08/04/2019
Morning

# Answers & Solutions

*for*

# JEE (MAIN)-2019 (Online) Phase-2

## (Physics, Chemistry and Mathematics)

**Time : 3 hrs.** **M.M. : 360**

### Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. *¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question.

# PART–A : PHYSICS

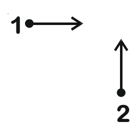
1. Two particles move at right angle to each other. Their de Broglie wavelengths are $\lambda_1$ and $\lambda_2$ respectively. The particles suffer perfectly inelastic collision. The de Broglie wavelength $\lambda$, of the final particle, is given by:

(1) $\lambda = \sqrt{\lambda_1 \lambda_2}$  (2) $\lambda = \frac{\lambda_1 + \lambda_2}{2}$

(3) $\frac{2}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$  (4) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$

**Answer (4)**

**Sol.**




$$p_1 = \frac{h}{\lambda_1}$$

$$p_2 = \frac{h}{\lambda_2}$$

$$\therefore \quad p_f = \sqrt{p_1^2 + p_2^2}$$

$$\Rightarrow \quad \frac{h}{\lambda} = \sqrt{\frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}}$$

$$\Rightarrow \quad \frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

2. In SI units, the dimensions of $\sqrt{\frac{\epsilon_0}{\mu_0}}$ is:

(1) $AT^2M^{-1}L^{-1}$  (2) $AT^{-3}ML^{3/2}$

(3) $A^{-1}TML^3$  (4) $A^2T^3M^{-1}L^{-2}$

**Answer (4)**

**Sol.** $\left[\sqrt{\frac{\epsilon_0}{\mu_0}}\right] = \left[\frac{\epsilon_0}{\sqrt{\mu_0 \epsilon_0}}\right] = [LT^{-1}] \times [\epsilon_0]$

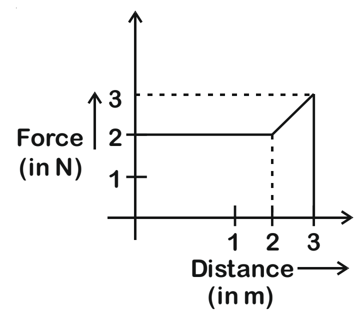
$$\because \quad F = \frac{q^2}{4\pi\epsilon_0 r^2}$$

$$\Rightarrow \quad [\epsilon_0] = \frac{[AT]^2}{[MLT^{-2}] \times [L^2]}$$

$$\therefore \quad \left[\sqrt{\frac{\epsilon_0}{\mu_0}}\right] = [LT^{-1}] \times [A^2M^{-1}L^{-3}T^4]$$

$$= [M^{-1}L^{-2}T^3A^2]$$

3. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is




(1) 4 J  (2) 2.5 J

(3) 5 J  (4) 6.5 J

**Answer (4)**

**Sol.** = Area under F–x graph

$$\Delta K.E = W = \frac{1}{2} \times (3+2) \times (3-2) + 2 \times 2$$

= 2.5 + 4

= 6.5 J

4. Radiation coming from transitions n = 2 to n = 1 of hydrogen atoms fall on $He^+$ ions in n = 1 and n = 2 states. The possible transition of helium ions as they absorb energy from the radiation is

(1) n = 2 $\rightarrow$ n = 4

(2) n = 2 $\rightarrow$ n = 5

(3) n = 2 $\rightarrow$ n = 3

(4) n = 1 $\rightarrow$ n = 4

**Answer (1)**

**Sol.** Energy released by hydrogen atom

$$\Delta E_1 = 13.6 \times \left(\frac{1}{1} - \frac{1}{4}\right) = \frac{3}{4} \times 13.6 \text{ eV}$$

= 10.2 eV

Also, energy absorbed by $H_e^+$ ion in transition n = 2 $\rightarrow$ n = 4

$$\Delta E_2 = 13.6 \times 4 \times \left(\frac{1}{4} - \frac{1}{16}\right) = 10.2 \text{ eV}$$

So, possible transition is n = 2 $\rightarrow$ n = 4

5. Water from a pipe is coming at a rate of 100 liters per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of : (density of water = 1000 kg/$m^3$, coefficient of viscosity of water = 1 mPa s)

(1) $10^2$ (2) $10^4$

(3) $10^3$ (4) $10^6$

**Answer (2)**

**Sol.** Flow rate of water (Q) = 100 lit/min

$$=\frac{100\times10^{-3}}{60}=\frac{5}{3}\times10^{-3}m^3$$

$\therefore$ Velocity of flow (v) = $\frac{Q}{A}=\frac{5\times10^{-3}}{3\times\pi\times(5\times10^{-2})^2}$

$$=\frac{10}{15\pi}=\frac{2}{3\pi}\text{ m/s}$$

= 0.2 m/s

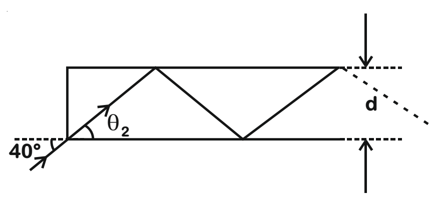
$\therefore$ Reynold number ($R_e$) $=\frac{Dv\rho}{\eta}$

$$=\frac{(10\times10^{-2})\times\frac{2}{3\pi}\times1000}{1}\simeq 2\times10^4$$

Order of $R_e = 10^4$

6. In figure, the optical fiber is l = 2 m long and has a diameter of d = 20 μm. If a ray of light is incident on one end of the fiber at angle $\theta_1$ = 40°, the number of reflections it makes before emerging from the other end is close to:

(refractive index of fiber is 1.31 and sin 40° = 0.64)




(1) 66000 (2) 55000

(3) 45000 (4) 57000

**Answer (4)**

**Sol.** 1 × sin 40° = 1.31 sinθ

$$\Rightarrow \sin\theta=\frac{0.64}{1.31}\Rightarrow \theta\simeq30^\circ$$

l = 20 μm × cotθ

$$\therefore N=\frac{2}{20\times10^{-6}\times\cot\theta}$$

$$=\frac{2\times10^6}{20\times\sqrt{3}}=57735$$

$N\approx57000$

7. A 200 Ω resistor has a certain color code. If one replaces the red color by green in the code, the new resistance will be:

(1) 400 Ω

(2) 500 Ω

(3) 300 Ω

(4) 100 Ω

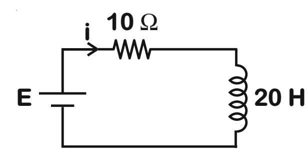
**Answer (2)**

**Sol.** 200 Ω = Red + Black + Brown

Green ≡ 5

So, Green + Black + Brown ≡ 500 Ω

8. A 20 Henry inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is:




(1) $\frac{2}{\ln 2}$

(2) 2 ln2

(3) ln2

(4) $\frac{1}{2}\ln 2$

**Answer (2)**

**Sol.** $P_R = i^2 \times R$

$P_B = V \times i$

$\therefore$ $P_L = Vi - i^2R$

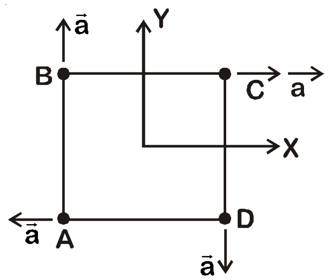
$\Rightarrow$ $Vi - i^2R = i^2R$

$$\Rightarrow i=\frac{V}{2R}\text{ and } i=\frac{V}{R}\cdot\left(1-e^{-t/\tau}\right)$$

$$\therefore \frac{V}{2R}=\frac{V}{R}\left(1-e^{-t/\tau}\right)$$

$$\Rightarrow t=\tau\ln(2)=\frac{20}{10}\ln(2)=2\ln(2)$$

9. Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is :




(1) Zero
(2) $a(\hat{i}+\hat{j})$
(3) $\frac{a}{5}(\hat{i}+\hat{j})$
(4) $\frac{a}{5}(\hat{i}-\hat{j})$

**Answer (4)**

**Sol.** $a_{CM} = \dfrac{(2m)a\hat{j} + 3m \times a\hat{i} + ma(-\hat{i}) + 4m \times a(-\hat{j})}{2m + 3m + 4m + m}$

$$= \frac{2a\hat{i} - 2a\hat{j}}{10} = \frac{a}{5}(\hat{i} - \hat{j})$$

10. A plane electromagnetic wave travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is $E = 6\ Vm^{-1}$ along y-direction. Its corresponding magnetic field component, B would be :

(1) $2 \times 10^{-8}$ T along y-direction
(2) $6 \times 10^{-8}$ T along z-direction
(3) $2 \times 10^{-8}$ T along z-direction
(4) $6 \times 10^{-8}$ T along x-direction

**Answer (3)**

**Sol.** $B_0 = \dfrac{E_0}{c} = \dfrac{6}{3 \times 10^8} = 2 \times 10^{-8}\,T$

Propagation direction = $\hat{E} \times \hat{B}$

$\hat{i} = \hat{j} \times \hat{B}$

$\Rightarrow \quad \hat{B} = \hat{k}$

11. Voltage rating of a parallel plate capacitor is 500 V. Its dielectric can withstand a maximum electric field of $10^6$ V/m. The plate area is $10^{-4}$ m$^2$. What is the dielectric constant if the capacitance is 15 pF?

(given $\varepsilon_0 = 8.86 \times 10^{-12}\ C^2/Nm^2$)

(1) 3.8
(2) 4.5
(3) 8.5
(4) 6.2

**Answer (3)**

**Sol.** $C = \dfrac{k \epsilon_0 A}{d}$

$$E = \frac{V}{d}$$

$$15 \times 10^{-12} = \frac{k \times 8.86 \times 10^{-12} \times 10^{-4} \times 10^{6}}{500}$$

k = 8.5

12. The wavelength of the carrier waves in a modern optical fiber communication network is close to :

(1) 600 nm
(2) 900 nm
(3) 1500 nm
(4) 2400 nm

**Answer (3)**

**Sol.** Fact Based

Wavelength of carrier waves in modern optical fiber communication is most widely used near about 1500 nm.

13. A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that $g = 3.1\,\pi\ ms^{-2}$, what will be the tensile stress that would be developed in the wire?

