FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 3:00 PM to 6:00 PM**

PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

1. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : Two identical balls A and B thrown with same velocity 'u' at two different angles with horizontal attained the same range R. If A and B reached the maximum height h_1 and h_2 respectively, then $R = 4\sqrt{h_1 h_2}$

Reason R: Product of said heights.

$$h_1 h_2 = \left(\frac{u^2 \sin^2\theta}{2g}\right) \cdot \left(\frac{u^2 \cos^2\theta}{2g}\right)$$

Choose the CORRECT answer :

(A) Both A and R are true and R is the correct explanation of A.

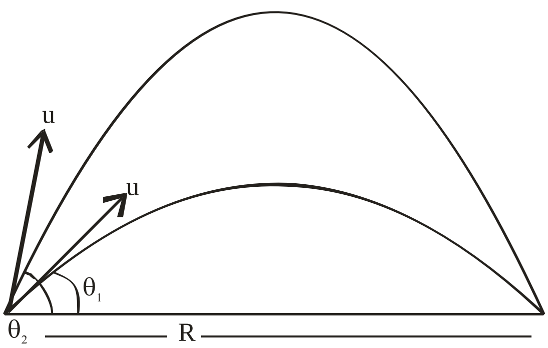
(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (A)

Sol. For same range $\theta_1 + \theta_2 = 90^0$



$$h_1 = \frac{u^2 \sin^2\theta_1}{2g} \qquad h_2 = \frac{u^2 \sin^2\theta_2}{2g}$$

$$h_1 h_2 = \frac{u^2 \sin^2\theta_1}{2g} \times \frac{u^2 \sin^2\theta_2}{2g}$$

$$\theta_2 = 90 - \theta_1$$

$$h_1 h_2 = \frac{u^2 \sin^2\theta_1}{2g} \cdot \frac{u^2 \cos^2\theta_1}{2g}$$

$$= \left[\frac{u^2 \sin\theta_1 \cos\theta_1}{2g}\right]^2$$

$$= \left[\frac{u^2 \sin\theta_1 \cos\theta_1}{2g} \times \frac{2}{2}\right]^2 = \frac{R^2}{16}$$

$$R = 4\sqrt{h_1 h_2}$$

So R is correct explanation of A

2. Two buses P and Q start from a point at the same time and move in a straight line and their positions are represented by $X_P(t) = \alpha t + \beta t^2$ and $X_Q(t) = ft - t^2$. At what time, both the buses have same velocity ?

(A) $\frac{\alpha - f}{1+\beta}$ (B) $\frac{\alpha + f}{2(\beta - 1)}$

(C) $\frac{\alpha + f}{2(1+\beta)}$ (D) $\frac{f - \alpha}{2(1+\beta)}$

Official Ans. by NTA (D)

Sol. $X_P(t) = \alpha t + \beta t^2$ $\quad X_Q = ft - t^2$

$V_P(t) = \alpha + 2\beta t$ $\quad V_Q = f - 2t$

$V_P = V_Q$

$\alpha + 2\beta t = f - 2t$

$$t = \frac{f - \alpha}{2\beta + 2}$$

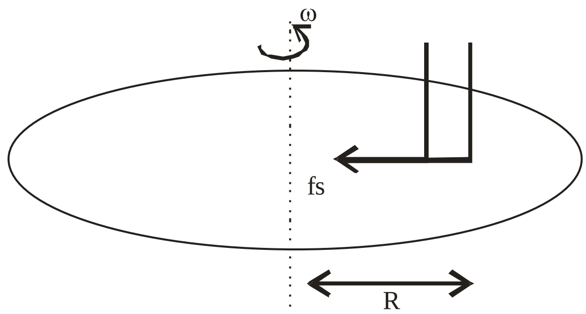
3. A disc with a flat small bottom beaker placed on it at a distance R from its center is revolving about an axis passing through the center and perpendicular to its plane with an angular velocity ω. The coefficient of static friction between the bottom of the beaker and the surface of the disc is μ. The beaker will revolve with the disc if :

(A) $R \le \frac{\mu g}{2\omega^2}$ (B) $R \le \frac{\mu g}{\omega^2}$

(C) $R \ge \frac{\mu g}{2\omega^2}$ (D) $R \ge \frac{\mu g}{\omega^2}$

Official Ans. by NTA (B)

Sol. For beaker to move with disc



$f_s = m\omega^2 R$

We know that $f_S \le f_{s\,max}$

$m\omega^2 R \le \mu mg$

$R \le \frac{\mu g}{\omega^2}$

4. A solid metallic cube having total surface area 24 m^2 is uniformly heated. If its temperature is increased by 10°C, calculate the increase in volume of the cube $\left(\text{Given} : \alpha = 5.0\times10^{-4}\,{}^\circ C^{-1}\right)$

(A) $2.4\times10^6\ cm^3$

(B) $1.2\times10^5\ cm^3$

(C) $6.0\times10^4\ cm^3$

(D) $4.8\times10^5\ cm^3$

Official Ans. by NTA (B)

Sol. Increase in volume $\Delta V = \gamma V_0 \Delta T$

$\gamma = 3\alpha$

So $\Delta V = (3\alpha) V_0 \Delta T$

Total surface area = $6a^2$, where a is side length

$24 = 6a^2$ a= 2m

Volume $V_0 = (2)^3 = 8m^3$

$\Delta V = \left(3\times5\times10^{-4}\right)(8)\times10$

$= 1.2\times10^5 cm^3$

5. A copper block of mass 5.0 kg is heated to a temperature of 500°C and is placed on a large ice block. What is the maximum amount of ice that can melt? [Specific heat of copper: $0.39\,J\,g^{-1}\,{}^\circ C^{-1}$ and latent heat of fusion of water : 335 J g^{-1}]

(A) 1.5 kg (B) 5.8 kg

(C) 2.9 kg (D) 3.8 kg

Official Ans. by NTA (C)

Sol. Heat given by block to get 0ºC temperature

$\Delta Q_1 = 5\times\left(0.39\times10^3\right)\times(500-0)$

$= 975\times10^3 J$

Heat absorbed by ice to melt m mass

$\Delta Q_2 = m\times\left(335\times10^3\right)J$

$\Delta Q_1 = \Delta Q_2$

$m\times\left(335\times10^3\right) = 975\times10^3$

$m = \frac{975}{335} = 2.910\,kg$

6. The ratio of specific heats $\left(\frac{C_P}{C_V}\right)$ in terms of degree of freedom (f) is given by:

(A) $\left(1+\frac{f}{3}\right)$ (B) $\left(1+\frac{2}{f}\right)$

(C) $\left(1+\frac{f}{2}\right)$ (D) $\left(1+\frac{1}{f}\right)$

Official Ans. by NTA (B)

Sol. Molar heat capacity at constant volume $C_V = \frac{fR}{2}$

where f is degree of freedom.

Molar heat capacity at constant pressure can be written as $C_P = R + C_V = R + \frac{fR}{2} = \left(1+\frac{f}{2}\right)R$

So $\frac{C_P}{C_v} = 1 + \frac{2}{f}$

7. For a particle in uniform circular motion, the acceleration $\vec{a}$ at any point $P(R,\theta)$ on the circular path of radius R is (when θ is measured from the positive x –axis and v is uniform speed) :

(A) $-\frac{v^2}{R}\sin\theta\hat{i} + \frac{v^2}{R}\cos\theta\hat{j}$

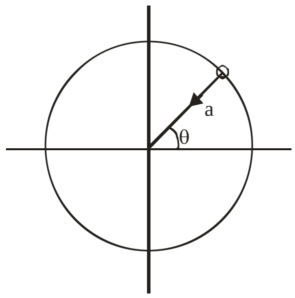
(B) $-\frac{v^2}{R}\cos\theta\hat{i} + \frac{v^2}{R}\sin\theta\hat{j}$

(C) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$

(D) $-\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

Official Ans. by NTA (C)

Sol. $a = |\vec{a}| = \frac{V^2}{R}$



$\vec{a} = -a\cos\theta\hat{i} - a\sin\theta\hat{j}$

$= -\frac{V^2}{R}\cos\theta\hat{i} - \frac{V^2}{R}\sin\theta\hat{j}$

8. Two metallic plates form a parallel plate capacitor. The distance between the plates is 'd'. A metal sheet of thickness $\frac{d}{2}$ and of area equal to area of each plate is introduced between the plates. What will be the ratio of the new capacitance to the original capacitance of the capacitor ?

(A) 2:1 (B) 1:2

(C) 1:4 (D) 4:1

Official Ans. by NTA (A)

Sol. $C_1 = \frac{\epsilon_0 A}{d}$

$C_2 = \frac{\epsilon_0 A}{\frac{d}{2} + \frac{d/2}{\infty}} = \frac{2\epsilon_0 A}{d}$

$\frac{C_2}{C_1} = \frac{2}{1}$

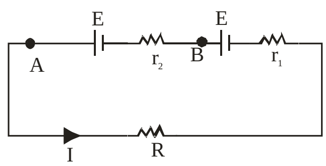
9. Two cells of same emf but different internal resistances r_1 and r_2 are connected in series with a resistance R. The value of resistance R, for which the potential difference across second cell is zero, is

(A) $r_2 - r_1$ (B) $r_1 - r_2$

(C) r_1 (D) r_2

Official Ans. by NTA (A)

Sol. $I = \frac{2E}{R + r_1 + r_2}$ **.....(i)**



But $V_A - V_B = E - Ir_2 = 0$

$\Rightarrow I = \frac{E}{r_2}$ ....(ii)

Comparing values of I from (i) and (ii)

$\frac{E}{r_2} = \frac{2E}{R + r_1 + r_2}$

$\Rightarrow R = r_2 - r_1$

10. Given below are two statements:

Statement – I : Susceptibilities of paramagnetic and ferromagnetic substances increase with decrease in temperature.

