JEE–MAIN EXAMINATION – JANUARY, 2023	
(Held On Thursday 25th January, 2023)	TIME : 3 : 00 PM to 6 : 00 PM

Physics

SECTION - A

1. According to law of equipartition of energy the molar specific heat of a diatomic gas at constant volume where the molecule has one additional vibrational mode is:-

(1) $\frac{5}{2}R$ (2) $\frac{9}{2}R$ (3) $\frac{7}{2}R$ (4) $\frac{3}{2}R$

Sol. **3**

(degree of freedom)

$\Rightarrow f = 3 + 2 + 2 = 7$

$C_V = \frac{fR}{2} = \frac{7R}{2}$

2. A wire of length 1 m moving with velocity 8 m/s at right angles to a magnetic field of 2 T. The magnitude of induced emf, between the ends of wire will be

(1) 20 V (2) 8 V (3) 12 V (4) 16 V

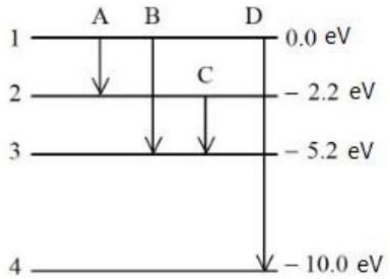
Sol. **4**

$e = B\vartheta l$

$e = 2 \times 8 \times 1$

$e = 16$ volt

3. The energy levels of an atom is shown in figure.



Which one of these transitions will result in the emission of a photon of wavelength 124.1 nm ?

Given ($h = 6.62 \times 10^{-34}$Js)

(1) D (2) B (3) C (4) A

Sol. **1**

$\underset{\text{(nm)}}{\lambda} = \frac{hc}{\Delta E} = \frac{1241}{\Delta E(\text{ev})} = \frac{1241}{10} = 124.1$

4. Given below are two statements :

Statement I: Stopping potential in photoelectric effect does not depend on the power of the light source.

Statement II: For a given metal, the maximum kinetic energy of the photoelectron depends on the wavelength of the incident light.

In the light of above statements, choose the most appropriate answer from the options given below

(1) Statement I is incorrect but statement II is correct

(2) Statement **I** is correct but statement **II** is incorrect

(3) Both Statement **I** and statement II are correct

(4) Both Statement I and Statement II are incorrect

Sol. **3**

Both statement I and statement II are correct

5. The distance travelled by a particle is related to time t as $x = 4t^2$. The velocity of the particle at t = 5 s is:-

(1) 40 ms^{-1} (2) 20 ms^{-1} (3) 8 ms^{-1} (4) 25 ms^{-1}

Sol. **1**

$v = \frac{dx}{dt} = 8t$

$v = 8 \times 5$

$v = 40$ m/s

6. Match List I with List II

LIST I		LIST II	
A.	Young's Modulus (Y)	I.	$[M L^{-1} T^{-1}]$
B.	Co-efficient of Viscosity (η)	II.	$[M L^{2} T^{-1}]$
C.	Planck's Constant (h)	III.	$[M L^{-1} T^{-2}]$
D.	Work Function (φ)	IV.	$[M L^{2} T^{-2}]$

Choose the correct answer from the options given below: options

(1) A-I, B-II, C-III, D-IV (2) A-II, B-III, C-IV, D-I

(3) A-I, B-III, C-IV, D-II (4) A-III, B-I, C-II, D-IV

Sol. **4**

$[Y] = \frac{F}{A}.\frac{\Delta L}{L} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$

$F = 6\pi \eta r v$

$[\eta] = \frac{F}{6\pi r v} = \frac{MLT^{-2}}{L\ LT^{-1}}$

$[\eta] = M L^{-1} T^{-1}$

$[h] = \frac{E}{f} = \frac{ML^2T^{-2}}{T^{-1}} = ML^2T^{-1}$

Work function $(\varphi) = ML^2T^{-2}$

7. Match List I with List II

LIST I		LIST II	
A.	Troposphere	I.	Approximate 65 – 75 km over Earth's surface
B.	E- Part of Stratosphere	II.	Approximate 300 km over Earth's surface
C.	F2- Part of Thermosphere	III.	Approximate 10 km over Earth's surface
D.	D- Part of Stratosphere	IV.	Approximate 100 km over Earth's surface

Choose the correct answer from the options given below:

(1) A-III, B-IV, C-II, D-I (2) A-III, B-II, C-I, D-IV

(3) A-I, B-IV, C-III, D-II (4) A-I, B-II, C-IV, D-III

Sol. **1**

By theory

8. The light rays from an object have been reflected towards an observer from a standard flat mirror, the image observed by the observer are:-

A. Real

B. Erect

C. Smaller in size then object

D. Laterally inverted

Choose the most appropriate answer from the options given below:

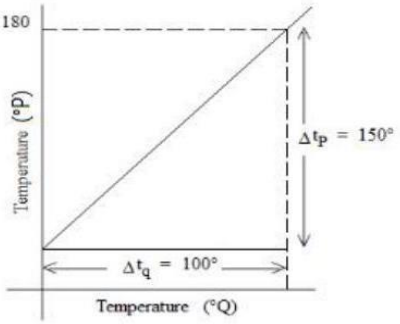
(1) A, C, and D Only (2) B and D Only

(3) A and D Only (4) B and C Only

Sol. **2**

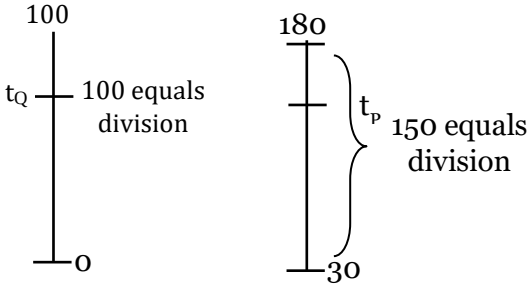
By theory

9. The graph between two temperature scales P and Q is shown in the figure. Between upper fixed point and lower fixed point there are 150 equal divisions of scale P and 100 divisions on scale Q. The relationship for conversion between the two scales is given by:-



(1) $\frac{t_P}{100} = \frac{t_Q - 180}{150}$ (2) $\frac{t_Q}{150} = \frac{t_P - 180}{100}$ (3) $\frac{t_p}{180} - \frac{t_Q - 40}{100}$ (4) $\frac{t_Q}{100} = \frac{t_P - 30}{150}$

Sol. **4**

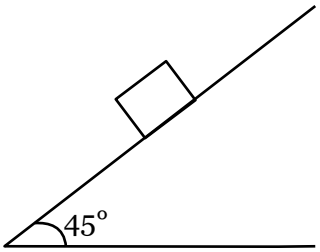


$$\frac{t_P - 30}{180 - 30} = \frac{t_Q - 0}{100 - 0}$$

$$\frac{t_P - 30}{150} = \frac{t_Q}{100}$$

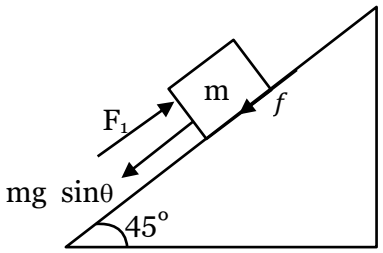
$$\frac{t_Q}{100} = \frac{t_P - 30}{150}$$

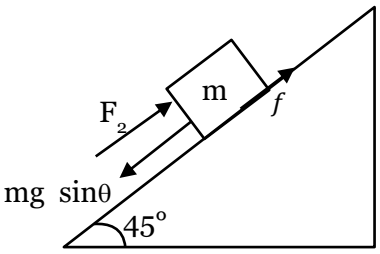
10. Consider a block kept on an inclined plane (inclined at 45°) as shown in the figure. If the force required to just push it up the incline is 2 times the force required to just prevent it from sliding down, the coefficient of friction between the block and inclined plane (μ) is equal to :



(1) 0.25 (2) 0.50 (3) 0.60 (4) 0.33

Sol. **4**





$F_1 = mg\sin\theta + \mu mg\cos\theta$
$F_1 = mg\sin45 + \mu mg\cos45$
$F_2 = mg\sin45 - \mu mg\cos45$

$F_1 = 2F_2$

$$mg\left(\frac{1}{\sqrt{2}} + \frac{\mu}{\sqrt{2}}\right) = 2mg\left(\frac{1}{\sqrt{2}} - \frac{\mu}{\sqrt{2}}\right)$$

$1 + \mu = 2 - 2\mu$
$3\mu = 1$
$\mu = \frac{1}{3} = 0.33$

11. Every planet revolves around the sun in an elliptical orbit:-
A. The force acting on a planet is inversely proportional to square of distance from sun.
B. Force acting on planet is inversely proportional to product of the masses of the planet and the sun.
C. The Centripetal force acting on the planet is directed away from the sun.
D. The square of time period of revolution of planet around sun is directly proportional to cube of semi-major axis of elliptical orbit.
Choose the correct answer from the options given below:
(1) B and C only (2) A and C Only (3) A and D only (4) C and D only

Sol. **3**

By Newton's law $F = \frac{Gm_1m_2}{r^2}$

By kepler's law $T^2 \alpha a^3$

12. For a moving coil galvanometer, the deflection in the coil is 0.05 rad when a current of 10 mA is passed through it. If the torsional constant of suspension wire is 4.0×10^{-5} N m rad $^{-1}$, the magnetic field is 0.01 T and the number of turns in the coil is 200 , the area of each turn (in cm^2) is :

(1) 1.0 (2) 2.0 (3) 1.5 (4) 0.5

Sol. **1**

$$\theta = \frac{NBA}{C} I$$

$$A = \frac{C\theta}{IBN}$$

$$= \frac{4\times10^{-5}\times.05}{10\times10^{-3}\times0.01\times200}$$

$A = 10^{-4}\ m^2$

$= 1\ cm^2$

13. Match List I with List II

LIST I		LIST II	
A.	Gauss's Law in Electrostatics	I.	$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$
B.	Faraday's Law	II.	$\oint \vec{B} \cdot d\vec{A} = 0$
C.	Gauss's Law in Magnetism	III.	$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_c + \mu_0 \in_0 \frac{d\phi_E}{dt}$
D.	Ampere-Maxwell Law	IV.	$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$

Choose the correct answer from the options given below:

