FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Sunday 26th June, 2022) **TIME : 3:00 PM to 06:00 PM**

PHYSICS | TEST PAPER WITH SOLUTION

SECTION-A

1. The dimension of mutual inductance is :

(A) $[ML^2T^{-2}A^{-1}]$ (B) $[ML^2T^{-3}A^{-1}]$

(C) $[ML^2T^{-2}A^{-2}]$ (D) $[ML^2T^{-3}A^{-2}]$

Official Ans. by NTA (C)

Sol. e_2 : induced emf in secondary coil

i_1 : Current in primary coil

M : Mutual inductance

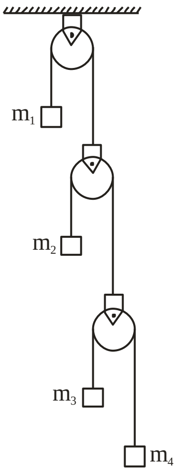
$$e_2 = -M\frac{di_1}{dt}$$

$$M = -\frac{e_2}{\frac{di_1}{dt}}$$

$$[M] = \frac{[e_2]}{\left[\frac{di_1}{dt}\right]} = \frac{\left[\frac{W}{q}\right]}{\left[\frac{di_1}{dt}\right]} = \frac{\frac{[ML^2T^{-2}]}{[AT]}}{[AT^{-1}]}$$

$= [ML^2T^{-2}A^{-2}]$

2. In the arrangement shown in figure a_1, a_2, a_3 and a_4 are the accelerations of masses m_1, m_2, m_3 and m_4 respectively. Which of the following relation is true for this arrangement?



(A) $4a_1 + 2a_2 + a_3 + a_4 = 0$

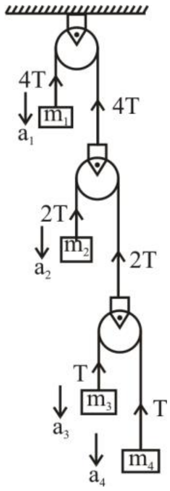
(B) $a_1 + 4a_2 + 3a_3 + a_4 = 0$

(C) $a_1 + 4a_2 + 3a_3 + 2a_4 = 0$

(D) $2a_1 + 2a_2 + 3a_3 + a_4 = 0$

Official Ans. by NTA (A)

Sol.



Using costraint

$$\sum \vec{T}.\vec{a} = 0$$

$-4Ta_1 - 2Ta_2 - Ta_3 - Ta_4 = 0$

$4a_1 + 2a_2 + a_3 + a_4 = 0$

3. Arrange the four graphs in descending order of total work done; where W_1, W_2, W_3 and W_4 are the work done corresponding to figure a, b, c and d respectively.

Figure-a

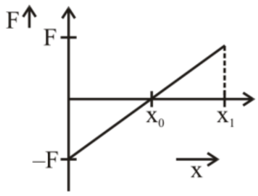


Figure-b

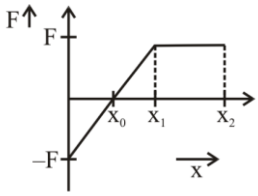


Figure-c

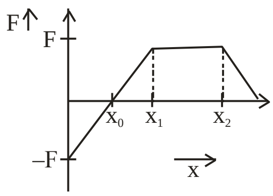
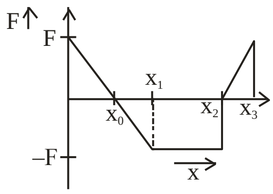


Figure-d



(A) $W_3 > W_2 > W_1 > W_4$

(B) $W_3 > W_2 > W_4 > W_1$

(C) $W_2 > W_3 > W_4 > W_1$

(D) $W_2 > W_3 > W_1 > W_4$

Official Ans. by NTA (A)

Sol. Work done = area under F – x curve. Area below x –axis is negative & area above x-axis is positive.

so

$W_3 > W_2 > W_1 > W_4$

4. Solid spherical ball is rolling on a frictionless horizontal plane surface about its axis of symmetry. The ratio of rotational kinetic energy of the ball to its total kinetic energy is :-

(A) $\frac{2}{5}$ (B) $\frac{2}{7}$ (C) $\frac{1}{5}$ (D) $\frac{7}{10}$

Official Ans. by NTA (B)

Sol. $K_{total} = K_{rotational} + K_{Translational}$

$$K_{total} = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mV_{cm}^2$$

$v_{cm} = R\omega$ for pure rolling

$$I_{cm} = \frac{2}{5}mR^2$$

$$K_{Rot} = \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}\times\frac{2}{5}mR^2\times\frac{v_{cm}^2}{R^2} = \frac{1}{5}mv_{cm}^2$$

$$K_{Total} = \frac{1}{5}mv_{cm}^2 + \frac{1}{2}mv_{cm}^2 = \frac{7}{10}mv_{cm}^2$$

$$\frac{K_{Rot}}{K_{Total}} = \frac{\frac{1}{5}mv_{cm}^2}{\frac{7}{10}mv_{cm}^2} = \frac{2}{7}$$

5. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : If we move from poles to equator, the direction of acceleration due to gravity of earth always points towards the center of earth without any variation in its magnitude.

Reason R : At equator, the direction of acceleration due to the gravity is towards the center of earth.

In the light of above statements, choose the correct answer from the options given below :

(A) Both A and R are true and R is the correct explanation of A.

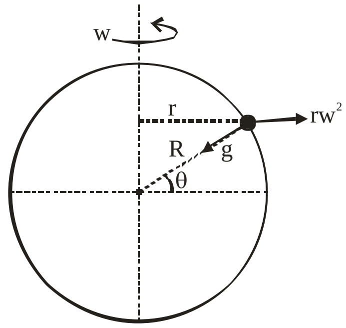
(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (D)

Sol.



Effective acceleration due to gravity is the resultant of g & rw^2 whose direction & magnitude depends upon θ. Hence assertion is false.

When θ = 0° (at equator), effective acceleration is radially inward.

6. If ρ is the density and η is coefficient of viscosity of fluid which flows with a speed v in the pipe of diameter d, the correct formula for Reynolds number R_e is :

(A) $R_e = \frac{\eta d}{\rho v}$ (B) $R_e = \frac{\rho v}{\eta d}$

(C) $R_e = \frac{\rho v d}{\eta}$ (D) $R_e = \frac{\eta}{\rho v d}$

Official Ans. by NTA (C)

Sol. Reynold's number is given by $\frac{\rho v d}{\eta}$

7. A flask contains argon and oxygen in the ratio of 3:2 in mass and the mixture is kept at 27°C. The ratio of their average kinetic energy per molecule respectively will be :

(A) 3 : 2 (B) 9 : 4

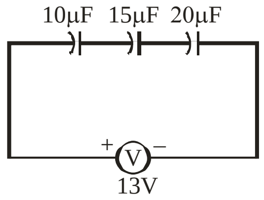
(C) 2 : 3 (D) 1 : 1

Official Ans. by NTA (D)

Sol. Average K.E./molecule $= \frac{f}{2}kT$

So, $\frac{K_{Ar}}{K_{O_2}} = \frac{\frac{3}{2}kT}{\frac{5}{2}kT} = \frac{3}{5}$

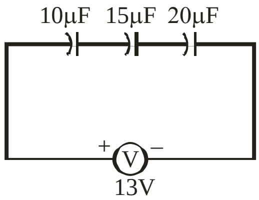
8. The charge on capacitor of capacitance 15μF in the figure given below is :



(A) 60μc (B) 130μc (C) 260 μc (D) 585 μc

Official Ans. by NTA (A)

Sol.



$$\frac{1}{C_{eq}}=\frac{1}{10}+\frac{1}{15}+\frac{1}{20}=\frac{12+8+6}{120}=\frac{26}{120}$$

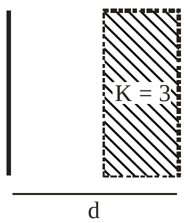
$$C_{eq}=\frac{60}{13}\mu F$$

$$Q=\frac{13\times 60}{13}=60\mu C$$

Charge on each capacitor is same

∵ they are in series.

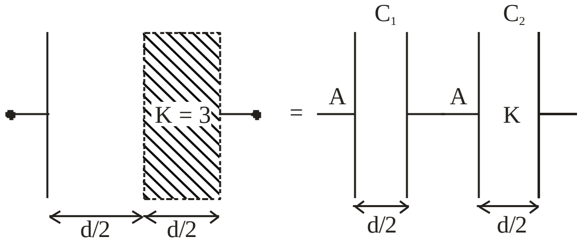
9. A parallel plate capacitor with plate area A and plate separation d=2 m has a capacitance of 4 μF. The new capacitance of the system if half of the space between them is filled with a dielectric material of dielectric constant K=3 (as shown in figure) will be :



(A) 2μF (B) 32μF (C) 6μF (D) 8μF

Official Ans. by NTA (C)

Sol. $C_{original}=\frac{A\varepsilon_0}{d}$



$$C_1=\frac{A\varepsilon_0}{d/2}=\frac{2A\varepsilon_0}{d}=C$$

$$C_2=\frac{KA\varepsilon_0}{d/2}=\frac{2KA\varepsilon_0}{d}=\frac{6A\varepsilon_0}{d}=3C$$

C_1 & C_2 are in series

$$C_{new}=\frac{C_1C_2}{C_1+C_2}=\frac{C\times 3C}{C+3C}=\frac{3C}{4}$$

$$=\frac{3}{4}\times\frac{2A\varepsilon_0}{d}=\frac{3}{2}\times\frac{A\varepsilon_0}{d}$$

$$C_{new}=\frac{3}{2}C_{original}$$

$$=\frac{3}{2}\times 4=6\mu F$$

10. Sixty four conducting drops each of radius 0.02 m and each carrying a charge of 5 μC are combined to form a bigger drop. The ratio of surface density of bigger drop to the smaller drop will be :

(A) 1 : 4 (B) 4 : 1 (C) 1 : 8 (D) 8 : 1

Official Ans. by NTA (B)

Sol. Let R = radius of combined drop

r = radius of smaller drop

Volume will remain same

$$\frac{4}{3}\pi R^3=64\times\frac{4}{3}\pi r^3$$

R = 4r

Q = 64q ;

q : charge of smaller drop

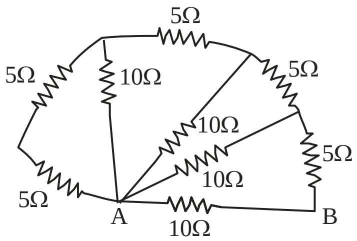
Q : Charge of combined drop

$$\frac{\sigma_{bigger}}{\sigma_{smaller}}=\frac{\frac{Q}{4\pi R^2}}{\frac{q}{4\pi r^2}}=\frac{Q}{q}\cdot\frac{r^2}{R^2}$$

$$=64\frac{r^2}{16r^2}=4$$

$$\frac{\sigma_{bigger}}{\sigma_{smaller}}=\frac{4}{1}$$

11. The equivalent resistance between points A and B in the given network is :

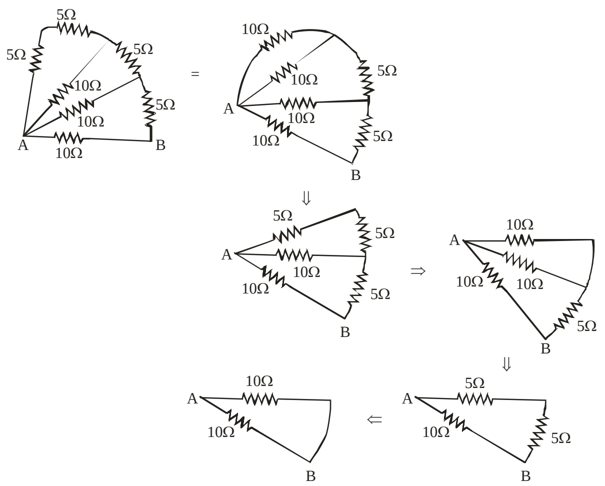


(A) 65Ω (B) 20Ω

(C) 5Ω (D) 2Ω

Official Ans. by NTA (C)

Sol.