(1) $4.8 \times 10^6\ Nm^{-2}$
(2) $3.1 \times 10^6\ Nm^{-2}$
(3) $5.2 \times 10^6\ Nm^{-2}$
(4) $6.2 \times 10^6\ Nm^{-2}$

**Answer (2)**

**Sol.** $\text{Stress} = \dfrac{F}{A} = \dfrac{4 \times 3.1\pi}{\pi \times (2 \times 10^{-3})^2} = 3.1 \times 10^6\ N/m^2$

14. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with $\rho_0$ as constant and r is the distance from its center. The moment of inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = a MR^2$. The value of the coefficient a is

(1) $\frac{3}{2}$
(2) $\frac{1}{2}$
(3) $\frac{3}{5}$
(4) $\frac{8}{5}$

**Answer (4)**

**Sol.** $M=\int_0^R \rho_0 r\times 2\pi r dr=\frac{2\pi\rho_0 R^3}{3}$

$$I_C=\int_0^R \rho_0 r\times 2\pi r dr\times r^2=\frac{2\pi\rho_0 R^5}{5}$$

$$\therefore\ I=I_C+MR^2=2\pi\rho_0 R^5\left(\frac{1}{3}+\frac{1}{5}\right)=\frac{16\pi\rho_0 R^5}{15}$$

$$=\frac{8}{5}\left[\frac{2}{3}\pi\rho_0 R^3\right]R^2=\frac{8}{5}MR^2$$

15. An alternating voltage v(t) = 220 sin $100\pi t$ volt is applied to a purely resistive load of 50 $\Omega$. The time taken for the current to rise from half of the peak value to the peak value is

(1) 2.2 ms (2) 7.2 ms

(3) 5 ms (4) 3.3 ms

**Answer (4)**

**Sol.** $I = I_m \sin(100\pi t)$

$$\Rightarrow \frac{I_m}{2}=I_m\sin(100\pi t_1)$$

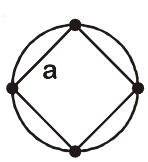
$$\Rightarrow \frac{\pi}{6}=100\pi t_1$$

$$\Rightarrow t_1=\frac{1}{600}\text{ s}$$

$$T=\frac{2\pi}{100\pi}=\frac{1}{50}\text{ s}$$

$$\therefore\ t_{req}=\frac{T}{4}-t_1=\frac{1}{200}-\frac{1}{600}=\frac{2}{600}=\frac{1}{300}\text{ s}=3.3\text{ ms}$$

16. Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of others' gravitational field in a circular orbit circumscribing the square?

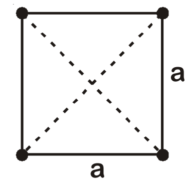



(1) $1.41\sqrt{\frac{GM}{a}}$ (2) $1.16\sqrt{\frac{GM}{a}}$

(3) $1.21\sqrt{\frac{GM}{a}}$ (4) $1.35\sqrt{\frac{GM}{a}}$

**Answer (2)**

**Sol.** $r=\frac{a}{\sqrt{2}}, F=\frac{GM^2}{a^2}\left(\sqrt{2}\right)+\frac{GM^2}{2a^2}$




$$\therefore\ \frac{Mv^2}{\left(\frac{a}{\sqrt{2}}\right)}=\frac{GM^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

$$\Rightarrow v^2=\frac{GM}{a}\left(1+\frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow v=\sqrt{\frac{GM}{a}\left(1+\frac{1}{2\sqrt{2}}\right)}=1.16\sqrt{\frac{GM}{a}}$$

17. The bob of a simple pendulum has mass 2 g and a charge of 5.0 $\mu$C. It is at rest in a uniform horizontal electric field of intensity 2000 V/m. At equilibrium, the angle that the pendulum makes with the vertical is

(take g = 10 m/s$^2$)

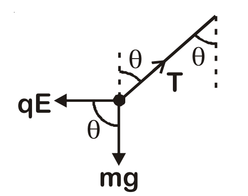
(1) $\tan^{-1}$ (0.2) (2) $\tan^{-1}$ (5.0)

(3) $\tan^{-1}$ (2.0) (4) $\tan^{-1}$ (0.5)

**Answer (4)**

**Sol.** $T\cos\theta = mg$

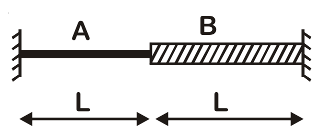
$T\sin\theta = qE$




$$\tan\theta=\frac{qE}{mg}$$

$$\tan\theta=\frac{5\times10^{-6}\times2000}{2\times10^{-3}\times10}=\frac{1}{2}$$

$$\Rightarrow \tan^{-1}\left(\frac{1}{2}\right)$$

18. 


A wire of length 2L, is made by joining two wires A and B of same length but different radii r and 2r and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q then the ratio p : q is

(1) 4 : 9 (2) 1 : 2

(3) 3 : 5 (4) 1 : 4

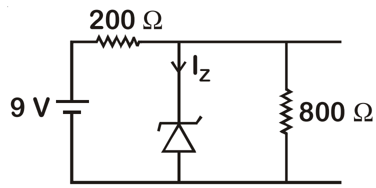
**Answer (2)**

**Sol.** $\dfrac{V_A}{V_B}=\sqrt{\dfrac{u_B}{u_A}}=\dfrac{r_B}{r_A}=2=\dfrac{\lambda_A}{\lambda_B}$

$\Rightarrow\ \lambda_A = 2\lambda_B$

$\Rightarrow\ \dfrac{p}{q}=\dfrac{1}{2}$

19. The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit.



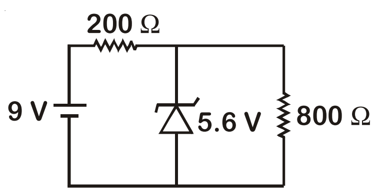

The current $I_Z$ through the Zener is

(1) 15 mA (2) 7 mA

(3) 10 mA (4) 17 mA

**Answer (3)**

**Sol.**

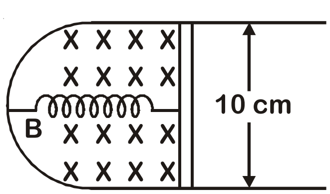



$I_{800\,\Omega}=\dfrac{5.6}{800}\text{ A}=7\text{ mA}$

$I_{200\,\Omega}=\dfrac{9-5.6}{200}=17\text{ mA}$

$\therefore\ I_Z = 17 - 7 = 10$ mA

20. A thin strip 10 cm long is on a U shaped wire of negligible resistance and it is connected to a spring of spring constant 0.5 $Nm^{-1}$(see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillations it performs before its amplitude decreases by a factor of e is N. If the mass of the strip is 50 grams, its resistance 10 Ω and air drag negligible, N will be close to




(1) 1000 (2) 5000

(3) 50000 (4) 10000

**Answer (2)**

**Sol.** $F=-kx-ilB=-kx-\dfrac{Blv}{R}\times lB$

$F=-kx-\dfrac{B^2l^2}{R}\times v$

So, it is case of damped oscillation

$\Rightarrow\ A=A_0e^{-\frac{bt}{2m}}$

$\Rightarrow\ \dfrac{A_0}{e}=A_0e^{-\frac{bt}{2m}}$

$\Rightarrow\ t=\dfrac{2m}{\left(\dfrac{B^2l^2}{R}\right)}=\dfrac{2\times50\times10^{-3}\times10}{0.01\times0.01}=10000\text{ s}$

Time period, $T=2\pi\sqrt{\dfrac{m}{k}}\simeq 2\text{ s}$

$\therefore$ Number, $N=\dfrac{10000}{2}=5000$

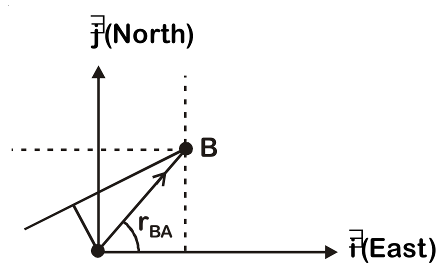
21. Ship A is sailing towards north-east with velocity $\vec{v}=30\hat{i}+50\hat{j}\text{ km/hr}$ where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in:

(1) 2.2 hrs. (2) 4.2 hrs.

(3) 3.2 hrs. (4) 2.6 hrs.

**Answer (4)**

**Sol.**




$\vec{v}_A=30i+50j\text{ km/hr}$

$\vec{v}_{BA}=(80\hat{i}+150\hat{j})\text{ km}$

$\vec{v}_B=(-10i)\text{ km/hr}$

$\vec{v}_{BA}=\vec{v}_B-\vec{v}_A=-10\hat{i}-30\hat{i}-50\hat{i}=-40\hat{i}-50\hat{j}$

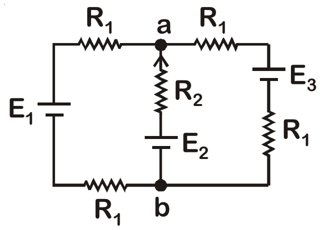
Projection of

$(\vec{r}_{BA})$ on $\vec{v}_{BA}=\dfrac{(\vec{r}_{BA})\cdot(\vec{v}_{BA})}{(\vec{v}_{BA})}$

$=\dfrac{(80\hat{i}+150\hat{j})(-40\hat{i}-50\hat{j})}{10\sqrt{41}}=\dfrac{\sqrt{mk}}{qB}$

$\therefore\ t=\dfrac{10\times107}{\sqrt{41}\times10\sqrt{41}}=\dfrac{107}{41}=2.6\text{ Hrs.}$

22. For the circuit shown, with $R_1$ = 1.0 Ω, $R_2$ = 2.0 Ω, $E_1$ = 2 V and $E_2$ = $E_3$ = 4 V, the potential difference between the points 'a' and 'b' is approximately (in V)




(1) 2.7 (2) 3.7

(3) 2.3 (4) 3.3

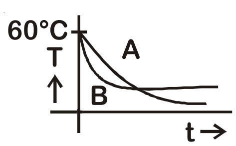
**Answer (4)**

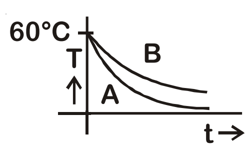
**Sol.** $v = \dfrac{E_1r_2r_3 + E_2r_3r_1 + E_3r_1r_2}{r_1r_2 + r_2r_3 + r_3r_1}$

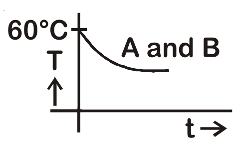
$= \dfrac{2(2\times2) + 4(2\times2) + 4(2\times2)}{4+4+4}$

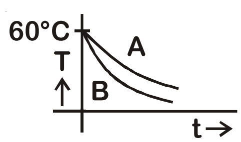
$= \dfrac{40}{12} = \dfrac{10}{3} = 3.3\text{ Volt}$

23. Two identical beakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of 8 × $10^2$ kg/m$^3$ and specific heat of 2000 Jkg$^{-1}$K$^{-1}$ while liquid in B has density of $10^3$ kgm$^{-3}$ and specific heat of 4000 Jkg$^{-1}$K$^{-1}$. Which of the following best describes their temperature versus time graph schematically? (assume the emissivity of both the beakers to be the same)

(1) 


(2) 


(3) 


(4) 


**Answer (2)**

**Sol.** $mS\left(-\dfrac{dT}{dt}\right) = e\sigma AT^4$

$-\dfrac{dT}{dt} = \dfrac{e\sigma\times A\times T^4}{\rho\times Vol.\times S}$

$\dfrac{\left(-\dfrac{dT}{dt}\right)_A}{\left(-\dfrac{dT}{dt}\right)_B} = \dfrac{\rho_B}{\rho_A}\times(2) > 1$

So, A cools down at faster rate.

