JEE(Advanced) – 2018 TEST PAPER - 2 WITH SOLUTION
(Exam Date: 20-05-2018)

PART-1 : PHYSICS

1. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true ?

(A) The force applied on the particle is constant

(B) The speed of the particle is proportional to time

(C) The distance of the particle from the origin increses linerarly with time

(D) The force is conservative

Ans. (A,B,D)

Sol. $\frac{dk}{dt} = \gamma t$ **as** $k = \frac{1}{2}mv^2$

$$\therefore \frac{dk}{dt} = mv\frac{dv}{dt} = \gamma t$$

$$\therefore m\int_0^v vdv = \gamma\int_0^t tdt$$

$$\frac{mv^2}{2} = \frac{\gamma t^2}{2}$$

$$v = \sqrt{\frac{\gamma}{m}}t \qquad .....(i)$$

$$a = \frac{dv}{dt} = \sqrt{\frac{\gamma}{m}} = \text{constant}$$

since F = ma

$$\therefore F = m\sqrt{\frac{\gamma}{m}} = \sqrt{\gamma m} = \text{constant}$$

2. Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity u_0. Which of the following statements is (are) true ?

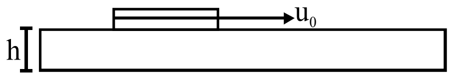
(A) The resistive force of liquid on the plate is inversely proportional to h

(B) The resistive force of liquid on the plate is independent of the area of the plate

(C) The tangential (shear) stress on the floor of the tank increases with u_0.

(D) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid.

Ans. (A,C,D)

Sol. 

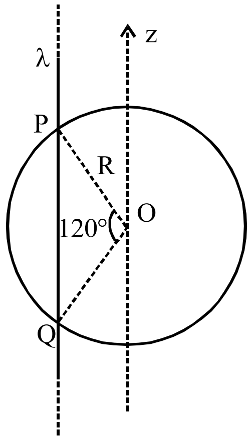
Viscous force is given by F = $-\eta A\dfrac{dv}{dy}$ since h is very small therefore, magnitude of viscous force is given by

$$F = \eta A\frac{\Delta v}{\Delta y}$$

$\therefore\ F = \dfrac{\eta A u_0}{h} \Rightarrow F \propto \eta\ \&\ F \propto u_0$; $\quad F \propto \dfrac{1}{h}$, $F \propto A$

Since plate is moving with constant velocity, same force must be acting on the floor.

3. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ε_0. Which of the following statements is (are) true ?



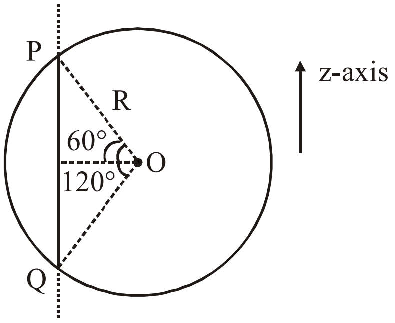
(A) The electric flux through the shell is $\sqrt{3}R\lambda/\varepsilon_0$

(B) The z-component of the electric field is zero at all the points on the surface of the shell

(C) The electric flux through the shell is $\sqrt{2}R\lambda/\varepsilon_0$

(D) The electric field is normal to the surface of the shell at all points

Ans. (A,B)

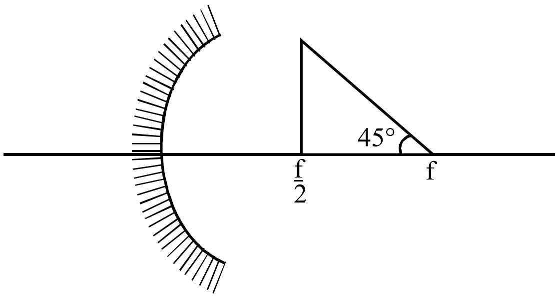
Sol. 