(1) A-IV, B-I, C-II, D-III (2) A-II, B-III, C-IV, D-I

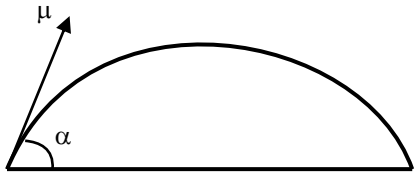
(3) A-III, B-IV, C-I, D-II (4) A-I, B-II, C-III, D-IV

Sol. **1**

14. Two objects are projected with same velocity 'u' however at different angles α and β with the horizontal. If $\alpha + \beta = 90°$, the ratio of horizontal range of the first object to the 2 nd object will be:

(1) 2: 1 (2) 1: 2 (3) 1: 1 (4) 4: 1

Sol. **3**



$$R_1 = \frac{u^2 \sin 2\alpha}{g}$$

$$R_2 = \frac{u^2 \sin 2\beta}{g} = \frac{u^2 \sin 2(90-\alpha)}{g}$$

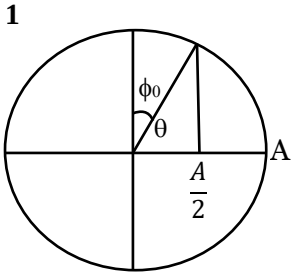
$$R_2 = \frac{u^2 \sin 2\alpha}{g} = R_1$$

$R_1 : R_2 = 1 : 1$

15. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. If time taken by particle to go from $x = 0$ to $\frac{A}{2}$ is 2 s; then time taken by particle in going from $x = \frac{A}{2}$ to A is

(1) 4 S (2) 1.5 S (3) 2 S (4) 3 S

Sol. **1**



$$\cos\theta = \frac{A}{2\times A} = \frac{1}{2} = \cos 60^\circ$$

$$\theta = 60 = \frac{\pi}{3}$$

$$\phi_0 = 30 = \frac{\pi}{6}$$

$$0 \rightarrow \frac{A}{2}\ , t = \frac{\frac{\pi}{6}}{\frac{2\pi}{T}} = \frac{T}{12} = 2$$

$$T = 24$$

$$\frac{A}{2} \rightarrow A\ ,\ t = \frac{\pi/3}{2\pi/T} = \frac{T}{6} = \frac{24}{6} = 4 \text{ sec}$$

16. Match List I with List II

LIST I		LIST II	
A.	Isothermal Process	I.	Work done by the gas decreases internal energy
B.	Adiabatic Process	II.	No change in internal energy
C.	Isochoric Process	III.	The heat absorbed goes partly to increase internal energy and partly to do work
D.	Isobaric Process	IV.	No work is done on or by the gas

Choose the correct answer from the options given below:

(1) A-I, B-II, C-III, D-IV (2) A-II, B-I, C-IV, D-III

(3) A-II, B-I, C-III, D-IV (4) A-I, B-II, C-IV, D-III

Sol. **2**

By theory

Isonormal $\rightarrow \Delta u = 0$ $A \rightarrow II$

Adiabatic $\rightarrow \Delta Q = 0$, $\Delta w(+)$ so $\Delta u\ (-) \downarrow$ $B \rightarrow I$

Isochoric = $\Delta V = 0$

$\Delta V = 0 \rightarrow \Delta w = 0$

$C \rightarrow IV$

Isobasic $\rightarrow P\Delta u \neq 0$

$\Delta v \neq 0$

$D \rightarrow III$

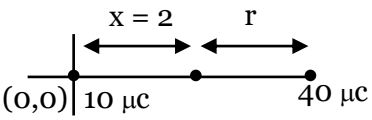
17. Statement I: When a Si sample is doped with Boron, it becomes P type and when doped by Arsenic it becomes N-type semi conductor such that P-type has excess holes and N-type has excess electrons.
Statement II: When such P-type and N-type semi-conductors, are fused to make a junction, a current will automatically flow which can be detected with an externally connected ammeter.
In the light of above statements, choose the most appropriate answer from the options given below
(1) Both Statement I and statement II are correct
(2) Statement **I** is incorrect but statement II is correct
(3) Both Statement I and Statement II are incorrect
(4) Statement I is correct but statement II is incorrect

Sol. **4**

By theory

18. A point charge of $10\mu C$ is placed at the origin. At what location on the X-axis should a point charge of $40\mu C$ be placed so that the net electric field is zero at $x = 2$ cm on the X-axis?
(1) $x = -4$ cm (2) $x = 6$ cm (3) $x = 4$ cm (4) $x = 8$ cm

Sol. **2**



$E_1 = E_2$

$$\frac{K \times 10}{(2)^2} = \frac{K \times 40}{4^2}$$

r = 4 cm

Distance from origin = 2 + 4 = 6 cm

19. The resistance of a wire is 5Ω. It's new resistance in ohm if stretched to 5 times of it's original length will be :
(1) 25 (2) 125 (3) 5 (4) 625

Sol. **2**

$R_{new} = n^2R$

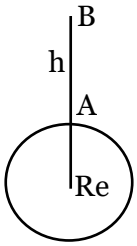
$= (5)^2 \times 5$

$= 125$

20. A body of mass is taken from earth surface to the height h equal to twice the radius of earth (R_e), the increase in potential energy will be:
(g = acceleration due to gravity on the surface of Earth)
(1) $3\ mgR_e$ (2) $\frac{1}{3}mgR_e$ (3) $\frac{2}{3}mgR_e$ (4) $\frac{1}{2}mgR_e$

Sol. **3**

$h = 2\ Re$



$\Delta U = U_B - U_A$

$$= \frac{-GM_em}{(R_e+h)} - \left(\frac{-GM_em}{R_e}\right)$$

$$= \frac{-GM_em}{R_e+2R_e} + \frac{GM_em}{R_e} = \frac{2}{3}\frac{GM_em}{R_e}$$

$$= \frac{2}{3}\frac{GM_em}{R_e^2}R_e$$

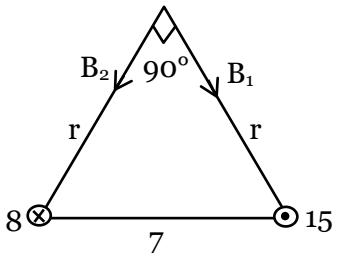
$$\Delta U = = \frac{2}{3}mgR_e$$

SECTION - B

21. Two long parallel wires carrying currents 8 A and 15 A in opposite directions are placed at a distance of 7 cm from each other. A point *P* is at equidistant from both the wires such that the lines joining the point *P* to the wires are perpendicular to each other. The magnitude of magnetic field at *P* is _____$\times 10^{-6}$ T

(Given : $\sqrt{2} = 1 \cdot 4$)

Sol. **60**



$$r = \frac{7}{\sqrt{2}}cm$$

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0 I_1}{2\pi r}\right)^2 + \left(\frac{\mu_0 I_2}{2\pi r}\right)^2}$$

$$= \frac{\mu_0}{2\pi r}\sqrt{8^2+15^2}$$

$$= \frac{4\pi\times10^{-7}\times17}{2\pi\times\frac{7}{\sqrt{2}}\times10^{-2}} = 68 \times 10^{-6}$$

$= 68$

22. A spherical drop of liquid splits into 1000 identical spherical drops. If u_i is the surface energy of the original drop and u_f is the total surface energy of the resulting drops, the (ignoring evaporation), $\frac{u_f}{u_i} = \left(\frac{10}{x}\right)$. Then value of x is _____.

Sol. **1**

$U_I = T\,4\pi R^2 = T\,4\pi\,(10r)^2 = 100 \times T \times 4\pi r^2$

$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$R = 10r$

$\frac{u_f}{u_i} = \frac{1000 \times T \times 4\pi r^2}{100 \times T \times 4\pi r^2} = 10$

$\therefore x = 1$

23. A nucleus disintegrates into two smaller parts, which have their velocities in the ratio 3: 2. The ratio of their nuclear sizes will be $\left(\frac{x}{3}\right)^{\frac{1}{3}}$. The value of ' x ' is:-

Sol. **2**

$0 = m_1\,3v - m_2\,2v$

$\frac{m_1}{m_2} = \frac{2}{3}$

$\frac{8v_1}{8v_2} = \frac{2}{3}$

$\frac{\frac{4}{3}\pi R_1^3}{\frac{4}{3}\pi R_2^3} = \frac{2}{3} = \frac{R_1}{R_2} = \left(\frac{2}{3}\right)^{\frac{1}{3}}$

$\therefore x = 2$

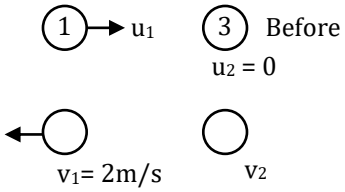
24. A train blowing a whistle of frequency 320 Hz approaches an observer standing on the platform at a speed of 66 m/s. The frequency observed by the observer will be (given speed of sound = 330 ms^{-1}) _____ Hz.

Sol. **400**

$f = \left(\frac{v \pm v_0}{v \pm v_s}\right) f_0 = \frac{330 \times 320}{330 - 66} = \frac{330 \times 320}{264} = 400$

25. A body of mass 1 kg collides head on elastically with a stationary body of mass 3 kg. After collision, the smaller body reverses its direction of motion and moves with a speed of 2 m/s. The initial speed of the smaller body before collision is _____ ms^{-1}

Sol. **4.00**



$p_i = p_f$

$u_1 + 0 = -1\times2 + 3v_2$

$u_1 = 3v_2 - 2$ ...(i)

$e = 1 = \dfrac{v_2 - (-2)}{u_1 - 0}$

$v_2 = u_1 - 2$ ...(2)

$u_1 = 3(u_1 - 2) - 2$

$2u_1 = 8,\ u_1 = 4$

26. A series LCR circuit is connected to an AC source of 220 V, 50 Hz. The circuit contains a resistance R = 80Ω, an inductor of inductive reactance $X_L = 70\Omega$, and a capacitor of capacitive reactance $X_C = 130\Omega$. The power factor of circuit is $\frac{x}{10}$. The value of x is:

Sol. **8.00**

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_C - X_L)^2}}$$

$$= \frac{80}{\sqrt{(80)^2 + (130-70)^2}} = \frac{80}{\sqrt{(80)^2 + (60)^2}}$$

$$\cos\phi = \frac{80}{100} = \frac{8}{10}$$

$x = 8$

27. If a solid sphere of mass 5 kg and a disc of mass 4 kg have the same radius. Then the ratio of moment of inertia of the disc about a tangent in its plane to the moment of inertia of the sphere about its tangent will be $\frac{x}{7}$. The the value of x is _____.

