

## TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Friday 11<sup>th</sup> JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

## MATHEMATICS

1. Let  $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$ . If  $AA^T = I_3$ , then  $|p|$

is :

(1)  $\frac{1}{\sqrt{2}}$

(2)  $\frac{1}{\sqrt{5}}$

(3)  $\frac{1}{\sqrt{6}}$

(4)  $\frac{1}{\sqrt{3}}$

**Ans. (1)****Sol.** A is orthogonal matrix

$$\Rightarrow 0^2 + p^2 + p^2 = 1 \Rightarrow |p| = \frac{1}{\sqrt{2}}$$

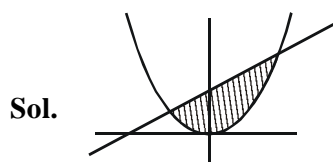
2. The area (in sq. units) of the region bounded by the curve  $x^2 = 4y$  and the straight line  $x = 4y - 2$  :-

(1)  $\frac{5}{4}$

(2)  $\frac{9}{8}$

(3)  $\frac{3}{4}$

(4)  $\frac{7}{8}$

**Ans. (2)**

$$\begin{aligned} x &= 4y - 2 \text{ \& } x^2 = 4y \\ \Rightarrow x^2 &= x + 2 \Rightarrow x^2 - x - 2 = 0 \\ x &= 2, -1 \end{aligned}$$

$$\text{So, } \int_{-1}^2 \left( \frac{x+2}{4} - \frac{x^2}{4} \right) dx = \frac{9}{8}$$

3. The outcome of each of 30 items was observed; 10 items gave an outcome  $\frac{1}{2} - d$  each, 10 items gave outcome  $\frac{1}{2}$  each and the remaining 10 items gave outcome  $\frac{1}{2} + d$  each. If the variance of this outcome data is  $\frac{4}{3}$  then  $|d|$  equals :-

(1) 2 (2)  $\frac{\sqrt{5}}{2}$  (3)  $\frac{2}{3}$  (4)  $\sqrt{2}$

**Ans. (4)**

**Sol.** Variance is independent of origin. So we shift the given data by  $\frac{1}{2}$ .

$$\text{so, } \frac{10d^2 + 10 \times 0^2 + 10d^2}{30} - (0)^2 = \frac{4}{3}$$

$$\Rightarrow d^2 = 2 \Rightarrow |d| = \sqrt{2}$$

4. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is  $\frac{27}{19}$ . Then the common ratio of this series is :

(1)  $\frac{4}{9}$  (2)  $\frac{2}{9}$   
(3)  $\frac{2}{3}$  (4)  $\frac{1}{3}$

**Ans. (3)**

**Sol.**  $\frac{a}{1-r} = 3 \dots(1)$

$$\frac{a^3}{1-r^3} = \frac{27}{19} \Rightarrow \frac{27(1-r)^3}{1-r^3} = \frac{27}{19}$$

$$\Rightarrow 6r^2 - 13r + 6 = 0$$

$$\Rightarrow r = \frac{2}{3} \text{ as } |r| < 1$$

5. Let  $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$ ,  $\vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$  and  $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$  be coplanar vectors.

Then the non-zero vector  $\vec{a} \times \vec{c}$  is :

- (1)  $-14\hat{i} - 5\hat{j}$  (2)  $-10\hat{i} - 5\hat{j}$   
(3)  $-10\hat{i} + 5\hat{j}$  (4)  $-14\hat{i} + 5\hat{j}$

Ans. (3)

Sol.  $[\vec{a} \ \vec{b} \ \vec{c}] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & \lambda^2 - 1 \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$$

$$\Rightarrow \lambda^2(\lambda - 2) - 9(\lambda - 2) = 0$$

$$\Rightarrow (\lambda - 3)(\lambda + 3)(\lambda - 2) = 0$$

$$\Rightarrow \lambda = 2, 3, -3$$

So,  $\lambda = 2$  (as  $\vec{a}$  is parallel to  $\vec{c}$  for  $\lambda = \pm 3$ )

$$\text{Hence } \vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix}$$

$$= -10\hat{i} + 5\hat{j}$$

6. Let  $\left(-2 - \frac{1}{3}i\right)^3 = \frac{x + iy}{27}$  ( $i = \sqrt{-1}$ ), where  $x$

and  $y$  are real numbers, then  $y - x$  equals :

