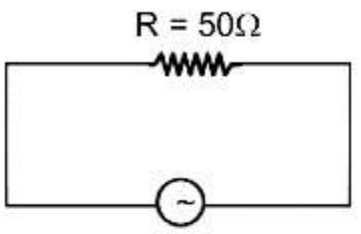
JEE Main 30 Jan 2024
(Shift-2)
(Memory Based)

The Actual Paper will be Updated with Solution After the Official Release

PART : PHYSICS

1. For a given electric circuit shown, find the time taken to change current from highest peak value to half of peak value

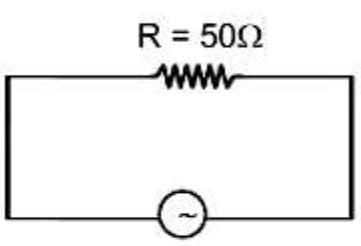


V = 220 sin (100 t)

(1) $\frac{\pi}{300}$ sec (2) $\frac{\pi}{200}$ sec (3) $\frac{\pi}{400}$ sec (4) $\frac{\pi}{100}$ sec

Ans. **(1)**

Sol.

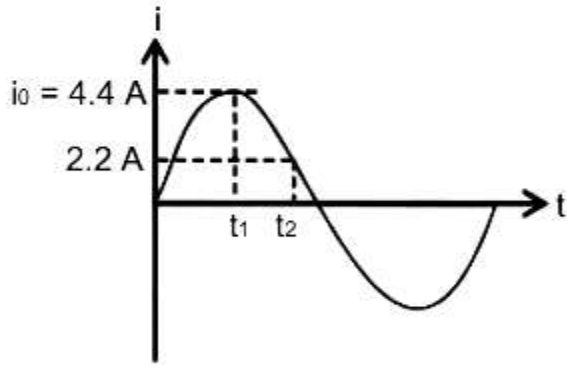


V = 220 sin (100 t)

$i = \frac{v}{R} \sin(100t)$

$= \frac{220}{50} \sin(100t)$

$i = 4.4 \sin(100t)$



For finding time t_1, taking i = 4.4 A

$4.4 = 4.4 \sin(100t_1)$

$\sin(100t_1) = 1$

$100\, t_1 = \frac{\pi}{2}$

$t_1 = \frac{\pi}{200}$ sec

For Finding time t_2, taking i = 2.2 A

$2.2 = 4.4\sin(100t_2)$

$\frac{1}{2} = \sin(100\ t_2)$

$\Rightarrow 100t_2 = \frac{5\pi}{6}$

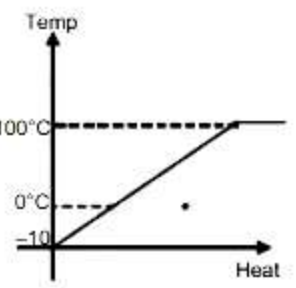
$t_2 = \frac{5\pi}{600}$ sec

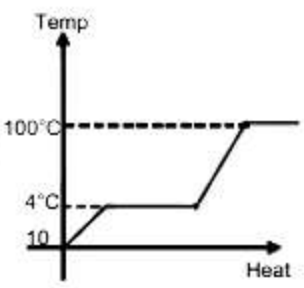
Time taken from peak value to half of peak value of current

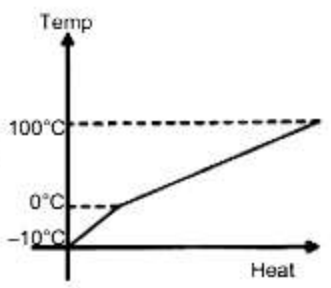
$\Delta t = t_2 = t_1 = \frac{5\pi}{600} - \frac{\pi}{200} \Rightarrow \frac{5\pi}{600} - \frac{3\pi}{600}$

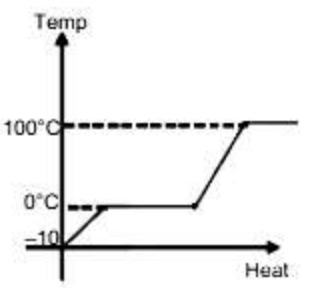
$= \frac{2\pi}{600} = \frac{\pi}{300}$ Ans

2. Draw true phase diagram for true temperature versus heat supplied when ice at (–10°C) converts into stream at 100°C.

(1) 

(2) 

(3) 

(4) 

Ans. **(4)**

3. A simple pendulum of length ℓ = 4 m is taken to height 'R' from earth surface. Calculate the time period of oscillation of simple pendulum at given height. Given : acceleration due to gravity at earth' surface $g = \pi^2$

(1) 4 sec (2) 8 sec (3) 6 sec (4) 10 sec

Ans. **(2)**

Sol. $T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$ where $g_{eff} = g\left[\frac{R}{R+h}\right]^2 = g\left[\frac{R}{R+R}\right]^2 = \frac{g}{4}$

$\therefore\ T = 2\pi\sqrt{\frac{4}{g/4}} = 2\pi\sqrt{\frac{16}{\pi^2}} = 8\,\text{sec}.$

4. If $m = K.c^x G^{\frac{1}{2}} .h^{\frac{1}{2}}$ then find x if symbols have general meaning and k is dimensionless (m = mass, c = speed of light, G = universal gravitation constant f h = plank's constant)

(1) 1 (2) $-\frac{1}{2}$ (3) $\frac{1}{2}$ (4) 2

Ans. **(3)**

Sol. $[M^1] = [L^x T^{-x}][L^3 M^{-1} T^{-2}]^{\frac{-1}{2}} [M^1 L^2 T^{-1}]^{\frac{1}{2}}$

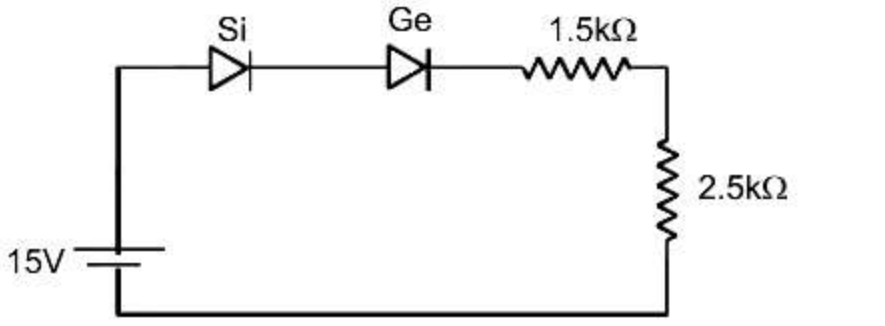
$\Rightarrow \quad [M^1] = \left[M^1 L^{\left(x-\frac{1}{2}\right)} T^{\left(\frac{1}{2}-x\right)}\right]$

$\Rightarrow$ After comparing powers

$x - \frac{1}{2} = 0, \quad \frac{1}{2} - x = 0$

$x = \frac{1}{2}, \quad x = \frac{1}{2}$

5. In the given circuit find the potential difference 2.5 k Ω. Given : knew voltages for silicon and germanium P-N junction is 0.7 volt and 0.3 Volt respectively



(1) 3.25 V (2) 4.75 (3) 9.50 (4) 8.75 V

Ans. **(4)**

Sol. $V_{Si} = 0.7$ V

$V_{Ge} = 0.3$ V

$i = \frac{15 - 0.3 - 0.7}{(2.5 + 1.5) \times 10^3} = \frac{14}{4}$ mA $= \frac{7}{2}$ mA

therefor potential difference across resistor

$V = iR$

$= \frac{7}{2} \times 10^3 \times 2.5 \times 10^3$

$= \frac{35}{4} \times 1$

$= 8.75$ V.

6. A Charge '–q' and mass 'm' is revolving in circular orbit of radius 'r' around infinite length wire with linear charge density 'λ'. Find the time period of revolution

(1) $(2\pi)^{1/2} r\sqrt{\frac{m\varepsilon_0}{\lambda q}}$ (2) $(2\pi)^{3/2}\sqrt{\frac{rm\varepsilon_0}{\lambda q}}$ (3) $(2\pi)^{3/2} r\sqrt{\frac{m\varepsilon_0}{\lambda q}}$ (4) $(2\pi)^{1/2}\sqrt{\frac{rm\varepsilon_0}{\lambda q}}$

Ans. **(3)**

Sol. $E = \frac{\lambda}{2\pi r \in_o}$

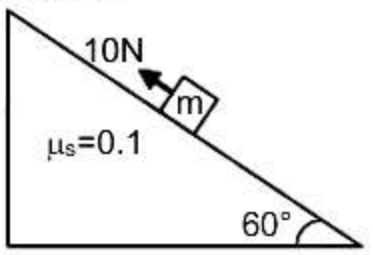
$\left|\vec{F_e}\right|$ = centripetal force required

$\left|\vec{F_e}\right| = \frac{\lambda q}{2\pi r \in_o} \quad \Rightarrow \frac{\lambda q}{2\pi r \in_o} = m\omega^2 r$

$\Rightarrow \quad \omega = \sqrt{\frac{\lambda q}{2\pi m r^2 \in_o}}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{\lambda q}{2\pi m r^2 \in_o}}} = (2\pi)^{3/2} r\sqrt{\frac{m \in_o}{\lambda q}}$

