QUESTION PAPER WITH SOLUTION

PHYSICS _ 2 Sep. _ SHIFT - 2

1. If momentum (P), area (A) and time (T) are taken to be the fundamental quantities then the dimensional formula for energy is:

(1) $[P^{1/2} AT^{-1}]$ (2) $[PA^{1/2}T^{-1}]$ (3) $[PA^{1/2}T^{-1}]$ (4) $[P^2AT^{-2}]$

Sol. (2)

$[P] = MLT^{-1} \leftarrow$ momentum

$[A] = M^0L^2T^0 \leftarrow$ Area

$[T] = M^0L^0T^1 \leftarrow$ Time

Let $[E] = P^xA^yT^z$

$ML^2T^{-2} = [MLT^{-1}]^x [L^2]^y [T]^z$

$= M^x L^{x+2y} T^{z-x}$

Comparing both sides :-

$x = 1$ .....(i)

$x + 2y = 2 \Rightarrow 1 + 2y = 2$ or, $y = \frac{1}{2}$ .....(ii)

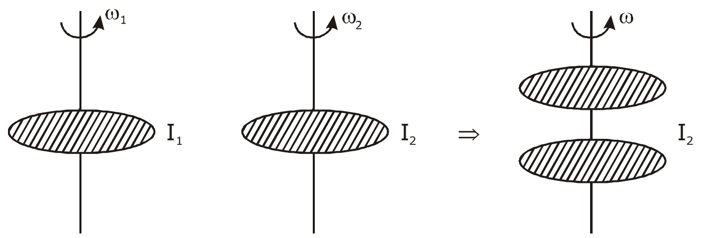
$z - x = -2 \Rightarrow z-1 = -2$ or $z=-1$ .....(iii)

$\therefore [E] = [P^1 A^{1/2} T^{-1}]$

2. Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg $-m^2$ and 10 rad s^{-1} respectively while those for the second one are 0.2 kg$-m^2$ and 5 rad s^{-1} respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The Kinetic energy of the combined system is:

(1) $\frac{2}{3}$ J (2) $\frac{10}{3}$ J (3) $\frac{5}{3}$ J (4) $\frac{20}{3}$ J

Sol. **(4)**



$I_1\omega_1 + I_2\omega_2 = (I_1+I_2)\omega$

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{0.1\times10+0.2\times5}{0.1+0.2} = \frac{1+1}{0.3} = \frac{2}{0.3}$$

$$\omega = \frac{20}{3}$$

Now find KE = $\frac{1}{2}I_1\omega^2 + \frac{1}{2}I_2\omega^2$

= $\frac{1}{2}(I_1 + I_2)\omega^2 = \frac{1}{2}\times 0.3\times\left(\frac{20}{3}\right)^2$

= $\frac{1}{2}\times\frac{3}{10}\times\frac{20}{3}\times\frac{20}{3}$

$\frac{0.3}{2}\times\frac{400}{9}$

$\boxed{(\text{K.E.})_f = \frac{20}{3}}$

3. A particle is moving 5 times as fast as an electron. The ratio of the de-Broglie wavelength of the particle to that of the electron is 1.878×10^{-4}. The mass of the particle is close to:
(1) 4.8×10^{-27} kg (2) 9.1×10^{-31} kg (3) 9.7×10^{-28} kg (4) 1.2×10^{-28}kg

Sol. **(3)**

$P = \frac{h}{\lambda}$

$\lambda = \frac{h}{P}$

$\frac{\lambda_{\text{Particle}}}{\lambda_e} = 1.878\times 10^{-4}$

$\Rightarrow \frac{h}{P_{\text{particle}}}\times\frac{Pe}{h} = 13.878\times 10^{-4} \Rightarrow \frac{Pe}{P_{\text{particle}}} = 1.878\times 10^{-4}$

$\Rightarrow \frac{M_e.V_e}{M_p.V_p} = 1.878\times 10^{-4} \quad \Rightarrow M_p = \frac{M_e}{1.878\times 10^{-4}}\times\left(\frac{V_e}{V_p}\right)$

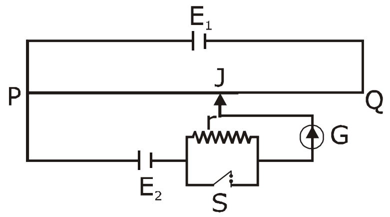
$= \frac{9.11\times 10^{-31}}{1.878\times 10^{-4}}\times\frac{1}{5}$

$= \frac{7.11\times 10}{1.878\times 10^{-4}}\times\frac{1}{5}$

= 0.97×10^{-27} kg

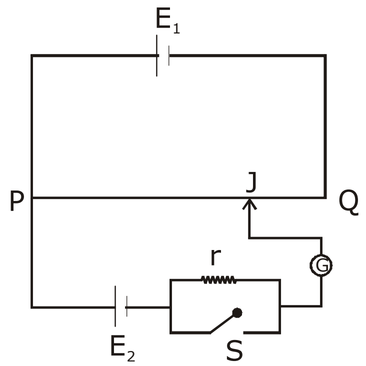
= 9.7×10^{-28} kg

4. A potentiometer wire PQ of 1 m length is connected to a standard cell E_1. Another cell E_2 of emf 1.02 V is connected with a resistance 'r' and switch S (as shown in figure). With switch S open, the null positon is obtained at a distance of 49 cm from Q. The potential gradient in the potentiometer wire is:



(1) 0.03V/cm (2) 0.02 V/cm (3) 0.04 V/cm (4) 0.01 V/cm

Sol. (2)



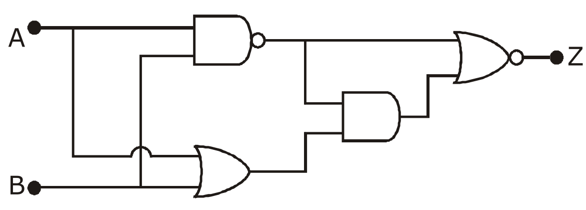
PQ = 1m

QJ = 49 cm

$\therefore$ PJ = 51 cm

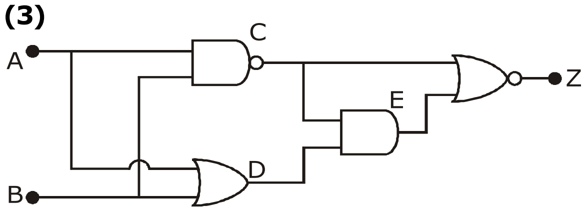
$\frac{v}{\ell} = \frac{1.02}{51} =$ 0.02 v/cm

5. In the following digital circuit, what will be the output at 'Z', when the input (A,B) are (1,0), (0,0), (1,1), (0,1):



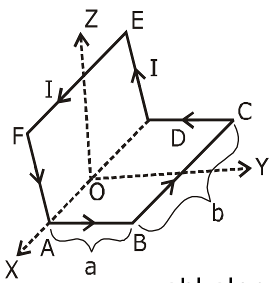
(1) 0,1,0,0 (2) 1,1,0,1 (3) 0,0,1,0 (4) 1,0,1,1

Sol. **(3)**



A	B	$C = \overline{A.B}$	$D = A + B$	$E = C.D$	$Z = \overline{C + E}$
1	0	1	1	1	0
0	0	1	0	0	0
0	0	1	0	0	0
1	1	0	1	0	1
0	1	1	1	1	0

6. A wire carrying current I is bent in the shape ABCDEFA as shown, where rectangle ABCDA and ADEFA are perpendicular to each other. If the sides of the rectangles are of lengths a and b, then the magnitude and direction of magnetic moment of the loop ABCDEFA is:



(1) $\sqrt{2}$ abI, along $\left(\frac{\hat{j}}{\sqrt{5}} + \frac{2\hat{k}}{\sqrt{5}}\right)$ (2) abI, along $\left(\frac{\hat{j}}{\sqrt{5}} + \frac{2\hat{k}}{\sqrt{5}}\right)$

(3) $\sqrt{2}$ abI, along $\left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}\right)$ (4) abI, along $\left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}\right)$

Sol. **(3)**

LOOP = ABCD

$\vec{M}_1 = (abI)\hat{k}$

Loop DEFA

$\vec{M}_2 = (abI)\hat{j}$

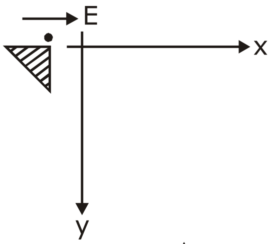
$\vec{M} = \vec{M}_1 + \vec{M}_2 = abI(\hat{j} + \hat{k})$

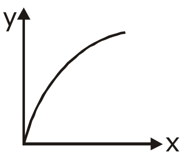
$|\vec{M}| = \sqrt{2}abI$

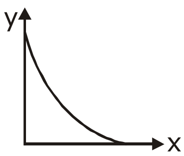
direction = along $\left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}\right)$

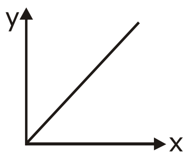
$\sqrt{2}$ abI, along $\left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}\right)$

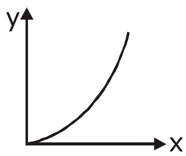
7. A small point mass carrying some positive charge on it, is released from the edge of a table. There is a uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass? (Curves are drawn schematically and are not to scale).



