$= \frac{3.5}{23}$ mol

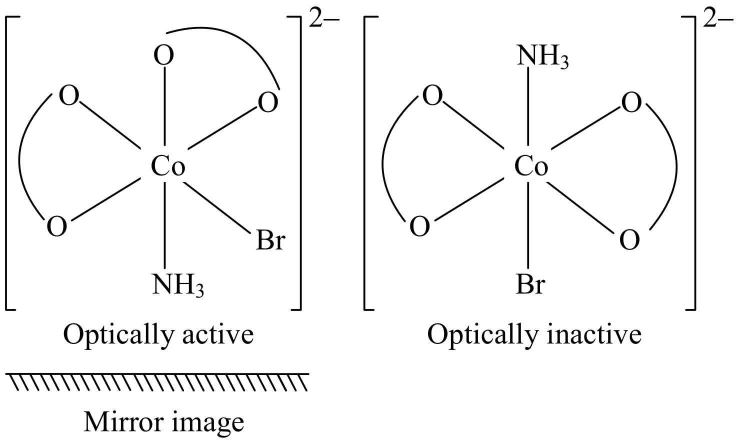
Mass of $NaNO_3$ = $\frac{3.5}{23} \times 85 = 12.934$

$\simeq$ 13gm Ans.

2. The number of stereoisomers possible for $[Co(ox)_2(Br)(NH_3)]^{2-}$ is ____________[ox = oxalate]

Ans. 3

Sol. $\left[Co(ox)_2 Br(NH_3)\right]^{2-}$



Total stereoisomer = 2 (OI) + 1 POE (pair of enantiomers) = 3

3. The average S–F bond energy in kJ mol^{-1} of SF_6 is __________. (Rounded off to the nearest integer)

[Given : The values of standard enthalpy of formation of $SF_6(g)$, S(g) and F(g) are - 1100, 275 and 80 kJ mol^{-1} respectively.]

Ans. 309

Sol. $SF_6(g) \longrightarrow S(g) + 6F(g)$

$$\Delta H^o_{reaction} = 6 \times E_{S-F} = \Delta H^o_f[S(g)] + 6 \times \Delta H^o_f[F(g)] - \Delta H^o_f[SF_6(g)]$$

$6 \times E_{S-F} = 275 + 6 \times 80 - (-1100)$

$= 275 + 480 + 1100$

$6 \times E_{S-F} = 1855$

$E_{S-F} = \dfrac{1855}{6} = 309.1667$

$\simeq 309$ kJ/mol Ans.

4. Emf of the following cell at 298 K in V is x $\times 10^{-2}$.

$Zn|Zn^{2+}$ (0.1 M)||Ag^{+}(0.01 M)| Ag

The value of x is __________. (Rounded off to the nearest integer)

[Given: $E^0_{Zn^{2+}/Zn} = -0.76V; E^0_{Ag^+/Ag} = +0.80V; \frac{2.303RT}{F} = 0.059$]

Ans. 147

Sol. $Zn(s)\left|Zn^{+2}(0.1M)\right|\left|Ag^{+}(0.01M)\right|Ag(s)$

$$Zn(s) + 2Ag^{+} \rightleftharpoons 2Ag(s) + Zn^{+2}$$

$$E^0 = 0.80 + 0.76 = 1.56 \ ; \quad Q = \left\{\frac{Zn^{2+}}{(Ag^{+})^2}\right\}$$

$$E = E^0 - \frac{0.059}{n}\log(Q)$$

$$E = 1.56 - \frac{0.059}{2}\log\left[\frac{0.1}{(0.01)^2}\right]$$

$$E = 1.56 - \frac{0.059}{2}\log\left[(10)^3\right]$$

$$E = 1.4715 = 147.15 \times 10^{-2}\text{volt}$$

$$= x \times 10^{-2}$$

X = 147.15 $\simeq$ 147 Ans.

5. A ball weighing 10g is moving with a velocity of $90ms^{-1}$. If the uncertainty in its velocity is 5%, then the uncertainty in its position is _________$\times 10^{-33}$m. (Rounded off to the nearest integer)

[Given : h = 6.63×10^{-34} Js]

Ans. 1

Sol. m = 10 g = 10^{-2} Kg

v = 90 m/sec.

$$\Delta v = v \times 5\% \quad = 90 \times \frac{5}{100} = 4.5\,m/sec$$

$$m.\Delta v.\Delta x \geq \frac{h}{4\pi}$$

$$10^{-2} \times 4.5 \times \Delta x \geq \frac{6.63 \times 3 \times 10^{-34}}{4 \times \frac{22}{7}}$$

$$\Delta x \geq \frac{6.63 \times 7 \times 2 \times 10^{-34}}{9 \times 4 \times 22 \times 10^{-2}}$$

$$\Delta x \geq 1.17 \times 10^{-33} = x \times 10^{-33}$$

$$x = 1.17 \simeq 1$$

6. In mildly alkaline medium, thiosulphate ion is oxidized by MnO_4^- to "A". The oxidation state of sulphur in "A" is________.

Ans. **6**

Sol. $S_2O_3^{2-} + MnO_4^- \xrightarrow[\text{Medium}]{\text{Alkaline}} A$

$A \rightarrow SO_4^{-2}$

$\therefore$ Oxidation no. of 'S' = +6 Ans.

7. When 12.2 g of benzoic acid is dissolved in 100g of water, the freezing point of solution was found to be –0.93°C (K_f (H_2O) = 1.86 K kg mol^{-1}). The number (n) of benzoic acid molecules associated (assuming 100% association) is_____________.

