JEE-Main-20-07-2021-Shift-2 (Memory Based)

PHYSICS

Question: In a series LCR circuit, $R = 5\Omega, L = 0.5\,\text{mH}, C = 2.5\,\mu\text{F}$. The RMS value of external voltage is 250 V. Find the power dissipated if circuit is in Resonance
Answer: 12500 W
Solution:

$$\text{Power} = V_{rms} I_{rms}\left(\frac{R}{Z}\right)$$

Z = R (at resonance)

$$\text{Power} = 250 \times \frac{250}{5} \times \frac{5}{5}$$

$$= 12500\,\text{W}$$

Question: The wavelength of sodium lamp is observed to be $2886\,\text{Å}$ from earth & original wavelength was $2880\,\text{Å}$. Find speed of galaxy.
Options:
(a) $3\times10^5\,\text{m}\,\text{s}^{-1}$
(b) $4\times10^5\,\text{m}\,\text{s}^{-1}$
(c) $6.25\times10^5\,\text{m}\,\text{s}^{-1}$
(d) None
Answer: (c)
Solution: $\Delta\lambda = 2886 - 2880 = 6\text{Å}$

Using doppler shift,

$$-\frac{\Delta\lambda}{\lambda} = -\frac{V_{\text{radial}}}{C}$$

$$\Rightarrow V_{\text{radial}} = C\left(\frac{\Delta\lambda}{\lambda}\right) = 3\times10^8 \times \left(\frac{6\times10^{-10}}{2880\times10^{-10}}\right)$$

$$= 6.25\times10^5\,\text{ms}^{-1}$$

Hence, speed of galaxy $= 6.25\times10^5\,\text{ms}^{-1}$.

Question: A body is under the influence of a force such that it delivers a constant power P. The variation of position with time of body as

Options:

(a) $t^{\frac{1}{2}}$

(b) $t^{\frac{3}{2}}$

(c) $t^{\frac{5}{2}}$

(d) None

Answer: (b)

Solution: Power = P

F v = P

$$m\left(\frac{dv}{dt}\right)\cdot v = P$$

$$\Rightarrow \int_0^v v\,dv = \int_0^t \frac{P}{m}\cdot dt$$

$$\Rightarrow v = \sqrt{\frac{2P}{m}t}$$

$$\Rightarrow \frac{dx}{dt} = \sqrt{\frac{2P}{m}t}\ [\text{assuming at } t = 0, x = 0\ \&\ v = 0]$$

$$\Rightarrow \int_0^x dx = \int_0^t \sqrt{\frac{2P}{m}t}\cdot dt$$

$$\Rightarrow t = x = \sqrt{\frac{2P}{m}}\frac{2}{3}t^{3/2}$$

$$\Rightarrow x \propto t^{3/2}$$

Question: When a metal is illuminated by light of wavelength λ ,the stopping potential is V_0 and for wavelength 2λ , it is $3V_0$. Then the threshold wavelength is?

Options:

(a) $\frac{2\lambda}{3}$

(b) $\frac{4\lambda}{5}$

(c) $\frac{\lambda}{3}$

(d) $\frac{5\lambda}{2}$

Answer: (b)

Solution: $eV_0 = \frac{hc}{\lambda} - \phi ...(1)$

$$3eV_0 = \frac{hc}{2\lambda} - \phi...(2)$$

Multiply by 3 in equation (1)

$$3eV = \frac{3hc}{\lambda} - 3\phi...(3)$$

Equation (3) – (2)

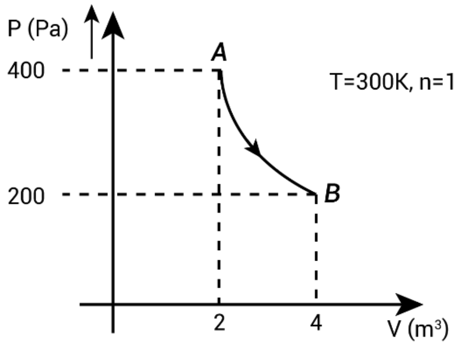
$$\frac{3hc}{\lambda} - \frac{hc}{2\lambda} - 2\phi = 0$$

$$\frac{5hc}{2\lambda} = 2\phi = \frac{2hc}{\lambda_0}$$

$$\frac{5}{4}\frac{hc}{\lambda} = \frac{hc}{\lambda_0}$$

$$\lambda_0 = \frac{4\lambda}{5}$$

Question: A gas is taken through an isothermal process as shown. Find the work done by the gas



Options:
(a) 240 J
(b) 360 J
(c) 560 J
(d) None
Answer: (c)

Solution: $\text{Work} = PV \ln\frac{v_2}{v_1}$

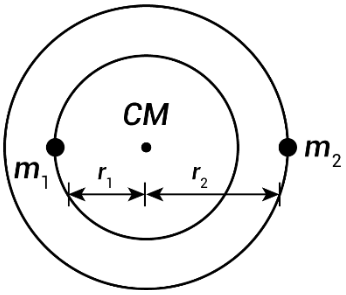
$$= 400 \times 2 \ln\frac{4}{2}$$

$$= 800 \ln 2$$

$$= 800 \times 0.7$$

$= 560\,\text{J}$

Question: Two stars of masses m_1 and m_2 form a binary system, revolving around each other in circular orbits of radii r_1 and r_2 respectively. Time period of revolution for this system is



Options:

(a) $2\pi\sqrt{\dfrac{(r_1+r_2)^3}{G(m_1+m_2)}}$

(b) $2\pi\sqrt{\dfrac{(r_1+r_2)r_2^2}{G(m_1+m_2)}}$

(c) $\dfrac{2\pi(r_1+r_2)^{\frac{3}{2}}}{\sqrt{G(m_1+m_2)}}$

(d) $\dfrac{2\pi(r_1+r_2)^2 r_1}{G(m_1+m_2)}$

Answer: (c)

Solution:

$$\frac{Gm_1m_2}{(r_1+r_2)^2} = \frac{m_1v_1^2}{r_1}$$

$$v_1^2 = \frac{Gm_2r_1}{(r_1+r_2)^2} \Rightarrow v_1 = \frac{\sqrt{Gm_2r_1}}{(r_1+r_2)}$$

$$T = \frac{2\pi r_1}{v_1} = \frac{2\pi\sqrt{r_1}\,(r_1+r_2)}{\sqrt{Gm_2}} \quad ...(1)$$

By using COM concept.

$$r_1 = \frac{m_2(r_1+r_2)}{m_1+m_2}$$

Put this value of r_1 in eq^n (1)

We get

$$T = \frac{2\pi\left(r_1 + r_2\right)^{3/2}}{\sqrt{G\left(m_1 + m_2\right)}}$$

Question: Tension in a spring is T_1 when length of the spring is L_1 and tension is T_2 when its length is L_2. The natural length of the spring is

Options:

(a) $\frac{T_2 l_2 + T_1 l_1}{T_2 + T_1}$

(b) $\frac{T_2 l_2 - T_1 l_1}{T_2 - T_1}$

(c) $\frac{T_2 l_1 + T_1 l_2}{T_2 + T_1}$

(d) $\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$

Answer: (d)

Solution:

Let the natural length of wire be l_0.