24. A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of –4 Q, the new potential difference between the same two surfaces is:

(1) –2 V (2) V

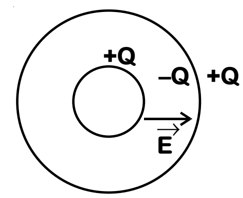
(3) 2 V (4) 4 V

**Answer (2)**

**Sol.** In case-1

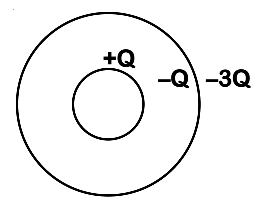
Electric field between spherical surface

$\vec{E} = \dfrac{KQ}{r^2}$




In case-2

Electric field between surfaces remain unchanged.




∴ Potential difference between them remain unchanged too.

25. A circular coil having N turns and radius r carries a current I. It is held in the XZ plane in a magnetic field $B\hat{i}$. The torque on the coil due to the magnetic field is:

(1) $\dfrac{B\pi r^2 I}{N}$ (2) $\dfrac{Br^2 I}{\pi N}$

(3) $B\pi r^2 I N$ (4) Zero

**Answer (3)**

**Sol.** $|\vec{\tau}| = |\vec{\mu}\times\vec{B}|$

= NIA × B [A = $\pi r^2$]

⇒ $\tau = NI\pi r^2 B$

26. An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be:

(1) 20 cm from the convergent mirror, twice the size of the object

(2) 20 cm from the convergent mirror, same size as the object

(3) 40 cm from the convergent lens, twice the size of the object

(4) 40 cm from the convergent mirror, same size as the object

**Answer (Bonus)**

**Sol.** $v_1 = \frac{40 \times 20}{(40-20)} = 40\text{ cm}$

$u_2 = 60 - 40 = 20$ cm

$\therefore \quad v_2 = \frac{20 \times 10}{(20-10)} = 20\text{ cm}$

$\therefore$ Image traces back to object itself as image formed by lens is a centre of curvature of mirror.

27. If $10^{22}$ gas molecules each of mass $10^{-26}$ kg collide with a surface (perpendicular to it) elastically per second over an area 1 $m^2$ with a speed $10^4$ m/s, the pressure exerted by the gas molecules will be of the order of:

(1) $10^8$ N/m$^2$ (2) $10^3$ N/m$^2$

(3) $10^{16}$ N/m$^2$ (4) $10^4$ N/m$^2$

**Answer (Bonus)**

**Sol.** $P = \frac{N \times (2mv)}{\Delta t \times A}$

$= \frac{10^{22} \times 2 \times 10^{-26} \times 10^4}{1 \times 1}$ = 2 N/m$^2$

28. A thermally insulated vessel contains 150 g of water at 0°C. Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closest to:

(Latent heat of vaporization of water = 2.10 × $10^6$ J kg$^{-1}$ and Latent heat of Fusion of water = 3.36 × $10^5$ J kg$^{-1}$)

(1) 130 g (2) 150 g

(3) 20 g (4) 35 g

**Answer (3)**

**Sol.** Let amount of water evaporated be m gram.

$\therefore \quad m \times L_v = (150 - m) \times L_s$

$m \times 540 = (150 - m) \times 80$

$\Rightarrow \quad m \simeq 20$ g

29. In an interference experiment the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2} = \frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be:

(1) 4 (2) 18

(3) 9 (4) 2

**Answer (1)**

**Sol.** $\frac{a_1}{a_2} = \frac{3}{1}$

$\therefore \quad \frac{I_{max}}{I_{min}} = \left(\frac{a_2 + a_1}{a_2 - a_1}\right)^2 = \left(\frac{3+1}{3-1}\right)^2 = 4$

30. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released, the stone flies off with a velocity of 20 ms$^{-1}$. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to:

(1) $10^4$ Nm$^{-2}$ (2) $10^3$ Nm$^{-2}$

(3) $10^8$ Nm$^{-2}$ (4) $10^6$ Nm$^{-2}$

**Answer (4)**

**Sol.** $\frac{1}{2} \cdot \left(\frac{YA}{L}\right)(\Delta l)^2 = \frac{1}{2} mv^2$

$\Rightarrow \quad Y = \frac{mv^2 L}{A(\Delta l)^2}$

$= \frac{0.02 \times 400 \times 0.42 \times 4}{\pi \times 36 \times 10^{-6} \times 0.04}$

$= 2.3 \times 10^6$ N/m$^2$

So, order is $10^6$.

# PART–B : CHEMISTRY

1. With respect to an ore, Ellingham diagram helps to predict the feasibility of its

   (1) Zone refining

   (2) Vapour phase refining

   (3) Thermal reduction

   (4) Electrolysis

**Answer (3)**

**Sol.** Ellingham diagram is used to select reducing agent so it help to predict feasibility of its thermal reduction.

2. Which one of the following equations does not correctly represent the first law of thermodynamics for the given processes involving an ideal gas? (Assume non-expansion work is zero)

   (1) Isothermal process : $q = -w$

   (2) Cyclic process : $q = -w$

   (3) Isochoric process : $\Delta U = q$

   (4) Adiabatic process : $\Delta U = -w$

**Answer (4)**

**Sol.** $\Delta U = q + W$

Adiabatic process $q = 0$

$\Delta U = W$

For isothermal, $\Delta U = 0$

For cyclic, $\Delta U = 0$

For isochoric, $W = 0$

3. The quantum number of four electrons are given below:

   (I) $n = 4, l = 2, m_l = -2, m_s = -\frac{1}{2}$

   (II) $n = 3, l = 2, m_l = 1, m_s = +\frac{1}{2}$

   (III) $n = 4, l = 1, m_l = 0, m_s = +\frac{1}{2}$

   (IV) $n = 3, l = 1, m_l = 1, m_s = -\frac{1}{2}$

   The correct order of their increasing energies will be:

   (1) IV < II < III < I

   (2) I < III < II < IV

   (3) IV < III < II < I

   (4) I < II < III < IV

**Answer (1)**

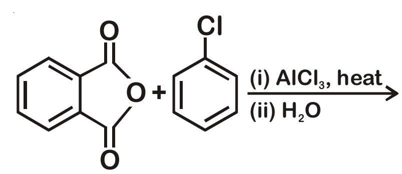
**Sol.**

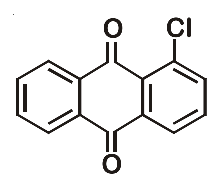
| | | | | n + l |
|---|---|---|---|---|
| (I) | n = 4 | l = 2 | 4d | 6 |
| (II) | n = 3 | l = 2 | 3d | 5 |
| (III) | n = 4 | l = 1 | 4p | 5 |
| (IV) | n = 3 | l = 1 | 3p | 4 |

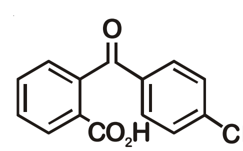
more is n + l value, more is energy

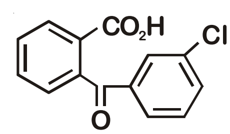
3p < 3d < 4p < 4d

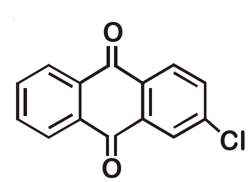
4. The major product of the following reaction is :



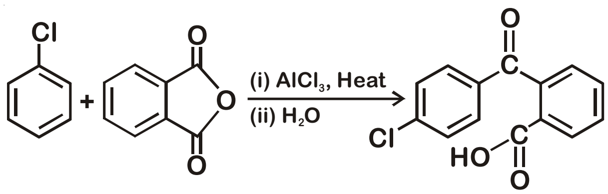

(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

5. The correct order of the spin-only magnetic moment of metal ions in the following low-spin complexes, $[V(CN)_6]^{4-}$, $[Fe(CN)_6]^{4-}$, $[Ru(NH_3)_6]^{3+}$ and $[Cr(NH_3)_6]^{2+}$, is:

(1) $V^{2+} > Cr^{2+} > Ru^{3+} > Fe^{2+}$

(2) $Cr^{2+} > V^{2+} > Ru^{3+} > Fe^{2+}$

(3) $V^{2+} > Ru^{3+} > Cr^{2+} > Fe^{2+}$

(4) $Cr^{2+} > Ru^{3+} > Fe^{2+} > V^{2+}$

Answer (1)

Sol.

| | | No. of unpaired electrons |
|---|---|---|
| $[V(CN)_6]^{4-}$ | $V^{+2}$ | 3 |
| $[Ru(NH_3)_6]^{3+}$ | $Ru^{+3}$ | 1 |
| $[Fe(CN)_6]^{4-}$ | $Fe^{+2}$ | 0 |
| $[Cr(NH_3)_6]^{2+}$ | $Cr^{+2}$ | 2 |