Statement – II: Diamagnetism is a result of orbital motions of electrons developing magnetic moments opposite to the applied magnetic field.

Choose the **CORRECT** answer from the options given below : -

(A) Both statement – I and statement -II are true.

(B) Both statement – I and Statement – II are false.

(C) Statement – I is true but statement – II is false.

(D) Statement-I is false but Statement-II is true.

Official Ans. by NTA (A)

Sol. According to curie's law, magnetic susceptibility is inversely proportional to temperature for a fixed value of external magnetic field i.e. $\chi = \frac{C}{T}$.

The same is applicable for ferromagnet & the relation is given as $\chi = \frac{C}{T - T_C}$ (T_C is curie temperature)

Diamagnetism is due to non-cooperative behaviour of orbiting electrons when exposed to external magnetic field.

Hence option (A).

11. A long solenoid carrying a current produces a magnetic field B along its axis. If the current is doubled and the number of turns per cm is halved, the new value of magnetic field will be equal to

(A) B (B) 2 B

(C) 4 B (D) $\frac{B}{2}$

Official Ans. by NTA (A)

Sol. $B_1 = \mu_0 nI$

$B_2 = \mu_0\left(\frac{n}{2}\right)(2I)$

$\Rightarrow B_1 = B_2$

12. A sinusoidal voltage V(t) = 210 sin 3000t volt is applied to a series LCR circuit in which L = 10 mH, C = 25 μF and R = 100Ω. The phase difference (Φ) between the applied voltage and resultant current will be :

(A) $\tan^{-1}$ (0.17) (B) $\tan^{-1}$ (9.46)

(C) $\tan^{-1}$ (0.30) (D) $\tan^{-1}$ (13.33)

Official Ans. by NTA (A)

Sol. $X_L = 10^{-2} \times 3000 = 30\Omega$

$X_C = \frac{1}{3000 \times 25 \times 10^{-6}} = \frac{40}{3}\Omega$

$X = X_L - X_C$

$= 30 - \frac{40}{3} = \frac{50}{3}$

$\tan\delta = \frac{X}{R} = \frac{50}{3 \times 100} = \frac{1}{6}$

$\delta = \tan^{-1}\left(\frac{1}{6}\right) = \tan^{-1}(0.17)$

13. The electromagnetic waves travel in a medium at a speed of 2.0×10^8 m/s.The relative permeability of the medium is 1.0. The relative permittivity of the medium will be:

(A) 2.25 (B) 4.25

(C) 6.25 (D) 8.25

Official Ans. by NTA (A)

Sol. $V = 2 \times 10^8 \,m/s$

$C = 3 \times 10^8 \,m/s$

$\frac{C}{V} = \sqrt{\mu_r \in_r}$

$\frac{9}{4} = 1 \times \in_r$

$\in_r = \frac{9}{4} = 2.25$

14. The interference pattern is obtained with two coherent light sources of intensity ratio 4 :1. And the ratio $\frac{I_{max}+I_{min}}{I_{max}-I_{min}}$ is $\frac{5}{x}$. Then, the value of x will be equal to :

(A) 3 (B) 4

(C) 2 (D) 1

Official Ans. by NTA (B)

Sol. $\frac{I_1}{I_2}=4$

$$\frac{I_{max}}{I_{min}}=\left[\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right]^2$$

$$\frac{I_{max}}{I_{min}}=\left[\frac{2\sqrt{I_2}+\sqrt{I_2}}{2\sqrt{I_2}-\sqrt{I_2}}\right]^2$$

$$\frac{I_{max}}{I_{min}}=9$$

$$\frac{I_{max}+I_{min}}{I_{max}-I_{min}}=\frac{10}{8}$$

$$\frac{5}{x}=\frac{10}{8}$$

x = 4

15. A light whose electric field vectors are completely removed by using a good Polaroid, allowed to incident on the surface of the prism at Brewster's angle. Choose the most suitable option for the phenomenon related to the prism.

(A) Reflected and refracted rays will be perpendicular to each other

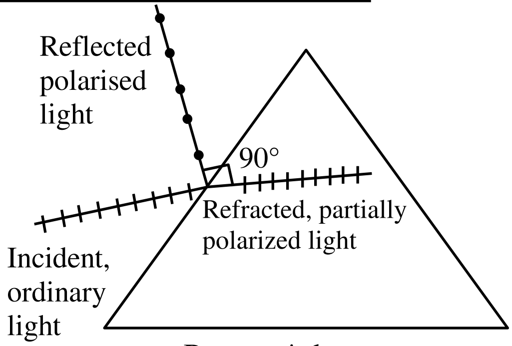
(B) Wave will propagate along the surface of prism

(C) No refraction, and there will be total reflection of light.

(D) No reflection and there will be total transmission of light.

Official Ans. by NTA (D)

Sol.



Brewster's law

But as the incident light electric field vectors are completely removed so there will be no reflection and there will be total transmission of light, explained by an experiment in NCERT.

[Reference NCERT Part-2 Pg-380, (A special case of total transmission)]

Note : Since direction of polarization is not mentioned hence most suitable option (D) corresponding to case in which electric field is absent perpendicular to plane consisting incident and normal.

16. A proton, a neutron, an electron and an α-particle have same energy. If $\lambda_p, \lambda_n, \lambda_e$ and λ_α are the de Broglie's wavelengths of proton, neutron, electron and α particle respectively, then choose the correct relation from the following :

(A) $\lambda_p=\lambda_n>\lambda_e>\lambda_\alpha$

(B) $\lambda_\alpha<\lambda_n<\lambda_p<\lambda_e$

(C) $\lambda_e<\lambda_p=\lambda_n>\lambda_\alpha$

(D) $\lambda_e=\lambda_p=\lambda_n=\lambda_\alpha$

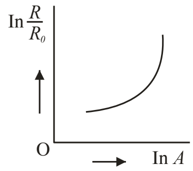
Official Ans. by NTA (B)

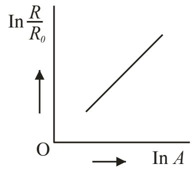
Sol. $\lambda=\frac{h}{\sqrt{2Em}}$

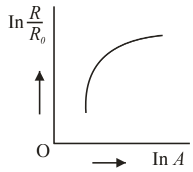
$$\lambda\propto\frac{1}{\sqrt{m}}$$

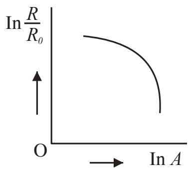
$\therefore$ $\lambda_e>\lambda_p>\lambda_n>\lambda_\alpha$

17. Which of the following figure represents the variation of $\ln\left(\frac{R}{R_0}\right)$ with ln A(If R = radius of a nucleus and A = its mass number)

(A)


(B)


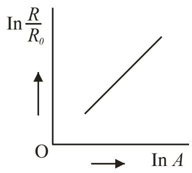
(C)


(D)


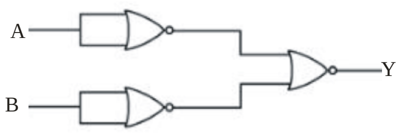
Official Ans. by NTA (B)

Sol. $R = R_0 A^{\frac{1}{3}}$

$$\ln\frac{R}{R_0} = \frac{1}{3}\ln A$$



18. Identify the logic operation performed by the given circuit :



(A) AND gate (B) OR gate

(C) NOR gate (D) NAND gate

Official Ans. by NTA (A)

Sol. $=\left[\overline{\left[\overline{A+A}\right]+\left[\overline{B+B}\right]}\right]$

$Y = \overline{\overline{A}+\overline{B}}$ (D' MORGAN LAW)

Y = AB

19. Match List I with List II

List -I		List – II	
A	Facsimile	I.	Static Document Image
B.	Guided media Channel	II.	Local Broadcast Radio
C.	Frequency Modulation	III.	Rectangular wave
D.	Digital Signal	IV.	Optical Fiber

Choose the correct answer from the following options :

(A) A –IV, B-III, C-II, D-I

(B)A-I, B-IV, C-II, D-III

(C) A –IV, B-II, C-III, D-I

(D) A-I, B-II, C-III, D-IV

Official Ans. by NTA (B)

Sol. Question based on the theory given in NCERT.

20. If n represents the actual number of deflections in a converted galvanometer of resistance G and shunt resistance S. Then the total current I when its figure of merit is K will be :

(A) $\frac{KS}{(S+G)}$ (B) $\frac{(G+S)}{nKS}$

(C) $\frac{nKS}{(G+S)}$ (D) $\frac{nK(G+S)}{S}$

Official Ans. by NTA (D)

Sol.

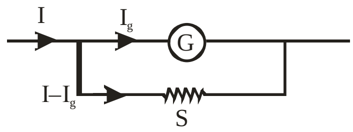


Figure of merit $\frac{I_g}{\theta} = K$

$I_g = Kn$

$I = \frac{I_g}{s}(G+S)$

$I = \frac{nK}{S}(G+S)$

SECTION-B

1. For $z = a^2x^3y^{\frac{1}{2}}$, where 'a' is a constant. If percentage error in measurement of 'x' and 'y' are 4% and 12%, respectively, then the percentage error for 'z' will be %.