Ans. **2**

Sol. n PhCOOH $\rightarrow$ $(PhCOOH)_n$

$N = \frac{1}{x} = i \left\{ As \quad \alpha = 1 \right\}$

$\Delta T_f = i \times k_f \times m$

$0.93 = \frac{1}{n} \times 1.86 \times \frac{12.2 \times 1000}{122 \times 100}$

n = 2

8. If the activation energy of a reaction is 80.9 kJ mol^{-1}, the fraction of molecules at 700K, having enough energy to react to form products is e^{-x}. The value of x is ______.

(Rounded off to the nearest integer)

[Use R = 8.31 JK^{-1} mol^{-1}]

Ans. **14**

Sol. $E_a = 80.9 kJ / mol$

Fraction of molecules able to cross energy barrier = $e^{-E_a/RT} = e^{-x}$

x = $\frac{E_a}{RT} = \frac{80.9 \times 1000}{8.31 \times 700} = 13.91$

$x \simeq 14$ Ans

9. The pH of ammonium phosphate solution, if pk_a of phosphoric acid and pk_b of ammonium hydroxide are 5.23 and 4.75 respectively, is_____________.

Ans. **7**

Sol. $(NH_4)_3PO_4 \rightleftharpoons 3NH_4^+ + PO_4^{3-}$

$$\left[H^+\right] = K_a \times \sqrt{\frac{kw}{k_a \times k_b}}$$

$$pH = pk_a + \frac{1}{2}\left\{pk_w - pk_a - pk_b\right\}$$

$$pH = 5.23 + \frac{1}{2}\{14 - 5.23 - 4.75\}$$

$$pH = 5.23 + \frac{1}{2}\ (4.02) = 7.24 = 7 \text{(Nearest integer)}$$

10. The number of octahedral voids per lattice site in a lattice is __________.

(Rounded off to the nearest integer)

Ans. **1**

Sol. Assuming FCC

No of lactice sites = 6 face centre + 8 corner = 14

No. of octahedral voids = 13

Ratio $= \frac{13}{14} = 0.92857 = 1$ (Nearest integer)

26^{th} Feb. 2021 | Shift - 2
MATHEMATICS

1. Let L be a line obtained from the intersection of two planes $x + 2y + z = 6$ and $y + 2z = 4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from (3, 2, 1) on L, then the value of $21(\alpha + \beta + \gamma)$ equals :

(1) 142

(2) 68

(3) 136

(4) 102

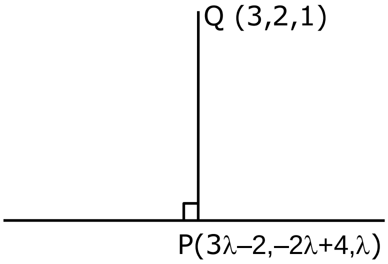
Ans. (4)

Sol. Dr's of line $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 3\hat{i} - 2\hat{j} + \hat{k}$

Dr/s :- $(3, -2, 1)$

Points on the line $(-2, 4, 0)$

Equation of the line $\dfrac{x+2}{3} = \dfrac{y-4}{-2} = \dfrac{z}{1} = \lambda$



Dr's of PQ ; $3\lambda-5, -2\lambda+2. \lambda-1$

Dr's of y lines are $(3, -2, 1)$

Since PQ $\perp$ line

$3(3\lambda - 5) - 2(-2\lambda + 2) + 1(\lambda - 1) = 0$

$\lambda = \dfrac{10}{7}$

$P\left(\dfrac{16}{7}, \dfrac{8}{7}, \dfrac{10}{7}\right)$

$21(\alpha + \beta + \gamma) = 21\left(\dfrac{34}{7}\right) = 102$

2. The sum of the series $\sum_{n=1}^{\infty}\frac{n^2+6n+10}{(2n+1)!}$ is equal to :

(1) $\frac{41}{8}$ e + $\frac{19}{8}$ e^{-1} – 10

(2) $-\frac{41}{8}$ e + $\frac{19}{8}$ e^{-1} – 10

(3) $\frac{41}{8}$ e – $\frac{19}{8}$ e^{-1} – 10

(4) $\frac{41}{8}$ e + $\frac{19}{8}$ e^{-1} + 10

Ans. (3)

Sol. $\sum_{n=1}^{\infty}\frac{n^2+6n+10}{(2n+1)!}$

Put 2n + 1 = r, where r = 3,5,7,...

$\Rightarrow n = \frac{r-1}{2}$

$$\frac{n^2-6n+10}{(2n+1)!}=\frac{\left(\frac{r-1}{2}\right)^2+3r-3+10}{r!}=\frac{r^2+10r+29}{4r!}$$

Now $\sum_{r=3,5,7}\frac{r(r-1)+11r+29}{4r!}=\frac{1}{4}\sum_{r=3,5,7,......}\left(\frac{1}{(r-2)!}+\frac{11}{(r-1)!}+\frac{29}{r!}\right)$

$$=\frac{1}{4}\left\{\left(\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+.....\right)+11\left(\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+....\right)+29\left(\frac{1}{3!}+\frac{1}{5!}+\frac{1}{7!}+.....\right)\right\}$$

$$=\frac{1}{4}\left\{\frac{e-\frac{1}{e}}{2}+11\left(\frac{e+\frac{1}{e}-2}{2}\right)+29\left(\frac{e-\frac{1}{e}-2}{2}\right)\right\}$$

$$=\frac{1}{8}\left\{e-\frac{1}{e}+11e+\frac{11}{e}-22+29e-\frac{29}{e}-58\right\}$$

$$=\frac{1}{8}\left\{41e-\frac{19}{e}-80\right\}$$

3. Let f(x) be a differentiable function at x = a with f′(a) = 2 and f(a) = 4. Then $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$ equals :