7. A 10 N Force is applied on mass 1 kg in upward direction as shown. Find the work done against friction force in taking it up by 10 m along inclined

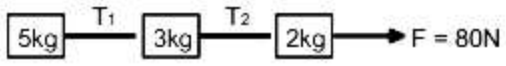


(1) 10 J (2) 5 J (3) 20 J (4) 25 J

Ans. **(2)**

Sol. $w = (\mu mg\cos 60°)10 = 5$ J

8. Find the value of tension T_1 and T_2 respectively in the given figure?

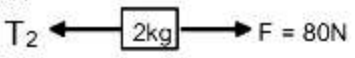


(1) 60N, 72 N (2) 72 N, 60N (3) 40 N, 64N (4) 64N, 40 N

Ans. **(3)**

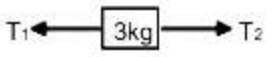
Sol. $M_{Total} = 5 + 3 + 2 = 10$ kg

$a = \frac{F}{M_{Total}} = \frac{80}{10} = 8 m/sec^2$

$\Rightarrow$ 

$80 - T_2 = 2 \times 8$

$T_2 = 80 - 16 = 64$ N

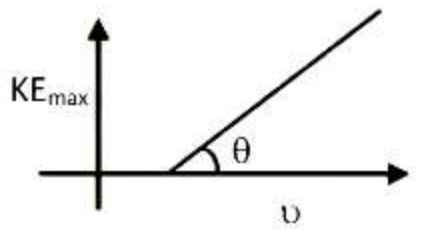
$\Rightarrow$ 

$T_2 - T_1 = 3 \times 8$

$T_1 = 64 - 3 \times 8$

$T_1 = 64 - 24 = 40$ N

9. Graph of maximum possible K.E. of photo-electron and frequency of incident photons is as shown in figure. Find slope of graph.



(1) h/e (2) h (3) e/h (4) 1/h

Ans. **(2)**

Sol. $k_{max} = h\upsilon - h\upsilon_{th}$

Slope = h.

10. Match the following :

A.	$\oint \dot{B}.\vec{dA} = 0$	P.	Faraday & Lens's law
B.	$\oint \dot{E}.\vec{dA} = \frac{Q_{in}}{\epsilon_o}$	Q.	Gauss law of on magnetism
C.	$\oint \dot{B}.\vec{dl} = \mu_o i_{enc}$	R.	Ampere's law
D.	$\oint \dot{E}.\vec{dl} = \frac{-d\phi_B}{dt}$	S.	gauss law of electrostatics

(1) (A – Q), (B – S), (C – R), (D – P)

(2) (A – S), (B – Q), (C – R), (D – P)

(3) (A – Q), (B – R), (C – S), (D – P)

(4) (A – Q), (B – S), (C – P), (D – R)

Ans. **(1)**

Sol. (A – Q), (B – S), (C – R), (D – P)

11. For a given planet $R_P = \frac{R_e}{3}$ & $M_P = \frac{M_e}{6}$, Then find the escape velocity for this planet if the escape velocity for earth is 11.2 km/sec. (R_e = radius of earth and Me = mass of earth)

(1) 7.92 km/sec (2) 11. 2 km/sec (3) 10.3 km/sec (4) 6.9 km/sec

Ans. **(1)**

Sol. $V_{es} = \sqrt{\frac{2GM}{R}}$

$$\frac{(V_{es})_p}{(V_{es})_e} = \sqrt{\frac{M_P}{M_e} \times \frac{R_e}{R_P}}$$

$$\frac{(V_{es})_p}{11.2} = \sqrt{\frac{3}{6}}$$

$(V_{es})_P$ = 7.92 km/sec.

12. 5A current is passing through a square of side 1m then find the magnetic field at the centre of this square.

(1) $8\sqrt{2}\times10^{-6}\,T$ (2) $4\sqrt{2}\times10^{-6}\,T$ (3) $2\sqrt{2}\times10^{-6}\,T$ (4) $6\sqrt{2}\times10^{-6}\,T$

Ans. **(2)**

Sol. $B_C = 4\times\frac{\mu_0 I}{4\pi d}(\sin45^\circ+\sin45^\circ)$

$$= 20\times\frac{\mu_0}{4\pi\times\frac{1}{2}}\times\sqrt{2}$$

$$= 4\sqrt{2}\times10^{-6}\,T$$

13. A vector which have magnitude same as of $3\hat{i}+4\hat{j}$ & direction along $4\hat{i}+3\hat{j}$, is $x\hat{i}+3\hat{j}$ then value x will be?

(1) 3 (2) 2 (3) 4 (4)10

Ans. **(3)**

Sol. $\hat{d} = \frac{4\hat{i}+3\hat{j}}{\sqrt{4^2+3^2}} = \frac{4}{5}\hat{i}+\frac{3}{5}\hat{j}$

$$\left|\vec{d}\right| = \sqrt{3^2+4^2} = \sqrt{25} = 5$$

$$\vec{d} = \left|\vec{d}\right|\ \vec{d}$$

$$\vec{d} = 5\ \left(\frac{4}{5}\hat{i}+\frac{3}{5}\hat{j}\right)$$

$$\vec{d} = \left(4\hat{i}+3\hat{j}\right)$$

14. An electron is revolving in n^{th} orbit of He^+ ion. Its magnetic moment depends on the radius of orbit as :

(1) r (2) $r^{1/2}$ (3) $r^{3/2}$ (4) r^2

Ans. **(2)**

Sol. $M = iA$

$$= \frac{e}{T}A$$

$$= \frac{ev}{2\pi r}\pi r^2$$

$$= \frac{evr}{2}$$

$$= \frac{er}{2}\left[\frac{nh}{2\pi rmr}\right] = \frac{neh}{4\pi m}$$

$\therefore$ $M \propto n$

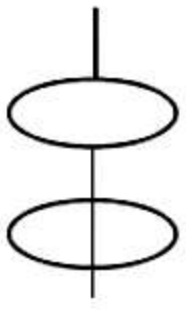
$M \propto \sqrt{r}$

15. A disc of moment of inertia 4 kg/m^2 is spinning freely with ω = 10 rad/s. A second disc of moment of inertia 2 kg/m^2 and spinning with ω = 4 rad/s in same direction, slides down on the spindle and combined slowly and start spinning together. What is the loss in kinetic energy?

(1) 12 J (2) 24 J (3) 36 J (4) 48 J

Ans. **(2)**

Sol. Angular momentum conservation



$I_1 = 4kg/\omega^2$
ω_1=10 rad/s

$I_2 = 2kg/\omega^2$
ω_2=4 rad/s

$L_i = L_f$

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

$4 \times 10 + 2 \times 4 = (4 + 2)\omega$

$40 + 8 = 6 \times \omega$

$$\frac{48}{6} = \omega$$

W = 8 rad/s

$$E_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$$

$$= \frac{1}{2} \times 4 \times (10)^2 + \frac{1}{2} \times 2 \times (4)^2$$

E_i = 200 + 16 = 216

$$E_f = -\frac{1}{2}(I_1 + I_2)\omega^2$$

$$E_f = \frac{1}{2}(4 + 2)8^2$$

$E_f = 3 \times 64$

E_f = 192.

Loss in K.E. = 216 – 192 = 24 J

16. Two polarizers are placed at 45° angle. The intensity of final light if unpolarised light of intensity I_0 is incident on one of polarizer.

(1) $\frac{I_0}{2}$ (2) $\frac{I_0}{8}$ (3) $\frac{I_0}{4}$ (4) $\frac{I_0}{16}$

Ans. **(3)**

Sol. $I = \frac{I_0}{2}\cos^2 45°$

$= \frac{I_0}{4}$

17. 3 moles of monoatomic gas is mixed with 2 moles of diatomic gas. find γ_{mix}

(1) 1.32 (2) 1.42 (3) 1.52 (4) 1.72

Ans. **(3)**

Sol. $\gamma_{mix} = \frac{C_{p_{mix}}}{Cv_{mix}} = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1C_{v_1} + n_2C_{v_2}}$

$= \frac{3\left(\frac{5}{2}R\right) + 2\left(\frac{7}{2}R\right)}{3\times\left(\frac{3}{2}R\right) + 2\left(\frac{5}{2}R\right)}$

$= \frac{15+14}{9+10}$

$= \frac{29}{19} = 1.52$

18. A step down transformer has primary voltage of $V_p = 3.2$ KV, number of turn in primary coil is 3000 with current 5 A. on secondary coil voltage is 320 V with number of turns Ns. If efficiency of transformer is 90% then find the current is secondary coil?

(1) 15 A (2) 30 A (3) 45 A (4) 60 A

Ans. **(3)**

Sol. Input power $= 3.2 \times 5 \times 10^3$

$= 16$ kw

$\text{eff.} = 90\% = \frac{\text{output power}}{\text{input power}} = \frac{P_{out}}{16KV} = \frac{9}{10}$

$P_{out} = \frac{9}{10} \times 16 \times 10^3$

$V_sI_s = \frac{9}{10} \times 16 \times 10^3$

$I_s = \frac{9\times16\times10^3}{10\times320} = 45A$

19. Heat developed in a wire of resistance R is W. If it is cut into two equal parts and connected into parallel with same source battery then heat produced in same time will be.