$R_{AB} = 5\Omega$

12. A bar magnet having a magnetic moment of $2.0 \times 10^5\ JT^{-1}$, is placed along the direction of uniform magnetic field of magnitude $B = 14 \times 10^{-5}$ T. The work done in rotating the magnet slowly through 60° from the direction of field is :

(A) 14 J (B) 8.4 J (C) 4 J (D) 1.4 J

Official Ans. by NTA (A)

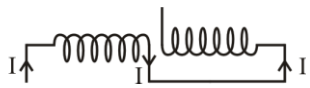
Sol. Work done = MB ($\cos\theta_1 - \cos\theta_2$)

$\theta_1 = 0°, \theta_2 = 60°$

$= 2 \times 10^5 \times 14 \times 10^{-5} (1 - 1/2)$

= 14 J

13. Two coils of self inductance L_1 and L_2 are connected in series combination having mutual inductance of the coils as M. The equivalent self inductance of the combination will be :



(A) $\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{M}$ (B) $L_1 + L_2 + M$

(C) $L_1 + L_2 + 2M$ (D) $L_1 + L_2 - 2M$

Official Ans. by NTA (D)

Sol. Current on both the inductor is in opposite direction.

Hence :

$L_{eq} = L_1 + L_2 - 2M$

14. A metallic conductor of length 1m rotates in a vertical plane parallel to east-west direction about one of its end with angular velocity 5 rad/s. If the horizontal component of earth's magnetic field is 0.2×10^{-4} T, then emf induced between the two ends of the conductor is :

(A) 5μV (B) 50μV (C) 5mV (D) 50mV

Official Ans. by NTA (B)

Sol. emf induced between the two ends = $\frac{B_H \omega l^2}{2}$

$\frac{0.2 \times 10^{-4} \times 5 \times 1}{2} = 0.5 \times 10^{-4} = 50 \times 10^{-6}\ V = 50\mu V$

15. Which is the correct ascending order of wavelengths?

(A) $\lambda_{visible} < \lambda_{X-ray} < \lambda_{gamma-ray} < \lambda_{microwave}$

(B) $\lambda_{gamma-ray} < \lambda_{X-ray} < \lambda_{visible} < \lambda_{microwave}$

(C) $\lambda_{X-ray} < \lambda_{gamma-ray} < \lambda_{visible} < \lambda_{microwave}$

(D) $\lambda_{microwave} < \lambda_{visible} < \lambda_{gamma-ray} < \lambda_{X-ray}$

Official Ans. by NTA (B)

Sol. From electromagnetic wave spectrum.

λ increases $\longrightarrow$

γ-ray	x-rays	ultra violet	visible	infrared	microwave	Radio wave

$\lambda_{\text{gamma-ray}} < \lambda_{X\text{-ray}} < \lambda_{\text{visible}} < \lambda_{\text{microwave}}$

16. For a specific wavelength 670 nm of light coming from a galaxy moving with velocity v, the observed wavelength is 670.7 nm.

The value of v is :

(A) $3 \times 10^8\ ms^{-1}$ (B) $3 \times 10^{10}\ ms^{-1}$

(C) $3.13 \times 10^5\ ms^{-1}$ (D) $4.48 \times 10^5\ ms^{-1}$

Official Ans. by NTA (C)

Sol. $\lambda_{\text{emitted}} = 670$ nm

$\lambda_{\text{obs}} = 670.7$ nm

$v = ?$

$c = 3 \times 10^8$ m/s

If $v << c$

$$\frac{\lambda_{\text{obs}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}} = \frac{v}{c}$$

$$\frac{670.7 - 670}{670} = \frac{v}{c}$$

$V = 3.13 \times 10^5$ m/s

17. A metal surface is illuminated by a radiation of wavelength 4500 Å. The ejected photo-electron enters a constant magnetic field of 2 mT making an angle of 90° with the magnetic field. If it starts revolving in a circular path of radius 2 mm, the work function of the metal is approximately :

(A) 1.36 eV (B) 1.69 eV (C) 2.78 eV (D) 2.23 eV

Official Ans. by NTA (A)

Sol. $\lambda = 4500$ Å

$B = 2mT$, $R = 2mm$

$$R = \frac{\sqrt{2Km}}{qB}$$

$$\frac{(qBR)^2}{2m} = K$$

$$\frac{\left(1.6\times10^{-19}\times2\times10^{-3}\times2\times10^{-3}\right)^2}{2\times9.1\times10^{-31}} = K$$

$$\frac{(6.4)^2}{2\times9.1}\times\frac{10^{-50}}{10^{-31}} = K$$

$K = 2.25 \times 10^{-19}$ J

$$= \frac{2.25\times10^{-19}}{1.6\times10^{-19}}\text{eV} = 1.40\text{eV}$$

$$E = \frac{12400}{4500} = 2.76\text{eV}$$

$\phi = E - K = (2.76 - 1.40)$ eV $= 1.36$ eV

18. A radioactive nucleus can decay by two different processes. Half-life for the first process is 3.0 hours while it is 4.5 hours for the second process. The effective half- life of the nucleus will be :

(A) 3.75 hours (B) 0.56 hours

(C) 0.26 hours (D) 1.80 hours

Official Ans. by NTA (D)

Sol. $\lambda_{eq} = \lambda_1 + \lambda_2$

$$\frac{\ln 2}{(t_{1/2})_{eq}} = \frac{\ln 2}{(t_{1/2})_1} + \frac{\ln 2}{(t_{1/2})_2}$$

$$(t_{1/2})_{eq} = \frac{(t_{1/2})_1 \times (t_{1/2})_2}{(t_{1/2})_1 + (t_{1/2})_2}$$

$$= \frac{3\times4.5}{3+4.5} = \frac{3\times4.5}{7.5} = \frac{3\times3}{5} = 1.8\text{ hr}$$

19. The positive feedback is required by an amplifier to act an oscillator. The feedback here means :

(A) External input is necessary to sustain ac signal in output.

(B) A portion of the output power is returned back to the input.

(C) Feedback can be achieved by LR network.

(D) The base-collector junction must be forward biased.

Official Ans. by NTA (B)

Sol. When the amplifier connects with positive feedback, it acts as the oscillator the feedback here is positive feedback which means some amount of voltage is given to the input.

20. A sinusoidal wave $y(t) = 40\sin(10 \times 10^6\ \pi t)$ is amplitude modulated by another sinusoidal wave $x(t) = 20\sin(1000\pi t)$. The amplitude of minimum frequency component of modulated signal is :

(A) 0.5 (B) 0.25 (C) 20 (D) 10

Official Ans. by NTA (D)

Sol. $y(t) = 40 \sin(10 \times 10^6\ \pi t)$

$x(t) = 20\sin(1000\pi t)$

$\Rightarrow \omega_c = 10^7\ \pi$

$\omega_m = 10^3\ \pi$

$A_C = 40$

$A_m = 20$

Equation of modulated wave = $(A_C + A_m \sin \omega_m t) \sin \omega_c t$

$$= A_c\left(1+\frac{A_m}{A_c}\sin\omega_m t\right)\sin\omega_c t$$

$$= A_c(1 + \mu \sin \omega_m t)\sin \omega_c t, \qquad \mu = \frac{A_m}{A_c}$$

$$= A_c \sin\omega_c t + \frac{\mu A_c}{2}\left[\cos(\omega_c-\omega_m)t - \cos(\omega_c+\omega_m)t\right]$$

Amplitude of minimum frequency = $\frac{\mu A_c}{2} = \frac{A_m}{A_c}\times\frac{A_c}{2} = \frac{A_m}{2} = 10$

SECTION-B

1. A ball is projected vertically upward with an initial velocity of 50 ms^{-1} at t = 0s. At t = 2s. another ball is projected vertically upward with same velocity. At t =________s, second ball will meet the first ball ($g = 10\ ms^{-2}$).

Official Ans. by NTA (6)

Sol. Let they meet at t = t

So first ball gets t sec.

& 2nd gets (t – 2) sec. & they will meet at same height

$$h_1 = 50t - \frac{1}{2}gt^2$$

$$h_2 = 50(t-2) - \frac{1}{2}g(t-2)^2$$

$h_1 = h_2$

$$50t - \frac{1}{2}gt^2 = 50(t-2) - \frac{1}{2}g(t-2)^2$$

$$100 = \frac{1}{2}g\left[t^2 - (t-2)^2\right]$$

$$100 = \frac{10}{2}[4t-4]$$

$5 = t - 1$

t = 6 sec.

2. A batsman hits back a ball of mass 0.4 kg straight in the direction of the bowler without changing its initial speed of 15 ms^{-1}. The impulse imparted to the ball is _______________Ns.

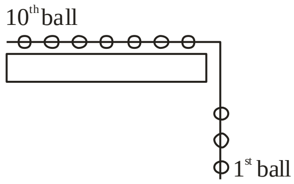
Official Ans. by NTA (12)

Sol. Impulse = change in momentum

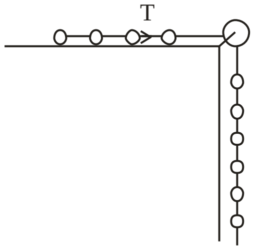
= m[v – (–v)] = 2 mv

= 2 × 0.4 × 15 = 12 Ns

3. A system to 10 balls each of mass 2 kg are connected via massless and unstretchable string. The system is allowed to slip over the edge of a smooth table as shown in figure. Tension on the string between the 7th and 8th ball is_____________N when 6th ball just leaves the table.



Official Ans. by NTA (36)

Sol. 

$$a = \frac{6mg}{10m} = \frac{6g}{10} = \frac{3g}{5}$$

taking 8,9,10 together 

T = 3 ma

$$= 3m \times \frac{3g}{5}$$

= 36 N

4. A geyser heats water flowing at a rate of 2.0 kg per minute from 30°C to 70°C. If geyser operates on a gas burner, the rate of combustion of fuel will be ___________ g min^{-1}

[Heat of combustion = 8 × 10^3 Jg^{-1}

Specific heat of water = 4.2 Jg^{-1} °C^{-1}]

Official Ans. by NTA (42)

Sol. m = 2000 gm/min

Heat required by water/min = $mS\Delta T$

= (2000) × 4.2 × 40 J/min

= 336000 J/min

The rate of combustion = $\left(\frac{dm}{dt}L\right) = 336000 J/min$

$$\frac{dm}{dt} = \frac{336000}{8 \times 10^3} g/min$$

= 42 gm/min

5. A heat engine operates with the cold reservoir at temperature 324 K.

The minimum temperature of the hot reservoir, if the heat engine takes 300 J heat from the hot reservoir and delivers 180 J heat to the cold reservoir per cycle, is ___________K.