(1) 2a + 4

(2) 2a – 4

(3) $4 - 2a$

(4) $a + 4$

Ans. **(3)**

Sol. By L-H rule

$$L = \lim_{x \to a} \frac{f(a) - af'(x)}{1}$$

$\therefore L = 4 - 2a$

4. Let A (1, 4) and B(1, −5) be two points. Let P be a point on the circle $(x - 1)^2 + (y - 1)^2 = 1$ such that $(PA)^2 + (PB)^2$ have maximum value, then the points, P, A and B lie on :

(1) a parabola

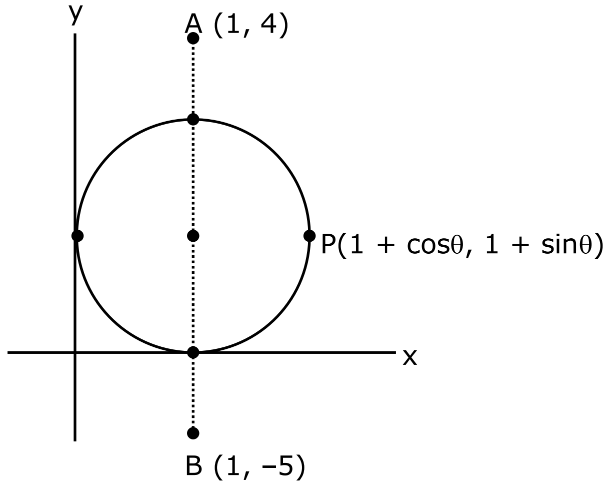
(2) a straight line

(3) a hyperbola

(4) an ellipse

Ans. **(2)**

Sol.



$\therefore PA^2 = \cos^2\theta + (\sin\theta - 3)^2 = 10 - 6\sin\theta$

$PB^2 = \cos^2\theta + (\sin\theta - 6)^2 = 37 - 12\sin\theta$

$PA^2 + PB^2 |_{max.} = 47 - 18\sin\theta|_{min.} \Rightarrow \theta = \dfrac{3\pi}{2}$

$\therefore$ P, A, B lie on a line $x = 1$

5. If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of the radius r, then r is equal to :

(1) $\frac{1}{4}$

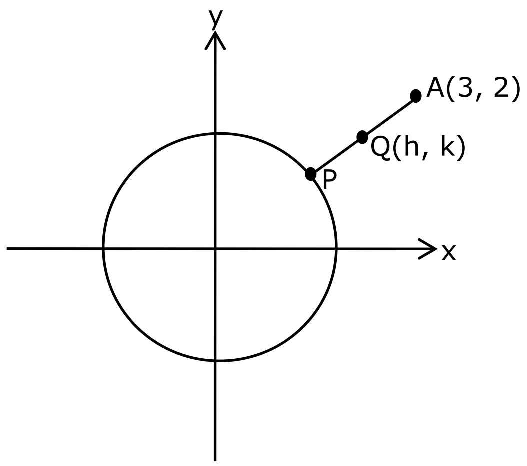
(2) $\frac{1}{2}$

(3)1

(4) $\frac{1}{3}$

Ans. (2)

Sol.



$\therefore$ P $\equiv$ (2h – 3, 2k – 2) $\rightarrow$on circle

$\therefore \left(h - \frac{3}{2}\right)^2 + (k-1)^2 = \frac{1}{4}$

$\Rightarrow$ radius = $\frac{1}{2}$

6. Let slope of the tangent line to a curve at any point P(x, y) be given by $\frac{xy^2 + y}{x}$. If the curve intersects the line x + 2y = 4 at x = – 2, then the value of y, for which the point (3, y) lies on the curve, is :

(1) $-\frac{18}{11}$

(2) $-\frac{18}{19}$

(3) $-\frac{4}{3}$

(4) $\frac{18}{35}$

Ans. (2)

Sol. $\frac{dy}{dx} = \frac{xy^2 + y}{x}$

$\Rightarrow \frac{xdy - ydx}{y^2} = xdx$

$\Rightarrow -d\left(\frac{x}{y}\right) = d\left(\frac{x^2}{2}\right)$

$\Rightarrow \frac{-x}{y} = \frac{x^2}{2} + C$

Curve intersect the line x + 2y = 4 at x = – 2

So, –2 + 2y = 4 $\Rightarrow$ y = 3

So the curve passes through (–2, 3)

$\Rightarrow \frac{2}{3} = 2 + C$

$\Rightarrow C = \frac{-4}{3}$

$\therefore$ curve is $\frac{-x}{y} = \frac{x^2}{2} - \frac{4}{3}$

It also passes through (3, y)

$\frac{-3}{y} = \frac{9}{2} - \frac{4}{3}$

$\Rightarrow \frac{-3}{y} = \frac{19}{6}$

$\Rightarrow y = -\frac{18}{19}$

7. Let A_1 be the area of the region bounded by the curves y = sinx, y = cos x and y-axis in the first quadrant. Also, let A_2 be the area of the region bounded by the curves y = sin x, y = cos x, x-axis and x= $\frac{\pi}{2}$ in the first quadrant. Then,

(1) $A_1 = A_2$ and $A_1 + A_2 = \sqrt{2}$

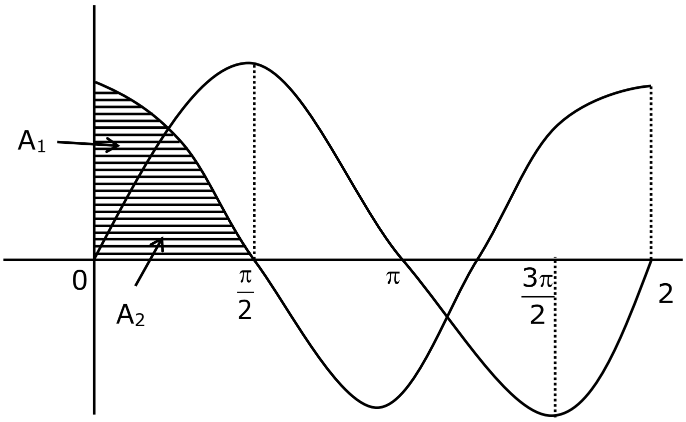
(2) $A_1 : A_2 = 1 : 2$ and $A_1 + A_2 = 1$