Using Hooke's law, $Y = \frac{Tl_0}{A\Delta l}$

Where, $\Delta l = l - l_0$

We get $l - l_0 = \frac{Tl_0}{AY}$

Case 1: Tension is T_1 and length of wire $l = l_1$

$$\therefore l_1 - l_0 = \frac{T_1 l_0}{AY} \quad ...(1)$$

Case 2: Tension is T_2 and length of wire $l = l_2$

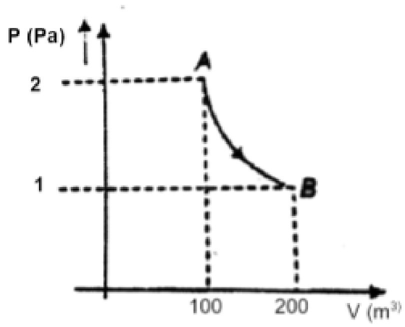
$$\therefore l_2 - l_0 = \frac{T_2 l_0}{AY} \quad ...(2)$$

Dividing both equations

$$\frac{l_1 - l_0}{l_2 - l_0} = \frac{T_1}{T_2}$$

$$l_0 = \frac{l_1 T_2 - l_2 T_2}{T_2 - T_1}$$

Question: Find work done in the process $A \rightarrow B$ (isothermal) by gas?



Options:

(a) $100 \ln 2$

(b) $-100 \ln 2$

(c) $200 \ln 2$

(d) $-200 \ln 2$

Answer: (c)

Solution:

Work done by gas in isothermal process is $\Rightarrow W = nRT \ln\left(\frac{V_2}{V_1}\right)$

$$W = P_1V_1 \ln\left(\frac{V_2}{V_1}\right)$$

$$W = 2 \times 100 \ln\left(\frac{200}{100}\right)$$

$$W_{gas} = 200 \ln 2$$

Question: A body rotating have an angular velocity of 300 rpm and its angular acceleration is $\frac{\pi}{20}\left(rad / s^2\right)$. Resolutions done by this body in 10 sec is

Options:

(a) $\frac{205}{4}(rev)$

(b) $\frac{307}{3}(rev)$

(c) $75(rev)$

(d) $\frac{189}{2}(rev)$

Answer: (a)

Solution:

$$\omega = 300\, rpm = 300 \times \frac{2\pi}{60} = 10\,\pi \text{ rad / sec}$$

$\alpha = \frac{\pi}{20} rad / s^2$

$\theta = \omega T + \frac{1}{2}\alpha T^2$

$\theta = 10\pi \times 10 + \frac{1}{2} \times \frac{\pi}{20} \times (10)^2 = 102.5\pi\, rad$

$\theta = \frac{102.5\pi}{2\pi} rev = \frac{205}{4} rev$

Question: A boy at the airport takes time t_1 to walk on escalator if the escalator is at rest and takes time t_2 if boy is at rest on moving escalator. Then find the time taken to walk on the escalator for same path?

Options:

(a) $|t_1 - t_2|$

(b) $\frac{t_1 + t_2}{2}$

(c) $\frac{2t_1t_2}{t_1 + t_2}$

(d) $\frac{t_1t_2}{t_1 + t_2}$

Answer: (d)

Solution:

Let distance to be traversed = x

Speed of escalator $= \frac{x}{t_1}$

Speed of boy walking $= \frac{x}{t_2}$

When the boy is walking on a moving escalator, speed is $= x\left(\frac{1}{t_2} + \frac{1}{t_1}\right) = x\left(\frac{t_1 + t_2}{t_1t_2}\right)$

Time taken is distance by speed $= \frac{x}{v} = \frac{t_1t_2}{t_1 + t_2}$

$T = \frac{t_1t_2}{t_1 + t_2}$

Question: A particle is performing SHM along x-axis, such that its velocity is v_1 , when its displacement from mean position is x_1 and v_2 when its displacement from mean position is x_2. Time period of oscillation is

Options:

(a) $\frac{1}{2\pi}\sqrt{\frac{x_2-x_1}{(v_1-v_2)}}$

(b) $2\pi\sqrt{\frac{(x_1^2+x_2^2)}{(v_2^2+v_1^2)}}$

(c) $2\pi\sqrt{\frac{(x_2^2-x_1^2)}{(v_1^2-v_2^2)}}$

(d) $2\pi\sqrt{\frac{(x_1x_2-x_1^2)}{(v_1v_2-v_1^2)}}$

Answer: (c)

Solution:

$v=\omega\sqrt{A^2-x^2}$

$v_1=\omega\sqrt{A^2-x_1^2}$

$\Rightarrow v_1^2=\omega^2\left(A^2-x_1^2\right) \qquad \ldots(i)$

$v_2=\omega\sqrt{A^2-x_2^2}$

$v_2^2=\omega^2\left(A^2-x_2^2\right)$

$\Rightarrow\left(\frac{v_1^2}{\omega^2}+x_2^2\right)=A^2 \qquad \ldots(ii)$

From (i) and (ii)

$v_1^2=\omega^2\left(\frac{v_2^2}{\omega^2}+x_2^2-x_1^2\right)$

$v_1^2=v_2^2+\omega^2\left(x_2^2-x_1^2\right)$

$\Rightarrow \omega^2=\frac{v_1^2-v_2^2}{x_2^2-x_1^2}$

$\Rightarrow \omega=\sqrt{\frac{v_1^2-v_2^2}{x_2^2-x_1^2}}$

We know that

$T=\frac{2\pi}{\omega}$

$\Rightarrow T=2\pi\sqrt{\frac{x_2^2-x_1^2}{v_1^2-v_2^2}}$

Question: An electron $\left(9\times10^{-31}\,kg,\ 1.6\times10^{-19}\,C\right)$ is accelerated by a voltage of 40 kV. What is the wavelength?

$h=6.6\times10^{-34}$ SI units.

Answer: 6.15×10^{-12}

Solution:

$$\lambda=\frac{h}{\sqrt{2M_eeV}}$$

$$\lambda=\frac{6.6\times10^{-34}}{\sqrt{2\times9\times10^{-31}\times1.6\times10^{-19}\times40\times10^{3}}}$$

$$\lambda=\frac{6.6\times10^{-34}}{\sqrt{1152\times10^{-47}}}$$

$$\lambda=0.615\times10^{-34+23}$$

$$\lambda=6.15\times10^{-12}\ m$$

Question: For a medium, the magnetic susceptibility is 499. The permeability of free space is $4\pi\times10^{-7}$ SI units. Then the permeability of the medium is?

Options:

(a) $2\pi\times10^{-4}$ SI units

(b) $2\pi\times10^{-7}$ SI units

(c) $\frac{5\pi}{4}\times10^{-7}$ SI units

(d) $\frac{4\pi}{5}\times10^{-4}$ SI units

Answer: (a)

Solution:

$$\chi_M=\mu_r-1$$

$$\mu_r=499+1=500$$

We know that

$$\mu=\mu_r\mu_0$$

$$\mu=500\times4\pi\times10^{-7}$$

$$\mu=2\pi\times10^{-4}\text{ SI units}$$

Question: A particle has 4 times its initial kinetic energy. Find the percentage change in momentum?