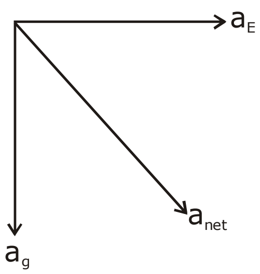
(1) 

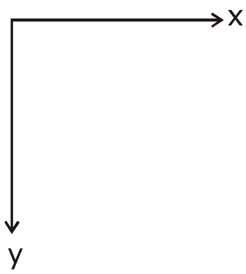
(2) 

(3) 

(4) 

Sol. **(3)**





Since it is released from rest.
And a_{net} is constant.
If will have straight line path along net 'a'.

8. In a plane electromagnetic wave, the directions of electric field and magnetic field are represented by k and $2\hat{i}-2\hat{j}$, respectively. What is the unit vector along direction of propagation of the wave.

(1) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ (2) $\frac{1}{\sqrt{5}}(2\hat{i}+\hat{j})$ (3) $\frac{1}{\sqrt{5}}(\hat{i}+2\hat{j})$ (4) $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$

Sol. **(1)**

$\vec{E}\times\vec{B}=\hat{k}\times(2\hat{i}-2\hat{j}) = 2\hat{k}\times\hat{i}-2\hat{k}\times\hat{j} = 2\hat{j}+2\hat{i}$

unit vector along $\vec{E}\times\vec{B}=\frac{1}{2\sqrt{2}}(2\hat{i}+2\hat{j}) = \frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$

$C = \frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$

9. An inductance coil has a reactance of 100 Ω. When an AC signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. The self-inductance of the coil is:

(1) 6.7×10^{-7} H (2) 5.5×10^{-5} H (3) 1.1×10^{-1} H (4) 1.1×10^{-2} H

Sol. (4)

L-R circuit

$\tan 45° = \frac{x_L}{R}$

$1 = \frac{x_L}{R} \Rightarrow x_L = R$

Now $Z = \sqrt{R^2 + x_L^2}$

or $Z = \sqrt{x_L^2 + x_L^2} = \sqrt{2x_L^2} = \sqrt{2}x_L$

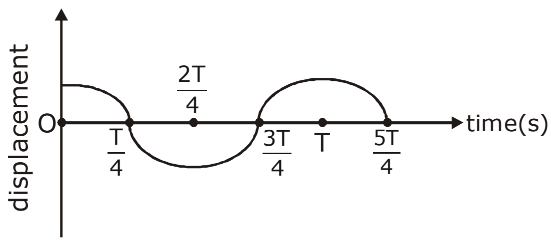
$100 = \sqrt{2}x_L$

$x_L = \frac{100}{\sqrt{2}}$

$\omega L = \frac{100}{\sqrt{2}} \Rightarrow L = \frac{100}{\sqrt{2}\omega} = \frac{100}{\sqrt{2} \times 2\pi f} = \frac{100}{\sqrt{2} \times 2 \times 3.14 \times 1000}$

$= 1.1 \times 10^{-2}$ H

10. This displacement time graph of a particle executing S.H.M. is given in figure : (sketch is schematic and not to scale)



Which of the following statements is/are true for this motion?

(A) The force is zero at $t = \frac{3T}{4}$

(B) The acceleration is maximum at t=T

(C) The speed is maximum at $t = \frac{T}{4}$

(D) The P.E. is equal to K.E. of the oscillation at $t = \frac{T}{2}$

(1) (B), (C) and (D) (2) (A), (B) and (D) (3) (A) and (D) (4) (A), (B) and (C)

Sol. **(A,B,C)**

(A) at $t = \frac{3T}{4}$

Particle is at mean position
a = 0
F = 0
(B) at t= T,
Particle is at extreme.
F is maximum
a = max

(C) at $t = \frac{T}{4}$; mean position

so, maximum velocity
(d) KE = PE

$$\frac{1}{2}k\left(A^2 - x^2\right) = \frac{1}{2}kx^2$$

$A^2 - x^2 = x^2$
$A^2 = 2x^2$
A = $\sqrt{2}x$

$$x = \frac{A}{\sqrt{2}} = A\cos\omega t$$

$$\cos\omega t = \frac{1}{\sqrt{2}}$$

$$\omega t = \frac{\pi}{4}$$

$$\frac{2\pi}{T}.t = \frac{\pi}{4} \Rightarrow t = \frac{T}{8}$$

11. In a Young's double slit experiment, 16 fringes are observed in a certain segment of the screen when light of wavelength 700 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen would be:
(1) 28 (2) 24 (3) 30 (4) 18

Sol. **(1)**

$$y = \frac{D\lambda}{d}$$

or $n_1 \frac{D\lambda_1}{d} = n_2 \frac{D\lambda_2}{d}$

$$n_1\lambda_1 = n_2\lambda_2$$

$$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}$$

$$n_2 = n_1.\frac{\lambda_1}{\lambda_2} \Rightarrow 16\times\frac{700}{400} = 28$$

12. A heat engine is involved with exchange of heat of 1915 J, −40J, + 125J and −QJ, during one cycle achieving an efficiency of 50.0%. The value of Q is:

(1) 980 J (2) 640 J (3) 40 J (4) 400 J

Sol. **(1)**

$$\eta = \frac{W}{\Sigma Q_+} = \frac{Q_1 + Q_2 + Q_3 + Q_4}{Q_1 + Q_3}$$

$$0.5 = \frac{1915 - 40 + 125 - Q}{1915 + 125}$$

1020 = 1915 − 40 + 125 − Q

Q = 2000 − 1020 = 980 J

13. In a hydrogen atom the electron makes a transition from $(n + 1)^{th}$ level to the n^{th} level. If $n>>1$, the frequency of radiation emitted is proportional to :

(1) $\frac{1}{n^2}$ (2) $\frac{1}{n}$ (3) $\frac{1}{n^3}$ (4) $\frac{1}{n^4}$

Sol. **(3)**

$$E_n = \frac{-Rhc}{n^2}$$

$$E_{n+1} = \frac{-Rhc}{(n+1)^2}$$

$$\Delta E = E_{n+1} - E_n$$

$$h\nu = RhC\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$$

$$\nu = Rc\left[\frac{(n+1)^2 - n^2}{n^2.(n+1)^2}\right]$$

$$= Rc\left[\frac{1+2n}{n^2(n+1)^2}\right]$$

if n>>1

$$\nu = \frac{2n}{n^2 \times n^2} = \frac{2n}{n^4} = \frac{2}{n^3}$$

$$\boxed{\nu \propto \frac{1}{n^3}}$$

14. When the temperature of a metal wire is increased from 0°C to 10°C, its length increases by 0.02%. The percentage change in its mass density will be closest to :

(1) 0.06 (2) 0.008 (3) 2.3 (4) 0.8

Sol. (A)

$$\Delta\ell = \ell \propto \Delta t$$

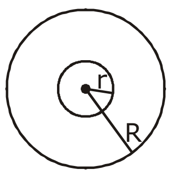
$$\alpha = \frac{\Delta\ell}{\ell\Delta t} = \frac{0.02}{100\times10} = 2\times10^{-5}$$

$$v = 3\alpha = 6\times10^{-5}$$

Now, $\frac{\Delta v}{v}\times100 = \gamma.\Delta t.100 = 6\times10^{-5}\times10\times100$

= 6×10^{-2} = 0.06

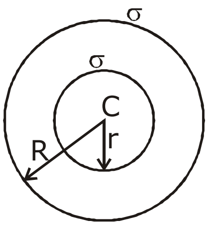
15. A charge Q is distributed over two concentric conducting thin spherical shells radii r and R (R > r). If the surface charge densities on the two shells are equal, the electric potential at the common centre is:



(1) $\frac{1}{4\pi\epsilon_0}\frac{(2R+r)}{(R^2+r^2)}Q$ (2) $\frac{1}{4\pi\epsilon_0}\frac{(R+r)}{(R^2+r^2)}Q$

(3) $\frac{1}{4\pi\epsilon_0}\frac{(R+r)}{2(R^2+r^2)}Q$ (4) $\frac{1}{4\pi\epsilon_0}\frac{(R+2r)Q}{2(R^2+r^2)}$

Sol. **(2)**



$Q_1 = \sigma.4\pi r^2$

$Q_2 = \sigma.4\pi R^2$

$Q = 4\pi\sigma\left(R^2 + r^2\right)$

$\Rightarrow \sigma = \dfrac{Q}{4\pi\left(R^2 + r^2\right)}$

$v_c = \dfrac{KQ_1}{r} + \dfrac{KQ_2}{R}$

$= \dfrac{K\sigma 4\pi r^2}{r} + \dfrac{K\sigma 4\pi R^2}{R} = K\sigma 4\pi\left(r + R\right)$

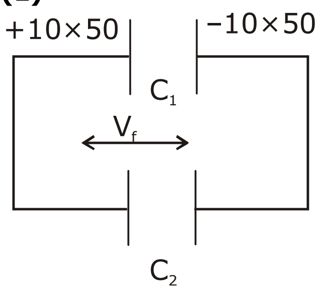
$= 4\pi K \dfrac{Q}{4\pi\left(R^2 + r^2\right)}\left(r + Q\right) = \dfrac{KQ\left(r + Q\right)}{\left(R^2 + r^2\right)}$

$\dfrac{1}{4\pi E_0} \dfrac{(R + r)}{(R^2 + r^2)} \times Q$

16. A 10 μF capacitor is fully charged to a potential difference of 50V. After removing the source voltage it is connected to an uncharged capacitor in parallel. Now the potential difference across them becomes 20 V. The capacitance of the second capacitor is:

(1) 15 μF (2) 20 μF (3) 10 μF (4) 30 μF

Sol. **(1)**



$C_1 = 10\,\mu F$

$v_f = 20$ v

$$\frac{500}{C_1 + C_2} = 20$$

$$\frac{500}{10 + C_2} = 20$$

$$10 + C_2 = \frac{500}{20} = 25$$

$$C_2 = 15\mu F$$

17. An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true?

(A) the mean free path of the molecules decreases.