$\therefore$ Order of spin magnetic moment

$V^{2+} > Cr^{2+} > Ru^{3+} > Fe^{2+}$

6. Which is wrong with respect to our responsibility as a human being to protect our environment?

(1) Using plastic bags

(2) Restricting the use of vehicles

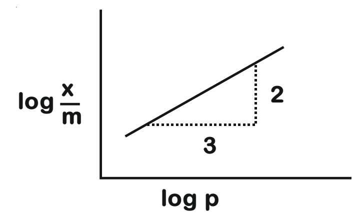
(3) Avoiding the use of floodlighted facilities

(4) Setting up compost tin in gardens

Answer (1)

Sol. Use of plastic bags is hazardous to our environment

7. Adsorption of a gas follows Freundlich adsorption isotherm. x is the mass of the gas adsorbed on mass m of the adsorbent. The plot of $\log \frac{x}{m}$ versus log p is shown in the given graph. $\frac{x}{m}$ is proportional to




(1) $p^{3/2}$ (2) $p^3$

(3) $p^{2/3}$ (4) $p^2$

Answer (3)

Sol. $\frac{x}{m} \propto p^{\frac{1}{n}}$ $\quad \frac{x}{m} = kp^{\frac{1}{n}}$

$$\text{Slope} = \frac{2}{3}$$

$$\log\frac{x}{m} = \log k + \frac{1}{n}\log p$$

$$\text{Slope} = \frac{1}{n} = \frac{2}{3}$$

$$\frac{x}{m} \propto p^{\frac{2}{3}}$$

8. Given that $E^{\ominus}_{O_2/H_2O} = +1.23\ V$;

$E^{\ominus}_{S_2O_8^{2-}/SO_4^{2-}} = 2.05\ V$

$E^{\ominus}_{Br_2/Br^-} = +1.09\ V$;

$E^{\ominus}_{Au^{3+}/Au} = +1.4\ V$

The strongest oxidizing agent is

(1) $Br_2$

(2) $Au^{3+}$

(3) $S_2O_8^{2-}$

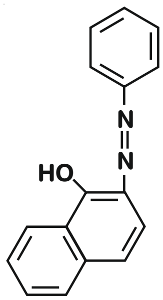
(4) $O_2$

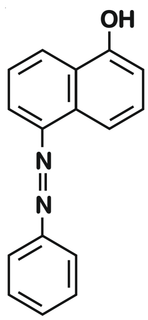
Answer (3)

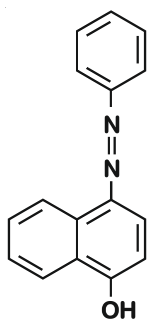
Sol. More positive is the reduction potential stronger is the oxidising agent.

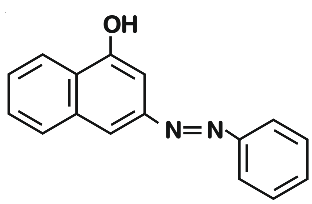
Reduction potential is maximum for $S_2O_8^{2-}$

9. Coupling of benzene diazonium chloride with 1-naphthol in alkaline medium will give

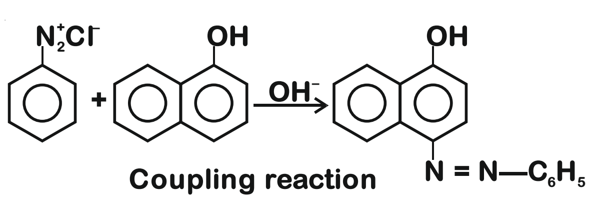
(1) 


(2) 


(3) 


(4) 


**Answer (3)**

**Sol.** 


10. Maltose on treatment with dilute HCl gives

(1) D-Galactose

(2) D-Glucose and D-Fructose

(3) D-Glucose

(4) D-Fructose

**Answer (3)**

**Sol.** Hydrolysis of maltose give glucose as maltose is composed of two $\alpha$-D glucose units.

11. The vapour pressures of pure liquids A and B are 400 and 600 mmHg, respectively at 298 K. On mixing the two liquids, the sum of their initial volumes is equal to the volume of the final mixture. The mole fraction of liquid B is 0.5 in the mixture. The vapour pressure of the final solution, the mole fractions of components A and B in vapour phase, respectively are :

(1) 500 mmHg, 0.4, 0.6

(2) 500 mmHg, 0.5, 0.5

(3) 450 mmHg, 0.4, 0.6

(4) 450 mmHg, 0.5, 0.5

**Answer (1)**

**Sol.** $P = x_B p_B^\circ + x_A p_A^\circ$

$= 0.5 \times 600 + 0.5 \times 400 = 300 + 200 = 500$

$p_B = y_B P_{Total}$

$$y_B = \frac{p_B}{P_{Total}} = \frac{300}{500} = \frac{3}{5} = 0.6$$

$$y_A = \frac{p_A}{P_{Total}} = \frac{200}{500} = \frac{2}{5} = 0.4$$

12. For the reaction $2A + B \rightarrow C$, the values of initial rate at different reactant concentrations are given in the table below. The rate law for the reaction is

| [A](mol $L^{-1}$) | [B](mol $L^{-1}$) | Initial Rate (mol $L^{-1}s^{-1}$) |
|---|---|---|
| 0.05 | 0.05 | 0.045 |
| 0.10 | 0.05 | 0.090 |
| 0.20 | 0.10 | 0.72 |

(1) Rate = $k[A]^2[B]^2$ (2) Rate = $k[A][B]$

(3) Rate = $k[A]^2[B]$ (4) Rate = $k[A][B]^2$

**Answer (4)**

**Sol.** $2A + B \longrightarrow P$

Rate = $k[A]^x [B]^y$

Exp-1, $0.045 = k[0.05]^x [0.05]^y$ ...(i)

Exp-2, $0.090 = k[0.1]^x [0.05]^y$ ...(ii)

Exp-3, $0.72 = k[0.2]^x [0.1]^y$ ...(iii)

Divide equation (i) by equation (ii)

$$\frac{0.045}{0.090} = \left(\frac{1}{2}\right)^x \Rightarrow x = 1$$

Divide equation (i) by equation (iii)

$$\frac{0.045}{0.72} = \left(\frac{0.05}{0.1}\right)^y \left(\frac{0.05}{0.2}\right)^1$$

$$\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^y \Rightarrow y = 2$$

Rate law = $k[A]^1 [B]^2$.

13. For silver, $C_p(JK^{-1}\ mol^{-1}) = 23 + 0.01T$. If the temperature (T) of 3 moles of silver is raised from 300 K to 1000 K at 1 atm pressure, the value of $\Delta H$ will be close to

(1) 21 kJ (2) 13 kJ

(3) 62 kJ (4) 16 kJ

**Answer (3)**

**Sol.** n = 3

$T_1 = 300$

$T_2 = 1000$

$C_p = 23 + 0.01T$

$$\Delta H = \int_{T_1}^{T_2} nC_p dT$$

$$= n \int_{300}^{1000} (23 + 0.01T)dT$$

$$= 3\left[23T + \frac{0.01T^2}{2}\right]_{300}^{1000}$$

$= 3[16100 + 4550]$

$= 3 \times 20650 = 61950$ J

$= 61.95$ kJ

$\approx 62$

14. The lanthanide ion that would show colour is

(1) $Gd^{3+}$ (2) $Lu^{3+}$

(3) $La^{3+}$ (4) $Sm^{3+}$

**Answer (4)**

**Sol.** $Sm^{+3}$ = Partially filled f orbital = $4f^5$

$Sm = 4f^6 6s^2$

$Sm^{+3}$ = Yellow.

$Lu^{+3} = 4f^{14}$ colourless.

15. The correct order of hydration enthalpies of alkali metal ions is

(1) $Na^+ > Li^+ > K^+ > Rb^+ > Cs^+$

(2) $Li^+ > Na^+ > K^+ > Cs^+ > Rb^+$

(3) $Na^+ > Li^+ > K^+ > Cs^+ > Rb^+$

(4) $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$

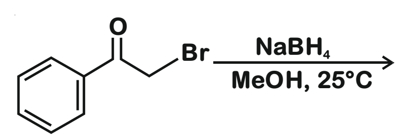
**Answer (4)**

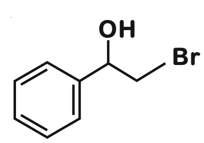
**Sol.** Smaller is size more is hydration energy.

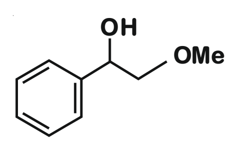
$Li^+ < Na^+ < K^+ < Rb^+ < Cs^+$ Size

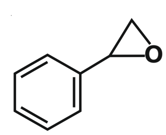
$Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$ Hydration energy

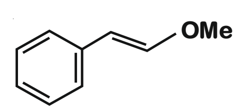
16. The major product of the following reaction is



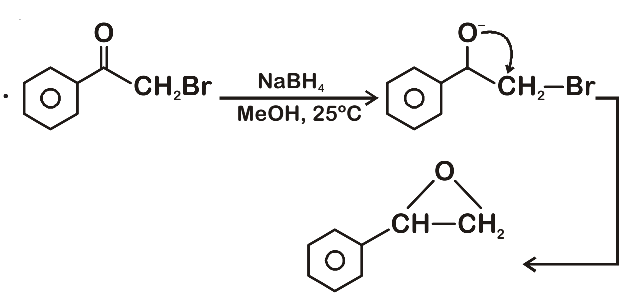

(1) 


(2) 


(3) 


(4) 


**Answer (3)**

**Sol.** 

17. Element 'B' forms ccp structure and 'A' occupies half of the octahedral voids, while oxygen atoms occupy all the tetrahedral voids. The structure of bimetallic oxide is

(1) $A_2B_2O$

(2) $AB_2O_4$

(3) $A_4B_2O$

(4) $A_2BO_4$

**Answer (2)**

**Sol.** Lattice formed by B(ccp) = 4

A = 50% of octahedral voids = 2

O = tetrahedral voids = 8

Formula = $AB_2O_4$

18. If solubility product of $Zr_3(PO_4)_4$ is denoted by $K_{sp}$ and its molar solubility is denoted by S, then which of the following relation between S and $K_{sp}$ is correct?