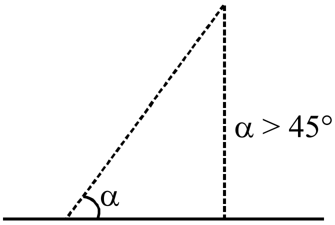
Field due to straight wire is perpendicular to the wire & radially outward. Hence $E_z = 0$

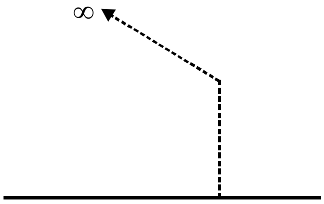
Length, PQ = 2R sin 60 = $\sqrt{3}R$ According to Gauss's law

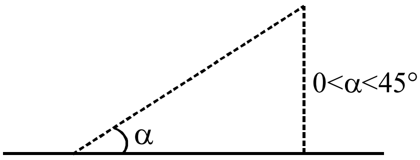
total flux = $\oint \vec{E}.\vec{ds} = \dfrac{q_{in}}{\epsilon_0} = \dfrac{\lambda\sqrt{3}R}{\epsilon_0}$

4. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire ? (These figures are not to scale.) ?



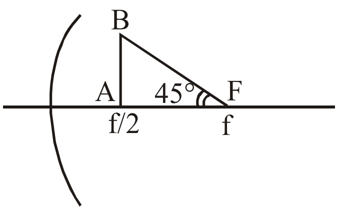
(A) 

(B) 

(C) 

(D) 

Ans. (D)

Sol. 

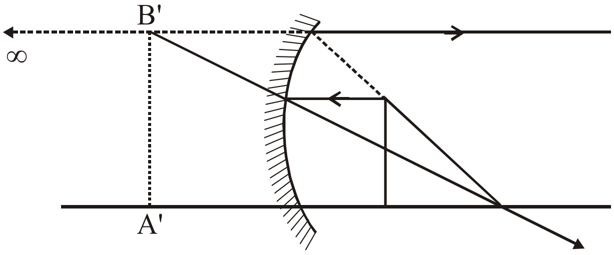
Distance of point A is f/2

Let A' is the image of A from mirror, for this image

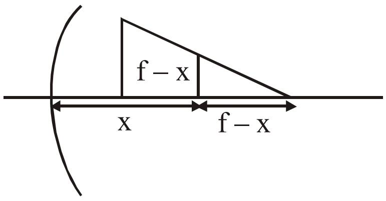
$$\frac{1}{v}+\frac{1}{-f/2}=\frac{1}{-f}$$

$$\frac{1}{v}=\frac{2}{f}-\frac{1}{f}=\frac{1}{f}$$

image of line AB should be perpendicular to the principle axis & image of F will form at infinity, therefor correct image diagram is



OR



$$\frac{f}{f-u}=\frac{h_2}{h_1}$$

$$h_2=\frac{-f(f-x)}{-f+x}$$

h_2 = f

5. In a radioactive decay chain, ${}^{232}_{90}\mathrm{Th}$ nucleus decays to ${}^{212}_{82}\mathrm{Pb}$ nucleus. Let N_α and N_β be the number of α and β^- particles, respectively, emitted in this decay process. Which of the following statements is (are) true ?

(A) N_α = 5 (B) N_α = 6 (C) N_β = 2 (D) N_β = 4

Ans. (A,C)

Sol. ${}^{232}_{90}\mathrm{Th}$ is converting into ${}^{212}_{82}\mathrm{Pb}$

Change in mass number (A) = 20

$\therefore$ no of α particle = $\frac{20}{4}=5$

Due to 5 α particle, z will change by 10 unit.

Since given change is 8, therefore no. of β particle is 2

6. In an experment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 Hz is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length 50.7 cm and 83.9 cm. Which of the following statements is (are) true ?

(A) The speed of sound determined from this experiment is 332 ms^{-1}

(B) The end correction in this experiment is 0.9 cm

(C) The wavelength of the sound wave is 66.4 cm

(D) The resonance at 50.7 cm corresponds to the fundamental harmonic

Ans. (A,C or A,B,C)

Sol. Let n_1 harmonic is corresponding to 50.7 cm & n_2 harmonic is corresponding 83.9 cm.

since both one consecutive harmonics.