(1) W (2) 2 W (3) 3 W (4) 4W

Ans. **(4)**

Sol. Head $W = \frac{V^2}{R}t$

$$\frac{W_2}{W_1} = \frac{R_1}{R_2} = \frac{R}{\left(\frac{R}{4}\right)} = 4$$

$W_2 = 4W_1 \Rightarrow W_{new} = 4W$

20. Number of spectral line in the spectrum of He^+, for transition from n = 5 to 1.

(1) 10 (2) 6 (3) 8 (4) 5

Ans. (1)

Sol. Number of spectral lines = ${}^nC_2$

$= {}^5C_2 = 10$

21. A mass is to be kept on the surface of curve $y = x^2 / 4$ such that it does not slip. Find the maximum height at witch it should be kept if $\mu = 0.5$

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) 1 (4) 2

Ans. **(2)**

Sol. For no slipping

$\mu\, mg \cos\theta \geq mg \sin\theta$

$\mu \geq \tan\theta$

$\mu \geq dy/dx$

$0.5 \geq \frac{x}{2}$

$1 \geq x$

$y = \frac{x^2}{4} \Rightarrow x^2 = 4y$

$x^2 \leq 1$

$4y \leq 1$

$y \leq \frac{1}{4}$

19. Heat developed in a wire of resistance R is W. If it is cut into two equal parts and connected into parallel with same source battery then heat produced in same time will be.

(1) W (2) 2 W (3) 3 W (4) 4W

Ans. **(4)**

Sol. Head $W = \frac{V^2}{R}t$

$$\frac{W_2}{W_1} = \frac{R_1}{R_2} = \frac{R}{\left(\frac{R}{4}\right)} = 4$$

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(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) 1 (4) 2

Ans. **(2)**

Sol. For no slipping

$\mu\, mg \cos\theta \geq mg \sin\theta$

$\mu \geq \tan\theta$

$\mu \geq dy/dx$

$0.5 \geq \frac{x}{2}$

$1 \geq x$

$y = \frac{x^2}{4} \Rightarrow x^2 = 4y$

$x^2 \leq 1$

$4y \leq 1$

$y \leq \frac{1}{4}$

22. 1000 drops , each have surface energy E_1 converted into 1 bigger drop of surface energy E_2 then $\frac{E_1}{E_2}$ will be

(1) $\frac{1}{110}$ (2) $\frac{1}{81}$ (3) $\frac{1}{100}$ (4) $\frac{1}{121}$

Ans. **(3)**

Sol. $E = S\times A$

$E_1 = S \times 4\pi r^2$ .... (i)

$E_2 = S \times 4\pi R^2$ .... (ii)

Vol. conservation

$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$10^3 \times r^3 = R^3$

$\frac{r}{R} = \frac{1}{10}$

$\frac{E_1}{E_2} = \frac{s\times 4\pi r^2}{s\times 4\pi R^2} = \left(\frac{r}{R}\right)^2$

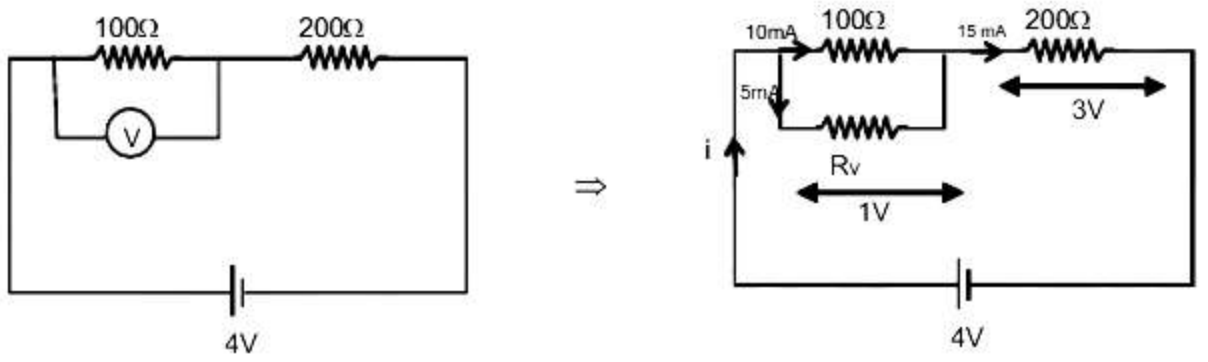
$\frac{E_1}{E_2} = \frac{1}{100}$

23. A 100Ω resistance and 200 Ω resistance is connected in a series with 4v battery. A voltmeter connected across 100 Ω reads 1V. Find internal resistance of voltmeter

(1) 150 (2) 200 (3) 190 (4) 220

Ans. **(2)**

Sol.



$V = I\,Rv$

$1 = 5 \times 10^{-3} \times Rv \Rightarrow Rv = \frac{10^3}{5} = 200\Omega$

24. 49 main scale divisions is equal to 50 vernier scale divisions. If one main scale divisions is 0.5 mm then find the value of vernier constant

(1) 0.01 mm (2) 0.1mm (3) 0.02 mm (4) 0.2 mm

Ans. **(1)**

Sol. vernier constant = L.C = 1MSD – 1VSD

$$= \left[1-\frac{49}{50}\right] \text{MSD}$$

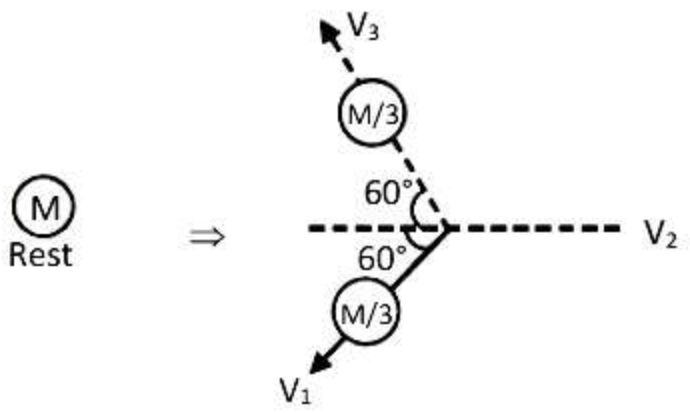
$$= \frac{1}{50} \times 0.5\,\text{mm} = 0.01\text{mm}$$

25. A parent nuclei of mass M is splits into three daughter nuclei of equal mass. Find speed of daughter nuclei if mass defect is Δm :

(1) $\sqrt{\frac{2\Delta M}{M}}$ (2) $\sqrt{\frac{2\Delta MC}{M}}$ (3) $\sqrt{\frac{3\Delta M}{M}}$ (4) $\frac{2\Delta MC^2}{m}$

Ans. **(1)**

Sol.

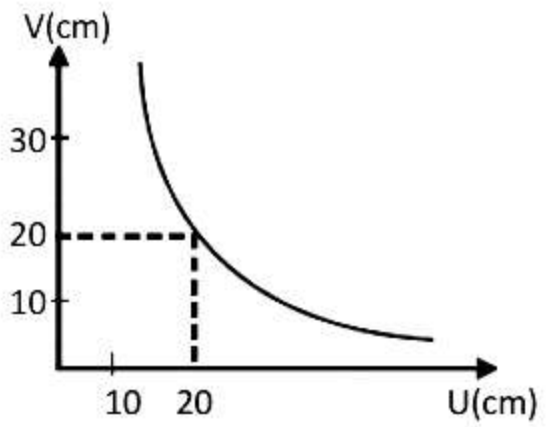


$V_1 = V_2 = V_3$ from symmetry or momentum conservation

Energy released $3\left(\frac{1}{2}\frac{M}{3}V^2\right) = \Delta mC^2$

$V = \sqrt{\frac{2\Delta m}{M}}.C$.

26. v-u graph is given for a concave mirror. Find focal length of concave mirror.



(1) 200 cm (2) 10 cm (3) 15 cm (4) 5 cm

Ans. **(2)**

Sol. $\frac{1}{V}+\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{-20}+\frac{1}{-20}=\frac{1}{f}$$

F = 10 cm

JEE Main 30 Jan 2024 (Shift-2) (Memory Based)

PART : CHEMISTRY

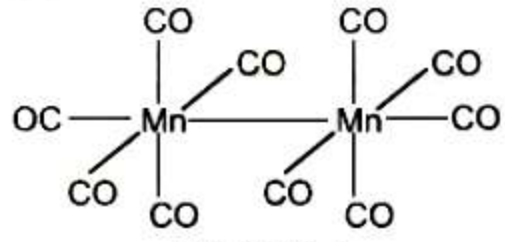
1. Geometry of Decacarbonyl dimanganese (0) is :

(1) Octahedral (2) Square planar

(3) Trigonal bipyramidal (4) Square pyramidal

Ans. **(1)**

Sol.



$[Mn_2(CO)_{10}]$

2. Which of the following species has square pyramidal geometry ?

(1) PCl_5 (2) BrF_5 (3) PF_5 (4) $[Ni(CN)_4]^{2-}$

Ans. **(2)**

Sol.

(1) PCl_5 sp^3d (trigonal bipyramidal (5 BP + 0 LP))

(2) BrF_5 sp^3d^2 (square pyramidal (5 BP + 1 LP))

(3) PF_5 sp^3d (trigonal bipyramidal (5 BP + 0 LP))

(4) $[Ni(CN)_4]^{2-}$ dsp^2 (square planar)

3. **Statement-1** : H_2Te is more acidic than H_2S.

