QUESTION PAPER WITH SOLUTION

PHYSICS _ 4 Sep. _ SHIFT - 2

1. A circular coil has moment of inertia 0.8 kg m^2 around any diameter and is carrying current to produce a magnetic moment of 20 Am^2. The coil is kept initially in a vertical position and it can rotate freely around a horizontal diameter. When a uniform magnetic field of 4 T is applied along the vertical,it starts rotating around its horizontal diameter. The angular speed the coil acquires after rotating by 60° will be:

(1) 10 π rad s^{-1} (2) 20 rad s^{-1} (3) 20 π rad s^{-1} (4) 10 rad s^{-1}

Sol. **4**

By energy conservation

$$U_i + K_i = U_f + K_f$$

$$-MB\cos 60° + 0 = -MB\cos 0° + \frac{1}{2}I\omega^2$$

$$-\frac{MB}{2} + MB = \frac{1}{2}I\omega^2$$

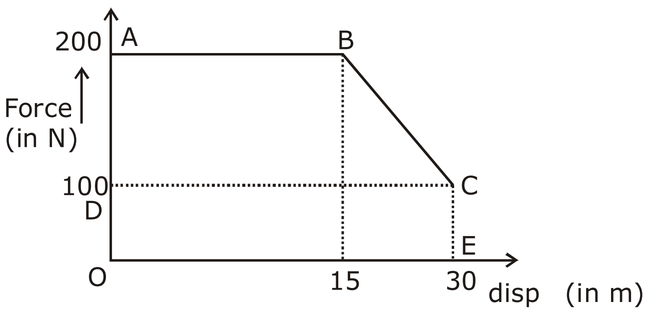
$$\frac{MB}{2} = \frac{1}{2}I\omega^2$$

$$\omega = \sqrt{\frac{MB}{I}} = \sqrt{\frac{20\times 4}{0.8}} = \sqrt{100}$$ = 10 rad/s

2. A person pushes a box on a rough horizontal plateform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement of the box?

(1) 5690 J (2) 5250 J (3) 2780 J (4) 3280 J

Sol. **2**



Work done = area of ABCEO

= area of trap. ABCD + area of rect. ODCE

= $\frac{1}{2}\times 45\times 30 + 100\times 30 = 5250J$

3. Match the thermodynamic processes taking place in a system with the correct conditions. In the table : ΔQ is the heat supplied, ΔW is the work done and ΔU is change in internal energy of the system.

Process	Condition
(I) Adiabatic	(1) $\Delta W = 0$
(II) Isothermal	(2) $\Delta Q = 0$
(III) Isochoric	(3) $\Delta U \neq 0$, $\Delta W \neq 0$, $\Delta Q \neq 0$
(IV) Isobaric	(4) $\Delta U = 0$

(1) (I) - (1), (II) - (1), (III) - (2), (IV) - (3)
(2) (I) - (1), (II) - (2), (III) - (4), (IV) - (4)
(3) (I) - (2), (II) - (4), (III) - (1), (IV) - (3)
(4) (I) - (2), (II) - (1), (III) - (4), (IV) - (3)

Sol. **3**

adiabatic, $\Delta Q = 0$
Isothermal, $\Delta U = 0$
Isochoric, $\int pdV = 0$

$\Delta W = 0$

Isobaric, $\Delta Q \neq 0, \Delta U \neq 0, \Delta W \neq 0$

4. The driver of a bus approaching a big wall notices that the frequency of his bus's horn changes from 420 Hz to 490 Hz when he hears it after it gets reflected from the wall. Find the speed of the bus if speed of the sound is 330 ms^{-1}.

(1) 81 kmh^{-1} (2) 91 kmh^{-1} (3) 71 kmh^{-1} (4) 61 kmh^{-1}

Sol. **2**

Freq received by wall,

$$f_w = \left(\frac{330}{330-v}\right)f_0$$

freq. after reflection, $f' = \left(\frac{330+v}{330}\right)f_w$

$$= \left(\frac{330+v}{330}\right) \times \left(\frac{330}{330-v}\right)f_0$$

$$490 = \left(\frac{330+v}{330-v}\right)420$$

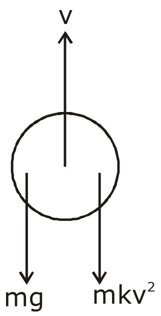
$\therefore v = 25.2\,m/s$

= 91 km/h

5. A small ball of mass m is thrown upward with velocity u from the ground. The ball experiences a resistive force mkv^2 where v is its speed. The maximum height attained by the ball is:

(1) $\frac{1}{k}\tan^{-1}\frac{ku^2}{2g}$ (2) $\frac{1}{2k}\ln\left(1+\frac{ku^2}{g}\right)$ (3) $\frac{1}{k}\ln\left(1+\frac{ku^2}{2g}\right)$ (4) $\frac{1}{2k}\tan^{-1}\frac{ku^2}{g}$

Sol. **2**



at H_m , V = 0

$F_{net} = ma$

$$-mg - mKv^2 = mv\frac{dv}{ds}$$

$$\int_{S=0}^{H} ds = (-1)\underbrace{\int_{v=u}^{v=0}\frac{vdv}{g+kv^2}}_{\int \frac{xdx}{a+bx^2}}$$

$$H_{max} = \frac{1}{2K}\ell n\left(\frac{g+ku^2}{g}\right)$$

$$\boxed{H_m = \frac{1}{2K}\ell n\left(1+\frac{Ku^2}{g}\right)}$$

6. Consider two uniform discs of the same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2, respectively, about their axes is $I_1 : I_2 = 1 : 16$ then the value of α is :

(1) $\sqrt{2}$ (2) 2 (3) $2\sqrt{2}$ (4) 4

Sol. **2**

Moment of inertia of disc, $I = \frac{MR^2}{2} = \frac{\left[\rho\left(\pi R^2\right)t\right]R^2}{2}$

$I = KR^4$

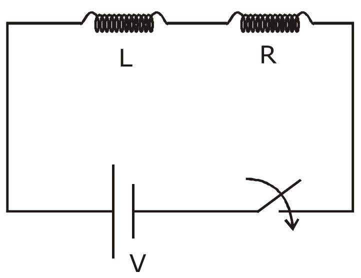
$$\frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^4$$

$$\frac{1}{16} = \left(\frac{R}{\alpha R}\right)^4 \Rightarrow \alpha = (16)^{\frac{1}{4}} = 2$$

7. A series L-R circuit is connected to a battery of emf V. If the circuit is switched on at t = 0, then the time at which the energy stored in the inductor reaches $\left(\frac{1}{n}\right)$ times of its maximum value, is :

(1) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)$ (2) $\frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$ (3) $\frac{L}{R}\ln\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)$ (4) $\frac{L}{R}\ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$

Sol. **2**



P.E. in inductor, $U = \frac{1}{2}LI^2$

$U \propto I^2$

$$\frac{U}{U_0} = \left(\frac{I}{I_0}\right)^2$$

$$\frac{1}{n} = \left(\frac{I}{I_0}\right)^2$$

$$I = \frac{I_0}{\sqrt{n}}$$

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\frac{I_0}{\sqrt{n}} = I_0\left(1 - e^{-\frac{R}{L}t}\right)$$

taking ℓn & solving we get,

$$\boxed{t = \frac{L}{R}\ell n\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)}$$

8. The electric field of a plane electromagnetic wave is given by

$$\vec{E} = E_0\left(\hat{x} + \hat{y}\right)\sin\left(kz - \omega t\right)$$

Its magnetic field will be given by :

(1) $\frac{E_0}{c}\left(\hat{x} + \hat{y}\right)\sin\left(kz - \omega t\right)$ (2) $\frac{E_0}{c}\left(\hat{x} - \hat{y}\right)\sin\left(kz - \omega t\right)$

(3) $\frac{E_0}{c}\left(\hat{x} - \hat{y}\right)\cos\left(kz - \omega t\right)$ (4) $\frac{E_0}{c}\left(-\hat{x} + \hat{y}\right)\sin\left(kz - \omega t\right)$

Sol. **(4)**

$\vec{E}\times\vec{B}$ should be in direction of $\vec{v}$

$$\therefore \vec{B} = \frac{E_0}{C}\left(-\hat{x} + \hat{y}\right)\sin\left(Kz - \omega t\right)$$

9. A cube of metal is subjected to a hydrostatic pressure of 4 GPa. The percentage change in the length of the side of the cube is close to :

(Given bulk modulus of metal, B = 8 × 10^{10} Pa)

(1) 0.6 (2) 20 (3) 1.67 (4) 5

Sol. **(3)**

$$(-)\frac{\Delta P}{\Delta V/V} = B$$

$$\Delta P = \left(\frac{\Delta V}{V}\right).B$$

$$= \frac{3\Delta L}{L}\times B$$

$\therefore \frac{\Delta L}{L} = \frac{\Delta P}{3B}$ $\therefore \%$ we get, $\frac{\Delta L}{L}\times 100\%$