Sol. **5.00**

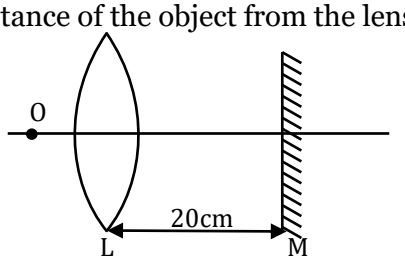
$$I_{ss} = \frac{2}{5}mR^2 + mR^2 = \frac{7}{5}mR^2 = \frac{7}{5}\times5\times R^2 = 7R^2$$

$$I_{Disc} = \frac{mR^2}{4} + mR^2 = \frac{5mR^2}{4} = \frac{5}{4}\times4\times R^2 = 5R^2$$

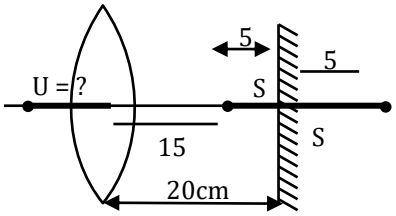
$$\frac{I_{Disc}}{I_{SS}} = \frac{5R^2}{7R^2} = \frac{5}{7}$$

$x = 5$

28. An object is placed on the principal axis of convex lens of focal length 10 cm as shown. A plane mirror is placed on the other side of lens at a distance of 20 cm. The image produced by the plane mirror is 5 cm inside the mirror. The distance of the object from the lens is cm



Sol. **30.00**



$\therefore$ for lens $v = 20 - 5 = 15$ cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{15} - \frac{1}{u} = \frac{1}{10}$$

$$\frac{1}{u} = \frac{1}{15} - \frac{1}{10} = \frac{2-3}{30} = \frac{1}{30}$$

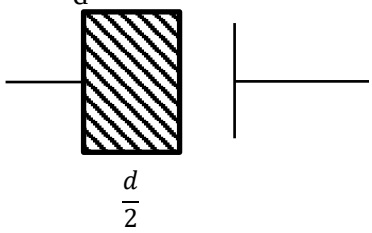
$u = -30$

$= 30$

29. A capacitor has capacitance $5\mu F$ when it's parallel plates are separated by air medium of thickness d. A slab of material of dielectric constant 1.5 having area equal to that of plates but thickness $\frac{d}{2}$ is inserted between the plates. Capacitance of the capacitor in the presence of slab will be μF.

Sol. **6**

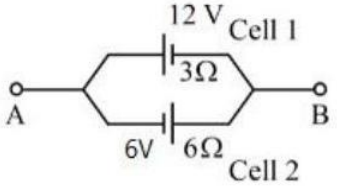
$$C_o = \frac{\epsilon_o A}{d} = 5$$



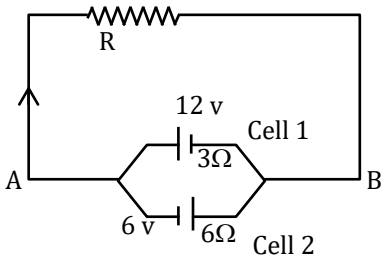
$$C_1 = \frac{\epsilon_o (1.5) A}{\frac{d}{2}} = 3C_o, \; C_2 = \frac{\epsilon_o A}{\frac{d}{2}} = 2Co$$

$$= \frac{3C_o \times 2C_o}{5C_o} = \frac{6}{5} \times 5 = 6\mu f$$

30. Two cells are connected between points A and B as shown. Cell 1 has emf of 12 V and internal resistance of 3Ω. Cell 2 has emf of 6 V and internal resistance of 6Ω. An external resistor R of 4Ω is connected across A and B. The current flowing through R will be _____ A.

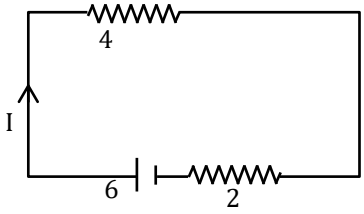


Sol. **1**



$$v = \frac{12\times 6 - 6\times 3}{6+3} = \frac{54}{9} = 6 \text{ volt}$$

$$r_{eq} = \frac{6\times 3}{6+3} = 2\Omega$$



$$I = \frac{6}{4+2} = 1A$$

Chemistry

SECTION - A

31. When the hydrogen ion concentration $[H^+]$ changes by a factor of 1000 , the value of pH of the solution

(1) increases by 2 units (2) increases by 1000 units

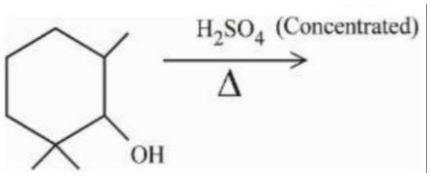
(3) decreases by 2 units (4) decreases by 3 units

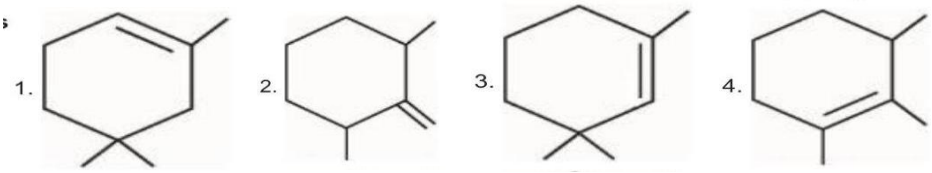
Sol. **4**

If $[H^+] \rightarrow 10^3$ times

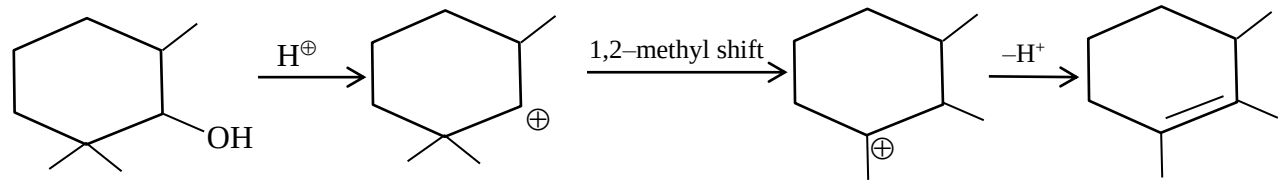
then pH decreases by 3 units.

32. Find out the major product from the following reaction.





Sol. **4**



33. Given below are two statements, one is labelled as Assertion **A** and the other is labelled as Reason **R**

Assertion A : Carbon forms two important oxides - CO and CO_2. CO is neutral whereas CO_2 is acidic in nature

Reason **R**: CO_2 can combine with water in a limited way to form carbonic acid, while CO is sparingly soluble in water

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) A is correct but R is not correct

(3) Both A and R are correct and R is the correct explanation of A

(4) A is not correct but R is correct

Sol. **3**

(i) CO_2 is acidic as it from carbonic acid

$$CO_2 + H_2O \rightarrow H_2CO_3$$

(ii) CO is almost insoluble in water

34. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason **R**
Assertion A : The alkali metals and their salts impart characteristic colour to reducing flame.
Reason R : Alkali metals can be detected using flame tests.
In the light of the above statements, choose the most appropriate answer from the options given below
(1) A is not correct but R is correct
(2) Both A and R are correct but R is NOT the correct explanation of A
(3) A is correct but R is not correct
(4) Both A and R are correct and R is the correct explanation of A

Sol. **1**

The alkali metal and their salts impart characteristic colour to an oxidizing flame. this is because the heat from the flame excites the outemost orbital electron to a higher energy level : when the excited electron comes back to the ground state, there is emission of radiation in the visible region.
Alkali metal can therefore, be detected by the respective flame test and can be determined by flame photometry or atomic absorption spectroscopy.

35. Potassium dichromate acts as a strong oxidizing agent in acidic solution. During this process, the oxidation state changes from
(1) +2 to +1 (2) +3 to +1 (3) +6 to +2 (4) +6 to +3

Sol. **4**

$$Cr_2O_7^{-2} + 14H^+ + 6e^- \longrightarrow 2Cr^{+3} + 7H_2O$$

$$+6 \longrightarrow +3$$

36. Match List I with List II

LIST I (Name of polymer)		LIST II (Uses)	
A.	Glyptal	I.	Flexible pipes
B.	Neoprene	II.	Synthetic wool
C.	Acrilan	III	Paints and Lacquers
D.	LDP	IV.	Gaskets

Choose the correct answer from the options given below:
(1) A-III, B-IV, C-I, D-II (2) A-III, B-II, C-IV, D-I
(3) A-III, B-I, C-IV, D-II (4) A-III, B-IV, C-II, D-I

Sol. **4**

(A) Glyptal $\rightarrow$ Paints and Lacquers (III)

(B) Neoprene $\rightarrow$ Gaskets (IV)

(C) Acrilan $\rightarrow$ Synthetic wool (II)

(D) LDP $\rightarrow$ Flexible pipes (I)

37. Which of the following represents the correct order of metallic character of the given elements ?

(1) $Si < Be < Mg < K$ (2) $Be < Si < K < Mg$

(3) $Be < Si < Mg < K$ (4) $K < Mg < Be < Si$

Sol. **1**

$Si < Be < Mg < K$

Si is having Non-metallic character.

38. Match List I with List II

LIST I		LIST II	
A.	Cobalt catalyst	I.	$(H_2 + Cl_2)$ production
B.	Syngas	II.	Water gas production
C.	Nickel catalyst	III.	Coal gasification
D.	Brine solution	IV.	Methanol production

Choose the correct answer from the options given below:

(1) A-IV, B-I, C-II, D-III (2) A-IV, B-III, C-II, D-I

(3) A-II, B-III, C-IV, D-I (4) A-IV, B-III, C-I, D-II

Sol. **2**

(a) Cobalt catalyst $\rightarrow$ methanol production.

(b) Syngas $\rightarrow$ coal gasification

(c) Nickel Catalyst $\rightarrow$ water gas production .

(d) Brine solution $\rightarrow$ H_2 + Cl_2 production.

