09/04/2019
Morning

Answers & Solutions

for

JEE (MAIN)-2019 (Online) Phase-2

(Physics, Chemistry and Mathematics)

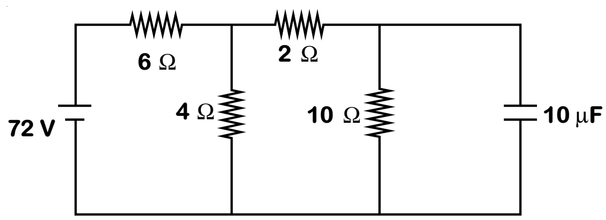
Time : 3 hrs. **M.M. : 360**

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. *¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question.

PART–A : PHYSICS

1. Determine the charge on the capacitor in the following circuit :



(1) 200 μC (2) 60 μC

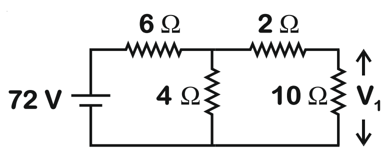
(3) 10 μC (4) 2 μC

Answer (1)

Sol. At steady state current through capacitor is zero.

$V_C = V_1$

$$V_1 = \frac{5}{6} \times V_0 \times \frac{3}{9}$$



$$V_1 = \frac{5 \times 72 \times 3}{6 \times 9} = 20 \text{ V}$$

$Q_1 = CV_1$

$= 200\ \mu C$

2. The magnetic field of a plane electromagnetic wave is given by :

$\vec{B} = \vec{B}_0 \hat{i}\ [\cos(kz - \omega t) + B_1 \hat{j} \cos(kz + \omega t)$

where $B_0 = 3 \times 10^{-5}$ T and $B_1 = 2 \times 10^{-6}$ T.

The rms value of the force experienced by a stationary charge $Q = 10^{-4}$ C at z = 0 is closest to:

(1) 0.6 N (2) 0.9 N

(3) 3×10^{-2} N (4) 0.1 N

Answer (1)

Sol. $|\vec{E}_1| = CB_1$

$|\vec{E}_2| = CB_2$

Also $\vec{E}_1 \perp E_2$

$$F_{net} = \frac{\theta}{\sqrt{2}} \sqrt{E_1^2 + E_2^2}$$

$$= \frac{10^{-4}}{\sqrt{2}} \times 3 \times 10^8 \times 30 \times 10^{-6}$$

$$= \frac{90 \times 10^8 \times 10^{-10}}{\sqrt{2}}$$

$\simeq 0.6$ N

3. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is:

(1) $\frac{2k}{I}\theta$ (2) $\frac{k}{I}\theta$

(3) $\frac{k}{2I}\theta$ (4) $\frac{k}{4I}\theta$

Answer (1)

Sol. $\tau = \frac{dE}{d\theta}$

$2k\theta = I\alpha$

$\alpha = \frac{2k\theta}{I}$

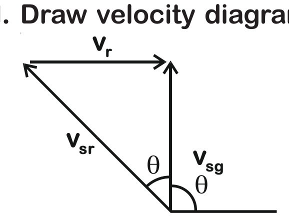
4. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight?

(1) 60° (2) 90°

(3) 150° (4) 120°

Answer (4)

Sol. Draw velocity diagram



$$\sin\theta = \frac{v_r}{v_{sr}} = \frac{1}{2}$$

$\theta = 30°$

$\phi = 90 + \theta = 120°$

5. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be:

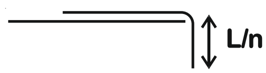
(1) $\frac{MgL}{n^2}$ (2) nMgL

(3) $\frac{MgL}{2n^2}$ (4) $\frac{2MgL}{n^2}$

Answer (3)

Sol. $m_1 = \frac{M}{n}$

$U_i = \frac{-M}{n} \times g \frac{L}{2n}$



$W = \frac{MgL}{2n^2}$

6. A signal $A\cos\omega t$ is transmitted using $v_0 \sin\omega_0 t$ as carrier wave. The correct amplitude modulated (AM) signal is:

(1) $v_0 \sin\omega_0 t + \frac{A}{2}\sin(\omega_0 - \omega)t + \frac{A}{2}\sin(\omega_0 + \omega)t$

(2) $(v_0 + A)\cos\omega t \sin\omega_0 t$

(3) $v_0 \sin\omega_0 t + A\cos\omega t$

(4) $v_0 \sin[\omega_0(1 + 0.01 A\sin\omega t)t]$

Answer (1)

Sol. A $= (v_o + A\cos\omega t)\sin\omega_o t$

$= v_0 \sin(\omega_0 t) + \frac{A}{2}[\sin(\omega_0 - \omega)t + \sin(\omega_0 + \omega)t]$

7. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane : (i) a ring of radius R, (ii) a solid cylinder of radius $\frac{R}{2}$ and (iii) a solid sphere of radius $\frac{R}{4}$. If, in each case, the speed of the center of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is :

(1) 14 : 15 : 20 (2) 10 : 15 : 7

(3) 4 : 3 : 2 (4) 2 : 3 : 4

Answer (Bonus)

Sol. $mgh = \frac{1}{2} I_P \omega^2$

For ring $\rightarrow h_1 = \frac{1}{2}\frac{(2mr'^2)}{mg}\frac{v^2}{(r')^2} = \frac{v^2}{g}$

For cylinder $\rightarrow h_2 = \frac{1}{2}\left(\frac{3}{2}mr'^2\right)\frac{v^2}{(r')^2} = \frac{3}{4}\frac{v^2}{g}$

For sphere $\rightarrow h_3 = \frac{1}{2} \times \frac{\frac{7}{5}m(r')^2}{g}\frac{v^2}{(r')^2} = \frac{7}{10}\frac{v^2}{g}$

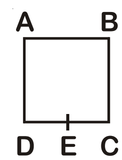
$h_1 : h_2 : h_3$

$2 : \frac{3}{2} : \frac{14}{10}$

20 : 15 : 14

8. A wire of resistance R is bent to form a square ABCD as shown in the figure. The effective resistance between E and C is

(E is mid-point of arm CD)



(1) $\frac{3}{4}R$ (2) R

(3) $\frac{1}{16}R$ (4) $\frac{7}{64}R$

Answer (4)

Sol. $R_1 = \frac{R}{8}$, $R_2 = \frac{7R}{8}$

$\frac{1}{R_{eq}} = \frac{8}{R} + \frac{8}{7R}$

$R_{eq} = \frac{7R}{64}$

9. In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is

(1) 0.31 kg/m^3 (2) 0.01 kg/m^3

(3) 0.10 kg/m^3 (4) 0.07 kg/m^3

Answer (Bonus)

Sol. $\rho = M/V = \dfrac{10}{(0.1)^3} = 10{,}000\ \text{kg/m}^3$

$$\frac{d\rho}{\rho} = \left[\frac{dM}{M} + \frac{dV}{V}\right]$$

$$\frac{d\rho}{10{,}000} = \frac{0.1}{10} + \frac{0.03}{0.1}$$

$d\rho$ = 3100 kg/m^3

10. A rectangular coil (Dimension 5 cm × 2.5 cm) with 100 turns, carrying a current of 3 A in the clock-wise direction, is kept centered at the origin and in the X-Z plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is

(1) 0.55 Nm (2) 0.27 Nm

(3) 0.42 Nm (4) 0.38 Nm

Answer (2)

Sol. $\tau = \vec{M} \times \vec{B}$

$$\tau = 100 \times 3 \times 5 \times 2.5 \times 10^{-4} \times 1 \times \frac{1}{\sqrt{2}}$$

$$= 0.27\ \text{Nm}$$

11. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\dfrac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is :

(1) $2T\sqrt{\dfrac{1}{14}}$ (2) $4T\sqrt{\dfrac{1}{15}}$

(3) $4T\sqrt{\dfrac{1}{14}}$ (4) $2T\sqrt{\dfrac{1}{10}}$

Answer (2)

Sol. $T = 2\pi\sqrt{\dfrac{l}{g_{eff}}}$

$$\frac{T'}{T} = \sqrt{\frac{g_{eff}}{g'_{eff}}}$$

$$g'_{eff} = g - \frac{g}{16} = \frac{15}{16}$$

$$\frac{T'}{T} = \sqrt{\frac{16}{15}}$$

12. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to

(1) 1/L (2) L

(3) 1/L^2 (4) L^2

Answer (1)

Sol. B = $\mu_0 ni$

$\phi = \mu_0 ni(nL)\ A$

(Inductance) $L' = \mu_0 n^2 L\ A$ $\left[\eta = \dfrac{N}{L}\right]$

$$L' = \mu_0 NA\left(\frac{N}{L}\right)$$

$$L' \propto \frac{1}{L}$$

13. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be:

(1) $100\sqrt{5}$ m/s (2) 100 m/s

(3) $80\sqrt{5}$ m/s (4) 80 m/s

Answer (1)

Sol. $V \propto \sqrt{T}$

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{200}{V_2} = \sqrt{\frac{400}{500}}$$

$$V_2 = 200\sqrt{\frac{5}{4}} = 100\sqrt{5}\ \text{m/s}$$

14. A capacitor with capacitance 5 μF is charged to 5 μC. If the plates are pulled apart to reduce the capacitance to 2 μF, how much work is done?