(3) $2A_1 = A_2$ and $A_1 + A_2 = 1 + \sqrt{2}$

(4) $A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$

Ans. **(4)**

Sol. $A_1 + A_2 = \int_0^{\pi/2} \cos x.dx = \sin x|_0^{\pi/2} = 1$



$A_1 = \int_0^{\pi/4} (\cos x - \sin x)\,dx = (\sin x + \cos x)|_0^{\pi/4} = \sqrt{2} - 1$

$\therefore\ A_2 = 1 - \left(\sqrt{2} - 1\right) = 2 - \sqrt{2}$

$\therefore\ \frac{A_1}{A_2} = \frac{\sqrt{2} - 1}{\sqrt{2}\left(\sqrt{2} - 1\right)} = \frac{1}{\sqrt{2}}$

8. If $0 < a, b < 1$, and $\tan^{-1} a + \tan^{-1} b = \frac{\pi}{4}$, then the value of

$(a + b) - \left(\frac{a^2 + b^2}{2}\right) + \left(\frac{a^3 + b^3}{3}\right) - \left(\frac{a^4 + b^4}{4}\right) + ..$ is :

(1) $\log_e 2$

(2) $\log_e \left(\frac{e}{2}\right)$

(3) e

(4) $e^2 - 1$

Ans. **(1)**

Sol. $\tan^{-1}\left(\frac{a + b}{1 - ab}\right) = \frac{\pi}{4} \Rightarrow a + b = 1 - ab \Rightarrow (1 + a)(1 + b) = 2$

Now, $(a + b) - \left(\frac{a^2 + b^2}{2}\right) + \left(\frac{a^3 + b^3}{3}\right) .... \infty$

$= \left(a - \frac{a^2}{2} + \frac{a^3}{3} ....\right) + \left(b - \frac{b^2}{2} + \frac{b^3}{3} ....\right)$

$\log_e (1 + a) + \log_e (1 + b) = \log_e (1 + a)(1 + b) = \log_e 2$

9. Let $F_1(A, B, C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \to \sim A)$ be two logical expressions. Then :

(1) F_1 is not a tautology but F_2 is a tautology

(2) F_1 is a tautology but F_2 is not a tautology

(3) F_1 and F_2 both area tautologies

(4) Both F_1 and F_2 are not tautologies

Ans. (1)

Sol. Truth table for F_1

A	B	C	~A	~B	~C	$A \vee \sim B$	$A \vee B$	$\sim C \vee (A \vee B)$	$[\sim C \wedge (A \vee B)] \vee \sim A$	$(A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$
T	T	T	F	F	F	F	T	F	F	F
T	T	F	F	F	T	F	T	T	T	T
T	F	T	F	T	F	T	T	F	F	T
T	F	F	F	T	T	T	T	T	T	T
F	T	T	T	F	F	F	T	F	T	T
F	T	F	T	F	T	F	T	T	T	T
F	F	T	T	T	F	F	F	F	T	T
F	F	F	T	T	T	F	F	F	T	T

Not a tautology

Truth table for F_2

A	B	$A \vee B$	$\sim A$	$B \to \sim A$	$(A \vee B) \vee (B \to \sim A)$
T	T	T	F	F	T
T	F	T	F	T	T
F	T	T	T	T	T
F	F	F	T	T	T

F_1 not shows tautology and F_2 shows tautology

10. Consider the following system of equations :

$x + 2y - 3z = a$

$2x + 6y - 11z = b$

$x - 2y + 7z = c$,

Where a, b and c are real constants. Then the system of equations :

(1) has a unique solution when $5a = 2b + c$

(2) has infinite number of solutions when $5a = 2b + c$

(3) has no solution for all a, b and c

(4) has a unique solution for all a, b and c

Ans. (2)

Sol. $D = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{vmatrix}$

$= 20 - 2(25) - 3(-10)$

$= 20 - 50 + 30 = 0$

$D_1 = \begin{vmatrix} a & 2 & -3 \\ b & 6 & -11 \\ c & -2 & 7 \end{vmatrix}$

$= 20a - 2(7b + 11c) - 3(-2b - 6c)$

$= 20a - 14b - 22c + 6b + 18c$

$= 20a - 8b - 4c$

$= 4(5a - 2b - c)$

$$D_2 = \begin{vmatrix} 1 & a & -3 \\ 2 & b & -11 \\ 1 & c & 7 \end{vmatrix}$$

$= 7b + 11c - a(25) - 3(2c - b)$

$= 7b + 11c - 25a - 6c + 3b$

$= -25a + 10b + 5c$

$= -5(5a - 2b - c)$

$$D_3 = \begin{vmatrix} 1 & 2 & a \\ 2 & 6 & b \\ 1 & -2 & c \end{vmatrix}$$

$= 6c + 2b - 2(2c - b) - 10a$

$= -10a + 4b + 2c$

$= -2(5a - 2b - c)$

for infinite solution

$D = D_1 = D_2 = D_3 = 0$

$\Rightarrow 5a = 2b + c$

11. A seven digit number is formed using digit 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is :

(1) $\frac{6}{7}$

(2) $\frac{4}{7}$

(3) $\frac{3}{7}$ (4) $\frac{1}{7}$

Ans. (3)

