JEE Main - 2018 (CBT)
ExamTest Date : 16/04/2018

Part - C (Mathematics)

1. If $x = \sqrt{2^{\operatorname{cosec}^{-1} t}}$ and $y = \sqrt{2^{\sec^{-1} t}}$ $(|t| \geq 1)$, then $\frac{dy}{dx}$ is equal to :

(1) $\frac{y}{x}$ (2*) $-\frac{y}{x}$ (3) $-\frac{x}{y}$ (4) $\frac{x}{y}$

Ans. (2)

Sol. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \dfrac{\frac{1}{2\sqrt{2^{\sec^{-1}t}}}\, 2^{\sec^{-1}t}\ell n 2\left(\frac{1}{t\sqrt{t^2-1}}\right)}{\frac{1}{2\sqrt{2^{\operatorname{cosec}^{-1}t}}}\, 2^{\operatorname{cosec}^{-1}t}\ell n 2\left(\frac{1}{t\sqrt{t^2-1}}\right)}$

$= -\frac{\sqrt{2^{\sec^{-1}t}}}{\sqrt{2^{\operatorname{cosec}^{-1}t}}} = -\frac{y}{x}$

2. Let N denote the set of all natural numbers. Define two binary relations on N as $R_1 = \{(x, y) \in N \times N : 2x + y = 10\}$ and $R_2 = \{(x, y) \in N \times N : x + 2y = 10\}$. Then

(1) Both R_1 and R_2 are transitive relations
(2) Range of R_2 is {1, 2, 3, 4}.
(3) Range of R_1 is {2, 4, 8}
(4) Both R_1 and R_2 are symmetric relations.

Ans. (2)

Sol. $R_1 = \{(1, 8), (2, 6), (3, 4), (9, 2)\}$
$R_2 = \{(8, 1), (6, 2), (4, 3), (2, 4)\}$
Range of $R_2 = \{1, 2, 3, 4\}$

3. The coefficient of x^2 in the expansion of the product $(2- x^2)\cdot ((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$ is :

(1) 107 (2) 108 (3) 155 (4*) 106

Ans. (4)

Sol. coefficient of x^2 = 2 coefficient of x^2 in $((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$ – constant term

$(1 + 2x + 3x^2)^6 = \sum_{r=0}^{6} {}^6c_r (2x+3x^2)^r$

$= {}^6C_0 + {}^6C_1 (2x + 3x^2) + {}^6C_2 (2x + 3x^2)^2 + \ldots.$

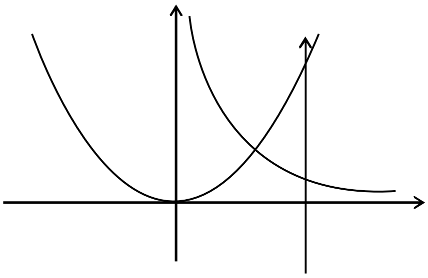
coefficient of $x^2 = 2(18 + 60 - 24) - 2$
$= 108 - 2 = 106$

4. If the area of the region bounded by the curves, $y = x^2$, $y = \frac{1}{x}$ and the lines $y = 0$ and $x = t$ $(t > 1)$ is 1 sq. unit, then t is equal to :

(1*) $e^{\frac{2}{3}}$ (2) $e^{\frac{3}{2}}$ (3) $\frac{3}{2}$ (4) $\frac{4}{3}$

Ans. **(1)**

Sol. $\int_x^1 x^2dx + \int + \frac{1}{x}dx = 1$



$\frac{1}{3} + \ell nt = 1 \quad \Rightarrow \quad t = e^{\frac{2}{3}}$

5. If the length of the latus rectum of an ellipse is 4 units and the distance between a focus and its nearest vertex on the major axis is $\frac{3}{2}$ units, then its eccentricity is :