Official Ans. by NTA (18)

Sol. $z = a^2x^3y^{1/2}$

$$\frac{\Delta z}{z} = \frac{2\Delta a}{a} + \frac{3\Delta x}{x} + \frac{1}{2}\frac{\Delta y}{y}$$

a is constant

$$\frac{\Delta z}{z} \times 100 = 3(4\%) + \frac{1}{2}(12\%) = 18\%$$

2. A curved in a level road has a radius 75m. The maximum speed of a car turning this curved road can be 30 m/s without skidding. If radius of curved road is changed to 48 m and the coefficient of friction between the tyres and the road remains same, then maximum allowed speed would be__ m/s.

Official Ans. by NTA (24)

Sol. $f_{s\max} = \frac{mv^2}{R}$

$\mu mg = \frac{mv^2}{R}$

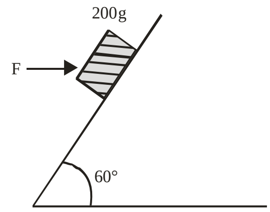
$v = \sqrt{\mu Rg}$

$\frac{v_2}{v_1} = \sqrt{\frac{R_2}{R_1}}$

$\frac{v_2}{30} = \sqrt{\frac{48}{75}}$

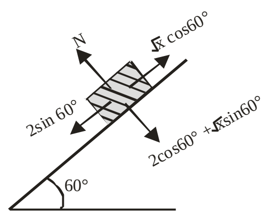
$v_2 = 24 m/s$

3. A block of mass 200 g is kept stationary on a smooth inclined plane by applying a minimum horizontal force $F = \sqrt{x}N$ as shown in figure. The value of x = ________.



Official Ans. by NTA (12)

Sol. mg = 2N



$\sqrt{x}\frac{1}{2} = \frac{2\sqrt{3}}{2}$

x = 12

4. Moment of Inertia (M.I.) of four bodies having same mass 'M' and radius '2R' are as follows:

I_1= M.I. of solid sphere about its diameter

I_2 = M.I. of solid cylinder about its axis

I_3 = M.I. of solid circular disc about its diameter

I_4 = M.I. of thin circular ring about its diameter

If $2(I_2 + I_3) + I_4 = x.\ I_1$ then the value of x will be ____

Official Ans. by NTA (5)

Sol. $I_1 = \frac{2}{5}M(2R)^2 = \frac{8}{5}MR^2$

$I_1 = \frac{1}{2}M(2R)^2 = 2MR^2$

$I_3 = \frac{M(2R)^2}{4} = MR^2$

$I_4 = \frac{M(2R)^2}{2} = 2MR^2$

$2(I_2 + I_3) + I_4 = x\ I_1$

$8MR^2 = x\frac{8}{5}MR^2$

$x = 5$

5. Two satellites S_1 and S_2 are revolving in circular orbits around a planet with radius R_1 = 3200 km and R_2 = 800 km respectively. The ratio of speed of satellite S_1 to the speed of satellite S_2 in their respective orbits would be $\frac{1}{x}$ where x =

Official Ans. by NTA (2)

Sol. $V = \frac{GM}{r} \Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{800}{3200}} = \frac{1}{2}$

6. When a gas filled in a closed vessel is heated by raising the temperature by 1°C, its pressure increase by 0.4%. The initial temperature of the gas is______ K.

Official Ans. by NTA (250)

Sol. $pV = nRT$

$\Delta P.V = nR\Delta T$

$\Rightarrow \quad \frac{\Delta P}{P} = \frac{\Delta T}{T} = \frac{0.4}{100}$

$\Rightarrow \quad T = \frac{100\times 1}{0.4} = 250K$

7. 27 identical drops are charged at 22V each. They combine to form a bigger drop. The potential of the bigger drop will be ____ V.

Official Ans. by NTA (198)

Sol. $q \to nq$

$n\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(r')^3$

$\Rightarrow \quad r' = n^{\frac{1}{3}}r$

$V = \frac{kq}{r} \propto \frac{n}{n^{1/3}} \propto n^{2/3} \propto 27^{2/3} \Rightarrow v' = 9V = 9\times 22 = 198$

8. The length of a given cylindrical wire is increased to double of its original length. The percentage increase in the resistance of the wire will be ____%.

Official Ans. by NTA (300)

Sol. V' = V

$\ell'A = \ell A$

$2\ell A' = \ell A$

$A' = \frac{A}{2}$

$R = \rho\frac{\ell}{A}$ ... (i)

$\ell' = 2\ell$

$A' = \frac{A}{2}$

$R' = \frac{\rho\ell'}{A'} = \frac{\rho 2\ell}{\frac{A}{2}}$

$R' = \frac{4\rho\ell}{A}$

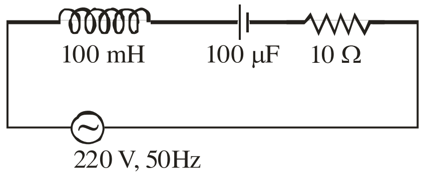
R' = 4R from equation (i)

% increase in resistance

$= \frac{R' - R}{R}\times 100 = \frac{4R - R}{R}\times 100$

= 300 %

9. In a series LCR circuit, the inductance, capacitance and resistance are L = 100mH, C = 100μF and R = 10Ω respectively. They are connected to an AC source of voltage 220V and frequency of 50 Hz. The approximate value of current in the circuit will be_____ A.



Official Ans. by NTA (22)

Sol. $X_L = \omega L = 2\pi \times 50 \times 10^{-1} = 10\pi$

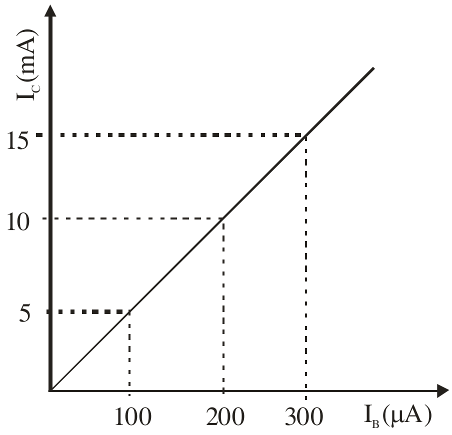
$$X_X = \frac{1}{\omega C} = \frac{1}{2\pi \times 50} \times 10^4 = \frac{100}{\pi}$$

$R = 10\Omega$

$$Z = \sqrt{\left(10\pi - \frac{100}{\pi}\right)^2 + 10^2} \simeq 10\Omega$$

$$i = \frac{E}{2} \simeq \frac{220}{10} \simeq 22\text{Amp}$$

10. In an experiment of CE configuration of n-p-n transistor, the transfer characteristics are observed as given in figure.



If the input resistance is 200Ω and output resistance is 60 Ω the voltage gain in this experiment will be ______

Official Ans. by NTA (15)

Sol. $$\text{Voltage Gain} = \frac{I_C}{I_B} \times \frac{R_0}{R_I} = \frac{10 \times 10^{-3}}{200 \times 10^{-6}} \times \frac{60}{200} = 15$$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY **TEST PAPER WITH SOLUTION**

SECTION-A

1. The minimum energy that must be possessed by photons in order to produce the photoelectric effect with platinum metal is:

[Given: The threshold frequency of platinum is $1.3 \times 10^{15}\ s^{-1}$ and $h = 6.6 \times 10^{-34}$ J s.]

(A) 3.21×10^{-14} J (B) 6.24×10^{-16} J

(C) 8.58×10^{-19} J (D) 9.76×10^{-20} J

Official Ans. by NTA (C)

Sol. $W = h\nu$

$= 6.6 \times 10^{-34} \times 1.3 \times 10^{15}$

$= 8.58 \times 10^{-19}$ J

2. At 25°C and 1 atm pressure, the enthalpy of combustion of benzene (1) and acetylene (g) are –3268 kJ mol^{-1} and –1300 kJ mol^{-1}, respectively. The change in enthalpy for the reaction $3\ C_2H_2(g) \rightarrow C_6H_6(l)$, is

(A) + 324 kJ mol^{-1} (B) +632 kJ mol^{-1}

(C) – 632 kJ mol^{-1} (D) – 732 kJ mol^{-1}

Official Ans. by NTA (C)

Sol. $\Delta H = \Sigma \Delta H_{Combustion}$ (Reactant) - $\Sigma \Delta H_{Combustion}$ (Product)

$= 3 \times (-1300) - [-3268]$

$= -632$ kJ mol^{-1}

3. Solute A associates in water. When 0.7 g of solute A is dissolved in 42.0 g of water, it depresses the freezing point by 0.2°C. The percentage association of solute A in water, is

[Given : Molar mass of A = 93 g mol^{-1}. Molal depression constant of water is 1.86 K kg mol^{-1}]

(A) 50 % (B) 60 %

(C) 70 % (D) 80 %

Official Ans. by NTA (D)