- (1) -85 (2) 85  
(3) -91 (4) 91

Ans. (4)

$$\text{Sol. } \left(-2 - \frac{i}{3}\right)^3 = -\frac{(6+i)^3}{27}$$

$$= \frac{-198 - 107i}{27} = \frac{x + iy}{27}$$

Hence,  $y - x = 198 - 107 = 91$

7. Let  $f(x) = \begin{cases} -1, & -2 \leq x < 0 \\ x^2 - 1, & 0 \leq x \leq 2 \end{cases}$  and

$g(x) = |f(x)| + f(|x|)$ . Then, in the interval

$(-2, 2)$ ,  $g$  is :-

- (1) differentiable at all points  
(2) not differentiable at two points  
(3) not continuous  
(4) not differentiable at one point

Ans. (4)

$$\text{Sol. } |f(x)| = \begin{cases} 1, & -2 \leq x < 0 \\ 1 - x^2, & 0 \leq x < 1 \\ x^2 - 1, & 1 \leq x \leq 2 \end{cases}$$

and  $f(|x|) = x^2 - 1, x \in [-2, 2]$

$$\text{Hence } g(x) = \begin{cases} x^2, & x \in [-2, 0) \\ 0, & x \in [0, 1) \\ 2(x^2 - 1), & x \in [1, 2] \end{cases}$$

It is not differentiable at  $x = 1$

8. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = \frac{x}{1 + x^2}$ ,

$x \in \mathbb{R}$ . Then the range of  $f$  is :

- (1)  $(-1, 1) - \{0\}$  (2)  $\left[-\frac{1}{2}, \frac{1}{2}\right]$   
(3)  $\mathbb{R} - \left[-\frac{1}{2}, \frac{1}{2}\right]$  (4)  $\mathbb{R} - [-1, 1]$

Ans. (2)

Sol.  $f(0) = 0$  &  $f(x)$  is odd.

Further, if  $x > 0$  then

$$f(x) = \frac{1}{x + \frac{1}{x}} \in \left(0, \frac{1}{2}\right]$$

$$\text{Hence, } f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

9. The sum of the real values of  $x$  for which the middle term in the binomial expansion of

$$\left(\frac{x^3}{3} + \frac{3}{x}\right)^8 \text{ equals } 5670 \text{ is :}$$

- (1) 6                      (2) 8                      (3) 0                      (4) 4

**Ans. (3)**

**Sol.**  $T_5 = {}^8C_4 \frac{x^{12}}{81} \times \frac{81}{x^4} = 5670$

$$\Rightarrow 70x^8 = 5670$$

$$\Rightarrow x = \pm\sqrt{3}$$

10. The value of  $r$  for which

${}^{20}C_r {}^{20}C_0 + {}^{20}C_{r-1} {}^{20}C_1 + {}^{20}C_{r-2} {}^{20}C_2 + \dots + {}^{20}C_0 {}^{20}C_r$  is maximum, is

- (1) 20                                      (2) 15  
(3) 11                                      (4) 10

**Ans. (1)**

**Sol.** Given sum = coefficient of  $x^r$  in the expansion of  $(1+x)^{20}(1+x)^{20}$ ,

which is equal to  ${}^{40}C_r$

It is maximum when  $r = 20$

11. Let  $a_1, a_2, \dots, a_{10}$  be a G.P. If  $\frac{a_3}{a_1} = 25$ , then

$\frac{a_9}{a_5}$  equals :

- (1)  $2(5^2)$                                       (2)  $4(5^2)$   
(3)  $5^4$                                       (4)  $5^3$

**Ans. (3)**

**Sol.**  $a_1, a_2, \dots, a_{10}$  are in G.P.,

Let the common ratio be  $r$

$$\frac{a_3}{a_1} = 25 \Rightarrow \frac{a_1 r^2}{a_1} = 25 \Rightarrow r^2 = 25$$

$$\frac{a_9}{a_5} = \frac{a_1 r^8}{a_1 r^4} = r^4 = 5^4$$

12. If  $\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x)(\sqrt{1-x^2})^m + C$ , for

a suitable chosen integer  $m$  and a function  $A(x)$ , where  $C$  is a constant of integration then  $(A(x))^m$  equals :

- (1)  $\frac{-1}{3x^3}$                                       (2)  $\frac{-1}{27x^9}$   
(3)  $\frac{1}{9x^4}$                                       (4)  $\frac{1}{27x^6}$