Options:

(a) 100%

(b) 200%

(c) 300%

(d) 400%

Answer: (a)

Solution:

Initial K.E. = k
Final K.E. = 4k

$$k = \frac{P^2}{2m}$$

$$\Rightarrow P = \sqrt{2mk}$$

$$\Delta P = \sqrt{2m(4k)} - \sqrt{2mk}$$

$$= 2\sqrt{2mk} - \sqrt{2mk}$$

$$= \sqrt{2mk}$$

$$\%\,\text{change} = \frac{\Delta P}{P} \times 100 = \frac{\sqrt{2mk}}{\sqrt{2mk}} \times 100 = 100\%$$

Question: Electrons with de-Broglie wavelength λ , fall on a target in an x-ray tube. The cut-off wavelength of emitted x-ray is

Options:

(a) $\frac{2mc\lambda^2}{h}$

(b) $\frac{2h}{mc}$

(c) $\frac{h}{mc}$

(d) None

Answer: (a)

Solution: de-Broglie wavelength $= \lambda$

$$\Rightarrow mv = \frac{h}{\lambda}$$

$$\Rightarrow \frac{1}{2}mv^2 = \left(\frac{h}{\lambda}\right)^2 \times \frac{1}{2m}$$

Energy of corresponding to cut-off wavelength is equal to

$$\frac{hc}{\lambda_0} = \frac{1}{2}mv^2 = \frac{h^2}{\lambda^2} \times \frac{1}{2m}$$

$$\Rightarrow \lambda_0 = \frac{\lambda^2(2m)c}{w}$$

Hence, cut-off wavelength $= \frac{2mc\lambda^2}{h}$

Question: An element has $\frac{1}{16}^{\text{th}}$ of initial activity in 20 sec. Half life of the nuclei is

Options:
(a) 5 sec
(b) $\frac{4}{3}$sec
(c) 2.5 sec
(d) 7.5 sec
Answer: (a)
Solution:
$N = N_0 e^{-\lambda t}$
$\Rightarrow \frac{N_0}{16} = N_0 e^{-\lambda t}$
$\Rightarrow 2^{-4} = e^{-\lambda t}$
$\Rightarrow -4 \ln 2 = -\lambda t$
$\Rightarrow -4 \frac{\ln 2}{\lambda} = -t$
$\Rightarrow t = 4\left(\frac{\ln 2}{\lambda}\right) = 4 \times t_{1/2}$
$\Rightarrow t_{1/2} = 5\,\text{sec}$.

Question: A solid cylinder and ring are released from rest in top of inclined plane. Find ratio of their velocities when they reach bottom, assuming pure rolling
Options:
(a) $\sqrt{\frac{3}{5}}$
(b) $\sqrt{\frac{5}{3}}$
(c) $\sqrt{\frac{7}{5}}$
(d) $\sqrt{\frac{4}{3}}$
Answer: (d)
Solution: Loss in PE in gain in K.E
$\Rightarrow mgh = \frac{1}{2} mv^2 + \frac{1}{2} I \frac{v^2}{R^2}$
$\Rightarrow v = \sqrt{\frac{2mgh}{m + \frac{I}{R^2}}}$

$$\Rightarrow v_{cylinder} = \sqrt{\frac{2mgh}{m+\frac{mR^2}{2R^2}}} = \sqrt{\frac{4gh}{3}} \text{ and } v_{ring} = \sqrt{\frac{2mgh}{m+\frac{mR^2}{R^2}}} = \sqrt{gh}$$

$$\Rightarrow \frac{v_{cylinder}}{v_{ring}} = \sqrt{\frac{4}{3}}$$

Question: The angle of Dip in a plane at an angle of 30º with Geographical meridian is 45º. The value of true Dip is

Options:

(a) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(b) $\tan^{-1}\left(\frac{1}{2}\right)$

(c) 30º

(d) 60º

Answer: (a)

Solution:

Angle of dip

$$\tan\theta = \frac{V}{H} \quad ...(1)$$

For 30 to meridian

$$\tan 45° = \frac{V}{H\cos 30°} \Rightarrow \frac{V}{H} = \cos 30° \quad ...(2)$$

$$\tan\theta = \cos 30° \qquad \text{(By comparing)}$$

$$\tan\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

JEE-Main-20-07-2021-Shift-2 (Memory Based)

CHEMISTRY

Question: Which gas retard the rate of photosynthesis?
Options:
(a) CO
(b) NO_2
(c) CO_2
(d) CFC
Answer: (b)
Solution: Higher concentration of NO_2 damage the leaves of plants and retard the rate of photosynthesis

Question: Sodium will not react normally with
Options:
(a) Ammonia gas
(b) Ethyne
(c) But-2-yne
(d) All of these
Answer: (c)
Solution: Sodium will not react with But-2-yne

Question: The hybridisation of NO_2^-, NO_2^+ and NH_4^+ are respectively
Options:
(a) sp^2, sp, sp^3
(b) sp, sp, sp^3
(c) sp^2, sp, sp
(d) sp^2, sp^2, sp^3
Answer: (a)
Solution:
Hybridisation of N in $NO_2^- = sp^2$
Hybridisation of N in $NO_2^+ = sp$
Hybridisation of N in $NH_4^+ = sp^3$

Question: What is added with HNO_3 in carius method?
Options:
(a) Silver nitrate
(b) Copper nitrate
(c) Copper sulphate
(d) None of these
Answer: (a)

Solution: In the Carius method, a known mass of the compound is heated with concentration nitric acid in the presence of silver nitrate in a hard glass tube. The hard glass is known as Carius tube.

Question: $4s^2\ 4p^1$: Diagonally next period in p-block. Identify element
Options:
(a) Al
(b) Sb
(c) Cd
(d) Sn
Answer: (d)
Solution: Ga has electronic configuration $4s^2\ 4p^1$
Sn is present diagonally to Ga in the next period in p-block.

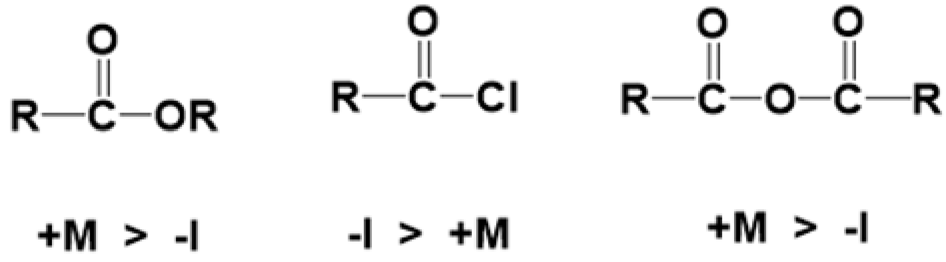
Question: Cu^{2+} salts reacts with KI and forms:
Options:
(a) Cu_2I_2
(b) CuI
(c) CuI_2
(d) None of these
Answer: (a)
Solution:
$$2Cu^{2+} + 4I^- \rightarrow \underset{(\text{white precipitate})}{Cu_2I_2} + I_2 \uparrow$$

Question: In nitration, HNO_3 and H_2SO_4 act as:
Options:
(a) Both acid
(b) Both base
(c) HNO_3 : Acid & H_2SO_4 : Base
(d) HNO_3 : Base & H_2SO_4 : Acid
Answer: (d)
Solution: Nitration is an electrophilic substitution reaction, in its first step HNO_3 takes a proton from sulphuric acid and then forms $-NO_2^+$.
So, in the nitrating mixture HNO_3 acts as a base, and H_2SO_4 acts as an acid.

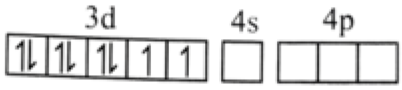
Question: The common monomer present in novolac and bakelite is:
Options:
(a) Acetaldehyde
(b) Methanal
(c) Phenol
(d) Ethylene glycol
Answer: (b)
Solution: Formaldehyde (Methanal) is the common monomer that is present in both novolac and bakelite.