39. Match List I with List II

LIST I (Amines)		LIST II ($\mathbf{pK_b}$)	
A.	Aniline	I.	3.25
B.	Ethanamine	II.	3.00
C.	N-Ethylethanamine	III	9.38
D.	N. N-Diethylethanamine	IV.	3.29

Choose the correct answer from the options given below:

(1) A-III, B-IV, C-II, D-I (2) A-III, B-II, C-I, D-IV

(3) A-I, B-IV, C-II, D-III (4) A-III, B-II, C-IV, D-I

Sol. **1**

Basicity order

$Et - NH - Et > Et - \underset{\underset{Et}{|}}{N} - Et > Et - NH_2 >$ Aniline

pKb : 3.00, pKb : 3.25 pKb : 3.29 pKb : 9.38

40. Match List I with List II

	LIST I		LIST II
	Isomeric pairs		Type of isomers
A.	Propanamine and N-Methylethanamine	I.	Metamers
B.	Hexan-2-one and Hexan-3-one	II.	Positional isomers
C.	Ethanamide and Hydroxyethanimine	III.	Functional isomers
D.	o-nitrophenol and p-nitrophenol	IV.	Tautomers

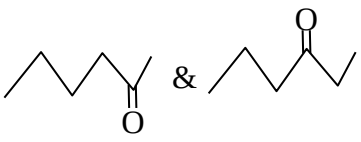
Choose the correct answer from the options given below:

(1) A-II, B-III, C-I, D-IV (2) A-III, B-I, C-IV, D-II

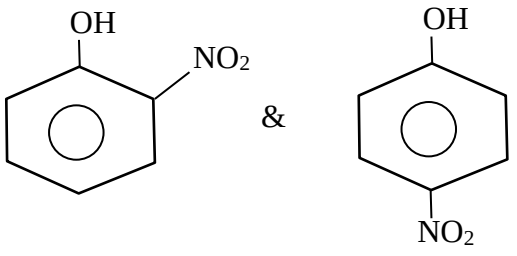
(3) A-III, B-IV, C-I, D-II (4) A-IV, B-III, C-I, D-II

Sol. **2**

(A) $C–C–C–NH_2$ & $C - NH - C - C$: functional isomer (III)

(B)  : Metamer (I)

(C) $CH_3 - \underset{\underset{O}{\|}}{C} - NH_2$ & $CH_3 - \underset{\underset{OH}{|}}{C} = NH$: Tautamer (IV)

(D)  : Position isomer (II)

41. What is the mass ratio of ethylene glycol ($C_2H_6O_2$, molar mass = 62 g/mol) required for making 500 g of 0.25 molal aqueous solution and 250 mL of 0.25 molal aqueous solution?

(1) 1 : 1 (2) 2 : 1 (3) 1 : 2 (4) 3 : 1

Sol. 2

Case I

x gm $C_2H_6O_2$ present

$$0.25 = \frac{x/62}{500 - x} \times 1000$$

$$125 = \left(\frac{1000}{62} + 0.25\right) x \qquad \text{.....(1)}$$

Case II

y gm $C_2H_6O_2$ is present.

$$0.25 = \frac{y/62}{250 - y} \times 1000$$

$$62.5 - 0.25y = \frac{1000}{62} y$$

$$62.5 = \left(\frac{1000}{62} + 0.25\right) y \qquad \text{.....(2)}$$

equation (1) ÷ equation (2)

$$\frac{x}{y} = \frac{125}{62.5} = \frac{2}{1}$$

42. Match list I with List II

LIST I Coordination entity		**LIST II** Wavelength of light absorbed in nm	
A.	$[CoCl(NH_3)_5]^{2+}$	I.	310
B.	$[Co(NH_3)_6]^{3+}$	II.	475
C.	$[Co(CN)_6]^{3-}$	III.	535
D.	$[Cu(H_2O)_4]^{2+}$	IV.	600

Choose the correct answer from the options given below:

(1) A-III, B-I, C-II, D-IV (2) A-IV, B-I, C-III, D-II

(3) A-III, B-II, C-I, D-IV (4) A-II, B-III, C-IV, D-I

Sol. **3**

$\Delta_o \uparrow \lambda \downarrow$

(splitting energy = $\frac{hc}{\lambda_{abs}}$)

43. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason **R**

Assertion A : Butylated hydroxy anisole when added to butter increases its shelf life.

Reason R : Butylated hydroxy anisole is more reactive towards oxygen than food.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) A is correct but R is not correct

(2) A is not correct but R is correct

(3) Both A and R are correct and R is the correct explanation of A

(4) Both A and R are correct but R is NOT the correct explanation of A

Sol. **3**

The molecule BHA = Butylated hydroxyanisole commonly used as food preservatives which normally acts as antifungal and antiviral BHA reduces the rancidity of oil and fat which halps in retaining the nutrients (Butter contains saturated fats).

44. The isomeric deuterated bromide with molecular formula C_4H_8DBr having two chiral carbon atoms is

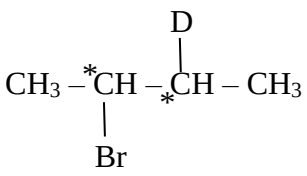
(1) 2 - Bromo - 2 - deuterobutane

(2) 2 - Bromo-1-deuterobutane

(3) 2 - Bromo - 1 - deutero - 2 - methylpropane

(4) 2 - Bromo -3 – deuterobutane

Sol. **4**



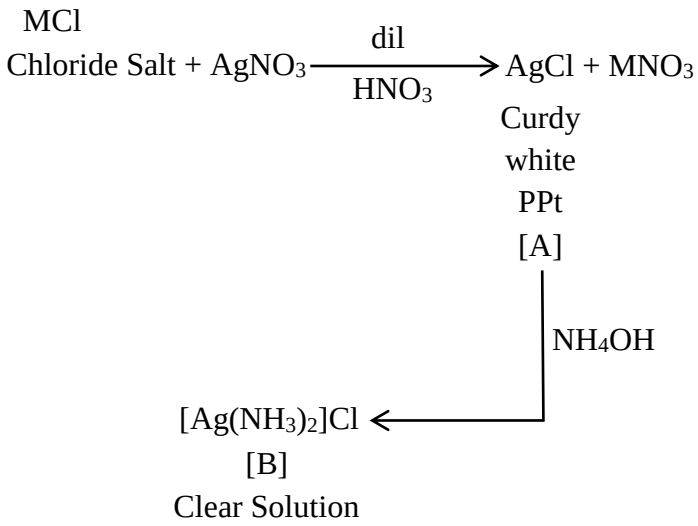
2 - Bromo -3 – deuterobutane

45. A chloride salt solution acidified with dil. HNO_3 gives a curdy white precipitate, [A], on addition of $AgNO_3$ · [A] on treatment with NH_4OH gives a clear solution, B. A and B are respectively

(1) $AgCl\&(NH_4)[Ag(OH)_2]$ (2) $AgCl\&[Ag(NH_3)_2]Cl$

(3) $H[AgCl_3]\&(NH_4)[Ag(OH)_2]$ (4)$H[AgCl_3]\&[Ag(NH_3)_2]Cl$

Sol. 2



46. Statement I : Dipole moment is a vector quantity and by convention it is depicted by a small arrow with tail on the negative centre and head pointing towards the positive centre.

Statement II : The crossed arrow of the dipole moment symbolizes the direction of the shift of charges in the molecules.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is incorrect but Statement II is correct

(2) Statement I is correct but Statement II is incorrect

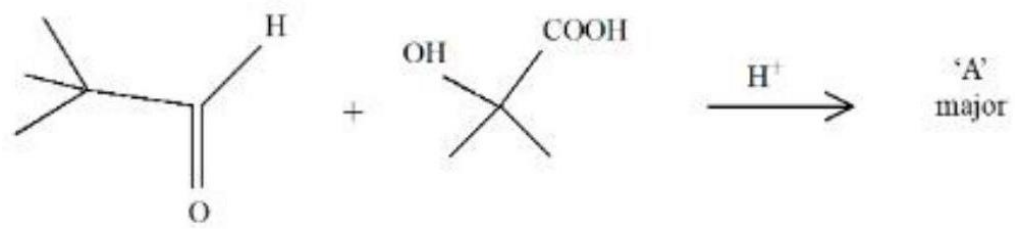
(3) Both Statement I and Statement II are incorrect

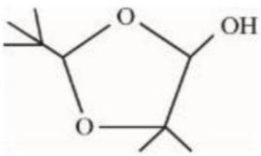
(4) Both Statement I and Statement II are correct

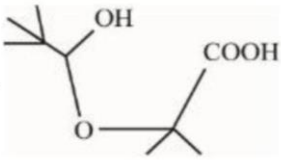
Sol. 2

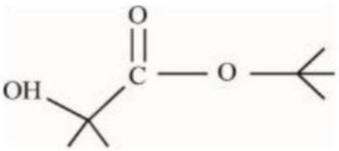
The crossed arrow of the dipole moment symbolizes the direction of the shift of electron density in the molecules.

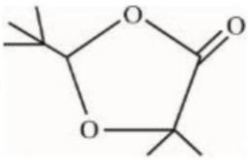
47. ' A ' in the given reaction is



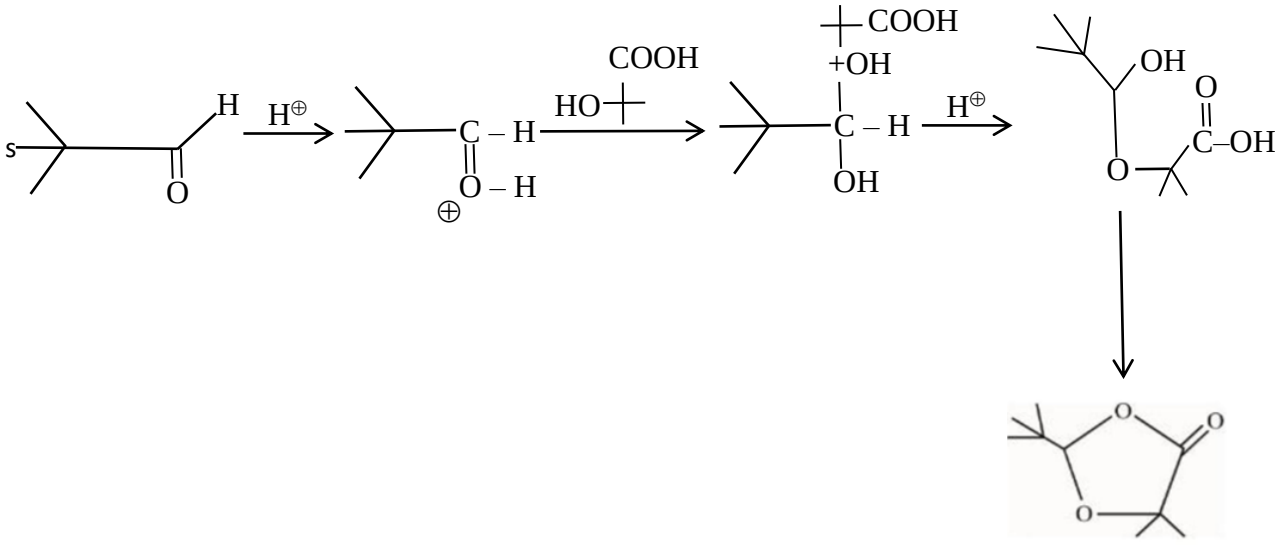
1. 