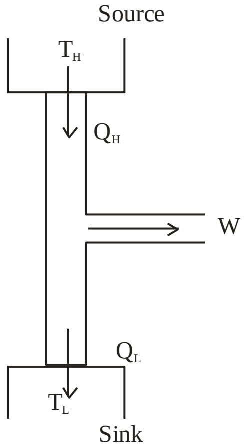
Official Ans. by NTA (540)

Sol. T_c = 324 k

T_H = ?

Q_H = 300 J

Q_L = 180 J



$$1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$$

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$$

$$T_H = \frac{Q_H}{Q_L} \times T_L = \frac{300}{180} \times 324 = 540 K$$

6. A set of 20 tuning forks is arranged in a series of increasing frequencies. If each fork gives 4 beats with respect to the preceding fork and the frequency of the last fork is twice the frequency of the first, then the frequency of last fork is_____Hz.

Official Ans. by NTA (152)

)

Sol. $f_1 = f$

$f_2 = f + 4$

$f_3 = f + 2 \times 4$

$f_4 = f + 3 \times 4$

$f_{20} = f + 19 \times 4$

$f + (19 \times 4) = 2 \times f$

$f = 76$ Hz.

Frequency of last tuning forks = 2f

= 152 Hz

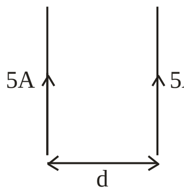
7. Two 10 cm long, straight wires, each carrying a current of 5A are kept parallel to each other. If each wire experienced a force of 10^{-5} N, then separation between the wires is _________cm.

Official Ans. by NTA (5)

Sol. It should be mentioned, 10 cm wire is part of long wire.

Force experienced by unit length of wire

$$= \frac{\mu_0 I_1 I_2}{2\pi d}, \ I_1 = I_2 = 5A$$



Force experienced by wires of length 10 cm

$$= \frac{\mu_0 I_1 I_2}{2\pi d} \times 10 \times 10^{-2}$$

$$10^{-5} = \frac{2 \times 10^{-7} \times 5 \times 5}{d} \times 10 \times 10^{-2}$$

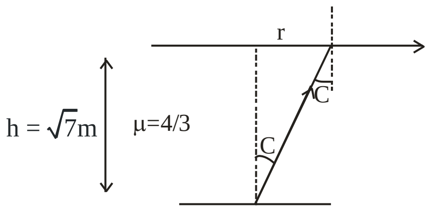
$d = 50 \times 10^{-3}$ m

$d = 50 \times 10^{-1}$ cm = 5 cm.

8. A small bulb is placed at the bottom of a tank containing water to a depth of $\sqrt{7}$ m. The refractive index of water is $\frac{4}{3}$. The area of the surface of water through which light from the bulb can emerge out is $x\pi$ m^2. The value of x is_______.

Official Ans. by NTA (9)

Sol. C : Criticle angle



$$\tan C = \frac{r}{h}$$

r = h tan C

$$\sin C = \frac{1}{\mu} = \frac{3}{4}$$

$$\tan C = \frac{3}{\sqrt{7}}$$

$$r = \sqrt{7} \times \frac{3}{\sqrt{7}} = 3$$

Area of surface = $\pi r^2 = 9\pi m^2$

9. A travelling microscope is used to determine the refractive index of a glass slab. If 40 divisions are there in 1 cm on main scale and 50 Vernier scale divisions are equal to 49 main scale divisions, then least count of the travelling microscope is ______$\times 10^{-6}$ m.

Official Ans. by NTA (5)

Sol. 50 VSD = 49 MSD

$$1VSD = \frac{49}{50} MSD$$

Least count = 1 MSD – 1 VSD

$$= \left(1 - \frac{49}{50}\right) MSD = \frac{1}{50} MSD$$

$$1MSD = \frac{1}{40} cm$$

$$\text{Least count} = \frac{1}{50 \times 40} \text{ cm}$$

$$= \frac{1}{2000} \text{ cm} = \frac{1}{2} \times 10^{-5} \text{m}$$

$= 0.5 \times 10^{-5}$ m

$= 5 \times 10^{-6}$ m

10. The stopping potential for photoelectrons emitted from a surface illuminated by light of wavelength 6630 Å is 0.42 V. If the threshold frequency is $x \times 10^{13}$/s, where x is ______ (nearest integer).

(Given, speed light = 3×10^8 m/s, Planck's constant = 6.63×10^{-34} Js)

Official Ans. by NTA (35)

Sol. Stopping potential $V_0 = 0.42$ V

$\lambda = 6630$ Å

$E = \phi + eV_0$

E : energy of incident photon

V_0 : Stopping potential

$\phi = E - eV_0$

$$E = \frac{12400}{6630} eV = 1.87 \text{ eV}$$

$\phi = (1.87 - 0.42) = 1.45$ eV

$\phi = h\nu_0$; ν_0 : threshold frequency

$1.45 \times 1.6 \times 10^{-19} = 6.63 \times 10^{-34} \times \nu_0$

$\nu_0 = 0.35 \times 10^{15}$

$= 35 \times 10^{13}$ sec^{-1}

= 35

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Sunday 26th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY **TEST PAPER WITH SOLUTION**

SECTION-A

1. The number of radial and angular nodes in 4d orbital are. respectively

(A) 1 and 2 (B) 3 and 2

(C) 1 and 0 (D) 2 and 1

Official Ans. by NTA (A)

Sol. Radial node = n – l – 1

= 4 – 2 – 1

= 1

Angular node (l) = 2

2. Match List I with List II.

List I Enzyme	List II Conversion of
A. Invertase	I. Starch into maltose
B. Zymase	II. Maltose into glucose
C. Diastase	III. Glucose into ethanol
D. Maltase	IV. Cane sugar into glucose

Choose the most appropriate answer from the options given below :

(A) A-III, B-IV. C-II. D-I

(B) A-III. B-II. C-I. D-IV

(C) A-IV, B-IIL C-I. D-II

(D) A-IV, B-II. C-III. D-I

Official Ans. by NTA (C)

Sol. Invertase : Cane sugar → Glucose and fructose

Zymase : Glucose → Ethanol and CO_2

Diastase : Starch → Maltose

Maltase : Maltose → Glucose

3. Which of the following elements in considered as a metalloid?

(A) Sc (B) Pb (C) Bi (D) Te

Official Ans. by NTA (D)

Sol. Sc, Pb, Bi are metals

Te is a metalloid

4. The role of depressants in Froth Flotation method* is to

(A) selectively prevent one component of the ore from coming to the froth.

(B) reduce the consumption of oil for froth formation.

(C) stabilize the froth.

(D) enhance non-wettability of the mineral particles.

Official Ans. by NTA (A)

Sol. Depressant prevent one component from coming to the froth.

For eg., in Galena ore, the depressant (NaCN) prevents impurity (ZnS) from coming to the froth.

5. Boiling of hard water is helpful in removing the temporary hardness by converting calcium hydrogen carbonate and magnesium hydrogen carbonate to

(A) $CaCO_3$ and $Mg(OH)_2$

(B) $CaCO_3$ and M_2CO_3

(C) $Ca(OH)_2$ and $MgCO_3$

(D) $Ca(OH)_2$ and $Mg(OH)_2$

Official Ans. by NTA (A)

Sol. $Mg(HCO_3)_2 \xrightarrow{\text{Boil}} Mg(OH)_2 + 2CO_2\uparrow$

$Ca(HCO_3)_2 \xrightarrow{\text{Boil}} CaCO_3 + H_2O + CO_2\uparrow$

6. s-block element which cannot be qualitatively confirmed by the flame test is

(A) Li (B) Na (C) Rb (D) Be

Official Ans. by NTA (D)

Sol.

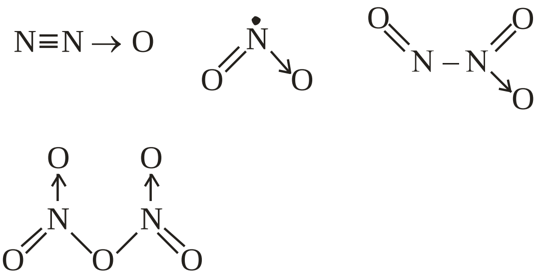
	Flame color
Li	Crimson Red
Na	Yellow
Rb	Red violet
Be	No color

7. The oxide which contains an odd electron at the nitrogen atom is

(A) N_2O (B) NO_2 (C) N_2O_3 (D) N_2O_5

Official Ans. by NTA (B)

Sol.



8. Which one of the following is an example of disproportionation reaction?

(A) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

(B) $MnO_4^{2-} + 4H^+ + 4e^- \rightarrow MnO_2 + 2H_2O$

(C) $10I^- + 2MnO_4^- + 16H^+ \rightarrow 2Mn^{2+} + 8H_2O + 5I_2$

(D) $8MnO_4^- + 3S_2O_3^{2-} + H_2O \rightarrow 8MnO_2 + 6SO_4^{2-} + 2OH^-$

Official Ans. by NTA (A)

Sol. Reduction

$$3\,\overset{+6}{Mn}O_4^{2-} + 4H^+ \rightarrow 2\,\overset{+7}{Mn}O_4^- + \overset{+4}{Mn}O_2 + 2H_2O$$

Oxidation

9. The most common oxidation state of Lanthanoid elements is +3. Which of the following is likely to deviate easily from +3 oxidation state?

(A) Ce (At. No. 58) (B) La (At. No. 57)

(C) Lu (At. No. 71) (D) Gd (At. No. 64)

Official Ans. by NTA (A)

Sol. Ce = [Xe] $4f^1\,5d^1\,6s^2$

Ce^{3+} = [Xe] $4f^1\,5d^0$

Ce^{+4} = [Xe] $4f^0\,5d^0$ (Noble gas configuration)

10. The measured BOD values for four different water samples (A-D) are as follows:

A = 3 ppm: B=18 ppm: C-21 ppm: D=4 ppm. The water samples which can be called as highly polluted with organic wastes, are

(A) A and B (B) A and D

(C) B and C (D) B and D

Official Ans. by NTA (C)

Sol. Clean water $\longrightarrow$ B.O.D. < 5 ppm

Highly polluted water $\longrightarrow$ B.O.D. > 17 ppm

11. The correct order of nucleophilicity is

(A) $F^- > OH^-$ (B) $H_2\ddot{\underset{..}{O}} > OH^-$

(C) $R\ddot{\underset{..}{O}}H > RO^-$ (D) $NH_2^- > NH_3$

Official Ans. by NTA (D)

Sol. Nucleophilicity $\propto$ electro density on donor atom

$\propto$ size of donor atom (in gas)

$\propto \dfrac{1}{\text{EN of atom}}$ (for period)

12. Oxidation of toluene to Benzaldehyde can be easily carried out with which of the following reagents?

(A) CrO_3/acetic acid, H_3O^+

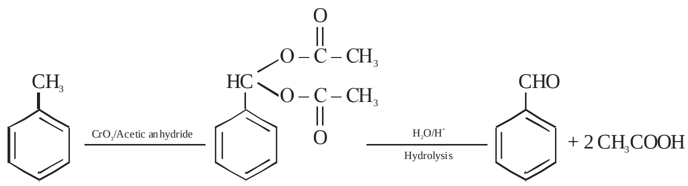
(B) CrO_3/acetic anhydride, H_3O^+

(C) $KMnO_4$/HCl, H_3O^+

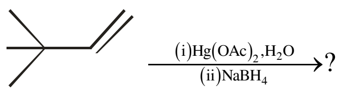
(D) CO/HCl, anhydrous $AlCl_3$

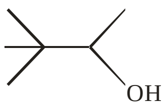
Official Ans. by NTA (B)

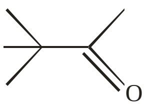
Sol.