Sol. $n(s) = \frac{7!}{2!3!2!}$

$n(E) = \frac{6!}{2!2!2!}$

$P(E) = \frac{n(E)}{n(S)} = \frac{6!}{7!} \times \frac{2!3!2!}{2!2!2!}$

$\frac{1}{7} \times 3 = \frac{3}{7}$

12. If vectors $\vec{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}_2 = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is :

(1) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

(2) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$

(3) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

(4) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$

Ans. (3)

Sol. $\frac{x}{1} = -\frac{1}{y} = \frac{1}{z} = \lambda(\text{let})$

Unit vector parallel to $x\hat{i} + y\hat{j} + z\hat{k} = \pm \frac{\left(\lambda\hat{i} - \frac{1}{\lambda}\hat{j} + \frac{1}{\lambda}\hat{k}\right)}{\sqrt{\lambda^2 + \frac{2}{\lambda^2}}}$

For $\lambda = 1$, it is $\pm\frac{\left(\hat{i} - \hat{j} + \hat{k}\right)}{\sqrt{3}}$

13. For $x>0$, if $f(x) = \int_1^x \frac{\log_e t}{(1+t)} dt$, then $f(e) + f\left(\frac{1}{e}\right)$ is equal to :

(1) $\frac{1}{2}$

(2) −1

(3) 1

(4) 0

Ans. (1)

Sol. $f(e) + f\left(\frac{1}{e}\right) = \int_1^e \frac{\ell nt}{1+t} dt + \int_1^{1/e} \frac{\ell nt}{1+t} dt = I_1 + I_2$

$I_2 = \int_1^{1/e} \frac{\ell nt}{1+t} dt$ put $t = \frac{1}{z}, dt = -\frac{dz}{z^2}$

$= \int_1^e -\frac{\ell nz}{1+\frac{1}{z}} \times \left(-\frac{dz}{z^2}\right) = \int_1^e \frac{\ell nz}{z(z+1)} dz$

$$f(e)+f\left(\frac{1}{e}\right)=\int_1^e \frac{\ell nt}{1+t}dt+\int_1^e \frac{\ell nt}{t(t+1)}\ dt=\int_1^e \frac{\ell nt}{1+t}+\frac{\ell nt}{t(t+1)}dt$$

$$=\int_1^e \frac{\ell nt}{t}dt\ \{\ln t=u, \frac{1}{t}dt\}$$

$$=du=\int_0^1 u\ du=\left.\frac{u^2}{2}\right|_0^1=\frac{1}{2}$$

14. Let f : R $\rightarrow$ R be defined as $f(x) = \begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), & \text{if } x<-1 \\ |ax^2+x+b|, & \text{if } -1 \le x \le 1 \\ \sin(\pi x) & \text{if } x>1 \end{cases}$

If f(x) is continuous on R, then a + b equals :
(1) 3
(2) −1
(3) −3
(4) 1

Ans. **(2)**
Sol. If f is continuous at x = −1, then
$f(-1^-) = f(-1)$

$\Rightarrow 2 = |a - 1 + b|$

$\Rightarrow |a + b - 1| = 2$ ..... (i)

similarly

$f(1^-) = f(1)$

$\Rightarrow |a + b + 1| = 0$

$\Rightarrow a + b = -1$

15. Let A = {1,2,3……,10}and f: A$\rightarrow$ A be defined as $f(k)=\begin{cases} k+1 & \text{if } k \text{ is odd} \\ k & \text{if } k \text{ is even} \end{cases}$ Then the number of possible functions g : A$\rightarrow$A such that gof = f is :
(1) 10^5
(2) ${}^{10}C_5$
(3)5^5
(4) 5!

Ans. (1)

Sol. $g(f(x)) = f(x)$

$\Rightarrow g(x) = x$, when x is even.

5 elements in A can be mapped to any 10

So, $10^5 \times 1 = 10^5$

16. A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$, $y > z$. Then the number of odd divisors of n, including 1, is :

(1) 11

(2) 6x

(3)12

(4) 6

Ans. (3)

Sol. $y + z = 5$ ...(1)

$$\frac{1}{y} + \frac{1}{z} = \frac{5}{6}$$

$$\Rightarrow \frac{y+z}{yz} = \frac{5}{6}$$

$$\Rightarrow \frac{5}{yz} = \frac{5}{6}$$

$\Rightarrow yz = 6$

Also $(y - z)^2 = (y + z)^2 - 4yz$

$\Rightarrow (y - z)^2 = (y + z)^2 - 4yz$

$\Rightarrow (y - z)^2 = 25 - 4(6) = 1$

$\Rightarrow y - z = 1$ ...(2)

from (1) and (2), $y = 3$ and $z = 2$

for calculating odd divisor of $p = 2^x.3^y.5^z$

x must be zero

$P = 2^0.3^3.5^2$

$\therefore$ total odd divisors must be $(3 + 1)(2 + 1) = 12$

17. Let f(x) = $\sin^{-1} x$ and g(x) = $\frac{x^2-x-2}{2x^2-x-6}$. If g(2) = $\lim_{x\to 2} g(x)$, then the domain of the function fog is :

(1) $(-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$

(2) $(-\infty, -1] \cup [2, \infty)$

(3) $(-\infty, -2] \cup [-1, \infty)$

(4) $(-\infty, -2] \cup \left[-\frac{3}{2}, \infty\right)$

Ans. (1)

Sol. $g(2) = \lim_{x\to 2} \frac{(x-2)(x+1)}{(2x+3)(x-2)} = \frac{3}{7}$

For domain of fog (x)

$\left|\frac{x^2-x-2}{2x^2-x-6}\right| \le 1 \Rightarrow (x+1)^2 \le (2x+3)^2 \Rightarrow 3x^2 + 10x + 8 \ge 0$