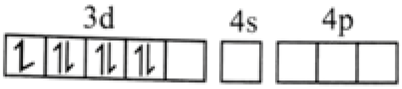
Question: Rate of hydrolysis : Ester, Acid chloride, Acid anhydride
Options:
(a) Acid chloride > Acid anhydride > Ester

(b) Ester > Acid chloride > Acid anhydride
(c) Acid anhydride > Ester > Acid chloride
(d) Acid chloride < Acid anhydride < Ester
Answer: (a)
Solution:

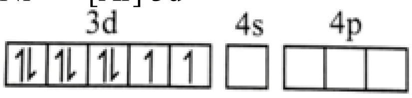


Question: What is the difference in the number of unpaired electrons in $[NiCl_4]^{2-}$ and $[Ni(CN)_4]^{2-}$
Options:
(a) 4
(b) 1
(c) 0
(d) 2
Answer: (d)
Solution:
$[Ni(CN)_4]^{2-}$ Ni(Z = 28) $\Rightarrow$ $3d^8\ 4s^2$ $Ni^{2+} = 3d^8$





Unpaired electrons = 0
$[NiCN_4]^{2-}$
$Ni^{2+} = [Ar]\ 3d^8$



Unpaired electrons = 2
Difference between unpaired electrons = 2

Question: Radioactive substance becomes 1/16th of original in 80 minutes, Find the half-life
Options:
(a) 20 min
(b) 40 min
(c) 60 min
(d) 80 min
Answer: (a)
Solution:

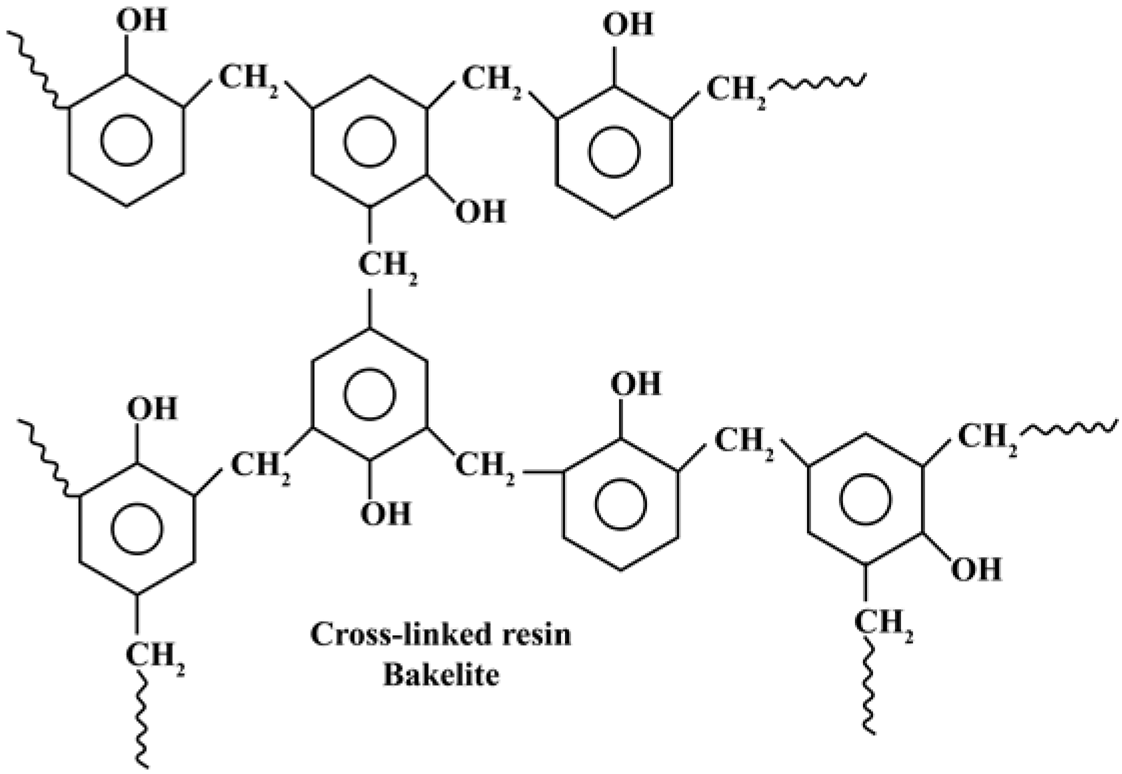
$$k = \frac{1}{t} \times 2.303 \log \frac{1}{1/16}$$

$$k = \frac{1}{80} \times 2.303 \log 16$$

$$k = \frac{1}{80} \times 2.303 \times 4 \log 2 = 0.035\,\text{min}^{-1}$$

$$t_{1/2} = \frac{0.693}{0.035\,\text{min}^{-1}} = 20\,\text{min}$$

Question: Bakelite is formed by cross linking of which of the following?
Options:
(a) Novolac
(b) Buna-s
(c) Dacron
(d) PHBV
Answer: (a)
Solution:



(Formed by Novolac)

Question: In FCC, 50 % tetrahedral void is filled. Find the effective number of atoms in the cell if made using the same atoms?
Options:
(a) 2
(b) 4
(c) 6
(d) 8
Answer: (d)
Solution:
Number of atoms in FCC unit cell = 4
Total number of tetrahedral voids = 2 x 4 = 8

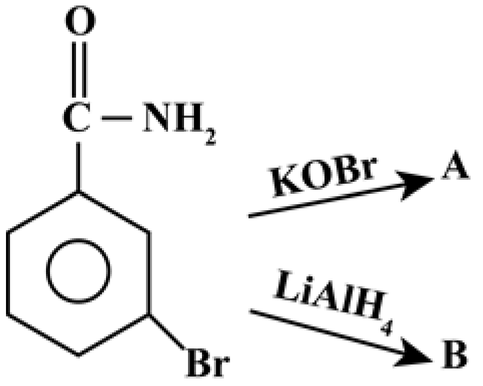
Given that 50% tetrahedral voids are occupied by atoms, i.e.,
Number of atoms present in tetrahedral voids = 50% of 8 = 4
Therefore, the effective number of atoms in the cell = 4 + 4 = 8

Question: Which of the following do not have magnetic moment of 1.73 B.M?
Options:
(a) O_2^+
(b) O_2^-
(c) $[Cu(NH_3)_4]Cl_2$
(d) CuI
Answer: (d)
Solution:
CuI : Cu^+ : $[Ar]3d^{10}$
No unpaired electrons

Question: If equimolar mixture of NaOH and Na_2CO_3 weight 4g then weight of NaOH is:
Options:
(a) 1.595
(b) 1.095
(c) 2.904
(d) 2.945
Answer: (b)
Solution:
Given equimolar mixture of NaOH and Na_2CO_3
Mass of mixture = 4g
Let mass of NaOH = w
$$\Rightarrow \frac{w}{40} = \frac{4-w}{106}$$
$$\Rightarrow w = \frac{160}{146} = 1.0958\,g$$

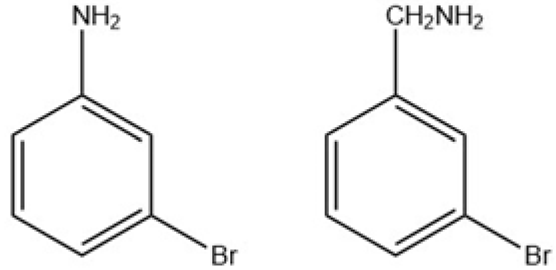
Question: $PCl_5 \longrightarrow PCl_3 + Cl_2$
The above first order reaction has initial moles as 10 and after 20 min final moles are 2. Find the rate constant. (Given: log 5 = 0.693)
Options:
(a) 0.08 min^{-1}
(b) 0.16 min^{-1}
(c) 0.24 min^{-1}
(d) 0.02 min^{-1}
Answer: (a)
Solution:
$$K = \frac{1}{t} \times 2.303 \log \frac{[A_o]}{[A_t]}$$
$$K = \frac{1}{20} \times 2.303 \log \frac{10}{2}$$
$$K = \frac{1}{20} \times 2.303 \log 5 = 0.08\,min^{-1}$$

Question: What are A and B respectively in the following reactions?