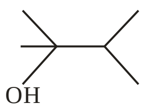


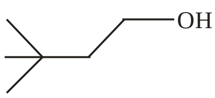
13. The major product in the following reaction



(A) 

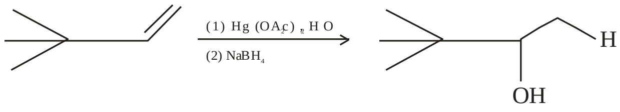
(B) 

(C) 

(D) 

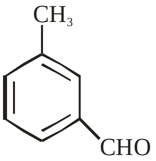
Official Ans. by NTA (A)

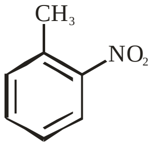
Sol.

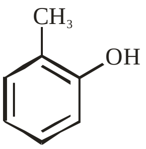


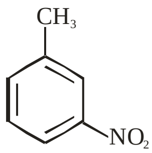
Oxymercuration – Demercuration
Addition of H_2O
Markovnikov's addition without rearrangement

14. Halogenation of which one of the following will yield m-substituted product with respect to methyl group as a major product?

(A) 

(B) 

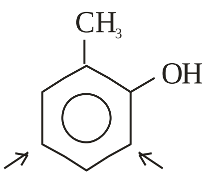
(C) 

(D) 

Official Ans. by NTA (C)

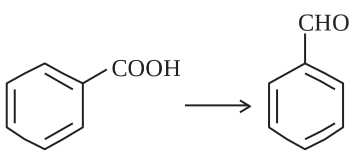
Sol. Electrophile will attack at ortho and para position with respect to better electron releasing group (ERG)

ERG : – OH > – CH_3



Para position with respect to – OH (+R) group and it will be meta position with respect to – CH_3 group.

15. The reagent, from the following, which converts benzoic acid to benzaldehyde in one step is



(A) $LiAlH_4$ (B) $KMnO_4$

(C) MnO (D) $NaBH_4$

Official Ans. by NTA (C)

Sol.

$$C_6H_5-\boxed{\overset{O}{\overset{\|}{C}}-OH+HO}-\overset{O}{\overset{\|}{C}}-C_6H_5 \xrightarrow[\Delta]{MnO} C_6H_5-\overset{O}{\overset{\|}{C}}-C_6H_5 + CO_2 + H_2O$$

$$C_6H_5-\boxed{\overset{O}{\overset{\|}{C}}-OH+HO}-\overset{O}{\overset{\|}{C}}-H \xrightarrow[\Delta]{MnO} C_6H_5-\overset{O}{\overset{\|}{C}}-H + CO_2 + H_2O$$

16. The final product 'A' in the following reaction sequence

$$CH_3CH_2-\underset{\underset{O}{\|}}{C}-CH_3 \xrightarrow{HCN} ? \xrightarrow[\text{Heat}]{95\%\ H_2SO_4} A$$

(A) $CH_3-CH=\overset{\overset{CH_3}{|}}{C}-COOH$

(B) $CH_3-CH=\underset{\underset{CH_3}{|}}{C}-CN$

(C) $CH_3-CH-\overset{\overset{OH}{|}}{\underset{\underset{CH_3}{|}}{C}}-COOH$

(D) $CH_3-CH=\underset{\underset{CH_3}{|}}{C}-CONH_2$

Official Ans. by NTA (A)

Sol.

$$CH_3CH_2-\overset{\overset{O}{\|}}{C}-CH_3 \xrightarrow{HCN} CH_3CH_2-\overset{\overset{CN}{|}}{\underset{\underset{OH}{|}}{C}}-CH_3 \xrightarrow[\text{Heat}]{95\%\ H_2SO_4}$$

$$CH_3-CH=\overset{\overset{CH_3}{|}}{C}-COOH$$

17. Which statement is NOT correct for p-toluenesulphonyl chloride?

(A) It is known as Hinsberg's reagent.

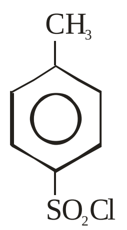
(B) It is used to distinguish primary and secondary amines.

(C) On treatment with secondary amine, it leads to a product, that is soluble in alkali.

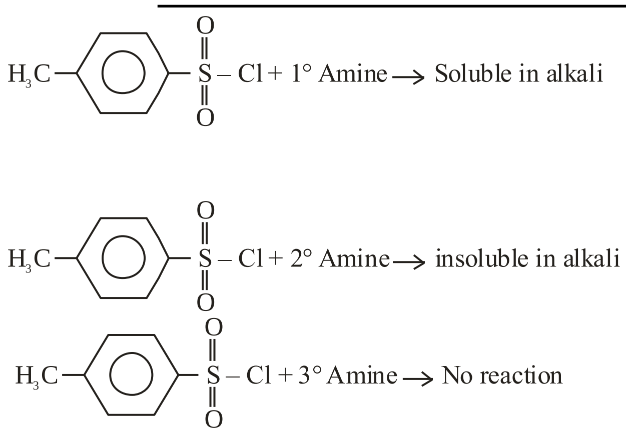
(D) It doesn't react with tertiary amines.

Official Ans. by NTA (C)

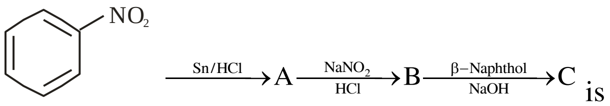
Sol.



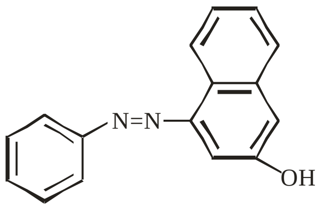
Hinsberg's reagent

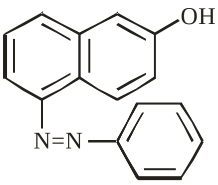


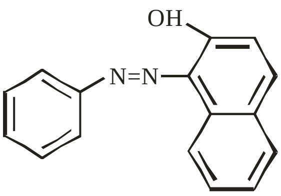
18. The final product 'C' is the following series series of reactions

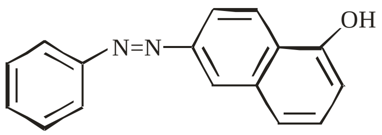


is

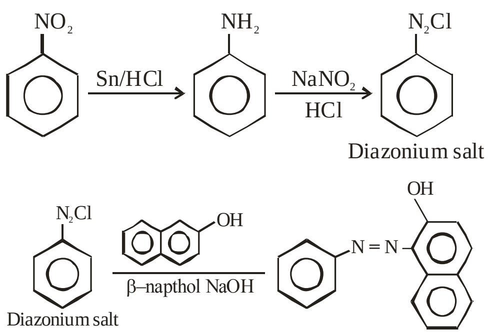
(A)


(B)


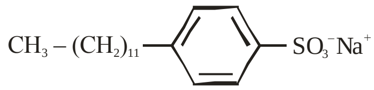
(C)


(D)


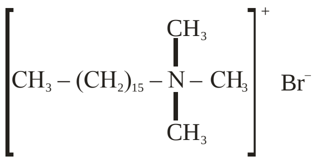
Official Ans. by NTA (C)

Sol.


19. Which of the following is NOT an example of synthetic detergent?

(A)


(B) $CH_3 – (CH_2)_{16} –COO^– Na^+$

(C)


(D) $CH_3(CH_2)_{16}COO(CH_2CH_2O)_nCH_2CH_2OH$

Official Ans. by NTA (B)

Sol. Refer NCERT (Page No. 452)

20. Which one of the following is a water soluble vitamin, that is not excreted easily?

(A) Vitamin B_2 (B) Vitamin B_1

(C) Vitamin B_6 (D) Vitamin B_{12}

Official Ans. by NTA (D)

Sol. Refer NCERT (Page No. 426)

SECTION-B

1. CNG is an important transportation fuel. When 100 g CNG is mixed with 208 oxygen in vehicles, it leads to the formation of CO_2 and H_2O and produces large quantity of heat during this combustion, then the amount of carbon dioxide, produced in grams is________. [nearest integer]

[Assume CNG to be methane]

Official Ans. by NTA (143)

Sol. $$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

Mole $\frac{100}{16}$ $\frac{208}{32}$

$= 6.25$ $= 6.5$

$$\frac{\text{Mole}}{\text{Stoi. Coeff.}} \quad \frac{6.25}{1} \quad \frac{6.5}{2} = 3.25$$

So, O_2 is limiting reagent

<u>Mole – Mole analysis</u>

$$\frac{n_{O_2}}{2} = \frac{n_{co_2}}{1}$$

$$\frac{6.5}{2} = n_{co_2}$$

Mass of $CO_2 = \frac{6.5}{2} \times 44 = 143$ gm

2. In a solid AB. A atoms are in ccp arrangement and B atoms occupy all the octahedral sites. If two atoms from the opposite faces are removed, then the resultant stoichiometry of the compound is A_xB_y. The value of x is_______. [nearest integer]

Official Ans. by NTA (3)

Sol. $A \rightarrow 4 - \left(2 \times \frac{1}{2}\right) = 3$

$B \rightarrow 12 \times \frac{1}{4} + 1 \times 1 = 4$

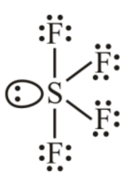
So, Compound is A_3B_4

The value of x is 3.

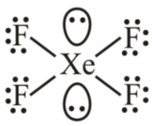
3. Amongst SF_4, XeF_4, CF_4 and H_2O, the number of species with two lone pairs of electrons _______ .

Official Ans. by NTA (2)

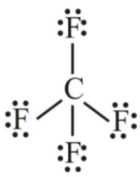
Sol.



Total lone pairs = 13

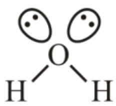


Total lone pairs = 14



Total lone pairs = 12

Total lone pairs = 2



4. A fish swimming in water body when taken out from the water body is covered with a film of water of weight 36 g. When it is subjected to cooking at 100°C, then the internal energy for vaporization in kJ mol^{-1} is _________.