Putting values we get = 1.67

10. A paramagnetic sample shows a net magnetisation of 6 A/m when it is placed in an external magnetic field of 0.4 T at a temperature of 4 K. When the sample is placed in an external magnetic field of 0.3 T at a temperature of 24 K, then the magnetisation will be:

(1) 4 A/m (2) 1 A/m (3) 0.75 A/m (4) 2.25 A/m

Sol. **(3)**

$$M = \frac{CB_{ext}}{T}$$

$$6 = \frac{C \times 0.4}{4}$$

$$\Rightarrow C = 60$$

$$\therefore \text{case - II :- } M = \frac{60 \times 0.3}{24} = \frac{60 \times 3}{240} = \frac{3}{4} = 0.75 \text{ A/m}$$

11. A body is moving in a low circular orbit about a planet of mass M and radius R. The radius of the orbit can be taken to be R itself. Then the ratio of the speed of this body in the orbit to the escape velocity from the planet is:

(1) 2 (2) $\sqrt{2}$ (3) 1 (4) $\frac{1}{\sqrt{2}}$

Sol. **(4)**

$$V_0 = \sqrt{\frac{GM}{r}}, \; V_e = \sqrt{\frac{2GM}{r}}$$

$$\frac{v_0}{v_e} = \sqrt{\frac{GM}{r} \times \frac{r}{2GM}} = \frac{1}{\sqrt{2}}$$

12. A particle of charge q and mass m is subjected to an electric field $E = E_0 (1 - ax^2)$ in the x-direction, where a and E_0 are constants. Initially the particle was at rest at x =0. Other than the initial position the kinetic energy of the particle becomes zero when the distance of the particle from the origin is:

(1) $\sqrt{\frac{2}{a}}$ (2) a (3) $\sqrt{\frac{3}{a}}$ (4) $\sqrt{\frac{1}{a}}$

Sol. **(3)**

$$W = \Delta KE$$

$$\int_0^x F dx = 0$$

$$\int_0^x qE dx = 0$$

$$q\int_0^x E_0\left(1-ax^2\right)dx = 0$$

$$qE_0\left[\int_0^x dx - a\int_0^x x^2 dx\right] = 0$$

$$qE_0\left[x-\frac{ax^3}{3}\right] = 0$$

$$x\left(1-\frac{ax^2}{3}\right) = 0$$

x=0, $1-\frac{ax^2}{3} = 0$

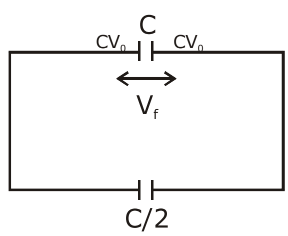
$$\frac{ax^2}{3} = 1$$

$$\boxed{x=\sqrt{\frac{3}{a}}}$$

13. A capacitor C is fully charged with voltage V_0. After disconnecting the voltage source, it is connected in parallel with another uncharged capacitor of capacitance $\frac{C}{2}$. The energy loss in the process after the charge is distributed between the two capacitors is:

(1) $\frac{1}{2}CV_0^2$ (2) $\frac{1}{4}CV_0^2$ (3) $\frac{1}{3}CV_0^2$ (4) $\frac{1}{6}CV_0^2$

Sol. **(4) Our Answer**
NTA Answer (2)



$$v_f = \frac{CV_0}{3\frac{c}{2}} = \frac{2V_0}{3}$$

$u_i = \frac{1}{2}cv_0^2$

$u_f = \frac{1}{2}\left(\frac{3c}{2}\right).\frac{4v_0^2}{9} = \frac{CV_0^2}{3}$

$u_i - u_f = \frac{1}{2}cv_0^2 - \frac{cv_0^2}{3}$

$= cv_0^2\left(\frac{1}{2}-\frac{1}{3}\right) = \frac{cv_0^2}{6}$

14. Find the Binding energy per neucleon for ${}^{120}_{50}Sn$. Mass of proton m_p = 1.00783 U, mass of neutron m_n = 1.00867 U and mass of tin nucleus m_{Sn} = 119.902199 U. (take 1U = 931 MeV)

(1) 8.0 MeV (2) 9.0 MeV (3) 7.5 MeV (4) 8.5 MeV

Sol. **(4)**

$B.E. = \Delta mc^2$

$= \Delta m \times 931$

$\Delta m = (50 \times 1.00783) + (70 \times 1.00867) - \{119.902199\}$

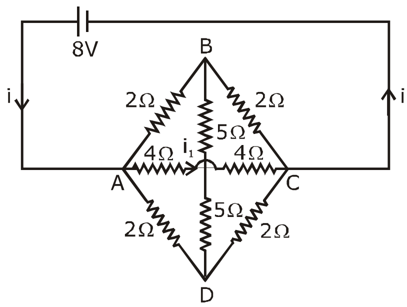
$= \{120.9984 - 119.902199\}$ U

= 1.1238 U

$BE = 1.1238 \times 931 = 1046.2578\,MeV$

BE per nucleon $\simeq$ 1046/120 $\approx$ 8.5 Mev

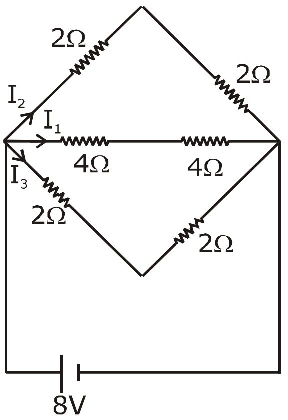
15. The value of current i_1 flowing from A to C in the circuit diagram is:



(1) 4 A (2) 5 A (3) 2 A (4) 1 A

Sol. **(4)**

eq circuit $\Rightarrow$

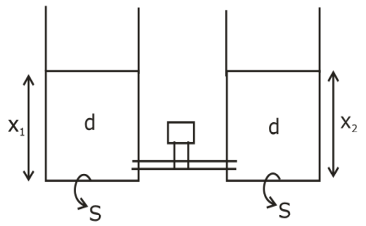


$$I_2 = \frac{8}{4+4} = 1\text{amp}$$

16. Two identical cylindrical vessels are kept on the ground and each contain the same liquid of density d. The area of the base of both vessels is S but the height of liquid in one vessel is x_1 and in the other, x_2. When both cylinders are connected through a pipe of negligible volume very close to the bottom, the liquid flows from one vessel to the other until it comes to equilibrium at a new height. The change in energy of the system in the process is:

(1) $gdS\,(x_2 + x_1)^2$ (2) $gdS\left(x_2^2 + x_1^2\right)$ (3) $\frac{1}{4}gdS(x_2 - x_1)^2$ (4) $\frac{3}{4}gdS(x_2 - x_1)^2$

Sol. **(3)**



$$u_i = \left[dsx_1.\frac{x_1}{2} + dsx_2.\frac{x_2}{2}\right]g \qquad \left\{dsx_1 \to m,\ \frac{x_1}{2} \to h(com)\right\}$$

$$u_f = \left[ds\left(\frac{x_1 + x_2}{2}\right) \times \left(\frac{x_1 + x_2}{4}\right) \times 2\right]g$$

$$u_i - u_f = dsg\left[\frac{x_1^2}{2} + \frac{x_2^2}{2} - \frac{(x_1 + x_2)^2}{4}\right]$$

$$= dsg\frac{(x_1 - x_2)^2}{4}$$

17. A quantity x is given by (IFv^2/WL^4) in terms of moment of inertia I, force F, velocity v, work W and Length L. The dimensional formula for x is same as that of :

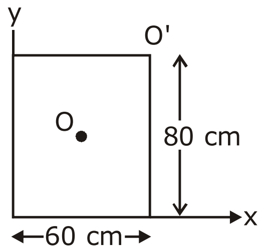
(1) coefficient of viscosity (2) energy density
(3) force constant (4) planck's constant

Sol. **(2)**

$$[x] = \frac{IFV^2}{WL^4} = \frac{(M^1L^2)(MLT^{-2})(LT^{-2})^2}{(ML^2T^{-2})L^4}$$

$= ML^{-1}T^{-2}$ = Energy density

18. For a uniform rectangular sheet shown in the figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through O (the centre of mass) and O' (corner point) is:



(1) 1/2 (2) 2/3 (3) 1/4 (4) 1/8

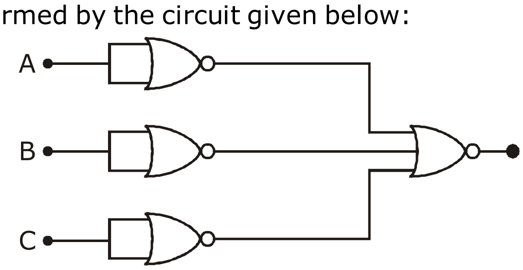
Sol. **(3)**

$$I_0 = \frac{M}{12}\left(a^2+b^2\right)$$

$$I_{0'} = \frac{M}{12}\left(a^2+b^2\right) + M\left(\frac{a^2}{4}+\frac{b^2}{4}\right)$$

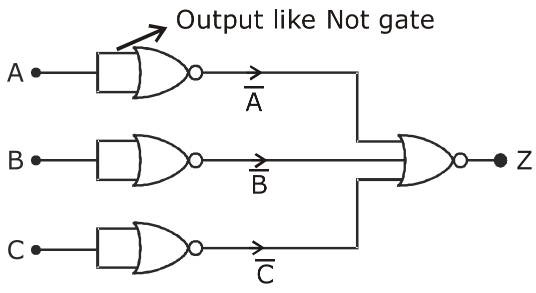
$$\frac{I_0}{I_{0'}} = \frac{\frac{M}{12}\left(a^2+b^2\right)}{\frac{M}{12}\left(a^2+b^2\right)+\frac{M}{4}\left(a^2+b^2\right)} = \frac{\frac{1}{12}}{\frac{1}{12}+\frac{1}{4}} = \frac{1}{12}\times\frac{3}{1} = \frac{1}{4}$$

19. Identify the operation performed by the circuit given below:



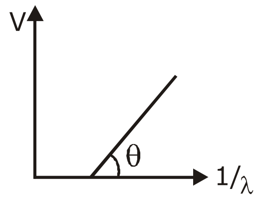
(1) NOT (2) OR (3) AND (4) NAND

19. **(3)**



$Z = \overline{\overline{A} + \overline{B} + \overline{C}} = A.B.C$ (AND gate)

20. In a photoelectric effect experiment, the graph of stopping potential V versus reciprocal of wavelength obtained is shown in the figure. As the intensity of incident radiation is increased:



(1) Straight line shifts to right
(2) Straight line shifts to left
(3) Slope of the straight line get more steep
(4) Graph does not change

Sol. **(4)**

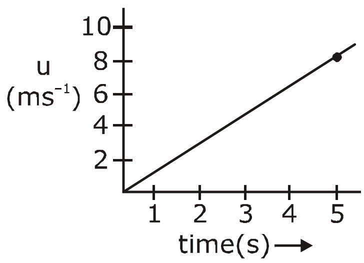
$ev = h\upsilon - w$ (w = work function)

$$v = \frac{h\upsilon}{e} - \frac{w}{e}$$

as $\frac{h}{e}$ & $\frac{w}{e} \rightarrow$ constant

Therefore no change in graph.

21. The speed verses time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval t = 0 to t = 5 s will be________.

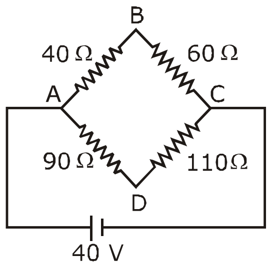


Sol. **20**

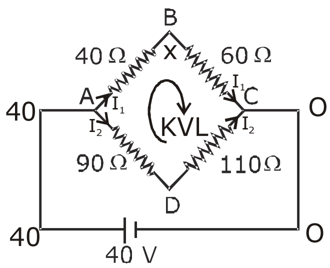
Distence = Area under speed – time graph

$= \frac{1}{2} \times 8 \times 5 = 20\text{m}$

22. Four resistances 40 Ω, 60 Ω, 90 Ω and 110 Ω make the arms of a quadrilateral ABCD. Across AC is a battery of emf 40 V and internal resistance negligible.The potential difference across BD in V is _______.



Sol. **2**



$I_1 = \frac{40}{100}$

$I_2 = \frac{40}{200}$

$V_B - \frac{40}{100} \times 60 + 110 \times \frac{40}{200}$

$= V_D$

$V_B - V_D = \frac{40 \times 60}{100} - \frac{100 \times 40}{200}$

= 24 – 22

= 2V

23. The change in the magnitude of the volume of an ideal gas when a small additional pressure ΔP is applied at a constant temperature, is the same as the change when the temperature is reduced by a small quantity ΔT at constant pressure. The initial temperature and pressure of the gas were 300 K and 2 atm. respectively. If $|\Delta T| = C|\Delta P|$ then value of C in (K/atm.) is ________.

Sol. **150**

1st case

PV = nRT

PdV + VdP = 0

$P\Delta V + V\Delta P = 0$ $\boxed{\Delta v = \frac{-\Delta P}{P} v}$

2nd case

$P\Delta V = -nR\Delta T$

$\boxed{\Delta V = -\frac{nR\Delta T}{P}}$

$-\frac{\Delta P}{P} V = \frac{-nR\Delta T}{P} \Rightarrow \Delta T = \Delta P \frac{v}{nQ}$

$\Rightarrow \frac{\Delta T}{\Delta P} = \frac{V}{nR}$

Now, given $|\Delta T| = C|\Delta P|$

$C = \frac{\Delta T}{\Delta P} = \frac{V}{nR}$

$C = \frac{T}{P} = \frac{300}{2} = 150$

24. Orange light of wavelength 6000×10^{-10} m illuminates a single slit of width 0.6×10^{-4} m. The maximum possible number of diffraction minima produced on both sides of the central maximum is ________.

Sol. **200**

For minima

$d \sin\theta = n\lambda$

or $\sin\theta = \frac{n\lambda}{d}$

$\because$ maximum value of $\sin\theta$ is 1

$\therefore \frac{n\lambda}{d} \le \perp$

$n \le \frac{d}{\lambda}$

$n \le \frac{0.6 \times 10^{-4}}{6000 \times 10^{-10}}$

$n \le 100$

for both sides 100 +100 = 200

25. The distance between an object and a screen is 100 cm. A lens can produce real image of the object on the screen for two different positions between the screen and the object. The distance between these two positions is 40 cm. If the power of the lens is close to $\left(\frac{N}{100}\right)$D where N is an integer, the value of N is __________.

Sol. **5**

$$\because \ f=\frac{D^2-d^2}{4D}=\frac{100^2-40^2}{400}$$

$$=\frac{10000-1600}{400}$$

$$=\frac{100-16}{4}=\frac{84}{4}=21$$

$$p=\frac{1}{f}=\frac{1}{21}=\frac{1}{21}\times\frac{100}{100}=\left(\frac{4.76}{100}\right)=\frac{N}{100}$$

$$\therefore \ \boxed{N\approx 5}$$

QUESTION PAPER WITH SOLUTION

CHEMISTRY _ 4 Sep. _ SHIFT - 2

1. The reaction in which the hybridisation of the underlined atom is affected is :

(1) $H_2\underline{S}O_4 + NaCl \xrightarrow{420\ K}$ (2) $\underline{Xe}F_4 + SbF_5 \longrightarrow$

(3) $\underline{N}H_3 \xrightarrow{H^+}$ (4) $H_3\underline{P}O_2 \xrightarrow{\text{Disproportionation}}$

Sol. **2**

1 $H_2SO_4 + NaCl \xrightarrow{420k} NaHSO_4 + HCl$

Sp^3 Sp^3

2 $XeF_4 + SbF_5 \longrightarrow (Xe\overset{+}{F}_3)(SbF_6)^-$

Sp^3d^2 Sp^3d

3 $NH_3 \xrightarrow{H^+} NH_4^+$

Sp^3 Sp^3

4 $H_3PO_2 \xrightarrow[\text{Reaction}]{\text{Disproportionation}} PH_3 + H_3PO_4$

Sp^3 Sp^3 Sp^3

2. The process that is NOT endothermic in nature is :

(1) $H_{(g)} + e^- \rightarrow H^-_{(g)}$ (2) $Na_{(g)} \rightarrow Na^+_{(g)} \rightarrow e^-$

(3) $Ar_{(g)} + e^- \rightarrow Ar^-_{(g)}$ (4) $O^-_{(g)} + e^- \rightarrow O^{2-}_{(g)}$

Sol. **1**

$H_{(g)} + e^{\ominus} \longrightarrow H^{\ominus}_{(g)}$

is an exothermic Rxn.

