# JEE Main 2020 Paper

Date: **8th January 2020 (Shift 2)**
Time: **2:30 P.M. to 5:30 P.M.**
Subject: **Mathematics**

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1. Solution set of $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$ contains
   a. exactly one element
   b. at least four elements
   c. two elements
   d. infinite elements

   **Answer**: $(a)$

   **Solution:**

   $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$

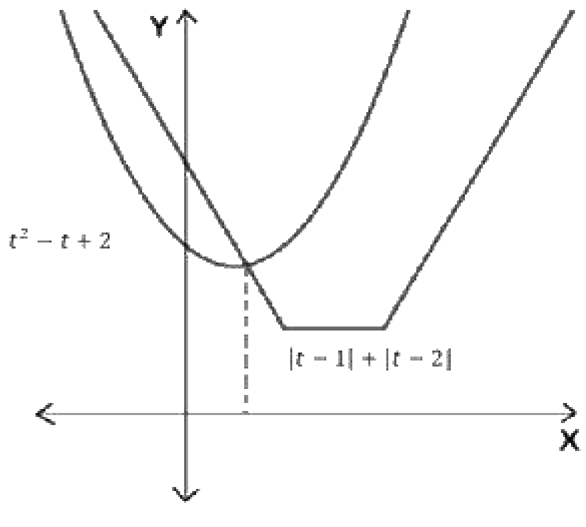
   Let $3^x = t$

   $t(t - 1) + 2 = |t - 1| + |t - 2|$

   $\Rightarrow t^2 - t + 2 = |t - 1| + |t - 2|$

   We plot $t^2 - t + 2$ and $|t - 1| + |t - 2|$

   As $3^x$ is always positive, therefore only positive values of $t$ will be the solution.




   Therefore, we have only one solution.

2. Which of the following is a tautology?
   a. $\sim(p \wedge \sim q) \rightarrow (p \vee q)$
   b. $(\sim p \vee q) \rightarrow (p \vee q)$
   c. $\sim(p \vee \sim q) \rightarrow (p \vee q)$
   d. $\sim(p \vee \sim q) \rightarrow (p \wedge q)$

   **Answer**: $(c)$

   **Solution:**

$\sim(p \vee \sim q) \rightarrow (p \vee q)$

$= (p \vee \sim q) \vee (p \vee q)$

$= (p \vee p) \vee (q \vee \sim q)$

$= p \vee T$

$= T$

3. If a hyperbola has vertices $(\pm 6,\ 0)$ and $P(10,\ 16)$ lies on it, then the equation of normal at $P$ is

a. $2x + 5y = 10$
b. $2x + 5y = 100$
c. $2x - 5y = 100$
d. $5x + 2y = 100$

**Answer:** $(b)$

**Solution:**

Vertex of hyperbola is $(\pm a, 0) \equiv (\pm 6, 0) \Rightarrow a = 6$

Let the equation of hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{36} - \frac{y^2}{b^2} = 1$

As $P(10, 16)$ lies on the hyperbola.

$\frac{100}{36} - \frac{256}{b^2} = 1$

$\Rightarrow \frac{64}{36} = \frac{256}{b^2} \Rightarrow b^2 = 144$

Equation of hyperbola becomes $\frac{x^2}{36} - \frac{y^2}{144} = 1$

Equation of normal is $\frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 + b^2$

$\Rightarrow \frac{36x}{10} + \frac{144y}{16} = 180$

$\Rightarrow \frac{x}{50} + \frac{y}{20} = 1$

$\Rightarrow 2x + 5y = 100$

4. If $y = mx + c$ is a tangent to the circle $(x-3)^2 + y^2 = 1$ and also perpendicular to the tangent to the circle $x^2 + y^2 = 1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then

a. $c^2 - 6c - 7 = 0$
b. $c^2 - 6c + 7 = 0$
c. $c^2 + 6c - 7 = 0$
d. $c^2 + 6c + 7 = 0$

**Answer:** $(d)$

**Solution:**

For circle, $x^2 + y^2 = 1$

$2x + 2yy' = 0 \Rightarrow y' = -\dfrac{x}{y}$

Slope of tangent to $x^2 + y^2 = 1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -1$