**Ans. (2)**

**Sol.**  $\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x)(\sqrt{1-x^2})^m + C$

$$\int \frac{|x| \sqrt{\frac{1}{x^2} - 1}}{x^4} dx,$$

$$\text{Put } \frac{1}{x^2} - 1 = t \Rightarrow \frac{dt}{dx} = \frac{-2}{x^3}$$

**Case-I**  $x \geq 0$

$$-\frac{1}{2} \int \sqrt{t} dt \Rightarrow -\frac{t^{3/2}}{3} + C$$

$$\Rightarrow -\frac{1}{3} \left( \frac{1}{x^2} - 1 \right)^{3/2}$$

$$\Rightarrow \frac{(\sqrt{1-x^2})^3}{-3x^2} + C$$

$$A(x) = -\frac{1}{3x^3} \text{ and } m = 3$$

$$(A(x))^m = \left( -\frac{1}{3x^3} \right)^3 = -\frac{1}{27x^9}$$

**Case-II**  $x \leq 0$

$$\text{We get } \frac{(\sqrt{1-x^2})^3}{-3x^3} + C$$

$$A(x) = \frac{1}{-3x^3}, \quad m = 3$$

$$(A(x))^m = \frac{-1}{27x^9}$$

13. In a triangle, the sum of lengths of two sides is  $x$  and the product of the lengths of the same two sides is  $y$ . If  $x^2 - c^2 = y$ , where  $c$  is the length of the third side of the triangle, then the circumradius of the triangle is :

(1)  $\frac{y}{\sqrt{3}}$  (2)  $\frac{c}{\sqrt{3}}$  (3)  $\frac{c}{3}$  (4)  $\frac{3}{2}y$

Ans. (2)

Sol. Given  $a + b = x$  and  $ab = y$

$$\text{If } x^2 - c^2 = y \Rightarrow (a + b)^2 - c^2 = ab$$

$$\Rightarrow a^2 + b^2 - c^2 = -ab$$

$$\Rightarrow \frac{a^2 + b^2 - c^2}{2ab} = -\frac{1}{2}$$

$$\Rightarrow \cos C = -\frac{1}{2}$$

$$\Rightarrow \angle C = \frac{2\pi}{3}$$

$$R = \frac{c}{2\sin C} = \frac{c}{\sqrt{3}}$$

14. The value of the integral  $\int_{-2}^2 \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$

(where  $[x]$  denotes the greatest integer less than  $20\pi$  or equal to  $x$ ) is :

(1) 4 (2)  $4 - \sin 4$   
(3)  $\sin 4$  (4) 0

Ans. (4)

Sol.  $I = \int_{-2}^2 \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$

$$I = \int_0^2 \left( \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} + \frac{\sin^2(-x)}{\left[-\frac{x}{\pi}\right] + \frac{1}{2}} \right) dx$$

$$\left( \left[\frac{x}{\pi}\right] + \left[-\frac{x}{\pi}\right] = -1 \text{ as } x \neq n\pi \right)$$

$$I = \int_0^2 \left( \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} + \frac{\sin^2 x}{-1 - \left[\frac{x}{\pi}\right] + \frac{1}{2}} \right) dx = 0$$

15. If the system of linear equations

$$2x + 2y + 3z = a$$

$$3x - y + 5z = b$$

$$x - 3y + 2z = c$$

where  $a, b, c$  are non-zero real numbers, has more than one solution, then :

(1)  $b - c - a = 0$  (2)  $a + b + c = 0$   
(3)  $b + c - a = 0$  (4)  $b - c + a = 0$

Ans. (1)

Sol.  $P_1 : 2x + 2y + 3z = a$

$$P_2 : 3x - y + 5z = b$$

$$P_3 : x - 3y + 2z = c$$

We find

$$P_1 + P_3 = P_2 \Rightarrow a + c = b$$

16. A square is inscribed in the circle

$x^2 + y^2 - 6x + 8y - 103 = 0$  with its sides parallel to the coordinate axes. Then the distance of the vertex of this square which is nearest to the origin is :-

(1) 13 (2)  $\sqrt{137}$   
(3) 6 (4)  $\sqrt{41}$

Ans. (4)

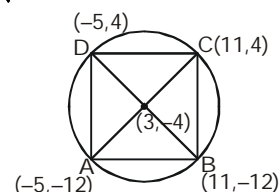
Sol.  $R = \sqrt{9 + 16 + 103} = 8\sqrt{2}$

$$OA = 13$$

$$OB = \sqrt{265}$$

$$OC = \sqrt{137}$$

$$OD = \sqrt{41}$$



17. Let  $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$  for  $k = 1, 2, 3, \dots$ . Then for all  $x \in \mathbb{R}$ , the value of  $f_4(x) - f_6(x)$  is equal to :-