[nearest integer]

[Assume steam to be an ideal gas. Given $A_{vap}H^{\ominus}$ for water at 373 K and 1 bar is 41.1 kJ mol^{-1} ; R = 8.31 $JK^{-1}mol^{-1}$]

Official Ans. by NTA (38)

Sol. $H_2O\ (l) \rightarrow H_2O(g)$

$$n = \frac{36}{18} = 2 \text{ mol}$$

$$\Delta U = \Delta H - \Delta n_g RT$$

$$= 41.1 - \frac{1 \times 8.31 \times 373}{1000} \text{kJ/mol}$$

$= 38$ kJ/mol

5. The osmotic pressure exerted by a solution prepared by dissolving 2.0 g of protein of molar mass 60 kg mol^{-1} in 200 mL of water at 27°C is _______ Pa. [integer value]

(use R = 0.083 L bar mol^{-1} K^{-1})

Official Ans. by NTA (415)

Sol. $\pi = iCRT$

$$= \frac{1 \times 2}{60000 \times 0.2} \times 0.083 \times 300$$

$= 0.00415$ bar $\quad (\because\ 1 \text{ bar} = 10^5 \text{ Pa})$

So, 0.00415×10^5 Pa = 415 Pa

6. 40° of HI undergoes decomposition to H_2 and I_2 at 300 K. $\Delta G^{\ominus}$ for this decompostion reaction at one atmosphere pressure is________ J mol^{-1}. [nearest integer]

(Use R = 8.31 J K^{-1} mol^{-1}; log 2 = 0.3010. In 10 = 2.3, log 3 = 0.477)

Official Ans. by NTA (2735)

Sol.

	HI	$\rightleftharpoons$	$\frac{1}{2}H_2$ +	$\frac{1}{2}I_2$
t_i	1			
teq	$1 - 0.4$		$\frac{0.4}{2}$	$\frac{0.4}{2}$

$$K_p = \frac{(0.2)^{\frac{1}{2}}(0.2)^{\frac{1}{2}}}{1 - 0.4} = \frac{0.2}{0.6} = \frac{1}{3}$$

$\Delta G = \Delta G° + RT \ \ln K = 0$

$\Delta G° = -RT \ \ln K \Rightarrow -8.31 \times 300 \times 2.3 \times \log\left(\frac{1}{3}\right)$

$= 2735$ J/mol

7. Cu(s) + Sn^{2+} (0.001M) $\rightarrow$ Cu^{2+} (0.01M) + Sn(s)

The Gibbs free energy change for the above reaction at 298 K is x $\times$ 10^{-1} kJ mol^{-1};

The value of x is______. [nearest integer]

$\left[\text{Given}: E^{\ominus}_{Cu^{2+}/Cu} = 0.34V; E^{\ominus}_{Sn^{2+}/Sn} = -0.14V; F = 96500C\ mol^{-1}\right]$

Official Ans. by NTA (983)

Sol. $Cu_{(s)}$ + Sn^{2+} (0.001 M) $\rightarrow$ Cu^{2+} (0.01 M) + $Sn_{(s)}$

$E°_{cell} = E°_{cathode} - E°_{anode}$

$= -0.14 - (0.34)$

$= -0.48$ V

$$E_{cell} = E°_{cell} - \frac{0.059}{2}\log\frac{[Cu^{2+}]}{[Sn^{2+}]}$$

$$= -0.48 - \frac{0.059}{2}\log\frac{0.01}{0.001}$$

$= -0.509$

$\Delta G = -nF\, E_{cell}$

$= -2 \times 96500 \times (-0.5095)$

$= 98333.5$ J/mol

$= 98.335$ kJ/mol

$= 983.35 \times 10^{-1}$ kJ/mol

Nearest Integer : 983

8. Catalyst A reduces the activation energy for a reaction by 10 kJ mol^{-1} at 300 K. The ratio of rate constants, $\frac{{}^{k}T,\text{Catalysed}}{{}^{k}T,\text{Uncatalysed}}$ is e^x. The value of x is _______. [nearest integer]

[Assume theat the pre-exponential factor is same in both the cases.

Given R = 8.31 J K^{-1} mol^{-1}]

Official Ans. by NTA (4)

Sol.

$$K = Ae^{\frac{-Ea}{RT}}$$

$$K_{cat} = Ae^{-\frac{E_a^{\,1}}{RT}}, \quad K_{uncat.} = Ae^{-\frac{Ea}{RT}}$$

$$\frac{K_{cat}}{K_{uncat.}} = e^{\frac{E_a - E_a^{\,1}}{RT}} = e^{\frac{10\times1000}{8.31\times300}} = e^{4.009} = e^x$$

$\therefore \ x = 4$

9. Reaction of $[Co(H_2O)_6]^{2+}$ with excess ammonia and in the presence of oxygen results into a diamagnetic product. Number of electrons present in t_{2g}–orbitals of the product is _______ .

Official Ans. by NTA (6)

Sol. $[Co(H_2O)_6]^{2+}$ +NH_3(excess) $\rightarrow$ $[Co(NH_3)_6]^{3+}$ + $6H_2O$

Diamagnetic

$\downarrow$

Low spin complex

$Co^{3+} \Rightarrow 3d^6\ 4s^0$

$\Rightarrow t_{2g}^{\ 6}\ e_g^{\ 0}$

Total number electrons = 6

10. The moles of methane required to produce 81 g of water after complete combustion is ______ $\times$ 10^{-2} mol. [nearest integer]

Official Ans. by NTA (225)

Sol. $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

POAC on H atom

$n_{CH4} \times 4 = n_{H2O} \times 2$

$$n_{CH_4} = \frac{81}{18} \times 2 \times \frac{1}{4} = \frac{81}{36}$$

$n_{CH_4} = 2.25$

$= 225 \times 10^{-2}$

Nearest Integers = 225

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Sunday 26th June, 2022) **TIME : 03 : 00 PM to 06 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let f : $\mathbb{R} \to \mathbb{R}$ be defined as f(x) = x-1 and $g : \mathbb{R} - \{1, -1\} \to \mathbb{R}$ be defined as $g(x) = \frac{x^2}{x^2-1}$.

Then the function fog is :

(A) one-one but not onto function

(B) onto but not one-one function

(C) both one-one and onto function

(D) neither one-one nor onto function

Official Ans. by NTA (D)

Sol. $f(x) = x - 1$; $g(x) = \frac{x^2}{x^2-1}$

$f(g(x)) = g(x) - 1$

$= \frac{x^2}{x^2-1} - 1 = \frac{x^2 - x^2 + 1}{x^2 - 1}$

$f(g(x)) = \frac{1}{x^2-1}$; $x \neq \pm 1$, even function

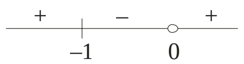
$\to$ Hence f(g(x)) is many one function

$y = \frac{1}{x^2-1}$

$y \cdot x^2 - y = 1$

$x^2 = \left(\frac{1+y}{y}\right)$

$\left(\frac{1+y}{y}\right) \geq 0$



Range:- $y \in (-\infty, -1] \cup (0, \infty)$

Hence, Range $\neq$ Co-domain $\Rightarrow$ f(g(x)) is into function

2. If the system of equations $\alpha x + y + z = 5$, $x + 2y + 3z = 4$, $x + 3y + 5z = \beta$,

has infinitely many solutions, then the ordered pair (α, β) is equal to :

(A) (1,–3) (B) (–1, 3)

(C) (1, 3) (D) (–1, –3)

Official Ans. by NTA (C)

For infinitely many solutions,

$\Delta = 0 = \Delta_x = \Delta_y = \Delta_z$

Sol. $\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix} = 0$

$\Rightarrow \alpha(10 - 9) - 1(5 - 3) + 1(3 - 2) = 0$

$\Rightarrow \alpha - 2 + 1 = 0$

$\Rightarrow \alpha = 1$

$\Delta_x = \begin{vmatrix} 5 & 1 & 1 \\ 4 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0$

$\Rightarrow 5(10 - 9) - 1(20 - 3\beta) + 1(12 - 2\beta)$

$\Rightarrow 5 - 20 + 3\beta + 12 - 2\beta$

$\Rightarrow -3 + \beta = 0$

$\Rightarrow \beta = 3$

3. If $A = \sum_{n=1}^{\infty} \frac{1}{\left(3+(-1)^n\right)^n}$ and $B = \sum_{n=1}^{\infty} \frac{(-1)^n}{\left(3+(-1)^n\right)^n}$, then $\frac{A}{B}$ is equal to :

(A) $\frac{11}{9}$ (B) 1 (C) $-\frac{11}{9}$ (D) $-\frac{11}{3}$

Official Ans. by NTA (C)

Sol. $A = \left(\frac{1}{2} + \frac{1}{4^2} + \frac{1}{2^3} + \frac{1}{4^4} + \ldots\ldots\infty\right)$

$A = \left(\frac{1}{2} + \frac{1}{2^3} + \ldots..\infty\right) + \left(\frac{1}{4^2} + \frac{1}{4^4} + \ldots\ldots\infty\right)$

$A = \left(\frac{\frac{1}{2}}{1-\frac{1}{4}} + \frac{\frac{1}{16}}{1-\frac{1}{16}}\right)$

$\Rightarrow A = \frac{1}{2} \times \frac{4}{3} + \frac{1}{16} \times \frac{16}{15} \Rightarrow A = \frac{11}{15}$

$B = \frac{-1}{2} + \frac{1}{4^2} + \frac{-1}{2^3} + \frac{1}{4^4} + \ldots..\infty$

$B = \left(\frac{-1}{2} + \frac{-1}{2^3} + \ldots.\infty\right) + \left(\frac{1}{4^2} + \frac{1}{4^4} + \ldots.\infty\right)$

$$B=\frac{-\frac{1}{2}}{1-\frac{1}{4}}+\frac{\frac{1}{16}}{1-\frac{1}{16}}$$

$$\Rightarrow B=-\frac{1}{2}\times\frac{4}{3}+\frac{1}{16}\times\frac{16}{15}$$

$$B=-\frac{9}{15}$$

$$\frac{A}{B}=\frac{11}{15}\times\frac{15}{(-9)}$$

$$\frac{A}{B}=-\frac{11}{9}$$

4. $\lim\limits_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$ is equal to :

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{12}$

Official Ans. by NTA (C)

Sol. $\lim\limits_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4};\left(\frac{0}{0}\right)$

$$\lim_{x\to 0}\left(\frac{2\cdot\sin\left(\frac{x+\sin x}{2}\right)\cdot\sin\left(\frac{x-\sin x}{2}\right)}{x^4}\right)$$

$$\lim_{x\to 0}2\left(\frac{\sin\left(\frac{x+\sin x}{2}\right)}{\left(\frac{x+\sin x}{2}\right)}\right)\left(\frac{\sin\left(\frac{x-\sin x}{2}\right)}{\left(\frac{x-\sin x}{2}\right)}\right)\left(\frac{\left(\frac{x+\sin x}{2}\right)}{x^4}\left(\frac{x-\sin x}{2}\right)\right)$$

$$\lim_{x\to 0}\left(\frac{x^2-\sin^2 x}{2x^4}\right):\left(\frac{0}{0}\right)$$

Apply L-Hopital Rule :

$$\lim_{x\to 0}\frac{2x-2\sin x\cos x}{2.4.x^3}$$

$\lim\limits_{x\to 0}\frac{2x-\sin 2x}{8x^3};\frac{0}{0}$: Again apply L-Hopital rule

$$\lim_{x\to 0}\frac{2-2\cos(2x)}{8(3)x^2}$$

$$\lim_{x\to 0}\frac{2(1-\cos(2x))}{24(4x^2)}\times 4\Rightarrow\frac{2}{24}\times\frac{1}{2}\times 4\Rightarrow\frac{1}{6}$$

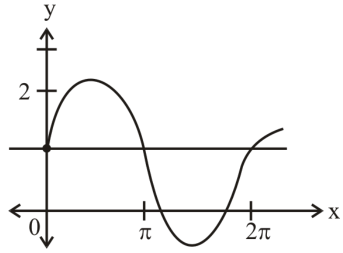
5. Let f (x) = min {1, 1 + x sin x}, $0\le x\le 2\pi$. If m is the number of points, where f is not differentiable and n is the number of points, where f is not continuous, then the ordered pair (m, n) is equal to

(A) (2, 0) (B) (1, 0)

(C) (1, 1) (D) (2, 1)

Official Ans. by NTA (B)

Sol.