Statement-2 : H_2Te has less bond strength than H_2S.

(1) Statement-1 and Statement-2 are correct.

(2) Statement-1 and Statement-2 are incorrect.

(3) Statement-1 is correct and Statement-2 is incorrect.

(4) Statement-1 is incorrect and Statement-2 is correct.

Ans. **(1)**

Sol. In H_2Te bond length increases and bond energy decreases therefore acid strength increases.

4. Among the following correct statement is :

(1) Stability of hydrides order: $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$

(2) Reducing strength order: $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$

(3) NH_3 is strongest reducing agent while BiH_3 is mild reducing agent

(4) Basicity of hydrides: $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$

Ans. **(2)**

Sol. Due to increase in bond length down the group reducing strength increases.

5. Given standard electrode potential of BrO_4^-, IO_4^- and ClO_4^- are 1.85 V, 1.65 V and 1.20 V respectively then select order of their oxidizing power :

(1) $ClO_4^- < BrO_4^- < IO_4^-$ (2) $IO_4^- < BrO_4^- < ClO_4^-$

(3) $ClO_4^- < IO_4^- < BrO_4^-$ (4) $BrO_4^- < ClO_4^- < IO_4^-$

Ans. **(3)**

Sol. More reduction potential, more is the oxidizing strength.

6. **Statement-1:** Since fluorine is more electronegative than nitrogen, the resultant dipole moment of NH_3 is greater than that of NF_3.

Statement-2: In case of NH_3 the orbital dipole due to lone pair is in the same direction as the resultant dipole moment of N-H bonds, whereas in NF_3 the orbital dipole is in the direction opposite to the resultant dipole moment of three N-F bonds.

(1) Statement I and Statement II are correct.

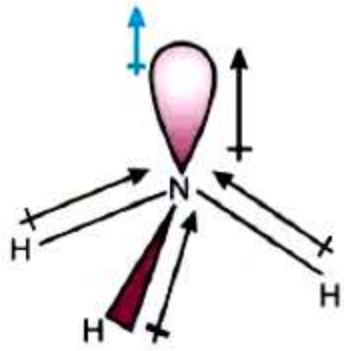
(2) Statement I is correct and Statement II is incorrect

(3) Statement I is incorrect and Statement II is correct

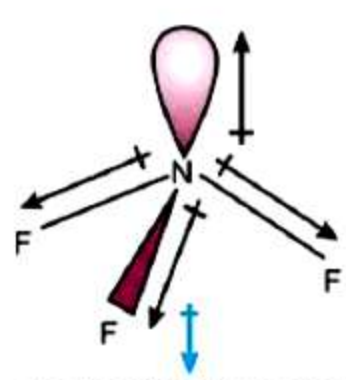
(4) Statement I and Statement II are incorrect

Ans. **(1)**

Sol. The orbital dipole because of lone pair decreases the effect of the resultant N – F bond moments, which results in the low dipole moment of NF_3 as represented below :



Resultant dipole moment in $NH_3 = 4.90 \times 10^{-30}$ C m



Resultant dipole moment in $NF_3 = 0.80 \times 10^{-30}$ C m

7. In a mixture of B & C, A is added. Given moles of A, B & C are respectively n_A, n_B & n_C then determine mole fraction of C.

(1) $\frac{n_C}{n_A + n_B + n_C}$ (2) $\frac{n_C}{n_A . n_B + n_C}$ (3) $\frac{n_C}{n_A . n_C + n_B}$ (4) $\frac{n_C}{n_A + n_B}$

Ans. **(1)**

Sol. Mole fraction = $\frac{\text{moles of substance}}{\text{total no. of moles}}$

8. The colour of $KMnO_4$ and $K_2Cr_2O_7$ is due to

(1) d-d transition (2) Metal to ligand charge transfer

(3) Ligand to metal charge transfer (4) F-Center

Ans. **(3)**

Sol. $KMnO_4$ $K_2Cr_2O_7$

(Purple) (Orange Red)

Colour of $KMnO_4$ & $K_2Cr_2O_7$ is due to ligand to metal charge transfer phenomenon.

9. $MnO_2 + KOH + O_2 \xrightarrow{\Delta} [X]$

$[X] \xrightarrow[\text{Process}]{\text{Electrolytic}} [Y]$

Select correct option

	[X]	[Y]
(1)	MnO_4^-	MnO_4^{-2}
(2)	MnO_4^{-2}	MnO_4^-
(3)	Mn_2O_3	Mn
(4)	Mn_2O_7	MnO_4^-

Ans. **(2)**

Sol. $2\,MnO_2 + 4KOH + O_2 \longrightarrow 2K_2MnO_4 + 2H_2O$

$MnO_4^{-2} \xrightarrow[\text{Oxidation}]{\text{Electrolytic}} MnO_4^-$

[X] [Y]

10. $CrO_2Cl_2 + NaOH \longrightarrow [X]$

$[X] + HCl + H_2O_2 \longrightarrow [Y]$

Select correct option

	[X]	[Y]
(1)	Na_2CrO_4	CrO_5
(2)	$Na_2Cr_2O_7$	Cr_2O_3
(3)	CrO_5	Na_2CrO_4
(4)	Cr_2O_3	$Na_2Cr_2O_7$

Ans. **(1)**

Sol. $CrO_2Cl_2 + NaOH \longrightarrow Na_2CrO_4 + NaCl + H_2O$

$CrO_4^{-2} \xrightarrow{H^+} Cr_2O_7^{-2} \xrightarrow{H^+ \,\&\, H_2O_2} CrO_5$

11. Which of the following solution have maximum depression in freezing point ?

(1) 180 g of glucose in water

(2) 180 g of Benzoic acid in Benzene

(3) 180 g of Acetic acid in Benzene

(4) 180 g of Acetic acid in water

Sol. $\Delta T_f = i K_f m$; $\Delta T_f \propto im$

$\Delta T_f \propto in$

$\Delta T_f \propto i\frac{W}{M}$

For Glucose i = 1 in Water

Benzoic acid i < 1 in Benzene

Acetic acid i < 1 in Benzene

Acetic acid i > 1 in Water

More the number of mole of solute greater is depression in freezing point.

12. Among the following how many are optical active

(i) cis $[Co(en)_2Cl_2]$

(ii) trans $[Co(en)_2Cl_2]$

(iii) cis $[Pt(en)_2Cl_2]^{+2}$

(iv) trans $[Pt(en)_2Cl_2]^{+2}$

(v) $[Pt(en)_3]^{+4}$

(vi) $[Pt(en)Cl_4]$

Ans. **(3)**

Sol.

Complex	**Optical Nature**
(i) cis $[Co(en)_2Cl_2]$	Optical active
(ii) trans $[Co(en)_2Cl_2]$	Optical inactive
(iii) cis $[Pt(en)_2Cl_2]^{+2}$	Optical active
(iv) trans $[Pt(en)_2Cl_2]^{+2}$	Optical inactive
(v) $[Pt(en)_3]^{+4}$	Optical active
(vi) $[Pt(en)Cl_4]$	Optical inactive

13. In He^+ ion an electron Jumps from 5^{th} excited state to 1^{st} excited state, then total number of spectral lines formed are _____.

Ans. **(10)**

Sol. 5^{th} excited state = $n_2 = 6$

1^{st} excited state = $n_1 = 2$

total spectral line

upto 5^{th} state = 1

4^{th} state = 2

3^{th} state = 3

2^{nd} state = 4

total line = 10

14. Using following reaction

(i) $2\,Fe_{(s)} + \frac{3}{2}O_2(g) \longrightarrow Fe_2O_{3(s)}$ $\Delta H^\circ_{rxn} = -822$ KJ\mole.

(ii) $C_{(s)} + \frac{1}{2}O_{2(g)} \longrightarrow CO_{(g)}$ $\Delta H^\circ_{rxn} = -110$ KJ/Mole.

The value of ΔH°_{rxn} for reaction $Fe_2O_{3(s)} + 3C_{(s)} \longrightarrow 2Fe_{(s)} + 3CO$ is _____ KJ [Nearest integer]

Ans. **(492)**

Sol. Target equation = 3 × equation 2 – equation 1

$\Delta H_{rxn} = 3 \times [-110] - [-822] = -330 + 822 = 492$ KJ

15. How many of following species can show redox disproportion reaction.

H_2O_2, P_4, NO_2, Cu^+, ClO_3^-, ClO_4^-, F_2, Cl_2

Ans. **(6)**

Sol. To show disproportion reaction element must show at least 3 oxidation state and element must present in intermediate oxidation state.

$\overset{-1}{H_2O_2}$, $\overset{0}{P_4}$, $\overset{+3}{NO_2}$, $\overset{+1}{Cu}$, $\overset{+5}{ClO_3^-}$, $\overset{0}{Cl_2}$

can show disproportion reaction.

16. How many of the following can show flame colour test ?

Be, Mg, Sr, Ba, Li, Cu

Ans. **(4)**

Sol. Be & Mg do not show flame colour test.

