FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Tuesday 11th April, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. If $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x+81)$, then λ, $\frac{\lambda}{3}$ are the roots of the equation

(1) $4x^2 + 24x - 27 = 0$ (2) $4x^2 - 24x + 27 = 0$

(3) $4x^2 + 24x + 27 = 0$ (4) $4x^2 - 24x - 27 = 0$

Official Ans. by NTA (2)

Sol. Put $x = 0$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9}{8} \times 81$$

$$\lambda^3 = \frac{9^3}{8} \therefore \lambda = \frac{9}{2}$$

$$\therefore \frac{\lambda}{3} = \frac{3}{2}$$

$$\therefore \text{Required equation is} : x^2 - x\left(\frac{9}{2} + \frac{3}{2}\right)x + \frac{27}{4} = 0$$

$$4x^2 - 24x + 27 = 0$$

2. Let the line passing through the points, P (2, – 1, 2) and Q (5, 3, 4) meet the plane $x - y + z = 4$ at the point R. Then the distance of the point R from the plane $x + 2y + 3z + 2 = 0$ measured parallel to the line $\frac{x-7}{2} = \frac{y+3}{2} = \frac{z-2}{1}$ is equal to

(1) $\sqrt{31}$ (2) $\sqrt{189}$

(3) $\sqrt{61}$ (4) 3

Official Ans. by NTA (4)

Sol. Line : $\frac{x-5}{3} = \frac{y-3}{4} = \frac{z-4}{2} = \lambda$

$R(3\lambda+5, 4\lambda+3, 2\lambda+4)$

$\therefore 3\lambda + 5 - 4\lambda - 3 + 2\lambda + 4 = 4$

$\lambda + 6 = 4 \therefore \lambda = -2$

$\therefore R \equiv (-1, -5, 0)$

Line : $\frac{x+1}{2} = \frac{y+5}{2} = \frac{z-0}{1} = \mu$

Point $T = (2\mu - 1, 2\mu - 5, \mu)$

It lies on plane

$2\mu - 1 + 2(2\mu - 5) + 3\mu + 2 = 0$

$\mu = 1$

$\therefore T = (1, -3, 1)$

$\therefore RT = 3$

3. If the 1011^{th} term from the end in the binomial expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ is 1024 times 1011^{th} term from the beginning, then $|x|$ is equal to

(1) 12 (2) 8

(3) 10 (4) 4

Official Ans. by NTA (3)

Sol. T_{1011} from beginning $= T_{1010+1}$

$= {}^{2022}C_{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{-5}{2x}\right)^{1010}$

T_{1011} from end

$= {}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$

Given : ${}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$

$= 2^{10} \cdot {}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1010}\left(\frac{4x}{5}\right)^{1012}$

$\left(\frac{-5}{2x}\right)^2 = 2^{10}\left(\frac{4x}{5}\right)^2$

$x^4 = \frac{5^4}{2^{16}}$

$|x| = \frac{5}{16}$

4. Let the function f : [0, 2] → R be defined as

$$f(x)=\begin{cases} e^{\min\{x^2,x-[x]\}}, & x\in[0,1) \\ e^{[x-\log_e x]}, & x\in[1,2] \end{cases}$$

where [t] denotes the greatest integer less than or equal to t. Then the value of the integral $\int_0^2 xf(x)dx$ is

(1) $2e-1$ (2) $1+\frac{3e}{2}$

(3) $2e-\frac{1}{2}$ (4) $(e-1)\left(e^2+\frac{1}{2}\right)$

Official Ans. by NTA (3)

Sol. Minimum $\{x^2, \{x\}\} = x^2;\ x\in[0,1)$

$$[x-\log_e x]=1;\ x\in[1,2)$$

$$\therefore f(x)=\begin{cases} e^{x^2}; x\in[0,1) \\ e\ ; x\in[1,2) \end{cases}$$

$$\int_0^2 xf(x)dx=\int_0^1 xe^{x^2}dx+\int_1^2 ex\,dx$$

$$=\frac{1}{2}(e-1)+\frac{1}{2}(4-1)e$$

$$=2e-\frac{1}{2}$$

5. Let y = y (x) be the solution of the differential equations $\frac{dy}{dx}+\frac{5}{x(x^5+1)}y=\frac{(x^5+1)^2}{x^7}, x>0.$ If y(1) = 2, then y(2) is equal to

(1) $\frac{637}{128}$ (2) $\frac{679}{128}$

(3) $\frac{693}{128}$ (4) $\frac{697}{128}$

Official Ans. by NTA (3)

Sol. $I.F=e^{\int\frac{5dx}{x(x^5+1)}}=e^{\int\frac{5x^{-6}dx}{(x^{-5}+1)}}$

Put, $1+x^{-5}=t\Rightarrow -5x^{-6}dx=dt$

$$\Rightarrow e^{\int\frac{-dt}{t}}=\frac{1}{t}=\frac{x^5}{1+x^5}$$

$$y\cdot\frac{x^5}{1+x^5}=\int\frac{x^5}{(1+x^5)}\times\frac{(1+x^5)^2}{x^7}dx$$

$$=\int x^3dx+\int x^{-2}dx$$

$$y\cdot\frac{x^5}{1+x^5}=\frac{x^4}{4}-\frac{1}{x}+c$$

Given that : x = 1 $\Rightarrow$ y = 2

$$2\cdot\frac{1}{2}=\frac{1}{4}-1+c$$

$$c=\frac{7}{4}$$

$$y\cdot\frac{x^5}{1+x^5}=\frac{x^4}{4}-\frac{1}{x}+\frac{7}{4}$$

Now put, x = 2

$$y\cdot\left(\frac{32}{33}\right)=\frac{21}{4}$$

$$y=\frac{693}{128}$$

6. If four distinct points with position vectors $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are coplanar; then $[\vec{a}\,\vec{b}\,\vec{c}]$ is equal to

(1) $[\vec{d}\,\vec{c}\,\vec{a}]+[\vec{b}\,\vec{d}\,\vec{a}]+[\vec{c}\,\vec{d}\,\vec{b}]$

(2) $[\vec{d}\,\vec{b}\,\vec{a}]+[\vec{a}\,\vec{c}\,\vec{d}]+[\vec{d}\,\vec{b}\,\vec{c}]$

(3) $[\vec{a}\,\vec{d}\,\vec{b}]+[\vec{d}\,\vec{c}\,\vec{a}]+[\vec{d}\,\vec{b}\,\vec{c}]$

(4) $[\vec{b}\,\vec{c}\,\vec{d}]+[\vec{d}\,\vec{a}\,\vec{c}]+[\vec{d}\,\vec{b}\,\vec{a}]$

Official Ans. by NTA (1)

Sol. a, b, c, d are coplanar points.

$\vec{b}-\vec{a},\vec{c}-\vec{a},\vec{d}-\vec{a}$ are coplanar vectors.

So, $[\vec{b}-\vec{a}\ \vec{c}-\vec{a}\ \vec{d}-\vec{a}]=0$

$$(\vec{b}-\vec{a})\cdot((\vec{c}-\vec{a})\times(\vec{d}-\vec{a}))=0$$

$$[\vec{b}\ \vec{c}\ \vec{d}]-[\vec{b}\ \vec{c}\ \vec{a}]-[\vec{b}\ \vec{a}\ \vec{d}]-[\vec{a}\ \vec{c}\ \vec{d}]=0$$

$$\Rightarrow[\vec{a}\,\vec{b}\,\vec{c}]=[\vec{c}\,\vec{d}\,\vec{b}]+[\vec{b}\,\vec{d}\,\vec{a}]+[\vec{d}\ \vec{c}\ \vec{a}]$$

7. If $f:\mathbb{R}\to\mathbb{R}$ be a continuous function satisfying

$$\int_0^{\pi/2} f(\sin 2x)\cdot\sin x dx+\alpha\int_0^{\pi/4} f(\cos 2x)\cdot\cos x\,dx=0,$$

then α is equal to

(1) $-\sqrt{3}$ (2) $\sqrt{2}$

(3) $\sqrt{3}$ (4) $-\sqrt{2}$

Official Ans. by NTA (4)

Sol. $I=\int_0^{\frac{\pi}{4}} f(\sin 2x)\sin x\,dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} f(\sin 2x)\sin x\,dx$

$$+\alpha\int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x\,dx=0$$

Apply king in first part and put $x-\frac{\pi}{4}=t$ in second part.

$$I=\int_0^{\frac{\pi}{4}} f(\cos 2x)\sin\left(\frac{\pi}{4}-x\right)dx+\int_0^{\frac{\pi}{4}} f(\cos 2t)\sin\left(\frac{\pi}{4}+t\right)dt$$

$$+\alpha\int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x\,dx = 0$$

$$I=\int_0^{\frac{\pi}{4}} f(\cos 2x)\left[2\sin\frac{\pi}{4}\cdot\cos x+\alpha\cos x\right]dx=0$$

$$I=\left(\alpha+\sqrt{2}\right)\int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x\,dx=0$$

$$\therefore \alpha=-\sqrt{2}$$

8. If the system of linear equations

$7x+11y+\alpha z=13$

$5x+4y+7z=\beta$

$175x+194y+57z=361$

has infinitely many solutions, then $\alpha+\beta+2$ is equal to

(1) 4 (2) 3

(3) 5 (4) 6

Official Ans. by NTA (1)