2. 

3. 

4. 

Sol. **4**



48. A. Ammonium salts produce haze in atmosphere.

B. Ozone gets produced when atmospheric oxygen reacts with chlorine radicals.

C. Polychlorinated biphenyls act as cleansing solvents.

D. 'Blue baby' syndrome occurs due to the presence of excess of sulphate ions in water.

Choose the correct answer from the options given below:

(1) A and D only (2) A, B and C only (3) A and C only (4) B and C only

Sol. 3

(i) Ammonium salt are major component of both atmospheric nitrogen aerosols and wet deposited.

(iii) PCB belongs to a broad family of man-made organic chemicals known. as chlorinated hydrocarbons.

49. Given below are two statements:

Statement I : In froth floatation method a rotating paddle agitates the mixture to drive air out of it.

Statement II : Iron pyrites are generally avoided for extraction of iron due to environmental reasons.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Sol. 1

The rotating paddle in the froth flotation process violently agitates the suspension of powdered ore in water, as well the collectors and froth stablisers, generating frothing.

50. Which one among the following metals is the weakest reducing agent?

(1) Li (2) K (3) Rb (4) Na

Sol. 4

Na metals is the weakest Reducing agent.

Section B

51. Total number of moles of AgCl precipitated on addition of excess of $AgNO_3$ to one mole each of the following complexes $[Co(NH_3)_4Cl_2]Cl$, $[Ni(H_2O)_6]Cl_2$, $[Pt(NH_3)_2Cl_2]$ and $[Pd(NH_3)_4]Cl_2$ is

Sol. **5**

(i) $\left[Co(NH_3)_4Cl_2\right]Cl + AgNO_3 \rightarrow \left[Co(NH_3)_4Cl_2\right]^+ + AgCl$

(ii) $\left[Ni(H_2O)_6\right]Cl_2 + AgNO_3 \rightarrow \left[Ni(H_2O)_6\right]^{+2} + 2AgCl$

(iii) $[Pt(NH_3)_2Cl_2] + AgNO_3 \rightarrow$ no AgCl mole are ppt

(iv) $[Pd(NH_3)_4]Cl_2 + AgNO_3 \rightarrow [Pd(NH_3)_4]^{+2} + 2AgCl$

Total 5 mole AgCl are formed.

52. The number of incorrect statement/s from the following is/are

A. Water vapours are adsorbed by anhydrous calcium chloride.

B. There is a decrease in surface energy during adsorption.

C. As the adsorption proceeds, ΔH becomes more and more negative.

D. Adsorption is accompanied by decrease in entropy of the system.

Sol. **2**

A & C are incorrect

$CaCl_2$ absorbs water vapour.

As adsorption proceeds,

ΔH becomes less negative.

53. Number of hydrogen atoms per molecule of a hydrocarbon A having 85.8% carbon is (Given: Molar mass of A $= 84$ g mol^{-1})

Sol. **12**

$C \rightarrow 85.8\%$

$H \rightarrow 14.2\ \%$

mass of H in one molecule = $84 \times \frac{14.2}{100} \approx 12$

No. of H– atoms = $\frac{12}{1}$

$= 12$

54. The number of given orbitals which have electron density along the axis is

$P_x, P_y, P_z, d_{xy}, d_{yz}, d_{xz}, d_z^2, d_{x^2} - y^2$

Sol. 5

$Px, Py, Pz, dz^2, dx^2 - y^2$ have Electron density along the axis.

55. 28.0 L of CO_2 is produced on complete combustion of 16.8 L gaseous mixture of ethene and methane at 25°C and 1 atm. Heat evolved during the combustion process is_____ kJ.

Given : $\Delta H_c(CH_4) = -900\,kJ\,mol^{-1}$

$\Delta H_c(C_2H_4) = -1400\ kJ\ mol^{-1}$

Sol. **847**

Moles of mixture = $\frac{Pv}{RT} = \frac{1\times16.8}{0.0821\times298} = 0.6866$ moles

Moles of CO_2 = $\frac{1\times28}{0.0821\times298} = 1.144$ mole

$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$

x x

$C_2H_4 + 3O_2 \longrightarrow 2CO_2 + 2H_2O$

$(0.6866 - x)$ $2(0.686 - x)$

Total CO_2 produced = 1.144

x + 2 (0.6866 – x) = 1.144

x = 1.3732 – 1.144

= 0.2292

Moles of CH_4 = 0.2292

Moles of C_2H_4 = 0.6866 – 0.2292

= 0.4574

Total Heat produced

= (900 × 0.2292) + (0.4574 × 1400)

= 206.28 + 640.36 = 846.64

56. $Pt(s)|H_2(g)(1bar)||H^+(aq)(1M) \parallel M^{3+}(aq), M^+(aq)|Pt(s)$

The E_{cell} for the given cell is 0.1115 V at 298 K when $\frac{[M^+(aq)]}{[M^{3+}(aq)]} = 10^a$

The value of a is

Given : $E^{\theta}M^{3+}/M^{+} = 0.2$ V

$\frac{2.303RT}{F} = 0.059$ V

Sol. **3**

Cell Reaction

$$H_2 + M^{3+} \longrightarrow 2H^+ + M^+$$

$$E_{cell} = E^o_{cell} - \frac{2.303RT}{2F}\log\frac{\left[M^+\right]\left[H^+\right]^2}{[M^{3+}]}$$

$$0.1115 = 0.2 - \frac{0.059}{2}\log 10^a$$

$$\frac{0.059}{2}\log 10^a = 0.0885$$

a = 3

57. The number of pairs of the solutions having the same value of the osmotic pressure from the following is (Assume 100% ionization)

A. 0.500 M C_2H_5OH (aq) and 0.25 M KBr (aq)

B. 0.100 M $K_4[Fe(CN)_6]$ (aq) and 0.100 M $FeSO_4(NH_4)_2SO_4$ (aq)

C. 0.05 M $K_4[Fe(CN)_6]$ (aq) and 0.25 M NaCl (aq)

D. 0.15 M NaCl(aq) and 0.1 M $BaCl_2$(aq)

E. 0.02 M $KCl \cdot MgCl_2 \cdot 6H_2O$(aq) and 0.05 M KCl(aq)

Sol. **4**

(a) $(ic)_{c_2H_5OH} = 0.5$

$(ic)_{kBr} = 2\times 0.25 = 0.5$

osmotic pressure will be same.

(b) $(ic)_{k_4[Fe(CN)_6]} = 0.1\times 5 = 0.5$

$(ic)_{FeSO_4.(NH_4)_2SO_4} = 0.1\times 5 = 0.5$

osmotic pressure will be same.

(c) $(ic)_{k_4[Fe(CN)_6]} = 5\times 0.05 = 0.25$

$(ic)_{NaCl} = 0.25\times 2 = 0.5$

osmotic pressure will not be same.

(d) $(ic)_{NaCl} = 0.15\times 2 = 0.3$

$(i\ c)_{BaCl_2} = 0.1 \times 3 = 0.3$

osmotic pressure will be same.

(e) $(i\ c)_{Kcl.MgCl.6H_2O} = 0.02 \times 5 = 0.1$

$(i\ c)_{Kcl} = 0.05 \times 2 = 0.1$

osmotic pressure will be same.

58. A first order reaction has the rate constant, $= 4.6 \times 10^{-3}\ s^{-1}$. The number of correct statement/s from the following is/are

Given: log 3 = 0.48

A. Reaction completes in 1000 s.

B. The reaction has a half-life of 500 s.

C. The time required for 10% completion is 25 times the time required for 90% completion.

D. The degree of dissociation is equal to $\left(1 - e^{-kt}\right)$

E. The rate and the rate constant have the same unit.

Sol. **1**

$k = 4.6 \times 10^{-3}\ sec^{-1}$

for Ist order :-

$t½ = \dfrac{0.693}{k} = \dfrac{0.693}{4.6 \times 10^{-3}} = 150.65 sec.$

$t_{completion} = \infty$

Degree of dissociation ($\propto$) = $\dfrac{x}{[A]_0} = \dfrac{[A]_0 - [A]_t}{[A]_0}$

$$= \frac{[A]_0 - [A]_0 e^{-kt}}{[A]_0} = 1 - e^{-kt}$$

rate and rate constant have different units

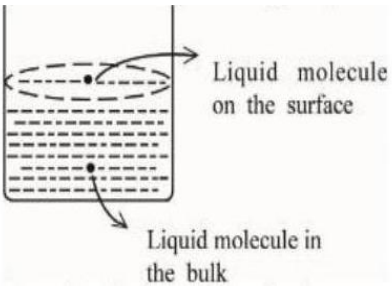
$t_{10\%} = \dfrac{1}{K} \ln \dfrac{100}{90}$

$t_{90\%} = \dfrac{1}{K} \ln \dfrac{100}{10}$

$$\frac{t_{10\%}}{t_{90\%}} = \frac{\log 10 - \log 9}{\log 10} = 0.045$$

$$t_{10\%} = 0.045 t_{90\%}$$

59. Based on the given figure, the number of correct statement/s is/are _____



A. Surface tension is the outcome of equal attractive and repulsive forces acting on the liquid molecule in bulk.

B. Surface tension is due to uneven forces acting on the molecules present on the surface.

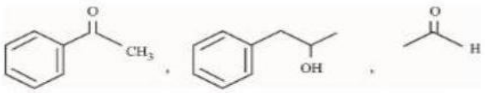
C. The molecule in the bulk can never come to the liquid surface.

D. The molecules on the surface are responsible for vapours pressure if system is a closed system.

Sol. 2

B & D option are correct.