(1)  $\frac{5}{12}$  (2)  $-\frac{1}{12}$  (3)  $\frac{1}{4}$  (4)  $\frac{1}{12}$

Ans. (4)

Sol.  $f_4(x) - f_6(x)$

$$= \frac{1}{4}(\sin^4 x + \cos^4 x) - \frac{1}{6}(\sin^6 x + \cos^6 x)$$

$$= \frac{1}{4} \left( 1 - \frac{1}{2} \sin^2 2x \right) - \frac{1}{6} \left( 1 - \frac{3}{4} \sin^2 2x \right) = \frac{1}{12}$$

18. Let  $[x]$  denote the greatest integer less than or equal to  $x$ . Then :-

$$\lim_{x \rightarrow 0} \frac{\tan(\pi \sin^2 x) + (|x| - \sin(x[x]))^2}{x^2}$$

- (1) equals  $\pi$   
 (2) equals 0  
 (3) equals  $\pi + 1$   
 (4) does not exist

**Ans. (4)**

**Sol.** R.H.L. =  $\lim_{x \rightarrow 0^+} \frac{\tan(\pi \sin^2 x) + (|x| - \sin(x[x]))^2}{x^2}$

(as  $x \rightarrow 0^+ \Rightarrow [x] = 0$ )

$$= \lim_{x \rightarrow 0^+} \frac{\tan(\pi \sin^2 x) + x^2}{x^2}$$

$$= \lim_{x \rightarrow 0^+} \frac{\tan(\pi \sin^2 x)}{(\pi \sin^2 x)} + 1 = \pi + 1$$

$$\text{L.H.L.} = \lim_{x \rightarrow 0^-} \frac{\tan(\pi \sin^2 x) + (-x + \sin x)^2}{x^2}$$

(as  $x \rightarrow 0^- \Rightarrow [x] = -1$ )

$$\lim_{x \rightarrow 0^+} \frac{\tan(\pi \sin^2 x)}{\pi \sin^2 x} \cdot \frac{\pi \sin^2 x}{x^2} + \left(-1 + \frac{\sin x}{x}\right)^2 \Rightarrow \pi$$

R.H.L.  $\neq$  L.H.L.

19. The direction ratios of normal to the plane through the points  $(0, -1, 0)$  and  $(0, 0, 1)$  and

making an angle  $\frac{\pi}{4}$  with the plane  $y - z + 5 = 0$  are:

- (1)  $2\sqrt{3}, 1, -1$   
 (2)  $2, \sqrt{2}, -\sqrt{2}$   
 (3)  $2, -1, 1$   
 (4)  $\sqrt{2}, 1, -1$

**Ans. (2, 4)**

**Sol.** Let the equation of plane be

$$a(x - 0) + b(y + 1) + c(z - 0) = 0$$

It passes through  $(0, 0, 1)$  then

$$b + c = 0 \quad \dots(1)$$

$$\text{Now } \cos \frac{\pi}{4} = \frac{a(0) + b(1) + c(-1)}{\sqrt{2}\sqrt{a^2 + b^2 + c^2}}$$

$$\Rightarrow a^2 = -2bc \text{ and } b = -c$$

we get  $a^2 = 2c^2$

$$\Rightarrow a = \pm\sqrt{2}c$$

$$\Rightarrow \text{direction ratio } (a, b, c) = (\sqrt{2}, -1, 1) \text{ or } (\sqrt{2}, 1, -1)$$

20. If  $x \log_e(\log_e x) - x^2 + y^2 = 4$  ( $y > 0$ ), then  $dy/dx$  at  $x = e$  is equal to :

(1)  $\frac{e}{\sqrt{4 + e^2}}$

(2)  $\frac{(1 + 2e)}{2\sqrt{4 + e^2}}$

(3)  $\frac{(2e - 1)}{2\sqrt{4 + e^2}}$

(4)  $\frac{(1 + 2e)}{\sqrt{4 + e^2}}$

**Ans. (3)**

**Sol.** Differentiating with respect to  $x$ ,

$$x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + \ln(\ln x) - 2x + 2y \cdot \frac{dy}{dx} = 0$$

at  $x = e$  we get

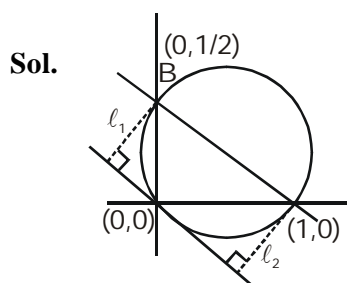
$$1 - 2e + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{2e - 1}{2y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2e - 1}{2\sqrt{4 + e^2}} \text{ as } y(e) = \sqrt{4 + e^2}$$