(1) 2.55 × 10^{-6} J (2) 6.25 × 10^{-6} J

(3) 3.75 × 10^{-6} J (4) 2.16 × 10^{-6} J

Answer (3)

Sol. $U_i = \frac{Q^2}{2C_1}$

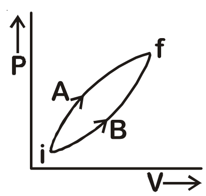
$U_f = \frac{Q^2}{2C_2}$

$W = \frac{Q^2}{2}\left[\frac{1}{C_2} - \frac{1}{C_1}\right]$

$= \frac{(5)^2}{2}\left[\frac{1}{2} - \frac{1}{5}\right]$

W = 3.75 μJ

15. Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies, respectively, then:



(1) $\Delta Q_A > \Delta Q_B$, $\Delta U_A = \Delta U_B$

(2) $\Delta Q_A = \Delta Q_B$; $\Delta U_A = \Delta U_B$

(3) $\Delta Q_A > \Delta Q_B$, $\Delta U_A > \Delta U_B$

(4) $\Delta Q_A < \Delta Q_B$, $\Delta U_A < \Delta U_B$

Answer (1)

Sol. $\Delta W_A > \Delta W_B$

$\Delta U_A > \Delta U_B$

From first law of thermodynamics,

$\Delta Q = \Delta U + \Delta W$

$\Delta Q_A > \Delta Q_B$

16. A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is:

(1) 0.32 m (2) 0.24 m

(3) 1.60 m (4) 0.16 m

Answer (1)

Sol. $\frac{1}{V} + \frac{1}{U} = -\frac{1}{40}$

$\frac{V}{U} = -5$

$-\frac{1}{5U} + \frac{1}{U} = -\frac{1}{40}$

U = – 32 cm

17. A solid sphere of mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness 2a and mass 2M. The gravitational field at distance '3a' from the centre will be:

(1) $\frac{GM}{9a^2}$ (2) $\frac{2GM}{9a^2}$

(3) $\frac{GM}{3a^2}$ (4) $\frac{2GM}{3a^2}$

Answer (3)

Sol. $E = \frac{GM}{(3a)^2} + \frac{2GM}{(3a)^2}$

$E = \frac{GM}{3a^2}$

18. A moving coil galvanometer has resistance 50 Ω and it indicates full deflection at 4 mA current. A voltmeter is made using this galvanometer and a 5 kΩ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to:

(1) 10 V (2) 20 V

(3) 15 V (4) 40 V

Answer (2)

Sol. $V = I_g (R_s + R_g)$

$= 4 \times 10^{-3}$ [5050]

≈ 20 V

19. The pressure wave, P = 0.01sin[1000 t – 3x]Nm^{-2}, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0°C. On some other day when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be 336 ms^{-1}. Approximate value of T is :

(1) 4°C (2) 12°C

(3) 15°C (4) 11°C

Answer (1)

Sol. $V_1 = \frac{1000}{3}$ m/s

$$V \propto \sqrt{T}$$

$$\frac{dV}{V} = \frac{1}{2}\frac{dT}{T}$$

$$\frac{8\times3}{3\times1000} = \frac{1}{2}\times\frac{dT}{273}$$

$$dT = \frac{273\times2\times8}{1000} = 4.36°C$$

20. Taking the wavelength of first Balmer line in hydrogen spectrum (n = 3 to n = 2) as 660 nm, the wavelength of the 2nd Balmer line (n = 4 to n = 2) will be :

(1) 889.2 nm (2) 488.9 nm

(3) 388.9 nm (4) 642.7 nm

Answer (2)

Sol. $\frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$

$$\frac{1}{\lambda_2} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$$

$$\frac{5\lambda_1}{36} = \frac{12\lambda_2}{4\times16}$$

$$\lambda_2 = \frac{5\times660\times64}{36\times12}$$

= 489 nm

21. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body?

(1) 1.5 kg (2) 1.8 kg

(3) 1.0 kg (4) 1.2 kg

Answer (4)

Sol. $2V = \frac{2V}{4} + m_2V_2$

$$V = V_2 - \frac{V}{4}$$

$$\frac{3V}{2} = m_2\left(V + \frac{V}{4}\right)$$

$$m_2 = \frac{6}{5}\text{ kg}$$

22. An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is $\bar{v}$, m is its mass and k_B is Boltzmann constant, then its temperature will be :

(1) $\frac{m\bar{v}^2}{5k_B}$

(2) $\frac{m\bar{v}^2}{6k_B}$

(3) $\frac{m\bar{v}^2}{7k_B}$

(4) $\frac{m\bar{v}^2}{3k_B}$

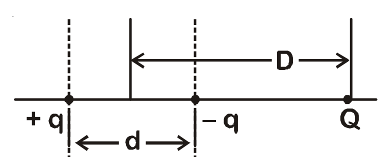
Answer (4)

Sol. $\bar{v} = \sqrt{\frac{3\,RT}{M}}$

$$\bar{v} = \sqrt{\frac{3\,RT}{mN_A}}$$

$$\bar{v} = \sqrt{\frac{3\,k_BT}{m}}$$

23. A system of three charges are placed as shown in the figure :



If D >> d, the potential energy of the system is best given by :

(1) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right]$

(2) $\frac{1}{4\pi\varepsilon_0}\left[+\frac{q^2}{d} + \frac{qQd}{D^2}\right]$

(3) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} + \frac{2qQd}{D^2}\right]$

(4) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{2D^2}\right]$

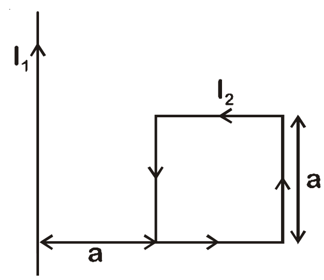
Answer (1)

Sol. $U = -\frac{Kq^2}{d} + QV$

$$V = -\frac{KP}{D^2} = -\frac{Kqd}{D^2}$$

$$U = -\frac{Kq^2}{d} - \frac{KQqd}{D^2}$$

24. A rigid square loop of side 'a' and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to the wire will be :



(1) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{4\pi}$

(2) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{2\pi}$

(3) Zero

(4) Attractive and equal to $\frac{\mu_0 I_1 I_2}{3\pi}$

Answer (1)

Sol. $F = I_2 a (B_1 - B_2)$

$$B_1 = \frac{\mu_0 I_1}{2\pi a}$$

$$B_2 = \frac{\mu_0 I_1}{4\pi a}$$

$$F = \frac{\mu_0 I_1 I_2}{4\pi}$$

25. A string is clamped at both the ends and it is vibrating in its 4^{th} harmonic. The equation of the stationary wave is $Y = 0.3 \sin(0.157x) \cos(200\pi t)$. The length of the string is : (All quantities are in SI units)

(1) 60 m

(2) 20 m

(3) 40 m

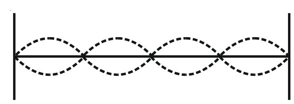
(4) 80 m

Answer (4)

Sol. $\lambda = 0.157$

$$= \frac{3.14}{20} = \frac{\pi}{20}$$

In 4^{th} harmonic



$L = 2\lambda$

$$\therefore \frac{2\pi}{\lambda} = \pi/20$$

$\lambda = 40$ m

$$\therefore L = \frac{4\lambda}{2} = 80\text{ m}$$

26. An NPN transistor is used in common emitter configuration as an amplifier with 1 kΩ load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and 15 μA change in the base current of the amplifier. The input resistance and voltage gain are :

(1) 0.33 kΩ, 1.5

(2) 0.33 kΩ, 300

(3) 0.67 kΩ, 200

(4) 0.67 kΩ, 300

Answer (4)

Sol. $\beta = \frac{I_C}{I_B} = 200$

$R_i = \frac{10\times10^{-3}}{15\times10^{-6}} = 0.67\ K\Omega$

Voltage Gain $= \beta\left(\frac{R_o}{R_i}\right) = 300$

27. The electric field of light wave is given as $\vec{E} = 10^{-3}\cos\left(\frac{2\pi x}{5\times10^{-7}} - 2\pi\times6\times10^{14}t\right)\hat{x}\ \frac{N}{C}$. This light falls on a metal plate of work function 2eV. The stopping potential of the photo-electrons is :