60. Number of compounds giving (i) red colouration with ceric ammonium nitrate and also (ii) positive iodoform test from the following is



Sol. **3**

Mathematics

SECTION - A

61. Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $(p \to q)\Delta(p \nabla q)$ is a tautology. Then

(1) $\Delta = \vee, \nabla = \vee$ (2) $\Delta = \vee, \nabla = \wedge$ (3) $\Delta = \wedge, \nabla = \vee$ (4) $\Delta = \wedge, \nabla = \wedge$

Sol. **(1)**

p	q	$p \to q$	$p \vee q$	$(p \to q) \vee (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

62. If the four points, whose position vectors are $3\hat{i} - 4\hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$, $-2\hat{i} - \hat{j} + 3\hat{k}$ and $5\hat{i} - 2\alpha\hat{j} + 4\hat{k}$ are coplanar, then α is equal to

(1) $\frac{73}{17}$ (2) $\frac{107}{17}$ (3) $\frac{-73}{17}$ (4) $\frac{-107}{17}$

Sol. **(1)**

$\underbrace{3\hat{i} - 4\hat{j} + 2\hat{k}}_{P}$, $\underbrace{\hat{i} + 2\hat{j} - \hat{k}}_{Q}$, $\underbrace{-2\hat{i} - \hat{j} + 3\hat{k}}_{R}$, $\underbrace{5\hat{i} - 2\alpha\hat{j} + 4\hat{k}}_{S}$

$\overrightarrow{PQ} = -2\hat{i} + 6\hat{j} - 3\hat{k}$

$\overrightarrow{QR} = -3\hat{i} - 3\hat{j} + 4\hat{k}$

$\overrightarrow{RS} = 7\hat{i} + (1 - 2\alpha)\hat{j} + \hat{k}$

$\left[\overrightarrow{PQ}\ \overrightarrow{QR}\ \overrightarrow{RS}\right] = 0$

$$\begin{vmatrix} -2 & 6 & -3 \\ -3 & -3 & 4 \\ 7 & 1-2\alpha & 1 \end{vmatrix} = 0$$

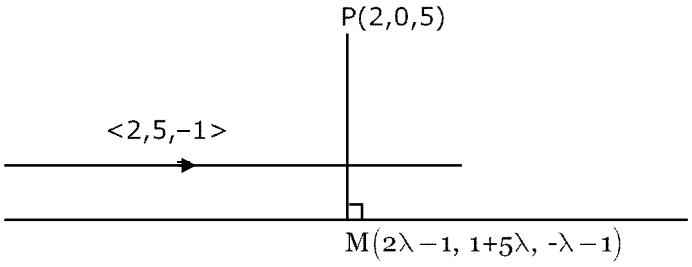
$-2(-3 + 8\alpha - 4) - 6(-31) - 3(6\alpha - 3 + 21) = 0$

$\alpha = \frac{73}{17}$

63. The foot of perpendicular of the point $(2,0,5)$ on the line $\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1}$ is (α, β, γ). Then, which of the following is NOT correct?

(1) $\frac{\beta}{\gamma} = -5$ (2) $\frac{\gamma}{\alpha} = \frac{5}{8}$ (3) $\frac{\alpha}{\beta} = -8$ (4) $\frac{\alpha\beta}{\gamma} = \frac{4}{15}$

Sol. **(1)**



$\overrightarrow{PM}(2,5,-1) = 0$

$(2\lambda - 3,\ 5\lambda + 1,\ -\lambda - 6)\cdot(2,5,-1) = 0$

$4\lambda - 6 + 25\lambda + 5 + \lambda + 6 = 0$

$\boxed{\lambda = -\frac{1}{6}}$

Now, $\alpha = 2\left(-\frac{1}{6}\right) - 1 = -\frac{4}{3}$

$\beta = \frac{1}{6}$

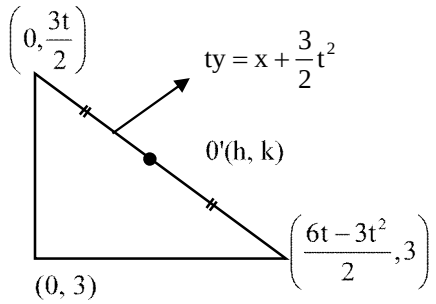
$\gamma = -\frac{5}{6}$

64. The equations of two sides of a variable triangle are x= 0 and y=3, and its third side is a tangent to parabola y^2=6x. The locus of its circumcentre is:

(1) $4y^2 - 18y - 3x - 18 = 0$ (2) $4y^2 - 18y - 3x + 18 = 0$

(3) $4y^2 - 18y + 3x + 18 = 0$ (4) $4y^2 + 18y + 3x + 18 = 0$

Sol. **(3)**



$2h = \frac{6t - 3t^2}{2}$

$4h = 6t - 3t^2$ .....(i)

& $2k = \frac{3t + 6}{2}$

$\boxed{\frac{4k - 6}{3} = t}$

$4h = 8k - 12 - \frac{1}{3}(16k^2 - 48k + 36)$

$12h = 24k - 36 - 16k^2 + 48k - 36$

$4y^2 - 18y + 3x + 18 = 0$

65. Let $f(x) = 2x^n + \lambda, \lambda \in \mathbb{R}, n \in \mathbb{N}$, and $f(4) = 133, f(5)255$.

Then the sum of all the positive integer divisors of $(f(3) - f(2))$ is

(1) 60 (2) 59 (3) 61 (4) 58

Sol. **(1)**

$133 = 2(4^n)+\lambda$

$255 = 2(5^n)+\lambda$

$122 = 2[5^n-4^n]$

$\boxed{5^n - 4^n = 61}$

$\Downarrow$

$\boxed{n = 3}$

Now,

$f(3) = 2(3)^3+ \lambda$

$f(2) = 2(2)^3+ \lambda$

$f(3) - f(2) = 38 = 2 \times 19$

$(2^0 +2^1) (19^0 + 19)$

$= 60$

66. $\sum_{k=0}^{6} {}^{51}C_3$ is equal to

(1) ${}^{51}C_4 - {}^{45}C_4$ (2) ${}^{52}C_3 - {}^{45}C_3$ (3) ${}^{52}C_4 - {}^{45}C_4$ (4) ${}^{51}C_3 - {}^{45}C_3$

Sol. **(3)**

${}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3$

add and subtract ${}^{45}C_4$

$\left({}^{45}C_4 + {}^{45}C_3\right) + {}^{46}C_3 + {}^{47}C_3 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3 - {}^{45}C_4 \quad \left({}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r\right)$

${}^{52}C_4 - {}^{45}C_4$

$\Rightarrow$ [C]

67. Let the function $f(x) = 2x^3 + (2p - 7)x^2 + 3(2p - 9)x - 6$ have a maxima for some value of $x < 0$ and a minima for some value of $x > 0$. Then, the set of all values of p is

(1) $\left(0, \frac{9}{2}\right)$ (2) $\left(-\infty, \frac{9}{2}\right)$ (3) $\left(-\frac{9}{2}, \frac{9}{2}\right)$ (4) $\left(\frac{9}{2}, \infty\right)$

Sol. **(2)**

$f(x) = 2x^3 + (2p - 7)x^2 + 3(2p - 9)x - 6$

$f'(x) = 6x^2 + (4p - 14)x + 6p - 27 = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$

let $\alpha > 0$ & $\beta < 0$

Products of roots $< 0 \Rightarrow$ (2)

68. Let $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$, where $i = \sqrt{-1}$.

If $M = A^TBA$, then the inverse of the matrix $AM^{2023} A^T$ is

(1) $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Sol. **(4)**

Now, $M^2 = \left(A^TBA\right)\left(A^TBA\right) = A^TB^2A$ $\boxed{AA^T = I}$

$\Rightarrow M^{2023} = A^TB^{2023}A$

Let $D = AM^{2023}A^T = A\,A^TB^{2023}AA^T$ $\boxed{AA^T = I}$

$D = B^{2023}$

Now, $B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$

Now, $B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

$D^{-1} = \begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

69. Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$. Then $\vec{a} - 6\vec{b}$ is equal to

(1) $3(\hat{i} - \hat{j} + \hat{k})$ (2) $(\hat{i} + \hat{j} - \hat{k})$ (3) $3(\hat{i} + \hat{j} + \hat{k})$ (4) $3(\hat{i} - \hat{j} - \hat{k})$

Sol. **(3)**

$$\vec{a} \times \left(\vec{a} \times \vec{b}\right) = \begin{vmatrix} i & j & k \\ -1 & -1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = \hat{i} + \hat{j} + 2\hat{k}$$

$\vec{a} - 3\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$

$-3\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$

$-6\vec{b} = 4\hat{i} + 4\hat{j} + 2\hat{k}$

Now, $\vec{a} - 6\vec{b} = 3\left(\hat{i} + \hat{j} + \hat{k}\right)$

70. The integral $16\int_1^2 \frac{dx}{x^3(x^2+2)^2}$ is equal to

(1) $\frac{11}{12} - \log_e 4$ (2) $\frac{11}{6} - \log_e 4$ (2) $\frac{11}{6} + \log_e 4$ (4) $\frac{11}{12} + \log_e 4$

Sol. **(2)**

$$16\int_1^2 \frac{dx}{x^3 x^4 \left(1+\frac{2}{x^2}\right)^2}$$

Let, $1 + \frac{2}{x^2} = t \Rightarrow -\frac{4}{x^3}dx = dt$

$$\frac{-4}{4}\int_3^{3/2} \frac{(t-1)^2}{t^2}dt = \int_{3/2}^{3} \frac{t^2 - 2t + 1}{t^2}dt$$

$$\Rightarrow \int_{3/2}^{3} \left(1 - \frac{2}{t} + \frac{1}{t^2}\right)dt$$

$$\Rightarrow 3 - \frac{3}{2} - 2\left(\ln 3 - \ln 3/2\right) - \frac{1}{3} + \frac{2}{3}$$

$$\Rightarrow \frac{11}{6} - 2\ln 2 \Rightarrow \frac{11}{6} - \ln 4$$

71. Let T and C respectively be the transverse and conjugate axes of the hyperbola $16x^2 - y^2 + 64x + 4y + 44 = 0$. Then the area of the region above the parabola $x^2 = y + 4$, below the transverse axis T and on the right of the conjugate axis C is:

(1) $4\sqrt{6} + \frac{28}{3}$ (2) $4\sqrt{6} - \frac{44}{3}$ (3) $4\sqrt{6} + \frac{44}{3}$ (4) $4\sqrt{6} - \frac{28}{3}$