Sol. $7x+11y+\alpha z=13$ …… (i)

$5x+4y+7z=\beta$ …… (ii)

$175x+194y+57z=361$ ….. (iii)

(i) × 10 + (ii) × 21 – (iii)

$z(10\alpha+147-57)=130+21\beta-361$

$\therefore 10\alpha+90=0$

$\alpha=-9$

$130-361+21\beta=0$

$\beta=11$

$\alpha+\beta+2=4$

9. The domain of the function

$f(x)=\dfrac{1}{\sqrt{[x]^2-3[x]-10}}$ is (where [x] denotes the greatest integer less than or equal to x)

(1) $(-\infty,-2)\cup(5,\infty)$ (2) $(-\infty,-3]\cup[6,\infty)$

(3) $(-\infty,-2)\cup[6,\infty)$ (4) $(-\infty,-3]\cup(5,\infty)$

Official Ans. by NTA (3)

Sol. $[x]^2-3[x]-10>0$

$[x]<-2 \text{ or } [x]>5$

10. Let P be the plane passing through the points (5, 3, 0), (13, 3, –2) and (1, 6, 2). For $\alpha\in N$, if the distances of the points A (3, 4, α) and B (2, α, a) from the plane P are 2 and 3 respectively, then the positive value of a is

(1) 6 (2) 4

(3) 3 (4) 5

Official Ans. by NTA (2)

Sol. $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 8 & 0 & -2\\ 4 & -3 & -2\end{vmatrix}=\hat{i}(-6)+8\hat{j}-24\hat{k}$

Normal of the plane = $3\hat{i}-4\hat{j}+12\hat{k}$

Plane : $3x-4y+12z=3$

Distance from A (3, 4, α)

$$\left|\frac{9-16+12\alpha-3}{13}\right|=2$$

$\alpha=3$

$\alpha=-8(\text{rejected})$

Distance from B (2, 3, a)

$$\left|\frac{6-12+12a-3}{13}\right|=3$$

a = 4

11. The converse of the statement $((\sim p)\wedge q)\Rightarrow r$ is

(1) $(\sim r)\Rightarrow p\wedge q$ (2) $(\sim r)\Rightarrow((\sim p)\wedge q)$

(3) $((\sim p)\vee q)\Rightarrow r$ (4) $(p\vee(\sim q))\Rightarrow(\sim r)$

Official Ans. by NTA (4)

Sol. Converse of $(\sim p)\wedge q)\Rightarrow r$

$\equiv r\Rightarrow(\sim p\wedge q)$

$\equiv\sim r\vee(\sim p\wedge q)$

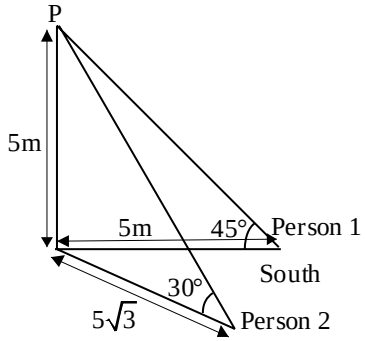
$\equiv\sim r\vee(p\vee\sim q)\equiv(p\vee\sim q)\Rightarrow\sim r$

12. The angle of elevation of the top P of a tower from the feet of one person standing due South of the tower is 45° and from the feet of another person standing due west of the tower is 30°. If the height of the tower is 5 meters, then the distance (in meters) between the two persons is equal to

(1) 10 (2) 5

(3) $5\sqrt{5}$ (4) $\frac{5}{2}\sqrt{5}$

Official Ans. by NTA (1)



Sol.

Distance = 10 (By Pythagoras theorem)

13. Let a, b, c and d be positive real numbers such that a + b + c + d = 11. If the maximum value of $a^5b^3c^2d$ is 3750β, then the value of β is

(1) 90 (2) 110

(3) 55 (4) 108

Official Ans. by NTA (1)

Sol. $$\frac{5\left(\frac{a}{5}\right)+3\left(\frac{b}{3}\right)+2\left(\frac{c}{2}\right)+d}{11} \geq \left(\frac{a^5b^3c^2d}{5^5 3^3 2^2}\right)^{1/11}$$

$$1 \geq \left(\frac{a^5b^3c^2d}{5^5 3^3 2^2}\right)^{1/11}$$

$$\beta = 90$$

14. If the radius of the largest circle with centre (2, 0) inscribed in the ellipse $x^2 + 4y^2 = 36$ is r, then $12r^2$ is equal to

(1) 72 (2) 115

(3) 92 (4) 69

Official Ans. by NTA (3)

Sol. $(x-2)^2 + y^2 = r^2$

Solving with ellipse, we get

$$(x-2)^2 + \frac{36-x^2}{4} = r^2$$

$$3x^2 - 16x + 52 - 4r^2 = 0$$

$$D = 0 \Rightarrow 4r^2 = \frac{92}{3}$$

15. Let the mean of 6 observation 1, 2, 4, 5, x and y be 5 and their variance be 10. Then their mean deviation about the mean is equal to

(1) $\frac{10}{3}$ (2) $\frac{7}{3}$

(3) 3 (4) $\frac{8}{3}$

Official Ans. by NTA (4)

Sol. x + y = 18 $\because$ mean = 5 ….. (i)

$$10 = \frac{1+4+16+25+x^2+y^2}{6} - 25$$

$$x^2 + y^2 = 164 \ldots\ldots \text{(ii)}$$

By solving (i) and (ii)

x = 8, y = 10

$$\text{M.D.}(\bar{x}) = \frac{\sum|x_i - \bar{x}|}{6} = \frac{8}{3}$$

16. The sum of the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+2}$, which are in the ratio 1 : 3 : 5, is equal to

(1) 25 (2) 63

(3) 41 (4) 92

Official Ans. by NTA (2)

Sol. ${}^{n+2}C_{r-1} : {}^{n+2}C_r : {}^{n+2}C_{r+1} = 1:3:5$

$$\frac{{}^{n+2}C_{r-1}}{{}^{n+2}C_r} = \frac{1}{3}$$

$$n = 4r - 3 \ldots\ldots \text{(i)}$$

$$\frac{{}^{n+2}C_r}{{}^{n+2}C_{r+1}} = \frac{3}{5}$$

$$8r - 1 = 3n \ldots\ldots \text{(ii)}$$

From, (i) and (ii)

r = 2 and n = 5

Required sum = 63

17. If the letters of the word MATHS are permuted and all possible words so formed are arranged as in a dictionary with serial numbers, then the serial number of the word THAMS is

(1) 103 (2) 104

(3) 101 (4) 102

Official Ans. by NTA (1)

Sol. $4 \times 4! + 1 \times 3! + 1 = 103$

18. For $a \in C$, let $A = \{z \in C : \text{Re}(a + \bar{z}) > \text{Im}(\bar{a} + z)\}$ and $B = \{z \in C : \text{Re}(a + \bar{z}) < \text{Im}(\bar{a} + z)\}$. Then among the two statements :

(S1) : If Re (A), Im (A) > 0, then the set A contains all the real numbers

(S2) : If Re (A), Im (A) < 0, then the set B contains all the real numbers,

(1) Only (S1) is true (2) both are false

(3) Only (S2) is true (4) Both are true

Official Ans. by NTA (2)

Sol. Let $a = x_1 + iy_1$ $z = x + iy$

Now $\mathrm{Re}(a + \bar{z}) > \mathrm{Im}(\bar{a} + z)$

$\therefore x_1 + x > -y_1 + y$

$x_1 = 2, y_1 = 10, x = -12, y = 0$

Given inequality is not valid for these values.

S1 is false.

Now $\mathrm{Re}(a + \bar{z}) < \mathrm{Im}(\bar{a} + z)$

$x_1 + x < -y_1 + y$

$x_1 = -2, y_1 = -10, x = 12, y = 0$

Given inequality is not valid for these values.

S2 is false.

19. Let A = {1, 3, 4, 6, 9} and B = {2, 4, 5, 8, 10}. Let R be a relation defined on A × B such that R = $\{((a_1, b_1),(a_2, b_2)) : a_1 \le b_2$ and $b_1 \le a_2\}$. Then the number of elements in the set R is

(1) 26 (2) 160

(3) 180 (4) 52

Official Ans. by NTA (2)

Sol. Let $a_1 = 1 \Rightarrow 5$ choices of b_2

$a_1 = 3 \Rightarrow 4$ choices of b_2

$a_1 = 4 \Rightarrow 4$ choices of b_2

$a_1 = 6 \Rightarrow 2$ choices of b_2

$a_1 = 9 \Rightarrow 1$ choices of b_2

For (a_1, b_2) 16 ways .