17. In buffer solution of benzoic acid and sodium benzoate pH of solution is 4.5. then ratio of moles of salt to moles of acid is _____ [Nearest Integer]

[Give pKa (Benzoic acid) = 4.5 and log2 = 0.3]

Ans. **(2)**

Sol. Benzoic acid + sodium benzoate.

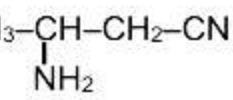
Acidic Buffer solution

$$pH = pKa + \log\frac{[Salt]}{[Acid]}$$

$$4.5 = 4.2 + \log\frac{[Salt]}{[Acid]}$$

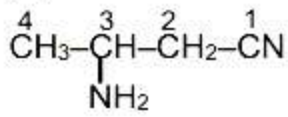
$$0.3 = \log\frac{[Salt]}{[Acid]}$$

$$\frac{[Salt]}{[Acid]} = 2 \quad \Rightarrow \frac{n_{salt}}{n_{acid}} = 2$$

18. IUPAC name of  is :

(1) 3-Aminobutanenitrile
(2) 3-Aminobutanecarbonitrile
(3) 2-Amino-1-cyanopropane
(4) 3-Aminebutanenitrile

Ans. **(1)**

Sol. 

19. Which reagent on reaction with phenol give salicyldehyde :

(1) CO_2, NaOH
(2) $CHCl_3$, NaOH
(3) CCl_4, NaOH
(4) H_2O, H^+

Ans. **(2)**

Sol. This reaction is known as Reimer-Tieman reaction.

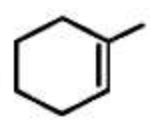
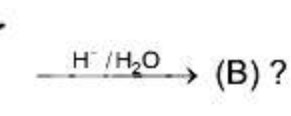
20. The correct order of stability for given carbocations is :

(I) $(CH_3)_3C^+$, (II) $(CH_3)_2CH^+$, (III) $CH_3CH_2^+$, (IV) CH_3^+

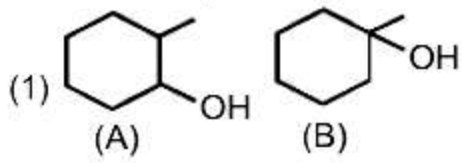
(1) II > I > III > IV
(2) I > II > III > IV
(3) IV > III > II > I
(4) I > II > IV > III

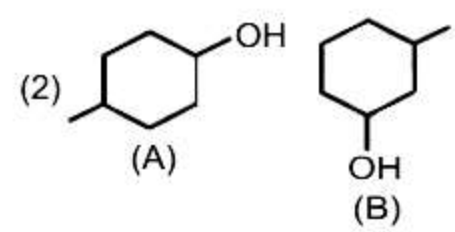
Ans. **(2)**

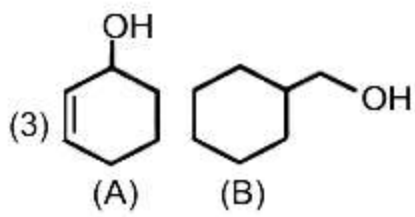
Sol. Greater the number of α-hydrogen, greater the hyperconjugation and stability of carbocation.

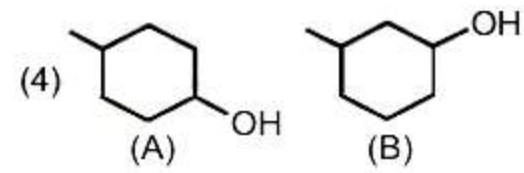
21. (A)   (B) ?

A & B are respectively

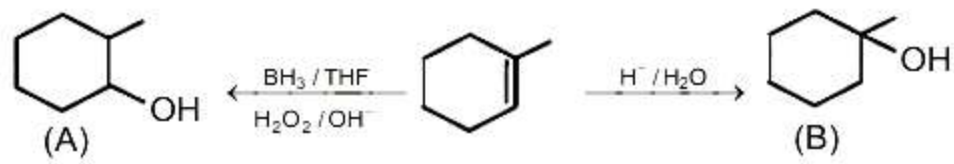
(1) 
(A) (B)

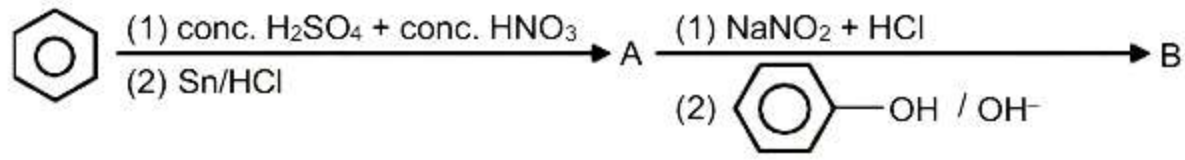
(2) 
(A) (B)

(3) 
(A) (B)

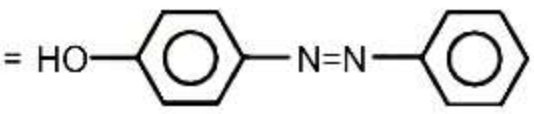
(4) 
(A) (B)

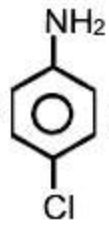
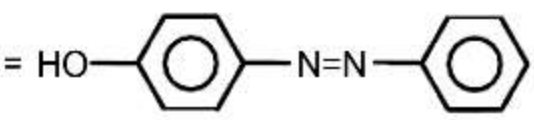
Ans. **(1)**

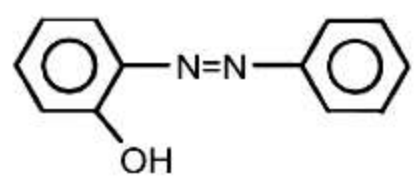
Sol. 
(A) (anti-Marconikov addition of water) (B) (Marconikov addition of water)

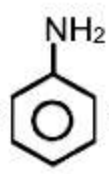
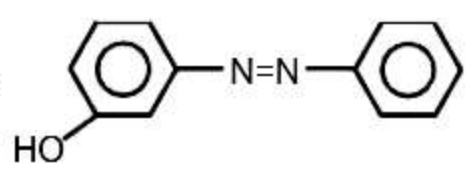
22. 

What is A and B

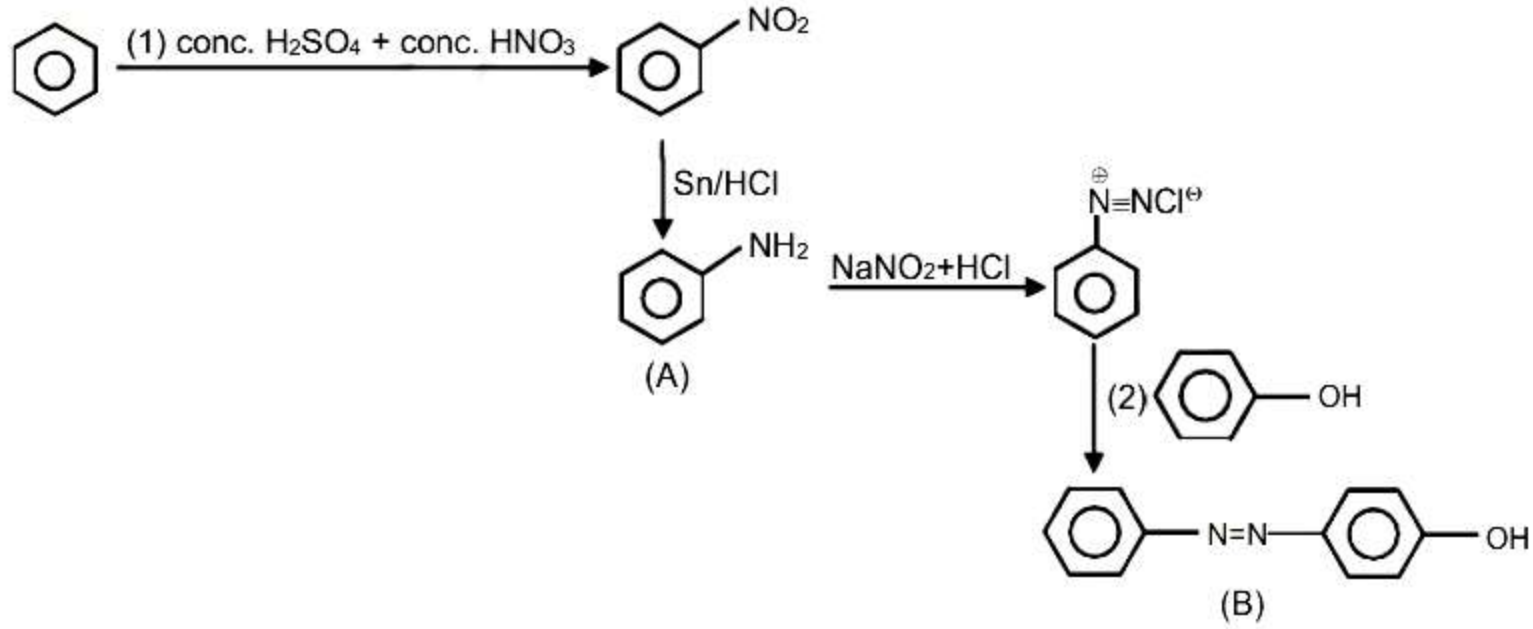
(1) A =  ; B = 

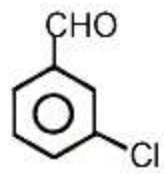
(2) A =  ; B = 

(3) A =  ; B = 

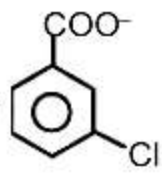
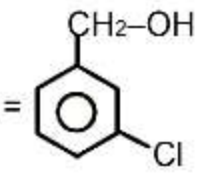
(4) A =  ; B = 

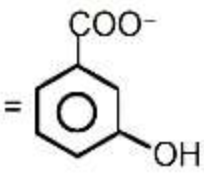
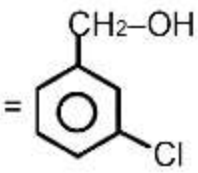
Ans. **(1)**

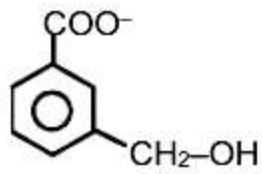
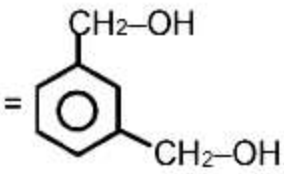
Sol. 