No. of non-differentiable points = 1 (m)

No. of not continuous points = 0 (n)

(m, n) = (1, 0)

6. Consider a cuboid of sides 2x, 4x and 5x and a closed hemisphere of radius r. If the sum of their surface areas is a constant k, then the ratio x : r, for which the sum of their volumes is maximum, is :

(A) 2 : 5 (B) 19:45 (C) 3 : 8 (D) 19 : 15

Official Ans. by NTA (B)

Sol. Surface area = $76x^2+3\pi r^2$ = constant (K)

$$V=40x^3+\frac{2}{3}\pi r^3$$

$[76x^2+3\pi r^2=K]$

$$r^2=\frac{K-76x^2}{3\pi}$$

$$r=\left(\frac{K-76x^2}{3\pi}\right)^{\frac{1}{2}}$$

$$V=40x^3+\frac{2}{3}\pi\left(\frac{K-76x^2}{3\pi}\right)^{\frac{3}{2}}$$

$$\frac{dV}{dx}=120x^2+\frac{2}{3}\pi.\frac{3}{2}\left(\frac{K-76x^2}{3\pi}\right)^{\frac{1}{2}}.\left(\frac{-76(2x)}{3\pi}\right)$$

Put

$$\frac{dV}{dx}=0\Rightarrow 120x^2+\frac{2}{3}\pi.\frac{3}{2}\left(\frac{K-76x^2}{3\pi}\right)^{\frac{1}{2}}.\left(\frac{-76(2x)}{3\pi}\right)=0$$

$$\Rightarrow 120x^2=\frac{152x}{3}\left(\frac{k-76x^2}{3\pi}\right)^{\frac{1}{2}}$$

$$\Rightarrow\frac{45}{19}x^2=x\left(\frac{k-76x^2}{3\pi}\right)^{\frac{1}{2}}\ ;x\ne 0$$

$$\Rightarrow\frac{45}{19}x=\left(\frac{k-76x^2}{3\pi}\right)^{\frac{1}{2}}\Rightarrow\left(\frac{45}{19}\right)^2x^2=\frac{k-76x^2}{3\pi}$$

$$\Rightarrow\left(\frac{45}{19}\right)^2x^2=r^2\Rightarrow\frac{x^2}{r^2}=\left(\frac{19}{45}\right)^2$$

$$\Rightarrow\frac{x}{r}=\frac{19}{45}$$

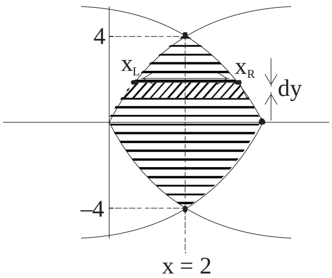
7. The area of the region bounded by $y^2 = 8x$ and $y^2 = 16(3 - x)$ is equal to :-

(A) $\frac{32}{3}$ (B) $\frac{40}{3}$ (C) 16 (D) 19

Official Ans. by NTA (C)

Sol. $y^2 = 8x$; $y^2 = 16(3 - x)$

$$y^2 = -16(x-3)$$



finding their intersection pts.

$y^2 = 8x$ & $y^2 = -16(x - 3)$

$8x = -16x + 48$

$24x = 48$

$x = 2$; $y = \pm 4$

$$A = 2.\int_0^4 (x_R - x_L)dy$$

Required Area

$$= 2.\int_0^4 \left(\underbrace{3 - \frac{y^2}{16}}_{(x_R} - \underbrace{\frac{y^2}{8}}_{x_L)}\right)dy$$

$$= 2\left(3y - \frac{y^3}{3\times16} - \frac{y^3}{3\times8}\right)_0^4$$

$$= 2\left(3\times4 - \frac{4\times4\times4}{3\times16} - \frac{4\times4\times4\times2}{3\times8\times2}\right)$$

$$= 2\left(12 - \frac{4}{3} - \frac{8}{3}\right) = 2\times12\left(1 - \frac{1}{3}\right) = 2\times12\times\frac{2}{3} = 16$$

8. If $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x) + c, g(1) = 0$, then $g\left(\frac{1}{2}\right)$ is equal to :

(A) $\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) + \frac{\pi}{3}$ (B) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \frac{\pi}{3}$

(C) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) - \frac{\pi}{3}$ (D) $\frac{1}{2}\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) - \frac{\pi}{6}$

Official Ans. by NTA (A)

Sol. $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x) + c$

Put $x = \cos 2\theta$

$dx = -2\sin 2\theta \cdot d\theta$

$$= \int \frac{1}{\cos 2\theta}\tan\theta(-4\sin\theta\cdot\cos\theta)d\theta$$

$$= \int \frac{1}{\cos 2\theta}(-4\sin^2\theta)d\theta$$

$$= -2\int \frac{1-\cos 2\theta}{\cos 2\theta}d\theta$$

$$= -\frac{2}{2}\ln|\sec 2\theta + \tan 2\theta| + 2\theta + c$$

$$= \ln|\sec 2\theta - \tan 2\theta| + 2\theta + c$$

$$= \ln\left|\frac{1-\sin 2\theta}{\cos 2\theta}\right| + \cos^{-1}x + c$$

$$= \underbrace{\ln\left|\frac{1-\sqrt{1-x^2}}{x}\right| + \cos^{-1}x}_{g(x)} + c$$

$\therefore g(1) = 0$

$$g(x) = \ln\left|\frac{1-\sqrt{1-x^2}}{x}\right| + \cos^{-1}x$$

$$g\left(\frac{1}{2}\right) = \ln\left|2-\sqrt{3}\right| + \frac{\pi}{3}$$

$$g\left(\frac{1}{2}\right) = \ln\left|\frac{\sqrt{3}-1}{\sqrt{3}+1}\right| + \frac{\pi}{3}$$

9. If $y = y(x)$ is the solution of the differential equation $x\frac{dy}{dx} + 2y = xe^x, y(1) = 0$, then the local maximum value of the function $z(x) = x^2y(x) - e^x$, $x \in R$ is :

(A) $1 - e$ (B) 0 (C) $\frac{1}{2}$ (D) $\frac{4}{e} - e$

Official Ans. by NTA (D)

Sol. $x\frac{dy}{dx} + 2y = xe^x$

$$\frac{dy}{dx} + \frac{2y}{x} = e^x$$

I.F. $= x^2$

$$y.x^2 = \int x^2e^x dx$$

$= \int e^x \left(x^2 + 2x - 2x - 2 + 2\right) dx$

$yx^2 = e^x (x^2 - 2x + 2) + c$

$y(1) = 0$

$0 = e(1 + 0) + c$

$c = -e$

$z(x) = x^2 y(x) - e^x$

$= e^x(x^2 - 2x + 2) - e - e^x$

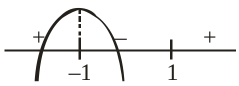
$= e^x (x - 1)^2 - e$

$$\frac{dz}{dx} = e^x.2(x-1) + e^x (x-1)^2 = 0$$

$x^x (x - 1) (2 + x - 1) = 0$

$e^x (x - 1) (x + 1) = 0$

$x = -1, 1$



$x = -1$ local maxima. Then maximum value is

$z(-1) = \frac{4}{e} - e$

10. If the solution of the differential equation

$\frac{dy}{dx} + e^x \left(x^2 - 2\right) y = \left(x^2 - 2x\right)\left(x^2 - 2\right) e^{2x}$ satisfies

$y(0) = 0$, then the value of $y(2)$ is ________ .

(A) –1 (B) 1 (C) 0 (D) e

Official Ans. by NTA (C)

Sol. $I.F. = e^{\int e^x(x^2-2)dx} = e^{\int e^x(x^2-2x+2x-2)dx}$

$= e^{e^x(x^2-2x)}$

$y.e^{e^x(x^2-2x)} = \int e^{e^x\left(x^2-2x\right)} e^x \left(x^2 - 2x\right)\left(x^2 - 2\right) e^x dx$

Let $e^x(x^2 - 2x) = t$

So, $y.e^{e^x\left(x^2-2x\right)} = \int e^t.t\ dt$

At $x = 0, t = 0$

$x = 2, t = 0$

$= t \cdot e^t - e^t + c$

$x = 0 ; 0 \cdot 1 = 0 - 1 + c \Rightarrow c = 1$

for $x = 2; y \cdot 1 = 0 - 1 + 1 = 0$

$y(2) = 0$

11. If m is the slope of a common tangent to the curves

$\frac{x^2}{16} + \frac{y^2}{9} = 1$ and $x^2+y^2 = 12$, then $12m^2$ is equal to :

(A) 6 (B) 9

(C) 10 (D) 12

Official Ans. by NTA (B)

Sol. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

equation of tangent to the ellipse is

$y = mx \pm \sqrt{a^2m^2 + b^2}$

$y = mx \pm \sqrt{16m^2 + 9}$ ....(i)

$x^2 + y^2 = 12$

equation of tangent to the circle is

$y = mx \pm \sqrt{12}\sqrt{1 + m^2}$ ....(ii)

for common tangent equate eq. (i) and (ii)

$\Rightarrow 16m^2 + 9 = 12(1 + m^2)$

$16\ m^2 - 12\ m^2 = 3$

$4\ m^2 = 3$

$12\ m^2 = 9$

12. The locus of the mid point of the line segment joining the point (4, 3) and the points on the ellipse $x^2 + 2y^2 = 4$ is an ellipse with eccentricity :

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2\sqrt{2}}$

(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

Official Ans. by NTA (C)

Sol. $\frac{x^2}{4} + \frac{y^2}{2} = 1$

$P(4,3)$ •——•——• $Q\left(2\cos\theta, \sqrt{2}\sin\theta\right)$

D

Coordinate of D is

$\left(\frac{2\cos\theta + 4}{2}, \frac{\sqrt{2}\sin\theta + 3}{2}\right) \equiv (h, k)$

$\frac{2h - 4}{2} = \cos\theta$ ....(i)

$\frac{2k - 3}{\sqrt{2}} = \sin\theta$ ....(ii)

$(i)^2 + (ii)^2$, then we get

$$\left(\frac{2h-4}{2}\right)^2+\left(\frac{2k-3}{\sqrt{2}}\right)^2=1 \Rightarrow \frac{(x-2)^2}{1}+\frac{\left(y-\frac{3}{2}\right)^2}{\left(\frac{1}{2}\right)}=1$$

$\therefore$ Required eccentricity is

$$e=\sqrt{1-\frac{1}{2}}=\frac{1}{\sqrt{2}}$$

13. The normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{9}=1$ at the point $\left(8,3\sqrt{3}\right)$ on it passes through the point :

(A) $\left(15,-2\sqrt{3}\right)$ (B) $\left(9,2\sqrt{3}\right)$

(C) $\left(-1,9\sqrt{3}\right)$ (D) $\left(-1,6\sqrt{3}\right)$

Official Ans. by NTA (C)

Sol. $\frac{x^2}{a^2}-\frac{y^2}{9}=1$: $\left(8,3\sqrt{3}\right)$ lie on Hyperbola then

$$\frac{64}{a^2}-\frac{27}{9}=1 \Rightarrow a^2=\frac{64}{4}=16$$

equation of normal at $\left(8,3\sqrt{3}\right)$:

$$\frac{16x}{8}+\frac{9y}{3\sqrt{3}}=16+9$$

$$2x+\sqrt{3}y=25$$

Check options.