Sol. $\Delta T = i\ k_f \times m$

$$0.2 = i \times 1.86 \times \frac{0.7}{93} \times \frac{1000}{42}$$

$$i = \frac{0.2 \times 93 \times 6}{1.86 \times 100}$$

$i = 0.60$

$$2A \rightleftharpoons A_2$$

$$1 - \alpha \qquad \frac{\alpha}{2}$$

$$i = 1 - \alpha + \frac{\alpha}{2}$$

$$i = 1 - \frac{\alpha}{2}$$

$$1 - \frac{\alpha}{2} = 0.60$$

$$1 - 0.60 = \frac{\alpha}{2}$$

$\alpha = 0.80$

4. The K_{sp} for bismuth sulphide (Bi_2S_3) is 1.08×10^{-73}. The solubility of Bi_2S_3 in mol L^{-1} at 298 K is

(A) 1.0×10^{-15} (B) 2.7×10^{-12}

(C) 3.2×10^{-10} (D) 4.2×10^{-8}

Official Ans. by NTA (A)

Sol. $$Bi_2S_3 \rightleftharpoons \underset{2s}{2Bi^{3+}} + \underset{3s}{3S^{2-}}$$

$k_{sp} = (2s)^2\ (3s)^3$

$= 4s^2 \times 27\ (s)^3$

$= 108\ (s)^5$

$$(s)^5 = \frac{1.08 \times 10^{-73}}{108}$$

$\Rightarrow s = 10^{-15}$

5. Match List I with List II.

List I	List II
A. Zymase	I. Stomach
B. Diastase	II. Yeast
C. Urease	III. Malt
D. Pepsin	IV. Soyabean

Choose the correct answer from the options given below:

(A) A-II, B-III, C-I, D-IV

(B) A-II, B-III, C-IV, D-I

(C) A-III, B-II, C-IV, D-I

(D) A-III, B-II, C-I, D-IV

Official Ans. by NTA (B)

Sol. Zymase naturally occurs in yeast.

Diastase is found in malt.

Urease is found in soyabean

Pepsin is found in stomach

6. The correct order of electron gain enthalpies of Cl, F, Te and Po is

(A) F < Cl < Te < Po (B) Po < Te < F < Cl

(C) Te < Po < Cl < F (D) Cl < F < Te < Po

Official Ans. by NTA (D)

Sol. As Cl has maximum electron affinity among all elements.

Element	**$\Delta_{eg}H$ (kJ/mol)**
F	–328
Cl	–349
Te	–190
Po	–174

7. Given below are two statements.

Statement I: During electrolytic refining, blister copper deposits precious metals

Statement II: In the process of obtaining pure copper by electrolysis method, copper blister is used to make the anode.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II is false.

(D) Statement I is false but Statement II is true.

Official Ans. by NTA (A)

Sol. In the electro-refining, impure metal (here blister copper) is used as an anode while precious metal like Au, Pt get deposited as anode mud.

8. Given below are two statements one is labelled as **Assertion A** and the other is labelled as **Reason R:**

Assertion A : The amphoteric nature of water is explained by using Lewis acid/base concept.

Reason R : Water acts as an acid with NH_3 and as a base with H_2S.

In the light of the above statements choose the correct answer from the options given below :

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Official Ans. by NTA (D)

Sol. $$\underset{\text{Acid}}{H_2S} + \underset{\text{Base}}{H_2O} \rightleftharpoons H_3O^+ + HS^-$$

$$\underset{\text{Acid}}{H_2O} + \underset{\text{Base}}{NH_3} \rightleftharpoons NH_4OH$$

9. The correct order of reduction potentials of the following pairs is

A. Cl_2/Cl^-

B. I_2/I^-

C. Ag^+/Ag

D. Na^+/Na

E. Li^+/Li

Choose the correct answer from the options given below.

(A) A > C > B > D > E

(B) A > B > C > D > E

(C) A > C > B > E > D

(D) A > B > C > E > D

Official Ans. by NTA (A)

Sol. $E^{\circ}_{Cl_2/Cl^-} = +1.36\,V$

$E^{\circ}_{I_2/I^-} = +0.54\,V$

$E^{\circ}_{Ag^+/Ag} = +0.80\,V$

$E^{\circ}_{Na^+/Na} = -2.71\,V$

$E^{\circ}_{Li^+/Li} = -3.05\,V$

10. The number of bridged oxygen atoms present in compound B formed from the following reactions is

$$Pb(NO_3)_2 \xrightarrow{673\text{ K}} A + PbO + O_2$$

$$A \xrightarrow{\text{Dimerise}} B$$

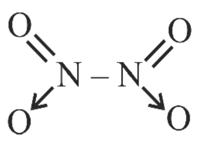
(A) 0 (B) 1

(C) 2 (D) 3

Official Ans. by NTA (A)

Sol. $$Pb(NO_3)_2 \xrightarrow[673\text{ K}]{\Delta} PbO + \underset{(A)}{NO_2} + O_2$$

$$\underset{A}{2NO_2} \xrightarrow{\text{Dimerise}} \underset{B}{N_2O_4}$$



(no bridged oxygen)

11. The metal ion (in gaseous state) with lowest spin-only magnetic moment value is

(A) V^{2+} (B) Ni^{2+}

(C) Cr^{2+} (D) Fe^{2+}

Official Ans. by NTA (B)

Sol. V^{2+} : $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^3$

$\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\ \ }\boxed{\ \ }$ (3d) (unpaired e^- = 3)

Ni^{2+} : $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^8$

$\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}$ (3d) (unpaired e^- = 2)

Cr^{2+} : $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^4$

$\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\ \ }$ (3d) (unpaired e^- = 4)

Fe^{2+} : $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^6$

$\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$ (3d) (unpaired e^- = 4)

12. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: Polluted water may have a value of BOD of the order of 17 ppm.

Reason R: BOD is a measure of oxygen required to oxidise both the biodegradable and non-biodegradable organic material in water.

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both A and R are correct and R is the correct explanation of A.

(B) Both A and R are correct but R is NOT the correct explanation of A.

(C) A is correct but R is not correct.

(D) A is not correct but R is correct.

Official Ans. by NTA (C)

Sol. Clean water have BOD less than 5 ppm while highly polluted water has BOD greater or equal to 17 ppm. So, assertion is correct.

BOD is measure of oxygen required to oxidise only bio-degradable organic matter. So, reason is false.

13. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A: A mixture contains benzoic acid and napthalene. The pure benzoic acid can be separated out by the use of benzene.

Reason R: Benzoic acid is soluble in hot water.

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Official Ans. by NTA (D)

Sol. Benzoic acid and Napthalene can be effectively separated by crystallization. Benzoic acid is soluble in hot water whereas Napthalene is insoluble.

Hence assertion is incorrect but reason is correct

14. During halogen test, sodium fusion extract is boiled with concentrated HNO_3 to

(A) remove unreacted sodium

(B) decompose cyanide or sulphide of sodium

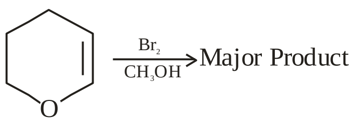
(C) extract halogen from organic compound

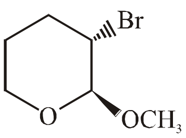
(D) maintain the pH of extract

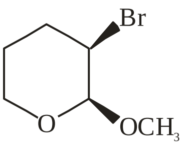
Official Ans. by NTA (B)

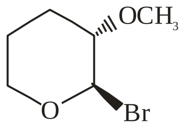
Sol. Sodium fusion extract is boiled with concentrated HNO_3 to remove sodium cyanide and sodium sulphide

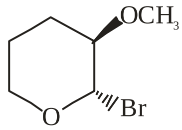
15. Amongst the following, the major product of the given chemical reaction is



(A)


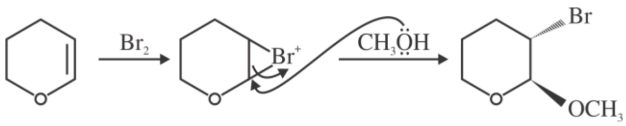
(B)


(C)


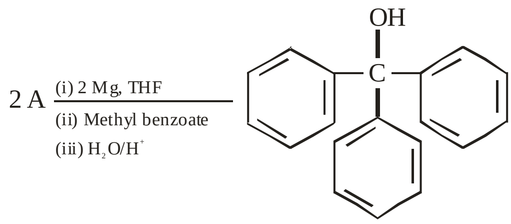
(D)


Official Ans. by NTA (A)

Sol.



16. In the given reaction



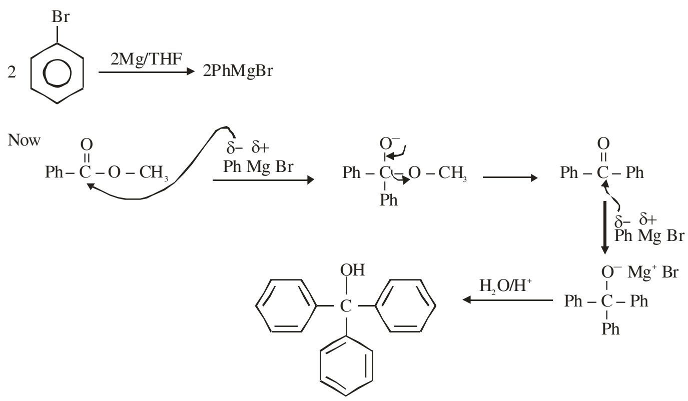
'A' can be

(A) benzyl bromide (B) bromobenzene

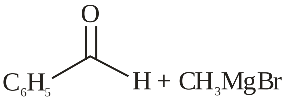
(C) cyclohexyl bromide (D) methyl bromide

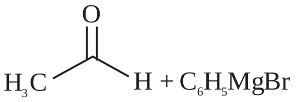
Official Ans. by NTA (B)

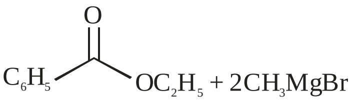
Sol.