(1) $\frac{2}{3}$ (2) $\frac{1}{2}$ (3) $\frac{1}{9}$ (4) $\frac{1}{3}$

Ans. **(4)**

Sol. $\frac{2b^2}{a} = 4 \Rightarrow \quad b^2 = 2a$

$b^2 = a^2(1 - e^2)$, $a(1 - e) = \frac{3}{2}$

$2 = a(1 - e)(1 + e)$

$2 = \frac{3}{2}(1 + e) \quad e = \frac{1}{3}$

6. The number of numbers between 2,000 and 5,000 that can be formed with the digits 0, 1, 2, 3, 4 (repetition of digits is not allowed) and are multiple of 3 is :

(1) 36 (2) 30 (3) 24 (4) 48

Ans. **(2)**

Sol. number can be formed y (0, 1, 2, 3) or (0, 2, 3, 4)

number of 4 digits number = 2 × 3 ! + 3 × 3 ! = 30

7. Two different families A and B are blessed with equal number of children. There are 3 tickests to be distributed amongst the children of these families so that no child gets more than one ticket. If the probability that all the tickets go to the children of the family B is $\frac{1}{12}$, then the number of children in each family is :

(1) 6 (2) 5 (3) 3 (4) 4

Ans. **(2)**

Sol. Let n number of children are there in each family

$\frac{1}{12} = \frac{{}^nC_3.3!}{{}^{2n}C_3.3!}$

$\frac{{}^nC_3}{{}^{2n}C_3} = \frac{1}{12}$ n = 5

8. $\lim\limits_{x\to 0} \dfrac{(27+x)^{\frac{1}{3}}-3}{9-(27+x)^{\frac{2}{3}}}$ equals :

(1) $-\dfrac{1}{6}$ (2) $\dfrac{1}{6}$ (3) $\dfrac{1}{3}$ (4) $-\dfrac{1}{3}$

Ans. **(1)**

Sol. $\lim\limits_{x\to 0} \dfrac{3\left[\left(1+\dfrac{x}{27}\right)^{\frac{1}{3}}-1\right]}{9\left[1-\left(1+\dfrac{x}{27}\right)^{\frac{2}{3}}\right]}$

$\lim\limits_{x\to 0} \dfrac{1}{3}\left[\dfrac{\dfrac{x}{81}}{-\dfrac{2}{3}\cdot\dfrac{x}{27}}\right] = \dfrac{-1}{6}$

9. Let p, q and r be real numbers ($p \neq q$, $r \neq 0$), such that the roots of the equation $\dfrac{1}{x+p}+\dfrac{1}{x+q}=\dfrac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to :

(1) $p^2 + q^2$ (2) $\dfrac{p^2+q^2}{2}$ (3) $2(p^2 + q^2)$ (4) $p^2 + q^2 + r^2$

Ans. **(1)**

Sol. $(2x + p + q)\, r = (x + p)(x + q)$

$x^2 + (p + q - 2r)\, x + pq - pr - qr = 0$

$p + q = 2r$ ……..(i)

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 0 - 2\,[pq - pr - qr] = -2pq + 2r\,(p + q) = -2pq + (p + q)^2 = p^2 + q^2$

10. Let $\dfrac{1}{x_1}, \dfrac{1}{x_2}, \ldots, \dfrac{1}{x_n}$ ($x_i \neq 0$ for i = 1, 2, …., n) be in A.P. such that $x_1 = 4$ and $x_{21} = 20$. If n is the least positive integer for which $x_n > 50$, then $\sum\limits_{i=1}^{n}\left(\dfrac{1}{x_i}\right)$ is equal to :

(1) 3 (2) $\dfrac{1}{8}$ (3) $\dfrac{13}{4}$ (4) $\dfrac{13}{8}$

Ans. **(3)**

Sol. $\dfrac{1}{4} + 20.d = \dfrac{1}{20}$

$d = \dfrac{-1}{100}$

$\dfrac{1}{x_n} < \dfrac{1}{50}$

$\dfrac{1}{4} - \dfrac{n-1}{100} < \dfrac{1}{50} \quad \Rightarrow \quad n > 24$

$n = 25$

$\sum\limits_{i=1}^{25}\left(\dfrac{1}{x_i}\right) = \dfrac{25}{2}\left[2\times\dfrac{1}{4}-\dfrac{1}{100}\times 24\right] = \dfrac{13}{4}$