$\Rightarrow$Slope of tangent to $(x-3)^2 + y^2 = 1$ is $1 \Rightarrow m = 1$

Tangent to $(x-3)^2 + y^2 = 1$ is $y = x + c$

Perpendicular distance of tangent $y = x + c$ from centre $(3, 0)$ is equal to radius $= 1$

$\left|\dfrac{3+c}{\sqrt{2}}\right| = 1$

$\Rightarrow c + 3 = \pm\sqrt{2}$

$\Rightarrow c^2 + 6c + 9 = 2$

$\Rightarrow c^2 + 6c + 7 = 0$

5. If $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{c}$ is non-zero vector and $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$, $\vec{a}.\vec{c} = 0$ then $\vec{b}.\vec{c}$ is equal to

a. $\frac{1}{2}$

b. $-\frac{1}{3}$

c. $-\frac{1}{2}$

d. $\frac{1}{3}$

**Answer:** $(c)$

**Solution:**

$\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$

$\Rightarrow \vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b} \times \vec{a})$

$\Rightarrow (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = (\vec{a}.\vec{a})\vec{b} - (\vec{a}.\vec{b})\vec{a}$

$\Rightarrow -(\vec{a}.\vec{b})\vec{c} = (\vec{a}.\vec{a})\vec{b} - (\vec{a}.\vec{b})\vec{a}$

$\Rightarrow -4\vec{c} = 6(\hat{\imath} - \hat{\jmath} + \hat{k}) - 4(\hat{\imath} - 2\hat{\jmath} + \hat{k})$

$\Rightarrow \vec{c} = -\frac{1}{2}(\hat{\imath} + \hat{\jmath} + \hat{k})$

$\therefore \vec{b}.\vec{c} = -\frac{1}{2}$.

6. If the coefficient of $x^4$ and $x^2$ in the expansion of $\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$ is $\alpha$ and $\beta$ ,then $\alpha-\beta$ is equal to

a. 48
b. $-60$
c. 60
d. $-132$

**Answer:** $(d)$

**Solution:**

$\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$

$=2[\,{}^6C_0x^6+{}^6C_2x^4(x^2-1)+{}^6C_4x^2(x^2-1)^2+{}^6C_6(x^2-1)^3]$

$=2[32x^6-48x^4+18x^2-1]$

$\Rightarrow \alpha=-96, \qquad \beta=36$

$\Rightarrow \alpha-\beta=-132$

7. Differential equation of $x^2=4b(y+b)$, where $b$ is a parameter, is

a. $x\left(\frac{dy}{dx}\right)^2=2y\left(\frac{dy}{dx}\right)+x$
b. $x\left(\frac{dy}{dx}\right)^2=2y\left(\frac{dy}{dx}\right)+x^2$
c. $x\left(\frac{dy}{dx}\right)^2=y\left(\frac{dy}{dx}\right)+x^2$
d. $x\left(\frac{dy}{dx}\right)^2=y\left(\frac{dy}{dx}\right)+2x^2$

**Answer**: $(a)$

**Solution:**

$x^2=4b(y+b)$ ... (1)

Differentiating both the sides w.r.t. $x$, we get

$\Rightarrow 2x=4by'$

$\Rightarrow b=\frac{x}{2y'}$

Putting the value of $b$ in $(1)$, we get

$\Rightarrow x^2=\frac{2x}{y'}\left(y+\frac{x}{2y'}\right)$

$\Rightarrow x^2=\frac{2xy}{y'}+\frac{x^2}{y'^2}$

$\Rightarrow xy'^2=2yy'+x$

$$\Rightarrow x\left(\frac{dy}{dx}\right)^2 = 2y\left(\frac{dy}{dx}\right) + x$$

8. Image of point $(1, 2, 3)$ w.r.t a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$ then which of the following points lie on this plane

a. $(1,\ 1,\ -1)$
b. $(-1,\ -1,\ 1)$
c. $(-1,\ 1,\ -1)$
d. $(-1,\ -1,\ -1)$

**Answer**: $(a)$

**Solution:**

Image of point $P(1, 2, 3)$ w.r.t. a plane $ax + by + cz + d = 0$ is $Q\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$

Direction ratios of $PQ$: $-\frac{10}{3}, -\frac{10}{3}, -\frac{10}{3} = 1, 1, 1$

Direction ratios of normal to plane is $1, 1, 1$

Mid-point of $PQ$ lies on the plane

$\therefore$ The mid-point of $PQ = \left(-\frac{2}{3},\ \frac{1}{3},\ \frac{4}{3}\right)$

$\therefore$ Equation of plane is $x + \frac{2}{3} + y - \frac{1}{3} + z - \frac{4}{3} = 0$

$\Rightarrow x + y + z = 1$

$(1,\ 1,\ -1)$ satisfies the equation of the plane.