(1) $S=\left(\frac{K_{sp}}{929}\right)^{\frac{1}{9}}$ (2) $S=\left(\frac{K_{sp}}{216}\right)^{\frac{1}{7}}$

(3) $S=\left(\frac{K_{sp}}{144}\right)^{\frac{1}{6}}$ (4) $S=\left(\frac{K_{sp}}{6912}\right)^{\frac{1}{7}}$

**Answer (4)**

**Sol.** $Zr_3(PO_4)_4 \rightleftharpoons \underset{3S}{3Zr^{+4}} + \underset{4S}{4PO_4^{3-}}$

$K_{sp} = \left[Zr^{+4}\right]^3\left[PO_4^{3-}\right]^4 = (3S)^3(4S)^4$

$K_{sp}$ = 6912 $S^7$

$S=\left(\frac{K_{sp}}{6912}\right)^{1/7}$

19. In order to oxidise a mixture of one mole of each of $FeC_2O_4$, $Fe_2(C_2O_4)_3$, $FeSO_4$ and $Fe_2(SO_4)_3$ in acidic medium, the number of moles of $KMnO_4$ required is

(1) 1.5

(2) 2

(3) 3

(4) 1

**Answer (2)**

**Sol.** $5e + MnO_4^- \longrightarrow Mn^{+2}$

(i) $FeC_2O_4 \longrightarrow Fe^{3+} + 2CO_2 + 3e$

1 mole of $FeC_2O_4$ react with $\frac{3}{5}$ moles of acidified $KMnO_4$

(ii) $Fe_2(C_2O_4)_3 \longrightarrow Fe^{3+} + CO_2 + 6e$

1 mole of $Fe_2(C_2O_4)_3$ react with $\frac{6}{5}$ moles of $KMnO_4$

(iii) $FeSO_4 \longrightarrow Fe^{3+} + e$

1 mole of $FeSO_4$ react with $\frac{1}{5}$ moles of $KMnO_4$

$\therefore$ Total moles required = $\frac{3}{5}+\frac{6}{5}+\frac{1}{5}$ = 2

20. 100 mL of a water sample contains 0.81 g of calcium bicarbonate and 0.73 g of magnesium bicarbonate. The hardness of this water sample expressed in terms of equivalents of $CaCO_3$ is

(molar mass of calcium bicarbonate is 162 g $mol^{-1}$ and magnesium bicarbonate is 146 g $mol^{-1}$)

(1) 5,000 ppm

(2) 100 ppm

(3) 10,000 ppm

(4) 1,000 ppm

**Answer (3)**

**Sol.** Moles of $Ca(HCO_3)_2$ = 0.005

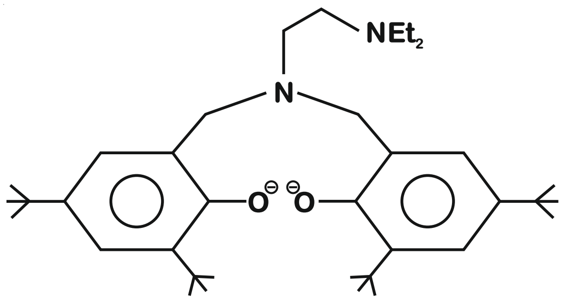
Moles of $Mg(HCO_3)_2$ = 0.005

Hardness in terms of $CaCO_3$ pm

$= \frac{(0.005+0.005)\times 100}{100}\times 10^6$

= $10^4$ ppm

21. The following ligand is




(1) Tetradentate (2) Tridentate

(3) Bidentate (4) Hexadentate

**Answer (1)**

**Sol.** It has four lone pairs but maximum it will be able to donate three lone pairs.

Maximum denticity is 3.

22. Diborane ($B_2H_6$) reacts independently with $O_2$ and $H_2O$ to produce, respectively:

(1) $H_3BO_3$ and $B_2O_3$ (2) $HBO_2$ and $H_3BO_3$

(3) $B_2O_3$ and $H_3BO_3$ (4) $B_2O_3$ and $[BH_4]^-$

**Answer (3)**

**Sol.** $B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$

$B_2H_6 + 6H_2O \longrightarrow 2H_3BO_3 + 6H_2$

23. The size of the iso-electronic species $Cl^-$, Ar and $Ca^{2+}$ is affected by

(1) Nuclear charge

(2) Principal quantum number of valence shell

(3) Azimuthal quantum number of valence shell

(4) Electron-electron interaction in the outer orbitals

**Answer (1)**

**Sol.** Iso-electronic species differ in size due to different effective nuclear charge.

24. The IUPAC name of the following compound is:

$$H_3C-\underset{\displaystyle CH_3}{\overset{|}{CH}}-\underset{\displaystyle OH}{\overset{|}{CH}}-CH_2-COOH$$

(1) 3-Hydroxy-4-methylpentanoic acid

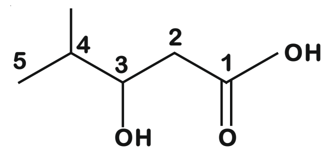
(2) 4-Methyl-3-hydroxypentanoic acid

(3) 2-Methyl-3-hydroxypentan-5-oic acid

(4) 4,4-Dimethyl-3-hydroxybutanoic acid

**Answer (1)**

**Sol.** IUPAC name




3-Hydroxy-4-methyl pentanoic acid

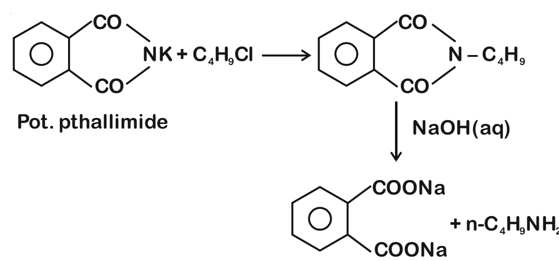
25. Which of the following amines can be prepared by Gabriel phthalimide reaction?

(1) Neo-pentylamine (2) n-butylamine

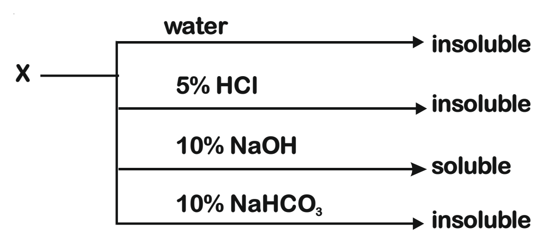
(3) t-butylamine (4) Triethylamine

**Answer (2)**

**Sol.** Primary amines are prepared by Gabriel pthallimide synthesis




26. An organic compound 'X' showing the following solubility profile is

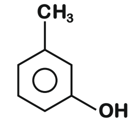


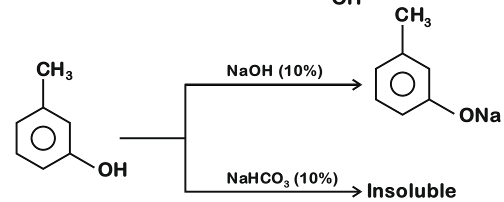

(1) Benzamide (2) Oleic acid

(3) o-Toluidine (4) m-Cresol

**Answer (4)**

**Sol.** X is Meta cresol

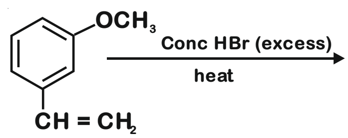


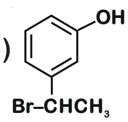




Benzamide is amphoteric

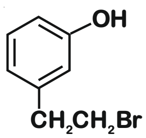
Oleic acid will dissolve in NaOH as well as $NaHCO_3$ due to acidic nature.

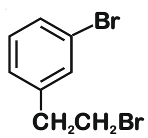
27. The major product of the following reaction is



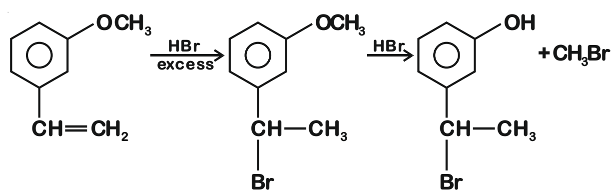
(1) 

(2) 

(3) 

(4) 

**Answer (1)**

**Sol.** 

28. Assertion : Ozone is destroyed by CFCs in the upper stratosphere.

Reason : Ozone holes increase the amount of UV radiation reaching the earth.

(1) Assertion and reason are both correct, and the reason is the correct explanation for the assertion.

(2) Assertion is false, but the reason is correct.

(3) Assertion and reason are correct, but the reason is not the explanation for the assertion.

(4) Assertion and reason are incorrect.

**Answer (3)**

**Sol.** CFC's are responsible for depletion of ozone layer

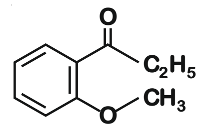
$$CF_2Cl_2 \xrightarrow{uv} \dot{C}l_{(g)} + F_2\dot{C}Cl$$

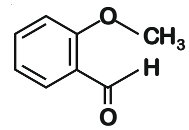
$$\dot{C}l + O_3 \longrightarrow Cl\dot{O} + O_2$$

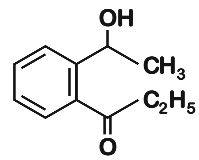
$$Cl\dot{O} + O \longrightarrow \dot{C}l + O_2$$

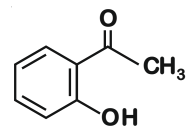
Both statements are correct.

29. An organic compound neither reacts with neutral ferric chloride solution nor with Fehling solution. It however, reacts with Grignard reagent and gives positive iodoform test. The compound is

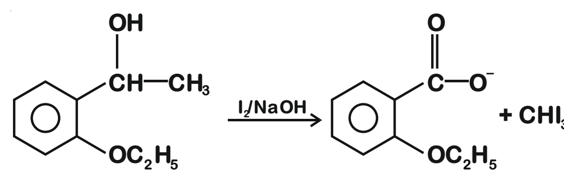
(1) 

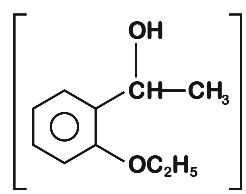
(2) 

(3) 

(4) 

**Answer (3)**

**Sol.** 



→ Reacts with Grignard's reagent due to acidic hydrogen.