17. Which of the following conditions or reaction sequence will NOT give acetophenone as the major product ?

(A) (a)  (b) $Na_2Cr_2O_7, H^+$

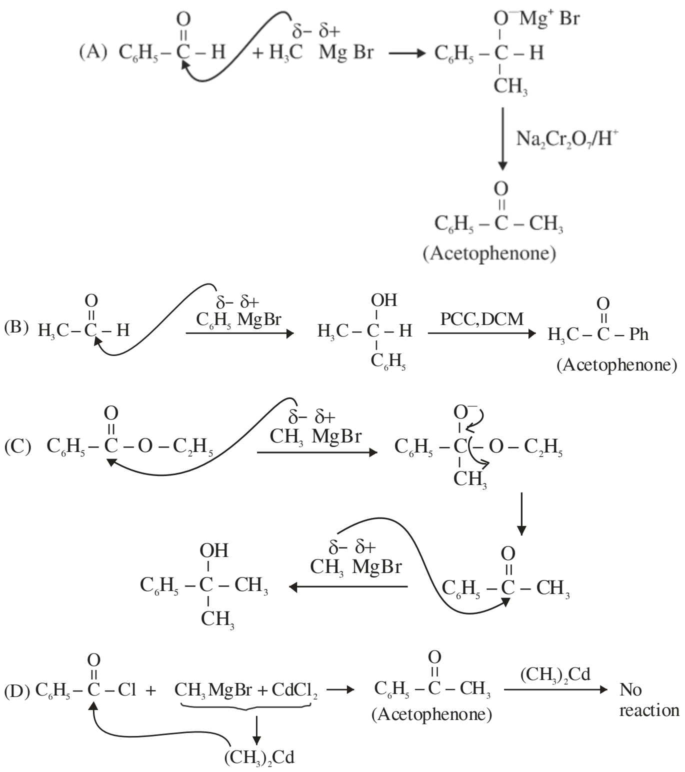
(B) (a)  (b) PCC, DCM

(C) 

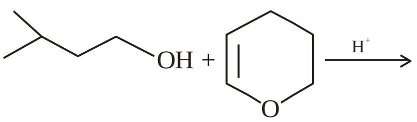
(D) 

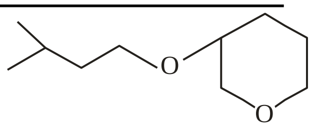
Official Ans. by NTA (C)

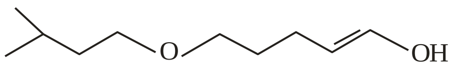
Sol.

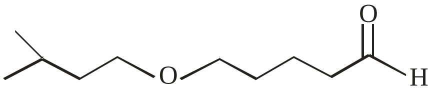


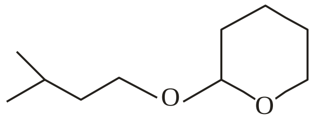
18. The major product formed in the following reaction, is



(A) 

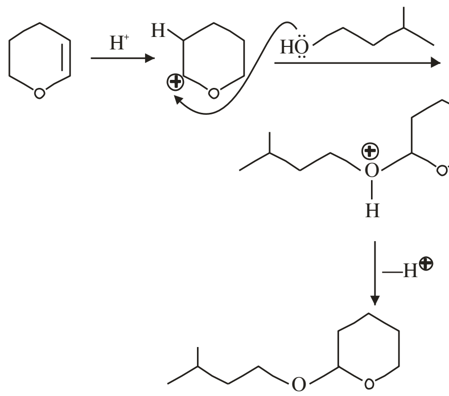
(B) 

(C) 

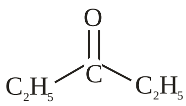
(D) 

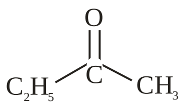
Official Ans. by NTA (D)

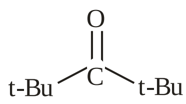
Sol.

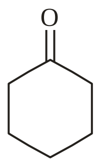


19. Which of the following ketone will NOT give enamine on treatment with secondary amines? [where t-Bu is $-C(CH_3)_3$]

(A) 

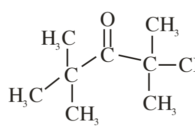
(B) 

(C) 

(D) 

Official Ans. by NTA (C)

Sol. Enamine formation is an example of nucleophilic addition elimination reaction

Since in ketone  Carbonyl

Group is highly sterically hindered hence attack of nucleophile will not be possible.

20. An antiseptic dettol is a mixture of two compounds 'A' and 'B' where A has 6π electrons and B has 2π electrons. What is 'B'?

(A) Bithionol

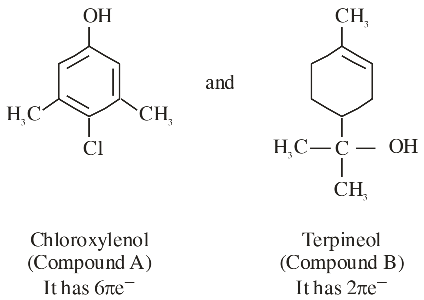
(B) Terpineol

(C) Chloroxylenol

(D) Chloramphenicol

Official Ans. by NTA (B)

Sol. Dettol is mixture of



Hence compound 'B' is Terpineol.

SECTION-B

1. A protein 'A' contains 0.30% of glycine (molecular weight 75). The minimum molar mass of the protein 'A' is________ $\times\ 10^3$ g mol^{-1} [nearest integer]

Official Ans. by NTA (25)

Sol. 0.30 % glycine is equal to 75

$$1\ \% \longrightarrow \frac{75}{0.30}$$

$$100\ \% \longrightarrow \frac{75}{0.30}\times 100$$

$$= 25000 \text{ g}$$

2. A rigid nitrogen tank stored inside a laboratory has a pressure of 30 atm at 06:00 am when the temperature is 27 °C. At 03:00 pm, when the temperature is 45°C, the pressure in the tank will be ____________atm. [nearest integer]

Official Ans. by NTA (32)

Sol. $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$$\frac{30}{300} = \frac{P_2}{318}$$

$$P_2 = \frac{30}{300}\times 318$$

$$= \frac{1}{10}\times 318$$

$$= 32$$

3. Amongst BeF_2, BF_3, H_2O, NH_3, CCl_4 and HCl, the number of molecules with non-zero net dipole moment is ___________.

Official Ans. by NTA (3)

Sol. BeF_2, BF_3 and $CCl_4 \Rightarrow \mu_{net} = 0$

H_2O, NH_3 and $HCl \Rightarrow \mu_{net} \neq 0$

4. At 345 K, the half life for the decomposition of a sample of a gaseous compound initially at 55.5 kPa was 340 s. When the pressure was 27.8 kPa, the half life was fund to be 170 s. The order of the reaction is_________. [integer answer]

Official Ans. by NTA (0)

Sol. $t_{1/2} \times \frac{1}{[P_0]^{n-1}}$

$$\frac{t_1}{t_2} = \frac{(P_2)^{n-1}}{(P_1)^{n-1}}$$

$$\frac{340}{170} = \left(\frac{27.8}{55.5}\right)^{n-1}$$

$$\Rightarrow\ 2 = \frac{1}{(2)^{n-1}}$$

$$n = 0$$

5. A solution of $Fe_2(SO_4)_3$ is electrolyzed for 'x' min with a current of 1.5 A to deposit 0.3482 g of Fe. The value of x is______. [nearest integer]

Given : 1 F = 96500 C mol^{-1}

Atomic mass of Fe = 56 g mol^{-1}

Official Ans. by NTA (20)

Sol. $Fe^{3+} + 3e^- \longrightarrow Fe$

$3F \longrightarrow$ 1 mole Fe is deposited

For 56 g $\longrightarrow 3 \times 96500$ (required charge)

For 1g $\longrightarrow \frac{3\times 96500}{56}$ (required charge)

For 0.3482 g $\longrightarrow \frac{3\times 96500}{56}\times 0.3482$

$= 1800.06$

$Q = it$

$1800.06 = 1.5\ t$

$t = 20$ min

6. Consider the following reactions :

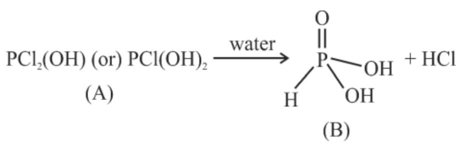
$PCl_3 + H_2O \longrightarrow A + HCl$

$A + H_2O \longrightarrow B + HCl$

number of ionisable protons present in the product B________.

Official Ans. by NTA (2)

Sol. $PCl_3 + H_2O \xrightarrow[\text{hydrolysis}]{\text{Partial}} PCl_2(OH)$ (or) $PCl(OH)_2 +$ HCl



no. of ionisable protons in B = 2

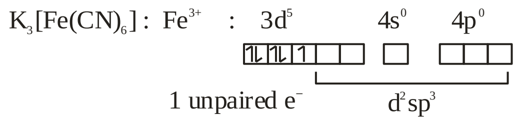
7. Amongst $FeCl_3.3H_2O$, $K_3[Fe(CN)_6]$ and $[Co(NH_3)_6]Cl_3$, the spin-only magnetic moment value of the inner-orbital complex that absorbs light at shortest wavelength is________ B.M. [nearest integer]

Official Ans. by NTA (2)

Sol. $[Fe(H_2O)_3Cl_3]$, $\underbrace{K_3[Fe(CN)_6], [Co(NH_3)_6]Cl_3}_{\text{inner orbital complexes}}$

$K_3[Fe(CN)_6]$ has more value of Δ_0 than that of $[Co(NH_3)_6]Cl_3$; as $\bar{C}N$ is stronger ligand.