9. $\lim_{x\to 0} \frac{\int_0^x t \sin 10t\, dt}{x}$ is equal to

a. 10
b. 0
c. 1
d. 5

**Answer**: $(b)$

**Solution:**

$$\lim_{x\to 0} \frac{\int_0^x t \sin 10t\, dt}{x}$$

Applying L'Hospital's Rule:

$$= \lim_{x\to 0} \frac{x \sin 10x}{1} = 0$$

10. Let $P$ be the set of points $(x,\ y)$ such that $(x^2 \leq y \leq -2x + 3)$. Then area bounded by points in $P$ is

a. $\frac{16}{3}$

b. $\frac{29}{3}$

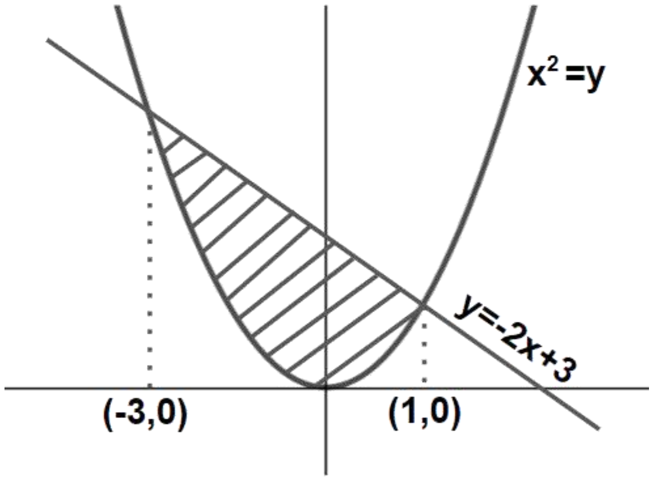
c. $\frac{20}{3}$

d. $\frac{32}{3}$

**Answer:** $(d)$

**Solution:**

We have $x^2 \le y \le -2x + 3$




For point of intersection of two curves -

$x^2 + 2x - 3 = 0$

$\Rightarrow x = -3, 1$

$\Rightarrow$ Area $= \int_{-3}^{1}\left((-2x + 3) - x^2\right)dx$

$= \left[-x^2 + 3x - \frac{x^3}{3}\right]_{-3}^{1} = \frac{32}{3}$ sq. units.

11. If $f(x) = \frac{x[x]}{x^2+1} : (1, 3) \to \boldsymbol{R}$, then the range of $f(x)$ is (where $[.]$ denotes greatest integer function)

a. $\left(0, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{7}{5}\right]$

b. $\left(\frac{2}{5}, 1\right) \cup \left(1, \frac{4}{5}\right]$

c. $\left(0, \frac{1}{3}\right) \cup \left(\frac{2}{5}, \frac{4}{5}\right]$

d. $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

**Answer**: $(d)$

**Solution:**

$f(x) = \frac{x[x]}{x^2+1}$

$$\Rightarrow f(x) = \begin{cases} \frac{x}{x^2+1} : 1 < x < 2 \\ \frac{2x}{x^2+1} : 2 \le x < 3 \end{cases}$$

$\Rightarrow$Range of $f(x)$ is $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$.

12. If $A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then the value of $10A^{-1}$ is
    a. $A - 4I$
    b. $A - 6I$
    c. $6I - A$
    d. $4I - A$

    **Answer:** $(b)$

    **Solution:**

    $A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}$

    $A^{-1} = -\frac{1}{10}\begin{bmatrix} 4 & -2 \\ -9 & 2 \end{bmatrix}$

    $\Rightarrow 10A^{-1} = \begin{bmatrix} -4 & 2 \\ 9 & -2 \end{bmatrix}$

    $\Rightarrow 10A^{-1} = A - 6I$

13. For 20 observations mean and variance is given as 10 and 4, later it was observed that by mistake 9 was taken in place of 11, then the correct variance is
    a. 3.98
    b. 3.99
    c. 4.01
    d. 4.02