Similarly, $b_1 = 2 \Rightarrow 4$ choices of a_2

$b_1 = 4 \Rightarrow 3$ choices of a_2

$b_1 = 5 \Rightarrow 2$ choices of a_2

$b_1 = 8 \Rightarrow 1$ choices of a_2

Required elements in R = 160

20. Let f and g be two functions defined by

$$f(x) = \begin{cases} x+1, & x < 0 \\ |x-1|, & x \ge 0 \end{cases} \text{ and } g(x) = \begin{cases} x+1, & x < 0 \\ 1, & x \ge 0 \end{cases}.$$

Then (gof) (x) is

(1) Differentiable everywhere

(2) Continuous everywhere but not differentiable exactly at one point

(3) Not continuous at x = – 1

(4) Continuous everywhere but not differentiable at x = 1

Official Ans. by NTA (2)

Sol. $f(x) = \begin{cases} x+1, & x < 0 \\ 1-x, & 0 \le x < 1 \\ x-1, & 1 \le x \end{cases}$

$g(x) = \begin{cases} x+1, & x < 0 \\ 1, & x \ge 0 \end{cases}$

$g(f(x)) = \begin{cases} x+2, & x < -1 \\ 1, & x \ge -1 \end{cases}$

$\therefore g(f(x))$ is continuous everywhere

$g(f(x))$ is not differentiable at x = – 1

Differentiable everywhere else

SECTION-B

21. The number of points, where the curve $f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1$, $x \in \mathbb{R}$ cuts x-axis, is equal to

Official Ans. by NTA (2)

Sol. Let $e^{2x} = t$

$\Rightarrow t^4 - t^3 - 3t^2 - t + 1 = 0$

$\Rightarrow t^2 + \frac{1}{t^2} - \left(t + \frac{1}{t}\right) - 3 = 0$

$\Rightarrow \left(t + \frac{1}{t}\right)^2 - \left(t + \frac{1}{t}\right) - 5 = 0$

$\Rightarrow t + \frac{1}{t} = \frac{1+\sqrt{21}}{2}$

Two real values of t.

22. Let the probability of getting head for a biased coin be $\frac{1}{4}$. It is tossed repeatedly until a head appears. Let N be the number of tosses required. If the probability that the equation $64x^2 + 5Nx + 1 = 0$ has no real root is $\frac{p}{q}$, where p and q are co-prime, then q – p is equal to

Official Ans. by NTA (27)

Sol. $64x^2+5Nx+1=0$

$D=25N^2-256<0$

$\Rightarrow N^2<\frac{256}{25}\Rightarrow N<\frac{16}{5}$

$\therefore N=1,2,3$

$\therefore \text{Probability}=\frac{1}{4}+\frac{3}{4}\times\frac{1}{4}+\frac{3}{4}\times\frac{3}{4}\times\frac{1}{4}=\frac{37}{64}$

$\therefore q-p=27$

23. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}-\hat{k}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c}=11$, $\vec{b}\cdot(\vec{a}\times\vec{c})=27$ and $\vec{b}\cdot\vec{c}=-\sqrt{3}|\vec{b}|$, then $|\vec{a}\times\vec{c}|^2$ is equal to

Official Ans. by NTA (285)

Sol. $\vec{a}=\hat{i}+2\hat{j}+3\hat{k},\ \vec{b}=\hat{i}+\hat{j}-\hat{k}$

$\vec{b}\cdot(\vec{a}\times\vec{c})=27,\ \vec{a}\cdot\vec{b}=0$

$\vec{b}\times(\vec{a}\times\vec{c})=-3\vec{a}$

Let θ be angle between $\vec{b}, \vec{a}\times\vec{c}$

Then $|\vec{b}|\cdot|\vec{a}\times\vec{c}|\sin\theta=3\sqrt{14}$

$|\vec{b}|\cdot|\vec{a}\times\vec{c}|\cos\theta=27$

$\Rightarrow \sin\theta=\frac{\sqrt{14}}{\sqrt{95}}$

$\therefore |\vec{b}|\times|\vec{a}\times\vec{c}|=3\sqrt{95}$

$\Rightarrow |\vec{a}\times\vec{c}|=\sqrt{3}\times\sqrt{95}$

24. Let $S=\left\{z\in C-\{i,2i\}:\frac{z^2+8iz-15}{z^2-3iz-2}\in R\right\}$. If $\alpha-\frac{13}{11}i\in S,\alpha\in\mathbb{R}-\{0\}$, then $242\alpha^2$ is equal to

Official Ans. by NTA (1680)

Sol. $\left(\frac{z^2+8iz-15}{z^2-3iz-2}\right)\in R$

$\Rightarrow 1+\frac{(11iz-13)}{(z^2-3iz-2)}\in R$

Put $z=\alpha-\frac{13}{11}i$

$\Rightarrow (z^2-3iz-2)$ is imaginary

Put $z=x+iy$

$\Rightarrow (x^2-y^2+2xyi-3ix+3y-2)\in$ Imaginary

$\Rightarrow \text{Re}(x^2-y^2+3y-2+(2xy-3x)i)=0$

$\Rightarrow x^2-y^2+3y-2=0$

$x^2=y^2-3y+2$

$x^2=(y-1)(y-2)\ \therefore z=\alpha-\frac{13}{11}i$

Put $x=\alpha,\ y=\frac{-13}{11}$

$\alpha^2=\left(\frac{-13}{11}-1\right)\left(\frac{-13}{11}-2\right)$

$\alpha^2=\frac{(24\times35)}{121}$

$242\alpha^2=48\times35=1680$

25. For $k\in\mathbb{N}$, if the sum of the series

$1+\frac{4}{k}+\frac{8}{k^2}+\frac{13}{k^3}+\frac{19}{k^4}+\ldots$ is 10, then the value of k is

Official Ans. by NTA (2)

Sol. $10 = 1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + ...\text{upto}\infty$

$9 = \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + ...\text{upto}\infty$

$\frac{9}{k} = \frac{4}{k^2} + \frac{8}{k^3} + \frac{13}{k^4} + ...\text{upto}\infty$

$S = 9\left(1 - \frac{1}{k}\right) = \frac{4}{k} + \frac{4}{k^2} + \frac{5}{k^3} + \frac{6}{k^4} + ...\text{upto}\infty$

$\frac{S}{k} = \quad \frac{4}{k^2} + \frac{4}{k^3} + \frac{5}{k^4} + ...\text{upto}\infty$

$\left(1 - \frac{1}{k}\right)S = \frac{4}{k} + \frac{1}{k^3} + \frac{1}{k^4} + \frac{1}{k^5} + ...\infty$

$9\left(1 - \frac{1}{k}\right)^2 = \frac{4}{k} + \frac{\frac{1}{k^3}}{\left(1 - \frac{1}{k}\right)}$

$9(k-1)^3 = 4k(k-1) + 1$

$k = 2$

26. Let A = {1, 2, 3, 4, 5} and B = {1, 2, 3, 4, 5, 6}. Then the number of functions $f : A \to B$ satisfying f(1) + f(2) = f(4) – 1 is equal to

Official Ans. by NTA (360)

Sol. $f(1) + f(2) + 1 = f(4) \le 6$

$f(1) + f(2) \le 5$

Case (i) $f(1) = 1 \Rightarrow f(2) = 1, 2, 3, 4 \Rightarrow 4\,\text{mappings}$

Case (ii) $f(1) = 2 \Rightarrow f(2) = 1, 2, 3 \Rightarrow 3\,\text{mappings}$

Case (iii) $f(1) = 3 \Rightarrow f(2) = 1, 2 \Rightarrow 2\,\text{mappings}$

Case (iv) $f(1) = 4 \Rightarrow f(2) = 1 \Rightarrow 1\,\text{mapping}$

$f(5)$ & $f(6)$ both have 6 mappings each

Number of functions $= (4 + 3 + 2 + 1) \times 6 \times 6 = 360$

27. Let the tangent to the parabola $y^2 = 12x$ at the point $(3, \alpha)$ be perpendicular to the line 2x + 2y = 3. Then the square of distance of the point (6, – 4) from the normal to the hyperbola $\alpha^2x^2 - 9y^2 = 9\alpha^2$ at its point $(\alpha - 1, \alpha + 2)$ is equal to

Official Ans. by NTA (116)

Sol. $\because P(3, \alpha)$ lies on $y^2 = 12x$

$\Rightarrow \alpha = \pm 6$

But, $\left.\frac{dy}{dx}\right|_{(3,\alpha)} = \frac{6}{\alpha} = 1 \Rightarrow \alpha = 6 (\alpha = -6 \text{ reject})$

Now, hyperbola $\frac{x^2}{9} - \frac{y^2}{36} = 1$, normal at $Q(\alpha - 1, \alpha + 2)$ is $\frac{9x}{5} + \frac{36y}{8} = 45$

$\Rightarrow 2x + 5y - 50 = 0$

Now, distance of (6, –4) from 2x + 5y – 50 = 0 is equal to

$\left|\frac{2(6) - 5(4) - 50}{\sqrt{2^2 + 5^2}}\right| = \frac{58}{\sqrt{29}}$

$\Rightarrow \text{Square of distance} = 116$

28. Let the line $\ell : x = \frac{1 - y}{-2} = \frac{z - 3}{\lambda}, \lambda \in \mathbb{R}$ meet the plane P : x + 2y + 3z = 4 at the point (α, β, γ). If the angle between the line ℓ and the plane P is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then $\alpha + 2\beta + 6\gamma$ is equal to

Official Ans. by NTA (11)

Sol. $\ell : x = \frac{y-1}{2} = \frac{z-3}{\lambda}, \lambda \in \mathbb{R}$

DR's of line ℓ (1, 2, λ)