More $\Delta_0 \Rightarrow$ smaller value of absorbed λ



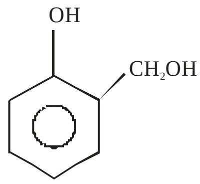
Spin only magnetic moment (μ) = $\sqrt{3}$ BM

= 1.732 BM

Rounding off $\Rightarrow$ 2

8. The Novolac polymer has mass of 963 g. The number of monomer units present in it are

Official Ans. by NTA (9)

Sol. Monomer unit of Novolac is  its molecular mass is 124 amu.

Upon considering molecular weight of polymer as 963 amu (In question its given as 963 gram)

Now if during formation of Novolac, (n–1) unit of water are removed then

$n\times 124 = 963 + [18\times(n-1)]$

n = 9

9. How many of the given compounds will give a positive Biuret test ______ ? Glycine, Glycylalanine, Tripeptide, Biuret

Official Ans. by NTA (2)

Sol. Biuret test is given by all proteins and peptides having atleast two peptide linkages.

Hence positive test must be given by tripeptide and Biuret.

10. The neutralization occurs when 10 mL of 0.1 M acid 'A' is allowed to react with 30 mL of 0.05 M base $M(OH)_2$. The basicity of the acid 'A' is______. [M is a metal]

Official Ans. by NTA (3)

Sol. Acid + Base $\longrightarrow$ Salt + H_2O

0.1 M	$M(OH)_2$	
10ml	0.05 M	
	30 ml	

at equivalence point

equivalent of acid = equivalent of base

$0.1 \times 10 \times n = 30 \times 0.05 \times 2$

n = 3

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let $A=\{x\in R:|x+1|<2\}$ and $B=\{x\in R:|x-1|\ge 2\}$. Then which one of the following statements is **NOT** true ?

(A) $A-B=(-1,1)$ (B) $B-A=R-(-3,1)$

(C) $A\cap B=(-3,-1]$ (D) $A\cup B=R-[1,3)$

Official Ans. by NTA (B)

Sol. $A: x\in(-3,1)$ $B: x\in(-\infty,-1]\cup[3,\infty)$

$B-A=(-\infty,-3]\cup[3,\infty)=R-(-3,3)$

2. Let $a,b\in R$ be such that the equation $ax^2-2bx+15=0$ has a repeated root α. If α and β are the roots of the equation $x^2-2bx+21=0$, then $\alpha^2+\beta^2$ is equal to:

(A) 37 (B) 58

(C) 68 (D) 92

Official Ans. by NTA (B)

Sol. $ax^2-2bx+15=0$

$2\alpha=\frac{2b}{a},\alpha^2=\frac{15}{a}$

$\frac{\alpha}{2}=\frac{15}{2b}$

$\alpha=\frac{15}{b}$

$x^2-2bx+21=0$

$\left(\frac{15}{b}\right)^2-2b\left(\frac{15}{b}\right)+21=0$

$b^2=25$

$\alpha+\beta=2b,\ \alpha\beta=21$

$\alpha^2+\beta^2=4b^2-42$

$=58$

3. Let z_1 and z_2 be two complex numbers such that $\overline{z}_1=i\overline{z}_2$ and $\arg\left(\frac{z_1}{\overline{z}_2}\right)=\pi$. Then

(A) $\arg z_2=\frac{\pi}{4}$ (B) $\arg z_2=-\frac{3\pi}{4}$

(C) $\arg z_1=\frac{\pi}{4}$ (D) $\arg z_1=-\frac{3\pi}{4}$

Official Ans. by NTA (C)

Sol. $\overline{z}_1=i\overline{z}_2$

$z_1=-iz_2$

$\arg\left(\frac{z_1}{\overline{z}_2}\right)=\pi$

$\arg\left(-i\frac{z_2}{\overline{z}_2}\right)=\pi$ $\arg(z_2)=\theta$

$-\frac{\pi}{2}+\theta+\theta=\pi$

$2\theta=\frac{3\pi}{2}$

$\arg(z_2)=\theta=\frac{3\pi}{4}$, $\arg z_1=\frac{\pi}{4}$

4. The system of equations

$-kx+3y-14z=25$

$-15x+4y-kz=3$

$-4x+y+3z=4$

is consistent for all k in the set

(A) R (B) $R-\{-11,13\}$

(C) $R-\{13\}$ (D) $R-\{-11,11\}$

Official Ans. by NTA (D)

Sol. $\Delta = \begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -k \\ -4 & 1 & 3 \end{vmatrix} = 121 - k^2$

$\Delta \neq 0 \qquad k \in R - \{11, -11\}$ (Unique sol.)

If k = 11

$\Delta_z = \begin{vmatrix} -11 & 3 & 25 \\ -15 & 4 & 3 \\ -4 & 1 & 4 \end{vmatrix} \neq 0$

No solution

If k = –11

$\Delta_z = \begin{vmatrix} 11 & 3 & 25 \\ -15 & 4 & 3 \\ -4 & 1 & 4 \end{vmatrix} \neq 0$

No solution

5. $\lim_{x \to \frac{\pi}{2}} \left(\tan^2 x \left(\left(2\sin^2 x + 3\sin x + 4\right)^{\frac{1}{2}} - \left(\sin^2 x + 6\sin x + 2\right)^{\frac{1}{2}} \right) \right)$

is equal to

(A) $\frac{1}{12}$ (B) $-\frac{1}{18}$

(C) $-\frac{1}{12}$ (D) $-\frac{1}{6}$

Official Ans. by NTA (A)

Sol.

$\lim_{x \to \frac{\pi}{2}} \tan^2 x \left[\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2} \right] =$

$\lim_{x \to \frac{\pi}{2}} \frac{\tan^2 x\,[\sin^2 x - 3\sin x + 2]}{\sqrt{9} + \sqrt{9}}$

$= \lim_{x \to \frac{\pi}{2}} \frac{\tan^2 x (\sin x - 1)(\sin x - 2)}{6}$

$= \frac{1}{6} \lim_{x \to \frac{\pi}{2}} \tan^2 x (1 - \sin x)$

$= \frac{1}{6} \lim_{x \to \frac{\pi}{2}} \frac{\sin^2 x\,(1 - \sin x)}{(1 - \sin x)(1 + \sin x)} = \frac{1}{12}$

6. The area of the region enclosed between the parabolas $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is

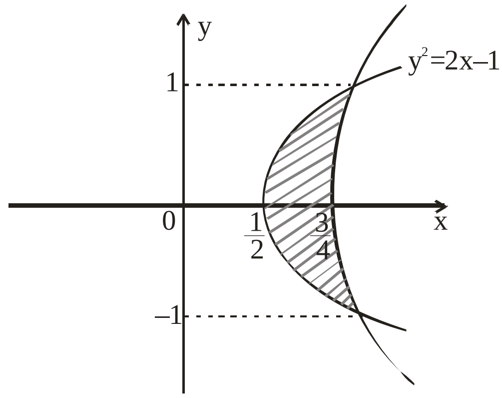
(A) $\frac{1}{3}$ (B) $\frac{1}{6}$

(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Official Ans. by NTA (A)

Sol. Required area = $2\int_0^1 \left(\frac{y^2 + 3}{4} - \frac{y^2 + 1}{2} \right) dy$

$= 2\int_0^1 \frac{1 - y^2}{4} dy = \frac{1}{2} \left| y - \frac{y^3}{3} \right|_0^1 = \frac{1}{3}$



7. The coefficient of x^{101} in the expression $(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + ...x^{500}$,

$x > 0$, is

(A) $^{501}C_{101}(5)^{399}$ (B) $^{501}C_{101}(5)^{400}$

(C) $^{501}C_{100}(5)^{400}$ (D) $^{500}C_{101}(5)^{399}$

Official Ans. by NTA (A)

Sol. $(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + ... + x^{500}$

$= \frac{(5+x)^{501} - x^{501}}{(5+x) - x} = \frac{(5+x)^{501} - x^{501}}{5}$

$\Rightarrow$ coefficient x^{101} in given expression

$= \frac{^{501}C_{101} 5^{400}}{5} = {}^{501}C_{101} 5^{399}$

8. The sum $1+2\cdot3+3\cdot3^2+.....+10\cdot3^9$ is equal to

(A) $\frac{2\cdot3^{12}+10}{4}$ (B) $\frac{19\cdot3^{10}+1}{4}$

(C) $5\cdot3^{10}-2$ (D) $\frac{9\cdot3^{10}+1}{2}$

Official Ans. by NTA (B)

Sol. $S=1\cdot3^0+2\cdot3^1+3\cdot3^2+.....+10.3^9$

$3S=1\cdot3^1+2.3^2.......................+9\times3^9+10\times3^{10}$

$-2S=(1\cdot3^0+3^1+3^2.....3^9)-10.3^{10}$

$S=5\times3^{10}-\left(\frac{3^{10}-1}{4}\right)$

$S=\frac{20.3^{10}-3^{10}+1}{4}=\frac{19.3^{10}+1}{4}$

9. Let P be the plane passing through the intersection of the planes

$\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})=5$ and $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=3$, and the point $(2,1,-2)$. Let the position vectors of the points X and Y be $\hat{i}-2\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+2\hat{k}$ respectively. Then the points