(B) the mean collision time between the molecules decreases.

(C) the mean free path remains unchanged.

(D) the mean collision time remains unchanged.

(1) (B) and (C) (2) (A) and (B) (3) (C) and (D) (4) (A) and (D)

Sol. **(1) B,C**

$$\lambda = \frac{1}{\sqrt{2}\left(\frac{N}{V}\right)\pi d^2}$$

$\lambda \rightarrow$ Mean free path

N $\rightarrow$ No. 7 molecules

v = volume of container

d = diameter of molecule

$\because$ N and V are constant

$\therefore$ Mean free path reamains unchanged.

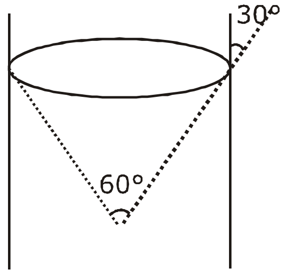
Now, If T ↑ no. of collisions increases.

18. A capillary tube made of glass of radius 0.15 mm is dipped vertically in a beaker filled with methylene iodide (surface tension = 0.05 Nm^{-1}, density = 667 kg m^{-3}) which rises to height h in the tube. It is observed that the two tangents drawn from liquid-glass interfaces (from opp. sides of the capillary) make an angle of 60° with one another. Then h is close to (g=10 ms^{-2}).

(1) 0.172 m (2) 0.049 m (3) 0.087 m (4) 0.137 m

Sol. (3)

$h = \frac{2T\cos\theta}{\rho g r}$ $\{\theta = 30°\}$



$= \frac{2 \times 0.05 \times \frac{1}{2}}{667 \times 10 \times 0.15 \times 10^{-3}}$

= 0.087 m option (3)

19. The height 'h' at which the weight of a body will be the same as that at the same depth 'h' from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected):

(1) $\frac{\sqrt{3}R - R}{2}$ (2) $\frac{\sqrt{5}}{2}R - R$ (3) $\frac{\sqrt{5}R - R}{2}$ (4) $\frac{R}{2}$

Sol. **(3)**

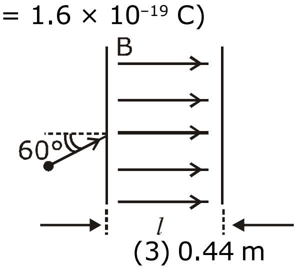
$$\frac{g_0}{\left(1+\frac{h}{R}\right)^2} = g_0\left(1-\frac{h}{R}\right)$$

$$\frac{R^2}{(R+h)^2} = \frac{(R-h)}{R}$$

$R^3 = (R - h)(R + h)^2$
$R^3 = (R - h)(R^2+2h+h^2)$
on solving we get,

$$h = \frac{\sqrt{5}R - R}{2}$$

20. The figure shows a region of length 'l' with a uniform magnetic field of 0.3 T in it and a proton entering the region with velocity 4×10^5 ms^{-1} making an angle 60° with the field. If the proton completes 10 revolution by the time it cross the region shown, 'l' is close to (mass of proton = 1.67×10^{-27} kg, charge of the proton = 1.6×10^{-19} C)



(1) 0.11 m (2) 0.22 m (3) 0.44 m (4) 0.88 m

Sol. **(3)**

ℓ = 10 × pitch

= 10 × vcos 60° × $\frac{2\pi m}{qB}$

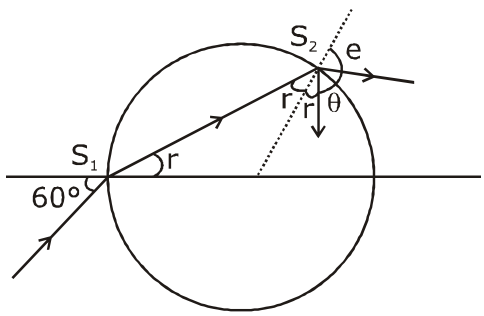
= 10 × v × $\frac{1}{2}\times\frac{2\pi m}{qB}$

$\boxed{\ell=\frac{10v\pi m}{qB}}=\frac{10\times4\times10^{5}\times3.14\times1.67\times10^{-27}}{1.6\times10^{-19}\times0.3}$

$\simeq$ 0.44 m

21. A light ray enters a solid glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence 60°. The ray is both reflected and refracted at the farther surface of the sphere. The angle (in degrees) between the reflected and refracted rays at this surface is ________.

21. **90**



At S_1

$1 \times \sin 60° = \sqrt{3}\sin r$

r = 30°

$\therefore$ r_1 = 30° {from geometry}

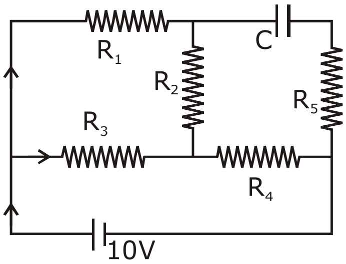
As S_2 $\sqrt{3}\sin r_1 = 1\sin e$

e = 60°

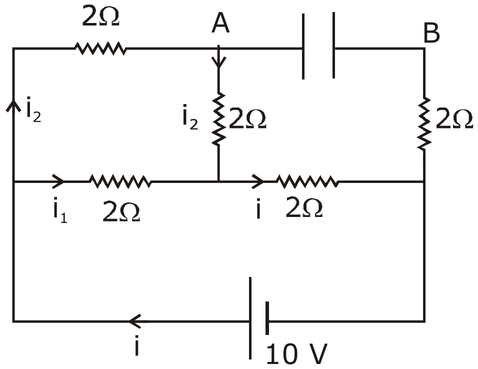
Now, $r_1 + \theta + e = 180°$

$\boxed{\theta = 90°}$

22. An ideal cell of emf 10 V is connected in circuit shown in figure. Each resistance is 2Ω. The potential difference (in V) across the capacitor when it is fully charged is ________.



Sol. **8**



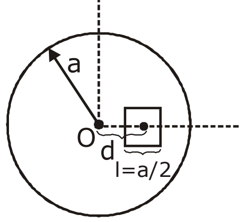
$$i = \frac{10}{\frac{4}{3}+2} = \frac{10\times 3}{10} = 3\text{Amp}$$

$i_1 = 2$ Amp

$i_2 = 1$ Amp

$V_{AB} = 1 \times 2 + 3 \times 2 = 8V$

23. A square shaped hole of side $l = \frac{a}{2}$ is carved out at a distance $d = \frac{a}{2}$ from the centre 'O' of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion from O is $-\frac{a}{X}$, value of X (to the nearest integer) is __________.



Sol. **23**

$$X_{cm} = \frac{\pi a^2 \times 0 - \frac{a^2}{4} \times \frac{a}{2}}{\pi a^2 - \frac{a^2}{4}}$$

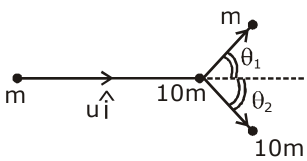
$$= \frac{-a}{2(4\pi - 1)} = \frac{-a}{8\pi - 2}$$

X = (8 π −2) = 8 × 3.14 −2

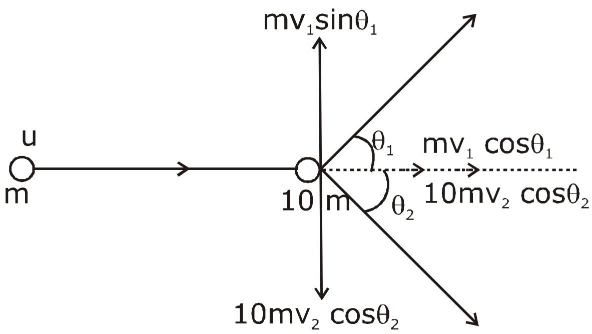
= 23.12

Nearest Integer = 23

24. A particle of mass m is moving along the x-axis with initial velocity $u\hat{i}$. It collides elastically with a particle of mass 10 m at rest and then moves with half its initial kinetic energy (see figure). If $\sin\theta_1 = \sqrt{n}\sin\theta_2$ then value of n is



24. **10**



$$\frac{1}{2}mv_1^2 = \frac{1}{2}\left(\frac{1}{2}mu^2\right)$$

$$v_1^2 = \frac{u^2}{2}$$

$$\boxed{v_1 = \frac{u}{\sqrt{2}}} \qquad \text{...(i)}$$

Also, $\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}\times(10m)\times v_2^2$

$$\frac{1}{2}\times 10m\times v_2^2 = \frac{1}{2}\times\frac{1}{2}mu^2$$

$v_2^2 = \frac{u^2}{20}$ or, $v_2 = \frac{u}{\sqrt{20}}$

now, $mv_1 \sin\theta_1 = 10mv_2 \sin\theta_2$

$$\frac{u}{\sqrt{2}}\sin\theta_1 = 10\times\frac{u}{\sqrt{20}}\sin\theta_2$$

$$\sin\theta_1 = \frac{10}{\sqrt{10}}\sin\theta_2$$

$$\sin\theta_1 = \sqrt{10}\sin\theta_2$$

$\therefore \boxed{n = 10}$ ans

25. A wire of density 9×10^{-3} kg cm^{-3} is stretched between two clamps 1 m apart. The resulting strain in the wire is 4.9×10^{-4}. The lowest frequency of the transverse vibrations in the wire is (Young's modulus of wire Y = 9×10^{10} Nm^{-2}), (to the nearest integer), _______.