14. If the plane 2x + y – 5z = 0 is rotated about its line of intersection with the plane 3x – y + 4z – 7 = 0 by an angle of $\frac{\pi}{2}$, then the plane after the rotation passes through the point :

(A) (2, –2, 0) (B) (–2, 2, 0)

(C) (1, 0, 2) (D) (–1, 0, –2)

Official Ans. by NTA (C)

Sol. (2x + y – 5z) + λ(3x – y + 4z – 7) = 0

Rotated by π/2

(2 + 3 λ)x + (1 – λ)y + (–5 + 4λ)z – 7λ = 0

2x + y – 5z = 0

2(2 + 3 λ) + (1 – λ) – 5(–5 + 4 λ) = 0

⇒ 4 + 6 λ + 1 – λ + 25 – 20 λ = 0

30 = 15 λ

λ = 2

Required plane :- 8x – y + 3z – 14 = 0

Check options

15. If the lines $\vec{r}=\left(\hat{i}-\hat{j}+\hat{k}\right)+\lambda\left(3\hat{j}-\hat{k}\right)$ and $\vec{r}=\left(\alpha\hat{i}-\hat{j}\right)+\mu\left(2\hat{i}-3\hat{k}\right)$ are co-planar, then distance of the plane containing these two lines from the point (•, 0, 0) is :

(A) $\frac{2}{9}$ (B) $\frac{2}{11}$

(C) $\frac{4}{11}$ (D) 2

Official Ans. by NTA (B)

Sol. $\vec{r}=\left(\hat{i}-\hat{j}+\hat{k}\right)+\lambda\left(3\hat{j}-\hat{k}\right)$ ….. L1

$\vec{r}=\left(\alpha\hat{i}-\hat{j}\right)+\mu\left(2\hat{i}-3\hat{k}\right)$ ….. L2

• L1 and L2 are coplanar

$$\therefore \begin{vmatrix} 0 & 3 & -1 \\ 2 & 0 & -3 \\ (1-\alpha) & 0 & 1 \end{vmatrix}=0$$

–3 (2 + 3 (1 – •)) = 0

2 + 3 – 3• = 0

• 3• = 5

$$\Rightarrow \alpha=\frac{5}{3}$$

Now,

$$\vec{n}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{vmatrix}=\hat{i}(-9)-\hat{j}(2)+k(-6)$$

= (9, 2, 6)

Equation of plane :

9 (x – 1) + 2(y + 1) + 6(z – 1) = 0

9x + 2y + 6z – 13 = 0

Perpendicular distance from (•, 0, 0)

$$=\left|\frac{\left(9\cdot\frac{5}{3}+0+0-13\right)}{\sqrt{81+36+4}}\right|=\frac{2}{\sqrt{121}}=\frac{2}{11}$$

16. Let $\vec{a}=\hat{i}+\hat{j}+2\hat{k}, \vec{b}=2\hat{i}-3\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+\hat{k}$ be three given vectors. Let $\vec{v}$ be a vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{2}{\sqrt{3}}$. If $\vec{v}.\hat{j}=7$, then $\vec{v}.\left(\hat{i}+\hat{k}\right)$ is equal to :

(A) 6 (B) 7 (C) 8 (D) 9

Official Ans. by NTA (D)

Sol. $\vec{v}=\lambda\vec{a}+\mu\vec{b}$

$\vec{v}=\lambda(1,1,2)+\mu(2,-3,1)$

$\vec{v}=(\lambda+2\mu,\lambda-3\mu,2\lambda+\mu)$

$\vec{v}.\hat{j}=7$ $\qquad$ $\vec{v}.\frac{\vec{c}}{|\vec{c}|}=\frac{2}{\sqrt{3}}$

$\lambda-3\mu=7$ $\qquad$ $\vec{v}.\vec{c}=2$

$\lambda+2\mu-\lambda+3\mu+2\lambda+\mu=2$

$2\lambda+6\mu=2$

$\lambda+3\mu=1$

$\lambda-3\mu=7$

$2\lambda=8$

$\lambda=4$

$\mu=-1$

We get $\vec{v}=(2,7,7)$

17. The mean and standard deviation of 50 observations are 15 and 2 respectively. It was found that one incorrect observation was taken such that the sum of correct and incorrect observations is 70. If the correct mean is 16, then the correct variance is equal to :

(A) 10 (B) 36 (C) 43 (D) 60

Official Ans. by NTA (C)

Sol. No. of observations: - 50

$\text{mean}(\bar{x})=15$

Standard deviation $(\sigma)=2$

Let incorrect observation is x_1 & correct observation is (x'_1)

Given $x_1+x'_1=70$

$$\bar{x}=\frac{x_1+x_2+....+x_{50}}{50}=15(\text{given})$$

$\Rightarrow x_1+x_2+.....x_{50}=750$ ....(i)

Now

Mean of correct observation is 16

$$\frac{x'_1+x_2+....+x_{50}}{50}=16$$

$x'_1+x_2+x_3+....x_{50}=16\times 50$ ....(ii)

eq. (ii) – eq. (i)

$\Rightarrow x'_1-x_1=16\times 50-15\times 50$

$x'_1-x_1=50$ & $x_1+x'_1=70$

$x'_1=60$

$x_1=10$

$$\Rightarrow 4=\frac{x_1^2+x_2^2+....+x_{50}^2}{50}-15^2 \quad ....(iii)$$

$$\Rightarrow \sigma^2=\frac{x'^2_1+x_2^2+....x_{50}^2}{50}-16^2 \quad ....(iv)$$

from (iii)

$$\Rightarrow 4=\frac{(10)^2}{50}+\frac{x_2^2+x_3^2+....+x_{50}^2}{50}-225$$

$$\Rightarrow 4=2-225+\frac{\left(x_2^2+x_3^2+....+x_{50}^2\right)}{50}$$

$$\Rightarrow 227=\frac{\left(x_2^2+x_3^2+....x_{50}^2\right)}{50}$$

From (iv)

$$\sigma^2=\frac{(60)^2}{50}+\left(\frac{x_2^2+x_3^2+....+x_{50}^2}{50}\right)-(16)^2$$

$$\sigma^2=\frac{60\times 60}{50}+227-256$$

$\sigma^2=72+227-256$

$\sigma^2=43$

18. 16sin(20°) sin(40°) sin(80°) is equal to :

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 3 (D) $4\sqrt{3}$

Official Ans. by NTA (B)

Sol. 16 sin20° sin 40° sin 80°

= 16 sin 40° sin 20° sin 80°

= 4(4 sin (60 – 20) sin (20) sin (60 + 20))

= 4 × sin (3 × 20°)

[∵ sin 3θ = 4 sin(60 – θ) × sin θ × sin (60 + θ)]

= 4 × sin 60°

$$=4\times\frac{\sqrt{3}}{2}=2\sqrt{3}$$

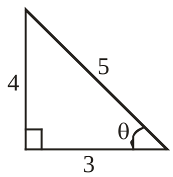
19. If the inverse trigonometric functions take principal values, then $\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)+\frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$ is equal to :

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Official Ans. by NTA (C)

Sol. Let

$$\tan^{-1}\frac{4}{3}=\theta \Rightarrow \tan\theta=\frac{4}{3}$$



$$E=\cos^{-1}\left(\frac{3}{10}\cos\theta+\frac{2}{5}\sin\theta\right)$$

$$=\cos^{-1}\left(\frac{3}{10}\times\frac{3}{5}+\frac{2}{5}\cdot\frac{4}{5}\right)$$

$$=\cos^{-1}\left(\frac{9}{50}+\frac{8}{25}\right)=\cos^{-1}\left(\frac{25}{50}\right)=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$$

20. Let r $\in$ {p, q, ~p, ~q} be such that the logical statement $r\vee(\sim p)\Rightarrow(p\wedge q)\vee r$ is a tautology. Then 'r' is equal to :

(A) p (B) q (C) ~p (D) ~q

Official Ans. by NTA (C)

Sol. By options

(1)

p=r	q	~p	$r\vee(\sim p)$	$(p\wedge q)$	$(p\wedge q)\vee r$	$r\vee(\sim p)\Rightarrow(p\wedge q)\vee r$
T	F	F	T	F	T	T
F	T	T	T	F	F	F
T	T	F	T	T	T	T
F	F	T	T	F	F	F

(2)

p	~p	$r\vee(\sim p)$	q=r	$(p\wedge q)$	$(p\wedge q)\vee r$	$r\vee(\sim p)\Rightarrow(p\wedge q)\vee r$
T	F	T	T	T	T	T
F	T	T	T	F	T	T
T	F	F	F	F	F	T
F	T	T	F	F	F	F

(3)

p	q	r = ~p	$r\vee(\sim p)$	$(p\wedge q)$	$(p\wedge q)\vee r$	$r\vee(\sim p)\Rightarrow(p\wedge q)\vee r$
T	T	F	F	T	T	T
F	T	T	T	F	T	T
T	F	F	F	F	F	T
F	F	T	T	F	T	T

(4)

~p	p	q	$r\vee(\sim p)$	r=~q	$(p\wedge q)$	$(p\wedge q)\vee r$	$r\vee(\sim p)\Rightarrow(p\wedge q)\vee r$
F	T	T	F	F	T	T	T
F	T	F	T	T	F	T	T
T	F	T	T	F	F	F	F
T	F	F	T	T	F	T	T

Now final answer is option no. 3.

SECTION-B

1. Let $f:\mathbb{R}\to\mathbb{R}$ satisfy $f(x + y) = 2^x f(y) + 4^y f(x), \forall x, y \in \mathbb{R}$. If f(2) = 3, then $14.\frac{f'(4)}{f'(2)}$ is equal to ______.

Official Ans. by NTA (248)

Sol. Put y = 2

$f(x + y) = 2^x.f(y) + 4^y.f(x).$

$f(x + 2) = 2^x.3 + 16f(x)$

$f'(x + 2) = 16f'(x) + 3.2^x \ln 2$

$f'(4) = 16f'(2) + 12\ln 2$ ….(i)

$f(y + 2) = 4f(y) + 3\cdot 4^y$

$f'(y+2) = 4f'(y) + 3.4^y \ln 4$

$f'(4) = 4f'(2) + 96 \ln 2$ ....(ii)

solving eq. (i) and (ii), we get

$f'(2) = 7 \ln 2$

from equation (i), we get

$f'(4) = 124 \ln 2$

Now, $\Rightarrow 14.\frac{f'(4)}{f'(2)}$

$14 \times \frac{124\ ln2}{7\ ln2}$

$= 248.$

2. Let p and q be two real numbers such that $p + q = 3$ and $p^4 + q^4 = 369$. Then $\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}$ is equal to ________.