21. The straight line  $x + 2y = 1$  meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is :

- (1)  $\frac{\sqrt{5}}{4}$   
 (2)  $\frac{\sqrt{5}}{2}$   
 (3)  $2\sqrt{5}$   
 (4)  $4\sqrt{5}$

Ans. (2)



Equation of circle

$$(x-1)(x-0) + (y-0)\left(y-\frac{1}{2}\right) = 0$$

$$\Rightarrow x^2 + y^2 - x - \frac{y}{2} = 0$$

Equation of tangent of origin is  $2x + y = 0$

$$\ell_1 + \ell_2 = \frac{2}{\sqrt{5}} + \frac{1}{2\sqrt{5}}$$

$$= \frac{4+1}{2\sqrt{5}} = \frac{\sqrt{5}}{2}$$

22. If q is false and  $p \wedge q \leftrightarrow r$  is true, then which one of the following statements is a tautology?

- (1)  $(p \vee r) \rightarrow (p \wedge r)$   
 (2)  $p \vee r$   
 (3)  $p \wedge r$   
 (4)  $(p \wedge r) \rightarrow (p \vee r)$

Ans. (4)

Sol. Given q is F and  $(p \wedge q) \leftrightarrow r$  is T  
 $\Rightarrow p \wedge q$  is F which implies that r is F  
 $\Rightarrow q$  is F and r is F  
 $\Rightarrow (p \wedge r)$  is always F  
 $\Rightarrow (p \wedge r) \rightarrow (p \vee r)$  is tautology.

23. If  $y(x)$  is the solution of the differential equation

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}, \quad x > 0,$$

where  $y(1) = \frac{1}{2}e^{-2}$ , then :

- (1)  $y(x)$  is decreasing in  $(0, 1)$   
 (2)  $y(x)$  is decreasing in  $\left(\frac{1}{2}, 1\right)$   
 (3)  $y(\log_e 2) = \frac{\log_e 2}{4}$   
 (4)  $y(\log_e 2) = \log_e 4$

Ans. (2)

Sol.  $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}$

$$\text{I.F.} = e^{\int \left(\frac{2x+1}{x}\right) dx} = e^{\int \left(2 + \frac{1}{x}\right) dx} = e^{2x + \ln x} = e^{2x} \cdot x$$

$$\text{So, } y(xe^{2x}) = \int e^{-2x} \cdot xe^{2x} + C$$

$$\Rightarrow xye^{2x} = \int x dx + C$$

$$\Rightarrow 2xye^{2x} = x^2 + 2C$$

It passes through  $\left(1, \frac{1}{2}e^{-2}\right)$  we get  $C = 0$

$$y = \frac{xe^{-2x}}{2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}e^{-2x}(-2x+1)$$

$$\Rightarrow f(x) \text{ is decreasing in } \left(\frac{1}{2}, 1\right)$$

$$y(\log_e 2) = \frac{(\log_e 2)e^{-2(\log_e 2)}}{2}$$

$$= \frac{1}{8} \log_e 2$$

24. The maximum value of the function  $f(x) = 3x^3 - 18x^2 + 27x - 40$  on the set

$$S = \{x \in \mathbb{R} : x^2 + 30 \leq 11x\} \text{ is :}$$

- (1) 122 (2) -222  
(3) -122 (4) 222

**Ans. (1)**

**Sol.**  $S = \{x \in \mathbb{R}, x^2 + 30 - 11x \leq 0\}$   
 $= \{x \in \mathbb{R}, 5 \leq x \leq 6\}$

Now  $f(x) = 3x^3 - 18x^2 + 27x - 40$

$\Rightarrow f'(x) = 9(x-1)(x-3)$ ,

which is positive in  $[5, 6]$

$\Rightarrow f(x)$  increasing in  $[5, 6]$

Hence maximum value  $= f(6) = 122$

25. If one real root of the quadratic equation  $81x^2 + kx + 256 = 0$  is cube of the other root, then a value of  $k$  is

- (1) -81 (2) 100 (3) -300 (4) 144

**Ans. (3)**

**Sol.**  $81x^2 + kx + 256 = 0$  ;  $x = \alpha, \alpha^3$

$$\Rightarrow \alpha^4 = \frac{256}{81} \Rightarrow \alpha = \pm \frac{4}{3}$$

Now  $-\frac{k}{81} = \alpha + \alpha^3 = \pm \frac{100}{27}$

$\Rightarrow k = \pm 300$

26. Two circles with equal radii are intersecting at the points  $(0, 1)$  and  $(0, -1)$ . The tangent at the point  $(0, 1)$  to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is :