Sol. **35**

$$f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{\rho A}} = \frac{1}{2\ell}\sqrt{\frac{Y\Delta\ell}{\rho\ell}}$$

$$f = \frac{1}{2\times 1}\sqrt{\frac{9\times 10^{10}\times 4.9\times 10^{-4}}{9000\times 1}}$$

$= \frac{1}{2}\sqrt{49\times 100}$ = 35 Hz ans

QUESTION PAPER WITH SOLUTION

CHEMISTRY - 2 Sep 2020 - SHIFT - 2

1. Cast iron is used for the manufacture of :

(1) Wrought iron and steel
(2) Wrought iron and pig iron
(3) Wrougth iron, pig iron and steel
(4) Pig iron, scrap iron and steel

Sol. **1**

Refer topic metallurgy

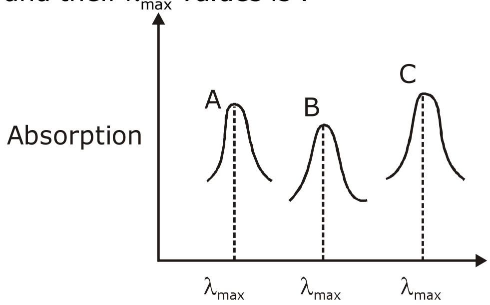
2. The shape/structure of $[XeF_5]^-$ and XeO_3F_2, respectively, are :

(1) Pentagonal planar and trigonal bipyramidal
(2) Trigonal bipyramidal and trigonal bipyramidal
(3) Octahedral and square pyramidal
(4) Trigonal bipyramidal and pentagonal planar

Sol. **1**

$[XeF_5]^-$ 5BP + 2LP = 7VSEP $\Rightarrow$ sp^3d^3 hybridisation
XeO_3F_2 5BP + 0LP = 5VSEP $\Rightarrow$ sp^3d hybridisation

3. Simplified absorption spectra of three complexes ((i), (ii) and (iii)) of M^{n+} ion are provided below; their λ_{max} values are marked as A, B and C respectively. The correct match between the complexes and their λ_{max} values is :



(i) $[M(NCS)_6]^{(-6+n)}$
(ii) $[MF_6]^{(-6+n)}$
(iii) $[M(NH_3)_6]^{n+}$

(1) A-(i), B-(ii), C-(iii)
(2) A-(iii), B-(i), C-(ii)
(3) A-(ii), B-(iii), C-(i)
(4) A-(ii), B-(i), C-(iii)

Sol. **2**

$$\Delta = \frac{hc}{\lambda_{absorbedf(max)}}$$

A $\rightarrow$ NH_3 comp (iii)
B $\rightarrow$ NCS comp (i)
C $\rightarrow$ F^- comp (ii)
using spectrochemical series of ligand
$F^- < NCS^- < NH_3$ order of Δ+e
crystal field spliting energy
So. NH_3 complex $\rightarrow$ A
F^- complex – C
NCS^- complex $\rightarrow$ B

4. The correct observation in the following reactions is :

$$\text{Sucrose} \xrightarrow[\text{(Hydrolysis)}]{\text{Glycosidic bond}}_{\text{Cleavage}} A + B \xrightarrow[\text{reagent}]{\text{Seliwanoff's}} ?$$

(1) Formation of red colour
(2) Formation of blue colour
(3) Formation of violet colour
(4) Gives no colour

Sol. **1**

$$\text{Sucrose} \xrightarrow[\text{Cleavage (Hydrolysis)}]{\text{Glycosidic bond}} \text{Glucose} + \text{Fructose} \xrightarrow[\text{reagent}]{\text{Seliwanoff's}} \text{Red Colour}$$

5. The results given in the below table were obtained during kinetic studies of the following reaction :
$2A + B \rightarrow C + D$

Experiment	[A]/ $molL^{-1}$	[B]/ $molL^{-1}$	Initial rate/ $molL^{-1}\, min^{-1}$
I	0.1	0.1	6.00×10^{-3}
II	0.1	0.2	2.40×10^{-2}
III	0.2	0.1	1.20×10^{-2}
IV	X	0.2	7.20×10^{-2}
V	0.3	Y	2.88×10^{-1}

X and Y in the given table are respectively :

(1) 0.4, 0.4
(2) 0.3, 0.4
(3) 0.4, 0.3
(4) 0.3, 0.3

Sol. **2**

$2A + B \longrightarrow C + D$

Exp. (I) $6 \times 10^{-3} = K\,(0.1)^p\,(0.1)^q$

(II) $2.4 \times 10^{-2} = K\,(0.1)^p\,(0.2)^q$

(III) $1.2 \times 10^{-2} = K\,(0.2)^p\,(0.1)^q$

$$\frac{\exp(I)}{\exp(II)} \qquad \frac{1}{4} = \left(\frac{1}{2}\right)^q \Rightarrow q = 2$$

$$\frac{Exp.(I)}{Exp.(III)} \qquad \frac{1}{2} = \left(\frac{1}{2}\right)^p \Rightarrow p = 1$$

exp. (I) $\div$ exp (IV)

$$\frac{0.6 \times 10^{-2}}{7.2 \times 10^{-2}} = \left(\frac{0.1}{x}\right)^1 \cdot \left[\frac{0.1}{0.2}\right]^2$$

$$\frac{1}{12} = \frac{0.1}{x} - \frac{1}{4}$$

[x] = 0.3

exp (I) $\div$ exp(V)

$$\frac{0.6 \times 10^{-2}}{2.88 \times 10^{-1}} = \left(\frac{0.1}{0.3}\right)^1 \times \left(\frac{0.1}{y}\right)^2$$

$$\frac{1}{48} = \frac{1}{3} \times \frac{10^{-2}}{y^2} \Rightarrow y^2 = 0.16$$

y = 0.4

Ans(2)

6. Match the type of interaction in column A with the distance dependence of their interaction energy in column B :

	A		**B**
(I)	ion-ion	(a)	$\frac{1}{r}$
(II)	dipole-dipole	(b)	$\frac{1}{r^2}$
(III)	London dispersion	(c)	$\frac{1}{r^3}$
		(d)	$\frac{1}{r^6}$

(1) (I)-(a), (II)-(b), (III)-(d)
(2) (I)-(a), (II)-(b), (III)-(c)
(3) (I)-(b), (II)-(d), (III)-(c)
(4) (I)-(a), (II)-(c), (III)-(d)

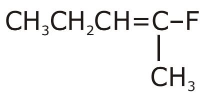
Sol. **4**

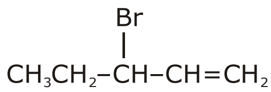
ion - ion α $\frac{1}{r}$

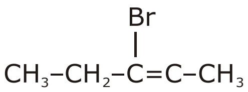
dipole – dipole α $\frac{1}{r^3}$

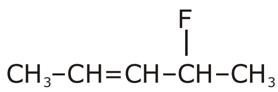
Londong dispersion α $\frac{1}{r^6}$

7. The major product obtained from E_2– elimination of 3-bromo-2-fluoropentane is :

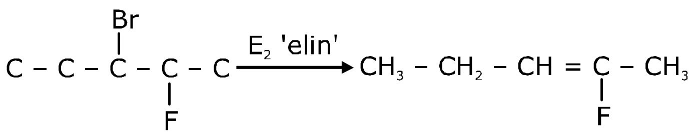
(1) 

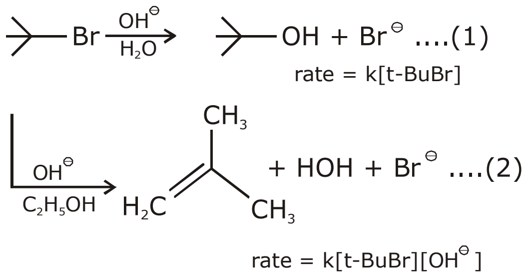
(2) 

(3) 

(4) 

Sol. **1**





Which of the following statements is true :

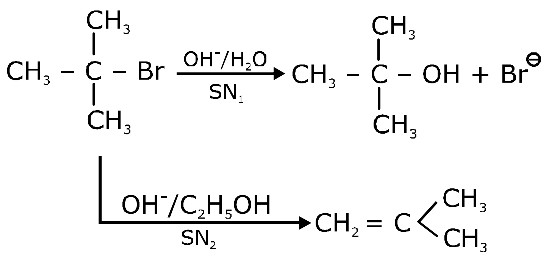
(1) Changing the concentration of base will have no effect on reaction (1).

(2) Doubling the concentration of base will double the rate of both the reactions.

(3) Changing the base from $OH^{\ominus}$ to $^{\ominus}OR$ will have no effect on reaction (2).

(4) Changing the concentration of base will have no effect on reaction (2).

Sol. **1**



9. The size of a raw mango shrinks to a much smaller size when kept in a concentrated salt solution. Which one of the following process can explain this ?