    **Answer**: (b)

    **Solution:**

    Mean $= 10 \Rightarrow \frac{\sum x_i}{20} = 10 \Rightarrow \sum x_i = 200$

    Variance $= 4 \Rightarrow \frac{\sum x\_i^2}{20} - 100 = 4 \Rightarrow \sum {x_i}^2 = 2080$

    New mean $= \frac{200-9+11}{20} = \frac{202}{20} = 10.1$

    New variance $= \frac{2080-81+121}{20} - (10.1)^2$

    $= 106 - 102.01$

    $= 3.99$

14. The correct option for the system of linear equations
    $\lambda x + 2y + 2z = 5$
    $2\lambda x + 3y + 5z = 8$
    $4x + \lambda y + 6z = 10$
    a. Infinite solutions when $\lambda = 2$
    b. Infinite solutions when $\lambda = 8$

c. No solutions when $\lambda = 2$

d. No solutions when $\lambda = 8$

**Answer**: (c)

**Solution:**

$$D = \begin{vmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix} = 18\lambda - 5\lambda^2 - 24\lambda + 40 + 4\lambda^2 - 24$$

$\Longrightarrow D = -\lambda^2 - 6\lambda + 16$

Now, $D = 0$

$\Rightarrow \lambda^2 + 6\lambda - 16 = 0$

$\Rightarrow \lambda = -8$ or $2$

For $\lambda = 2$

$$D_1 = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix} = 40 + 4 - 28 \neq 0$$

$\therefore$ Equations have no solution for $\lambda = 2$.

15. In an A. P. if $T_{10} = \frac{1}{20}; T_{20} = \frac{1}{10}$, then the sum of first 200 terms is

a. $100\frac{1}{2}$

b. $101\frac{1}{2}$

c. $201\frac{1}{2}$

d. $301\frac{1}{2}$

**Answer**: $(a)$

**Solution:**

$T_{10} = \frac{1}{20}, T_{20} = \frac{1}{10}$

$T_{20} - T_{10} = 10d$

$\Rightarrow \frac{1}{20} = 10d$

$d = \frac{1}{200}$

$\therefore a = \frac{1}{200}$

$$S_{200} = \frac{200}{2}\left[2\left(\frac{1}{100}\right) + 199\left(\frac{1}{200}\right)\right]$$

$= 100\frac{1}{2}$

16. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$ and $a = (1+\alpha)\sum_{k=0}^{100} \alpha^{2k}$ , $b = \sum_{k=0}^{100} \alpha^{3k}$ where $a$ and $b$ are the roots of the quadratic equation then the quadratic equation will be

a. $x^2 - 102x + 101 = 0$
b. $x^2 + 102x + 100 = 0$
c. $x^2 - 101x + 100 = 0$
d. $x^2 + 101x + 100 = 0$

**Answer:** $(a)$

**Solution:**

$$\alpha = \frac{-1+i\sqrt{3}}{2} = \omega$$

$$a = (1+\alpha)\sum_{k=0}^{100} \alpha^{2k}$$

$$\Rightarrow a(1+\alpha)[1+\alpha^2+\alpha^4+\cdots.+\alpha^{200}]$$

$$\Rightarrow a = (1+\alpha)\left[\frac{1-(\alpha^2)^{101}}{1-\alpha^2}\right]$$

$$\Rightarrow a = \left[\frac{1-(\omega^2)^{101}}{1-\omega}\right] = \left[\frac{1-\omega}{1-\omega}\right] = 1$$

$$b = \sum_{k=0}^{100} \alpha^{3k} = 1+\alpha^3+\alpha^6+\cdots..+\alpha^{300}$$

$$\Rightarrow b = 1+\omega^3+\omega^6+....+\omega^{300}$$

$$\Rightarrow b = 101$$

Required equation is $x^2 - 102x + 101 = 0$

17. If $f(x)$ is a three-degree polynomial for which $f'(-1) = 0, f''(1) = 0, f(-1) = 10, f(1) = 6$ then the local minima of $f(x)$ will be at