→ Fehling solution test is negative as there is no – CHO group.

→ Neutral $FeCl_3$ test is negative as there is no phenolic group

30. In the following compounds, the decreasing order of basic strength will be

(1) $NH_3 > C_2H_5NH_2 > (C_2H_5)_2NH$

(2) $C_2H_5NH_2 > NH_3 > (C_2H_5)_2NH$

(3) $(C_2H_5)_2NH > NH_3 > C_2H_5NH_2$

(4) $(C_2H_5)_2NH > C_2H_5NH_2 > NH_3$

**Answer (4)**

**Sol.** Correct order of $K_b$ value

$$(C_2H_5)_2\ddot{N} > (C_2H_5)_3N > NH_3$$

In aqueous medium sec. amines are most basic.

3°amines are more basic than $NH_3$ as +I factor dominate over steric factor.

# PART–C : MATHEMATICS

1. The shortest distance between the line y = x and the curve $y^2 = x - 2$ is :

(1) $\frac{11}{4\sqrt{2}}$ (2) $\frac{7}{8}$

(3) 2 (4) $\frac{7}{4\sqrt{2}}$

**Answer (4)**

**Sol.** The shortest distance between line y = x and parabola = the distance between line y = x and tangent of parabola having slope 1.

Let equation of tangent of parabola having slope 1 is,

$y = m(x-2) + \frac{a}{m}$

where m = 1 and $a = \frac{1}{4}$

Equation of tangent $y = x - \frac{7}{4}$

Distance between the line y = x and the tangent

$$= \left|\frac{\frac{7}{4} - 0}{\sqrt{1^2 + 1^2}}\right| = \frac{7}{4\sqrt{2}}$$

2. The sum of the co-efficients of all even degree terms in x in the expansion of $(x + \sqrt{x^3 - 1})^6 + (x - \sqrt{x^3 - 1})^6, (x > 1)$ is equal to :

(1) 24 (2) 32

(3) 26 (4) 29

**Answer (1)**

**Sol.** $\left(x + \sqrt{x^3 - 1}\right)^6 + \left(x - \sqrt{x^3 - 1}\right)^6$

$$= 2\left[{}^6C_0 x^6 + {}^6C_2 x^4 (x^3 - 1) + {}^6C_4 x^2 (x^3 - 1)^2 + {}^6C_6 (x^3 - 1)^3\right]$$

$=2[x^6 + 15x^7 - 15x^4 + 15x^8 - 30x^5 + 15x^2 + x^9 - 3x^6 + 3x^3 - 1]$

Sum of coefficients of even powers of x = 2[1 – 15 + 15 + 15 – 3 – 1] = 24

3. If $S_1$ and $S_2$ are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25, x \in R$, then :

(1) $S_1 = \{-2\}; S_2 = \{0, 1\}$

(2) $S_1 = \{-2, 1\}; S_2 = \{0\}$

(3) $S_1 = \{-1\}; S_2 = \{0, 2\}$

(4) $S_1 = \{-2, 0\}; S_2 = \{1\}$

**Answer (2)**

**Sol.** $f(x) = 9x^4 + 12x^3 - 36x^2 + 72$

$f'(x) = 36[x^3 + x^2 - 2x] = 36x(x-1)(x+2)$

| – | + | – | + |
|---|---|---|---|
| | –2 | 0 | 1 |

Whenever derivative changes sign from negative to positive, we get local minima, and whenever derivative changes sign from positive to negative, we get local maxima (while moving left to right on x-axis)

$S_1 = \{-2, 1\}$

$S_2 = \{0\}$

4. If $2y = \left(\cot^{-1}\left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right)^2, x \in \left(0, \frac{\pi}{2}\right)$ then $\frac{dy}{dx}$ is equal to :

(1) $2x - \frac{\pi}{3}$ (2) $x - \frac{\pi}{6}$

(3) $\frac{\pi}{3} - x$ (4) $\frac{\pi}{6} - x$

**Answer (2)**

**Sol.** $$2y = \left[\cot^{-1}\left(\frac{\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x}{\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x}\right)\right]^2$$

$$\Rightarrow 2y = \left[\cot^{-1}\left(\frac{\cos\left(\frac{\pi}{6} - x\right)}{\sin\left(\frac{\pi}{6} - x\right)}\right)\right]^2$$

$$\Rightarrow\quad 2y=\left[\cot^{-1}\left(\cot\left(\frac{\pi}{6}-x\right)\right)\right]^2 \quad \because \quad \frac{\pi}{6}-x\in\left(-\frac{\pi}{3},\frac{\pi}{6}\right)$$

$$\Rightarrow\quad 2y=\begin{cases}\left(\frac{7\pi}{6}-x\right)^2 & \text{if } \frac{\pi}{6}-x\in\left(-\frac{\pi}{3},0\right)\\ \left(\frac{\pi}{6}-x\right)^2 & \text{if } \frac{\pi}{6}-x\in\left(0,\frac{\pi}{6}\right)\end{cases}$$

$$\Rightarrow\quad \frac{dy}{dx}=\begin{cases}x-\frac{7\pi}{6} & \text{if } x\in\left(\frac{\pi}{6},\frac{\pi}{2}\right)\\ x-\frac{\pi}{6} & \text{if } x\in\left(0,\frac{\pi}{6}\right)\end{cases}$$

Note: Only one given option is correct.

5. A point on the straight line, $3x + 5y = 15$ which is equidistant from the coordinate axes will lie only in :

(1) $4^{th}$ quadrant

(2) $1^{st}$ quadrant

(3) $1^{st}$, $2^{nd}$ and $4^{th}$ quadrants

(4) $1^{st}$ and $2^{nd}$ quadrants

**Answer (4)**

**Sol.** A point which is equidistant from both the axes lies on either $y = x$ and $y = -x$.

As it is given that the point lies on the line $3x + 5y = 15$

So the required point is :

$$\begin{array}{l}3x+5y=15\\ \underline{x+y=0}\\ x=-\frac{15}{2},\quad y=\frac{15}{2}\Rightarrow\left(\frac{15}{2},\frac{15}{2}\right)\left\{2^{nd}\text{ quadrant}\right\}\end{array}$$

or

$$\begin{array}{l}3x+5y=15\\ \underline{x=y}\\ x=\frac{15}{8},\quad y=\frac{15}{8}\Rightarrow\left(\frac{15}{8},\frac{15}{8}\right)\left\{1^{st}\text{ quadrant}\right\}\end{array}$$

6. $\int\frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ is equal to :

(where c is a constant of integration)

(1) $2x + \sin x + 2\sin 2x + c$

(2) $x + 2\sin x + 2\sin 2x + c$

(3) $x + 2\sin x + \sin 2x + c$

(4) $2x + \sin x + \sin 2x + c$

**Answer (3)**

**Sol.** $\int\frac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{x}{2}\right)}dx$

$$=\int\frac{2\cos\frac{x}{2}.\sin\frac{5x}{2}}{2\cos\frac{x}{2}.\sin\frac{x}{2}}dx$$

$$=\int\frac{\sin 3x+\sin 2x}{\sin x}dx$$

Now use $\sin 2x = 2\sin x\cos x$ and $\sin 3x = 3\sin x - 4\sin^3 x$

$$=\int(1+2\cos x+2\cos 2x)dx$$

$= x + 2\sin x + \sin 2x + c$

7. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points (1, 2) and (a, b) are perpendicular to each other, then $a^2$ is equal to

(1) $\frac{64}{17}$ (2) $\frac{2}{17}$

(3) $\frac{4}{17}$ (4) $\frac{128}{17}$

**Answer (2)**

**Sol.** Equation of tangent at A(1, 2);

$4x + 2y = 8 \Rightarrow 2x + y = 4$

So tangent at B(a, b) can be assumed as

$$x-2y=c\quad\Rightarrow\quad y=\frac{1}{2}x-\frac{c}{2}$$

Condition for tangency;

$$-\frac{c}{2}=\pm\sqrt{2\left(\frac{1}{2}\right)^2+8}=\pm\sqrt{\frac{17}{2}}$$

$$\Rightarrow\quad c=\pm\sqrt{34}$$

Equation of tangent; $x - 2y = \pm\sqrt{34}$ ...(i)

Equation of tangent at P(a, b); $4ax + by = 8$...(ii)

Comparing both the equations ;

$$\frac{4a}{1}=\frac{b}{-2}=\frac{8}{\pm\sqrt{34}}$$

$$\Rightarrow\quad a=\pm\frac{2}{\sqrt{34}}\quad\Rightarrow\quad a^2=\frac{2}{17}$$

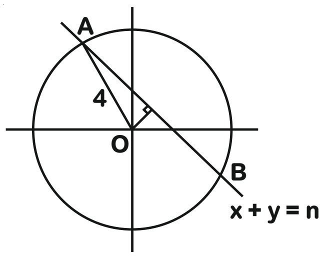
8. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is

(1) 105 (2) 160

(3) 320 (4) 210

**Answer (4)**

**Sol.**




Let the chord $x + y = n$ cuts the circle $x^2 + y^2 = 16$ at A and B length of perpendicular from O on $AB = \left|\frac{0+0-n}{\sqrt{1^2+1^2}}\right| = \frac{n}{\sqrt{2}}$

Length of chord $AB = 2\sqrt{4^2 - \left(\frac{n}{\sqrt{2}}\right)^2}$

$$= 2\sqrt{16 - \frac{n^2}{2}}$$

Here possible values of n are 1, 2, 3, 4, 5.