$\therefore$ their difference = $\frac{\lambda}{2}$

$\therefore$ $\frac{\lambda}{2}$ = (83.9 – 50.7) cm

$$\frac{\lambda}{2} = 33.2 \text{ cm.}$$

$$\lambda = 66.4 \text{ cm}$$

$$\therefore \frac{\lambda}{4} = 16.6 \text{ cm}$$

length corresponding to fundamental mode must be close to $\frac{\lambda}{4}$ & 50.7 cm must be closed to an odd multiple of this length as $16.6 \times 3 = 49.8$ cm. therefore 50.7 is 3rd harmonic

If end correction is e, then

$$e + 50.7 = \frac{3\lambda}{4}$$

$$e = 49.8 - 50.7 = -0.9 \text{ cm}$$

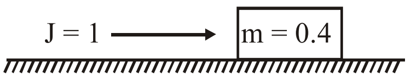
speed of sound, $v = f\lambda$

$$\therefore v = 500 \times 66.4 \text{ cm/sec} = 332.000 \text{ m/s}$$

7. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass m = 0.4kg is at rest on this surface. An impulse of 1.0 N s is applied to the block at time to t = 0 so that it starts moving along the x-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and $\tau = 4$ s. The displacement of the block, in metres, at $t = \tau$ is.................. Take $e^{-1} = 0.37$?

Ans. 6.30

Sol.



$$v = v_0 e^{-t/\tau}$$

$$v_0 = \frac{J}{m} = 2.5 \text{ m/s}$$

$$v = v_0 e^{-t/\tau}$$

$$\frac{dx}{dt} = v_0 e^{-t/\tau}$$

$$\int_0^x dx = v_0 \int_0^\tau e^{-t/\tau} dt \qquad \int e^{-x} dx = \frac{e^{-x}}{-1}$$

$$x = v_0 \left[\frac{e^{-t/\tau}}{-\frac{1}{\tau}} \right]_0^\tau$$

$$x = 2.5 \, (-4) \, (e^{-1} - e^{0})$$

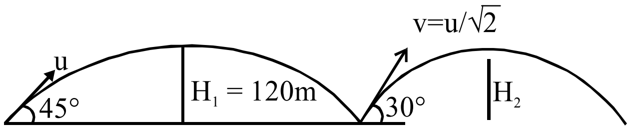
$$x = 25 \, (-4) \, (0.37 - 1)$$

$$x = 6.30 \text{ ans.}$$

8. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is................

Ans. 30.00

Sol.



$$H_1 = \frac{u^2 \sin^2 45}{2g} = 120$$

$$\Rightarrow \frac{u^2}{4g} = 120 \quad ....(i)$$

when half of kinetic energy is lost $v = \frac{u}{\sqrt{2}}$

$$H_2 = \frac{\left(\frac{u}{\sqrt{2}}\right)^2 \sin^2 30}{2g} = \frac{u^2}{16g} \quad .....(ii)$$

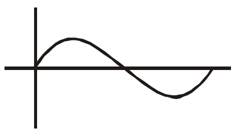
from (i) & (ii)

$H_2 = \frac{H_1}{4}$ = 30 m on 30.00

9. A particle, of mass 10^{-3} kg and charge 1.0 C, is initially at rest. At time t = 0, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin \omega t \, \hat{i}$ where E_0 = 1.0 N C^{-1} and $\omega=10^3$ rad s^{-1}. Consider the effect of only the electrical force on the particle. Then the maximum speed, in ms^{-1}, attained by the particle at subsequent times is..........................

Ans. 2.00

Sol. n = 10^{-3} kg q = 1C t = 0

$E = E_0 \sin \omega t \equiv$ 

Force on particle will be

$F = qE = qE_0 \sin \omega t$

at v_{max}, a, F = 0 $\qquad qE_0 \sin \omega t = 0$

$F = qE_0 \sin\omega t$

$$\frac{dv}{dt} = q\frac{E_0}{m}\sin\omega t$$

$$\int_0^v dv = \int_0^{\pi/\omega}\frac{qE_0}{m}\sin\omega t\,dt$$

$$v - 0 = \frac{qE_0}{m\omega}[-\cos\omega t]_0^{\pi/\omega}$$

$$v - 0 = \frac{qE_0}{m\omega}[(-\cos\pi)-(-\cos 0)]$$

$$v = \frac{1\times 1}{10^{-3}10^{3}}\times 2 = 2 \text{ m/s}$$

Ans. 2. 00

10. A moving coil galvanometer has 50 turns and each turn has an area 2×10^{-4} m^2. The magnetic field porduced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is 10^{-4} N m rad^{-1}. When a current flows through the galvanometer, a full scale deflection occurs if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0 – 1.0 A. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in ohms, is...........