Given, E (in eV) = $\frac{12375}{\lambda\ (\text{in Å})}$

(1) 0.48 V

(2) 2.48 V

(3) 0.72 V

(4) 2.0 V

Answer (1)

Sol. $\lambda = 5 \times 10^{-7}$ m

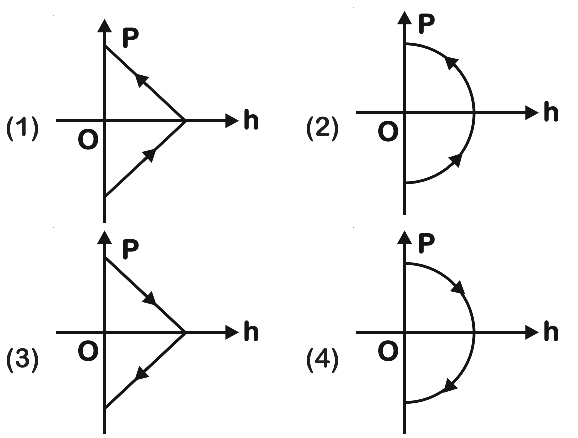
$\nu = \frac{3\times10^{8}}{5\times10^{-7}} = 6\times10^{14}\ Hz$

$E = \frac{12375}{5000} = 2.475\ eV$

$K_{max} = E - \phi$

$= 0.48$ eV

28. A ball is thrown vertically up (taken as + z-axis) from the ground. The correct momentum-height (p-h) diagram is:



Answer (4)

Sol. $V = \sqrt{V_0^2 - 2gh}$

Direction of velocity changes at top most point

29. If 'M' is the mass of water that rises in a capillary tube of radius 'r', then mass of water which will rise in a capillary tube of radius '2r' is

(1) 2 M

(2) M

(3) 4 M

(4) $\frac{M}{2}$

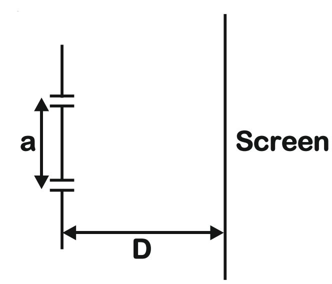
Answer (1)

Sol. $h \propto 1/r$

$M \propto \pi r^2 h$

$M \propto r$

30. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gets shifted by a distance equal to n fringe widths. If the wavelength of light used is λ, t will be



(1) $\frac{2nD\lambda}{a(\mu-1)}$

(2) $\frac{2D\lambda}{a(\mu-1)}$

(3) $\frac{nD\lambda}{a(\mu-1)}$

(4) $\frac{D\lambda}{a(\mu-1)}$

Answer (Bonus)

Sol. $(\mu - 1)\ t = n\ \lambda$

$t = \frac{n\lambda}{\mu-1}$

PART–B : CHEMISTRY

1. C_{60}, an allotrope of carbon contains
   (1) 16 hexagons and 16 pentagons
   (2) 18 hexagons and 14 pentagons
   (3) 20 hexagons and 12 pentagons
   (4) 12 hexagons and 20 pentagons

Answer (3)

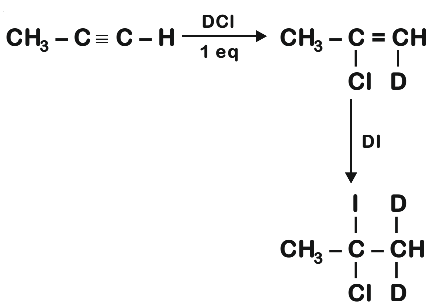
Sol. Fullerene (C_{60}) contains 20 six membered rings and 12 five membered rings.

2. The major product of the following reaction is

$$CH_3C\equiv CH \xrightarrow[\text{(ii) DI}]{\text{(i) DCl (1 equiv.)}}$$

   (1) $CH_3C(I)(Cl)CHD_2$ (2) $CH_3CD(I)CHD(Cl)$
   (3) $CH_3CD(Cl)CHD(I)$ (4) $CH_3CD_2CH(Cl)(I)$

Answer (1)

Sol. 

Both addition follow Markownikov's rule.

3. The standard Gibbs energy for the given cell reaction in kJ mol^{-1} at 298 K is

   $Zn(s) + Cu^{2+}(aq) \longrightarrow Zn^{2+}(aq) + Cu(s)$,

   $E° = 2$ V at 298 K

   (Faraday's constant, F = 96000 C mol^{-1})

   (1) 192
   (2) 384
   (3) –384
   (4) –192

Answer (3)

Sol. $\Delta G° = -\,nFE°_{cell}$

$= -2 \times (96000) \times 2$ V

$= -384$ kJ/mole

4. The number of water molecule(s) not coordinated to copper ion directly in $CuSO_4 \cdot 5H_2O$, is
   (1) 4 (2) 1
   (3) 2 (4) 3

Answer (2)

Sol. In $CuSO_4 \cdot 5H_2O$, four H_2O molecules are directly coordinated to the central metal ion while one H_2O molecule is hydrogen bonded.

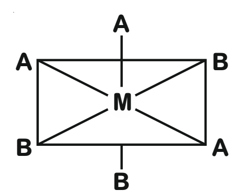
5. Magnesium powder burns in air to give
   (1) $Mg(NO_3)_2$ and Mg_3N_2
   (2) MgO and $Mg(NO_3)_2$
   (3) MgO and Mg_3N_2
   (4) MgO only

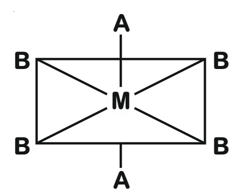
Answer (3)

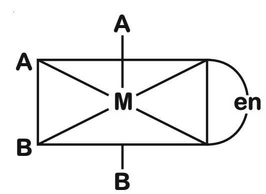
Sol. Mg burn in air and produces a mixture of nitride and oxide.

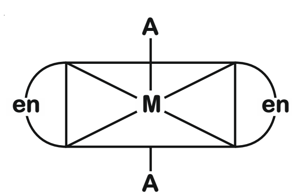
So, Mg_3N_2 and MgO are formed.

6. The one that will show optical activity is (en = ethane-1,2-diamine)

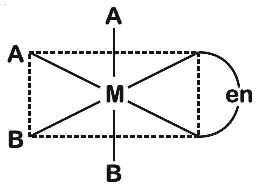
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

has no plane of symmetry or centre of symmetry, hence it is optically active.

7. Liquid 'M' and liquid 'N' form an ideal solution. The vapour pressures of pure liquids 'M' and 'N' are 450 and 700 mmHg, respectively, at the same temperature. Then correct statement is

(x_M = Mole fraction of 'M' in solution;

x_N = Mole fraction of 'N' in solution;

y_M = Mole fraction of 'M' in vapour phase;

y_N = Mole fraction of 'N' in vapour phase)

(1) $\frac{x_M}{x_N} = \frac{y_M}{y_N}$ (2) $\frac{x_M}{x_N} > \frac{y_M}{y_N}$

(3) $\frac{x_M}{x_N} < \frac{y_M}{y_N}$ (4) $(x_M - y_M) < (x_N - y_N)$

Answer (2)

Sol. $P_M^o = 450$ mmHg, $P_N^o = 700$ mmHg

$P_M = P_M^o X_M = Y_M P_T$

$\Rightarrow \quad P_M^o = \frac{Y_M}{X_M}(P_T)$

Similarly, $P_N^o = \frac{Y_N}{X_N}(P_T)$

Given, $P_M^o < P_N^o$

$\Rightarrow \quad \frac{Y_M}{X_M} < \frac{Y_N}{X_N}$

$\Rightarrow \quad \frac{Y_M}{Y_N} < \frac{X_M}{X_N}$

8. The major product of the following reaction is

$$CH_3CH = CHCO_2CH_3 \xrightarrow{LiAlH_4}$$

(1) $CH_3CH_2CH_2CH_2OH$

(2) $CH_3CH_2CH_2CHO$

(3) $CH_3CH_2CH_2CO_2CH_3$

(4) $CH_3CH = CHCH_2OH$

Answer (4)

Sol. $LiAlH_4$ reduces esters to alcohols but does not reduce C = C.

$$CH_3 - CH = CH - CO_2Me \xrightarrow{LiAlH_4} CH_3 - CH = CH - CH_2 - OH$$

9. For any given series of spectral lines of atomic hydrogen, let $\Delta\bar{\nu} = \bar{\nu}_{max} - \bar{\nu}_{min}$ be the difference in maximum and minimum frequencies in cm^{-1}. The ratio $\Delta\bar{\nu}_{Lyman} / \Delta\bar{\nu}_{Balmer}$ is