Sol. **(1)**

$16(x^2 + 4x) - (y^2 - 4y) + 44 = 0$

$16\{(x+2)^2 - 4\} - (y-2)^2 + 4 + 44 = 0$

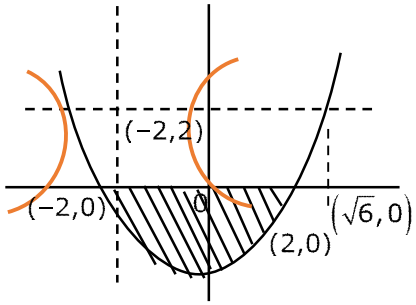
$16(x+2)^2 - (y-2)^2 = 16$

$$\frac{(x+2)^2}{1} - \frac{(y-2)^2}{16}$$

Area = $\int_{-2}^{\sqrt{6}} (y_2 - y_1)\,dx$

$= \int_{-2}^{\sqrt{6}} \left(2 - \left(x^2 - 4\right)\right) dx$

$= 4\sqrt{6} + \frac{28}{3}$



72. Let N be the sum of the numbers appeared when two fair dice are rolled and let the probability that $N-2,\ \sqrt{3N},\ N+2$ are in geometric progression be $\frac{k}{48}$. Then the value of k is

(1) 8 (2) 16 (3) 2 (4) 4

Sol. **(4)**

$3N = N^2 - 4$

$N^2 - 3N - 4 = 0$

$\boxed{N = 4}$

Sum should be equal to 4 so possible outcomes are {(1,3), (2,2), (3,1)}

$\Rightarrow$ Prob = $\frac{3}{36} = \frac{1}{12} = \frac{k}{48}$

$\boxed{K = 4}$

73. If the function $f(x) = \begin{cases} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}} & , 0 < x < \frac{\pi}{2} \\ \mu & , x = \frac{\pi}{2} \\ \frac{\cot 6x}{e^{\cot 4x}} & \frac{\pi}{2} < x < \pi \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then $9\lambda + 6\log_e \mu + \mu^6 - e^{6\lambda}$ is equal to

(1) 10 (2) $2e^4 + 8$ (3) 11 (4) 8

Sol. $f\left(\frac{\pi^+}{2}\right) = e^{\lim_{h\to 0}\frac{\cot 6h}{\cot 4h}} \Rightarrow 2/3$

$f\left(\frac{\pi^-}{2}\right) = \lim_{h\to 0}(1+\sin h)^{\frac{\lambda}{\sin h}}$

$= \frac{\lambda}{0}$

$\Rightarrow$ limit DNE (does not exist)

74. The number of functions $f : \{1,2,3,4\} \to \{a \in : \mathbb{Z} \mid a| \le 8\}$ satisfying $f(n) + \frac{1}{n} f(n+1) = 1, \forall n \in \{1,2,3\}$ is

(1) 1 (2) 4 (3) 2 (4) 3

Sol. **(2)**

$f:\{1,2,3,4\}\to\{a\in z:|a|\le 8\}$

$f(n)+\frac{1}{n}f(n+1)=1\ \forall n\in\{1,2,3\}$

$$f(n+1)=n(1-f(n))$$

Put n = 1, $f(2)=1-f(1)$

Put n = 2, $f(3)=2(1-f(2))=2f(1)$

Put n = 3, $f(4)=3(1-f(3))=3(1-2f(1))$

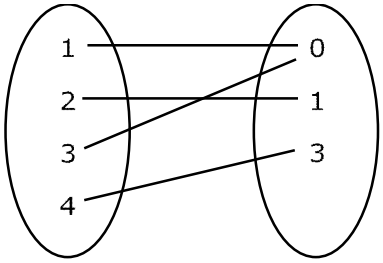
$$f(4)=3-6f(1)$$

Now : $f(2)=1-f(1)$

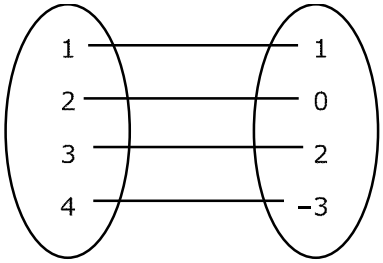
$$f(3)=2f(1)$$

$$f(4)=3-6f(1)$$

Case – I Take f(1) = 0



Case – II Take f(1) = 1



No. of function = 2

Ans : 4

75. Let y = y(t) be a solution of the differential equation $\frac{dy}{dt}+\alpha y=\gamma e^{-\beta t}$ where, $\alpha > 0, \beta > 0$ and $\gamma > 0$.

Then $\lim\limits_{t\to\infty} y(t)$

(1) is -1 (2) is 1 (3) does not exist (4) is 0

Sol. **(4)**

$$\frac{dy}{dt}+\alpha y=\gamma e^{-\beta t}$$

L.D.E (Linear differential equation)

I.F. = $e^{\int\alpha\cdot dt}=e^{\alpha t}$

$$y(e^{\alpha t})=\int\gamma e^{-\beta t}\cdot e^{\alpha t}\cdot dt$$

$$\Rightarrow ye^{\alpha t} = \gamma \frac{e^{(\alpha-\beta)t}}{\alpha-\beta} + C$$

$$\Rightarrow y(t) = \frac{\gamma}{\alpha-\beta} e^{-\beta t} + C \cdot e^{-\alpha t}$$

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty} \left\{ \frac{\gamma}{\alpha-\beta} e^{-\beta t} + c \cdot e^{-\alpha t} \right\}$$

$= 0 + 0$

$$\Rightarrow \lim_{t\to\infty} y(t) = 0$$

76. Let z be a complex number such that $\left|\frac{z-2i}{z+i}\right| = 2, z \neq -i$. Then z lies on the circle of radius 2 and centre

(1) (2,0) (2) (0,2) (3) $(0,-2)$ (4) (0,0)

Sol. $$\left|\frac{x+i(y-2)}{x+i(y+1)}\right| = 2$$

$$x^2 + (y-2)^2 = 4\left(x^2 + (y+1)^2\right)$$

$$3x^2 + 4y^2 + 4 + 8y - y^2 - 4 + 4y = 0$$

$$3\left(x^2 + y^2\right) + 12y = 0$$

$$x^2 + y^2 + 4y = 0$$

C(0,–2)

77. Let A, B, C be 3 × 3 matrices such that A is symmetric and B and C are skew-symmetric.

Consider the statements

(S1) $A^{13} B^{26} - B^{26} A^{13}$ is symmetric

(S2) $A^{26}C^{13} - C^{13} A^{26}$ is symmetric

Then,

(1) Only S2 is true (2) Both S1 and S2 are false

(3) Only S1 is true (4) Both S1 and S2 are true

Sol. **(1)**

$A^T = A$, $B^T = -B$, $C^T = -C$

(S_1) : $(A^{13}B^{26} - B^{26}A^{13})^T$

$= (A^{13}B^{26})^T - (B^{26}A^{13})^T$

$= (B^T)^{26} (A^T)^{13} - (A^T)^{13} (B^T)^{26}$

$= (-B)^{26} (A)^{13} - (A)^{13} (-B)^{26}$

$= B^{26} A^{13} - A^{13} B^{26}$

$= -(A^{13} B^{26} - B^{26} A^{13})$

($S_1 \rightarrow$ false)

(S_2) : $(A^{26} C^{13} - C^{13} A^{26})^T$

$= (A^{26} C^{13})^T - (C^{13} A^{26})^T$

$= (C^T)^{13} (A^T)^{26} - (A^T)^{26} (C^T)^{13}$

$= -C^{13}\,A^{26} - A^{26}\,(-C)^{13}$

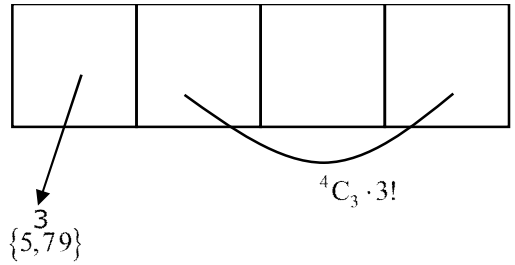
$= A^{26}\,C^{13} - C^{13}\,A^{26}$

$(S_2 \rightarrow \text{True})$

78. The number of numbers, strictly between 5000 and 10000 can be formed using the digits 1,3,5,7,9 without repetition, is

(1) 12 (2) 120 (3) 72 (4) 6

Sol. **(3)**



No. of ways = $3.4 \times 3! = 3.4! = 72$

79. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by

$f(x) = \log_{\sqrt{m}}\{\sqrt{2}(\sin x - \cos x) + m - 2\}$, for some m, such that the range of f is [0,2]. Then the value of m is

(1) 5 (2) 4 (3) 3 (4) 2

Sol. **(1)**

$\because -\sqrt{2} \le \sin x - \cos x \le \sqrt{2}$

$\Rightarrow -2 \le \sqrt{2}(\sin x - \cos x) \le 2$

$\Rightarrow m - 4 \le \sqrt{2}(\sin x - \cos x) + m - 2 \le m$

$\Rightarrow \log_{\sqrt{m}}^{(m-4)} \le \log_{\sqrt{m}}^{\{\sqrt{2}(\sin x - \cos x)+m-2\}} \le \log_{\sqrt{m}}^{m}$

$\Downarrow$

0

$\Rightarrow \log_{\sqrt{m}}^{(m-4)} = 0$

$\Rightarrow \boxed{m = 5}$

80. The shortest distance between the lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ is

(1) $\frac{3}{2}$ (2) 2 (3) $\frac{5}{2}$ (4) 3

Sol. **(2)**

$\frac{x+1}{1} = \frac{y}{\frac{1}{2}} = \frac{z}{\frac{-1}{12}}, \qquad \frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{\frac{1}{6}}$

$d = \left|\frac{(\vec{b} - \vec{a}) \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}\right|$

$\vec{a} = (-1,0,0), \qquad \vec{b} = (0,-2,1)$

$$\vec{p}=\left(1,\frac{1}{2},\frac{-1}{12}\right), \qquad \vec{q}=\left(1,1,\frac{1}{6}\right)$$

$$\vec{b}-\vec{a}=\hat{i}-2\hat{j}+\hat{k}$$

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \frac{1}{2} & -\frac{1}{12}\\ 1 & 1 & \frac{1}{6}\end{vmatrix}$$

$$=\hat{i}\left(\frac{1}{12}+\frac{1}{12}\right)-\hat{j}\left(\frac{1}{6}+\frac{1}{12}\right)+\hat{k}\left(1-\frac{1}{2}\right)$$

$$=\frac{\hat{i}}{6}-\frac{\hat{j}}{4}+\frac{\hat{k}}{2}$$

$$\left|\vec{p}\times\vec{q}\right|=\sqrt{\frac{1}{36}+\frac{1}{16}+\frac{1}{4}}=\frac{7}{12}$$

$$d=\left|\frac{\left(\hat{i}-2\hat{j}+\hat{k}\right)\cdot\left(\frac{\hat{i}}{6}-\frac{\hat{j}}{4}+\frac{\hat{k}}{2}\right)}{\frac{7}{12}}\right|$$

$$d=\left|\frac{\frac{1}{6}+\frac{1}{2}+\frac{1}{2}}{\frac{7}{12}}\right|=\frac{\frac{7}{6}}{\frac{7}{12}}=2$$

SECTION - B

81. 25% of the population are smokers. A smoker has 27 times more chances to develop lung cancer than a non smoker. A person is diagnosed with lung cancer and the probability that this person is a smoker is $\frac{k}{10}$. Then the value of k is.