Sum of square of length of chords

$$= \sum_{n=1}^{5} 4\left(16 - \frac{n^2}{2}\right)$$

$$= 64 \times 5 - 2.\frac{5\times 6\times 11}{6} = 210$$

9. If $f(x) = \log_e\left(\frac{1-x}{1+x}\right), |x| < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to :

(1) $2f(x)$ (2) $2f(x^2)$

(3) $-2f(x)$ (4) $(f(x))^2$

**Answer (1)**

**Sol.** $\because \quad f(x) = \ln\left(\frac{1-x}{1+x}\right)$

$$f\left(\frac{2x}{1+x^2}\right) = \ln\left(\frac{1-\frac{2x}{1+x^2}}{1+\frac{2x}{1+2x^2}}\right)$$

$$= \ln\left(\frac{1+x^2-2x}{1+x^2+2x}\right)$$

$$= \ln\left(\frac{1-x}{1+x}\right)^2$$

$$= 2\ln\left(\frac{1-x}{1+x}\right)$$

$$= 2f(x)$$

10. Let $y = y(x)$ be the solution of the differential equation, $(x^2+1)^2\frac{dy}{dx} + 2x(x^2+1)y = 1$ such that $y(0) = 0$. If $\sqrt{a}\,y(1) = \frac{\pi}{32}$, then the value of 'a' is

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) 1 (4) $\frac{1}{16}$

**Answer (4)**

**Sol.** $(1+x^2)\frac{2dy}{dx} + 2x(1+x^2)y = 1$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{1}{(1+x^2)^2}$$

It is a linear differential equation

$$I{\cdot}F{\cdot} = e^{\int\frac{2x}{1+x^2}dx} = e^{\ln(1+x^2)} = 1+x^2$$

$$\Rightarrow y\cdot(1+x^2) = \int\frac{dx}{1+x^2} + c$$

$\Rightarrow$ $y(1 + x^2) = \tan^{-1} x + c$

If $x = 0$ then $y = 0$

So, $0 = 0 + c$

$\Rightarrow$ $c = 0$

$\Rightarrow$ $y(1 + x^2) = \tan^{-1} x$

put $x = 1$

$$2y = \frac{\pi}{4}$$

$$\Rightarrow 2\left(\frac{\pi}{32\sqrt{a}}\right) = \frac{\pi}{4}$$

$$\Rightarrow \sqrt{a} = \frac{1}{4}$$

$$\Rightarrow a = \frac{1}{16}$$

11. If $\alpha = \cos^{-1}\left(\frac{3}{5}\right)$, $\beta = \tan^{-1}\left(\frac{1}{3}\right)$, where $0 < \alpha, \beta < \frac{\pi}{2}$, then $\alpha - \beta$ is equal to

(1) $\tan^{-1}\left(\frac{9}{14}\right)$ (2) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

(3) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (4) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

**Answer (3)**

**Sol.** $\because \quad \cos\alpha = \frac{3}{5}$

$\Rightarrow \quad \tan\alpha = \frac{4}{3}$

$\Rightarrow$ and $\tan\beta = \frac{1}{3}$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \cdot \tan\beta}$$

$$= \frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{9}} = \frac{1}{\frac{13}{9}}$$

$$= \frac{9}{13}$$

$$\alpha - \beta = \tan^{-1}\left(\frac{9}{13}\right) = \sin^{-1}\left(\frac{9}{5\sqrt{10}}\right) = \cos^{-1}\left(\frac{13}{5\sqrt{10}}\right)$$

12. Let $f : [0, 2] \to R$ be a twice differentiable function such that $f''(x) > 0$, for all $x \in (0, 2)$. If $\phi(x) = f(x) + f(2 - x)$, then $\phi$ is

(1) Decreasing on (0, 2)

(2) Increasing on (0, 2)

(3) Decreasing on (0, 1) and increasing on (1, 2)

(4) Increasing on (0, 1) and decreasing on (1, 2)

**Answer (3)**

**Sol.** $\phi(x) = f(x) + f(2 - x)$

differentiating w.r.t. x

$\phi'(x) = f'(x) - f'(2 - x)$

For $\phi(x)$ to be increasing $\phi'(x) > 0$

$\Rightarrow \quad f'(x) > f'(2 - x)$

($\because \quad f''(x) > 0$ then $f'(x)$ is an increasing function)

$\Rightarrow \quad x > 2 - x$

$\Rightarrow \quad x > 1$

So $\phi(x)$ is increasing in (1, 2) and decreasing in (0, 1)

13. The sum of the series $2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + \ldots + 62.{}^{20}C_{20}$ is equal to

(1) $2^{23}$ (2) $2^{25}$

(3) $2^{24}$ (4) $2^{26}$

**Answer (2)**

**Sol.** $2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + \ldots\ldots 62.{}^{20}C_{20}$

$$= \sum_{r=0}^{20}(3r+2)\,{}^{20}C_r = 3\sum_{r=0}^{20} r.\,{}^{20}C_r + 2\sum_{r=0}^{20}{}^{20}C_r$$

$$= 60\sum_{r=1}^{20}{}^{19}C_{r-1} + 2\sum_{r=0}^{20}{}^{20}C_r$$

$= 60 \times 2^{19} + 2 \times 2^{20}$

$= 2^{21}\,[15 + 1] = 2^{25}$

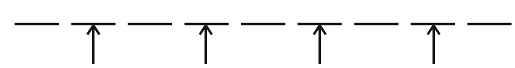
14. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is :

(1) 180 (2) 175

(3) 162 (4) 160

**Answer (1)**

**Sol.** There are total 9 digits; out of which only 3 digits are odd.



Number of ways to arrange odd digits first

$$= {}^4C_3 \cdot \frac{\underline{|3}}{\underline{|2}}$$

Total number of 9 digit numbers $= \left({}^4C_3 \cdot \frac{\underline{|3}}{\underline{|2}}\right) \cdot \frac{\underline{|6}}{\underline{|2}\,\underline{|4}}$

$= 180$

15. The greatest value of $c \in R$ for which the system of linear equations

$x - cy - cz = 0$

$cx - y + cz = 0$

$cx + cy - z = 0$

has a non-trivial solution, is :

(1) –1 (2) 0

(3) 2 (4) $\frac{1}{2}$

**Answer (4)**

**Sol.** If the system of equations has non-trivial solutions, then

$$\begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0$$

$\Rightarrow \ (1-c^2)+c(-c-c^2)-c(c^2+c)=0$

$\Rightarrow \ (1+c)(1-c)-2c^2(1+c)=0$

$\Rightarrow \ (1+c)(1-c-2c^2)=0$

$\Rightarrow \ (1+c)^2(1-2c)=0$

$\Rightarrow \ c=-1 \text{ or } \dfrac{1}{2}$

16. Let O(0, 0) and A(0, 1) be two fixed points. Then the locus of a point P such that the perimeter of $\Delta$AOP is 4, is :

(1) $8x^2-9y^2+9y=18$ (2) $9x^2+8y^2-8y=16$

(3) $9x^2-8y^2+8y=16$ (4) $8x^2+9y^2-9y=18$

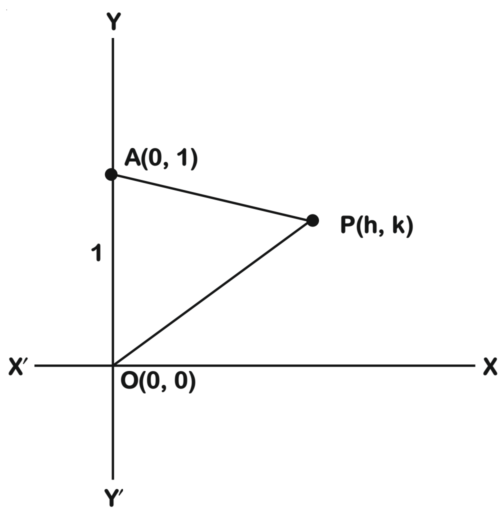
**Answer (2)**

**Sol.** Let point P(h, k)

$\because$ OA = 1

So, OP + AP = 3

$\sqrt{h^2+k^2}+\sqrt{h^2+(k-1)^2}=3$




$\Rightarrow \ h^2+(k-1)^2=9+h^2+k^2-6\sqrt{h^2+k^2}$

$\Rightarrow \ 6\sqrt{h^2+k^2}=2k+8$

$\Rightarrow \ 9(h^2+k^2)=k^2+16+8k$

$\Rightarrow \ 9h^2+8k^2-8k-16=0$

Locus of point P will be,

$9x^2+8y^2-8y-16=0$

17. Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct?

(1) $P(A|B)=P(B)-P(A)$

(2) $P(A|B)\le P(A)$

(3) $P(A|B)\ge P(A)$

(4) $P(A|B)=1$

**Answer (3)**

**Sol.** $\because \ A \subset B$; so $A \cap B = A$

Now, $P\left(\dfrac{A}{B}\right)=\dfrac{P(A\cap B)}{P(B)}$

$\Rightarrow \ P\left(\dfrac{A}{B}\right)=\dfrac{P(A)}{P(B)}$

$\because \ P(B)\le 1$

So, $P\left(\dfrac{A}{B}\right)\ge P(A)$

18. If $\alpha$ and $\beta$ be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\dfrac{\alpha}{\beta}\right)^n=1$ is :

(1) 4 (2) 5

(3) 3 (4) 2

**Answer (1)**

**Sol.** $x^2-2x+2=0$

roots of this equation are $\dfrac{2\pm\sqrt{-4}}{2}=1\pm i$

Then $\dfrac{\alpha}{\beta}=\dfrac{1+i}{1-i}=\dfrac{(1+i)^2}{1-i^2}=i$

or $\dfrac{\alpha}{\beta}=\dfrac{1-i}{1+i}=\dfrac{(1-i)^2}{1-i^2}=-i$

So, $\dfrac{\alpha}{\beta}=\pm i$

Now, $\left(\dfrac{\alpha}{\beta}\right)^n=1 \Rightarrow (\pm i)^n=1$

$\Rightarrow$ n must be a multiple of 4.

minimum value of n = 4

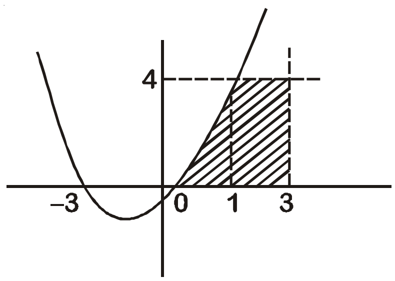
19. The area (in sq. units) of the region $A = \{(x, y) \in R \times R | 0 \le x \le 3,\ 0 \le y \le 4,\ y \le x^2 + 3x\}$ is:

(1) $\frac{26}{3}$ (2) $\frac{59}{6}$

(3) 8 (4) $\frac{53}{6}$

**Answer (2)**

**Sol.** $y \le x^2 + 3x$ represents region below the parabola.




Area of the required region

$$= \int_0^1 (x^2 + 3x)dx + \int_1^3 4 \cdot dx$$

$$= \frac{1}{3} + \frac{3}{2} + 8$$

$$= \frac{59}{6}$$

20. If $f(x) = \frac{2 - x\cos x}{2 + x\cos x}$ and $g(x) = \log_e x$, $(x > 0)$ then the value of the integral $\int_{-\pi/4}^{\pi/4} g(f(x))dx$ is :

(1) $\log_e 1$ (2) $\log_e 3$

(3) $\log_e 2$ (4) $\log_e e$

**Answer (1)**

**Sol.** $g(f(x)) = \ln\left(\frac{2 - x\cos x}{2 + x\cos x}\right)$

Let $I = \int_{-\pi/4}^{\pi/4} \ln\left(\frac{2 - x\cos x}{2 + x\cos x}\right)dx$ ...(i)

$\left(\text{Using property } \int_a^b f(x)dx = \int_a^b f(a + b - x)dx\right)$

$I = \int_{-\pi/4}^{\pi/4} \ln\left(\frac{2 + x\cos x}{2 - x\cos x}\right)dx$ ...(ii)

Adding (i) and (ii)

$$2I = \int_{-\pi/2}^{\pi/2} \ln(1)\,dx = 0$$

$\Rightarrow$ I = 0 = ln 1

21. The equation of a plane containing the line of intersection of the planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and passing through the point (1, 1, 0) is :

(1) $2x - z = 2$ (2) $x - 3y - 2z = -2$

(3) $x - y - z = 0$ (4) $x + 3y + z = 4$

**Answer (3)**

**Sol.** Let the equation of required plane be;

$(2x - y - 4) + \lambda(y + 2z - 4) = 0$

$\because$ This plane passes through (1, 1, 0) then

$(2 - 1 - 4) + \lambda(1 + 0 - 4) = 0 \Rightarrow \lambda = -1$

Equation of required plane will be

$(2x - y - 4) - (y + 2z - 4) = 0$

$\Rightarrow 2x - 2y - 2z = 0$

$\Rightarrow x - y - z = 0$

22. If $\cos(\alpha + \beta) = \frac{3}{5}, \sin(\alpha - \beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to :

(1) $\frac{21}{16}$ (2) $\frac{63}{52}$

(3) $\frac{33}{52}$ (4) $\frac{63}{16}$

**Answer (4)**

**Sol.** $\because$ $\alpha + \beta$ and $\alpha - \beta$ both are acute angles.

$$\cos(\alpha + \beta) = \frac{3}{5}$$

$$\Rightarrow \tan(\alpha + \beta) = \frac{4}{3}$$

$$\text{And } \sin(\alpha - \beta) = \frac{5}{13}$$

$$\Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$$

Now, $\tan 2\alpha = \tan((\alpha + \beta) + (\alpha - \beta))$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)}$$

$$= \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \cdot \frac{5}{12}} = \frac{63}{16}$$

23. Let $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, $(\alpha \in R)$ such that $A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Then a value of $\alpha$ is :

(1) $\frac{\pi}{32}$ (2) $\frac{\pi}{64}$

(3) 0 (4) $\frac{\pi}{16}$

**Answer (2)**

**Sol.** $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$$A^2=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\cos 2\alpha & -\sin 2\alpha\\ \sin 2\alpha & \cos 2\alpha\end{bmatrix}$$

Then $A^4=A^2.A^2=\begin{bmatrix}\cos 4\alpha & -\sin 4\alpha\\ \sin 4\alpha & \cos 4\alpha\end{bmatrix}$

similarly $A^8=A^4.A^4\begin{bmatrix}\cos 8\alpha & -\sin 8\alpha\\ \sin 8\alpha & \cos 8\alpha\end{bmatrix}$

and so on $A^{32}=\begin{bmatrix}\cos 32\alpha & -\sin 32\alpha\\ \sin 32\alpha & \cos 32\alpha\end{bmatrix}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

So $\sin 32\alpha = 1$ and $\cos 32\alpha = 0$

$\Rightarrow \quad 32\alpha=2n\pi+\dfrac{\pi}{2} \Rightarrow \alpha=\dfrac{n\pi}{16}+\dfrac{\pi}{64}$ where $n\in Z$

put n = 0, $\alpha=\dfrac{\pi}{64}$

24. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is:

(1) If you are born in India, then you are not a citizen of India.

(2) If you are not born in India, then you are not a citizen of India.

(3) If you are a citizen of India, then you are born in India.

(4) If you are not a citizen of India, then you are not born in India.

**Answer (4)**

**Sol.** S: "If you are born in India, then you are a citizen of India."

Contrapositive of $p \to q$ is $\sim q \to \sim p$

So contrapositive of statement S will be :

"If you are not a citizen of India, then you are not born in India."

25. The sum of the solutions of the equation $\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0, (x>0)$ is equal to:

(1) 4 (2) 10

(3) 9 (4) 12

**Answer (2)**

**Sol.** Let $\sqrt{x}=t$

$|t-2| + t(t-4) + 2 = 0$

$\Rightarrow |t-2|+t^2-4t+4-2=0$

$\Rightarrow |t-2|+(t-2)^2-2=0$

Let $|t-2| = z$ (Clearly $z \ge 0$)

$\Rightarrow z+z^2-2=0$

$\Rightarrow z = 1$ or $-2$ (rejected)

$\Rightarrow |t-2| = 1 \Rightarrow t = 1, 3$

If $\sqrt{x}=1 \Rightarrow x=1$

If $\sqrt{x}=3 \Rightarrow x=9$

Sum of solutions = 10

26. The magnitude of the projection of the vector $2\hat{i}+3\hat{j}+\hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$, is:

(1) $\sqrt{\dfrac{3}{2}}$ (2) $3\sqrt{6}$

(3) $\dfrac{\sqrt{3}}{2}$ (4) $\sqrt{6}$

**Answer (1)**

**Sol.** Let $\bar{a}=\hat{i}+\hat{j}+\hat{k}$ and $\bar{b}=\hat{i}+2\hat{j}+3\hat{k}$ vector perpendicular to $\bar{a}$ and $\bar{b}$ is $\bar{a}\times\bar{b}$

$$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & 2 & 3\end{vmatrix}=\hat{i}-2\hat{j}+\hat{k}$$

Projection of vector $\bar{c}=2\hat{i}+3\hat{j}+\hat{k}$ on $\bar{a}\times\bar{b}$ is

$$=\left|\frac{\bar{c}.(\bar{a}\times\bar{b})}{|\bar{a}\times\bar{b}|}\right|=\left|\frac{2-6+1}{\sqrt{6}}\right|$$

$$=\frac{3}{\sqrt{6}}=\sqrt{\frac{3}{2}}$$

27. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is:

(1) 45 (2) 40

(3) 48 (4) 49

**Answer (3)**

**Sol.** Let the remaining numbers are x and y

$$\text{Mean }(\bar{x})=\frac{\sum x_i}{N}=\frac{2+4+10+12+14+x+y}{7}=8$$

$\Rightarrow x + y = 14$ ...........(i)

$$\text{Variance }(\sigma^2)=\frac{\sum x_i^2}{N}-(\bar{x})^2=16$$

$$\Rightarrow \frac{2^2+4^2+10^2+12^2+14^2+x^2+y^2}{7}-(8)^2=16$$

$\Rightarrow x^2 + y^2 = 100$ ...........(ii)

From (i) and (ii) (x, y) = (6, 8) or (8, 6)

xy = 48

28. $\lim\limits_{x\to 0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals:

(1) $\sqrt{2}$ (2) $2\sqrt{2}$

(3) 4 (4) $4\sqrt{2}$

**Answer (4)**

**Sol.** $\lim\limits_{x\to 0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}=\lim\limits_{x\to 0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{2\cos^2\frac{x}{2}}}$ $\left[\frac{0}{0}\right]$

$$=\lim_{x\to 0}\frac{\sin^2 x}{\sqrt{2}\left[1-\cos\frac{x}{2}\right]}$$

$$=\lim_{x\to 0}\frac{\sin^2 x}{2\sqrt{2}\sin^2\frac{x}{4}}$$

$$=\lim_{x\to 0}\frac{\left(\frac{\sin x}{x}\right)^2.16}{2\sqrt{2}\left(\frac{\sin\frac{x}{4}}{\frac{x}{4}}\right)^2}$$

$$=\frac{16}{2\sqrt{2}}=4\sqrt{2}$$

29. The sum of all natural numbers 'n' such that 100 < n < 200 and H.C.F. (91, n) > 1 is :

(1) 3303 (2) 3121

(3) 3203 (4) 3221

**Answer (2)**

**Sol.** $\because$ 91 = 13 × 7

So the required numbers are either divisible by 7 or 13

Sum of such numbers = Sum of no. divisible by 7 + sum of the no. divisible by 13 – Sum of the numbers divisible by 91

= (105 + 112 + ...... + 196) + (104 + 117 + ...... + 195) – 182

= 2107 + 1196 – 182

= 3121

30. The length of the perpendicular from the point (2, –1, 4) on the straight line,

$\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}$ is :

(1) Greater than 3 but less than 4

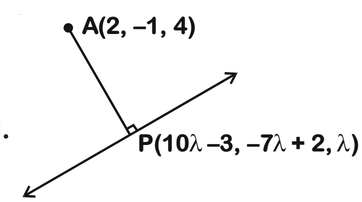
(2) Greater than 2 but less than 3

(3) Greater than 4

(4) Less than 2

**Answer (1)**

**Sol.**




Let P be the foot of perpendicular from point A(2, –1, 4) on the given line. So P can be assumed as P(10λ – 3, –7λ + 2, λ)

DR's of AP ∝ to 10λ – 5, –7λ + 3, λ – 4

$\because$ AP and given line are perpendicular, so

10(10λ – 5) – 7(–7λ + 3) + 1(λ – 4) = 0

$\Rightarrow \lambda=\frac{1}{2}$

$$AP=\sqrt{(10\lambda-5)^2+(-7\lambda+3)^2+(\lambda-4)^2}$$

$$=\sqrt{0+\frac{1}{4}+\frac{49}{4}}$$

$$=\sqrt{12.5}\,;\ \sqrt{12.5}\in(3,4)$$