- (1) 1 (2)  $\sqrt{2}$   
(3)  $2\sqrt{2}$  (4) 2

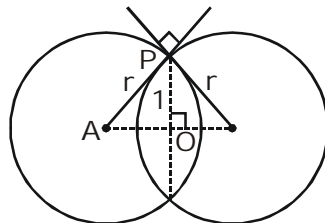
**Ans. (4)**

**Sol.** In  $\triangle APO$

$$\left(\frac{\sqrt{2}r}{2}\right)^2 + 1^2 = r^2$$

$$\Rightarrow \boxed{r = \sqrt{2}}$$

So distance between centres  $= \sqrt{2}r = 2$



27. Equation of a common tangent to the parabola  $y^2 = 4x$  and the hyperbola  $xy = 2$  is :

- (1)  $x + 2y + 4 = 0$   
(2)  $x - 2y + 4 = 0$   
(3)  $x + y + 1 = 0$   
(4)  $4x + 2y + 1 = 0$

**Ans. (1)**

**Sol.** Let the equation of tangent to parabola

$$y^2 = 4x \text{ be } y = mx + \frac{1}{m}$$

It is also a tangent to hyperbola  $xy = 2$

$$\Rightarrow x\left(mx + \frac{1}{m}\right) = 2$$

$$\Rightarrow x^2m + \frac{x}{m} - 2 = 0$$

$$D = 0 \Rightarrow m = -\frac{1}{2}$$

So tangent is  $2y + x + 4 = 0$

28. The plane containing the line  $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$

and also containing its projection on the plane

$2x + 3y - z = 5$ , contains which one of the following points ?

- (1)  $(2, 0, -2)$  (2)  $(-2, 2, 2)$   
(3)  $(0, -2, 2)$  (4)  $(2, 2, 0)$

**Ans. (1)**

**Sol.** The normal vector of required plane

$$= (2\hat{i} - \hat{j} + 3\hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$$

$$= -8\hat{i} + 8\hat{j} + 8\hat{k}$$

So, direction ratio of normal is  $(-1, 1, 1)$

So required plane is

$$-(x-3) + (y+2) + (z-1) = 0$$

$$\Rightarrow -x + y + z + 4 = 0$$

Which is satisfied by  $(2, 0, -2)$

29. If tangents are drawn to the ellipse  $x^2 + 2y^2 = 2$  at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinate axes lie on the curve :

$$(1) \frac{x^2}{2} + \frac{y^2}{4} = 1 \quad (2) \frac{x^2}{4} + \frac{y^2}{2} = 1$$

$$(3) \frac{1}{2x^2} + \frac{1}{4y^2} = 1 \quad (4) \frac{1}{4x^2} + \frac{1}{2y^2} = 1$$

**Ans. (3)**

**Sol.** Equation of general tangent on ellipse

$$\frac{x}{a \sec \theta} + \frac{y}{b \operatorname{cosec} \theta} = 1$$

$$a = \sqrt{2}, b = 1$$

$$\Rightarrow \frac{x}{\sqrt{2} \sec \theta} + \frac{y}{\operatorname{cosec} \theta} = 1$$

Let the midpoint be (h, k)

$$h = \frac{\sqrt{2} \sec \theta}{2} \Rightarrow \cos \theta = \frac{1}{\sqrt{2}h}$$

$$\text{and } k = \frac{\operatorname{cosec} \theta}{2} \Rightarrow \sin \theta = \frac{1}{2k}$$

$$\therefore \sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \frac{1}{2h^2} + \frac{1}{4k^2} = 1$$

$$\Rightarrow \frac{1}{2x^2} + \frac{1}{4y^2} = 1$$

30. Two integers are selected at random from the set  $\{1, 2, \dots, 11\}$ . Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is :

$$(1) \frac{2}{5}$$

$$(2) \frac{1}{2}$$

$$(3) \frac{3}{5}$$

$$(4) \frac{7}{10}$$

**Ans. (1)**

**Sol.** Since sum of two numbers is even so either both are odd or both are even. Hence number of elements in reduced samples space

$$= {}^5C_2 + {}^6C_2$$

$$\text{so required probability} = \frac{{}^5C_2}{{}^5C_2 + {}^6C_2}$$