11. The differential equation representing the family of ellipses having foci either on the x-axis or on the y-axis, centre at the origin and passing through the point (0, 3) is :

(1*) $xy\,y' - y^2 + 9 = 0$ (2) $xy\,y'' + x\,(y')^2 - y\,y' = 0$

(3) $xy\,y' + y^2 - 9 = 0$ (4) $x + y\,y'' = 0$

Ans. **(1)**

Sol. Equation of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Passes (0, 3) $\Rightarrow$ $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ ………(1) $\frac{x}{a^2} = -\frac{y}{9}\frac{dy}{dx}$

$\frac{2x}{a^2} + \frac{2y}{9}\frac{dy}{dx} = 0$ ………(2) $\frac{1}{a^2} = -\frac{y}{9x}y^1$

By (1) & (2) D. equation is

$$-\frac{xy}{9}y^1 + \frac{y^2}{9} = 1$$

$\Rightarrow$ $xy\,y^1 - y^2 + 9 = 0$

12. The sum of the intercepts on the coordinate axes of the plane passing trhough the point (–2, –2, 2) and containing the line joining the points (1, –1, 2) and (1, 1, 1), is :

(1) 4 (2) 12 (3) –8 (4) –4

Ans. **(4)**

Sol. Equation plane

$$\begin{vmatrix} x+2 & y+2 & z-2 \\ -3 & -1 & 0 \\ -3 & -3 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $-(x+2) + 3(y+2) + 6(z-2) = 0$

$\Rightarrow$ $x - 3y - 6z + 8 = 0$

sum of intercepts $= -8 + \frac{8}{3} + \frac{8}{6} = -4$

13. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^{20}$. Then the sum of the elements of the first column of B is :

(1) 210 (2) 211 (3) 251 (4) 231

Ans. **(4)**

Sol. $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$; $A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 10 & 4 & 1 \end{bmatrix}$ ….. $A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 20 & 1 & 0 \\ 210 & 20 & 1 \end{bmatrix}$

Sum of the elements of first column = 231

14. Let A, B and C be three events, which are pair-wise independent and $\bar{E}$ denotes the complement of an event E. If $P(A \cap B \cap C) = 0$ and $P(C) > 0$, then $P[(\bar{A} \cap \bar{B}) | C]$ is equal to :

(1) $P(\bar{A}) - P(B)$ (2) $P(\bar{A}) - P(\bar{B})$ (3) $P(\bar{A}) + P(\bar{B})$ (4) $P(A) + P(\bar{B})$

Ans. (1)

Sol. $P[(\bar{A} \cap \bar{B})|C] = \dfrac{P[(\bar{A} \cup \bar{B}) \cap C]}{P(C)}$

$= \dfrac{P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}{P(C)}$

$= \dfrac{P(C) - P(A)P(C) + P(B)P(C)}{P(C)}$

$= 1 - P(A) - P(B)$

$= P(\bar{A}) - P(B)$ or $P(\bar{B}) - P(A)$

15. If $p \to (\sim p \vee \sim q)$ is false, then the truth values of p and q are respectively :

(1) F, F (2) T, T (3) F, T (4) T, F

Ans. (2)

Sol. $P \to (\sim p \vee \sim q)$

p	q	$\sim p \vee \sim q$	$p \to (\sim p \vee \sim q)$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

16. If the function f defined as $f(x) = \dfrac{1}{x} - \dfrac{k-1}{e^{2x}-1}$, $x \neq 0$, is continuous at x = 0, then the ordered pair (k, f(0)) is equal to :

(1) (2, 1) (2) (3, 1) (3) (3, 2) (4) $\left(\dfrac{1}{3}, 2\right)$

Ans. (2)