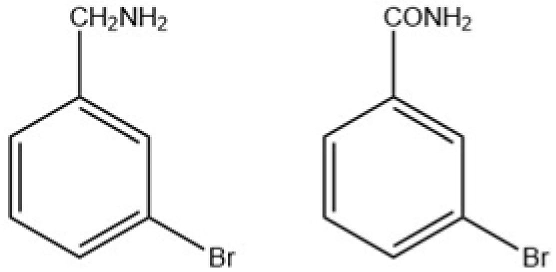


Options:

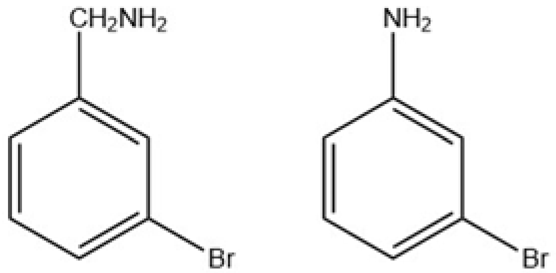
(a)



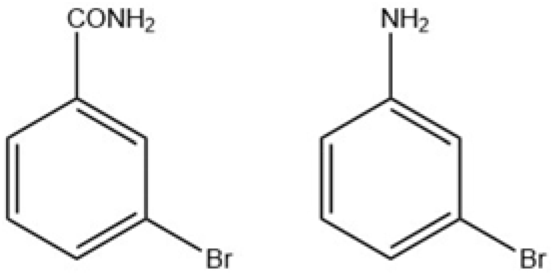
(b)



(c)

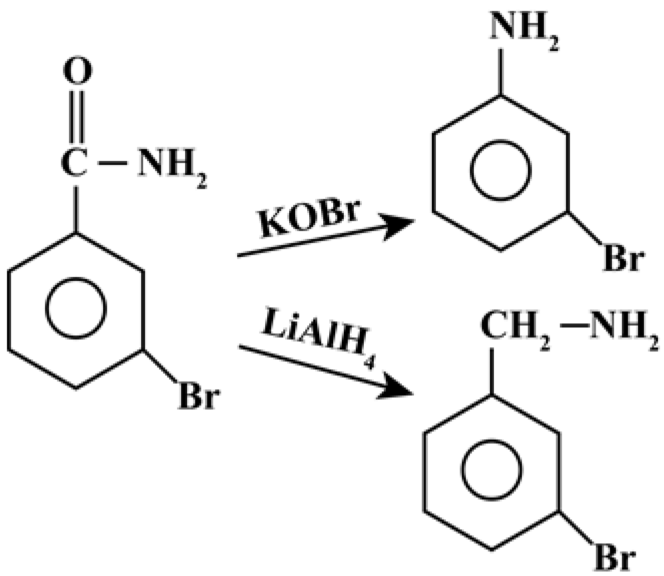


(d)



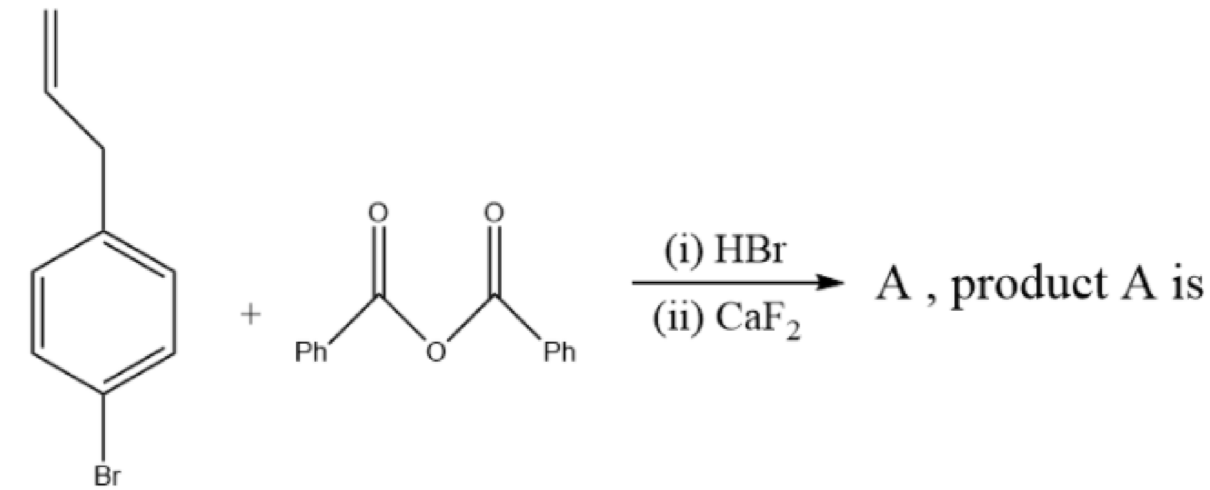
Answer: (a)

Solution:

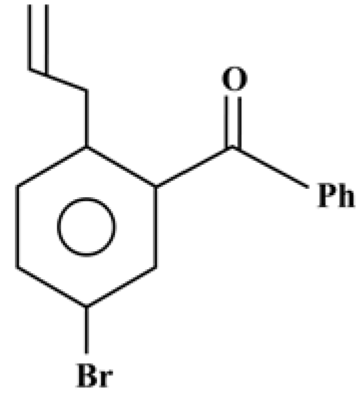


Question: Spin only magnetic moment of Fe^{2+} with weak field ligand is
Options:
(a) 4.90
(b) 1.73
(c) 2.80
(d) 0
Answer: (a)
Solution: Fe^{2+} (with weak field ligand) has 4 unpaired electrons ($t_{2g}^4\ e_g^2$), therefore spin only magnetic moment is 4.90 BM

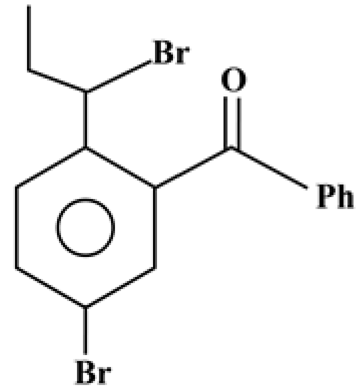
Question:



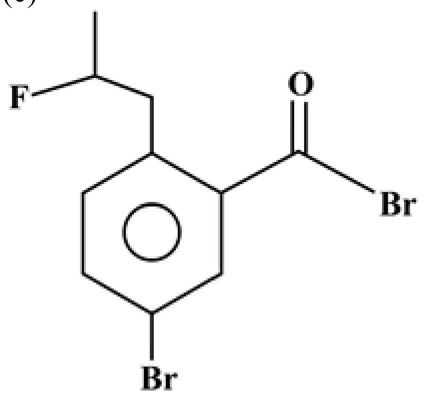
Options:
(a)