(1) 9 : 4 (2) 27 : 5

(3) 4 : 1 (4) 5 : 4

Answer (1)

Sol. $\bar{\nu} \propto \Delta E$

For H-atom

$$\bar{\nu} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For Lyman series,

$$\bar{\nu}(max) \propto 13.6\left(1 - \frac{1}{\infty}\right)$$

$$\bar{\nu}(min) \propto 13.6\left(1 - \frac{1}{4}\right)$$

$$\therefore \quad \bar{\nu}_{max} - \bar{\nu}_{min} \propto 13.6\left(\frac{1}{4}\right)$$

For Balmer series,

$$\bar{\nu}(max) \propto 13.6\left(\frac{1}{4} - \frac{1}{\infty}\right)$$

$$\bar{\nu}(min) \propto 13.6\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\therefore \quad \bar{\nu}_{min} - \bar{\nu}_{min} \propto 13.6\left(\frac{1}{9}\right)$$

$$\frac{\nu_{Lyman}}{\nu_{Balmer}} = \frac{9}{4}$$

10. Among the following, the set of parameters that represents path functions, is

(A) q + w (B) q

(C) w (D) H – TS

(1) (A), (B) and (C) (2) (B) and (C)

(3) (B), (C) and (D) (4) (A) and (D)

Answer (2)

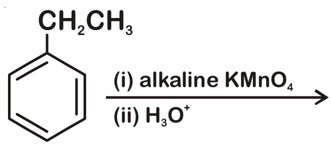
Sol. (A) $q + w = \Delta U$, state function

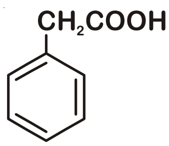
(B) q, path function

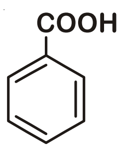
(C) w, path function

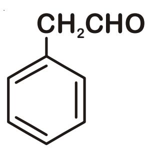
(D) $H - TS = G$, state function

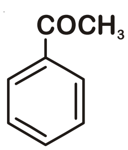
11. The major product of the following reaction is



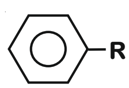
(1) 

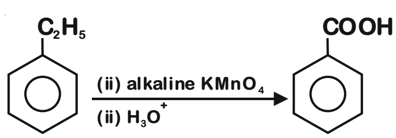
(2) 

(3) 

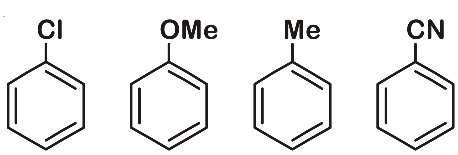
(4) 

Answer (2)

Sol. Alkaline $KMnO_4$ converts  with a benzylic hydrogen into benzoic acid.



12. The increasing order of reactivity of the following compounds towards aromatic electrophilic substitution reaction is



A B C D

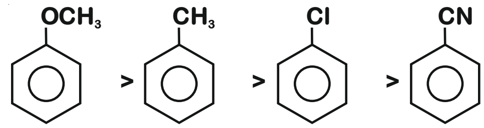
(1) D < B < A < C

(2) D < A < C < B

(3) B < C < A < D

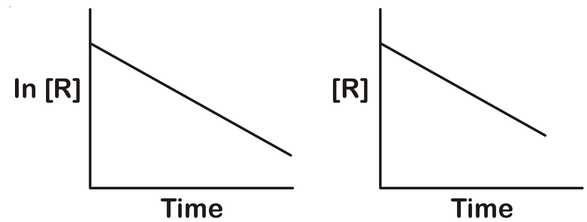
(4) A < B < C < D

Answer (2)

Sol. 

(+M) (+H) (–I > +M) (–M, –I)

13. The given plots represent the variation of the concentration of a reactant R with time for two different reactions (i) and (ii). The respective orders of the reactions are



(i) (ii)

(1) 0, 1 (2) 1, 1

(3) 1, 0 (4) 0, 2

Answer (3)

Sol. Graph-(i) : ln[Reactant] vs time is linear

Hence, 1st order

Graph-(ii) : [Reactant] vs time is linear

Hence, zero order

14. The osmotic pressure of a dilute solution of an ionic compound XY in water is four times that of a solution of 0.01 M $BaCl_2$ in water. Assuming complete dissociation of the given ionic compounds in water, the concentration of XY (in mol L^{-1}) in solution is

(1) 16×10^{-4} (2) 4×10^{-4}

(3) 6×10^{-2} (4) 4×10^{-2}

Answer (3)

Sol. $\pi_{XY} = 4\pi_{BaCl_2}$

$\therefore$ $2[XY] = 4 \times (0.01) \times 3$

$[XY] = 0.06$

$$= 6 \times 10^{-2}\ \frac{mol}{L}$$

15. Consider the van der Waals constants, a and b, for the following gases.

Gas	Ar	Ne	Kr	Xe
a/(atm dm^6 mol^{-2})	1.3	0.2	5.1	4.1
b/(10^{-2} dm^3 mol^{-1})	3.2	1.7	1.0	5.0

Which gas is expected to have the highest critical temperature?

(1) Ne (2) Kr

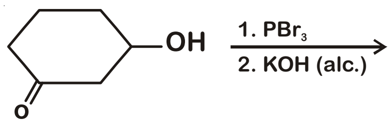
(3) Xe (4) Ar

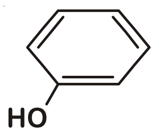
Answer (2)

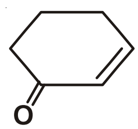
Sol. Critical temperature = $\frac{8a}{27Rb}$

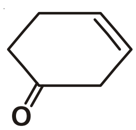
So, species with greatest value of $\frac{a}{b}$ has greatest value of critical temperature i.e. Kr.

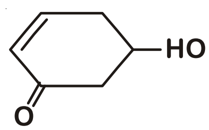
16. The major product of the following reaction is



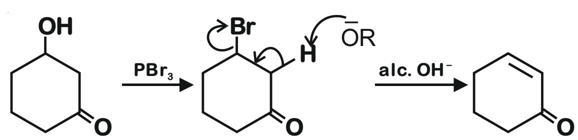
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

More stable product due to conjugation.

17. Among the following, the molecule expected to be stabilized by anion formation is

C_2, O_2, NO, F_2

(1) F_2 (2) NO

(3) C_2 (4) O_2

Answer (3)

Sol. C_2 has s-p mixing and the HOMO is $\pi 2p_x = \pi 2p_y$ and LUMO is $\sigma 2p_z$. So, the extra electron will occupy bonding molecular orbital and this will lead to an increase in bond order.

C_2^- has more bond order than C_2.

18. Excessive release of CO_2 into the atmosphere results in

(1) Depletion of ozone

(2) Polar vortex

(3) Formation of smog

(4) Global warming

Answer (4)

Sol. CO_2 causes global warming.

19. For a reaction,

$$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g);$$

Identify dihydrogen (H_2) as a limiting reagent in the following reaction mixtures.

(1) 35 g of N_2 + 8 g of H_2

(2) 28 g of N_2 + 6 g of H_2

(3) 56 g of N_2 + 10 g of H_2

(4) 14 g of N_2 + 4 g of H_2

Answer (3)

Sol. 28 g N_2 react with 6 g H_2

$$\underset{\substack{\text{1 mole}\\ \text{28 g}}}{N_2} + \underset{\substack{\text{3 moles}\\ \text{6 g}}}{3H_2} \longrightarrow 2NH_3$$

For 56 g of N_2, 12 g of H_2 is required.

20. Match the catalysts (Column I) with products (Column II).

Column I Catalyst	Column II Product
(A) V_2O_5	(i) Polyethylene
(B) $TiCl_4/Al(Me)_3$	(ii) Ethanal
(C) $PdCl_2$	(iii) H_2SO_4
(D) Iron Oxide	(iv) NH_3

(1) (A)-(iv); (B)-(iii); (C)-(ii); (D)-(i)

(2) (A)-(iii); (B)-(iv); (C)-(i); (D)-(ii)

(3) (A)-(iii); (B)-(i); (C)-(ii); (D)-(iv)

(4) (A)-(ii); (B)-(iii); (C)-(i); (D)-(iv)

Answer (3)

Sol. (A) V_2O_5 → Preparation of H_2SO_4 in contacts process

(B) $TiCl_4$ + $Al(Me)_3$ → Polyethylene (Ziegler-Natta catalyst)

(C) $PdCl_2$ → Ethanal (Wacker's process)

(D) Iron oxide → NH_3 in Haber's process

21. The element having greatest difference between its first and second ionization energies, is

(1) K (2) Sc

(3) Ca (4) Ba

Answer (1)

Sol. Alkali metals have high difference in the first ionisation and the second ionisation energy as they achieve stable noble gas configuration after first ionisation.