Sol. $f(x) = \dfrac{1}{x} - \dfrac{k-1}{e^{2x}-1}$; $x \neq 0$

f(x) is continuous at x = 0

$\Rightarrow f(0) = \lim\limits_{x \to 0} \dfrac{1}{x} - \dfrac{k-1}{e^{2x}-1}$

$= \lim\limits_{x \to 0} \dfrac{(1 + (2x) + \frac{1}{2!}(2x)^2 + ...(-1 - x(k-1)}{2x^2\left(\dfrac{e^{2x}-1}{2x}\right)}$

Clearly k = 3 and f(0) = 1

17. If the angle between the lines, $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{5-x}{-2}=\frac{7y-14}{p}=\frac{z-3}{4}$ is $\cos^{-1}\left(\frac{2}{3}\right)$, then p is equal to :

(1) $\frac{2}{7}$ (2) $\frac{7}{2}$ (3) $-\frac{4}{7}$ (4) $-\frac{7}{4}$

Ans. **(2)**

Sol. $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x-5}{2}=\frac{y-2}{P/7}=\frac{z-3}{4}$

Angle between both lines is $\cos^{-1}\left(\frac{2}{3}\right)=\cos^{-1}\left(\frac{4+\frac{2P}{7}+4}{3.\sqrt{4+\frac{P^2}{49}+16}}\right)$

$\Rightarrow \frac{2}{3}=\frac{56+2P}{3\sqrt{P^2+980}} \Rightarrow \sqrt{P^2+980}=P+28 \Rightarrow P^2+980=P^2+56P+784 \Rightarrow 56P=196 \Rightarrow P=\frac{7}{2}$

18. The locus of the point of intersection of the lines, $\sqrt{2}x-y+4\sqrt{2}k=0$ and $\sqrt{2}kx+ky-4\sqrt{2}=0$ (k is any non-zero real parameter), is :

(1) an ellipse whose eccentricity is $\frac{1}{\sqrt{3}}$.

(2) a hyperbola whose eccentricity is $\sqrt{3}$

(3) a hyperbola with length of its transverse axis $8\sqrt{2}$

(4) an ellipse with length of its major axis $8\sqrt{2}$.

Ans. **(3)**

Sol. $\sqrt{2}x-y+4\sqrt{2}k=0$ ....(i)

$\sqrt{2}kx+ky-4\sqrt{2}=0$ ....(ii)

Eliminating k by (i) and (ii)

$\left(\sqrt{2}x+y\right)\left(\frac{\sqrt{2}x-y}{-4\sqrt{2}}\right)=4\sqrt{2}$

$2x^2-y^2=-32$

$\frac{y^2}{32}-\frac{x^2}{16}=1$ Hyperbola

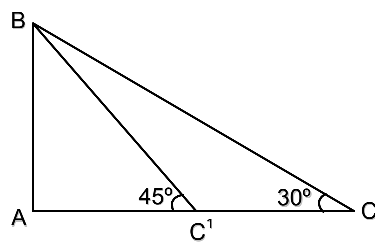
$e=\sqrt{1+\frac{16}{32}}=\sqrt{\frac{3}{2}}$ and length of transverse axis = $8\sqrt{2}$

19. A man on the top of a vertical tower observes a car moving at a uniform speed towards the tower on a horizonatal road. If it takes 18 min. for the angle of depression of the car to change from 30° to 45° ; then after this, the time taken (in min.) by the car to reach the foot of the tower, is :

(1) $\frac{9}{2}(\sqrt{3}-1)$ (2) $18(1+\sqrt{3})$ (3) $18(\sqrt{3}-1)$ (4) $9(1+\sqrt{3})$

Ans. **(4)**

Sol. Let length of tower = h



$\Rightarrow AC' = AB = h$

and $AC = AB \cot 30^\circ = \sqrt{3}\,h \Rightarrow CC' = (\sqrt{3}-1)\,h$

Time taken by car form C to C' = 18 min

$\Rightarrow$ time take by car to reach the foot of the tower = $\frac{18}{\sqrt{3}-1}$ min.