DR's of normal vector of plane P : x + 2y + 3z = 4 are (1, 2, 3)

Now, angle between line ℓ and plane P is given by

$$\sin\theta = \left|\frac{1+4+3\lambda}{\sqrt{5+\lambda^2}\cdot\sqrt{14}}\right| = \frac{3}{\sqrt{14}}\left(\text{given}\cos\theta = \sqrt{\frac{5}{14}}\right)$$

$$\Rightarrow \lambda = \frac{2}{3}$$

Let variable point on line ℓ is $\left(t, 2t+1, \frac{2}{3}t+3\right)$ lies on plane P.

$$\Rightarrow t = -1$$

$$\Rightarrow \left(-1, -1, \frac{7}{3}\right) \equiv (\alpha, \beta, \gamma)$$

$$\Rightarrow \alpha + 2\beta + 6\gamma = 11$$

29. If the line $\ell_1 : 3y - 2x = 3$ is the angular bisector of the lines $\ell_2 : x - y + 1 = 0$ and $\ell_3 : \alpha x + \beta y + 17 = 0$, then $\alpha^2 + \beta^2 - \alpha - \beta$ is equal to

Official Ans. by NTA (348)

Sol. Point of intersection of $\ell_1 : 3y - 2x = 3$

$\ell_2 : x - y + 1 = 0$ is $P \equiv (0,1)$

Which lies on $\ell_3 : \alpha x + \beta y + 17 = 0$,

$$\Rightarrow \boxed{\beta = -17}$$

Consider a random point $Q \equiv (-1, 0)$

on $\ell_2 : x - y + 1 = 0$, image of Q about

$\ell_2 : x - y + 1 = 0$ is $Q' \equiv \left(\frac{-17}{13}, \frac{6}{13}\right)$ which is calculated by formulae

$$\frac{x-(-1)}{2} = \frac{y-0}{-3} = -2\left(\frac{-2+3}{13}\right)$$

Now, Q′ lies on $\ell_3 : \alpha x + \beta y + 17 = 0$

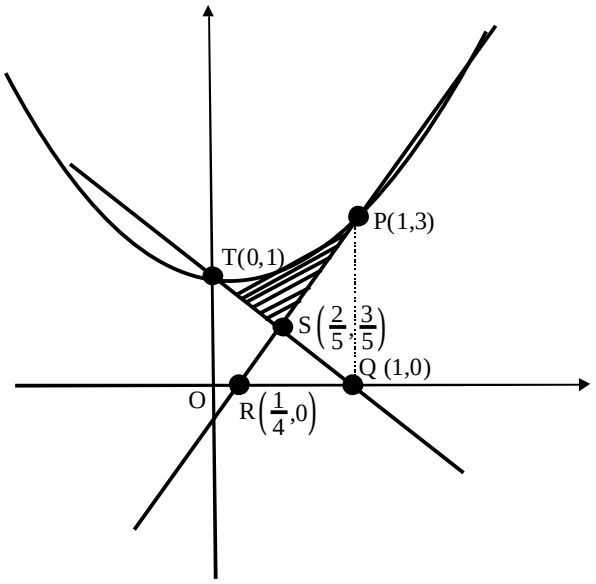
$$\Rightarrow \boxed{\alpha = 7}$$

Now, $\alpha^2 + \beta^2 - \alpha - \beta = 348$

30. If A is the area in the first quadrant enclosed by the curve $C : 2x^2 - y + 1 = 0$, the tangent to C at the point (1, 3) and the line x + y = 1, then the value of 60A is

Official Ans. by NTA (16)

Sol.



$y = 2x^2 + 1$

Tangent at (1, 3)

y = 4x – 1

$$A = \int_0^1 \left(2x^2 + 1\right)dx - \text{area of } (\Delta QOT) - \text{area of } (\Delta PQR) + \text{area of } (\Delta QRS)$$

$$A = \left(\frac{2}{3} + 1\right) - \frac{1}{2} - \frac{9}{8} + \frac{9}{40} = \frac{16}{60}$$

SECTION-A

31. Eight equal drops of water are falling through air with a steady speed of 10 cm/s. If the drops coalesce, the new velocity is:-

(1) 10 cm/s (2) 40 cm/s

(3) 16 cm/s (4) 5 cm/s

Official Ans. by NTA (2)

Sol. $v \propto r^2$

$$\frac{v_1}{v_2} = \left(\frac{r}{R}\right)^2$$

$$8.\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$R = 2r$

$$\frac{10}{v_2} = \left(\frac{1}{2}\right)^2$$

$v_2 = 40$ cm/s

32. A car P travelling at 20 ms^{-1} sounds its horn at a frequency of 400 Hz. Another car Q is travelling behind the first car in the same direction with a velocity 40 ms^{-1}. The frequency heard by the passenger of the car Q is approximately [Take, velocity of sound = 360 ms^{-1}]

(1) 514 Hz (2) 421 Hz

(3) 485 Hz (4) 471 Hz

Official Ans. by NTA (2)

Sol. $$f = f_0\left(\frac{c + v_0}{c + v_s}\right)$$

$$f = 400\left(\frac{360 + 40}{360 + 20}\right)$$

$f = 421$ Hz

33. A plane electromagnetic wave of frequency 20 MHz propagates in free space along x-direction. At a particular space and time, $\vec{E} = 6.6\hat{j}$ V / m. What is $\vec{B}$ at this point ?

(1) $-2.2\times10^{-8}\hat{i}$ T (2) $2.2\times10^{-8}\hat{k}$ T

(3) $-2.2\times10^{-8}\hat{k}$ T (4) $2.2\times10^{-8}\hat{i}$ T

Official Ans. by NTA (2)

Sol. $\vec{E} = 6.6\hat{j}$

$\nu = 20$ MHz

$\vec{c} = 3\times10^8\hat{i}$

$$|\vec{B}| = \frac{|\vec{E}|}{c} = 2.2\times10^{-8}\,T$$

$\hat{E}\times\hat{B} = \hat{c}$

$\vec{B} = 2.2\times10^{-8}\hat{k}$ T

34. A capacitor of capacitance C is charged to a potential V. The flux of the electric field through a closed surface enclosing the positive plate of the capacitor is:

(1) $\frac{CV}{2\varepsilon_0}$ (2) $\frac{2CV}{\varepsilon_0}$

(3) $\frac{CV}{\varepsilon_0}$ (4) Zero

Official Ans. by NTA (3)

Sol. $$\phi = \frac{q_{in}}{\epsilon_0}$$

$$= \frac{Q}{\epsilon_0}$$

$$= \frac{CV}{\epsilon_0}$$

35. If force (F), velocity (V) and time (T) are considered as fundamental physical quantity, then dimensional formula of density will be:

(1) $FV^{-2}T^2$ (2) $FV^{-4}T^2$

(3) FV^4T^{-6} (3) $F^2V^{-2}T^6$

Official Ans. by NTA (2)

Sol. $[ML^{-3}] = [MLT^{-2}]^a[LT^{-1}]^b[T]^c$

$= [M^aL^{a+b}T^{-2a-b+c}]$

$a=1,$

$a + b = -3,$

$\Rightarrow b= -4,$

also $-2a - b + c=0$

$c = -2$

36. In satellite communication, the uplink frequency band used is:

(1) 3.7 – 4.2 GHz

(2) 5.925 – 6.425 GHz

(2) 76 – 88 MHz

(4) 420 – 890 MHz

Official Ans. by NTA (2)

Sol. Conceptual

37. If V is the gravitational potential due to sphere of uniform density on it's surface, then it's value at the center of sphere will be:-

(1) $\frac{3V}{2}$ (2) V

(3) $\frac{4}{3}V$ (4) $\frac{V}{2}$

Official Ans. by NTA (1)

Sol. $V = \frac{GM}{2R^3}\left(3R^2 - r^2\right)$ at $r = R \Rightarrow V = \left(\frac{GM}{R}\right)$

at $r = 0,\ V_0 = \frac{3GM}{2R} = \left(\frac{3V}{2}\right)$

38. A body of mass 500 g moves along x-axis such that it's velocity varies with displacement x according to the relation $v = 10\sqrt{x}$ m/s the force acting on the body is:-

(1) 166 N (2) 25 N

(3) 125 N (4) 5 N

Official Ans. by NTA (2)

Sol. $v = 10\sqrt{x} \Rightarrow v^2 = 100x$

$2v\frac{dv}{dx} = 100 \Rightarrow a = 50\,m/s^2$

F= 25 N

39. A projectile is projected at 30° from horizontal with initial velocity 40 ms^{-1}. The velocity of the projectile at t = 2 s from the start will be:

(Given $g = 10\ m/s^2$)

(1) $20\sqrt{3}\,ms^{-1}$ (2) $40\sqrt{3}\,ms^{-1}$

(3) 20 ms^{-1} (4) Zero

Official Ans. by NTA (1)

Sol. At t = 2 particle is at maximum height

moving with velocity $V = 40\cos 30° = 20\sqrt{3}\,ms^{-1}$.