$\Rightarrow (3x+4)(x+2) \ge 0$

$x \in (-\infty, -2] \cup \left(-\frac{4}{3}, \infty\right]$

18. If the mirror image of the point (1,3,5) with respect to the plane $4x-5y+2z = 8$ is (α, β, γ), then $5(\alpha+\beta+\gamma)$ equals:

(1) 47

(2) 39

(3) 43

(4) 41

Ans. (1)

Sol. Image of (1, 3, 5) in the plane $4x - 5y + 2z = 8$ is (α, β, γ)

$\Rightarrow \frac{\alpha-1}{4} = \frac{\beta-3}{-5} = \frac{\gamma-5}{2} = -2\,\frac{(4(1)-5(3)+2(5)-8)}{4^2+5^2+2^2} = \frac{2}{5}$

$\therefore \alpha = 1 + 4\left(\frac{2}{5}\right) = \frac{13}{5}$

$\beta = 3 - 5\left(\frac{2}{5}\right) = 1 = \frac{5}{5}$

$\gamma = 5 + 2\left(\frac{2}{5}\right) = \frac{29}{5}$

Thus, $5(\alpha+\beta+\gamma) = 5\left(\frac{13}{5}+\frac{5}{5}+\frac{29}{5}\right) = 47$

19. Let $f(x)=\int_0^x e^t f(t)dt + e^x$ be a differentiable function for all $x \in R$. Then f(x)equals.

(1) $2e^{(e^x-1)}-1$

(2) $e^{(e^x-1)}$

(3) $2e^{e^x}-1$

(4) $e^{e^x}-1$

Ans. **(1)**

Sol. Given, $f(x)=\int_0^x e^t f(t)dt + e^x$ ...(1)

Differentiating both sides w.r.t x

$f'(x) = e^x \cdot f(x) + e^x$ (Using Newton Leibnitz Theorem)

$\Rightarrow \frac{f'(x)}{f(x)+1} = e^x$

Integrating w.r.t x

$\int \frac{f'(x)}{f(x)+1}dx = \int e^x dx$

$\Rightarrow \ell n\ (f(x) + 1) = e^x + c$

Put x = 0

$\ell n\ 2 = 1 + c$ ($\because f(0) = 1$, from equation (1))

$\therefore \ell n(f(x) + 1) = e^x + \ell n 2 - 1$

$\Rightarrow f(x) + 1 = 2.\ e^{e^x-1}$

$\Rightarrow f(x) = 2e^{e^x-1} - 1$

20. The triangle of maximum area that can be inscribed in a given circle of radius 'r' is:

(1) A right angle triangle having two of its sides of length 2r and r.

(2) An equilateral triangle of height $\frac{2r}{3}$.

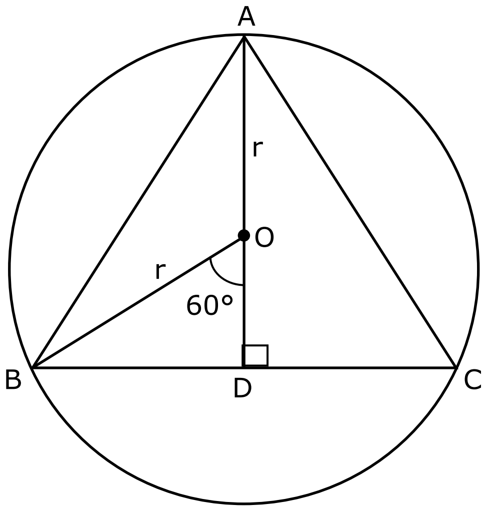
(3) An isosceles triangle with base equal to 2r.

(4) An equilateral triangle having each of its side of length $\sqrt{3}$ r.

Ans. **(4)**

Sol. Triangle of maximum area that can be inscribed in a circle is an equilateral triangle.

Let ΔABC be inscribed in circle,



Now, in ΔOBD

$OD = r\cos 60° = \frac{r}{2}$

Height $= AD = \frac{3r}{2}$

Again in ΔABD

Now $\sin 60° = \frac{3\frac{r}{2}}{AB}$

$\Rightarrow AB = \sqrt{3}\,r$

Section - B

1. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is

Ans. 1000

Sol. Since, required number has G.C.D with 18 as 3. It must be odd multiple of '3' but not a multiple of '9'.

(i) Now, 4-digit number which are odd multiple of '3' are,
1005,1011,1017, ... 9999 $\rightarrow$ 1499

(ii) 4–digit number which are odd multiple of 9 are,
1017, 1035, ... 9999 $\rightarrow$ 499

$\because$ Required numbers = 1499–499 = 1000

2. Let α and β be two real numbers such that $\alpha+\beta=1$ and $\alpha\beta = -1$. Let $P_n = (\alpha)^n + (\beta)^n$, $P_{n-1} = 11$ and $P_{n+1} = 29$ for some integer $n \geq 1$. Then, the value of P_n^2 is _______________.

Ans. 324

Sol. Given, $\alpha + \beta = 1$, $\alpha\beta = -1$

$\therefore$ Quadratic equation with roots α,β is $x^2-x-1 = 0$

$\Rightarrow \alpha^2 = \alpha + 1$

Multiplying both sides by α^{n-1}

$\alpha^{n+1} = \alpha^n + \alpha^{n-1}$ ________ (1)

Similarly,
$\beta^{n+1} = \beta^n + \beta^{n-1}$ ________ (2)
Adding (1) & (2)
$\alpha^{n+1} + \beta^{n+1} = (\alpha^n+\beta^n) + (\alpha^{n-1}+\beta^{n-1})$
$\Rightarrow P_{n+1} = P_n + P_{n-1}$
$\Rightarrow 29 = P_n + 11$ (Given, $P_{n+1} = 29$, $P_{n-1} = 11$)
$\Rightarrow P_n = 18$
$\therefore P_n^2 = 18^2 = 324$

3. Let X_1, X_2,.......... X_{18} be eighteen observation such that $\sum_{i=1}^{18}(X_i-\alpha)=36$ and $\sum_{i=1}^{18}(X_i-\beta)^2=90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is ______________.