$= 9(\sqrt{3}+1)$ min

20. If an angle A of a ΔABC satisfies $5\cos A + 3 = 0$, then the roots of the quadratic equaiton, $9x^2 + 27x + 20 = 0$ are :

(1) sec A, cotA (2) secA, tanA (3) tanA, cosA (4) sinA, secA

Ans. **(2)**

Sol. $5\cos A + 3 = 0 \Rightarrow \cos A = -\frac{3}{5}$ clearly $A \in (90^\circ, 180^\circ)$

Now roots of equation $9x^2 + 27x + 20 = 0$ are $-\frac{5}{3}$ and $-\frac{4}{3}$

$\Rightarrow$ Roots secA and tanA

21. If a circle C, whose radius is 3, touches externally the circle, $x^2 + y^2 + 2x - 4y - 4 = 0$ at the point (2, 2), then the length of the intercept cut by this circle C, on the x-axis is equal to :

(1) $2\sqrt{3}$ (2) $\sqrt{5}$ (3) $3\sqrt{2}$ (4) $2\sqrt{5}$

Ans. **(4)**

Sol. Centre of given circle = (–1, 2)

and radius = $\sqrt{1+4+4} = 3$

centre of required circle (5,2)

or (–4,2)

length of intercept on x-axis will be square in both circle

so one required circle $(x-5)^2 + (y-2)^2 = 3^2$

$x^2 + y^2 - 10x - 4y + 20 = 0$

Length of x intercept = $2\sqrt{g^2 - c}$

$= 2\sqrt{25-20} = 2\sqrt{5}$

22. Let P be a point on the parabola, $x^2 = 4y$. If the distance of P from the centre of the circle, $x^2 + y^2 + 6x + 8 = 0$ is minimum, then the equation of the tangent to the parabola at P, is :

(1) $x + y + 1 = 0$ (2) $x + 4y - 2 = 0$ (3) $x + 2y = 0$ (4) $x - y + 3 = 0$

Ans. **1**

Sol. Let P $(2t, t^2)$

equation normal at P to $x^2 = 4y$ be

$y - t^2 = -\frac{1}{t}(x - 2t)$

it passes through (–3,0)

$0 - t^2 = -\frac{1}{t}(-3 - 2t)$

$t^3 + 2t + 3 = 0$

$(t + 1)(t^2 - t + 3) = 0$

$\Rightarrow t = -1$

Point P is (–2,1)

equation of tangent to $x^2 = 4y$ at (–2,1)

$x(-2) = 2(y + 1)$

$x + y + 1 = 0$

23. If $f(x) = \int_0^x t(\sin x - \sin t)\,dt$ then :

(1) $f'''(x) - f''(x) = \cos x - 2x \sin x$
(2) $f'''(x) + f''(x) - f'(x) = \cos x$
(3) $f'''(x) + f''(x) = \sin x$
(4) $f'''(x) + f'(x) = \cos x - 2x \sin x$

Ans. **4**

Sol. $f(x) = \int_0^x t(\sin x - \sin t)dt$

$f(x) = \sin x \int_0^x t\,dt - \int_0^x t \sin t\,dt$

$f'(x) = (\sin x)\,x + \cos x \int_0^x t\,dt - x \sin x$

$f'(x) = \cos x \int_0^x t\,dt$

$f''(x) = (\cos x)\,x - (\sin x) \int_0^x t\,dt$

$f'''(x) = x(-\sin x) + \cos x - (\sin x)x - (\cos x) \int_0^x t\,dt$

$f'''(x) + f'(x) = \cos x - 2x \sin x$

24. The number of values of k for which the system of linear equations,
$(k + 2)x + 10y = k$
$kx + (k + 3)y = k - 1$ has no soution, is :