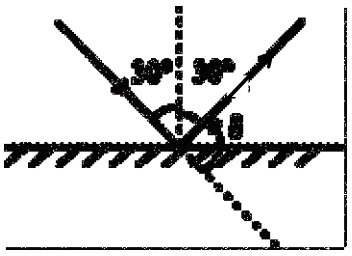
40. When one light ray is reflected from a plane mirror with 30° angle of reflection, the angle of deviation of the ray after reflection is:

(1) 140° (2) 120°

(3) 110° (4) 130°

Official Ans. by NTA (2)

Sol.



$\delta = 180° - 60° = 120°$

41. A spaceship of mass 2 × 10 kg is launched into a circular orbit close to the earth surface. The additional velocity to be imparted to the spaceship in the orbit to overcome the gravitational pull will be (if g = 10 m/s^2 and radius of earth = 6400 km)

(1) $11.2\left(\sqrt{2}-1\right)\text{km/s}$

(2) $7.9\left(\sqrt{2}-1\right)\text{km/s}$

(3) $8\left(\sqrt{2}-1\right)\text{km/s}$

(4) $7.4\left(\sqrt{2}-1\right)\text{km/s}$

Official Ans. by NTA (3)

Sol. $v_{orbit}=\sqrt{\frac{GM}{R}}=\sqrt{gR};$

$v_{escape}=\sqrt{\frac{2GM}{R}}=\sqrt{2gR}$

$\Delta v=\left(\sqrt{2}-1\right)\sqrt{gR}=8\left(\sqrt{2}-1\right)\text{km/s}$

42. The ratio of the de-Broglie wavelengths of proton and electron having same kinetic energy:

(Assume $m_p = m_e \times 1849$)

(1) 1 : 43 (2) 1 : 30

(3) 1 : 62 (4) 2 : 43

Official Ans. by NTA (1)

Sol. $\lambda \propto \frac{1}{\sqrt{m}} \Rightarrow \frac{\lambda_p}{\lambda_e}=\sqrt{\frac{m_e}{m_p}}=1:43$

43. The thermodynamic process, in which internal energy of the system remains constant is

(1) Isochoric (2) Isothermal

(3) Adiabatic (4) Isobaric

Official Ans. by NTA (2)

Sol. T = constant $\Rightarrow$ U = constant

44. The energy of He ion in its first excited state is. (The ground state energy for the Hydrogen atom is –13.6 eV):

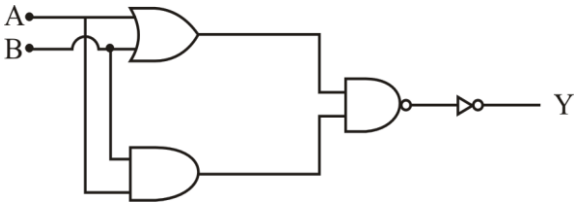
(1) –3.4 eV (2) –54.4 eV

(3) –13.6 eV (4) –27.2 eV

Official Ans. by NTA (3)

Sol. $E_n=\frac{-13.6Z^2}{n^2}=\frac{-13.6\times 4}{4}=-13.6\text{ eV}$

45. The logic operations performed by the given digital circuit is equivalent to:

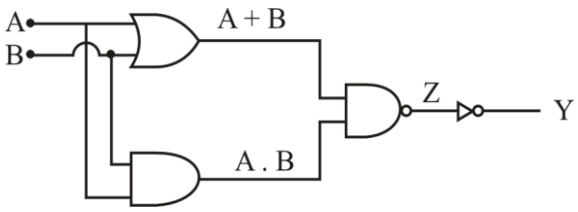


(1) AND (2) NOR

(3) OR (4) NAND

Official Ans. by NTA (1)

Sol.



$Z=\overline{(A+B).(A.B)}$

$Y=\overline{Z}=(A+B).(A.B)$

Y = A. B

$\therefore$ It is an AND Gate

46. The root mean square speed of molecules of nitrogen gas at 27°C is approximately:

(Given mass of a nitrogen molecule =4.6×10^{-26}kg and take Boltzmann constant $k_B = 1.4 \times 10^{-23}$ JK^{-1})

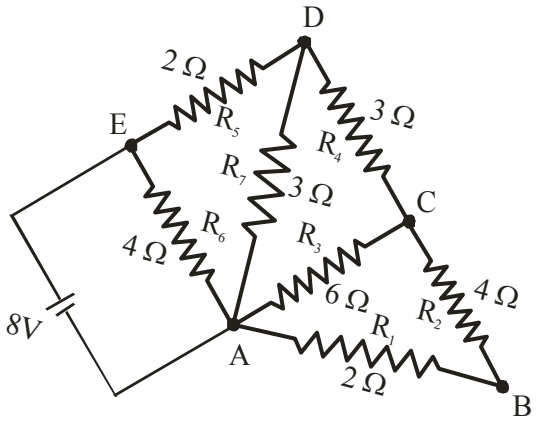
(1) 523 m/s (2) 1260 m/s

(3) 91 m/s (4) 27.4 m/s

Official Ans. by NTA (1)

Sol. $V_{rms}=\sqrt{\frac{3k_BT}{m}}=\sqrt{\frac{3\times1.4\times10^{-23}\times300}{4.6\times10^{-26}}}=523\text{ m/s}$

47.



The current flowing through R_2 is:

(1) $\frac{2}{3}$A

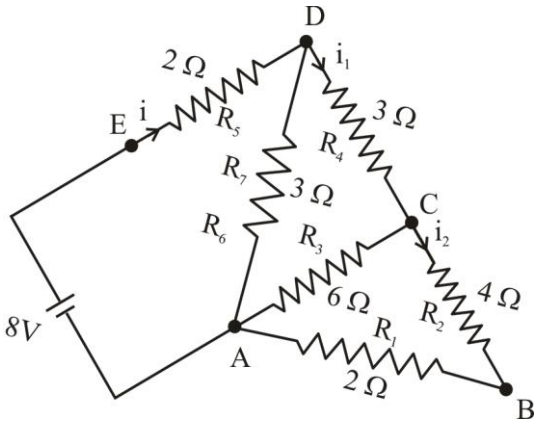
(2) $\frac{1}{4}$A

(3) $\frac{1}{2}$A

(4) $\frac{1}{3}$A

Official Ans. by NTA (4)

Sol.



$R_{eq} = 4\Omega$

$i = \frac{8}{4} = 2A$

$i_1 = \frac{2\times 3}{3+6} = \frac{2}{3}A$

$i_2 = \frac{2/3}{2} = \frac{1}{3}A$

48. When vector $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ is subtracted from vector $\vec{B}$, it gives a vector equal to $2\hat{j}$. Then the magnitude of vector $\vec{B}$ will be:

(1) $\sqrt{13}$

(2) 3

(3) $\sqrt{6}$

(4) $\sqrt{5}$

Official Ans. by NTA (BONUS)

Sol. $\vec{B} - \vec{A} = 2j$

$\vec{B} = 2\hat{i} + 5\hat{j} + 2\hat{k}$

$|\vec{B}| = \sqrt{33}$

49. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: A bar magnet dropped through a metallic cylindrical pipe takes more time to come down compared to a non–magnetic bar with same geometry and mass.

Reason R: For the magnetic bar, Eddy currents are produced in the metallic pipe which oppose the motion of the magnetic bar.

In the light of the above statements, choose the correct answer from the options given below

(1) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(2) **A** is true but **R** is false

(3) Both **A** and **R** are true and **R** is the correct explanation of **A**

(4) **A** is false but **R** is true

Official Ans. by NTA (3)

Sol. Conceptual

50. An electron is allowed to move with constant velocity along the axis of current carrying straight solenoid.

A. The electron will experience magnetic force along the axis of the solenoid.

B. The electron will not experience magnetic force.

C. The electron will continue to move along the axis of the solenoid.

D. The electron will be accelerated along the axis of the solenoid.

E. The electron will follow parabolic path–inside the solenoid.

Choose the correct answer from the options given below:

(1) B, C and D only (2) B and C only

(3) A and D only (4) B and E only

Official Ans. by NTA (2)

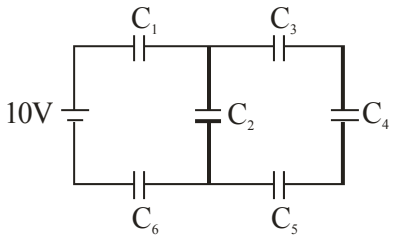
Sol. $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$ as angle between $\vec{v}$ and $\vec{B}$ is 0°

$\vec{F} = 0$

SECTION-B

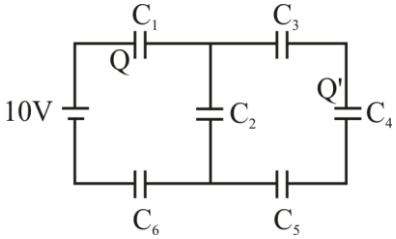
51. In the given circuit,

$C_1 = 2\ \mu F$, $C_2 = 0.2\ \mu F$, $C_3 = 2\ \mu F$, $C_4 = 4\ \mu F$, $C_5 = 2\ \mu F$, $C_6 = 2\ \mu F$, the charge stored on capacitor C_4 is _____ μC.



Official Ans. by NTA (4)

Sol.