Ans. 5.55

Sol. $n = 50$ turns $\quad A = 2 \times 10^{-4}$ m^2

$B = 0.02$ T $\quad K = 10^{-4}$

$Q_m = 0.2$ rad $\quad R_g = 50\ \Omega$

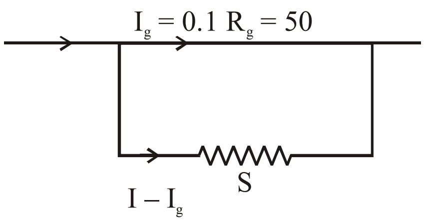
$I_A = 0 - 1.0$ A $\quad \tau = MB = C\theta$, $M = nIA$

$BINA = C\theta$

$0.02 \times 1 \times 50 \times 2 \times 10^{-4} = 10^{-4} \times 0.2\ 10$

$I_g = 0.1$ A

For galvanometer, resistance is to be connected to ammeter in shunt.



$I_g \times R_g = (I - I_g)S$

$0.1 \times 50 = (1 - 0.1)$ S

$$S = \frac{50}{9} = 5.55$$

11. A steel wire of diameter 0.5 mm and Young's modulus 2×10^{11} N m^{-2} carries a load of mass M. The length of the wire with the load is 1.0 m. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 mm, is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2kg, the vernier scale division which coincides with a main scale division is....... Take g = 10 ms^{-2} and $\pi = 3.2$.

Ans. **3.00**

Sol. d = 0.5 mm $\qquad$ Y = 2×10^{11} $\qquad$ ℓ = 1m

$$\Delta\ell = \frac{F\ell}{Ay} = \frac{mg\ell}{\frac{\pi d^2}{4}y} = \frac{1.2\times10\times1}{\frac{\pi}{4}\times(5\times10^{-4})^2\times2\times10^{11}}$$

$$\Delta\ell = \frac{1.2\times10}{\frac{3.2}{4}\times25\times10^{-8}\times2\times10^{11}}$$

$$= \frac{12}{0.8\times25\times2\times10^3} = \frac{12}{40\times10^3} = 0.3\text{mm}$$

so 3rd division of vernier scale will coincicle with main scale.

12. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant R = 8.0 J mol^{-1} K^{-1}, the decrease in its internal energy, in Joule, is............ .

Ans. **900**

Sol. $v_i = v$

$v_F = 8v$

For adiabatic process $\left\{\gamma = \frac{5}{3}\right.$ for monoatomic process

$$T_1V_1^{\gamma-1} = T_2.V_2^{\gamma-1}$$

$$100(v)^{2/3} = T_2(8v)^{2/3}$$

T_2 = 25 k

$$\Delta U = nc_V\Delta T = 1\left(\frac{FR}{2}\right)[100 - 25] = 12 \times 75 = 900 \text{ Joule}$$

13. In a photoelectric experiment a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficinecy is 100% A potential difference of 500 V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force F = n × 10^{-4} N due to the impact of the electrons. The value of n is........ Mass of the electron $m_e = 9 \times 10^{-31}$ kg and 1.0 eV = 1.6×10^{-19} J.?

Ans. 24

Sol. Power = nhν   n = number of photons per second

Since KE = 0, hν = ϕ

200 = n[6.25 × 1.6 × 10^{-19} Joule]

$$n = \frac{200}{1.6\times10^{-19}\times6.25}$$

As photon is just above threshold frequency KE_{max} is zero and they are accelrated by potential difference of 500V.

$KE_f = q\Delta V$

$$\frac{P^2}{2m} = q\Delta V \Rightarrow P = \sqrt{2mq\Delta V}$$

Since efficiency is 100%, number of electrons = number of photons per second

As photon is completely absorbed force exerted = nmv

$$= \frac{200}{6.25\times1.6\times10^{-19}}\times\sqrt{2\left(9\times10^{-31}\right)\times1.6\times10^{-19}\times500}$$

$$= \frac{3\times200\times10^{-25}\times\sqrt{1600}}{6.25\times1.6\times10^{-19}} = \frac{2\times40}{6.25\times1.6}\times10^{-4}\times3 = 24$$

14. Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the n = 2 to n = 1 transition has energy 74.8 eV higher than the photon emitted in the n = 3 to n = 2 transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of Z is.............. .