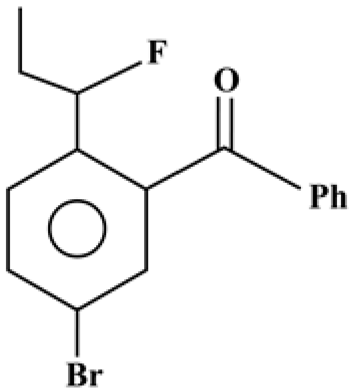
(b)



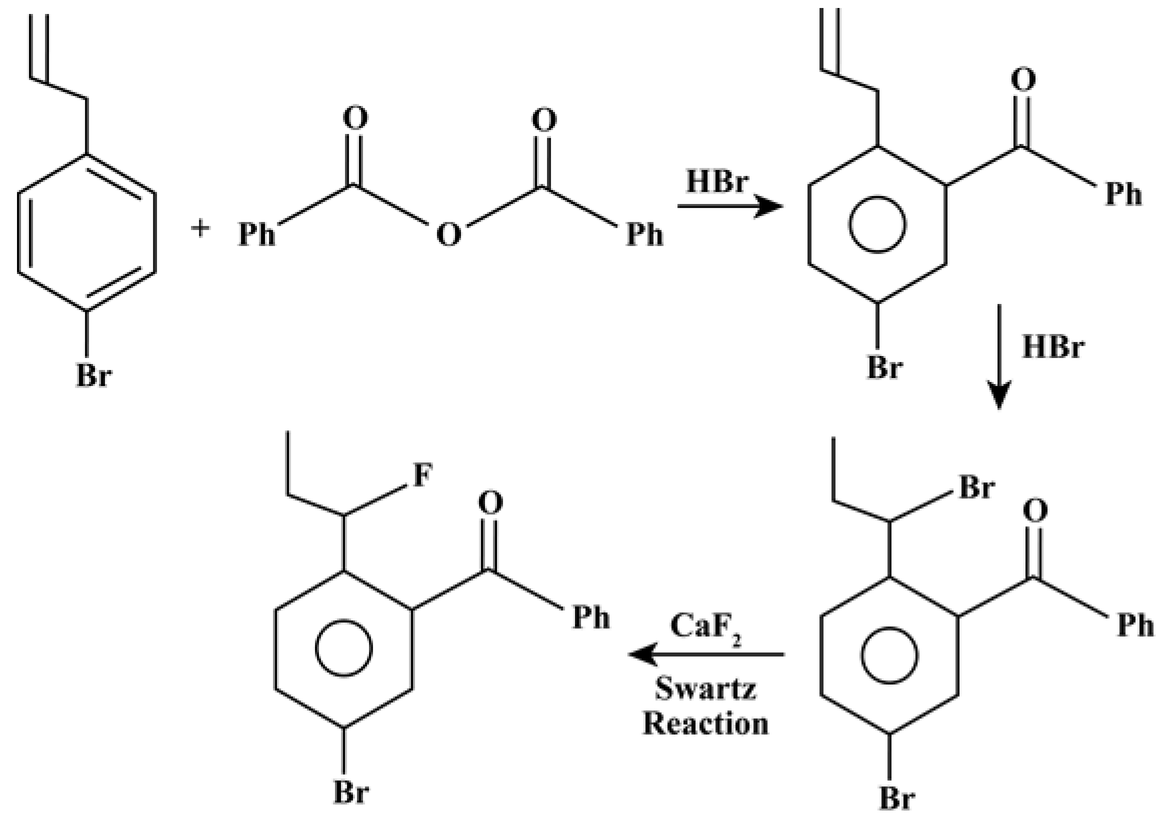
(c)



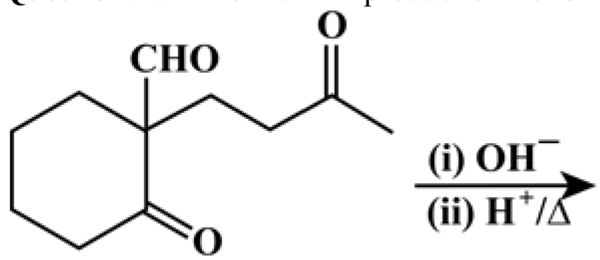
(d)



Answer: (d)
Solution:

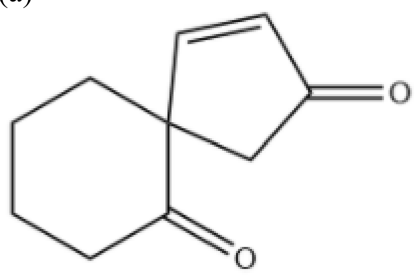


Question: What is the final product of the following reaction?

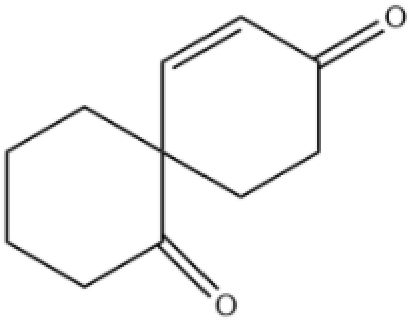


Options:

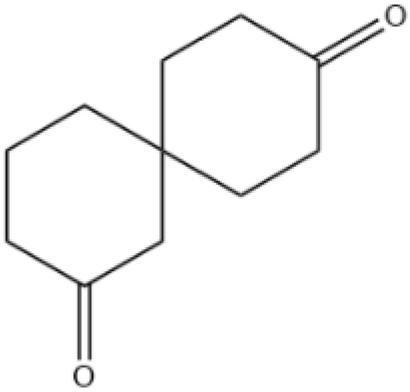
(a)



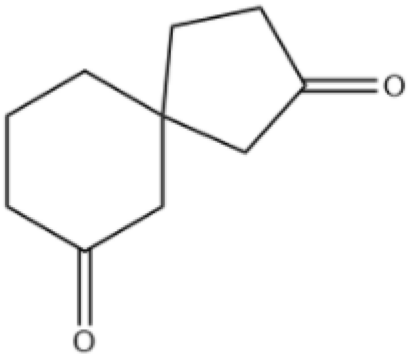
(b)



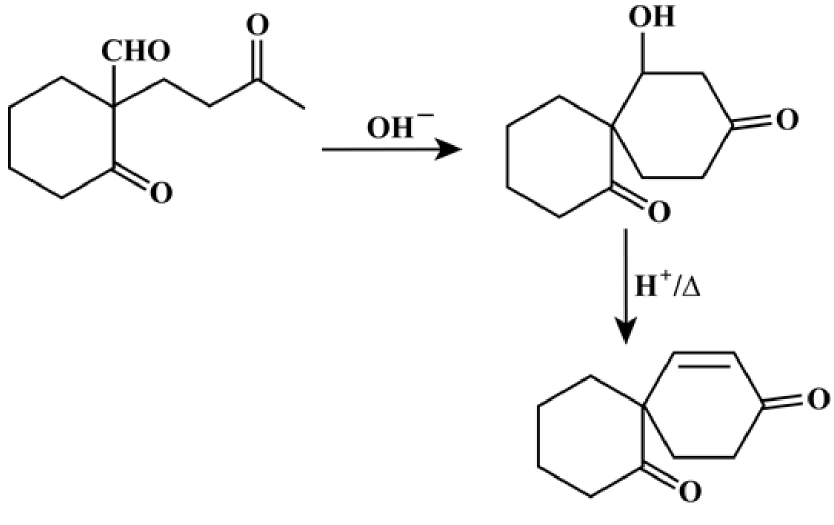
(c)



(d)



Answer: (b)
Solution:



JEE-Main-20-07-2021-Shift-2 (Memory Based)

MATHEMATICS

Question: Probability of only one of A and B is $1-k$
Probability of only one of A and C is $1-2k$
Probability of only one of B and C is $1-k$
$P(A\cap B\cap C)=K^2,\ K\in(0,1)$.