$C_{eq} = 0.5\ \mu F$

$Q = 0.5 \times 10 = 5\ \mu C$

$$Q' = \frac{5\mu C \times 0.8}{0.8 + 0.2} = 4\mu C$$

52. A circular plate is rotating in horizontal plane, about an axis passing through its center and perpendicular to the plate, with an angular velocity ω. A person sits at the center having two dumbbells in his hands. When he stretches out his hands, the moment of inertia of the system becomes triple. If E be the initial Kinetic energy of the system, then final Kinetic energy will be $\frac{E}{x}$.

The value of x is

Official Ans. by NTA (3)

Sol. $$KE = \frac{L^2}{2I} \Rightarrow \frac{KE_{final}}{KE_{initial}} = \frac{I_{initial}}{I_{final}} \Rightarrow \frac{KE_{final}}{E} = \frac{1}{3}$$

$$\Rightarrow KE_{final} = \frac{E}{3}$$

53. A nucleus disintegrates into two nuclear parts, in such a way that ratio of their nuclear sizes is $1 : 2^{1/3}$. Their respective speed have a ratio of n : 1. The value of n is _____

Official Ans. by NTA (2)

Sol. $$\frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{A_2}{A_1} = \frac{2}{1}$$

54. Two identical cells each of emf 1.5 V are connected in series across a 10 Ω resistance. An ideal voltmeter connected across 10 Ω resistance reads 1.5 V. The internal resistance of each cell is _____Ω.

Official Ans. by NTA (5)

Sol. $V = I \times 10$

$$1.5 = \left(\frac{3}{10 + 2r}\right) \times 10$$

$r = 5\ \Omega$

55. A block of mass 5 kg starting from rest pulled up on a smooth incline plane making an angle of 30° with horizontal with an affective acceleration of 1 ms^{-2}. The power delivered by the pulling force at t = 10 s from the start is _____ W.

[Use g = 10 ms^{-2}]

(calculate the nearest integer value)

Official Ans. by NTA (300)

Sol. $F - 5g\sin30° = 5a \Rightarrow F = 5 + 25 = 30N$

$V_{10} = u + at \Rightarrow v_{10} = 0 + 1(10) = 10$ m/s

$P_{10} = Fv = 300$ W

56. A coil has an inductance of 2H and resistance of 4Ω. A 10 V is applied across the coil. The energy stored in the magnetic field after the current has built up to its equilibrium value will be ____$\times 10^{-2}$J

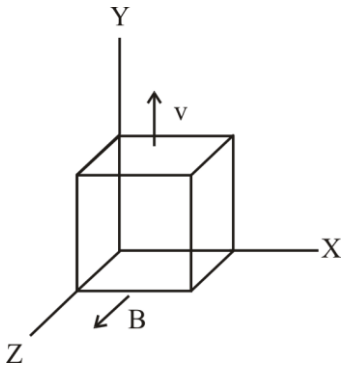
Official Ans. by NTA (625)

Sol. $I = \frac{V}{R} = \frac{5}{2}A$

$$E = \frac{1}{2}LI^2 = \frac{1}{2} \times 2 \times \left(\frac{5}{2}\right)^2$$

$$E = 625 \times 10^{-2} J$$

57. A metallic cube of side 15 cm moving along y–axis at a uniform velocity of 2 ms^{-1}. In a region of uniform magnetic field of magnitude 0.5 T directed along z-axis. In equilibrium the potential difference between the faces of higher and lower potential developed because of the motion through the field will be _____ mV.



Official Ans. by NTA (150)

Sol. $\Delta V = (v \times B)d$

$\Delta V = (2 \times 1/2)0.15$

$\Delta V = 150$ mV

58. A wire of density 8×10^3 kg/m^3 is stretched between two clamps 0.5 m apart. The extension developed in the wire is 3.2×10^{-4} m. If $Y = 8 \times 10^{10}$ N/m^2, the fundamental frequency of vibration in the wire will be _____ Hz.

Official Ans. by NTA (80)

Sol. $$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2L}\sqrt{\frac{YA\Delta L}{\rho AL}}$$

$f = 80$ Hz

59. The surface tension of soap solution is 3.5×10^{-2} Nm^{-1}. The amount of work done required to increase the radius of soap bubble from 10 cm to 20 cm is _____$\times 10^{-4}$ J.

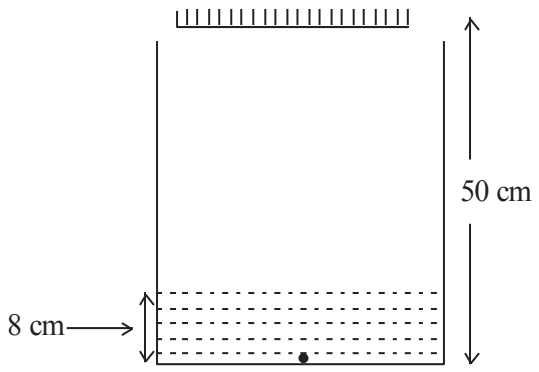
Official Ans. by NTA (264)

Sol. $W = T.(\Delta A)$

$$W = T\left(8\pi\left(r_2^2 - r_1^2\right)\right)$$

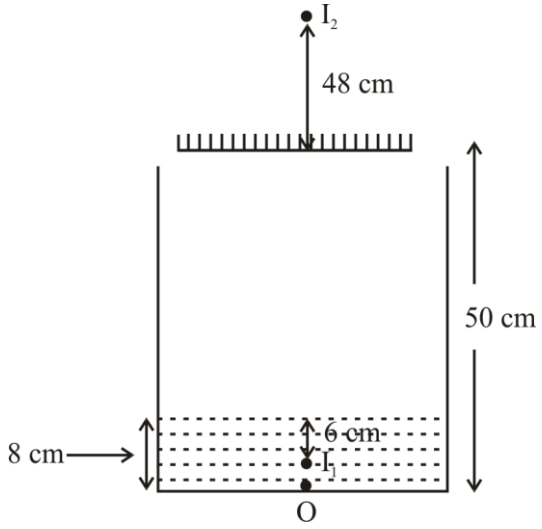
$W = 264 \times 10^{-4}$ J

60. As shown in the figure, a plane mirror is fixed at a height of 50 cm from the bottom of tank containing water $\left(\mu = \frac{4}{3}\right)$. The height of water in the tank is 8 cm. A small bulb is placed at the bottom of the water tank. The distance of image of the bulb formed by mirror from the bottom of the tank is _____ cm.



Official Ans. by NTA (98)

Sol.



Apparent depth of $O = \frac{d}{\mu} = 6$

Distance between O and $I_2 = 48 + 50 = 98$ cm

SECTION-A

61. Which hydride among the following is less stable?

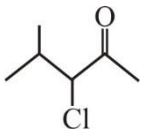
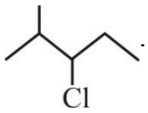
(1) BeH_2 (2) NH_3

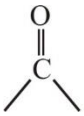
(3) HF (4) LiH

Official Ans. by NTA (1)

Solution : BeH_2 is hypovalent

62. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A :  can be subjected to Wolff-Kishner reduction to give  .

Reason R : Wolff-Kishner reduction is used to convert  into  .

In the light of the above statements, choose the correct answer from the options given below :

(1) Both A and R are true but R is NOT the correct explanation of A.

(2) A is true but R is false.

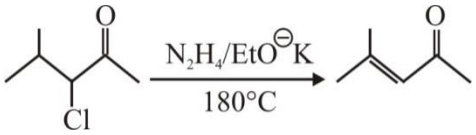
(3) A is false but R is true.

(4) Both A and R are true and R is the correct explanation of A.

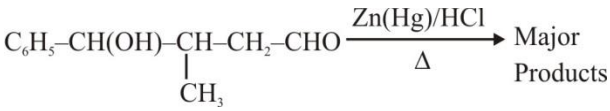
Official Ans. by NTA (3)

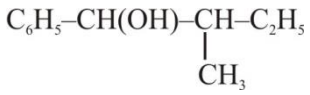
Solution :

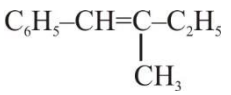
Wolff-Kishner reduction is not suitable for base sensitive group.

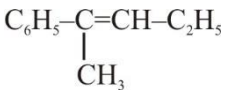


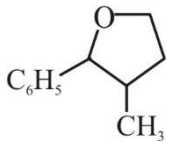
63. The major product formed in the following reaction is:



(A) 

(B) 

(C) 

(D) 

Choose the correct answer from the options given below :

(1) A only

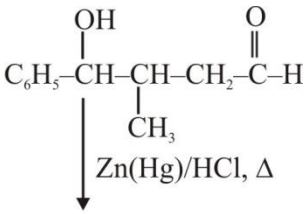
(2) B only

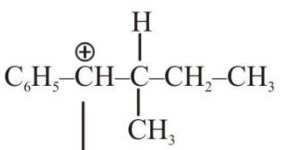
(3) C only

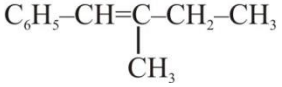
(4) D only

Official Ans. by NTA (2)

Solution :







64. Which of the following compounds is an example of Freon?

(1) $C_2Cl_2F_2$

(2) C_2HF_3

(3) $C_2H_2F_2$

(4) C_2F_4

Official Ans. by NTA (1)

Solution : Freons are chlorofluoro carbon.