(1) Diffusion
(2) Osmosis
(3) Reverse osmosis
(4) Dialysis

Sol. **2**

Theoritical

Ans. Osmosis

Option (2)

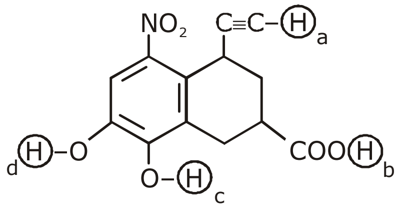
10. If you spill a chemical toiled cleaning liquid on your hand, your first aid would be :

(1) Aqueous NH_3
(2) Aqueous $NaHCO_3$
(3) Aqueous NaOH
(4) Vinegar

Sol. **2**

Fact

11. Arrange the followig labelled hydrogens in decreasing order of acidity :

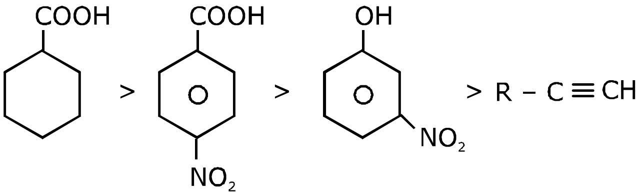


(1) b > a > c > d (2) b > c > d > a

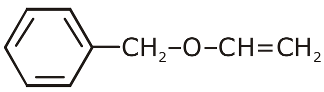
(3) c > b > d > a (4) c > b > a > d

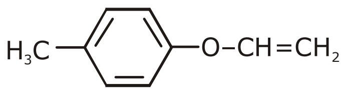
Sol. **2**

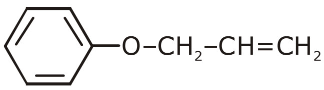
Order of acidic strength

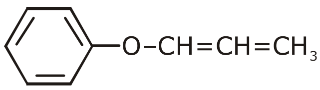


12. An organic compound 'A' ($C_9H_{10}O$) when treated with conc. HI undergoes cleavage to yield compounds 'B' and 'C'. 'B' gives yellow precipitate with $AgNO_3$ where as 'C' tautomerizes to 'D'. 'D' gives positive iodoform test. 'A' could be :

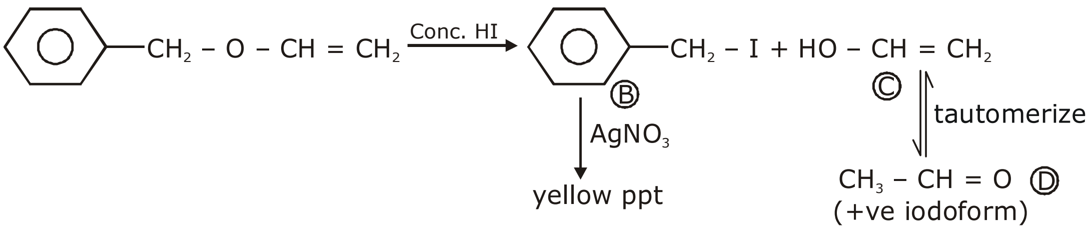
(1) 

(2) 

(3) 

(4) 

Sol. **1**



13. Two elements A and B have similar chemical properties. They don't form solid hydrogencarbonates, but react with nitrogen to form nitrides. A and B, respectively, are :

(1) Na and Ca (2) Cs and Ba

(3) Na and Rb (4) Li and Mg

Sol. **4**

$LiHCO_3$ & $Mg(HCO_3)_2$ does not exist in solid form but both forms nitrides with nitrogen gas

14. The number of subshells associated with n = 4 and m = –2 quantum numbers is :

(1) 4 (2) 8 (3) 2 (4) 16

Sol. **3**

n = 4

$\ell = 0$ m = 0

$\ell = 1$ m = – 1, 0, +1

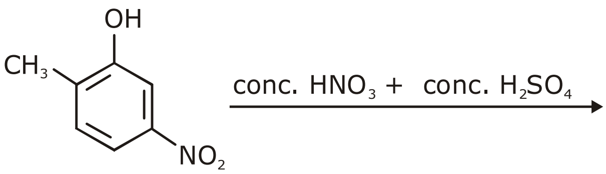
$\ell = 2$ m = – 2, + 2, –1, +1, 0

$\ell = 3$ $m = \pm 3, \pm 2, \pm 1, 0$

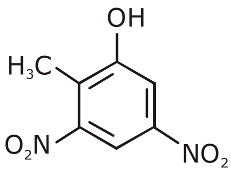
Ans. '2' Subshells

Option (3)

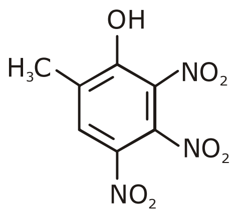
15. The major product of the following reaction is :



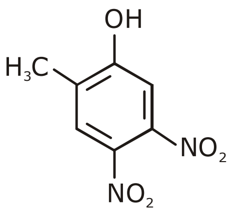
(1)



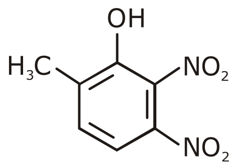
(2)



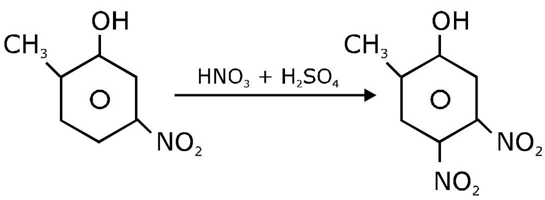
(3)



(4)



Sol. **3**



16. Two compounds A and B with same molecular formula (C_3H_6O) undergo Grignard's reaction with methylmagnesium bromide to give products C and D. Products C and D show following chemical tests.

Test	C	D
Ceric ammonium nitrate Test	Positive	Positive
Lucas Test	Turbidity obtained after five minutes	Turbidity obtained immediately
Iodoform Test	Positive	Negative

C and D respectively are :

(1) $C = H_3C-CH_2-CH_2-CH_2-OH$;

$$D = H_3C-CH_2-\underset{\underset{OH}{|}}{C}H-CH_3$$

(2) $$C = H_3C-CH_2-\overset{\overset{OH}{|}}{C}H-CH_3;$$

$$D = H_3C-\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}-OH$$

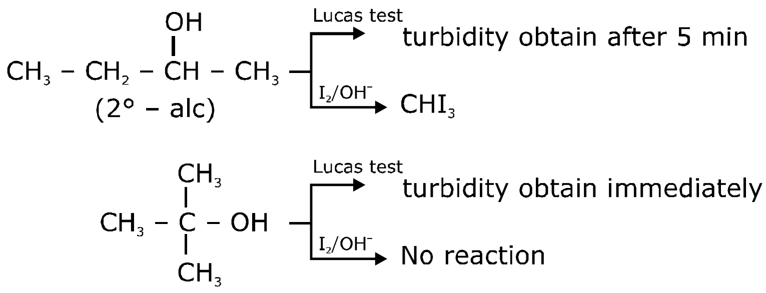
(3) $C = H_3C-CH_2-CH_2-CH_2-OH$;

$$D = H_3C-\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}-OH$$

(4) $$C = H_3C-\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}-OH$$

$$D = H_3C-CH_2-\underset{\underset{OH}{|}}{C}H-CH_3$$

Sol. **2**



17. Three elements X, Y and Z are in the 3rd peroid of the periodic table. The oxides of X, Y and Z, respectively, are basic, amphoteric and acidic, The correct order of the atomic numbers of X, Y and Z is :

(1) X < Y < Z (2) Y < X < Z
(3) Z < Y < X (4) X < Z < Y

Sol. **1**

x <	y <	z
Mg	Al	Si
Basic oxide	amphoteric	acidic oxide

18. The one that is not expected to show isomerism is :

(1) $[Ni(NH_3)_4(H_2O)_2]^{2+}$ (2) $[Ni(en)_3]^{2+}$
(3) $[Pt(NH_3)_2Cl_2]$ (4) $[Ni(NH_3)_2Cl_2]$

Sol. **4**

$[Ni(NH_3)_2Cl_2]$ Ni^{2+} is sp^3 hybridised & such tetrahedral complex does not show either of geometrical or optical isomerism

$[Ni(en)_3]^{2+}$ shows only optical isomers while other three shows geometrical isomerism

19. Amongst the following statements regarding adsorption, those that are valid are :

(a) ΔH becomes less negative as adsorption proceeds.
(b) On a given adsorbent, ammonia is adsorbed more than nitrogen gas.
(c) On adsorption, the residual force acting along the surface of the adsorbent increases.
(d) With increase in temperature, the equilibrium concentration of adsorbate increases.

(1) (b) and (c) (2) (c) and (d)
(3) (a) and (b) (4) (d) and (a)

Sol. Statement 'a' & 'b'

20. The molecular geometry of SF_6 is octahdral. What is the geometry of SF_4 (including lone pair(s) of electrons, if any) ?

(1) Pyramidal (2) Trigonal bipyramidal
(3) Tetrahedral (4) Square planar

Sol. **2**

SF_4 is Sp^3d hybridised in which hybrid orbitals have TBP arrangement but its shape is sea-saw

21. The ratio of the mass percentages of 'C & H' and 'C & O' of a saturated acyclic organic compound 'X' are 4 : 1 and 3 : 4 respectively. Then, the moles of oxygen gas required for complete combustion of two moles of organic compound 'X' is ____________.