22. The correct order of the oxidation states of nitrogen in NO, N_2O, NO_2, and N_2O_3 is

(1) $NO_2 < NO < N_2O_3 < N_2O$

(2) $N_2O < NO < N_2O_3 < NO_2$

(3) $NO_2 < N_2O_3 < NO < N_2O$

(4) $N_2O < N_2O_3 < NO < NO_2$

Answer (2)

Sol.

(oxide)	(oxidation state)
N_2O	+ 1
NO	+ 2
N_2O_3	+ 3
NO_2	+ 4

So, $N_2O < NO < N_2O_3 < NO_2$

23. Which of the following statements is not true about sucrose?

(1) The glycosidic linkage is present between C_1 of α-glucose and C_1 of β-fructose

(2) On hydrolysis, it produces glucose and fructose

(3) It is a non-reducing sugar

(4) It is also named as invert sugar

Answer (1)

Sol. Sucrose contains glycosidic link between C_1 of α-D glucose and C_2 of β-D-Fructose.

$$C_{12}H_{22}O_{11} + H_2O \longrightarrow \text{Glucose} + \text{Fructose}$$

24. The ore that contains the metal in the form of fluoride is

(1) malachite (2) sphalerite

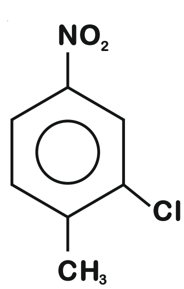
(3) magnetite (4) cryolite

Answer (4)

Sol.

Magnetite	Fe_3O_4
Sphalerite	ZnS
Cryolite	Na_3AlF_6
Malachite	$CuCO_3 \cdot Cu(OH)_2$

25. The correct IUPAC name of the following compound is



(1) 3-chloro-4-methyl-1-nitrobenzene

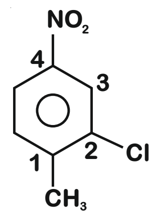
(2) 5-chloro-4-methyl-1-nitrobenzene

(3) 2-methyl-5-nitro-1-chlorobenzene

(4) 2-chloro-1-methyl-4-nitrobenzene

Answer (4)

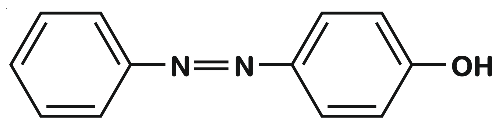
Sol.

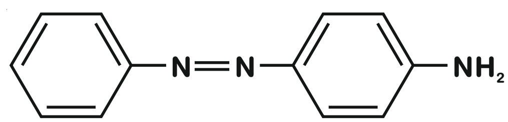


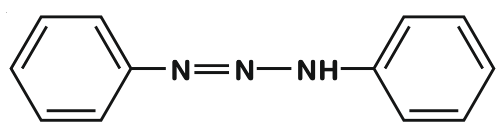
All Groups attached are to be treated as substituents and lowest set of locant rule is followed.

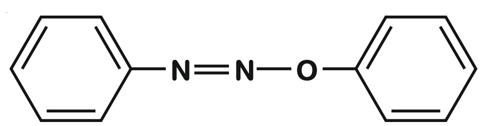
2-Chloro-1-methyl-4-nitrobenzene

26. Aniline dissolved in dilute HCl is reacted with sodium nitrite at 0°C. This solution was added dropwise to a solution containing equimolar mixture of aniline and phenol in dil. HCl. The structure of the major product is

(1) 

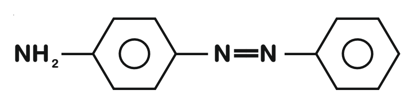
(2) 

(3) 

(4) 

Answer (2)

Sol. In acidic medium aniline is more reactive than phenol that's why electrophilic aromatic substitution of $Ph-\overset{+}{N}_2$ takes place with aniline



27. The aerosol is a kind of colloid in which

(1) solid is dispersed in gas

(2) gas is dispersed in solid

(3) liquid is dispersed in water

(4) gas is dispersed in liquid

Answer (1)

Sol. In aerosol, the dispersion medium is gas while the dispersed phase can be both solid or liquid.

28. The degenerate orbitals of $[Cr(H_2O)_6]^{3+}$ are

(1) d_{xz} and d_{yz}

(2) $d_{x^2-y^2}$ and d_{xy}

(3) d_{z^2} and d_{xz}

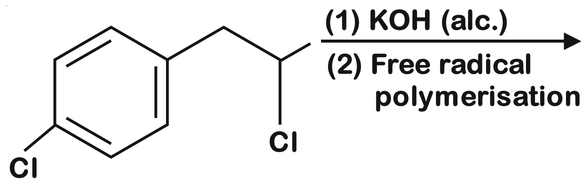
(4) d_{yz} and d_{z^2}

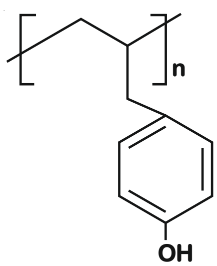
Answer (1)

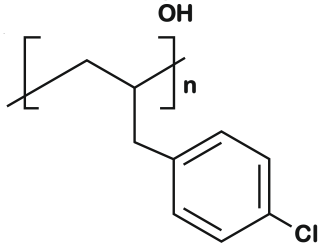
Sol. Cr^{3+} has d^3 configuration and forms an octahedral inner orbitals complex.

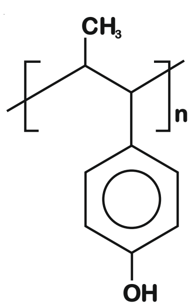
The set of degenerate orintals are (d_{xy}, d_{yz} and d_{xz}) and ($d_{x^2-y^2}$ and d_{z^2}).

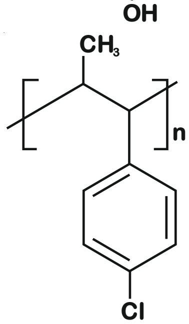
29. The major product of the following reaction is



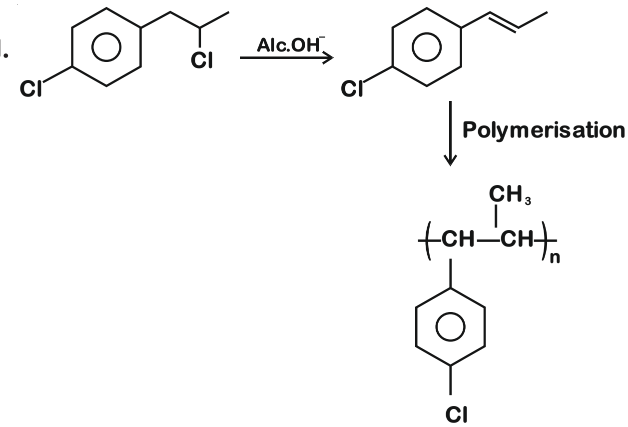
(1) 

(2) 

(3) 

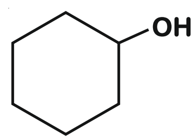
(4) 

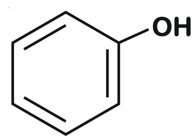
Answer (4)

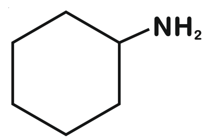
Sol. 

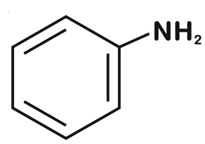
30. The organic compound that gives following qualitative analysis is

Test	Inference
(a) Dil. HCl	Insoluble
(b) NaOH solution	Soluble
(c) Br_2/water	Decolourization

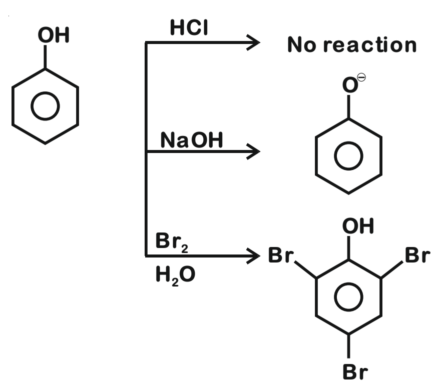
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

PART–C : MATHEMATICS

1. Let p, q $\in$ R. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then :
   (1) $q^2 - 4p - 16 = 0$ (2) $p^2 - 4q + 12 = 0$
   (3) $p^2 - 4q - 12 = 0$ (4) $q^2 + 4p + 14 = 0$

Answer (3)