Ans (1)

3. If the equilibrium constant for $A \rightleftharpoons B + C$ is $K^{(1)}_{eq}$ and that of $B + C \rightleftharpoons P$ is $K^{(2)}_{eq}$, the equilibrium constant for $A \rightleftharpoons P$ is :

(1) $K^{(1)}_{eq}\ K^{(2)}_{eq}$ (2) $K^{(2)}_{eq} - K^{(1)}_{eq}$ (3) $K^{(1)}_{eq} + K^{(2)}_{eq}$ (4) $K^{(1)}_{eq} / K^{(2)}_{eq}$

Sol. **1**

$A \rightleftharpoons B + C$ $K_{eq}{}^{(1)}$

$\dfrac{B + C \rightleftharpoons P}{A \rightleftharpoons P}$ $K_{eq}{}^{(2)}$

$K_{eq} = K_{eq}{}^{(1)} \times K_{eq}{}^{(2)}$

Ans.(1)

4. A sample of red ink (a colloidal suspension) is prepared by mixing eosin dye, egg white, HCHO and water. The component which ensures stability of the ink sample is :

(1) HCHO (2) Water (3) Eosin dye (4) Egg white

Sol. **4**

Surface theoritical eggwhite

5. The one that can exhibit highest paramagnetic behaviour among the following is :

gly = glycinato; bpy = 2, 2′-bipyridine

(1) $\left[Ti\left(NH_3\right)_6\right]^{3+}$ (2) $\left[Co\left(OX\right)_2\left(OH\right)_2\right]^{-}\left(\Delta_0 > P\right)$

(3) $\left[Pd\left(gly\right)_2\right]$ (4) $\left[Fe\left(en\right)\left(bpy\right)\left(NH_3\right)_2\right]^{2+}$

Sol. **2**

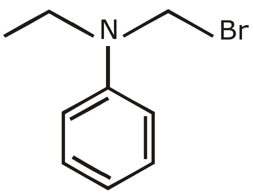
1. $(Ti(NH_3)_6)^{3+} \Rightarrow Ti^{3+}\ (3d^1) \Rightarrow \mu = \sqrt{3}$
2. $[Co(OX_2)(OH_2)^{-}\ (\Delta_0 > P) \Rightarrow Co^{+5}\ (3d^4) \Rightarrow t_2g^4\ eg^0$

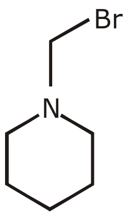
   $n = 2,\ \mu = \sqrt{8}$
3. $(Pd\ (gly)_2) \Rightarrow pd^{2+}\ (4d^8) \rightarrow$ Square planar

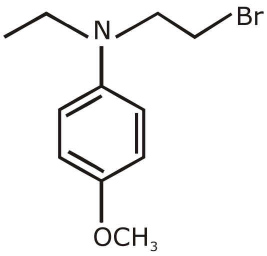
   $n = 0,\ \mu = 0$ diamagentic
4. $(Fe\ (en)\ (bpy)\ (NH_3)_2)^{2+}$

   $Fe^{2+} \Rightarrow 3d^6\ (t_2g^6\ eg^0) \Rightarrow n = 0,\ \mu = 0$

6. Which of the following compounds will form the precipitate with aq. $AgNO_3$ solution most readily?

(1) 

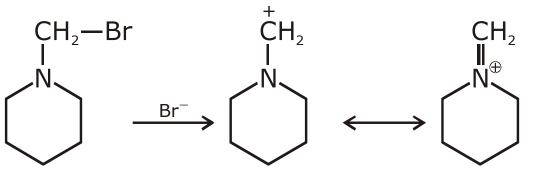
(2) 

(3) 

(4) 

Sol. **2**

Rate of reaction α stability of carbocation.



Highly stable due to back bonding

7. Five moles of an ideal gas at 1 bar and 298 K is expanded into vacuum to double the volume. The work done is :

(1) zero (2) $C_V(T_2 - T_1)$ (3) $-RT(V_2 - V_1)$ (4) $-RT\ln V_2/V_1$

Sol. **1**

As it is free expansion against zero ext. pressure

$\therefore$ Work Done = zero

Ans. (1)

8. 250 mL of a waste solution obtained from the workshop of a goldsmith contains 0.1 M $AgNO_3$ and 0.1 M AuCl. The solution was electrolyzed at 2 V by passing a current of 1 A for 15 minutes. The metal/metals electrodeposited will be:

$\left(E^0_{Ag^+/Ag} = 0.80\text{ V},\ E^0_{Au^+/Au} = 1.69\text{ V}\right)$

(1) Silver and gold in proportion to their atomic weights

(2) Silver and gold in equal mass proportion

(3) only silver

(4) only gold

Sol. **1**

Amount of charge transfered = $\dfrac{1\times15\times60}{96500} = \dfrac{9}{965} \simeq 10\times10^{-3}$

moles of gold deposited = $\dfrac{0.1\times250}{1000} = 25\times10^{-3}$

Both wil be deposited

Ans.(1)

9. The mechanism of action of "Terfenadine" (Seldane) is :

(1) Helps in the secretion of histamine (2) Activates the histamine receptor

(3) Inhibits the secretion of histamine (4) Inhibits the action of histamine receptor

Sol. **4**

The mechanism of action of "Terfenadine" (Seldane) is to inhibit the action of histamine receptor.

10. The shortest wavelength of H atom in the Lyman series is λ_1. The longest wavelength in the Balmer series of He^+ is : :

(1) $\frac{9\lambda_1}{5}$ (2) $\frac{27\lambda_1}{5}$ (3) $\frac{36\lambda_1}{5}$ (4) $\frac{5\lambda_1}{9}$

Sol. **1**

$$\frac{1}{\lambda_1} = R_4 \times (1)^2 \times \left\{1 \times \frac{1}{\infty^2}\right\} = R_H$$

$$\frac{1}{\lambda_2} = R_4 \times (2)^2 \times \left\{\frac{1}{4} - \frac{1}{a}\right\} = R_H \left\{\frac{5}{9}\right\}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{9}{5}$$

$$\lambda_2 = \frac{9}{5}\lambda_1$$

Ans. (1)

11. The major product [B] in the following reactions is :

$$CH_3-CH_2-\overset{\overset{CH_3}{|}}{C}H-CH_2-OCH_2-CH_3 \xrightarrow[Heat]{HI} [A]\ \text{alcohol} \xrightarrow[\Delta]{H_2SO_4} [B]$$

(1) $CH_3-CH_2-CH=CH-CH_3$

(2) $CH_3-CH_2-\overset{\overset{CH_3}{|}}{C}\quad CH_2$

(3) $CH_3-CH\quad \overset{\overset{CH_3}{|}}{C}-CH_3$

(4) $CH_2=CH_2$

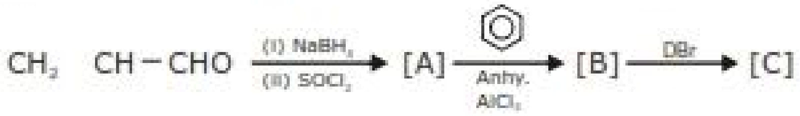
Sol. **3**

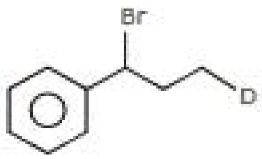
$$CH_3-CH_2-\overset{\overset{CH_3}{|}}{C}H-CH_2-O-CH_2-CH_3 \xrightarrow{HI} CH_3-CH_2-\underset{\underset{OH}{|}}{\overset{\overset{CH_3}{|}}{C}}-CH_3 + Et-I$$

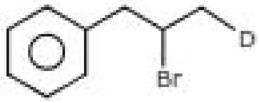
$$\Big\downarrow H_2SO_4/\Delta$$

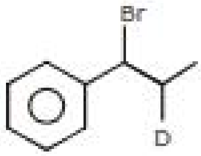
$$CH_3-C=C\begin{matrix} CH_3 \\ CH_3 \end{matrix}$$

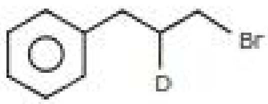
12. The major product [C] of the following reaction sequence will be :



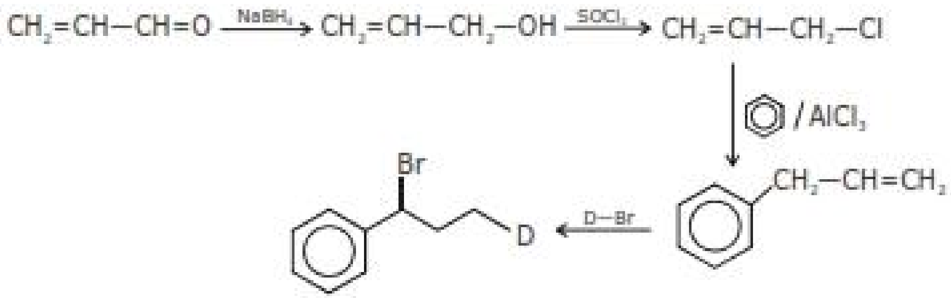
(1) 

(2) 

(3) 

(4) 