Sol. Mass ratio of C : H is 4 : 1 $\Rightarrow$ 12 : 3
& C : O is 3 : 4 $\Rightarrow$ 12 : 16
So,

	mass	mole	mole ratio
C	12	1	1
H	3	3	3
O	16	1	1

Empirical formula $\Rightarrow CH_3O$
as compound is satured a cyclic so,molecular formula is $C_2H_6O_2$.

$$\underset{2\,mole}{C_2H_6O_2} + \underset{5\ mole}{\frac{5}{2}O_{2(g)}} \longrightarrow 2CO_{2(g)} + 3H_2O_{(g)}$$

So, required moles of O_2 is $\Rightarrow$ 5

22. For the disproportionation reaction $2Cu^+(aq) \rightleftharpoons Cu(s) + Cu^{2+}(aq)$ at K, ln K (where K is the equilibrium constant) is _________ $\times 10^{-1}$.
Given :

$(E^0_{Cu^{2+}/Cu^+} = 0.16\,V$

$E^0_{Cu^+/Cu} = 0.52\,V$

$\frac{RT}{F} = 0.025)$

Sol. **144**

$2Cu^+ \rightleftharpoons Cu(s) + Cu^{+2}$

$E^o = 0.52 - 0.16$
$= 0.36$

$E^o = \frac{RT}{nF} \ln(k_{eq})$

$\ln(k_{eq}) = \frac{0.36}{0.025} \times \frac{1}{1}$

$= \frac{360}{25} = 14.4$

$= 144 \times 10^{-1}$

Ans. 144

23. The work function of sodium metal is 4.41×10^{-19}J. If photons of wavelength 300 nm are incident on the metal, the kinetic energy of the ejected electrons will be ($h = 6.63 \times 10^{-34}$ J s; $c = 3 \times 10^{8}$ m/s) __________ $\times 10^{-21}$ J.

Sol. **222**

$\phi = 4.41 \times 10^{-19}$ J

$\lambda = 300$ nm

$$KE_{max} = \frac{hc}{\lambda} - \phi$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{300 \times 10^{-9}} - 4.41 \times 10^{-19}$$

$= 6.63 \times 10^{-19} - 4.41 \times 10^{-19}$

$= 222 \times 10^{-21}$

Ans. 222

24. The oxidation states of transition metal atoms in $K_2Cr_2O_7$, $KMnO_4$ and K_2FeO_4, respectively, are x, y and z. The sum of x, y and z is ________.

Sol. **19**

$K_2\overset{+6}{Cr_2}O_7 \qquad K\overset{+7}{Mn}O_4 \qquad K_2[\overset{+6}{Fe}O_4]$

25. The heat of combustion of ethanol into carbon dioxide and water is −327 kcal at constant pressure. The heat evolved (in cal) at constant volume and 27ºC (if all gases behave ideally) is (R = 2 cal mol^{-1} K^{-1}) __________.

Sol. $\Delta H_c^0\,[C_2H_5OH] = -327$ kcal

$C_2H_5OH(l) + 3O_2(g) \longrightarrow 2CO_2(g) + 3(H_2O)(l)$

$\Delta E_c^0 \quad = \Delta H_c^0 - \Delta ngRT$

$= -\,327 \times 1000 - (-1) \times 2 \times 300$

$= -\,327000 + 600$

$= -\,326400$

QUESTION PAPER WITH SOLUTION

MATHEMATICS - 2 Sep 2020 - SHIFT - 2

Q.1 Let $f : R \to R$ be a function which satisfies $f(x+y) = f(x) + f(y)\ \forall x, y \in R$. If f(1) = 2 and $g(n) = \sum_{k=1}^{(n-1)} f(k), n \in N$ then the value of n, for which g(n) = 20, is:

(1) 9 (2) 5 (3) 4 (4) 20

Sol. **(2)**

f(1) = 2 ; f(x+y) =f(x)+f(y)

x = y = 1 $\Rightarrow$ f(2) = 2+2 = 4

x = 2, y=1 $\Rightarrow$ f(3)=4+2 = 6

g(n)=f(1)+f(2)+.........+f(n-1)

= 2 + 4 + 6 + ...+ 2(n-1)

= 2 $\sum$(n-1)

$= 2\,\frac{(n-1).n}{2}$

$= n^2 - n$

Given g(n) = 20 $\Rightarrow n^2 - n = 20$

$n^2 - n - 20 = 0$

n = 5

Q.2 If the sum of first 11 terms of an A.P., a_1, a_2, a_3, ... is 0($a_1 \neq 0$) then the sum of the A.P., a_1, a_3, a_5, ..., a_{23} is ka_1, where k is equal to:

(1) $-\frac{121}{10}$ (2) $-\frac{72}{5}$ (3) $\frac{72}{5}$ (4) $\frac{121}{10}$

Sol. **(2)**

$\sum_{k=1}^{11} a_k = 0 \Rightarrow 11a + 55d = 0$

a + 5d = 0

Now $a_1 + a_3 + ... + a_{23} = ka_1$

12a + d (2+4+6+.....+22) = ka

12a + 2d. 66 = ka

12(a+11d) = ka

$12\left(a + 11\left(-\frac{a}{5}\right)\right) = ka$

$12\left(1 - \frac{11}{5}\right) = k$

$k = -\frac{72}{5}$

Q.3 Let E^C denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1)>0$ and $P(E_1 \cap E_2 \cap E_3) = 0$. Then $P\left(E_2^C \cap E_3^C / E_1\right)$ is equal to:

(1) $P\left(E_3^C\right) - P\left(E_2^C\right)$ (2) $P\left(E_3\right) - P\left(E_2^C\right)$

(3) $P\left(E_3^C\right) - P\left(E_2\right)$ (4) $P\left(E_2^C\right) + P\left(E_3\right)$

Sol. **(3)**

$$P\left(E_2^c \cap E_3^c / E_1\right) = \frac{P(E_2^c \cap E_3^c \cap E_1)}{P(E_1)}$$

$$= \frac{P(E_1) - P(E_1 \cap E_2) - P(E_1 \cap E_3) + P(E_1 \cap E_2 \cap E_3)}{P(E_1)}$$

$$= \frac{P(E_1) - P(E_1).P(E_2) - P(E_1).P(E_3) + 0}{P(E_1)} = 1 - P(E_2) - P(E_3)$$

$$= P\left(E_3^c\right) - P\left(E_2\right)$$

Q.4 If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval:

(1) $\left(-\frac{1}{2}, -\frac{1}{4}\right]$ (2) $\left[-1, -\frac{1}{2}\right]$ (3) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$ (4) $\left(-\frac{5}{4}, -1\right)$

Sol. **(2)**

$\cos^4\theta + \sin^4\theta + \lambda = 0$

$$\lambda = -\left\{1 - \frac{1}{2}\sin^2 2\theta\right\}$$

$2(\lambda+1) = \sin^2 2\theta$

$0 \leq 2(\lambda+1) \leq 1$

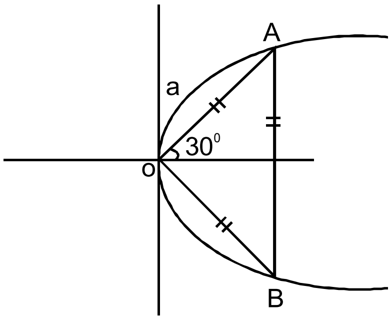
$$0 \leq \lambda + 1 \leq \frac{1}{2}$$

$$\boxed{-1 \leq \lambda \leq -\frac{1}{2}}$$

Q.5 The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is:

(1) $128\sqrt{3}$ (2) $192\sqrt{3}$ (3) $64\sqrt{3}$ (4) $256\sqrt{3}$

Sol. **(2)**



A : (a cos 30⁰, a sin 30⁰)
lies on parabola

$$\frac{a^2}{4} = 8.\frac{a.\sqrt{3}}{2}$$

$$\boxed{a = 16\sqrt{3}}$$

Area of equilateral $\Delta = \frac{\sqrt{3}}{4}a^2$

$$\Delta = \frac{\sqrt{3}}{4}.\ 16.\ 16.\ 3$$

$$\Delta = 192\sqrt{3}$$

Q.6 The imaginary part of $\left(3+2\sqrt{-54}\right)^{1/2} - \left(3-2\sqrt{-54}\right)^{1/2}$ can be :

(1) $\sqrt{6}$ (2) $-2\sqrt{6}$ (3) 6 (4) $-\sqrt{6}$

Sol. **(2)**

$$\left(3+2i\sqrt{54}\right)^{1/2} - \left(3-2i\sqrt{54}\right)^{1/2}$$

$$= \left(9+6i^2+2.3i\sqrt{6}\right)^{1/2} - \left(9+6i^2-2.3i\sqrt{6}\right)^{1/2}$$

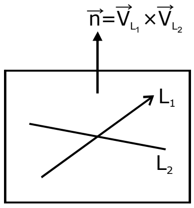
$$= \left(\left(3+\sqrt{6}i\right)^2\right)^{1/2} - \left(\left(3-\sqrt{6}i\right)^2\right)^{1/2}$$

$$= \pm\left(3+\sqrt{6}i\right) \mp \left(3-\sqrt{6}i\right) = -2\sqrt{6}\ i$$

Q.7 A plane passing through the point (3,1,1) contains two lines whose direction ratios are 1,−2,2 and 2,3, −1 respectively. If this plane also passes through the point (α,−3,5), then α is equal to:

(1) −5 (2) 10 (3) 5 (4) −10

Sol. **(3)**



$\vec{n}_p = (-4, 5, 7)$

Equation of plane :

P : −4(x-3)+5(y-1)+7(z-1)=0

P : −4x + 5y + 7z + 12 - 5 - 7 = 0

P : 4x - 5y - 7z = 0

Pass (α, -3,5)

$4\alpha + 15 - 35 = 0$

$4\alpha = 20$

$\alpha=5$

Q.8 Let A={X=(x, y, z)^T : PX=0 and $x^2+y^2+z^2 = 1$} ,where $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$, then the set A:

(1) contains more than two elements (2) is a singleton.