Ans. 3

Sol. $$\Delta E_{2-1} = 13.6\times z^2\left[1-\frac{1}{4}\right] = 13.6\times z^2\left[\frac{3}{4}\right]$$

$$\Delta E_{3-2} = 13.6\times z^2\left[\frac{1}{4}-\frac{1}{9}\right] = 13.6\times z^2\left[\frac{5}{36}\right]$$

$\Delta E_{2-1} = \Delta E_{3-2} + 74.8$

$$13.6\times z^2\left[\frac{3}{4}\right] = 13.6\times z^2\left[\frac{5}{36}\right]+74.8$$

$$13.6\times z^2\left[\frac{3}{4}-\frac{5}{36}\right] = 74.8$$

$z^2 = 9$

z = +3 ans

15. The electric field E is measured at a point P(0,0,d) generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.

List–I	**List–II**
P. E is indpendent of d	**1.** A point charge Q at the origin
Q. $E \propto \frac{1}{d}$	2. A small dipole with point charges Q at $(0,0,\ell)$ and – Q at $(0,0,-\ell)$. Take $2\ell << d$
R. $E \propto \frac{1}{d^2}$	3. An infinite line charge coincident with the x-axis, with uniform linear charge density λ.
S. $E \propto \frac{1}{d^3}$	4. Two infinite wires carrying uniform linear Charge density parallel to the x - axis. The one along $(y = 0, z = \ell)$ has a charge density $+\lambda$ and the one along $(y = 0, z = -\ell)$ has a charge density $-\lambda$. Take $2\ell << d$
	5. Infinite plane charge coincident with the xy-plane with uniform surface charge density

(A) $P \to 5$; $Q \to 3, 4$; $R \to 1$; $S \to 2$
(B) $P \to 5$; $Q \to 3,$; $R \to 1,4$; $S \to 2$
(C) $P \to 5$; $Q \to 3,$; $R \to 1,2$; $S \to 4$
(D) $P \to 4$; $Q \to 2, 3$; $R \to 1$; $S \to 5$

Ans. (B)

Sol. (i) $E = \frac{KQ}{d^2} \Rightarrow E \propto \frac{1}{d^2}$

(ii) Dipole

$$E = \frac{2kp}{d^3}\sqrt{1+3\cos^2\theta}$$

$E \propto \frac{1}{d^3}$ for dipole

(iii) For line charge

$$E = \frac{2k\lambda}{d}$$

$$E \propto \frac{1}{d}$$

(iv) $E = \frac{2K\lambda}{d-\ell} - \frac{2K\lambda}{d+\ell}$

$$= 2K\lambda\left[\frac{d+\ell-d+\ell}{d^2-\ell^2}\right]$$

$$E = \frac{2K\lambda(2\ell)}{d^2\left[1-\frac{\ell^2}{d^2}\right]}$$

$$E \propto \frac{1}{d^2}$$

(v) Electric field due to sheet

$$\in = \frac{\sigma}{2\in_0}$$

$\in$ = v is independent of r

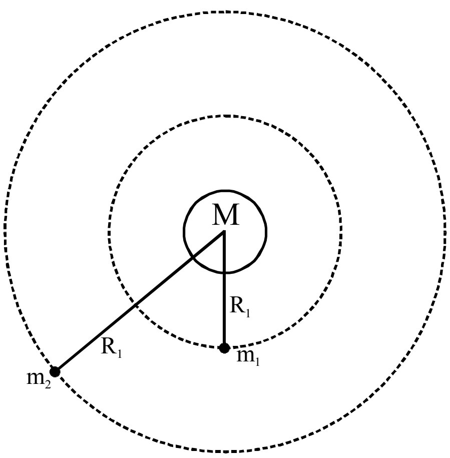
16. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1 ; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II.

List–I	List–II
P. $\frac{v_1}{v_2}$	1. $\frac{1}{8}$
Q. $\frac{L_1}{L_2}$	2. 1
R. $\frac{K_1}{K_2}$	3. 2
S. $\frac{T_1}{T_2}$	4. 8

(A) P $\rightarrow$ 4 ; Q $\rightarrow$ 2 ; R $\rightarrow$ 1 ; S $\rightarrow$ 3
(B) P $\rightarrow$ 3 ; Q $\rightarrow$ 2 ; R $\rightarrow$ 4 ; S $\rightarrow$ 1
(C) P $\rightarrow$ 2 ; Q $\rightarrow$ 3 ; R $\rightarrow$ 1 ; S $\rightarrow$ 4
(D) P $\rightarrow$ 2 ; Q $\rightarrow$ 3 ; R $\rightarrow$ 4 ; S $\rightarrow$ 1