(A) X and X + Y are on the same side of P

(B) Y and Y – X are on the opposite sides of P

(C) X and Y are on the opposite sides of P

(D) X + Y and X – Y are on the same side of P

Official Ans. by NTA (C)

Sol. $P_1+\lambda P_2=0$

$\Rightarrow(x+3y-z-5)+\lambda(2x-y+z-3)=0$

$(2,1,-2)$ lies on this plane

$\therefore\lambda=1\Rightarrow$ plane is $3x+2y-8=0$

10. A circle touches both the y-axis and the line x + y = 0. Then the locus of its center is

(A) $y=\sqrt{2}x$ (B) $x=\sqrt{2}y$

(C) $y^2-x^2=2xy$ (D) $x^2-y^2=2xy$

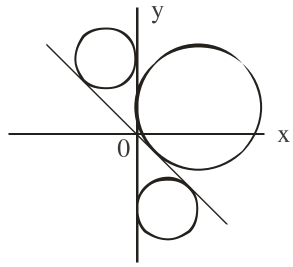
Official Ans. by NTA (D)

Sol. Let (h, k) is centre of circle

$\left|\frac{h-k}{\sqrt{2}}\right|=|h|$

$k^2-h^2+2hk=0$

$\therefore$ Equation of locus is $y^2-x^2+2xy=0$



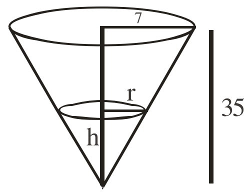
11. Water is being filled at the rate of $1\,cm^3/sec$ in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10 cm, the rate $(\text{in } cm^2/sec)$ at which the wet conical surface area of the vessel increases is

(A) 5 (B) $\frac{\sqrt{21}}{5}$

(C) $\frac{\sqrt{26}}{5}$ (D) $\frac{\sqrt{26}}{10}$

Official Ans. by NTA (C)

Sol. From figure $\frac{r}{h}=\frac{7}{35}\Rightarrow h=5r$



Given $\frac{dV}{dt}=1\Rightarrow\frac{d}{dt}\left(\frac{\pi r^2h}{3}\right)=1$

$\Rightarrow\frac{d}{dt}\left(\frac{5\pi}{3}r^3\right)=1\Rightarrow r^2\frac{dr}{dt}=\frac{1}{5\pi}$

Let wet conical surface area = S

$=\pi r\ell=\pi r\sqrt{h^2+r^2}$

$=\sqrt{26}\pi r^2\Rightarrow\frac{dS}{dt}=2\sqrt{26}\pi r\frac{dr}{dt}$

When h = 10 then r = 2 $\Rightarrow\frac{dS}{dt}=\frac{2\sqrt{26}}{10}$

12. If $b_n = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx}{\sin x} dx, n \in N$, then

(A) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in an A.P. with common difference -2

(B) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference 2

(C) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in a G.P.

(D) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference -2

Official Ans. by NTA (D)

Sol. $b_n = \int_0^{\pi/2} \frac{1+\cos 2nx}{\sin x} dx$

$b_{n+1} - b_n = \int_0^{\pi/2} \frac{\cos^2(n+1)x - \cos^2 nx}{\sin x} dx$

$= \int_0^{\pi/2} \frac{-\sin(2n+1)x \sin x}{\sin x} dx$

$= \left(\frac{\cos(2n+1)x}{2n+1}\right)_0^{\pi/2} = \frac{-1}{2n+1}$

$\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in A.P. with c.d.$= -2$

13. If y = y(x) is the solution of the differential equation $2x^2 \frac{dy}{dx} - 2xy + 3y^2 = 0$ such that $y(e) = \frac{e}{3}$, then y(1) is equal to

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) 3

Official Ans. by NTA (B)

Sol. $\frac{dy}{dx} - \frac{y}{x} = -\frac{3}{2}\left(\frac{y}{x}\right)^2$ $\quad y = vx$

$\frac{dv}{v^2} = -\frac{3dx}{2x}$

$-\frac{1}{v} = -\frac{3}{2}\ell n|x| + C$

$-\frac{x}{y} = \frac{-3}{2}\ell n|x| + C$

$x = e,\ y = \frac{e}{3}$

$C = -\frac{3}{2}$

When x = 1, y = $\frac{2}{3}$

14. If the angle made by the tangent at the point (x_0, y_0) on the curve x = 12(t + sin t cos t), $y = 12(1+\sin t)^2, 0 < t < \frac{\pi}{2}$, with the positive x-axis is $\frac{\pi}{3}$, then y_0 is equal to

(A) $6\left(3+2\sqrt{2}\right)$ (B) $3\left(7+4\sqrt{3}\right)$

(C) 27 (D) 48

Official Ans. by NTA (C)

Sol. $\frac{dy}{dx} = \frac{2(1+\sin t)\times\cos t}{1+\cos 2t}$

$\Rightarrow \frac{2(1+\sin t)\cos t}{2\cos^2 t} = \sqrt{3}$

$\Rightarrow t = \frac{\pi}{6}, y_0 = 27$

15. The value of 2sin(12°) – sin(72°) is :

(A) $\frac{\sqrt{5}\left(1-\sqrt{3}\right)}{4}$ (B) $\frac{1-\sqrt{5}}{8}$

(C) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{2}$ (D) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{4}$

Official Ans. by NTA (D)

Sol. $\sin 12° + \sin 12° - \sin 72°$

$= \sin 12° - 2\cos 42° \sin 30°$

$= \sin 12° - \sin 48°$

$= -2\cos 30° \sin 18°$

$= -2 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{5}-1}{4}$

$= \frac{\sqrt{3}}{4}(1-\sqrt{5})$

16. A biased die is marked with numbers 2,4, 8, 16, 32, 32 on its faces and the probability of getting a face with mark n is $\frac{1}{n}$. If the die is thrown thrice, then the probability, that the sum of the numbers obtained is 48, is

(A) $\frac{7}{2^{11}}$ (B) $\frac{7}{2^{12}}$

(C) $\frac{3}{2^{10}}$ (D) $\frac{13}{2^{12}}$

Official Ans. by NTA (D)

Sol. $P(n) = \frac{1}{n}$

$P(2) = \frac{1}{2}$ $P(8) = \frac{1}{8}$

$P(4) = \frac{1}{4}$ $P(16) = \frac{1}{16}$

$P(32) = \frac{2}{32}$

Possible cases

16, 16, 16 and 32, 8, 8

Probability = $\frac{1}{16^3} + \frac{2}{32} \times \frac{1}{8} \times \frac{1}{8} \times 3 = \frac{13}{16^3}$

17. The negation of the Boolean expression $((\sim q) \wedge p) \Rightarrow ((\sim p) \vee q)$ is logically equivalent to

(A) $p \Rightarrow q$ (B) $q \Rightarrow p$

(C) $\sim(p \Rightarrow q)$ (D) $\sim(q \Rightarrow p)$

Official Ans. by NTA (C)

Sol. $\sim p \vee q \equiv p \to q$

$\sim q \wedge p \equiv \sim (p \to q)$

Negation of $\sim (p \to q) \to (p \to q)$

is $\sim (p \to q) \wedge (\sim (p \to q))$ i.e. $\sim (p \to q)$

18. If the line $y = 4 + kx$, $k > 0$, is the tangent to the parabola $y = x - x^2$ at the point P and V is the vertex of the parabola, then the slope of the line through P and V is :

(A) $\frac{3}{2}$ (B) $\frac{26}{9}$

(C) $\frac{5}{2}$ (D) $\frac{23}{6}$

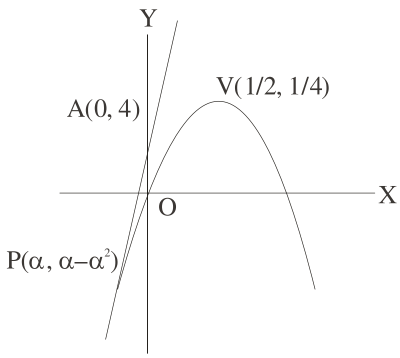
Official Ans. by NTA (C)

Sol. Slope of tangent at P = Slope of line AP

$y'|_P = 1 - 2\alpha = \frac{\alpha - \alpha^2 - 4}{\alpha}$

Solving $\alpha = -2 \Rightarrow P(-2, -6)$

Slope of PV = $\frac{5}{2}$



19. The value of $\tan^{-1}\left(\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\left(\frac{\pi}{4}\right)}\right)$ is equal to

(A) $-\frac{\pi}{4}$ (B) $-\frac{\pi}{8}$

(C) $-\frac{5\pi}{12}$ (D) $-\frac{4\pi}{9}$

Official Ans. by NTA (B)

Sol. $\tan^{-1}\left[\dfrac{\cos\left(4\pi-\dfrac{\pi}{4}\right)-1}{\sin\dfrac{\pi}{4}}\right] \Rightarrow \tan^{-1}\left(\dfrac{\cos\dfrac{\pi}{4}-1}{\sin\dfrac{\pi}{4}}\right)$