Find $P(A\cup B\cup C)$ is

Options:

(a) $>\frac{1}{2}$

(b) $\left[\frac{1}{8},\frac{1}{4}\right]$

(c) $<\frac{1}{4}$

(d)

Answer: (a)

Solution:

$$P(A)+P(B)-2P(A\cap B)=1-k \quad ....(1)$$

$$P(A)+P(C)-2P(A\cap C)=1-2k \quad ....(2)$$

$$P(B)+P(C)-2P(B\cap C)=1-k \quad ....(3)$$

$$P(A\cap B\cap C)=k^2$$

$$P(A\cap B\cap C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)+P(C\cap A)+P(A\cap B\cap C)$$

Add (1), (2) & 3

$$2\left(P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)\right)$$

$$=3-4k$$

$$P(A\cup B\cup C)=\frac{3-4k}{2}+k^2$$

$$=k^2-2k+\frac{3}{2}$$

$$=(k-1)^2+\frac{1}{2}>\frac{1}{2}$$

Question: If $a, b, 7, 10, 11, 15,$ Mean = 10 and Variance $= \frac{20}{3}$. Find a and b.

Options:
(a)
(b)
(c)
(d)
Answer: 8, 9
Solution:

$$\frac{a+b+43}{6} = 10 \Rightarrow a+b=17$$

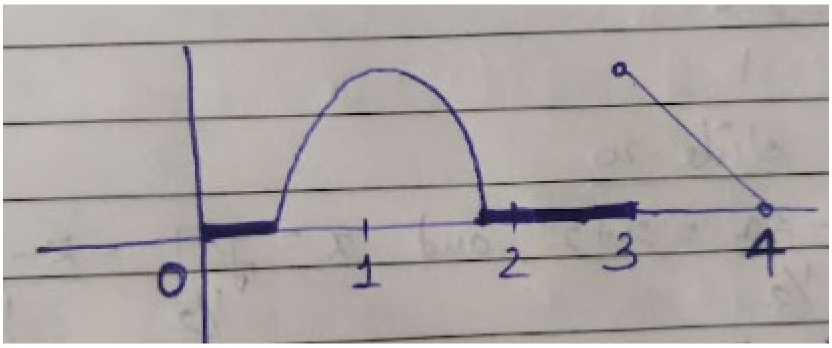
$$\frac{a^2+b^2+495}{6} - 100 = \frac{20}{3} \Rightarrow a^2+b^2=145$$

$$\Rightarrow (a,b) = (9,8) \text{ or } (8,9)$$

Question: $g(t) = \begin{cases} \max\left(t^3+6t^2+9t-3, 0\right); & t \in [0, 3] \\ 4-t; & t \in (3, 4) \end{cases}$ Find points of non-differentiability.

Options:
(a)
(b)
(c)
(d)
Answer: ()
Solution:

$$g(t) = \begin{cases} \max\left(t^3+6t^2+9t-3, 0\right); & t \in [0, 3] \\ 4-t; & t \in (3, 4) \end{cases}$$



$\therefore$ Point of non-differentiability is 3

Question: If ΔABC is right angled triangle with sides a, b and c and smallest angle θ. If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are also the sides of right angled triangle, then find $\sin\theta$

Options:
(a)
(b)

(c)
(d)
Answer: ()
Solution:
Let a be smallest side & c be largest side

$\therefore c^2 = a^2 + b^2$ and $\dfrac{a}{\sin\theta} = c \quad .....(1)$

Also, $\dfrac{1}{a^2} = \dfrac{1}{b^2} + \dfrac{1}{c^2} \Rightarrow b^2 = \dfrac{a^2c^2}{c^2 - a^2} = c^2 - a^2$

$\Rightarrow a^2c^2 = \left(c^2 - a^2\right)^2 = a^4 + c^4 - 2a^2c^2$

$\Rightarrow a^4 + c^4 - 3a^2c^2 = 0$

$\Rightarrow \left(\dfrac{a}{c}\right)^4 - 3\left(\dfrac{a}{c}\right)^2 + 1 = 0$

$\Rightarrow \left(\left(\dfrac{a}{c}\right)^2 - 1\right)^2 = \left(\dfrac{a}{c}\right)^2$

$\Rightarrow \left(\dfrac{a}{c}\right)^2 - \left(\dfrac{a}{c}\right) - 1 = 0$

$\Rightarrow \dfrac{a}{c} = \dfrac{1+\sqrt{5}}{2} = \sin\theta$

Question: If $\log x_{9^{\left(\frac{1}{2}\right)}} + \log x_{9^{\left(\frac{1}{3}\right)}} + \ldots$ upto 21 terms $= 504$. Find x.

Options:
(a)
(b)
(c)
(d)
Answer: ()
Solution:

$\log_9 x\left(2+3+....22\right) = 504$

$\log_9 x\left(\dfrac{21}{2}\left(4+20\right)\right) = 504$

$\log_9 x\left(21 \times 12\right) = 504$

$\log_9 x = \dfrac{504}{21 \times 12} = 2$

$x = 9^2 = 81$

Question: $\lim\limits_{x\to 0}\dfrac{\alpha e^{x}+\beta\ln(1+x)+\gamma e^{-x}}{x\sin^{2}x}=10,\ \alpha+\beta+\gamma=?$

Options:
(a)
(b)
(c)
(d)
Answer: ()
Solution:

$$\lim_{x\to 0}\frac{\alpha e^{x}+\beta\ln(1+x)+\gamma e^{-x}}{x^{2}}=10\Rightarrow\alpha+\gamma=0\quad ....(1)$$

$$\lim_{x\to 0}\frac{\alpha e^{x}+\dfrac{\beta}{1+x}-\gamma e^{-x}}{2x}=10\Rightarrow\alpha-\beta-\gamma=0\quad ....(2)$$

$$\lim_{x\to 0}\frac{\alpha e^{x}-\dfrac{\beta}{(1+x)^{2}}+\gamma e^{-x}}{2}=10\Rightarrow\alpha-\beta+\gamma=20\quad ....(3)$$

$$\therefore\ \beta=-20,\ \alpha=10,\ \gamma=-10$$

$$\therefore\ \alpha+\beta+\gamma=-20$$

Question: $\mathrm{Re}\left[(1+\cos\theta+2\sin\theta)^{-1}\right]=\dfrac{1}{5}$. Find θ.