Sol. p, q are rational numbers

$\therefore\ 2+\sqrt{3}$ in the other root

Now, p = –4, q = 1

$\Rightarrow\ p^2 - 4q - 12 = 16 - 4 - 12$

$= 0$

Note:- (Erratum) p, q, should be given as rational numbers instead of real numbers

2. If the line, $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ meets the plane, $x + 2y + 3z = 15$ at a point P, then the distance of P from the origin is :
   (1) $2\sqrt{5}$ (2) $\frac{9}{2}$
   (3) $\frac{7}{2}$ (4) $\frac{\sqrt{5}}{2}$

Answer (2)

Sol. Let point on line is p(2r + 1, 3r – 1, 4r + 2)

It lies on the plane x + 2y + 3z = 15

$\therefore\ 2r + 1 + 6r - 2 + 12r + 6 = 15$

$\Rightarrow\ r=\frac{1}{2}$

$\therefore\ P=\left(2,\frac{1}{2},4\right)$

$\therefore\ OP=\sqrt{4+\frac{1}{4}+16}=\sqrt{\frac{81}{4}}=\frac{9}{2}$

3. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is :
   (1) $x^2 + y^2 - 16x^2y^2 = 0$
   (2) $x^2 + y^2 - 2x^2y^2 = 0$
   (3) $x^2 + y^2 - 4x^2y^2 = 0$
   (4) $x^2 + y^2 - 2xy = 0$

Answer (3)

Sol. Let any tangent to circle $x^2 + y^2 = 1$ is

$x\cos\theta + y\sin\theta = 1$

$\therefore\ P\left(\frac{1}{\cos\theta},0\right);\ \ Q\left(0,\frac{1}{\sin\theta}\right)$

$\therefore$ Mid-point of PQ let $M\left(\frac{1}{2\cos\theta},\frac{1}{2\sin\theta}\right)=(h, k)$

$\Rightarrow\ \cos\theta=\frac{1}{2h};\ \ \sin\theta=\frac{1}{2k}$

$\therefore$ On squaring and adding

$\frac{1}{h^2}+\frac{1}{k^2}=4\ \ \Rightarrow x^2 + y^2 = 4x^2y^2$

4. If f(x) is a non-zero polynomial of degree four, having local extreme points at x = –1, 0, 1; then the set

   $S = \{x \in R : f(x) = f(0)\}$

   contains exactly :
   (1) Four irrational numbers
   (2) Four rational numbers
   (3) Two irrational and one rational number
   (4) Two irrational and two rational numbers

Answer (3)

Sol. $f'(x) = A(x + 1)\,x\,(x - 1)$

$= A(x^3 - x)$

$\Rightarrow\ f(x)=A\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+C$

Now f(0) = C

$\therefore\ f(x) = f(0)\ \ \Rightarrow\ A\left(\frac{x^4}{4}-\frac{x^2}{2}\right)=0$

$\Rightarrow\ \frac{x^2}{2}\left(\frac{x^2}{2}-1\right)=0$

$\Rightarrow\ x=0, 0,-\sqrt{2},\sqrt{2}$

$\therefore\ S=\{0,-\sqrt{2},\sqrt{2}\}$

5. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where k > 0, then k is equal to :

(1) $\sqrt{6}$ (2) $2\sqrt{6}$

(3) $2\sqrt{\frac{10}{3}}$ (4) $4\sqrt{\frac{5}{3}}$

Answer (2)

Sol. Mean of given observation = $\frac{k}{4}$

$\therefore$ $\sigma^2 = 5$(given) ...(i)

Also $\sigma^2 = \dfrac{\left(\frac{k}{4}+1\right)^2+\left(\frac{k}{4}\right)^2+\left(\frac{k}{4}-1\right)^2+\left(\frac{3k}{4}\right)^2}{4}$ ...(ii)

$\therefore$ from (i) and (ii)

$$\frac{\frac{12k^2}{16}+2}{4}=5 \quad \Rightarrow \frac{12k^2}{16}=18$$

$\Rightarrow$ $k^2 = 24$ $\Rightarrow k = 2\sqrt{6}$

6. If the fourth term in the Binomial expansion of $\left(\frac{2}{x}+x^{\log_8 x}\right)^6$ (x > 0) is 20×8^7, then a value of x is :

(1) 8^3 (2) 8

(3) 8^{-2} (4) 8^2

Answer (4)

Sol. $T_4 = 20\times 8^7 = {}^6C_3\left(\frac{2}{x}\right)^3\times\left(x^{\log_8 x}\right)^3$

$$\Rightarrow 8\times 20\times\left(\frac{x^{\log_8 x}}{x}\right)^3 = 20\times 8^7$$

$$\Rightarrow \left(\frac{x^{\log_8 x}}{x}\right)^3 = \left(8^2\right)^3$$

$\Rightarrow \frac{x^{\log_8 x}}{x} = 64$ Take $\log_8$ both side

$\Rightarrow$ $(\log_8 x)^2 - (\log_8 x) = 2$

$\Rightarrow$ $\log_8 x = -1$ or $\log_8 x = 2$

$\Rightarrow$ $x = \frac{1}{8}$ or $x = 8^2$

7. The integral $\int \sec^{2/3}x\,\text{cosec}^{4/3}x\,dx$ is equal to :

(Here C is a constant of integration)

(1) $-3\cot^{-1/3}x + C$

(2) $-3\tan^{-1/3}x + C$

(3) $-\frac{3}{4}\tan^{-4/3}x + C$

(4) $3\tan^{-1/3}x + C$

Answer (2)

Sol. $I = \int \sec^{\frac{2}{3}}x\cdot\text{cosec}^{\frac{4}{3}}dx$

$I = \int \dfrac{\sec^2 x\,dx}{\tan^{\frac{4}{3}}x}$ Put tan x = t

$\Rightarrow \sec^2 x\,dx = dt$

$\Rightarrow$ $I = \dfrac{t^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)}+C$ $\Rightarrow I = -3(\tan x)^{\frac{-1}{3}}+C$

8. If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1, 4), then the length of this focal chord is :

(1) 24

(2) 20

(3) 22

(4) 25

Answer (4)

Sol. $\because$ $y^2 = 16x$ $\Rightarrow a = 4$

One end of focal chord (1, 4) $\therefore$ 2 at = 4

$\Rightarrow t = \frac{1}{2}$

Length of focal chord $= a\left(t+\frac{1}{t}\right)^2$

$= 4\times\left(2+\frac{1}{2}\right)^2$

= 25

9. If the function f defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by

$$f(x) = \begin{cases} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}, & x \neq \frac{\pi}{4} \\ k, & x = \frac{\pi}{4} \end{cases}$$

is continuous, then k is equal to :

(1) 1 (2) $\frac{1}{2}$

(3) $\frac{1}{\sqrt{2}}$ (4) 2

Answer (2)

Sol. $\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} = k$ $\therefore$ By L hospital rule.

$\Rightarrow \lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\sin x}{\text{cosec}^2 x} = k$ $\Rightarrow k = \frac{1}{2}$

10. All the points in the set

$$S = \left\{\frac{\alpha + i}{\alpha - i} : \alpha \in R\right\} (i = \sqrt{-1})$$

lie on a :

(1) Straight line whose slope is 1

(2) Circle whose radius is $\sqrt{2}$

(3) Circle whose radius is 1

(4) Straight line whose slope is –1

Answer (3)

Sol. $\because$ $S = \frac{\alpha + i}{\alpha - i}$ Let S = x + iy

$\Rightarrow x + iy = \frac{(\alpha + i)^2}{\alpha^2 + 1}$ (by rationalisation)

$\Rightarrow x + iy = \frac{(\alpha^2 - 1)}{\alpha^2 + 1} + \frac{i(2\alpha)}{\alpha^2 + 1}$ (On comparing both sides)

$\Rightarrow x = \frac{\alpha^2 - 1}{\alpha^2 + 1}$ ... (i) $y = \frac{2\alpha}{\alpha^2 + 1}$ ... (ii)

By squaring and adding

$\Rightarrow x^2 + y^2 = 1$

11. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to :

(1) $\frac{1}{2}(3\hat{i} - 9\hat{j} + 5\hat{k})$ (2) $\frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$

(3) $-3\hat{i} + 9\hat{j} + 5\hat{k}$ (4) $3\hat{i} - 9\hat{j} - 5\hat{k}$

Answer (2)

Sol. $\vec{\beta} = \vec{\beta_1} - \vec{\beta_2}$ ...(i)

$\therefore$ $\vec{\beta_2} \cdot \vec{\alpha} = 0$

and Let $\vec{\beta_1} = \lambda\vec{\alpha}$

$\vec{\alpha} \cdot \vec{\beta} = \vec{\alpha} \cdot \vec{\beta_1} - \vec{\alpha} \cdot \vec{\beta_2}$