(3) contains exactly two elements (4) is an empty set.

Sol. **(3)**

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ & $x^2 + y^2 + z^2 = 1$

Px = O

$$\begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

x + 2y + z = 0........(1)

-2x + 3y − 4z = 0....(2)

x + 9y -z = 0..........(3)

from (1) & (3)

$\Rightarrow$ 2x+11y =0

from (1) & (2)

$\Rightarrow$ 2x + 11y = 0

from (2) & (3)
$-6x - 33y = 0$
$\Rightarrow 2x + 11y = 0$
put in (1)
$-7y + 2z = 0$

Now $\left(\frac{11y}{2}\right)^2 + y^2 + \left(\frac{7y}{2}\right)^2 = 1$

$y^2(121 + 1 + 49) = 4$
$y^2(171) = 4$

$y = \pm \frac{2}{\sqrt{171}} \Rightarrow x = \pm \frac{7}{\sqrt{171}} \Rightarrow z = \mp \frac{11}{\sqrt{171}} \Rightarrow$ Only two pair possible

Q.9 The equation of the normal to the curve $y=(1+x)^{2y} + \cos^2(\sin^{-1}x)$ at $x=0$ is:
(1) $y+4x=2$ (2) $2y+x=4$ (3) $x+4y=8$ (4) $y=4x+2$

Sol. **(3)**
at $x = 0 \Rightarrow y = 1 + \cos^2(0)=2$
$p : (0,2)$

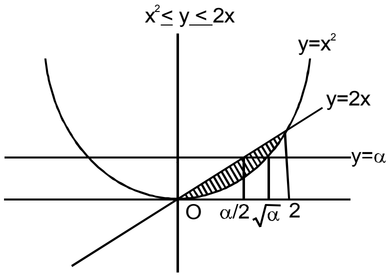
Now $y^1 = (1+x)^{2y}\left\{\frac{2y}{1+x} + \ln(1+x).2y\right\} - \sin 2(\sin^{-1}x) . \frac{1}{\sqrt{1-x^2}}$

$y\Big|_{(0,2)} = 4 - 0$

$N_o : y - 2 = -\frac{1}{4}(x-0)$
$N_o : 4y - 8 = -x$
$N_o : \boxed{x + 4y = 8}$

Q.10 Consider a region $R = \{(x, y) \in R^2 : x^2 \le y \le 2x\}$. If a line $y = \alpha$ divides the area of region R into two equal parts, then which of the following is true.?
(1) $\alpha^3 - 6\alpha^2 + 16 = 0$ (2) $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$
(3) $\alpha^3 - 6\alpha^{3/2} - 16$ (4) $3\alpha^2 - 8\alpha + 8 = 0$

Sol. **(2)**



$A = \text{Area} = \int_0^2 (2x - x^2)\,dx = 4 - \frac{8}{3} = \boxed{\frac{4}{3}}$

Now $\int_0^{\alpha/2} (2x - x^2)\,dx + \int_{\alpha/2}^{\sqrt{\alpha}} (\alpha - x^2)\,dx = \frac{1}{2}A$

$$\frac{\alpha^2}{4} - \frac{\alpha^3}{24} + \alpha\left(\sqrt{\alpha} - \alpha/2\right) - \left(\frac{\alpha\sqrt{\alpha}}{3} - \frac{\alpha^3}{24}\right) = \frac{4}{6}$$

$$\frac{\alpha^2}{4} + \alpha\sqrt{\alpha} - \frac{\alpha^2}{2} - \frac{\alpha\sqrt{\alpha}}{3} = \frac{4}{6}$$

$-3\alpha^2 + 8\alpha\sqrt{\alpha} = 8$

$3\alpha^2 - 8\alpha\sqrt{\alpha} + 8 = 0$

Q.11 Let $f:(-1,\infty) \to R$ be defined by f(0)=1 and $f(x) = \frac{1}{x}\log_e(1+x), x \neq 0$. Then the function f:

(1) increases in $(-1,\infty)$
(2) decreases in $(-1,0)$ and increases in $(0,\infty)$
(3) increases in $(-1,0)$ and decreases in $(0,\infty)$
(4) decreases in $(-1,\infty)$.

Sol. **(4)**

$f(x) = \frac{1}{x}\ell n(1+x)$

$$f' = \frac{x - \frac{1}{1+x} - \text{In}(1+x)}{x^2}$$

$$f' = \frac{1 - \frac{1}{1+x} - \ln(1+x)}{x^2}$$

$f' < 0 \quad \forall x \in (-1,\infty)$

Q.12 Which of the following is a tautology?

(1) $(p \to q) \wedge (q \to p)$
(2) $(\sim p) \wedge (p \vee q) \to q$
(3) $(q \to p) \vee \sim(p \to q)$
(4) $(\sim q) \vee (p \wedge q) \to q$

Sol. **(2)**

$\sim p$	pvq	$\sim p \wedge (p \vee q)$	$\sim p \wedge (p \vee q) \to q$
F	T	F	T
F	T	F	T
T	T	T	T
T	F	F	T

Q.13 Let f(x) be a quadratic polynomial such that f(−1)+f(2)=0. If one of the roots of f(x)=0 is 3, then its other roots lies in:

(1) (0,1) (2) (1,3) (3) (−1,0) (4) (−3,−1)

Sol. **(3)**

Let $f(x) = a(x-3)(x-\alpha)$

$f(-1)+f(2)=0$

$a[(-1-3)(-1-\alpha)+(2-3)(2-\alpha)]=0$

$a[4+4\alpha-2+\alpha]=0$

$5\alpha+2=0$

$\boxed{\alpha=-\frac{2}{5}}$

Q.14 Let S be the sum of the first 9 terms of the series :

$\{x+ka\}+\{x^2+(k+2)a\}+\{x^3+(k+4)a\}+\{x^4+(k+6)a\}+...$ where $a \neq 0$ and $a \neq 1$.

If $S=\frac{x^{10}-x+45a(x-1)}{x-1}$, then k is equal to:

(1) 3 (2) −3 (3) 1 (4) −5

Sol. **(2)**

$S = \{x+ka\} + \{x^2+(k+2)a\} + \{x^3+(k+4)a\}$ up to 9 term

$S = (x+x^2+....+x^9)+a\{k+(k+2)+(k+4)+......$up to 9 term$)\}$

$S = \frac{x(1-x^9)}{1-x} + a\{9k+2.36\}$

$S = \frac{x^{10}-x}{x-1} + 9ak + 72a$

$S = \frac{x^{10}-x+45a(x-1)}{x-1} = \frac{x^{10}-x+(9k+72)a(x-1)}{x-1}$

$= 45 = 9k + 72$

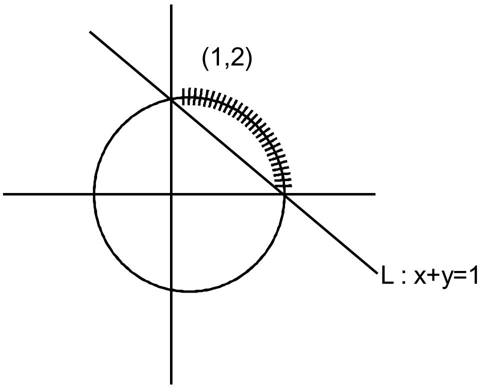
$9k = -27$

$k = -3$

Q.15 The set of all possible values of θ in the interval $(0,\pi)$ for which the points (1,2) and $(\sin\theta,\cos\theta)$ lie on the same side of the line x+y=1 is:

(1) $\left(0,\frac{\pi}{4}\right)$ (2) $\left(0,\frac{\pi}{2}\right)$ (3) $\left(0,\frac{3\pi}{4}\right)$ (4) $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$

Sol. **(2)**



(sinθ, cosθ) lie on $x^2+y^2=1$

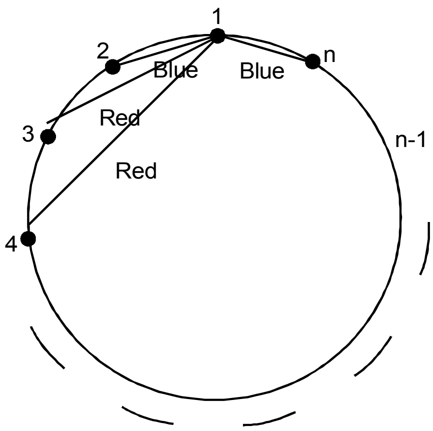
Shaded points satisfy

$\Rightarrow\ \theta \in (0, \pi/2)$

Q.16 Let n>2 be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is:

(1) 201 (2) 199 (3) 101 (4) 200

Sol. **(1)**



Red line = 99 blue line

$^nC_2 - n = 99\,n$

$\frac{n(n-1)}{2} = 100\,n$

$n-1 = 200$

$\boxed{n = 201}$

Q.17 If a curve y=f(x), passing through the point (1,2) is the solution of the differential equation, $2x^2dy=(2xy+y^2)dx$, then $f\left(\frac{1}{2}\right)$ is equal to:

(1) $\frac{-1}{1+\log_e 2}$ (2) $1+\log_e 2$ (3) $\frac{1}{1+\log_e 2}$ (4) $\frac{1}{1-\log_e 2}$

Sol. **(3)**

$2\frac{dy}{dx}=2\frac{y}{x}+\left(\frac{y}{x}\right)^2 \rightarrow$HDE

$\because$ y = vx

$2\left(v+x\frac{dv}{dx}\right) = 2v+v^2$

$2\frac{dv}{v^2}=\frac{dx}{x}$

$-\frac{2}{v} = \ln x + c$

$-\frac{2x}{y} = \ln x + c$

$\downarrow$ (1,2)

c = -1

$c : \ln x + \frac{2x}{y} = 1$

For $f(1/2) \Rightarrow \ln\left(\frac{1}{2}\right)+\frac{2}{2y}=1$

$y = \frac{1}{1+\ln 2}$

Q.18 For some $\theta \in \left(0,\frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2-y^2\sec^2\theta=10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2\sec^2\theta+y^2=5$, then the length of the latus rectum of the ellipse, is:

(1) $\frac{4\sqrt{5}}{3}$ (2) $\frac{2\sqrt{5}}{3}$ (3) $2\sqrt{6}$ (4) $\sqrt{30}$

Sol. **(1)**

H : $x^2 - y^2 \sec^2\theta = 10$

E : $x^2\sec^2\theta + y^2 = 5$

$$\sqrt{1+\frac{10\cos^2\theta}{10}}=\sqrt{5}\sqrt{1-\frac{5\cos^2\theta}{5}}$$

$1 + \cos^2\theta = 5 - 5\cos^2\theta$

$6\cos^2\theta = 4$

$$\cos\theta = \pm\sqrt{\frac{2}{3}}$$

$$l(LR)\text{ of ellipse} = \frac{2.5\cos^2\theta}{\sqrt{5}}$$

$$= 2\sqrt{5}.\frac{2}{3} = \boxed{\frac{4\sqrt{5}}{3}}$$

Q.19 $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$ is equal to:

(1) e (2) e^2 (3) 2 (4) 1

Sol. **(2)**

$$\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}\ \left(1^{\infty}\right)=e^{L}$$

$$L = \lim_{x\to 0}\frac{\tan\left(\frac{\pi}{4}+x\right)-1}{x}$$

$$L = \lim_{x\to 0}\frac{\frac{1+\tan x}{1-\tan x}-1}{x}$$

$$L = \lim_{x\to 0}2\left(\frac{\tan x}{x}\right).\left(\frac{1}{1-\tan x}\right)$$

$L = +2$

Ans. e^2

Q.20 Let a, b, c $\in$ R be all non-zero and satisfy $a^3+b^3+c^3=2$. If the matrix

$$A=\begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$$

satisfies $A^TA=I$, then a value of abc can be:

(1) $\frac{2}{3}$ (2) 3 (3) $-\frac{1}{3}$ (4) $\frac{1}{3}$

Sol. **(4)**

$a^3+b^3+c^3=2$

$A^TA = I$

$$\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= a^2+b^2+c^2 = 1$

& $ab + bc + ca = 0$

Now $(a+b+c)^2 = \sum a^2 + 2\sum ab$

$\left(\sum a\right)^2 = 1 + 0 \Rightarrow \left(\sum a\right)^2 = 1 \Rightarrow \sum a = \pm 1$

Now $\sum a^3 - 3abc = \left(\sum a\right)\left(\sum a^2 - \sum ab\right)$

$2 - 3\,abc = \pm 1\ (1-0)$

$2 - 3abc = \pm 1$

(+) (-)

$3\,abc = 1$ | $3\,abc = 3$

$\boxed{abc = \frac{1}{3}}$ | $\boxed{abc = 1}$

Q.21 Let the position vectors of points 'A' and 'B' be $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio $\lambda:1(\lambda>0)$. If O is the origin and $\vec{OB}.\vec{OP}-3\left|\vec{OA}\times\vec{OP}\right|^2=6$, then λ is equal to_____

Sol. **0.8**

$\overrightarrow{OA} = \langle 1,1,1 \rangle$, $\overrightarrow{OB} = \langle 2,1,3 \rangle$

λ : l

A p B

$\overrightarrow{OP} = \left(\frac{2\lambda+1}{\lambda+1}, 1, \frac{3\lambda+1}{\lambda+1}\right)$

$\overrightarrow{OB}.\overrightarrow{OP} = \frac{2(2\lambda+1)}{\lambda+1} + 1 + \frac{3(3\lambda+1)}{\lambda+1}$

$= \frac{14\lambda+6}{\lambda+1}$

$$\left|\overrightarrow{OA} \times \overrightarrow{OP}\right|^2 = |\overrightarrow{OA}|^2 |\overrightarrow{OP}|^2 - \left(\overrightarrow{OA}.\overrightarrow{OP}\right)^2$$

$$3.\left(\frac{(2\lambda+1)^2+(\lambda+1)^2+(3\lambda+1)^2}{(\lambda+1)^2}\right) - \left(\frac{2\lambda+1+\lambda+1+3\lambda+1}{\lambda+1}\right)^2$$

$$= \frac{1}{(\lambda+1)^2}\left\{3\left(14\lambda^2+12\lambda+3\right)-\left(6\lambda+3\right)^2\right\}$$

$$= \frac{1}{\left(\lambda+1\right)^2}\left\{6\lambda^2\right\}$$

Now $\frac{14\lambda+6}{\lambda+1} - 3\left(\frac{6\lambda^2}{\left(\lambda+1\right)^2}\right) = 6$

$(14\lambda+6)(\lambda+1) - 18\lambda^2 = 6(\lambda+1)^2$

$-4\lambda^2 + 20\lambda + 6 = 6\lambda^2 + 12\lambda + 6$

$10\lambda^2 - 8\lambda = 0$

$\lambda(10\lambda - 8) = 0$

$\because \lambda > 0$

$\boxed{\lambda = .8}$

Q.22 Let [t] denote the greatest integer less than or equal to t. Then the value of $\int_1^2 \left|2x-[3x]\right| dx$ is ____

Sol. **1**

$$\int_1^2 |2x - \left[3x\right]| dx$$

$3x = t$

$$= \frac{1}{3}\int_3^6 \left|\frac{2t}{3} - [t]\right| dt$$

$$= \frac{1}{9}\left[\int_3^6 |2t - 3[t]|\right] dt$$

$$= \frac{1}{9}\left[\int_3^4 |2t-9| + \int_4^5 |2t-12| + \int_5^6 |2t-15|\right] dt$$

$$= \frac{1}{9}\left[\int_3^4 (9-2t) + \int_4^5 (12-2t) + \int_5^6 (15-2t)\right] dt$$

$=\frac{1}{9}\left[9.1+12.1+15.1-\left[4^2-3^2\right]-\left[5^2-4^2\right]-\left[6^2-5^2\right]\right]$

$=\frac{1}{9}\left[36-\left[4^2-3^2+5^2-4^2+6^2-5^2\right]\right]$

$=\frac{1}{9}\left[36-36+9\right]=1$

Q.23 If $y=\sum_{k=1}^{6} k\cos^{-1}\left\{\frac{3}{5}\cos kx-\frac{4}{5}\sin kx\right\}$, then $\frac{dy}{dx}$ at x=0 is _____

Sol. **91**

$y=\sum_{k=1}^{6} k\cos^{-1}\left\{\cos\left(kx+\theta\right)\right\}$

where $\tan\theta=\frac{4}{3}$

$y=\cos^{-1}(\cos(x+\theta)+2\cos^{-1}(\cos(2x+\theta))\ldots\ldots+6\cos^{-1}(\cos(6x+\theta))$

$\left.\frac{dy}{dx}\right|_{x=0}=\frac{\sin(\theta)}{\sqrt{1-\cos^2\theta}}+\ldots\ldots..$

$=1.1+2.2+3.3+\ldots..+6.6$

$=\sum 6^2=\frac{6.7.13}{6}=91$

Q.24 If the variance of the terms in an increasing A.P., b_1, b_2, b_3, ..., b_{11} is 90, then the common difference of this A.P. is _____

Sol. **3**

$\text{Var}(x)=\frac{\sum bi^2}{11}-\left(\frac{\sum bi}{11}\right)^2$

$90=\frac{a^2+(a+d)^2+(a+2d)^2+\ldots.+(a+10d)^2}{11}$

$\left(\frac{a+a+d+a+2d+\ldots+(a+10d)}{11}\right)^2$

$10890=11\left\{11a^2+385d^2+110ad\right\}-\left\{11a+55d\right\}^2$

$10890=1210d^2$

$d=3$

Q.25 For a positive integer n, $\left(1+\frac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2:5:12, then n is equal to _____

Sol. **118**

Let 3 consecutive coH are

$^nC_{r-1} : {}^nC_r : {}^nC_{r+1} :: 2:5:12$

$\frac{^nC_{r-1}}{^nC_r} = \frac{2}{5}$ & $\frac{^nC_r}{^nC_{r+1}} = \frac{5}{12}$

$\frac{r}{n-r+1} = \frac{2}{5}$ & $\frac{r+1}{(n-r)} = \frac{5}{12}$

7r = 2n + 2 & 17r = 5n-12

$\Rightarrow \frac{2n+2}{7} = \frac{5n-12}{17}$

= 34n + 34 = 35n — 84

$\Rightarrow$ n = 118