Options:
(a)
(b)
(c)
(d)
Answer: ()
Solution:

$$\mathrm{Re}(1+\cos\theta+2\sin\theta)^{-1}=\frac{1}{5}$$

$$\left(\frac{1}{1+\cos\theta+2\sin\theta}\right)\frac{(1+\cos\theta-2i\sin\theta)}{(1+\cos\theta-2i\sin\theta)}$$

Real part = $\dfrac{1+\cos\theta}{(1+\cos\theta)^{2}+4\sin^{2}\theta}=\dfrac{1}{5}$

$$\Rightarrow 5(1+\cos\theta)=\left[(1+\cos^{2}\theta+2\cos\theta)+4(1-\cos^{2}\theta)\right]$$

$$\Rightarrow 5+5\cos\theta=5-3\cos^{2}\theta+2\cos\theta$$

$\Rightarrow 3\cos^2\theta + 3\cos\theta = 0$

$\Rightarrow \cos\theta(\cos\theta + 1) = 0$

$\cos\theta = 0$ or -1

$\theta = (2n+1)\dfrac{\pi}{2}$ or $(2n+1)\pi$

Question: In a ΔABC, we have $AB = 3, AC = 7, BC = 5$ then find projection of $\overline{AC}$ on $\overline{BC}$

Options:
(a)
(b)
(c)
(d)
Answer: ()
Solution:

$\cos c = \dfrac{7^2 + 5^2 - 3^2}{2 \cdot 7 \cdot 5} = \dfrac{49 + 25 - 9}{70}$

$= \dfrac{65}{70} = \dfrac{13}{14}$

Projection = $AC\cos\theta = \dfrac{7 \times 13}{14} = \dfrac{13}{2}$

Question: If $\tan\left(2\tan^{-1}\left(\dfrac{3}{5}\right) + \sin^{-1}\left(\dfrac{5}{13}\right)\right) = ?$

Options:
(a)
(b)
(c)
(d)
Answer: ()
Solution:

$\tan\left[2\tan^{-1}\left(\dfrac{3}{5}\right) + \sin^{-1}\left(\dfrac{5}{13}\right)\right]$

$= \tan\left[\tan^{-1}\left(\dfrac{\frac{6}{5}}{1 - \frac{9}{25}}\right) + \tan^{-1}\left(\dfrac{5}{12}\right)\right]$

$$= \tan\left[\tan^{-1}\left(\frac{15}{8}\right) + \tan^{-1}\left(\frac{5}{12}\right)\right] = \frac{\frac{15}{8} + \frac{5}{12}}{1 - \left(\frac{15}{8}\right)\left(\frac{5}{12}\right)}$$

$$= \frac{180 + 40}{96 - 75} = \frac{220}{21}$$

Question: If $y = \frac{5x+3}{6x+a}$ and $f\left(f\left(x\right)\right) = x$ then find a.

Options:
(a)
(b)
(c)
(d)
Answer: ()
Solution:

$$f\left(f\left(x\right)\right) = \frac{5\left(\frac{5x+3}{6x+a}\right) + 3}{6\left(\frac{5x+3}{6x+a}\right) + a} = \frac{25x + 15 + 18x + 3a}{30x + 18 + 6ax + a^2}$$

$$= \frac{33x + 15 + 3a}{x\left(30 + 6a\right) + 18 + a^2}$$

For $a = -5$ above expression is x

Question: The lines $x = ay - 3 = z + 2$ and $x = 2y - 2 = bz - 2$ are coplanr, find a and b?

Options:
(a)
(b)
(c)
(d)
Answer: ()
Solution:

$$x = \frac{y - \frac{3}{a}}{\frac{1}{a}} = z + 2 \text{ and } x = \frac{y - 1}{\frac{1}{2}} = \frac{z - \frac{2}{b}}{\frac{1}{b}}$$

$$\therefore \begin{vmatrix} 0 & 1-\frac{3}{a} & \frac{2}{b}+2 \\ 1 & \frac{1}{a} & 1 \\ 1 & \frac{1}{2} & \frac{1}{b} \end{vmatrix} = 0$$

$$\Rightarrow \left[\left(1-\frac{3}{a}\right)-\frac{1}{a}\left(\frac{2}{b}+2\right)\right]=\left[\frac{1}{b}\left(1-\frac{3}{a}\right)-\frac{1}{2}\left(\frac{2}{b}+2\right)\right]$$

$$\Rightarrow 1-\frac{3}{a}-\frac{2}{ab}-\frac{2}{a}=\frac{1}{b}-\frac{3}{ab}-\frac{1}{b}-1$$

$$\Rightarrow 2-\frac{5}{a}+\frac{1}{ab}=0$$

$$\Rightarrow 2ab-5b+1=0$$

Question: (α, β) is the point on $y^2=6x,$ that is closest to $\left(3, \frac{3}{2}\right)$ find $2(\alpha, \beta)$

Answer: 9

Solution:

$$\beta^2 = 6\alpha \quad \text{....}(1)$$

$$\left(\frac{\frac{3}{2}-\beta}{3-\alpha}\right)\times\left(\frac{3}{\beta}\right)=-1\Rightarrow 2\alpha\beta=9 \quad \text{....}(2)$$

From (1) and (2),

$$\beta^2=\frac{27}{\beta}$$

$$\Rightarrow \beta=3, \alpha=\frac{3}{2}$$

$$\therefore 2(\alpha+\beta)=9$$

Question: $f(x)=x+1.$ Find $\lim\limits_{x\to\infty}\frac{1}{n}\left(1+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+\text{.....}+f\left(\frac{5(n-1)}{n}\right)\right)$

Answer: $\frac{7}{2}$

Solution:

$$f(x)=x+1$$

$$\therefore \lim_{n\to\infty}\frac{1}{n}\left[1+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+....f\left(\frac{5(n-1)}{n}\right)\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[1+1+\frac{5}{n}+1+\frac{10}{n}+...1+\frac{5(n-1)}{n}\right]$$

$$=\lim_{n\to\infty}1+\frac{5}{n^2}\{1+2+....(n-1)\}$$

$$=\lim_{n\to\infty}1+\frac{5}{2}=\frac{7}{2}$$

Question: $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\left[[x]+\sin x\right]dx=?$

Answer: $-\pi$

Solution:

$$\left[[x]+\sin x\right]=[x]+[\sin x]$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[[x]+\sin x\right]dx=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[x]dx+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[\sin x]dx$$

$$\int_{-\frac{\pi}{2}}^{-1}[x]dx+\int_{-1}^{0}[x]dx+\int_{0}^{1}[x]dx+\int_{1}^{\frac{\pi}{2}}[x]dx+\int_{-\frac{\pi}{2}}^{0}[\sin x]dx+\int_{0}^{\frac{\pi}{2}}[\sin x]dx$$

$$=\int_{-\frac{\pi}{2}}^{-1}-2dx+\int_{-1}^{0}-dx+\int_{0}^{1}0\,dx+\int_{1}^{\frac{\pi}{2}}dx+\int_{-\frac{\pi}{2}}^{0}-dx+\int_{0}^{\frac{\pi}{2}}0\,dx$$

$$=-2x\Big|_{-\frac{\pi}{2}}^{-1}-x\Big|_{-1}^{0}+x\Big|_{1}^{\frac{\pi}{2}}-x\Big|_{-\frac{\pi}{2}}^{0}$$

$$=-2\left(-1+\frac{\pi}{2}\right)-(0+1)+\left(\frac{\pi}{2}-1\right)-\left(0+\frac{\pi}{2}\right)$$

$$=2-\pi-1+\frac{\pi}{2}-1-\frac{\pi}{2}$$

$$=-\pi$$