(1) 1 (2) 2 (3) 3 (4) 4

Ans. **(1)**

Sol. For no solution

$\frac{k+2}{k} = \frac{10}{k+3} \neq \frac{k}{k-1}$

$(k + 2)(k + 3) = 10k$

$k^2 - 5k + 6 = 0 \Rightarrow k = 2,3$

$k \neq 2$ for $k = 2$ both lines identical

so $k = 3$ only

so number of values of k is 1

25. If $\int \frac{\tan x}{1 + \tan x + \tan^2 x} dx = x - \frac{K}{\sqrt{A}} \tan^{-1}\left(\frac{K \tan x + 1}{\sqrt{A}}\right) + C$, (C is a constant of integration), then the ordered pair (K, A) is equal to

(1) (2, 1) (2) (2, 3) (3) (–2, 1) (4) (–2, 3)

Ans. **(2)**

Sol. $I = \int \frac{\tan}{1 + \tan x + \tan^2 x} dx$

$\int \left(1 - \frac{\sec^2 x}{1 + \tan x + \tan^2 x}\right) dx$

$= x - \int \frac{dt}{1 + t + t^2}$, where $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$= \int \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}} = x - \frac{1}{\sqrt{3}/2} \tan^{-1}\left(\frac{t + \frac{1}{2}}{\sqrt{3}/2}\right) + C = x - \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2\tan x + 1}{\sqrt{\sqrt{3}}}\right) + C$

$= k = 2$, $A = 3$.

26. The least positive integer n for which $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n = 1$, is

(1) 2 (2) 5 (3) 6 (4) 3

Ans. **(4)**

Sol. $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n = 1$

$\left(\frac{-2\omega^2}{-2\omega}\right)^n = 1$

$\omega^n = 1$

least positive integer value of n is 3.

27. The sum of the first 20 terms of the series $1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \frac{31}{16} + \ldots$, is :

(1) $39 + \frac{1}{2^{19}}$ (2) $38 + \frac{1}{2^{20}}$ (3) $38 + \frac{1}{2^{19}}$ (4) $39 + \frac{1}{2^{20}}$

Ans. **(3)**

Sol. $1+\frac{3}{2}+\frac{7}{4}+\frac{15}{8}+\frac{31}{16}+\ldots\ldots$

$= (2-1) + \left(2-\frac{1}{2}\right)+\left(2-\frac{1}{4}\right)+\left(2-\frac{1}{8}\right)+\left(2-\frac{1}{16}\right)+ \ldots\ldots\ldots\text{upto 20 terms}$

$= 40 - \left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots\text{up to 20 terms}\right)$

$= 40 - \left(\frac{1-\left(\frac{1}{2}\right)^{20}}{1-\frac{1}{2}}\right) = 40 - 2 + \frac{1}{2^{19}} = 38 + \frac{1}{2^{19}}$

28. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{j} - \hat{k}$ and a vector $\vec{b}$ be such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 3$. Then $|\vec{b}|$ equals :

(1) $\frac{11}{3}$ (2) $\frac{11}{\sqrt{3}}$ (3) $\sqrt{\frac{11}{3}}$ (4) $\frac{\sqrt{11}}{3}$

Ans. **(3)**

Sol. $\vec{a}\times\vec{b} = \vec{c}$

$\vec{a}\times\left(\vec{a}\times\vec{b}\right) = \vec{a}\times\vec{c}$

$\left(\vec{a}.\vec{b}\right)\vec{a} - \left(\vec{a}.\vec{a}\right)\vec{b} = \vec{a}\times\vec{c}$

$3\vec{a} - 3\vec{b} = -2\hat{i} + \hat{j} + \hat{k}$

$3i + 3j + 3\hat{k} - 3\hat{b} = -2\hat{i} + \hat{j} + \hat{k}$

$\vec{b} = \frac{1}{3}\left(5\hat{i} + 2\hat{j} + 2\hat{k}\right)$

$|\vec{b}| = \frac{\sqrt{25+4+4}}{3}$

$|\vec{b}| = \sqrt{\frac{11}{3}}$