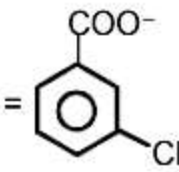
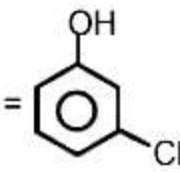
23.  $\xrightarrow{50\%\ KOH}$ A + B ?

A & B are respectively

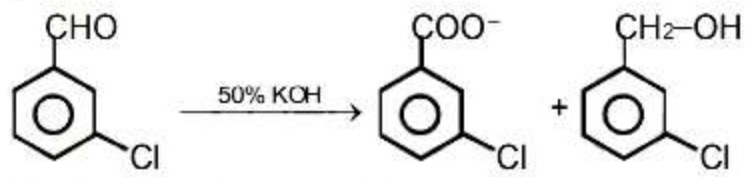
(1) A =  ; B = 

(2) A =  ; B = 

(3) A =  ; B = 

(4) A =  ; B = 

Ans. **(1)**

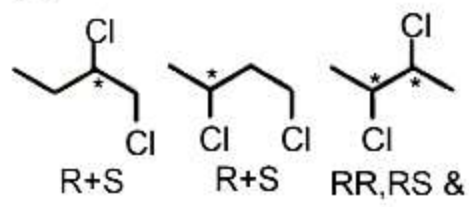
Sol. 

This is a cannizaro reaction.

24. Total number of optical isomer formed is

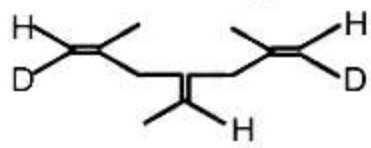
Chlorobutane + $Cl_2 \longrightarrow$ dichlorobutane

Ans. **(7)**

Sol. 

R+S R+S RR,RS & SS

25. Total number of geometrical isomer possible for given compound is ?



Ans. **(4)**

JEE Main 30 Jan 2024 (Shift-2) (Memory Based)

PART : MATHEMATICS

1. Let S_K denotes sum of first k terms of sequence 3, 7, 11, 15, ... Value of n, if

$$40 < \frac{6}{n(n+1)}\sum_{k=1}^{n} S_k < 42 \text{, is } \_\_\_\_\_$$

Ans. **(9)**

Sol. $S_K = 3 + 7 + 11 + \ldots\ldots$ up to k terms

$$= \frac{k}{2}[6 + (k-1)4] = k(2k+1)$$

$$\sum_{k=1}^{n}\left(2k^2 + k\right) = \frac{2n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$40 < 2(2n+1) + 3 < 42$

$35 < 4n < 37$

$8.75 < n < 9.25$

$n = 9$

2. Let $f(x) = \dfrac{x}{\left(1+x^4\right)^{\frac{1}{4}}}$ & $g(x) = f(f(f(f(x))))$ then $\displaystyle\int_0^{\sqrt{2\sqrt{5}}} x^2 g(x)dx$ is

(1) $\frac{13}{6}$ (2) $\frac{6}{13}$ (3) $\frac{2}{5}$ (4) $\frac{7}{2}$

Ans. **(1)**

Sol. $\dfrac{\left(1+x^4\right)^{\frac{1}{4}}}{\left(1+\frac{x^4}{1+x^4}\right)^{\frac{1}{4}}} = \dfrac{x}{\left(1+2x^4\right)^{\frac{1}{4}}}$

So $f(f(f(f(x)))) = \dfrac{x}{\left(1+4x^4\right)^{\frac{1}{4}}} = g(x)$

$$\int_0^{\sqrt{2\sqrt{5}}} x^2 g(x)dx = \int_0^{\sqrt{2\sqrt{5}}} \frac{x^3}{\left(1+4x^4\right)^{\frac{1}{4}}}dx$$

$$\int_1^3 \frac{\frac{1}{4}t^3 dt}{t} = \frac{1}{4}\int_1^3 t^2 dt = \frac{1}{12}(27-1) = \frac{26}{12} = \frac{13}{6}$$

$1 + 4x^4 = t^4$

$16x^3dx = 4t^3\,dt$

$x^3dx = \frac{1}{4}t^3dt$

3. $\bar{a}.\bar{b} = 3\sqrt{2}$ & $|\bar{b}|^2 = 6$ such that $\bar{a} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$. If angle between $\bar{a}$ & $\bar{b}$ is $\frac{\pi}{4}$ then value of $(\alpha^2+\beta^2)\,|\bar{a}\times\bar{b}|^2$ is _____.

Ans. **(90)**

Sol. $\bar{a}.\bar{b} = 3\sqrt{2}$

$$|\bar{a}||\bar{b}|\cos\frac{\pi}{4} = 3\sqrt{2}$$

$$|\bar{a}|\sqrt{6}.\frac{1}{\sqrt{2}} = 3\sqrt{2}$$

$$|\bar{a}| = \sqrt{6} = \sqrt{1+\alpha^2+\beta^2}$$

$\alpha^2 + \beta^2 = 5$

$$|\bar{a}\times\bar{b}|^2 = |\bar{a}|^2|\bar{b}|^2\sin^2\frac{\pi}{4} = 6\times6\times\frac{1}{2} = 18$$

So $\left(\alpha^2+\beta^2\right)|\bar{a}\times\bar{b}|^2 = 5 \times 18 = 90$

4. $R = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$, $x\sin\theta = y\sin\left(0+\frac{2\pi}{3}\right) = z\sin\left(0+\frac{4\pi}{3}\right) \neq 0$

Statement – 1 : Trace (R) = 0

Statement – 2 : Trace (adj(adj (R)) = 0

(1) Statement –1 is true and statement – 2 is false (2) Statement –1 is false and statement – 2 is false

(3) Statement –1 is false and statement – 2 is true (4) Statement –1 is true and statement – 2 is true

Ans. **(2)**

Sol. $$y = \frac{x\sin\theta}{\sin\left(\theta+\frac{2\pi}{3}\right)},\quad z = \frac{x\sin\theta}{\sin\left(\theta+\frac{4\pi}{3}\right)}$$

$$x + y + z = \frac{-3x}{4\sin\left(\theta+\frac{2\pi}{3}\right)\sin\left(\theta+\frac{4\pi}{3}\right)} \neq 0$$

$\Rightarrow$ Statement–1 is wrong

$$R = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix},\ \text{adj } R = \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$\text{adj(adj,R)} = \begin{bmatrix} x^2yz & 0 & 0 \\ 0 & y^2xz & 0 \\ 0 & 0 & z^2xy \end{bmatrix} \Rightarrow \text{Tr (adj (adj R) = } xyz(x + y + z) \neq 0$$

5. If $\left|\vec{a}\times\vec{b}\right| = 2$, and $\left|\vec{a}\right| = 1$, then $\left|\left(\vec{a}\times\vec{b}\right)-\vec{a}\right|^2$ is

Ans. **(5)**

Sol. $\left|\vec{a}\times\vec{b}\right| = 2$ and $\left|\vec{a}\right| = 1 = \left|\left(\vec{a}\times\vec{b}\right)-\vec{a}\right|^2 = \left|\vec{a}\times\vec{b}\right|^2 + \left|\vec{a}\right|^2 - 2\vec{a}.\left(\vec{a}\times\vec{b}\right) = (2)^2 + 1 - 0 = 5$

6. If 11^{th} term of G.P., whose 1^{st} term is 'a' and 3^{rd} term is 'b', is equal to p^{th} term of G.P. whose first term is 'a' and 5^{th} term is 'b', then value of p is

Ans. **(21)**

Sol. ($T_1 = a$, $T_3 = ar_1^2 = b \Rightarrow r_1^2 = b/a$) and ($T_1 = a$, $T_5 = ar_2^4 = b$

11^{th} term of first GP = p^{th} term of second G.P.