Sol. **9**

$P(\text{smoker}) = \frac{1}{4}$

$P(\text{non smoker}) = \frac{3}{4}$

Probability that a smoker has lung cancer

$P\left(\frac{C}{S}\right) = 27\ P\left(\frac{C}{NS}\right)$

Probability that a person is smoker when he has lung cancer

$$= \frac{P(S)\cdot P\left(\frac{C}{S}\right)}{P(S)\cdot P\left(\frac{C}{S}\right) + P(NS)\cdot P\left(\frac{C}{NS}\right)}$$

$$= \frac{\frac{1}{4}\times P\left(\frac{C}{S}\right)}{\frac{1}{4}\times P\left(\frac{C}{S}\right) + \frac{3}{4}P\left(\frac{C}{NS}\right)}$$

$$= \frac{\frac{1}{4}\times 27P\left(\frac{C}{NS}\right)}{\frac{1}{4}\times 27P\left(\frac{C}{NS}\right) + \frac{3}{4}P\left(\frac{C}{NS}\right)}$$

$\frac{27}{30} = \frac{k}{10}$

$\boxed{k = 9}$

82. The remainder when $(2023)^{2023}$ is divided by 35 is

Sol. **7**

$2023 = 289 \times 7$

2023 is a multiple of 7

$n = (2023)^{2023}$ is multiple of 7

and $(2023)^{2023} = (-2)^{2023} = -2(2^2)^{1011}$

$= -2(5-1)^{1011}$

$= -2\left[{}^5C_0 5^{1011} - {}^5C_1 5^{1010} + ... - {}^{1011}C_{1011}\right]$

$(2023)^{2023}$ when divided by 5

gives remainder 2

If $n=(2023)^{2023}$ divided by $35 = 7 \times 5$

$n = 7k$

$n - 7 = 7(k-1) \rightarrow n\ 7$ is multiple of 7

and $n = 5m + 2$

so $n - 7 = 5m - 5$ = multiple of 5

so $n - 7$ is multiple of 35 so when n is divided by 35, reminder = 7

83. Let $a \in \mathbb{R}$ and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$

If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is

Sol. **(45)**

$\alpha + \beta = -60^{\frac{1}{4}}$ and $\alpha\beta = a$

$\alpha^2 + \beta^2 = 60^{\frac{1}{2}} - 2a$

$\alpha^4 + \beta^4 + 2\alpha^2\beta^2 = 60\ 4a^2 - 4a \cdot 60^{\frac{1}{2}}$

$-30 + 2a^2 = 60 + 4a^2 - 4a\sqrt{60}$

$a^2 - 2a\sqrt{60} + 45 = 0$

$\boxed{\text{Product} = 45}$

84. For the two positive numbers a, b is a, b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}$, 10 and $\frac{1}{b}$ are in an arithmetic progression, then 16a + b is equal to

Sol. **(3)**

$$b^2 = \frac{a}{18}$$

$$20 = \frac{1}{a} + \frac{1}{b}$$

$$a = \frac{b}{20b-1}$$

$$b^2 = \frac{1}{18} \times \frac{b}{20b-1}$$

$360b^2 - 18b - 1 = 0$

$360b^2 - 30b + 12b - 1 = 0$

$(12b - 1)(30b + 1) = 0$

$b = \frac{1}{12}, \frac{-1}{30}$ (rejected)

$$a = \frac{1}{8}$$

$16a + 12b = 2 + 1 = 3$

85. If m and n respectively are the numbers of positive and negative values of q in the interval [–p, p] that satisfy the equation $\cos 2\theta\cos\frac{\theta}{2} = \cos 3\theta\cos\frac{9\theta}{2}$, then mn is equal to

Sol. **25**

$$2\cos 2\theta\cos\frac{\theta}{2} = 2\cos 3\theta\cos\frac{9\theta}{2}$$

$$\cos\frac{5\theta}{2} + \cos\frac{3\theta}{2} = \cos\frac{15\theta}{2} + \cos\frac{3\theta}{2}$$

$$\cos\frac{5\theta}{2} - \cos\frac{15\theta}{2} = 0$$

$$\sin 5\theta = 0 \text{ or } \sin\frac{5\theta}{2} = 0$$

$$\theta = \frac{n\pi}{5} \text{ or } \frac{2n\pi}{5}$$

$$\theta = 0, \pm\frac{\pi}{5}, \pm\frac{2\pi}{5}, \pm\frac{3\pi}{5}, \pm\frac{4\pi}{5}, \pm\pi$$

m = n = 5

$$\boxed{mn = 25}$$

86. If the shortest distance between the line joining the points $(1,2,3)$ and $(2,3,4)$, and the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$ is a, then $28a^2$ is equal to

Sol. **18**

A (1, 2, 3) B (2, 3, 4)

Equation of line AB

$$\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$$

Given line

$$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$$

$$\text{shortest distance} = \left|\frac{\left(\vec{a}_2 - \vec{a}_1\right)\cdot\left(\vec{b}_2 \times \vec{b}_1\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right|$$

$$= \left|\frac{\left(3\hat{j} - \hat{k}\right)\cdot\left(\hat{i} + 2\hat{j} - 3\hat{k}\right)}{\sqrt{1+4+9}}\right|$$

$$\alpha = \frac{3}{\sqrt{14}}$$

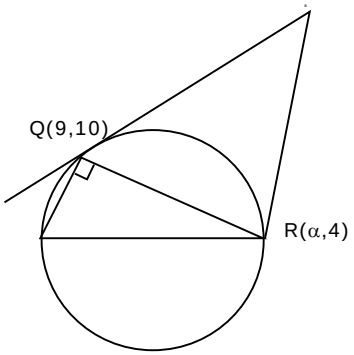
$$28\alpha^2 = 28 \times \frac{9}{14} = 18$$

87. Points P(–3,2),Q(9,10) and (a,4) lie on a circle C with PR as its diameter, The tangents to C at the points Q and R intersect at the point S. If S lies on the line $2x - ky = 1$, then k is equal to

Sol. **(3)**

Equation of circle is

$(x + 3)(x - \alpha) + (y - 2)(y - 4) = 0$



Q lies on it

$12(9 - \alpha) + 8 \times 6 = 0$

$\boxed{\alpha = 13}$

$x^2 + y^2 - 10x - 6y - 31 = 0$

Equation of Tangent at Q

$x.9 + y.10 - 5(x + 9) - 3(y + 10) - 31 = 0$

$4x + 7y = 106$ …….(1)

Equation of Tangent at R

$x.13 + y.4 - 5(x + 13) - 3(y + 4) - 31 = 0$

$8x + y = 108$ …….(2)

Solution (1) and (2)

$s = \left(\frac{25}{2}, 8\right)$

which lies on $2x - ky = 1$

$\boxed{k = 3}$

88. Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples and 8 oranges. If in the selected 5 fruits, at least 2 oranges, at least one red apple and at least one white apple must be given, then the number of ways, Anil's mother can offer 5 fruits to Anil is

Sol. **6860**

Three cases are possible

1R 1W 3O + 2R 1W 2O + 1R 2W 2O

$^{7}C_1 \cdot {}^{5}C_1 \cdot {}^{8}C_3 + {}^{7}C_2 \cdot {}^{5}C_1 \cdot {}^{8}C_2 + {}^{7}C_1 \cdot {}^{5}C_2 \cdot {}^{8}C_2$

= 6860

89. If $\int_{\frac{1}{3}}^{3}|\log_e x|dx = \frac{m}{n}\log_e\left(\frac{n^2}{e}\right)$, where m and n are coprime natural numbers, then $m^2 + n^2 - 5$ is equal to

Sol. **20**

$$\int_{\frac{1}{3}}^{3}|\log_e x|dx$$

$$= \int_{\frac{1}{3}}^{1}(-\ln x)dx + \int_{1}^{3}(\ln x)dx$$

$$-\left[x\ln x - x\right]_{\frac{1}{3}}^{1} + \left[x\ln x - x\right]_{1}^{3}$$

$$= \frac{4}{3}\ln\left(\frac{9}{e}\right) = \frac{m}{n}\ln\left(\frac{n^2}{e}\right)$$

m = 4 and n = 3

so $m^2 + n^2 - 5 = 16 + 9 - 5 = 20$

90. A triangle is formed by X- axis, Y-axis and the line 3x + 4y = 4y = 60. Then the number of points P(a, b) which lie strictly inside the triangle, where a is an integer and b is a multiple of a, is

Sol. **31**

3x + 4y = 60

x = 1, 4y = 57, y=14.2

x = 1, y = 1, 2, 3,…….14 $\rightarrow$ 14 points

x = 2, 4y = 54, y=13.5

x = 2, y = 2, 4, 6, 8, 10, 12 $\rightarrow$ 6 points

x = 3, y = 3, 6, 9, 12 $\rightarrow$ 4 points

x = 4, y = 4, 8 $\rightarrow$ 2 points

x = 5, y = 5, 10 $\rightarrow$ 2 points

x = 6, y = 6 $\rightarrow$ 1 points

x = 7, y = 7 $\rightarrow$ 1 points

x = 8, y = 8 $\rightarrow$ 1 points

x = 9, 4y = 23, y=5.7 × no point

Total points = 14 + 6 + 4 + 2 + 2 + 1 + 1 + 1 = 31