Official Ans. by NTA (4)

Sol. $p + q = 3 \quad p^4 + q^4 = 369$

$\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}$

$(p+q)^2 = 9$

$p^2 + q^2 = 9 - 2pq$

$$\frac{1}{\left(\frac{1}{p}+\frac{1}{q}\right)^2} = \frac{(qp)^2}{(q+p)^2} = \frac{(qp)^2}{9}$$

$p^4 + q^4 = (p^2+q^2)^2 - 2p^2q^2$

$369 = (9 - 2pq)^2 - 2(pq)^2$

$369 = 81 + 4p^2q^2 - 36pq - 2p^2q^2$

$288 = 2p^2q^2 - 36pq$

$144 = p^2q^2 - 18pq$

$(pq)^2 - 2 \times 9 \times pq + 9^2 = 144 + 9^2$

$(pq - 9)^2 = 225$

$pq - 9 = \pm 15$

$pq = \pm 15 + 9$

$pq = 24, -6$

(24 is rejected because $p^2 + q^2 = 9 - 2pq$ is negative)

$$\frac{(qp)^2}{9} = \frac{1(-6)^2}{9} = 4$$

3. If $z^2 + z + 1 = 0$, $z \in \mathbb{C}$, then $\left|\sum_{n=1}^{15}\left(Z^n + (-1)^n \frac{1}{Z^n}\right)^2\right|$ is equal to ______ .

Official Ans. by NTA (2)

Sol. $z^2 + z + 1 = 0 \Rightarrow z = w, w^2$

$$\left|\sum_{n=1}^{15}\left(z^n + (-1)\frac{1}{z^n}\right)^2\right| = \left|\sum_{n=1}^{15}\left(z^{2n} + \frac{1}{z^{2n}} + 2(-1)^n\right)\right|$$

$$= \left|\sum_{n=1}^{15} w^{2n} + \frac{1}{w^{2n}} + 2(-1)^n\right|$$

$$= \left|\frac{w^2(1-w^{30})}{1-w^2} + \frac{\frac{1}{w^2}\left(1-\frac{1}{w^{30}}\right)}{1-\frac{1}{w^2}} + 2(-1)\right|$$

$$= \left|\frac{w^2(1-1)}{1-w^2} + \frac{\frac{1}{w^2}(1-1)}{1-\frac{1}{w^2}} - 2\right|$$

$= |0 + 0 - 2| = 2$

4. Let $X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, Y = \alpha I + \beta X + \gamma X^2$ and $Z = \alpha^2 I - \alpha\beta X + (\beta^2 - \alpha\gamma)X^2, \alpha, \beta, \gamma \in \mathbb{R}$. If $Y^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$, then $(\alpha - \beta + \gamma)^2$ is equal to _____ .

Official Ans. by NTA (100)

Sol. $X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, X^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$Y = \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix}, Z = \begin{bmatrix} \alpha^2 & -\alpha\beta & \beta^2 - \alpha\gamma \\ 0 & \alpha^2 & -\alpha\beta \\ 0 & 0 & \alpha^2 \end{bmatrix}$

$Y \cdot Y^{-1} = I$

$$\begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\frac{\alpha}{5} = 1 \Rightarrow \alpha = 5$

$-\frac{2}{5}\alpha + \frac{\beta}{5} = 0 \Rightarrow \beta = 10$

$\frac{\alpha}{5} - \frac{2\beta}{5} + \frac{\gamma}{5} = 0 \Rightarrow \gamma = 15$

$\Rightarrow (\alpha - \beta + \gamma)^2 = (5 - 10 + 15)^2 = 100$

5. The total number of 3–digit numbers, whose greatest common divisor with 36 is 2, is _______ .

Official Ans. by NTA (150)

Sol. $36 = 2 \times 2 \times 3 \times 3$

Number should be odd multiple of 2 and does not having factor 3 and 9

Odd multiple of 2 are

102, 106, 110, 114 ……998 (225 no.)

No. of multiples of 3 are

102, 114, 126 ……. 990 (75 no.)

Which are also included multiple of 9

Hence,

Required = 225 – 75 = 150

6. If $\left({}^{40}C_0\right) + \left({}^{41}C_1\right) + \left({}^{42}C_2\right) + \ldots + \left({}^{60}C_{20}\right) = \frac{m}{n}\,{}^{60}C_{20}$, m and n are coprime, then m + n is equal to _____ .

Official Ans. by NTA (102)

Sol. ${}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + \ldots\ldots {}^{59}C_{19} + {}^{60}C_{20}$

$\left(\frac{1}{41} + 1\right){}^{41}C_1 + {}^{42}C_2 + \ldots\ldots$

$\left[\frac{42}{41}\left(\frac{2}{42}\right) + 1\right]{}^{42}C_2 + {}^{43}C_3 + \ldots\ldots$

$\left(\frac{2}{41} + 1\right){}^{42}C_2 + {}^{43}C_3 + \ldots\ldots$

$\left(\frac{43}{41} \times \frac{3}{43} + 1\right){}^{43}C_3 + {}^{44}C_4 + \ldots\ldots$

$\frac{3+41}{41}.{}^{43}C_3 + \ldots\ldots$

Similarly :

$\frac{20+41}{41}$

$\Rightarrow m = 61 ; n = 41$

m + n = 102

7. If $a_1 (> 0), a_2, a_3, a_4, a_5$ are in a G.P., $a_2 + a_4 = 2a_3 + 1$ and $3a_2 + a_3 = 2a_4$, then $a_2 + a_4 + 2a_5$ is equal to _____.

Official Ans. by NTA (40)

Sol. $a_1 > 0, a_2, a_3, a_4, a_5 \rightarrow$ G.P.

$3a_2 + a_3 = 2a_4$

$3ar + ar^2 = 2ar^3$

$3 + r = 2r^2$

$2r^2 - r - 3 = 0$

$r = -1 \ \&\ r = \frac{3}{2}$

$a_2 + a_4 = 2a_3 + 1$

$ar + ar^3 = 2ar^2 + 1$

$a(r + r^3 - 2r^2) = 1$

$a\left(\frac{3}{2} + \frac{27}{8} - \frac{18}{4}\right) = 1$

$a = \frac{8}{3}$

When $r = -1$, $a = -\frac{1}{4}$ (rejected, $a_1 > 0$)

$r = \frac{2}{3}, a = \frac{8}{3}$ (selected)

Now

$a_2 + a_4 + 2a_5$

$= \frac{8}{3} \times \frac{3}{2} + \frac{8}{3} \times \frac{27}{8} + 2 \times \frac{8}{3} \times \frac{81}{16}$

$= 4 + 9 + 27 = 40$

8. The integral $\frac{24}{\pi}\int_0^{\sqrt{2}} \frac{(2 - x^2)dx}{(2 + x^2)\sqrt{4 + x^4}}$ is equal to ____.

Official Ans. by NTA (3)

Sol. $\frac{24}{\pi}\int_0^{\sqrt{2}} \frac{(2 - x^2)}{(x^2 + 2)\sqrt{4 + x^4}}dx$

$\frac{24}{\pi}\int_0^{\sqrt{2}} \frac{x^2\left(\frac{2}{x^2} - 1\right)dx}{x\left(x + \frac{2}{x}\right) \times x\sqrt{\frac{4}{x^2} + x^2}}$

$$\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{\left(\frac{2}{x^2}-1\right)dx}{\left(x+\frac{2}{x}\right)\sqrt{\left(x+\frac{2}{x}\right)^2-4}}$$

$$x+\frac{2}{x}=t$$

$$dt=\left(1-\frac{2}{x^2}\right)dx$$

$$I=-\frac{24}{\pi}\int\frac{dt}{t\sqrt{t^2-4}}$$

$$=-\frac{24}{\pi}\times\frac{1}{2}\sec^{-1}\left.\left(\frac{x+\frac{2}{x}}{2}\right)\right|_0^{\sqrt{2}}$$

$$=-\frac{12}{\pi}\left[\sec^{-1}\left(\frac{2\sqrt{2}}{2}\right)-\sec^{-1}(\infty)\right]$$

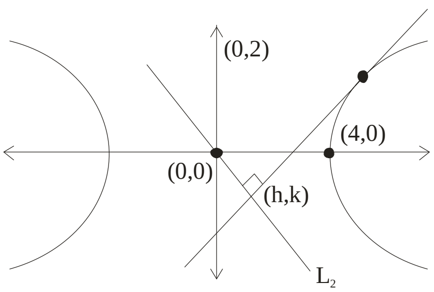
$$=-\frac{12}{\pi}\left[\frac{\pi}{4}-\frac{2\pi}{2\times2}\right]=-\frac{12}{\pi}\left[-\frac{\pi}{4}\right]$$

$= 3$

9. Let a line L_1 be tangent to the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ and let L_2 be the line passing through the origin and perpendicular to L_1. If the locus of the point of intersection of L_1 and L_2 is $(x^2 + y^2)^2 = \alpha x^2 + \beta y^2$, then $\alpha + \beta$ is equal to _______ .

Official Ans. by NTA (12)

Sol.



$$\frac{x\sec\theta}{4}-\frac{y\tan\theta}{2}=1$$

$$m_1=\frac{\sec\theta\times2}{4(\tan\theta)}=\frac{\sec\theta}{2\tan\theta}$$

$$m_2=\frac{k}{h}$$

$$m_1m_2=-1$$

$$\frac{k}{h}\cdot\frac{\sec\theta}{2\tan\theta}=-1$$

$$\frac{k}{2h\sin\theta}=-1$$

$$\sin\theta=\frac{-k}{2h}\qquad\cos\theta=\frac{\sqrt{4h^2-k^2}}{2h}$$

also

$$\frac{h\sec\theta}{4}-\frac{k\tan\theta}{2}=1$$

$$\frac{h}{4}\frac{2h}{\sqrt{4h^2-k^2}}-\frac{k}{2}\left(\frac{-k}{\sqrt{4h^2-k^2}}\right)=1$$

$$h^2+k^2=2\sqrt{4h^2-k^2}$$

$$(x^2+y^2)^2=4(4x^2-y^2)$$

$$(x^2+y^2)^2=16x^2-4y^2$$

$\alpha = 16, \beta = -4$

$\alpha + \beta = 16 - 4 = 12$

10. If the probability that a randomly chosen 6-digit number formed by using digits 1 and 8 only is a multiple of 21 is p, then 96 p is equal to _____ .

Official Ans. by NTA (33)

Sol. $2\times2\times2\times2\times2\times2=64$

Divisible by 21 when divided by 3.

Case – I : All 1 $\rightarrow$ (1)

Case – II : All 8 $\rightarrow$ (1)

Case – III : 3 ones & 3 eights

$$\frac{6!}{3!\times3!}=20$$

Required probability $\therefore p=\frac{22}{64}$

$$96p=96\times\frac{22}{64}=33$$