$a(r_1)^{10} = a(r_2)^{p-1}$

$$\left(\left(\frac{b}{a}\right)^{1/2}\right)^{10} = \left(\left(\frac{b}{a}\right)^{1/4}\right)^{p-1} \Rightarrow 5 = \frac{p-1}{4} \Rightarrow p = 21$$

7. Let $f(x) = \begin{cases} x^2+3x+a\,; & x \le 1 \\ bx+2\,; & x > 1 \end{cases}$ is differentiable everywhere. The value of $\int_2^2 f(x)dx$ is

(1) $\frac{37}{2}$ (2) $\frac{36}{2}$ (3) $\frac{37}{4}$ (4) $\frac{36}{4}$

Ans. **(1)**

Sol. f(x) is continuous at x = 1

$f(1^-) = f(1) = f(1^{+1})$

$a + 4 = b + 2 \Rightarrow b = a + 2$

f(x) is differential at x = 1

$f'(1^-) = f'(1^+)$

$5 = b \Rightarrow a = 3$

Now $\int_2^2 f(x)dx = \int_1^2 \left(x^2+3x+3\right)dx + \int_2^1 (5x+2)dx$

$$= \left(\frac{x^3}{3}+\frac{3x^2}{2}+3x\right)_{-2}^{1} + \left(\frac{5x^2}{2}+2x\right)_{1}^{2} = \left(\frac{1}{3}(1+8)+\frac{3}{2}(1-4)+9\right)+\frac{5}{2}(4-1)+2$$

$$= 3-\frac{9}{2}+9+\frac{15}{2}+2 = 14+\frac{9}{2} = \frac{37}{2}$$

8. Bag A contains 3 white & 7 red balls and bag B contains 2 white & 3 Red balls. If a ball is picked up randomly then what is the probability that the ball picked is white from bag A

(1) $\frac{3}{20}$ (2) $\frac{2}{20}$ (3) $\frac{3}{10}$ (4) $\frac{4}{20}$

Ans. **(1)**

Sol. Let A is event that bag A is selected

B is event that red bag B is selected

W is event that white ball is drawn

$P(A) = P(B) = \frac{1}{2}$

$P(A \cap W) = P(A)P\left(\frac{W}{a}\right)\left(\frac{W}{a}\right) = \frac{1}{2}.\frac{3}{10} = \frac{3}{20}$

9. f(x) is double differentiable function. Tangent at (1, f(1)) and (3,(f(3)) cuts positive x-axis at an angle $\frac{\pi}{6}$ and $\frac{\pi}{4}$ then value of $\int_1^3 \left((f'(x))^2+1\right)f''(x)dx$ is:

(1) $\frac{4}{3}-\frac{10}{9\sqrt{3}}$ (2) $\frac{3}{4}-\frac{10}{9\sqrt{3}}$ (3) $\frac{4}{3}-\frac{10}{\sqrt{3}}$ (4) $\frac{4}{3}-\frac{1}{9\sqrt{3}}$

Ans. **(1)**

Sol. $f'(1) = \tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}, f'(3)=1$

$$I = \int_1^3 \left((f'(x))^2+1\right)f''(x)dx$$

Let $f'(x) = t$

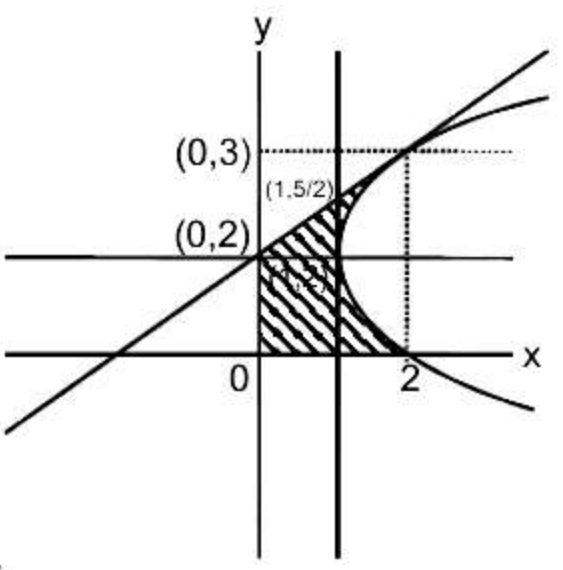
$f''(x)\,dx = dt$

$$I = \int_{\frac{1}{\sqrt{3}}}^{1} \left(t^2+1\right)dt = \left(\frac{t^3}{3}+t\right)_{\frac{1}{\sqrt{3}}}^{1} = \frac{4}{3}-\frac{10}{9\sqrt{3}}$$

10. Area bounded by $(y-2)^2 = x-1$, $x-2y+4=0$ and positive coordinate axes, is

Ans. **(5)**

Sol. Solving $y-2=\frac{x}{2}$ and $(y-2)^2 = x-1$



$x^2 = 4x-4$

$(x-2)^2 = 0$

$x = 2$

$y = 3$

$$\text{Area} = \int_0^3 (x_P - x_L)dy$$

$$= \int_0^3 \left((y-2)^2+1-2y+4\right)dy = \int_0^3 \left(y^2-6y+9\right)dy = \int_0^3 (y-3)^2 dy$$

$$\left(\frac{(y-3)^3}{3}\right)_0^3 = 0+\frac{27}{3} = 9$$

required area $= 9-\frac{1}{2}\times 4\times 2 = 5$

11. Let $f(x) = ae^{2x} + be^x + cx$, where $f(0) = -1$, $f'(\log_e 2) = 21$ and $\int_0^{\log_e 4} (f(x) - cx)dx = \frac{39}{2}$ then the value of $|a + b + c|$ is _____.

Ans. **(8)**

Sol. $f(0) = a + b = -1$ _____(1)

$f'(x) = 2ae^{2x} + be^x + c$

$f'(\log_e 2) = 8a + 2b + c = 21$ _____(2)

$$\int_0^{\log_e 4} (f(x) - cx)dx = \int_0^{\log_e 4} (ae^{2x} + be^x)dx$$

$$= \frac{a}{2}(16 - 1) + b(4 - 1)$$

$$= \frac{15a}{2} + 3b = \frac{39}{2} \quad \text{_____(3)}$$

$$\Rightarrow \frac{9a}{2} + 3(a + b) = \frac{39}{2} \Rightarrow \frac{9a}{2} = \frac{45}{2} \Rightarrow a = 5$$

From (2) $b = -6$ also $c = -7$

$|a + b + c| = |5 - 6 - 7| = 8$

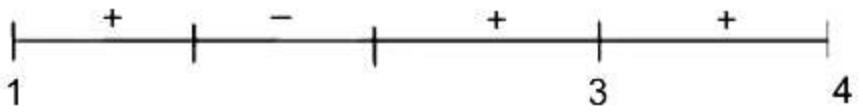
12. If $f(x) = (x - 2)^2 (x - 3)^3$ for $x \in [1, 4]$. If M and m denote maximum and minimum values respectively then M–m is

Ans. **(12)**

Sol. $f'(x) = 2(x - 2)(x - 3)^3 + 3(x - 2)^2 (x - 3)^2$

$= (x - 2)(x - 3)^2 (2x - 6 + 3x - 6)$

$= (5x - 12)(x - 2)(x - 3)^2$



Min max min max

Local max at $x = 2$ and $x = 4$

Local min at $x = \frac{12}{5}$ and $x = 1$

$f(1) = 1 \times (-8) = -8$

$$\begin{cases} f(2) = 0 \\ f\left(\frac{12}{5}\right) = \frac{-12}{125} \\ f(4) = 4 \times 1 = 4 \end{cases}$$

$M = 4$

$m = -8$

$M - m = 4 - (-8) = 12$

13. A question paper has three sections A,B, C having 8, 6, 4 questions respectively. If the student has to answer 15 questions attempting atleast four from each section. Find the number of ways the paper can be answered by student.

(1) 342 (2) 344 (3) 374 (4) 340

Ans. **(2)**

Sol.

8	**6**	**4**
A	B	C
7	4	$4 \rightarrow {}^8C_7 \times {}^6C_4 \times {}^4C_4$
6	5	$4 \rightarrow {}^8C_6 \times {}^6C_5 \times {}^4C_4$
5	6	$4 \rightarrow {}^8C_5 \times {}^6C_6 \times {}^4C_4$

An adding all cases we get

$({}^8C_7 \times {}^6C_4 \times {}^4C_4) + ({}^8C_6 \times {}^6C_5 \times {}^4C_4) + ({}^8C_5 \times {}^6C_6 \times {}^4C_4)$

$= \left(8 \times \frac{6 \times 5}{2} \times 1\right) + \left(\frac{8 \times 7}{2} \times 6 \times 1\right) + \left(\frac{8 \times 7 \times 6}{3 \times 2}\right)$

$= 120 + 168 + 56 = 344$

14. Consider the system of equations

$x + y + z = 5$

$x + 2y + \lambda^2 z = 9$

$x + 3y + \lambda z = \mu$

(1) system has unique solution for $\lambda = 1, \mu \neq 13$

(2) system has infinite solution for $\lambda = 1, \mu = 13$

(3) system is inconsistent for $\lambda = 1, \mu \in R$

(4) system has infinite solution for $\lambda = 1, \mu \neq 13$

Ans. **(2)**

Sol. $x + y + z = 5$ .....(1)

$x + 2y + \lambda^2 z = 9$ .....(2)

$x + 3y + \lambda z = \mu$ .....(3)

from (1) + (3) – 2(2)