a. $x = -1$
b. $x = 1$
c. $x = 2$
d. $x = 3$

**Answer:** $(d)$

**Solution:**

Let the polynomial be

$$f(x) = ax^3 + bx^2 + cx + d$$

$$\Rightarrow f'(x) = 3ax^2 + 2bx + c$$

$$\Rightarrow f''(x) = 6ax + 2b$$

$$f''(1) = 0 \Rightarrow 6a + 2b = 0 \Rightarrow b = -3a$$

$$f'(-1) = 0 \Rightarrow 3a - 2b + c = 0$$

$\Rightarrow c = -9a$

$f(-1) = 10 \Rightarrow -a + b - c + d = 10$

$\Rightarrow -a - 3a + 9a + d = 10$

$d = -5a + 10$

$f(1) = 6 \Rightarrow a + b + c + d = 6$

$\Rightarrow a - 3a - 9a - 5a + 10 = 6$

$\Rightarrow a = \frac{1}{4}$

$\therefore f'(x) = \frac{3}{4}x^2 - \frac{6}{4}x - \frac{9}{4} = \frac{3}{4}(x^2 - 2x - 3)$

For $f'(x) = 0 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow x = 3, -1$

Minima exists at $x = 3$

18. Let $I = \int_1^2 \frac{dx}{\sqrt{2x^3-9x^2+12x+4}}$ then

a. $\frac{1}{9} < I^2 < \frac{1}{8}$

b. $\frac{1}{9} < I < \frac{1}{8}$

c. $\frac{1}{3} < I^2 < \frac{1}{2}$

d. $\frac{1}{3} < I < \frac{1}{2}$

**Answer:** $(a)$

**Solution:**

$f(x) = \frac{1}{\sqrt{2x^3-9x^2+12x+4}}$

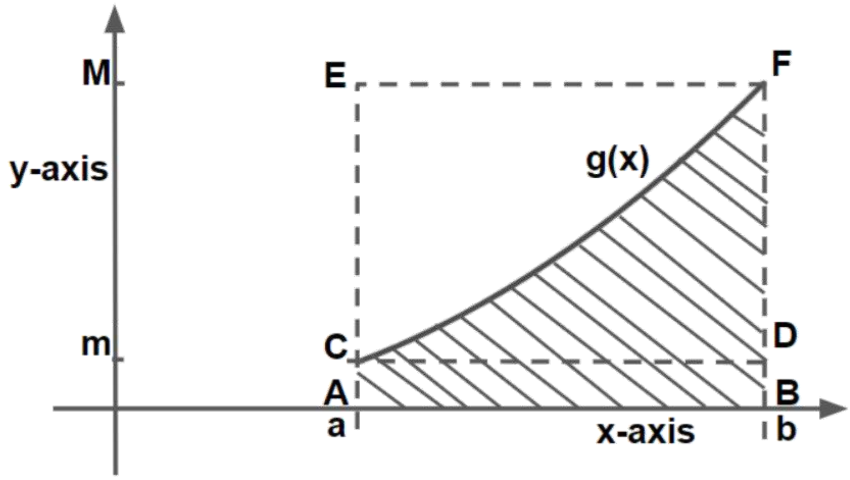
Differentiating w.r.t $x$

$$f'(x) = -\frac{1}{2} \times \frac{(6x^2-18x+12)}{(2x^3-9x^2+12x+4)^{3/2}}$$

$$= \frac{-6(x-1)(x-2)}{2(2x^3-9x^2+12x+4)^{3/2}}$$

Here $f(x)$ is increasing in $(1,2)$

At $x = 1$, $f(1) = \frac{1}{3}$ and $x = 2$, $f(2) = \frac{1}{\sqrt{8}}$




Let $g(x)$be a function such that it is increasing in $(a, b)$ and $m \leq g(x) \leq M$, then