65. For a chemical reaction A + B $\rightarrow$ Product, the order is 1 with respect to A and B.

Rate mol L^{-1} s^{-1}	[A] mol L^{-1}	[B] mol L^{-1}
0.10	20	0.5
0.40	*x*	0.5
0.80	40	*y*

What is the value of *x* and *y*?

(1) 80 and 2

(2) 40 and 4

(3) 160 and 4

(4) 80 and 4

Official Ans. by NTA (1)

Solution :

$r = K[A]^1[B]^1$

$0.1 = K(20)^1 (0.5)^1$ …(i)

$0.40 = K(x)^1 (0.5)^1$ …(ii)

$0.80 = K(40)^1 (y)^1$ …(iii)

From (i) and (ii)

$x = 80$

From (i) and (iii)

$y = 2$

66. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : $[CoCl(NH_3)_5]^{2+}$ absorbs at lower wavelength of light with respect to $[Co(NH_3)_5(H_2O)]^{3+}$

Reason R : It is because the wavelength of the light absorbed depends on the oxidation state of the metal ion.

In the light of the above statements, choose the correct answer from the options given below :

(1) A is false but R is true.

(2) A is true but R is false.

(3) Both A and R are true and R is the correct explanation of A.

(4) Both A and R are true and R is NOT the correct explanation of A.

Official Ans. by NTA (1)

Solution : Since H_2O is strong field ligand compared to chloride and Co^{3+} ion is present.

$\therefore$ CFSE is higher for $[Co(NH_3)_5H_2O]^{+3}$, hence it will absorb at lower wavelength.

67. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : A solution of the product obtained by heating a mole of glycine with a mole of chlorine in presence of red phosphorous generates chiral carbon atom.

Reason R : A molecule with 2 chiral carbons is always optically active.

In the light of the above statements, choose the correct answer from the options given below :

(1) A is false but R is true.

(2) A is true but R is false.

(3) Both A and R are true and R is the correct explanation of A.

(4) Both A and R are true and R is NOT the correct explanation of A.

Official Ans. by NTA (2)

Solution :

(1) $\underset{\text{Glycine}}{H_2N-CH_2-COOH} \xrightarrow[(HVZ)]{\text{Red P}} NH_2-\overset{*}{C}H(Cl)-COOH$

Optically active

(2) Meso compound are optically inactive.

68. $H_3C–CH_2–\underset{\underset{OH}{|}}{CH}–CH_3 \xrightarrow[(iii)\ D_2O]{(i)\ NaI,\ H_3PO_4 \\ (ii)\ Mg,\ Dry\ ether} [X]$ Product

Product [X] formed in the above reaction is :

(1) $H_3C–CH_2–\underset{\underset{D}{|}}{CH}–CH_3$

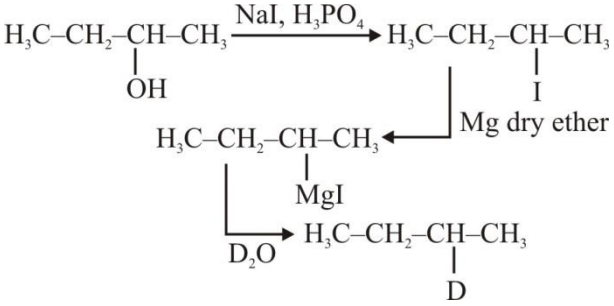
(2) $H_3C–CH_2–\overset{\overset{OH}{|}}{\underset{\underset{H}{|}}{C}}–CH_3$

(3) $H_3C—CH_2—CH=CH_2$

(4) $H_3C—CH=CH—CH_3$

Official Ans. by NTA (1)

Solution :



69. Given below are two statements :

Statements I : Ethene at 333 to 343K and 6-7 atm pressure in the presence of $AlEt_3$ and $TiCl_4$ undergoes addition polymerization to give LDP.

Statement II : Caprolactam at 533-543K in H_2O through step growth polymerizes to give Nylon 6.

In the light of the above Statements, chose the correct answer from the options given below :

(1) Both Statement I and Statement II are true.

(2) Statement I is false but Statement II is true.

(3) Statement I is true but Statement II is false.

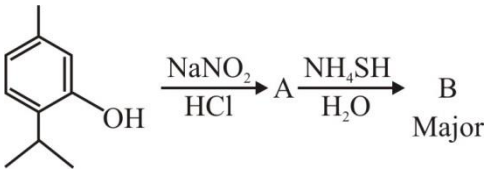
(4) Both Statement I and Statement II are false.

Official Ans. by NTA (2)

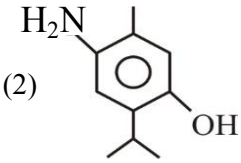
Solution : $S_1 \Rightarrow$ HDPE is formed by $TiCl_4$ & $Al(Et)_3$.

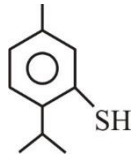
$\mathbf{S_2} \Rightarrow$ Nylon-6 is formed by caprolactam.

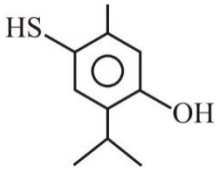
70. Compound 'B' is



(1) 

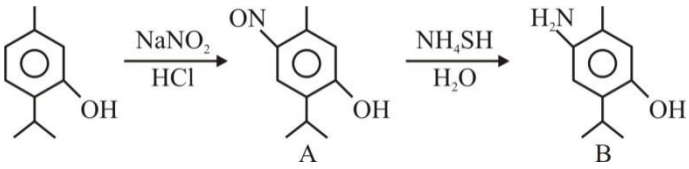
(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Solution :



71. Which one of the following pairs is an example of polar molecular solids?

(1) $SO_2(s)$, $NH_3(s)$

(2) $SO_2(s)$, $CO_2(s)$

(3) HCl(s), AlN(s)

(4) MgO(s), $SO_2(s)$

Official Ans. by NTA (1)

Solution : SO_2 and NH_3 are polar molecules. They are constituent particles of polar molecular solids.

72. One mole of P_4 reacts with 8 moles of $SOCl_2$ to give 4 moles of A, *x* mole of SO_2 and 2 moles of B. A, B and *x* respectively are

(1) PCl_3, S_2Cl_2 and 4

(2) $POCl_3$, S_2Cl_2 and 4

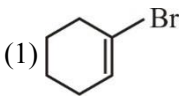
(3) PCl_3, S_2Cl_2 and 2

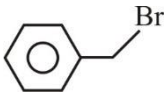
(4) $POCl_3$, S_2Cl_2 and 2

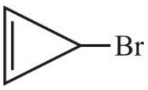
Official Ans. by NTA (1)

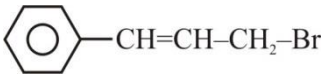
Solution : $P_4 + 8\ SOCl_2 \rightarrow 4PCl_3 + 2S_2Cl_2 + 4SO_2$

73. Compound from the following that will not produce precipitate on reaction with $AgNO_3$ is :

(1) 

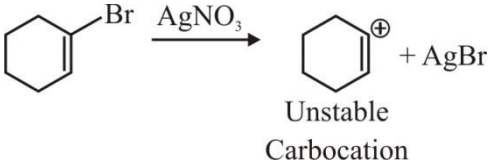
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Solution :



74. A solution is prepared by adding 2g of "X" of 1 mole of water. Mass percent of "X" in the solution is :

(1) 20%

(2) 5%

(3) 2%

(4) 10%

Official Ans. by NTA (4)

Solution : Solute (X) = 2 g

Solvent (H_2O) = 1 mole = 18 g

Total mass = 2 + 18 = 20 g

% mass of X = $\frac{2}{20} \times 100 = 10\%$

75. Given below are two statements :

Statement-I : In the metallurgy process, sulphide ore is converted to oxide before reduction.

Statement-II : Oxide ores in general are easier to reduce.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both Statement I and Statement II are correct.

(2) Statement I is correct but Statement II is incorrect.

(3) Both Statement I and Statement II are incorrect.

(4) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (1)

Solution : $2ZnS + 3O_2 \rightarrow 2ZnO + 2SO_2$

Oxides on carbon reduction forms CO_2 while sulphide on carbon reduction gives CS_2.

CO_2 is more volatile compared to CS_2 therefore oxides are easy to reduce.

76. Alkali metal from the following with least melting point is :

(1) Rb

(2) K

(3) Na

(4) Cs

Official Ans. by NTA (4)

Solution : On moving down the group in alkali metals melting point decreases.