$z(\lambda + 1 - 2\lambda^2) = (\mu - 13)$

for infinite solution $\lambda = -1/2, 1$ and $\mu = 13$

15. Let $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, $3\sin(\alpha + \beta) = 2\sin(\alpha - \beta)$ and $\tan\alpha = k\tan\beta$ then the value of k is

(1) – 5 (2) 5 (3) 3 (4) – 3

Ans. **(1)**

Sol. $3\sin(\alpha+\beta) = 2\sin(\alpha-\beta)$

$$\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{2}{3}$$

$$\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\sin(\alpha+\beta)-\sin(\alpha-\beta)} = \frac{5}{-1}$$

$$\frac{2\sin\alpha\cos\beta}{2\cos\alpha\sin\beta} = \frac{5}{-1}$$

$$\frac{\tan\alpha}{\tan\beta} = -5$$

$k = -5$

16. $L_1 \equiv \hat{i} - \hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j} + 2\hat{k})$ $\lambda \in R$

$L_2 \equiv \hat{j} - \hat{k} + \mu(3\hat{i} + \hat{j} + p\hat{k})$

$L_3 \equiv s(l\hat{i} + m\hat{j} + n\hat{k})$

L_1 is perpendicular to L_2

L_3 is perpendicular to L_1 & L_2

then (ℓ, m, n) can be

(1) (–1, 7, 4) (2) (4, –1, 7) (3) (7, 4, –1) (4) (7, –1, 4)

Ans. **(1)**

Sol. Since L_1 is perpendicular to L_2

$\Rightarrow (\hat{i} - \hat{j} + 2\hat{k}).(3\hat{i} + \hat{j} + p\hat{k}) = 0$

$3 - 1 + 2p = 0 \Rightarrow p = -1$

$L_3 \perp L_1$ & L_2

$\Rightarrow L_3$ is parallel to $(\hat{i} - \hat{j} + 2\hat{k}) \times (3\hat{i} + \hat{j} + p\hat{k})$

$$(\hat{i} - \hat{j} + 2\hat{k}) \times (3\hat{i} + \hat{j} + p\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & -1 \end{vmatrix} = -\hat{i} + 7\hat{j} + 4\hat{k}$$

$\ell = -1$, m = 7, n = 4

17. If A = {1, 2,3,4} then find number of symmetric relation on A which is not reflexive is

Ans. **(960)**

Sol. If set A has n elements then number of symmetric relation = $2^{\frac{n^2+n}{2}} = 2^{10} (\because n = 4)$

number of reflexive and symmetric relation = $2^{\frac{n^2-n}{2}} = 2^6$ ($\because$ n = 4)

number of relation which is symmetric but not reflexive = $2^{10} - 2^6 = 960$

18. Find number of common roots of the equations $Z^{1901} + Z^{100} + 1 = 0$ and $Z^3 + 2Z^2 + 2Z + 1 = 0$

(1) 2 (2) 3 (3) 4 (4) 6

Ans. **(1)**

Sol. $Z^3 + 2Z^2 + 2Z + 1 = 0$

$(Z+1)(Z^2 - Z + 1) + 2Z(Z+1) = 0$

$(Z+1)(Z^2 + Z + 1) = 0$

$\Rightarrow Z = -1, \quad Z = \omega, \ \omega^2$

$Z = -1$ does not satisfy $Z^{1901} + Z^{100} + 1 = 0$

If $Z = \omega \Rightarrow Z^{1901} + Z^{100} + 1 = \omega^{1901} + \omega^{100} + 1$

$= \omega^{1899} \cdot \omega^2 + \omega^{99} \times \omega + 1$

$= \omega^2 + \omega + 1 = 0$

If $Z = \omega^2 \Rightarrow Z^{1901} + Z^{100} + 1 = \omega^{2(1901)}) + \omega^{100} + 1$

$= \omega + \omega^2 + 1 = 0$

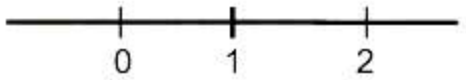
Number common roots = 2

19. Find number of real solutions of the equation $x(x^2+|x|+5|x-1|-6|x-2|)=0$

(1) 2 (2) 3 (3) 4 (4) 6

Ans. **(2)**

Sol.



Case-I : If $x \geq 2$

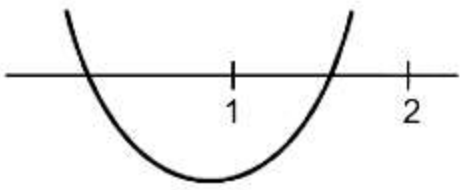
$x(x^2+x+5x-5-6x+12)=0$

$x(x^2+7)=0 \Rightarrow x=0$ (Rejected)

Case –II : if $1 \leq x < 2$

$x(x^2+x+5x-5+6x-12)=0$

$x(x^2+12x-17)=0 \Rightarrow$ one solution in [1,2)



Case-III: If $0 \leq x < 1$

$x(x^2+x-5x+5+6x-12)=0$

$x(x^2+2x-7)=0 \Rightarrow x=0$ is only solution in [0,1)

Case –IV : If $x < 0$

$x(x^2-x-5x+5+6x-12)=0$

$x(x^2-7)=0 \Rightarrow x=0, \sqrt{7}, -\sqrt{7}$

$x=-\sqrt{7}$ is only the solution

So number of required solution is 3

20. $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}$ and $f'(1)=2024$, then

(1) $x f'(x)-2024 f(x)=0$ (2) $2024 f'(x)=f(x)$

(3) $f'(x)-2024 f(x)=0$ (4) None of these

Ans. **(1)**

Sol. Put $x=y=1$

$f(1)=1$

Now put $x=1 \Rightarrow f\left(\frac{1}{y}\right)=\frac{1}{f(y)}$

$f(y)=\pm y^n$

but $f(1)=1 \Rightarrow f(y)=y^n$

$f'(y)=ny^{n-1} \Rightarrow f'(1)=n=2024$

$f(x)=x^{2024}$

$f'(x)=2024\,x^{2023}$

$xf'(x)=2024\,f(x) \Rightarrow xf'(x)-2024f(x)=0$

21. $\sum_{r=0}^{n}\frac{{}^{n}C_r\cdot{}^{n}C_r}{r+1}=\alpha,\ \sum_{r=0}^{n}\frac{{}^{n}C_r\cdot{}^{n+1}C_r}{r+1}=\beta$. If $4\beta = 7\alpha$ then n is

Ans. **(6)**

Sol. $\alpha=\frac{1}{n+1}\sum_{r=0}^{n}{}^{n+1}C_{r+1}\ {}^{n}C_r$

$$\alpha=\frac{1}{n+1}\left({}^{2n+1}C_n\right)$$

$$\beta=\frac{1}{n+1}\sum_{r=0}^{n}{}^{n+1}C_{r+1}\ {}^{n+1}C_r$$

$$\beta=\frac{1}{n+1}\left({}^{2n+2}C_n\right)$$

Now $4\beta = 7\alpha$

$$\frac{4}{n+1}\left({}^{2n+2}C_n\right)=7\left(\frac{{}^{2n+1}C_n}{n+1}\right)$$

$$\frac{4}{n+1}\ {}^{2n+2}C_{n+2}=\frac{7}{n+1}\ {}^{2n+1}C_{n+1}$$

$$4\left(\frac{2n+2}{n+2}\right)=7$$

n = 6

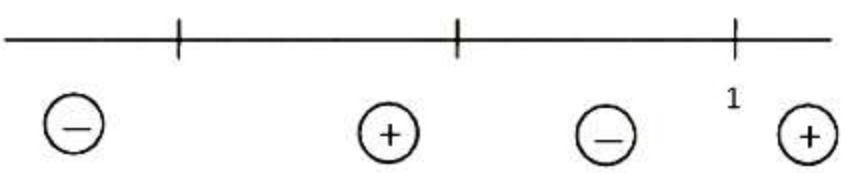
22. If the domain of $f(x)=\ell n\left(\frac{2x+3}{4x^2-x-3}\right)+\cos^{-1}\left(\frac{2x+1}{x+2}\right)$ is (α, β) find $5\alpha - 4\beta$.

(1) -2 (2) 2 (3) 4 (4) -4

Ans. **(1)**

Sol. For $\ell n\left(\frac{2x+3}{4x^2-x-3}\right)$ to be defined

$$\frac{2x+3}{4x^2-x-3}>0\Rightarrow\frac{(2x+3)}{(4x+3)(x-1)}>0$$



$$x\in\left(-\frac{3}{2},-\frac{3}{4}\right)\cup(1,\alpha) \qquad \text{.....(1)}$$

for $\cos^{-1}\left(\dfrac{2x+1}{x+2}\right)$ to be defined

$$-1 \le \frac{2x+1}{x+2} \le 1 \Rightarrow \frac{3(x+1)}{x+2} \ge 0 \ \& \ \frac{x-1}{x+2} \le 0 \Rightarrow x \in [-1, 1] \qquad \text{.....(2)}$$

(1) $\cap$ (2)

$$x \in \left[-1, -\frac{3}{4}\right]$$

$$\alpha = -1, \beta = -\frac{3}{4}$$

$$5\alpha = -5, \quad 4\beta = -3$$

$$5\alpha - 4\beta = -5 - (-3) = -2$$