$\Rightarrow$ 5 = $\lambda\alpha^2$

$\Rightarrow$ 5 = $\lambda \times 10$

$\Rightarrow$ $\lambda = \frac{1}{2}$

$\therefore$ $\boxed{\vec{\beta_1} = \frac{\vec{\alpha}}{2}}$

Cross product with $\vec{\beta_1}$ in equation (i)

$\Rightarrow$ $\vec{\beta} \times \vec{\beta_1} = -\vec{\beta_2} \times \vec{\beta_1}$

$\Rightarrow$ $\boxed{\vec{\beta} \times \vec{\beta_1} = \vec{\beta_1} \times \vec{\beta_2}} = \frac{(\vec{\beta} \times \vec{\alpha})}{2}$

$$\Rightarrow \vec{\beta_1} \times \vec{\beta_2} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 3 & 1 & 0 \end{vmatrix}$$

$$= \frac{1}{2}\left[-3\hat{i} - \hat{j}(-9) + \hat{k}(5)\right]$$

$$= \frac{1}{2}\left[-3\hat{i} + 9\hat{j} + 5\hat{k}\right]$$

12. Let $\sum_{k=1}^{10} f(a + k) = 16(2^{10} - 1)$, where the function f satisfies f(x + y) = f(x) f(y) for all natural numbers x, y and f(1) = 2. Then the natural number 'a' is :

(1) 2

(2) 3

(3) 16

(4) 4

Answer (2)

Sol. $\because$ $f(x + y) = f(x) \cdot f(y)$

$\therefore$ Let $\boxed{f(x) = b^x}$

$\because$ $f(1) = 2$

$\therefore$ $b' = 2$

$\Rightarrow$ $\boxed{f(x) = 2^x}$

Now, $\sum_{k=1}^{10} 2^{a+k} = 16(2^{10} - 1)$

$\Rightarrow$ $2^a \sum_{k=1}^{10} 2^k = 16(2^{10} - 1)$

$\Rightarrow$ $2^a \times \frac{((2^{10}) - 1) \times 2}{(2 - 1)} = 16 \times (2^{10} - 1)$

$\Rightarrow$ $2^a = 8$

$\Rightarrow$ $\boxed{a = 3}$

13. If $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \cdots\cdots \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$,

then the inverse of $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ is :

(1) $\begin{bmatrix} 1 & 0 \\ 13 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & -12 \\ 0 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 0 \\ 12 & 1 \end{bmatrix}$

Answer (2)

Sol. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix} 1 & 1+2+3+\ldots.+(n-1) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ $\frac{(n-1)n}{2} = 78$

$\Rightarrow$ $n = 13$

Now, inverse of $\begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$

14. Let $S = \{\theta \in [-2\pi, 2\pi]\ ;\ 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is :

(1) π (2) 2π

(3) $\frac{13\pi}{6}$ (4) $\frac{5\pi}{3}$

Answer (2)

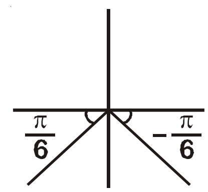
Sol. $\because$ $2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow$ $2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow$ $(2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow$ $\sin\theta = -\frac{1}{2}$; $\boxed{\sin\theta = 2} \rightarrow$ Not Possible

$\Rightarrow$



$\therefore$ Sum of all solutions in $[-2\pi, 2\pi]$ is

$= \left(\pi + \frac{\pi}{6}\right) + \left(2\pi - \frac{\pi}{6}\right) + \left(-\frac{\pi}{6}\right) + \left(-\pi + \frac{\pi}{6}\right) = 2\pi$

15. Let $f(x) = 15 - |x - 10|$; $x \in R$. Then the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is :

(1) (10, 15)

(2) {5, 10, 15, 20}

(3) {10}

(4) {5, 10, 15}

Answer (4)

Sol. Given $f(x) = 15 - |(10 - x)|$

$\Rightarrow$ $f(f(x)) = 15 - ||10 - x| - 5|$

$\therefore$ Non-differentiable at points where

$10 - x = 0$ and $|10 - x| = 5$

$\Rightarrow$ $x = 10$ and $x - 10 = \pm 5$

$\Rightarrow$ $x = 10$ and $x = 15, 5$

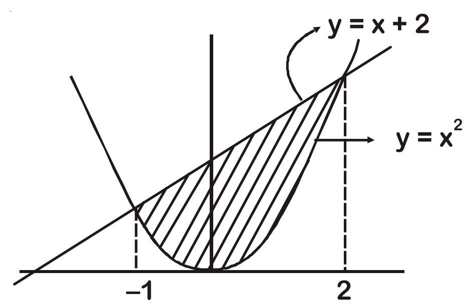
16. The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is :

(1) $\frac{31}{6}$ (2) $\frac{10}{3}$

(3) $\frac{9}{2}$ (4) $\frac{13}{6}$

Answer (3)

Sol.



$\therefore$ Required area $= \int_{-1}^{2}\left((x+2)-x^2\right)dx$

$=\left(\frac{x^2}{2}+2x-\frac{x^3}{3}\right)_{-1}^{2}$

$=\left(2+4-\frac{8}{3}\right)-\left(+\frac{1}{2}-2+\frac{1}{3}\right)$

$=8-3-\frac{1}{2}$

$=5-\frac{1}{2}=\frac{9}{2}$

17. If the line $y=mx+7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24}-\frac{y^2}{18}=1$, then a value of m is :

(1) $\frac{2}{\sqrt{5}}$ (2) $\frac{3}{\sqrt{5}}$

(3) $\frac{\sqrt{15}}{2}$ (4) $\frac{\sqrt{5}}{2}$

Answer (1)

Sol. $mx-y+7\sqrt{3}$ is normal to hyperbola $\frac{x^2}{24}-\frac{y^2}{18}=1$

then $\frac{24}{m^2}-\frac{18}{(-1)^2}=\frac{(24+18)^2}{\left(7\sqrt{3}\right)^2}$

$\Rightarrow \frac{24}{m^2}-18=\frac{42\times 42}{7\times 7\times 3}$

$\Rightarrow m=\frac{2}{\sqrt{5}}$

If lx + my + n = 0 is a normal to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$,

then $\frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{\left(a^2+b^2\right)^2}{n^2}$

18. Let α and β be the roots of the equation $x^2+x+1=0$. Then for $y \neq 0$ in R,

$\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to :

(1) $y(y^2-1)$ (2) y^3-1

(3) $y(y^2-3)$ (4) y^3

Answer (4)

Sol. Let $\alpha=\omega$ and $\beta=\omega^2$ are roots of $x^2+x+1=0$

$\therefore \begin{vmatrix} y+\omega & 1 & \omega^2 \\ 1 & y+\omega^2 & \omega \\ \omega^2 & \omega & 1+y \end{vmatrix}$ operate $c_1 \to c_1+c_2+c_3$

$=y\begin{vmatrix} 1 & 1 & \omega^2 \\ 1 & y+\omega^2 & \omega \\ 1 & \omega & 1+y \end{vmatrix}$ $\left(\text{By } \begin{matrix} R_2 \to R_2-R_1 \\ R_3 \to R_3-R_1 \end{matrix}\right)$

$=y\begin{vmatrix} 1 & 1 & \omega^2 \\ 0 & y+\omega^2-1 & \omega-\omega^2 \\ 0 & \omega-1 & 1+y-\omega^2 \end{vmatrix}$

$=y\{(y+\omega^2-1)(1+y-\omega^2)-\omega(\omega-1)(1-\omega)\}$

$=y(y^2-(\omega^2-1)^2)+y\omega(\omega-1)^2$

$=y^3+y(\omega-1)^2(\omega-(\omega+1)^2)=y^3$

19. The value of $\cos^2 10°-\cos 10°\cos 50°+\cos^2 50°$ is :

(1) $\frac{3}{4}$ (2) $\frac{3}{2}(1+\cos 20°)$

(3) $\frac{3}{2}$ (4) $\frac{3}{4}+\cos 20°$

Answer (1)

Sol. $\left(\frac{1+\cos 20°}{2}\right)+\left(\frac{1+\cos 100°}{2}\right)-\frac{1}{2}(2\cos 10°\cos 50°)$

$=1+\frac{1}{2}(\cos 20°+\cos 100°)-\frac{1}{2}[\cos 60°+\cos 40°]$

$=\left(1-\frac{1}{4}\right)+\frac{1}{2}[\cos 20°+\cos 100°-\cos 40°]$

$=\frac{3}{4}+\frac{1}{2}[2\cos 60°\times\cos 40°-\cos 40°]$

$=\frac{3}{4}$

20. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then :

(1) m = n = 68 (2) m + n = 68

(3) m = n = 78 (4) n = m – 8

Answer (3)