$\tan^{-1}\left(\dfrac{1-\sqrt{2}}{1}\right)=-\dfrac{\pi}{8}$

20. The line $y=x+1$ meets the ellipse $\dfrac{x^2}{4}+\dfrac{y^2}{2}=1$ at two points P and Q. If r is the radius of the circle with PQ as diameter then $(3r)^2$ is equal to

(A) 20 (B) 12

(C) 11 (D) 8

Official Ans. by NTA (A)

Sol. Ellipse $x^2+2y^2=4$

Line y = x + 1

Point of intersection

$x^2+2(x+1)^2=4$

$3x^2+4x-2=0$

$|x_1-x_2|=\dfrac{\sqrt{40}}{3}$

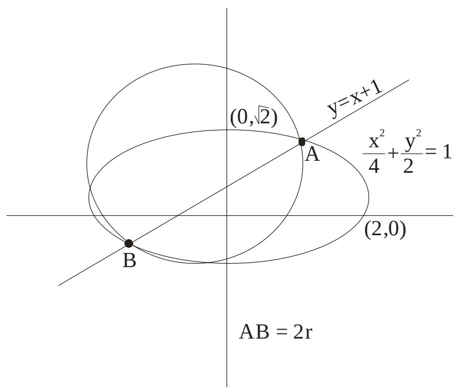
$AB=2r=|x_1-x_2|\sqrt{1+m^2}$,

m is slope of given line

$AB=\dfrac{\sqrt{40}}{3}\sqrt{1+1}$

$2r=\dfrac{\sqrt{80}}{3}\Rightarrow r=\dfrac{\sqrt{80}}{6}$

$(3r)^2=\left(3\times\dfrac{\sqrt{80}}{6}\right)^2=\dfrac{80}{4}=20$



SECTION-B

1. Let $A=\begin{pmatrix}2 & -2\\ 1 & -1\end{pmatrix}$ and $B=\begin{pmatrix}-1 & 2\\ -1 & 2\end{pmatrix}$. Then the number of elements in the set

$\{(n,m): n,m\in\{1,2,\ldots\ldots,10\}$ and $nA^n+mB^m=I\}$

is _____

Official Ans. by NTA (1)

Sol. $A^2=A$ and $B^2=B$

Therefore equation $nA^n+mB^m=I$ becomes

nA + mB = I, which gives m = n = 1

Only one set possible

2. Let $f(x)=[2x^2+1]$ and $g(x)=\begin{cases}2x-3, & x<0\\ 2x+3, & x\ge 0\end{cases}$,

where [t] is the greatest integer $\le t$. Then, in the open interval (–1, 1), the number of points where fog is discontinuous is equal to _____

Official Ans. by NTA (62)

Sol. $f(g(x))=\left[2g^2(x)\right]+1$

$=\begin{cases}\left[2(2x-3)^2\right]+1;\ x<0\\ \left[2(2x+3)^2\right]+1; x\ge 0\end{cases}$

$\therefore$ fog is discontinuous whenever $2(2x-3)^2$ or $2(2x+3)^2$ belongs to integer except x = 0.

$\therefore$ 62 points of discontinuity.

3. The value of b > 3 for which

$12\displaystyle\int_3^b \frac{1}{(x^2-1)(x^2-4)}dx=\log_e\left(\frac{49}{40}\right)$, is equal to

Official Ans. by NTA (6)

Sol. $\frac{12}{3}\left[\int_3^b\left(\frac{1}{x^2-4}-\frac{1}{x^2-1}\right)dx\right]=\log\frac{49}{40}$

$\frac{12}{3}\cdot\left[\frac{1}{4}\ell n\left|\frac{x-2}{x+2}\right|-\frac{1}{2}\ell n\left|\frac{x-1}{x+1}\right|\right]_3^b=\log\frac{49}{40}$

$\ell n\frac{(b-2)(b+1)^2}{(b+2)(b-1)^2}=\ell n\frac{49}{50}$

b = 6

4. If the sum of the coefficients of all the positive even powers of x in the binomial expansion of $\left(2x^3+\frac{3}{x}\right)^{10}$ is $5^{10}-\beta\cdot3^9$, then β is equal to _____

Official Ans. by NTA (83)

Sol. $T_{r+1}={}^{10}C_r(2x^3)^{10-r}\left(\frac{3}{x}\right)^r$

$={}^{10}C_r 2^{10-r}3^r x^{30-4r}$

Put r = 0, 1, 2, ... 7 and we get $\beta = 83$

5. If the mean deviation about the mean of the numbers 1, 2, 3, ..., n, where n is odd, is $\frac{5(n+1)}{n}$, then n is equal to _____

Official Ans. by NTA (21)

Sol. Mean deviation about mean of first n natural numbers is $\frac{n^2-1}{4n}$

$\therefore n=21$

6. Let $\vec{b}=\hat{i}+\hat{j}+\lambda\hat{k},\lambda\in R$. If $\vec{a}$ is a vector such that $\vec{a}\times\vec{b}=13\hat{i}-\hat{j}-4\hat{k}$ and $\vec{a}\cdot\vec{b}+21=0$, then $\left(\vec{b}-\vec{a}\right)\cdot\left(\hat{k}-\hat{j}\right)+\left(\vec{b}+\vec{a}\right)\cdot\left(\hat{i}-\hat{k}\right)$ is equal to

Official Ans. by NTA (14)

Sol. $\left(\vec{a}\times\vec{b}\right).\vec{b}=0$

$\Rightarrow 13-1-4\lambda=0\Rightarrow\lambda=3$

$\Rightarrow\vec{b}=\hat{i}+\hat{j}+3\hat{k}\ \Rightarrow\ \vec{a}\times\vec{b}=13\hat{i}-\hat{j}-4\hat{k}$

$\Rightarrow\left(\vec{a}\times\vec{b}\right)\times\vec{b}=\left(13\hat{i}-\hat{j}-4\hat{k}\right)\times\left(\hat{i}+\hat{j}+3\hat{k}\right)$

$\Rightarrow -21\vec{b}-11\vec{a}=\hat{i}-43\hat{j}+14\hat{k}$

$\Rightarrow\ \vec{a}=-2\hat{i}+2\hat{j}-7\hat{k}$

Now $\left(\vec{b}-\vec{a}\right).\left(\hat{k}-\hat{j}\right)+\left(\vec{b}+\vec{a}\right).\left(\hat{i}-\hat{k}\right)$=14

7. The total number of three-digit numbers, with one digit repeated exactly two times, is

Official Ans. by NTA (243)

Sol. If 0 taken twice then ways = 9

If 0 taken once then ${}^9C_1\times2=18$

If 0 not taken then ${}^9C_1\,{}^8C_1.3=216$

Total = 243

8. Let $f(x)=\left|(x-1)(x^2-2x-3)\right|+x-3,\ x\in R$. If m and M are respectively the number of points of local minimum and local maximum of f in the interval (0, 4), then m + M is equal to ____

Official Ans. by NTA (3)

Sol. $f(x)=\begin{cases}\left(x^2-1\right)\left(x-3\right)+\left(x-3\right), x\in(0,1]\cup[3,4)\\ -\left(x^2-1\right)\left(x-3\right)+\left(x-3\right), x\in[1,3]\end{cases}$

$\Rightarrow f'(x)=\begin{cases}3x^2-6x,\ x\in(0,1)\cup(3,4)\\ -3x^2+6x+2,\ x\in(1,3)\end{cases}$

f(x) is non-derivable at x = 1 and x = 3

also f'(x) = 0 at $x=1+\sqrt{\frac{5}{3}}$ $\Rightarrow$ m + M = 3

9. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $\frac{5}{4}$. If the equation of the normal at the point $\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ on the hyperbola is $8\sqrt{5}x+\beta y=\lambda$, then $\lambda-\beta$ is equal to

Official Ans. by NTA (85)

Sol. $e^2=1+\frac{b^2}{a^2}=\frac{25}{16}\Rightarrow\frac{b^2}{a^2}=\frac{9}{16}$ **......(1)**

$A\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ satisfies $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\Rightarrow\frac{64}{5a^2}-\frac{144}{25b^2}=1$ **......(2)**

Solving (1) & (2) $b=\frac{6}{5}$ $a=\frac{8}{5}$

Normal at A is $\frac{\sqrt{5}a^2x}{8}+\frac{5b^2y}{12}=a^2+b^2$

Comparing it $8\sqrt{5}x+\beta y=\lambda$

Gives $\lambda=100,\ \beta=15$

$\lambda-\beta=85$

10. Let l_1 be the line in xy-plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively, and l_2 be the line in zx-plane with x and z intercepts $-\frac{1}{8}$ and $-\frac{1}{6\sqrt{3}}$ respectively. If d is the shortest distance between the line l_1 and l_2, then d^{-2} is equal to

Official Ans. by NTA (51)

Sol. $8x+4\sqrt{2}y=1,\ z=0$

$$\Rightarrow\frac{x-\frac{1}{8}}{1}=\frac{y-0}{-\sqrt{2}}=\frac{z-0}{0}=\lambda$$

$-8x-6\sqrt{3}z=1,\ y=0$

$$\Rightarrow\frac{x+\frac{1}{8}}{3\sqrt{3}}=\frac{y-0}{0}=\frac{z-0}{-4}$$

$$\begin{vmatrix}\frac{1}{4} & 0 & 0\\ 1 & -\sqrt{2} & 0\\ 3\sqrt{3} & 0 & -4\end{vmatrix}=\sqrt{2}$$

$d=\frac{1}{\sqrt{51}}$

$\frac{1}{d^2}=51$