Sol. **1**



13. The Crystal Field Stabilizion Energy (CFSE) of $[CoF_3(H_2O)_3]$ $(\Delta_o < P)$ is:

(1) $-0.8\ \Delta_0$ (2) $-0.8\ \Delta_0 + 2P$ (3) $-0.4\ \Delta_0 + P$ (4) $-0.4\ \Delta_0$

Sol. **4**

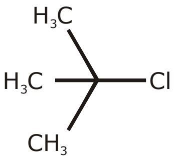
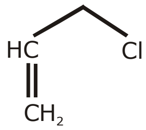
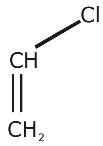
$[CoF_3(H_2O)_3]$ $(\Delta_0 < P)$

CO^{3+} $(3d^6) = t_2g^4\ eg^2$

$$\text{CFSE} = \left(-\frac{2}{5} \times 4 + \frac{3}{5} \times 2\right)\Delta_0$$

$$= -0.4\ \Delta_0$$

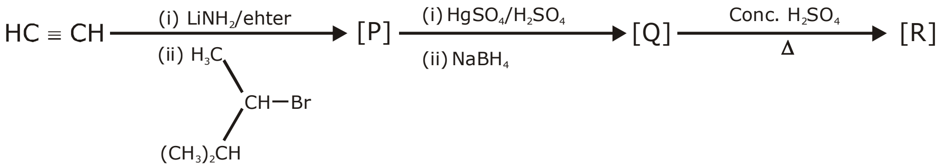
14. Among the following compounds, which one has the shortest $C-Cl$ bond?

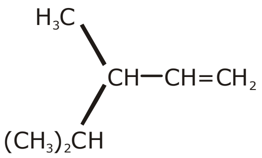
(1)  (2) H_3C-Cl (3)  (4) 

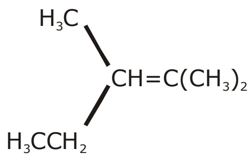
Sol. **4**

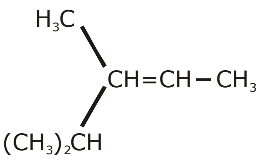
$CH_2=CH-\ddot{Cl}: \longleftrightarrow \overset{\ominus}{C}H_2-CH=\overset{\oplus}{Cl} \equiv \overset{\delta^-}{C}H_2=CH=\overset{\delta^+}{Cl}$

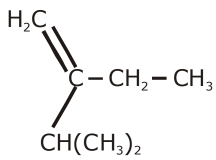
15. The major product [R] in the following sequence of reactions is :



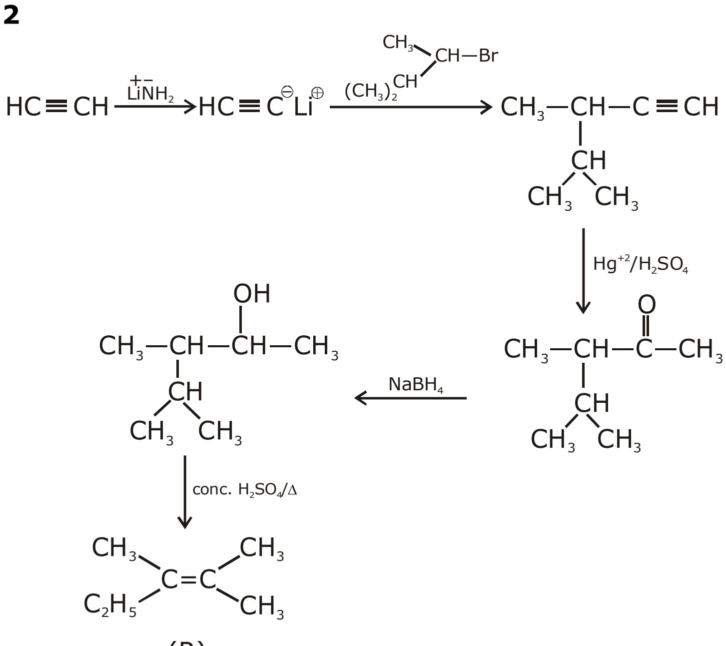
(1) 

(2) 

(3) 

(4) 

Sol. **2**



(R)

16. The molecule in which hybrid MOs involve only one d-orbital of the central atom is :

(1) $\left[CrF_6\right]^{3-}$ (2) XeF_4 (3) BrF_5 (4) $\left[Ni(CN)_4\right]^{2-}$

Sol. **4**

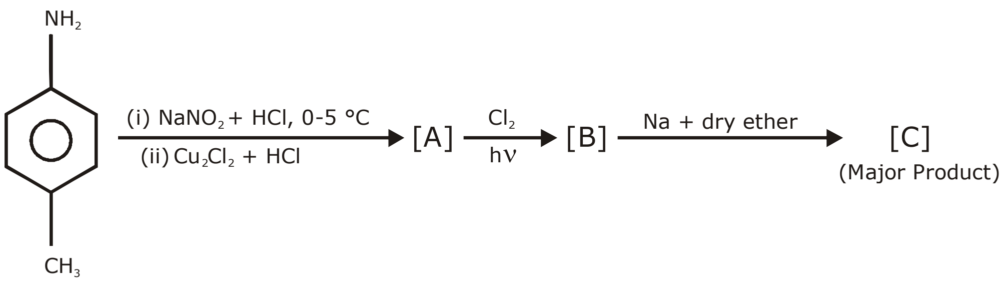
(1) $(CrF_6)^{3-}$ – d^2Sp^3

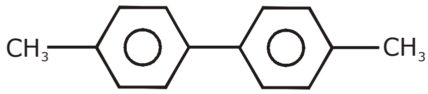
(2) XeF_4 – Sp^3d^2

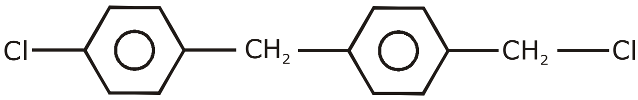
(3) BrF_5 – Sp^3d^2

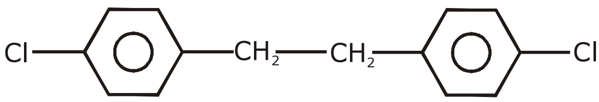
(4) $[Ni(CN)_4]^{2-} \rightarrow dsp^2$

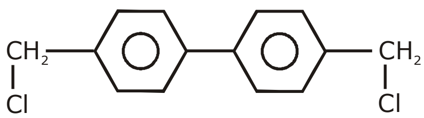
17. In the following reaction sequence, [C] is :



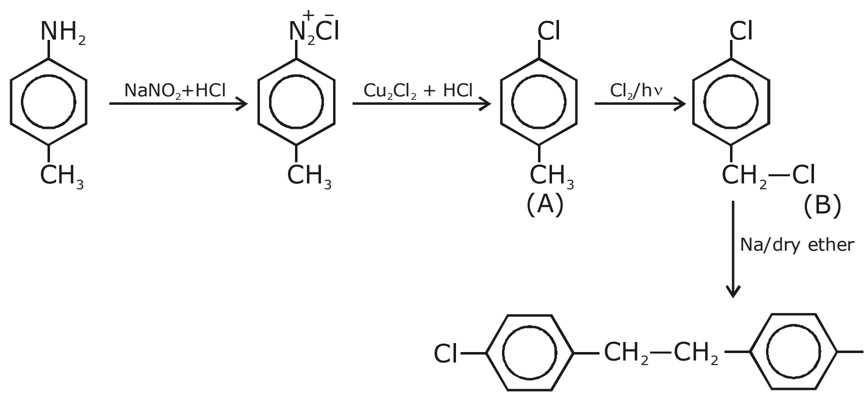
(1) 

(2) 

(3) 

(4) 

Sol. **3**



18. The processes of calcination and roasting in metallurgical industries, respectively, can lead to :

(1) Photochemical smog and ozone layer depletion
(2) Photochemical smog and global warming
(3) Global warming and photochemical smog
(4) Global warming and acid rain

Sol. **4**

Environmental
Calcination Releases $\rightarrow CO_2 \rightarrow$ Global warming
Roasting Releases $\rightarrow SO_2 \rightarrow$ Acid Rain
Ans. (4)

19. The incorrect statement(s) among (a) - (c) is (are) :

(a) W(VI) is more stable than Cr(VI).
(b) in the presence of HCl, permanganate titrations provide satisfactory results.
(c) some lanthanoid oxides can be used as phosphors.