Ans. **4**

Sol. Given, $\sum_{i=1}^{18}(X_i-\alpha)=36$

$\Rightarrow \sum x_i - 18\alpha = 36$

$\Rightarrow \sum x_i - 18(\alpha+2)$ ...(1)

Also, $\sum_{i=1}^{18}(X_i-\beta)^2=90$

$\Rightarrow \sum x_i^2 + 18\beta^2 - 2\beta\sum x_i = 90$

$\Rightarrow \sum x_i^2 + 18\beta^2 + 2\beta\times 18(\alpha+2) = 90$ (using equation (1))

$\Rightarrow \sum x_i^2 = 90 - 18\beta^2 + 36\beta(\alpha+2)$

$\sigma^2 = 1 \Rightarrow \frac{1}{18}\sum x_i^2 - \left(\frac{\sum x_i}{18}\right)^2 = 1$ ($\because \sigma = 1$, given)

$\Rightarrow \frac{1}{18}(90 - 18\beta^2 + 36\alpha\beta + 72\beta) - \left(\frac{18(\alpha+2)}{18}\right)^2 = 1$

$\Rightarrow 90 - 18\beta^2 + 36\alpha\beta + 72\beta - 18(\alpha+2)^2 = 18$

$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - (\alpha+2)^2 = 1$

$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - \alpha^2 - 4 - 4\alpha = 1$

$\Rightarrow \alpha^2 - \beta^2 + 2\alpha\beta + 4\beta - 4\alpha = 0$

$\Rightarrow (\alpha-\beta)(\alpha-\beta+4) = 0$

$\Rightarrow \alpha - \beta = -4$

$\therefore |\alpha-\beta| = 4$ $(\alpha \neq \beta)$

4. In $I_{m,n} = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$, for m, n $\geq$ 1 and $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx = \alpha I_{m,n}$, $\alpha \in R$, then α equals_______________ .

Ans. **1**

Sol. $I_{m,n} = \int_0^1 x^{m-1}.(1-x)^{n-1}\,dx$

Put $x = \frac{1}{y+1} \Rightarrow dx = \frac{-1}{(y+1)^2}dy$

$1 - x = \frac{y}{y+1}$

$\therefore\ I_{m,n} = \int_\infty^0 \frac{y^{n-1}}{(y+1)^{m+n}}(-1)\,dy = \int_0^\infty \frac{y^{n-1}}{(y+1)^{m+n}}\,dy$ ...(i)

Similarly $I_{m,n} = \int_0^1 x^{n-1}.(1-x)^{m-1}\,dx$

$\Rightarrow I_{m,n} = \int_0^\infty \frac{y^{m-1}}{(y+1)^{m+n}}dy$ ...(ii)

From (i) & (ii)

$2I_{m,n} = \int_0^\infty \frac{y^{m-1}+y^{n-1}}{(y+1)^{m+n}}\,dy$

$\Rightarrow 2I_{m,n} = \underbrace{\int_0^1 \frac{y^{m-1}+y^{n-1}}{(y+1)^{m+n}}\,dy}_{I_1} + \underbrace{\int_1^\infty \frac{y^{m-1}+y^{n-1}}{(y+1)^{m+n}}\,dy}_{I_2}$

Put $y = \frac{1}{z}$ in I_2

$dy = -\frac{1}{z^2}\,dz$

$\Rightarrow 2I_{m,n} = \int_0^1 \frac{y^{m-1}+y^{n-1}}{(y+1)^{m+n}}\,dy + \int_1^0 \frac{z^{m-1}+z^{n-1}}{(z+1)^{m+n}}(-dz)$

$\Rightarrow\ I_{m,n} = \int_0^1 \frac{y^{m-1}+y^{n-1}}{(y+1)^{m+n}}\,dy \Rightarrow \alpha = 1$

5. Let L be a common tangent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is _______________.

Ans. **3**

Sol. $E : \frac{x^2}{9} + \frac{y^2}{4} = 1$ $\qquad C : x^2 + y^2 = \frac{31}{4}$

equation of tangent to ellipse is

$y = mx \pm \sqrt{9m^2+4}$ ...(i)

equation of tangent to circle is

$y = mx \pm \sqrt{\frac{31}{4}m^2 + \frac{31}{4}}$ ...(ii)

Comparing equation (i) & (ii)

$9m^2 + 4 = \frac{31}{4}m^2 + \frac{31}{4}$

$\Rightarrow 36m^2 + 16 = 31m^2 + 31$

$\Rightarrow 5m^2 = 15$

$\Rightarrow m^2 = 3$

6. If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satisfies the equation $A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for some real numbers α and β, then $\beta - \alpha$ is equal to_______________.

Ans. **4**

Sol. $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 3 & 0 & -1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

.
.
.
.
.

$A^{19} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{19} & 0 \\ 3 & 0 & -1 \end{bmatrix}, A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\text{L.H.S} = A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1+\alpha+\beta & 0 & 0 \\ 0 & 2^{20}+\alpha 2^{19}+2\beta & 0 \\ 3\alpha+3\beta & 0 & 1-\alpha-\beta \end{bmatrix}$$

R.H.S $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \alpha+\beta = 0$ and $2^{20} + \alpha 2^{19} + 2\beta = 4$

$\Rightarrow 2^{20} + \alpha\,(2^{19} - 2) = 4$

$\Rightarrow \alpha = \dfrac{4-2^{20}}{2^{19}-2} = -2$

$\Rightarrow \beta = 2$

$\therefore \beta - \alpha = 4$

7. If the arithmetic mean and geometric mean of the p^{th} and q^{th} terms of the sequence −16, 8, −4, 2, ………… satisfy the equation $4x^2 - 9x+5 = 0$, then p+q is equal to ______________.