Ans. (B)

Sol.



$$\left.\begin{array}{l}\frac{m_1}{m_2} = 2 \\ \frac{R_1}{R_2} = \frac{1}{4}\end{array}\right\}\text{given}$$

$$\frac{GMm_1}{R_1^2} = \frac{m_1 v_1^2}{R_1}$$

$$v_1^2 = \frac{GM}{R_1}\ ,\ v_2^2 = \frac{GM}{R_2}$$

$$\frac{v_1^2}{v_2^2} = \frac{R_2}{R_1} = 4$$

(P) $\frac{v_1}{v_2} = 2$

(Q) $L = mvR$

$$\frac{L_1}{L_2} = \frac{m_1 v_1 R_1}{m_2 v_2 R_2} = 2 \times 2 \times \frac{1}{4} = 1$$

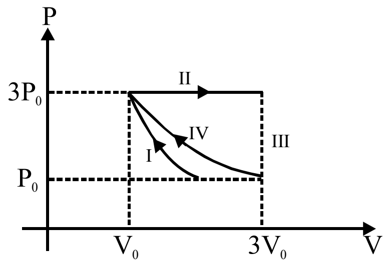
(R) $K = \frac{1}{2}mv^2$

$$\frac{K_1}{K_2} = \frac{m_1 v_1^2}{m_2 v_2^2} = 2 \times (2)^2 = 8$$

(S) $T = 2\pi R/V$

$$\frac{T_1}{T_2} = \frac{R_1}{v_1} \times \frac{v_2}{R_2} = \frac{R_1}{R_2} \times \frac{v_2}{v_1} = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

17. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List–II.



List–I		List–II	
P.	In process I	1.	Work done by the gas is zero
Q.	In process II	2.	Temperature of the gas remains unchanged
R.	In process III	3.	No heat is exchanged between the gas and its surroundings
S.	In process IV	4.	Work done by the gas is $6\ P_0V_0$

(A) $P \rightarrow 4\ ;\ Q \rightarrow 3\ ;\ R \rightarrow 1\ ;\ S \rightarrow 2$

(B) $P \rightarrow 1\ ;\ Q \rightarrow 3\ ;\ R \rightarrow 2\ ;\ S \rightarrow 4$

(C) $P \rightarrow 3\ ;\ Q \rightarrow 4\ ;\ R \rightarrow 1\ ;\ S \rightarrow 2$

(D) $P \rightarrow 3\ ;\ Q \rightarrow 4\ ;\ R \rightarrow 2\ ;\ S \rightarrow 1$

Ans. **(C)**

Sol. Process – I is an adiabatic process

$\Delta Q = \Delta U + W \qquad \Delta Q = 0$

$W = -\Delta U$

Volume of gas is decreasing $\Rightarrow W < 0$

$\Delta U > 0$

$\Rightarrow$ Temperatuer of gas increases.

$\Rightarrow$ No heat is exchanged between the gas and surrounding.

Process – II is an isobaric process

(Pressure remain constant)

$W = P\,\Delta V = 3P_0[3V_0 - V_0] = 6P_0V_0$

Process - III is an isochoric process

(Volume remain constant)

$\Delta Q = \Delta U + W$

$W = 0$

$\Delta Q = \Delta U$

Process – IV is an isothermal process

(Temperature remains constant)

$\Delta Q = \Delta U + W$

$\Delta U = 0$

18. In the List-I below, four different paths of a particle are given as functions of time. In these functions, α and β are positive constants of appropriate dimensions and $\alpha \neq B$. In each case, the force acting on the particle is either zero or conservative. In List–II, five physical quantities of the particle are mentioned; $\vec{p}$ is the linear momentum $\vec{L}$ is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List–I with those quantities in List–II, which are conserved for that path

	List–I		**List–II**
P.	$\vec{r}(t) = \alpha t\,\hat{i} + \beta t\,\hat{j}$	1.	$\vec{p}$
Q.	$\vec{r}(t) = \alpha\cos\omega t\,\hat{i} + \beta\sin\omega t\,\hat{j}$	2.	$\vec{L}$
R.	$\vec{r}(t) = \alpha(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j})$	3.	K
S	$\vec{r}(t) = \alpha t\,\hat{i} + \frac{\beta}{2}t^2\,\hat{j}$	4.	U
		5.	E