(1) (a) only (2) (b) and (c) only
(3) (a) and (b) only (4) (b) only

Sol. **4**

Fact

20. An alkaline earth metal 'M' readily forms water soluble sulphate and water insoluble hydroxide. Its oxide MO is very stable to heat and does not have rock-salt structure. M is :

(1) Ca (2) Be (3) Mg (4) Sr

Sol. **2**

Fact

21. The osmotic pressure of a solution of NaCl is 0.10 atm and that of a glucose solution is 0.20 atm.

The osmotic pressure of a solution formed by mixing 1 L of the sodium chloride solution with 2 L of the glucose solution is $x \times 10^{-3}$ atm. x is _______. (nearest integer)

Sol. **167**

$$\frac{0.1 \times 1 + 0.2 \times 2}{3}$$

$$= \frac{0.5}{3} = \frac{500}{3} \times 10^{-3} = 167 \text{ Ans.}$$

22. The number of molecules with energy greater than the threshold energy for a reaction increases five fold by a rise of temperature from 27 °C to 42 °C. Its energy of activation in J/mol is ________. (Take ln 5 = 1.6094 ; R = 8.314 J $mol^{-1}K^{-1}$)

Sol. $\frac{1}{5} = \frac{e^{-Ea/300R}}{e^{-Ea/315R}}$

$$5 = e^{\frac{Ea}{R}\left(\frac{1}{300}-\frac{1}{315}\right)}$$

$$\frac{Ea}{R}\left(\frac{15}{300 \times 315}\right) = \ln(5)$$

$E_a = 1.6094 \times 315 \times 20 \times 8.314$

$E_a = 84297.47$ J/mol Ans.

23. A 100 mL solution was made by adding 1.43 g of $Na_2CO_3 . xH_2O$. The normality of the solution is 0.1 N. The value of x is ________. (The atomic mass of Na is 23 g/mol).

Sol. $\frac{0.1}{2} \times \frac{100}{1000} = \frac{1.43}{1.6 + 18x}$

$106 + 18x = 286$

$18x = 180 \Rightarrow x = 10$ Ans.

24. Consider the following equations :

$2\ Fe^{2+} + H_2O_2 \rightarrow x\ A + y\ B$ (in basic medium)

$2\ MnO_4^- + 6\ H^+ + 5\ H_2O_2 \rightarrow x'C + y'D + z'E$ (in acidic medium).

The sum of the stoichiometric coeficients x, y, x′,y′ and z′ for products A, B, C, D and E, respectively, is __________.

Sol. **19**

$2Fe^{2+} + H_2O_2 \longrightarrow xA + yB \longrightarrow 2Fe^{3+} + 2OH^-$

$2MnO_4^- + 6H^{\oplus} + 5H_2O_2 \longrightarrow x`C + y`D + Z`E \longrightarrow 2Mn^{+2} + 5O_2 + 8H_2O$

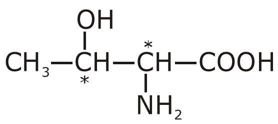
$x = 2$; $y = 2$; $x' = 2$, $y' = 5$, $z' = 8$

$2 + 2 + 2 + 5 + 8 = 19$

Ans. 19

25. The number of chiral centres present in threonine is ________.

Sol. **2**



QUESTION PAPER WITH SOLUTION

MATHEMATICS _ 4 Sep. _ SHIFT - 2

Q.1 Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, Ax=b when the vector b on the right side is equal to b_1, b_2 and b_3 respectively. if $x_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}, x_2 = \begin{bmatrix}0\\2\\1\end{bmatrix}, x_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}, b_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}, b_2 = \begin{bmatrix}0\\2\\0\end{bmatrix} \text{and } b_3 = \begin{bmatrix}0\\0\\2\end{bmatrix}$, then the determinant of A is equal to

(1) 2 (2) $\frac{1}{2}$ (3) $\frac{3}{2}$ (4) 4

Sol. **(1)**

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}_{3\times 3}$$

$$a_1 + a_2 + a_3 = 1 \quad 2a_2 + a_3 = 0$$
$$a_4 + a_5 + a_6 = 0 \quad 2a_5 + a_6 = 2$$
$$a_7 + a_8 + a_9 = 0 \quad 2a_8 + a_9 = 0$$

$$a_3 = 0, a_6 = 0, a_9 = 2$$

$$\therefore a_8 = -1, a_5 = 1, \quad a_2 = 0 \Rightarrow a_1 = \phi, a_4 = -1, \quad a_7 = -1$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix}$$

$|A| = 2(1) = 2$

Q.2 If a and b are real numbers such that $(2+\alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1+i\sqrt{3}}{2}$ then a+b is equal to:

(1) 33 (2) 57 (3) 9 (4) 24

Sol. **(3)**

$$(2+\alpha)^4 = a + b\alpha$$

$$\left(2 + \frac{\sqrt{3}i - 1}{2}\right)^4 = a + b\alpha$$

$$\left(\frac{3+\sqrt{3}i}{2}\right)^4 = 9\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^4$$

$9\left\{e^{i\pi/6}\right\}^4 = 9e^{i2\pi/3} = 9\left(\frac{-1}{2}+\frac{\sqrt{3}i}{2}\right) = \frac{-9}{2}+\frac{9\sqrt{3}}{2}$i

$-\frac{9}{2}+\frac{9\sqrt{3}}{2}i = a+b\left(\frac{-1}{2}+\frac{i\sqrt{3}}{2}\right)$

$= a-\frac{b}{2}+\frac{bi\sqrt{3}}{2}$

$\therefore \frac{b\sqrt{3}}{2}=\frac{9\sqrt{3}}{2} \Rightarrow b=9$

a = 0 $\therefore$ a + b = 9

Q.3 The distance of the point (1, −2, 3) from the plane x−y+z=5 measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ is:

(1) $\frac{1}{7}$ (2) 7 (3) $\frac{7}{5}$ (4) 1

Sol. (4)

Equation of line through (1,-2,3) whose dr's are (2,3,-6)

$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}=\lambda$

any point on line $(2\lambda+1, 3\lambda-2, -6\lambda+3)$

$\text{put in } (x-y+z=5)$

$2\lambda+1-3\lambda+2-6\lambda+3=5$

$-7\lambda=-1$

$\lambda=\frac{1}{7}$

distance $=\sqrt{(2\lambda)^2+(3\lambda)^2+(6\lambda)^2}$

$\sqrt{4\lambda^2+9\lambda^2+36\lambda^2}=7\lambda=1$

Q.4 Let $f:(0,\infty)\to(0,\infty)$ be a differentiable function such that f(1) = e and $\lim_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x}=0$.

If f(x)=1,then x is equal to :

(1) e (2) 2e (3) $\frac{1}{e}$ (4) $\frac{1}{2e}$

Sol. **(3)**

$f(1)=e$

$$\lim_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x}$$

L' Hospital

$$\lim_{t\to x}\left(2tf^2(x)-2x^2f(t)\cdot f'(t)\right)$$

$$\Rightarrow 2xf^2(x)-2x^2f(x)\cdot f'(x)=0$$

$$2xf(x)\left\{f(x)-xf'(x)\right\}=0$$

$$=)\quad \frac{f'(x)}{f(x)}=\frac{1}{x}$$

$$\ln f(x)=\ln x+\ln c$$

$$f(x)=cx$$

if x = 1, e = c

y = ex

$\therefore$ if f(x) = 1 $\Rightarrow$ x = $\frac{1}{e}$

Q.5 Contrapositive of the statement :

'If a function f is differentiable at a, then it is also continuous at a', is:

(1) If a function f is not continuous at a, then it is not differentiable at a.

(2) If a function f is continuous at a, then it is differentiable at a.

(3) If a function f is continuous at a, then it is not differentiable at a.

(4) If a function f is not continuous at a, then it is differentiable at a.

Sol. **(1)**

Contrapositive of $P\to q$ = $\sim q\to\sim p$

Q.6 The minimum value of $2^{\sin x}+2^{\cos x}$ is:

(1) $2^{1-\sqrt{2}}$ (2) $2^{1-\frac{1}{\sqrt{2}}}$ (3) $2^{-1+\sqrt{2}}$ (4) $2^{-1+\frac{1}{\sqrt{2}}}$

Sol. **(2)**

y = $2^{\sin x}+2^{\cos x}$

by Am $\geq$ GM

$$\frac{2^{\sin x}+2^{\cos x}}{2} \geq \sqrt{2^{\sin x+\cos x}}$$

$$2^{\sin x} + 2^{\cos x} \geq 2^1.2^{\frac{\sin x+\cos x}{2}}$$

$$2^{\sin x} + 2^{\cos x} \geq 2^{\frac{2+\sin x+\cos x}{2}} \quad \therefore (2^{\sin x} + 2^{\cos x})_{\min} = 2^{\frac{2-\sqrt{2}}{2}} = 2^{\frac{-1}{\sqrt{2}}+1}$$

Q.7 If the perpendicular bisector of the line segment joining the points P(1 ,4) and Q(k, 3) has y-intercept equal to –4, then a value of k is:

(1) –2 (2) $\sqrt{15}$ (3) $\sqrt{14}$ (4) –4

Sol. (4)

$$m_{PQ} = \frac{4-3}{1-k} \Rightarrow m_{\perp} = k-1$$

$$mid\ point\ of\ PQ = \left(\frac{k+1}{2}, \frac{7}{2}\right)$$

equation of perpendicular bisector

$$y - \frac{7}{2} = (k-1)\left(x - \frac{k+1}{2}\right)$$

for y intercept put x = 0

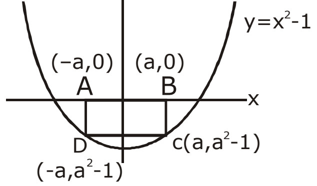
$$y = \frac{7}{2} - \left(\frac{k^2-1}{2}\right) = -4$$

$$\frac{k^2-1}{2} = \frac{15}{2} \Rightarrow k = \pm 4$$

Q.8 The area (in sq. units) of the largest rectangle ABCD whose vertices A and B lie on the x-axis and vertices C and D lie on the parabola, $y=x^2-1$ below the x-axis, is:

(1) $\frac{2}{3\sqrt{3}}$ (2) $\frac{4}{3}$ (3) $\frac{1}{3\sqrt{3}}$ (4) $\frac{4}{3\sqrt{3}}$

Sol. (4)



Area $= 2a\left(a^2-1\right)$

$A = 2a^3 - 2a$

$\frac{dA}{da} = 6a^2 - 2 = 0$

$a = \pm 1\sqrt{3}$

$A_{\max} = \frac{-2}{3\sqrt{3}} + \frac{2}{\sqrt{3}} = \frac{-2+6}{3\sqrt{3}} = \frac{4}{3\sqrt{3}}$

Q.9 The integral $\int_{\pi/6}^{\pi/3} \tan^3 x . \sin^2 3x(2\sec^2 x . \sin^2 3x + 3\tan x . \sin 6x)dx$ is equal to:

(1) $\frac{9}{2}$ (2) $-\frac{1}{18}$ (3) $-\frac{1}{9}$ (4) $\frac{7}{18}$

Sol. **(2)**

I = $\int_{\pi/6}^{\pi/3} 2.\tan^3 x \sec^2 x \sin^4 3x$ + $3\tan^4 x \sin^2 3x. 2\sin 3x \cos 3x\, dx$

$= \frac{1}{2}\int_{\pi/6}^{\pi/3} 4\tan^3 x \sec^2 x \sin^4 3x$ + $3.4\tan^4 x \sin^3 3x \cos 3x dx$

$= \frac{1}{2}\int_{\pi/6}^{\pi/3} \frac{d}{dx}\left(\tan^4 x \sin^4 3x\right)dx$

$= \frac{1}{2}\left[\tan^4 x \sin^4 3x\right]_{\pi/6}^{\pi/3}$

$= \frac{1}{2}\left[9.(0) - \frac{1}{3}.\frac{1}{3}(1)\right] = -\frac{1}{18}$

Q.10 If the system of equations

x+y+z=2

2x+4y–z=6

3x+2y+λz=μ

has infinitely many solutions, then

(1) $\lambda - 2\mu = -5$ (2) $2\lambda + \mu = 14$ (3) $\lambda + 2\mu = 14$ (4) $2\lambda - \mu = 5$

Sol. **(2)**

D = 0 $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0$

$$(4\lambda+2)-1(2\lambda+3)+1(4-12)=0$$

$$4\lambda+2-2\lambda-3-8=0$$

$$2\lambda=9 \Rightarrow \lambda=\frac{9}{2}$$

$$D_x = \begin{vmatrix} 2 & 1 & 1 \\ 6 & 4 & -1 \\ \mu & 2 & -9/2 \end{vmatrix} = 0$$

$\Rightarrow \mu = 5$

Now check option

$2\lambda + \mu = 14$

Q.11 In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws total a of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six. The game stops as soon as either of the players wins. The probability of A winning the game is:

(1) $\frac{5}{31}$ (2) $\frac{31}{61}$ (3) $\frac{30}{61}$ (4) $\frac{5}{6}$

Sol. **2**

sum total 7 = (1,6)(2,5)(3,4)(4,3)(5,2)(6,1)

P(sum) = $\frac{6}{36}$

sum total 6 $\Rightarrow$ (1,5)(2,4)(3,3)(4,2)(5,1)

P(sum 6) = $\frac{5}{36}$

$P(A_{win})$ = P(6) + $P(\overline{6}).P(\overline{7}).P(6)+....$

$$= \frac{5}{36}+\frac{31}{36}\times\frac{30}{36}\times\frac{5}{36}+.....$$

$$= \frac{\frac{5}{36}}{1-\frac{31\times 30}{36\times 36}} \Rightarrow \frac{5\times 36}{36\times 36-31\times 30} \Rightarrow \frac{5\times 36}{1296-930} = \frac{5\times 36}{366} \Rightarrow \frac{30}{61}$$

Q.12 If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+5}$ are in the ratio 5:10:14, then the largest coefficient in this expansion is :

(1) 792 (2) 252 (3) 462 (4)330

Sol. **3**

$T_r : T_{r+1} : T_{r+2}$

$^{n+5}C_{r-1} : ^{n+5}C_r : ^{n+5}C_{r+1} = 5:10:14$

$$\frac{(n+5)!}{(r-1)!-(n+6-r)!} : \frac{(n+5)!}{r!(n+5-r)!} = \frac{5}{10}$$

$$\frac{r}{n+6-r} = \frac{1}{2} \qquad \frac{(r+1)!(n+4-r)!}{r!(n+5-r)!} = \frac{5}{7}$$

2r = n + 6 – r

3r = n + 6 ...(1)

$$\frac{r+1}{n+5-r} = \frac{5}{7}$$

7r+7 = 5n + 25-5r

12r = 5n + 18 ..(2)

$\therefore$ 4(n + 6) = 5n + 18

n = 6

$\therefore$ (1 + x) largest coeff = $^{11}C_5$ = 462

Q.13 The function $f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1} x, & |x| \le 1 \\ \frac{1}{2}(|x|-1), & |x| > 1 \end{cases}$ is :

(1) both continuous and differentiable on R–{–1}

(2) continuous on R–{–1} and differentiable on R–{–1,1}

(3) continuous on R–{1} and differentiable on R–{–1,1}

(4) both continuous and differentiable on R–{1}

Sol. **(3)**

$$f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1} x & x \in [-1,1] \\ \frac{1}{2}(x-1) & x > 1 \\ \frac{1}{2}(-x-1) & x < -1 \end{cases}$$

at x = 1

$f(1) = \frac{\pi}{2}$ $\quad\quad$ $f(1^+) = 0$

$\therefore$ discontinuous $\Rightarrow$ non diff.

at x = – 1

$f(-1) = 0$ $\quad\quad$ $f(-1^-) = \frac{1}{2}\{+1-1\} = 0$

cont. at x = – 1

$$f'(x) = \begin{cases} \frac{1}{1+x^2} & x \in [-1,1] \\ \frac{1}{2} & x > 1 \\ -\frac{1}{2} & x < -1 \end{cases}$$

Q.14 The solution of the differential equation $\frac{dy}{dx} - \frac{y + 3x}{\log_e(y + 3x)} + 3 = 0$ is:

(where c is a constant of integration)

(1) $x - \log_e(y+3x) = C$ $\quad\quad$ (2) $x - \frac{1}{2}(\log_e(y+3x))^2 = C$

(3) $x - 2\log_e(y+3x) = C$ $\quad\quad$ (4) $y + 3x - \frac{1}{2}(\log_e x)^2 = C$

Sol. **(2)**

$$\frac{dy}{dx} - \frac{y+3x}{\ell n(y+3x)} + 3 = 0$$

Let ln(y + 3x) = t

$$\frac{1}{y+3x}\cdot\left(\frac{dy}{dx} + 3\right) = \frac{dt}{dx}$$

$$\Rightarrow \left(\frac{dy}{dx}+3\right)=\frac{y+3x}{\ell n(y+3x)}$$

$$\therefore (y+3x)\frac{dt}{dx}=\frac{y+3x}{t}$$

$\Rightarrow$ tdt = dx

$$\frac{t^2}{2}=x+c$$

$\frac{1}{2}\left(\ln(y+3x)\right)^2$ = x + c

Q.15 Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation, $x^2 + x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to:

(1) 27 (2) 9 (3) 18 (4) 36

Sol. **(3)**

$x^2 - x + 2\lambda = 0$ (α,β)

$$3x^2-10x+27\lambda=0(\alpha,\gamma)$$

$$3x^2-3x+6\lambda=0$$

$$\begin{array}{l} - \quad + \quad - \\ \hline -7x+21\lambda=0 \end{array}$$

$\therefore \alpha = 3\lambda$ Put in equation

$9\lambda^2 - 3\lambda + 2\lambda = 0$

$9\lambda^2 - \lambda = 0 \Rightarrow \lambda=\frac{1}{9} \Rightarrow \alpha=\frac{1}{3}$