77. What weight of glucose must be dissolved in 100 g of water to lower the vapour pressure by 0.20 mm Hg?
(Assume dilute solution is being formed)
Given : Vapour pressure of pure water is 54.2 mm Hg at room temperature. Molar mass of glucose is 180 g mol^{-1}.
(1) 4.69 g (2) 3.59 g
(3) 2.59 g (4) 3.69 g
Official Ans. by NTA (4)

Solution : $\frac{P^0 - P_s}{P^0} = \frac{n}{N}$ (for dilute solution)

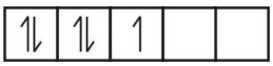
$\frac{0.2}{54.2} = \frac{n \times 18}{100}$

$n = \frac{100}{271 \times 18}$

$w = \frac{100 \times 180}{271 \times 18}$; $w = 3.69g$

78. The magnetic moment is measured in Bohr Magneton (BM).
Spin only magnetic moment of Fe in $[Fe(H_2O)_6]^{3+}$ and $[Fe(CN)_6]^{3-}$ complexes respectively is :
(1) 6.92 B.M. in both
(2) 4.89 B.M. and 6.92 B.M.
(3) 3.87 B.M. and 1.732 B.M.
(4) 5.92 B.M. and 1.732 B.M
Official Ans. by NTA (4)

Solution : $[Fe(H_2O)_6]^{3+}$
$Fe^{3+} \Rightarrow [Ar]\ 3d^5\ 4s^0$
No pairing 
$\therefore$ Unpaired $e^- = 5$
$\mu = \sqrt{n(n+2)}$
$= \sqrt{5(5+2)}$
$\mu = \sqrt{35} = 5.92\,B.M.$
$[Fe(CN)_6]^{-3}$
$Fe^{3+} \Rightarrow [Ar]\ 3d^5\ 4s^\circ$

Pairing occur due to strong field ligand CN^-

$\therefore$ Unpaired $e^- \Rightarrow 1$
$\mu = \sqrt{n(n+2)}$
$= \sqrt{1(1+2)} = \sqrt{3} = 1.732\,B.M.$

79. Match List I with List II.

List I Complex		List II Colour	
A.	$Mg(NH_4)PO_4$	I.	Brown
B.	$K_3[Co(NO_2)_6]$	II.	White
C.	$MnO(OH)_2$	III.	Yellow
D.	$Fe_4[Fe(CN)_6]_3$	IV.	blue

Choose the correct answer from the options given below :
(1) A-II, B-III, C-I, D-IV
(2) A-III, B-IV, C-II, D-I
(3) A-II, B-IV, C-I, D-III
(4) A-II, B-III, C-IV, D-I
Official Ans. by NTA (1)

Solution : $Mg(NH_4)PO_4 \Rightarrow$ White
$K_3[Co(NO_2)_6] \Rightarrow$ Yellow
$MnO(OH)_2 \Rightarrow$ Brown
$Fe_4[Fe(CN)_6]_3 \Rightarrow$ Blue

80. If Ni^{2+} is replaced by Pt^{2+} in the complex $[NiCl_2Br_2]^{2-}$, which of the following properties are expected to get changed?
A. Geometry
B. Geometrical isomerism
C. Optical isomerism
D. Magnetic properties
(1) A, B and C
(2) A, B and D
(3) A and D
(4) B and C
Official Ans. by NTA (2)

Solution : $[NiBr_2Cl_2]^{2-} \rightarrow$ This complex species is tetrahedral as $Br^\ominus$ & $Cl^\ominus$ are weak field ligands.
$[PtBr_2Cl_2]^{2-} \rightarrow$ As Pt belongs to 5d series. This complex species is square planar.
Both the complex species are optically inactive.
$[NiBr_2Cl_2]^{2-}$, being tetrahedral does not show Geometrical Isomerism.
$[PtBr_2Cl_2]^{2-}$ shows two Geometrical Isomers.

SECTION-B

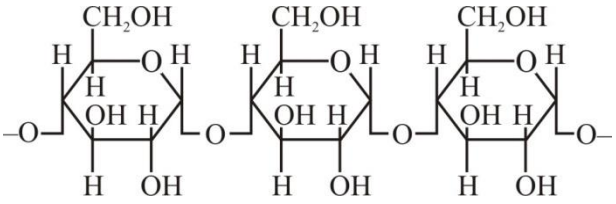
81. Number of compounds from the following which will not produce orange red precipitate with Benedict solution is …….

Glucose, maltose, sucrose, ribose, 2-deoxyribose, amylose, lactose.

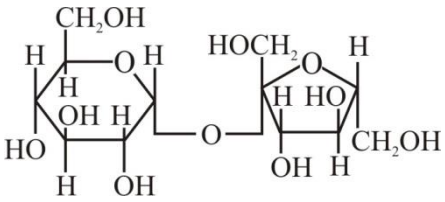
Official Ans. by NTA (3)

Solution :

Amylose



Sucrose :



Both Amylose and Sucrose does not give Benedict's test.

82. 4.5 moles each of hydrogen and iodine is heated in a sealed ten litre vessel. At equilibrium, 3 moles of HI were found. The equilibrium constant for $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$ is …….

Official Ans. by NTA (1)

Solution :

	$H_{2(g)}$	+ $I_{2(g)}$	$\rightleftharpoons$ $2HI_{(g)}$
t = 0	4.5	4.5	–
t_{eq}	3	3	3

$$K_c = \frac{[HI]^2}{[H_2][I_2]} = \frac{(3)^2}{3\times 3} = \frac{9}{9} = 1$$

83. The number of correct statements about modern adsorption theory of heterogeneous catalysis from the following is …….

A. The catalyst is diffused over the surface of reactants.

B. Reactants are adsorbed on the surface of the catalyst.

C. Occurrence of chemical reaction on the catalyst's surface through formation of an intermediate.

D. It is a combination of intermediate compound formation theory and the old adsorption theory.

E. It explains the action of the catalyst as well as those of catalytic promoters and poisons.

Official Ans. by NTA (3)

Solution : B, C and D are correct.
(NCERT – Surface Chemistry)

84. The number of correct statements from the following _____

A. For 1s orbital, the probability density is maximum at the nucleus.

B. For 2s orbital, the probability density first increases to maximum and then decreases sharply to zero.

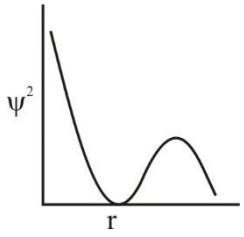
C. Boundary surface diagrams of the orbitals encloses a region of 100% probability of finding the electron.

D. p and d-orbitals have 1 and 2 angular nodes respectively.

E. Probability density of p-orbital is zero at the nucleus.

Official Ans. by NTA (3)

Solution : A, D and E statements are correct.



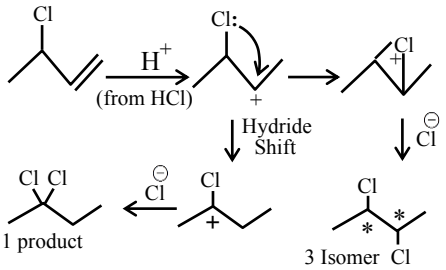
For 2s orbital, the probability density first decreases and then increases.

At any distance from nucleus the probability density of finding electron is never zero and it always have some finite value.

85. The number of possible isomeric products formed when 3-chloro-1-butene reacts with HCl through carbocation formation is _____

Official Ans. by NTA (4)

Solution :



Total Possible Isomeric product = 1+3 = 4

86. $Mg(NO_3)_2 \cdot XH_2O$ and $Ba(NO_3)_2 \cdot YH_2O$, represent formula of the crystalline forms of nitrate salts. Sum of X and Y is _____

Official Ans. by NTA (6)

Solution : $Mg(NO_3)_2 \cdot 6H_2O$ is a hydrated salt whereas $Ba(NO_3)_2$ is a anhydrous salt.

$\therefore$ x + y = 6

87. The total number of intensive properties from the following is _____

Volume, Molar heat capacity, Molarity, E^{θ} cell, Gibbs free energy change, Molar mass, Mole

Official Ans. by NTA (4)

Solution : Extensive $\Rightarrow$ Mole, Volume, Gibbs free energy.

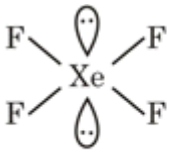
Intensive $\Rightarrow$ Molar mass, Molar heat capacity, Molarity, E^{θ} cell.

88. The maximum number of lone pairs of electrons on the central atom from the following species is ___

ClO_3^-, XeF_4, SF_4 and I_3^-

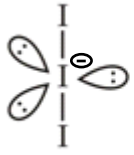
Official Ans. by NTA (3)

Solution :

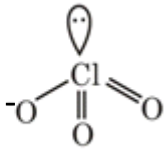


[2 lone pair]

[1 lone pair]



[3 lone pair]



[1 lone pair]

89. The volume of hydrogen liberated at STP by treating 2.4 g of magnesium with excess of hydrochloric acid is _____ $\times 10^{-2}$ L.

Given: Molar volume of gas is 22.4 L at STP.

Molar mass of magnesium is 24 g mol^{-1}.

Official Ans. by NTA (224)

Solution :

$Mg + 2HCl \rightarrow MgCl_2 + H_2 \uparrow$

w = 2.4 g

$N = \frac{2.4}{24} = 0.1$ mole

1 mole of gas at STP $\Rightarrow$ 22.4 lit.

$\therefore$ 0.1 mole of gas = 0.1 × 22.4

= 2.24 lit. = 224 × 10^{-2} litre

90. The number of correct statements from the following is :

A. E_{cell} is an intensive parameter.

B. A negative E^{Θ} means that the redox couple is a stronger reducing agent than the H^+/H_2 couple.

C. The amount of electricity required for oxidation or reduction depends on the stoichiometry of the electrode reaction.

D. The amount of chemical reaction which occurs at any electrode during electrolysis by a current is proportional to the quantity of electricity passed through the electrolyte.

Official Ans. by NTA (4)

Solution : Given statements A, B, C and D are correct.