$\text{ar}(ABCD) < \int_a^b g(x)\, dx < \text{ar}(ABEF)$

$m(b-a) < \int_a^b g(x)\, dx < M(b-a)$

Thus, $\frac{1}{3} < \int_1^2 f(x)dx \ < \frac{1}{\sqrt{8}}$

$\frac{1}{3} < I \ < \frac{1}{\sqrt{8}}$,

or $\frac{1}{9} < I^2 < \frac{1}{8}$

19. Normal at $(2,\ 2)$ to curve $x^2 + 2xy - 3y^2 = 0$ is $L$. Then perpendicular distance from origin to line $L$ is

a. $2\sqrt{2}$
b. $4$
c. $4\sqrt{2}$
d. $2$

**Answer:** $(a)$

**Solution:**

Given curve: $x^2 + 2xy - 3y^2 = 0$

$\Rightarrow x^2 + 3xy - xy - 3y^2 = 0$

$\Rightarrow (x + 3y)(x - y) = 0$

Equating we get,

$x + 3y = 0$ or $x - y = 0$

$(2,\ 2)$ lies on $x - y = 0$

$\therefore$ Equation of normal will be $x + y = \lambda$

It passes through $(2, 2)$

$\therefore \lambda = 4$

$L : x + y = 4$

Distance of $L$ from the origin $= \left|\frac{-4}{\sqrt{2}}\right| = 2\sqrt{2}$

20. If $A$ and $B$ are two events such that $P$(exactly one)$= \frac{2}{5}, P(A \cup B) = \frac{1}{2}$ then $P(A \cap B)$ is

a. $\frac{1}{8}$

b. $\frac{1}{10}$

c. $\frac{1}{12}$

d. $\frac{2}{9}$

**Answer:** $(b)$

**Solution:**

$P$(exactly one of $A$ or $B$)$= \frac{2}{5}$

$$\Rightarrow P(A) - P(A \cap B) + P(B) - P(A \cap B) = \frac{2}{5}$$

$$\Rightarrow P(A \cup B) - P(A \cap B) = \frac{2}{5}$$

$$\Rightarrow P(A \cap B) = \frac{1}{10}$$

21. The number of four-letter words that can be made from the letters of word "EXAMINATION" is

**Answer**: (2454)

**Solution:**

Word "EXAMINATION" consists of $2A, 2I, 2N, E, X, M, T, O$

Case I: All different letters are selected

Number of words formed $= {}^{8}C_4 \times 4! = 1680$

Case II: 2 letters are same and 2 are different

Number of words formed $= {}^{3}C_1 \times {}^{7}C_2 \times \frac{4!}{2!} = 756$

Case III: 2 pair of letters are same

Number of words formed $= {}^{3}C_2 \times \frac{4!}{2! \times 2!} = 18$

Total number of words formed $= 1680 + 756 + 18 = 2454$

22. Line $y = mx$ intersects the curve $y^2 = x$ at point $P$. The tangent to $y^2 = x$ at $P$ intersects $x$ −axis at $Q$. If area $\Delta OPQ = 4$, find $m$, $(m > 0)$

**Answer**: (0.5)

**Solution:**

Let the co-ordinates of $P$ be $(t^2,\ t)$

Equation of tangent at $P(t^2,\ t)$ is $y - t = \frac{1}{2t}(x - t^2)$

Therefore, co-ordinates of $Q$ will be $(-t^2,\ 0)$

Area of $\Delta OPQ = 4$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{vmatrix} = \pm 4$$

$\Rightarrow t^3 = 8 \Rightarrow t = \pm 2 \Rightarrow t = 2 \quad$ as $t > 0$

$$m = \frac{1}{t} = \frac{1}{2}$$

23. $\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$ is equal to

**Answer:** (504)

**Solution:**

$$\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$$

$$= \frac{1}{4}\sum_{n=1}^{7}(2n^3 + 3n^2 + n)$$

$$= \frac{1}{4}\left[2\sum_{n=1}^{7} n^3 + 3\sum_{n=1}^{7} n^2 + \sum_{n=1}^{7} n\right]$$

$$= \frac{1}{4}\left[2 \times \left(\frac{7 \times 8}{2}\right)^2 + 3 \times \frac{7 \times 8 \times 15}{6} + \frac{7 \times 8}{2}\right]$$

$$= \frac{1}{4}[2 \times 784 + 420 + 28] = 504$$

24. Let $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$, where $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$. Then $\tan(\alpha + 2\beta)$ is equal to

**Answer:** (1)

**Solution:**

$$\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7} \Rightarrow \frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \frac{1}{7} \Rightarrow \tan\alpha = \frac{1}{7}$$

$$\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}} \Rightarrow \frac{\sqrt{2}sin\beta}{\sqrt{2}} = \frac{1}{\sqrt{10}} \Rightarrow \sin\beta = \frac{1}{\sqrt{10}} \Rightarrow \tan\beta = \frac{1}{3}$$

$$\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{\frac{2}{3}}{1-\frac{1}{9}} = \frac{3}{4}$$

$$\tan(\alpha + 2\beta) = \frac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha\tan 2\beta} = \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\times\frac{3}{4}} = 1$$