$\alpha.\beta = \frac{2}{9} \Rightarrow \beta = \frac{2}{3}$

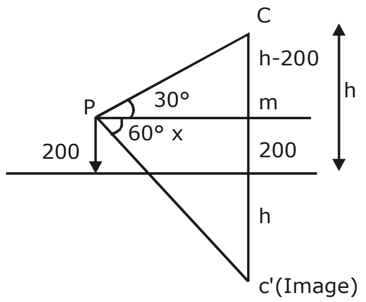
$\alpha.\gamma = 1 \Rightarrow \gamma = 3$

$\therefore \frac{\beta.r}{\lambda} \Rightarrow \frac{\frac{2}{3}.3}{\frac{1}{9}}$ = 18

Q.16 The angle of elevation of a cloud C from a point P, 200 m above a still lake is 30°. If the angle of depression of the image of C in the lake from the point P is 60°,then PC (in m) is equal to :

(1) $200\sqrt{3}$ (2) $400\sqrt{3}$ (3) 400 (4) 100

Sol. **(3)**



$$\frac{h-200}{x} = \tan 30^\circ \qquad \frac{h+200}{x} = \tan 60^\circ$$

$$\frac{h+200}{h-200} = 3$$

h + 200 = 3h – 600

2h = 800

h = 400

$$\therefore \frac{h-200}{PC} = \sin 30^\circ$$

PC = 400 m

Q.17 Let $\bigcup\limits_{i=1}^{50} X_i = \bigcup\limits_{i=1}^{n} Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i's and exactly 6 of sets Y_i's, then n is equal to :

(1) 15 (2) 30 (3) 50 (4) 45

Sol. **(2)**

$$\frac{50\times 10}{20} = \frac{n\times 5}{6}$$

$$\frac{50}{2}\times\frac{6}{5} = n \Rightarrow n = 30$$

Q.18 Let x=4 be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\frac{1}{2}$. If $P(1, \beta), \beta > 0$ is a point on this ellipse, then the equation of the normal to it at P is :

(1) 8x–2y=5 (2) 4x–2y=1 (3) 7x–4y=1 (4) 4x–3y=2

Sol. **(2)**

$e = \frac{1}{2}$ $x = \frac{a}{e} = 4$

$\Rightarrow a = 2$

$e^2 = 1 - \frac{b^2}{a^2} \Rightarrow \frac{1}{4} = 1 - \frac{b^2}{4}$

$\frac{b^2}{4} = \frac{3}{4} \Rightarrow b^2 = 3$

$\therefore$ Ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$P(1,\beta)$

$x = 1$; $\frac{1}{4} + \frac{\beta^2}{3} = 1$

$\frac{\beta^2}{3} = \frac{3}{4} \Rightarrow \beta = \frac{3}{2}$

$\Rightarrow P\left(1, \frac{3}{2}\right)$

Equation of normal $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$

$\frac{4x}{1} - \frac{3y}{\frac{3}{2}} = 4 - 3$

4x – 2y = 1

Q.19 Let $a_1, a_2, \ldots, a_n$ be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + \ldots + a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \leq n \leq 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to:

(1) (2480,248) (2) (2480,249) (3) (2490,249) (4) (2490,248)

Sol. **4**

$a_1 = 1$, $a_n = 300$, $15 \leq n \leq 50$

$300 = 1 + (n - 1)d$

$(n-1) = \frac{299}{d}$

d can 23 or 13

if n – 1 = 13

n = 14

reject

or d = 13

n – 1 = 23

n = 24

$S_{20} = \frac{20}{2}\{2+19.13\}$ $\quad\quad$ $a_{20} = 1 + 19.13$

$a_{20} = 248$

= 10{249} = 2490

$(S_{20}, a_{20}) = (2490, 248)$

Q.20 The circle passing through the intersection of the circles, $x^2+y^2-6x=0$ and $x^2+y^2-4y=0$, having its centre on the line, $2x-3y+12=0$, also passes through the point:

(1) (–1,3) (2) (1,–3) (3) (–3,6) (4) (–3,1)

Sol. **(3)**

$S_1 + \lambda(S_1 - S_2) = 0$

$x^2 + y^2 - 6x + \lambda(4y - 6x) = 0$

$x^2 + y^2 - 6x(1 + \lambda) + 4\lambda y = 0$

Centre $(3(1 + \lambda), -2\lambda)$ put in $2x - 3y + 12 = 0$

$6 + 6\lambda + 6\lambda + 12 = 0$

$12\lambda = -18$

$\lambda = -3/2$

$\therefore$ Circle is $x^2 + y^2 + 3x - 6y = 0$

Check options

Q.21 Let {x} and [x] denote the fractional part of x and the greatest integer $\leq$ x respectively of a real number x. If $\int_0^n \{x\}dx, \int_0^n [x]dx$ and $10(n^2-n), (n \in \mathrm{N}, n > 1)$ are three consecutive terms of a G.P., then n is equal to_____

Sol. **21**

$$\int_0^n \{x\}\,dx = n\int_0^1 x\,dx = n\left(\frac{x^2}{2}\right) = \frac{n}{2}$$

$$\int_0^n [x]\,dx = \int_0^1 0 + \int_1^2 1\,dx + \int_2^3 2\,dx .... + \int_{n-1}^n (n-1)\,dx$$

$$= 1 + 2 + ... + n - 1 \Rightarrow \frac{n(n-1)}{2}$$

$= \frac{n}{2}, \frac{n(n-1)}{2}, 10(n^2-n) \rightarrow G.P$

$= \frac{n^2(n-1)^2}{4} = \frac{n}{2}.10.n(n-1)$

n – 1 = 20 ; n = 21

Q.22 A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is ______

Sol. **135**

$^6C_4 \times 1 \times 3^2 = 15 \times 9 = 135$

Q.23 If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, then the value of $\left|\hat{i} \times (\vec{a} \times \hat{i})\right|^2 + \left|\hat{j} \times (\vec{a} \times \hat{j})\right|^2 + \left|\hat{k} \times (\vec{a} \times \hat{k})\right|^2$ is equal to____

Sol. **18**

$\left|\hat{i} \times (\vec{a} \times \hat{i})\right|^2 = \left|\vec{a} - (a.\hat{i})\hat{i}\right|^2$

$= \left|\hat{j} + 2\hat{k}\right|^2$ = 1 + 4 = 5

Similarly

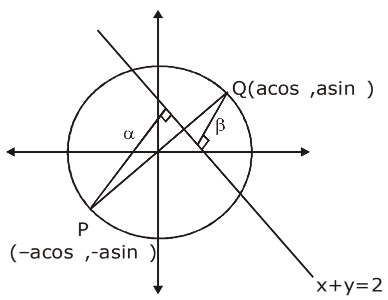
$\left|\hat{j} \times (\vec{a} \times \hat{j})\right|^2 = \left|2\hat{i} + 2\hat{k}\right|^2 = 4 + 4 = 8$

$\left|\hat{k} \times (\vec{a} \times \hat{k})\right|^2 = \left|2\hat{i} + \hat{j}\right|^2 = 4 + 1 = 5$

$\Rightarrow$ 5 + 8 + 5 = 18

Q.24 Let PQ be a diameter of the circle $x^2+y^2=9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line, x+y=2 respectively, then the maximum value of $\alpha\beta$ is _____

Sol. **7**



$$\alpha = \left|\frac{3\cos\theta + 3\sin\theta - 2}{\sqrt{2}}\right|$$

$$\beta = \left|\frac{+3\cos\theta + 3\sin\theta + 2}{\sqrt{2}}\right|$$

$$\alpha\beta = \left|\frac{(3\cos\theta + 3\sin\theta)^2 - 4}{2}\right| \Rightarrow \alpha\beta = \left|\frac{9 + 9\sin 2\theta - 4}{2}\right| \Rightarrow \alpha\beta = \left|\frac{5 + 9\sin 2\theta}{2}\right|$$

$$\alpha\beta_{\max} = \frac{9+5}{2} = 7$$

Q.25 If the variance of the following frequency distribution :

Class	:	10–20	20–30	30–40
Frequency	:	2	x	2

is 50, then x is equal to____

Sol. **4**

$$6^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$$

x_i	f_i	$x - \overline{x}$	$(x - \overline{x})^2$	$f_i(x - \overline{x})^2$
15	2	−10	100	200
25	x	0	0	0
35	2	10	100	200
	$4 + x$			400

$$\overline{x} = \frac{100 + 25x}{4 + x}$$

$\overline{x}$ = 25

$\therefore \dfrac{400}{4 + x}$ = 50

x = 4