(A) $P \rightarrow 1,2,3,4,5$; $Q \rightarrow 2,5$; $R \rightarrow 2,3,4,5$; $S \rightarrow 5$

(B) $P \rightarrow 1,2,3,4,5$; $Q \rightarrow 3,5$; $R \rightarrow 2,3,4,5$; $S \rightarrow 2,5$

(C) $P \rightarrow 2,3,4$; $Q \rightarrow 5$; $R \rightarrow 1,2,4$; $S \rightarrow 2,5$

(D) $P \rightarrow 1,2,3,5$; $Q \rightarrow 2,5$; $R \rightarrow 2,3,4,5$; $S \rightarrow 2,5$

Ans. (A)

Sol. (P) $\vec{r}(t) = \alpha t\hat{i} + \beta t\hat{j}$

$$\vec{v} = \frac{d\vec{r}(t)}{dt} = \alpha\hat{i} + \beta\hat{j} \text{ \{constant\}}$$

$$\vec{a} = \frac{\overrightarrow{dv}}{dt} = 0$$

$\vec{P} = m\vec{v}$ (remain constant)

$$k = \frac{1}{2}mv^2 \text{ \{remain constant\}}$$

$$\vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{i}\right] = 0$$

$\Rightarrow$ U $\rightarrow$ constant

E = K + U

$$\frac{d\vec{L}}{dt} = \vec{\tau} = \vec{r} \times \vec{F} = 0$$

$\vec{L}$ = constant

(Q) $\vec{r} = \alpha\cos(\omega t)\hat{i} + \beta\sin(\omega t)\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = -\alpha\omega\sin(\omega t)\hat{i} + \beta\omega\cos(\omega t)\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -\alpha\omega^2\cos(\omega t)\hat{i} - \beta\omega^2\sin(\omega t)\hat{j}$$

$= -\omega^2\left[\alpha\cos(\omega t)\hat{i} + \beta\sin(\omega t)\hat{j}\right]$

$\vec{a} = -\omega^2\vec{r}$

$\vec{\tau} = \vec{r} \times \vec{F} = 0$ {$\vec{r}$ and $\vec{F}$ are parallel}

$$\Delta U = -\int \vec{F}.dr = +\int_0^r m\omega^2.r.dr$$

$$\Delta U = m\omega^2\left[\frac{r^2}{2}\right]$$

$U \propto r^2$

r = $\sqrt{\alpha^2\cos^2(\omega t) + \beta^2\sin^2(\omega t)}$

r is a function of time (t)

U depends on r hence it will change with time

Total energy remain constant because force is central.

(R) $\vec{r}(t) = \alpha\left(\cos\omega t\hat{i} + \sin(\omega t)\hat{j}\right)$

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \alpha\left[-\omega\sin(\omega t)\hat{i} + \omega\cos(\omega t)\hat{j}\right]$$

$|\vec{v}| = \alpha\omega$ (Speed remains constant)

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \alpha\left[-\omega^2\cos(\omega t)\hat{i} - \omega^2\sin(\omega t)\hat{j}\right]$$

$$= -\alpha\omega^2\left[\cos(\omega t)\hat{i} + \sin(\omega t)\hat{j}\right]$$

$$\vec{a}(t) = -\omega^2(\vec{r})$$

$$\vec{\tau} = \vec{F} \times \vec{r} = 0$$

$|\vec{r}| = \alpha$ (remain constant)

Force is central in nature and distance from fixed point is constant.

Potential energy remains constant

Kinetic energy is also constant (speed is constant)

(S) $\vec{r} = \alpha t\hat{i} + \frac{\beta}{2}t^2\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = \alpha t\,\hat{i} + \beta t\,\hat{j}$ (speed of particle depends on 't')

$$\vec{a} = \frac{d\vec{v}}{dt} = \beta\hat{j}\ \{\text{constant}\}$$

$$\vec{F} = m\vec{a}\ \{\text{constant}\}$$

$$\Delta U = -\int \vec{F}.d\vec{r} = -m\int_0^t \beta\hat{j}.\left(\alpha\hat{i} + \beta t\hat{j}\right)dt$$

$$U = \frac{-m\beta^2t^2}{2}$$

$$k = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\alpha^2 + \beta^2t^2\right)$$

$$E = k + U = \frac{1}{2}m\alpha^2\ [\text{remain constant}]$$