Sol. Here m = $^8C_6 \cdot {}^5C_5 + {}^8C_7 \cdot {}^5C_4 + {}^8C_8 \cdot {}^5C_3 = 78$

n = $^5C_3 \cdot {}^8C_8 + {}^5C_4 \cdot {}^8C_7 + {}^5C_5 \cdot {}^8C_6 = 78$

So m = n = 78

21. Slope of a line passing through P(2, 3) and intersecting the line, x + y = 7 at a distance of 4 units from P, is :

(1) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$ (2) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$

(3) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$ (4) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$

Answer (2)

Sol. Point at 4 units from P(2, 3) will be

A(4cosθ + 2, 4sinθ + 3) will satisfy x + y = 7

$\Rightarrow \cos\theta+\sin\theta=\frac{1}{2}$ on squaring

$\Rightarrow \boxed{\sin 2\theta=\frac{-3}{4}} \Rightarrow \frac{2\tan\theta}{1+\tan^2\theta}=-\frac{3}{4}$

$\Rightarrow 3\tan^2\theta + 8\tan\theta + 3 = 0$

$\Rightarrow \tan\theta=\frac{-8\pm 2\sqrt{7}}{6}$ (Ignoring –ve sign)

$\Rightarrow \tan\theta=\frac{-8+2\sqrt{7}}{6} = \frac{1-\sqrt{7}}{1+\sqrt{7}}$

22. The value of $\int_0^{\pi/2}\frac{\sin^3 x}{\sin x+\cos x}dx$ is :

(1) $\frac{\pi-2}{4}$ (2) $\frac{\pi-2}{8}$

(3) $\frac{\pi-1}{4}$ (4) $\frac{\pi-1}{2}$

Answer (3)

Sol. $I=\int_0^{\frac{\pi}{2}}\frac{\sin^3 x\,dx}{\sin x+\cos x}$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos^3 x\,dx}{\sin x+\cos x}$

$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}\left(1-\frac{1}{2}\sin(2x)\right)dx$

$\Rightarrow I=\frac{1}{2}\left[x+\frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{2}}$

$I=\frac{1}{2}\left(\frac{\pi-1}{2}\right)=\frac{\pi-1}{4}$

23. Let S be the set of all values of x for which the tangent to the curve y = f(x) = $x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points (1, f(1)) and (–1, f(–1)), then S is equal to :

(1) $\left\{\frac{1}{3},-1\right\}$ (2) $\left\{-\frac{1}{3},1\right\}$

(3) $\left\{-\frac{1}{3},-1\right\}$ (4) $\left\{\frac{1}{3},1\right\}$

Answer (2)

Sol. y = f(x) = $x^3 - x^2 - 2x$

$\frac{dy}{dx}=3x^2-2x-2$

f(1) = 1 – 1 – 2 = –2, f(–1) = –1 – 1 + 2 = 0

According to question, $3x^2 - 2x - 2$

$= \frac{f(1)-f(-1)}{1-(-1)}$

$\Rightarrow 3x^2 - 2x - 2 = \frac{-2-0}{2}$

$\Rightarrow 3x^2 - 2x - 1 = 0$

$\Rightarrow x=\frac{2\pm 4}{6}=1,\frac{-1}{3}$

So, $S=\left\{\frac{-1}{3},1\right\}$

24. A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane y – z + 5 = 0, also passes through the point :

(1) $\left(\sqrt{2},1,4\right)$ (2) $\left(\sqrt{2},-1,4\right)$

(3) $\left(-\sqrt{2},-1,-4\right)$ (4) $\left(-\sqrt{2},1,-4\right)$

Answer (1)

Sol. Let the required plane be $\frac{x}{a}+\frac{y}{-1}+\frac{z}{1}=1$ given plane is y – z + 5 = 0

$$\therefore \quad \cos\frac{\pi}{4}=\frac{-1-1}{\sqrt{\frac{1}{a^2}+1+1}\ \sqrt{2}}$$

$$\Rightarrow \quad a^2=\frac{1}{2}$$

$$\Rightarrow \quad \boxed{\frac{1}{a}=\pm\sqrt{2}}$$

$$\Rightarrow \quad \boxed{\pm\sqrt{2}x-y+z=1}$$

$\therefore \quad \left(\sqrt{2},1,4\right)$ satisfies $-\sqrt{2}\,x-y+z=1$

25. The solution of the differential equation $x\frac{dy}{dx}+2y=x^2(x\neq 0)$ with y(1) = 1, is :

(1) $y=\frac{4}{5}x^3+\frac{1}{5x^2}$ (2) $y=\frac{x^3}{5}+\frac{1}{5x^2}$

(3) $y=\frac{x^2}{4}+\frac{3}{4x^2}$ (4) $y=\frac{3}{4}x^2+\frac{1}{4x^2}$

Answer (3)

Sol. $\frac{dy}{dx}+\frac{2}{x}y=x$ y(1) = 1 (given)

$$I\cdot F=e^{\int\frac{2}{x}dx}=x^2$$

$$y\times x^2=\int x^3\,dx$$

$$\Rightarrow \quad yx^2=\frac{x^4}{4}+c$$

$\because$ at x = 1 ; y = 1

$$\Rightarrow \quad c=\frac{3}{4}$$

$$\therefore \quad y=\frac{x^2}{4}+\frac{3}{4x^2}$$

26. Let the sum of the first n terms of a non-constant A.P., a_1, a_2, a_3, ... be 50 n + $\frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to :

(1) (50, 50 + 46A) (2) (A, 50 + 45A)

(3) (A, 50 + 46A) (4) (50, 50 + 45A)

Answer (3)

Sol. $\because \quad S_n=\left(50-\frac{7A}{2}\right)n+n^2\times\frac{A}{2}$

$\therefore$ Common difference = $\frac{A}{2}\times 2=\boxed{A}$

a_{50} = a_1 + 49 × d

= (50 – 3A) + 49 A

= 50 + 46 A

So, $\left(d,a_{50}\right)=\left(A,50+46\,A\right)$

27. For any two statements p and q, the negation of the expression p ∨ (~ p ∧ q) is :

(1) ~ p ∧ ~ q (2) ~ p ∨ ~ q

(3) p ∧ q (4) p ↔ q

Answer (1)

Sol. ~(p v (~p ∧ q)) = ~(~p ∧ q) ∧~p

= (~q v p) ∧~p

= ~p ∧ (p v ~q)

= (~q ∧ ~p) v (p ∧ ~p)

= (~p ∧ ~q)

28. If the tangent to the curve, y = x^3 + ax – b at the point (1, –5) is perpendicular to the line, –x + y + 4 = 0, then which one of the following points lies on curve?

(1) (–2, 1) (2) (2, –2)

(3) (2, –1) (4) (–2, 2)

Answer (2)

Sol. f(x) = x^3 + ax – b ⇒ f′(x) = $3x^2$ + a

f(1) = –5 and f′(1) = 3 + a

⇒ 1 + a – b = –5

⇒ a – b = –6 ...(1)

Also slope of tangent P(1, –5) = –1 = f′(1)

∴ 3 + a = –1

⇒ a = –4

$\Rightarrow$ b = 2

$\therefore$ Equation of the curve is $f(x) = x^3 - 4x - 2$

$\therefore$ (2, –2) lies on the curve

29. If the function $f : R - \{1, -1\} \rightarrow A$ defined by $f(x) = \dfrac{x^2}{1-x^2}$, is surjective, then A is equal to :

(1) $[0, \infty)$ (2) $R - \{-1\}$

(3) $R - (-1, 0)$ (4) $R - [-1, 0)$

Answer (4)

Sol. $f(x) = \dfrac{x^2}{1-x^2}$

$$\Rightarrow f(-x) = \frac{x^2}{1-x^2} = f(x)$$

$$\Rightarrow f'(x) = \frac{2x}{(1-x^2)^2}$$

$\therefore$ f(x) increases in $x \in (0, \infty)$

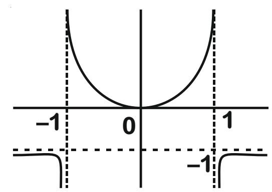
Also f(0) = 0

$\lim\limits_{x \to \pm\infty} f(x) = -1$

and F(x) is even function

$\therefore$ Set $A \rightarrow R - [-1, 0)$

$\therefore$ Graph of function



30. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is :

(1) $\dfrac{7}{32}$ (2) $\dfrac{25}{192}$

(3) $\dfrac{1}{192}$ (4) $\dfrac{25}{32}$

Answer (4)

Sol. P(at least one) = 1 – P(none)

$$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{7}{8}$$

$$= 1 - \frac{7}{32} = \boxed{\frac{25}{32}}$$

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