Ans. 10

Sol. Given, $4x^2 - 9x + 5 = 0$

$\Rightarrow (x - 1)(4x - 5) = 0$

$\Rightarrow$ A.M = $\dfrac{5}{4}$, G.M = 1 (∵ A.M > G.M)

Again, for the series

−16, 8, −4, 2 ......

p^{th} term $t_p = -16\left(\dfrac{-1}{2}\right)^{p-1}$

q^{th} term $t_p = -16\left(\dfrac{-1}{2}\right)^{q-1}$

Now, A.M = $\dfrac{t_p + t_q}{2} = \dfrac{5}{4}$ & $\text{G.M} = \sqrt{t_p t_q} = 1$

$\Rightarrow 16^2\left(-\dfrac{1}{2}\right)^{p+q-2} = 1$

$\Rightarrow (-2)^8 = (-2)^{(p+q-2)}$

$\Rightarrow p + q = 10$

8. Let the normals at all the points on a given curve pass through a fixed point (a, b). If the curve passes through (3, -3) and $\left(4,-2\sqrt{2}\right)$, and given that $a-2\sqrt{2}$ b = 3, then (a^2+b^2+ab) is equal to______________.

Ans. **9**

Sol. Let the equation of normal is $Y - y = -\frac{1}{m}(X - x)$, where, m = $\frac{dy}{dx}$

As it passes through (a, b)

$b - y = -\frac{1}{m}(a - x) = -\frac{dx}{dy}(a - x)$

$\Rightarrow (b - y)dy = (x - a)dx$

by $-\frac{y^2}{2} = \frac{x^2}{2} - ax + c$ ...(i)

It passes through (3,-3) & $(4, -2\sqrt{2})$

$\therefore \quad -3b - \frac{9}{2} = \frac{9}{2} - 3a + c$

$\Rightarrow -6b - 9 = 9 - 6a + 2c$

$\Rightarrow 6a - 6b - 2c = 18$

$\Rightarrow 3a - 3b - c = 9$ ...(ii)

Also

$-2\sqrt{2}b - 4 = 8 - 4a + c$

$4a - 2\sqrt{2}\,b - c = 12$ ...(iii)

Also $a - 2\sqrt{2}\,b = 3$ ...(iv) (given)

(ii) − (iii) $\Rightarrow -a + \left(2\sqrt{2} - 3\right)b = -3$ ....(v)

(iv) + (v) $\Rightarrow b = 0, \quad a = 3$

$\therefore a^2 + b^2 + ab = 9$

9. Let z be those complex number which satisfy

$|z+5| \le 4$ and $z(1+i) + \bar{z}(1-i) \ge -10, i = \sqrt{-1}$.

If the maximum value of $|z+1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is ______________.

Ans. **48**

Sol. Given, $|z + 5| \le 4$

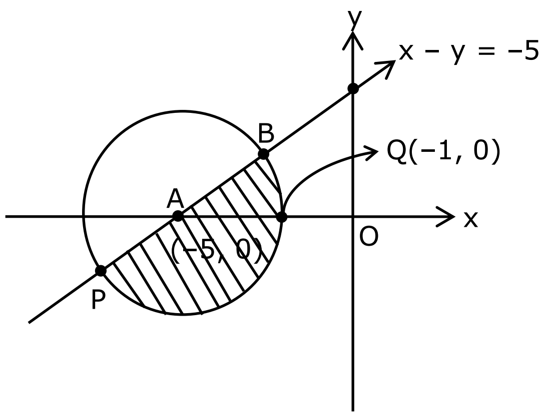
$\Rightarrow (x + 5)^2 + y^2 \le 16$ ...(1)

Also, $z(1+i) + \bar{z}(1-i) \ge -10$.

$\Rightarrow x - y \ge -5$ ...(2)

From (1) and (2)

Locus of z is the shaded region in the diagram.



$|z + 1|$ represents distance of 'z' from Q(−1, 0)

Clearly 'p' is the required position of 'z' when $|z + 1|$ is maximum.

$\therefore P \equiv \left(-5-2\sqrt{2}, -2\sqrt{2}\right)$

$\therefore (PQ)^2\Big|_{max} = 32 + 16\sqrt{2}$

$\Rightarrow \alpha = 32$

$\Rightarrow \beta = 16$

Thus, $\alpha + \beta = 48$

10. Let a be an integer such that all the real roots of the polynomial $2x^5+5x^4+10x^3+10x^2+10x+10$ lie in the interval (a, a + 1).
Then, |a| is equal to______________.

Ans. 2

Sol. Let, $f(x) = 2x^5+5x^4+10x^3+10x^2+10x+10$

$\Rightarrow f'(x) = 10\,(x^4 + 2x^3 + 3x^2 + 2x + 1)$

$$= 10\left(x^2 + \frac{1}{x^2} + 2\left(x + \frac{1}{x}\right) + 3\right)$$

$$= 10\left(\left(x + \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) + 1\right)$$

$$= 10\left(\left(x + \frac{1}{x}\right) + 1\right)^2 > 0; \forall x \in R$$

$\therefore$ f(x) is strictly increasing function. Since it is an odd degree polynomial it will have exactly one real root.

Now, by observation

$f(-1) = 3 > 0$

$f(-2) = -64 + 80 - 80 + 40 - 20 + 10$

$= -34 < 0$

$\Rightarrow$ f(x) has at least one root in $(-2,-1) \equiv (a, a + 1)$

$\Rightarrow a = -2$

$\Rightarrow |a| = 2$