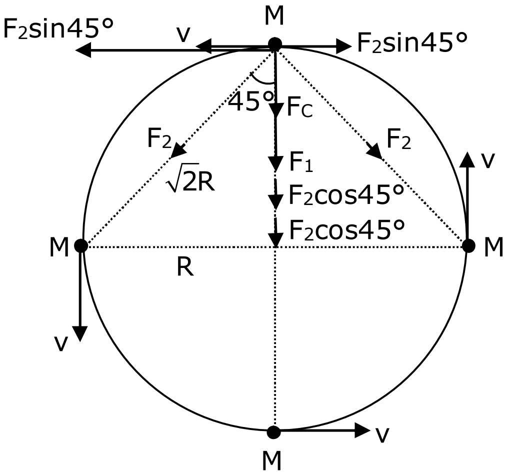
24th Feb. 2021 | Shift - 1
PHYSICS

Section - A

1. Four identical particles of equal masses 1 kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be -

(1) $\frac{\sqrt{(1+2\sqrt{2})G}}{2}$ (2) $\sqrt{G(1+2\sqrt{2})}$ (3) $\sqrt{\frac{G}{2}\left(2\sqrt{2}-1\right)}$ (4) $\sqrt{\frac{G}{2}\left(1+2\sqrt{2}\right)}$

Sol. **(1)**



$\Rightarrow$ By resolving force F_2, we get

$\Rightarrow F_1 + F_2 \cos 45° + F_2 \cos 45°$

$\Rightarrow F_1 + 2F_2 \cos 45° = F_c$

F_c = centripital force = $\frac{MV^2}{R}$

$$\Rightarrow \frac{GM^2}{(2R)^2} + \left[\frac{2GM^2}{\left(\sqrt{2}R\right)^2}\cos 45°\right] = \frac{MV^2}{R}$$

$$\Rightarrow \frac{GM^2}{4R^2} + \frac{2GM^2}{2\sqrt{2}R^2} = \frac{MV^2}{R}$$

$$\Rightarrow \frac{GM}{4R} + \frac{GM}{\sqrt{2}.R} = V^2$$

$$\Rightarrow V = \sqrt{\frac{GM}{4R} + \frac{GM}{\sqrt{2}.R}}$$

$$\Rightarrow V = \sqrt{\frac{GM}{R}\left[\frac{1+2\sqrt{2}}{4}\right]}$$

$$\Rightarrow V = \frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$$

(given : mass = 1 kg, radius = 1 m)

$$\Rightarrow v = \frac{1}{2}\sqrt{G(1+2\sqrt{2})}$$

2. Consider two satellites S_1 and S_2 with periods of revolution 1 hr. and 8 hr. respectively revolving around a planet in circular orbits.The ratio of angular velocity of satellite S_1 to the angular velocity of satellite S_2 is -

(1)8 : 1 (2)1 : 8 (3)2 : 1 (4) 1 : 4

Sol. (1)

We know that $\omega = \frac{2\pi}{T}$

given : Ratio of time period

$\frac{T_1}{T_2} = \frac{1}{8}$

$\Rightarrow \omega \propto \frac{1}{T}$

$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$

$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{8}{1}$

$\Rightarrow \omega_1 : \omega_2 = 8 : 1$

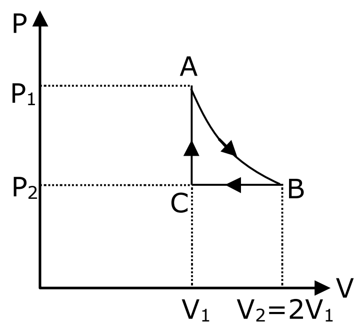
3. n mole of a perfect gas undergoes a cyclic process ABCA (see figure) consisting of the following processes –

$A \rightarrow B$: Isothermal expansion at temperature T so that the volume is doubled from V_1 to $V_2 = 2V_1$ and pressure changes from P_1 to P_2.

$B \rightarrow C$: Isobaric compression at pressure P_2 to initial volume V_1.

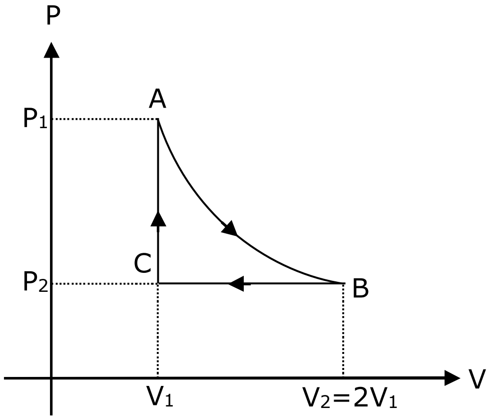
$C \rightarrow A$: Isochoric change leading to change of pressure from P_2 to P_1.

Total workdone in the complete cycle ABCA is –



(1)0 (2)$nRT\left(\ln 2 + \frac{1}{2}\right)$ (3)$nRT\ln 2$ (4) $nRT\left(\ln 2 - \frac{1}{2}\right)$

Sol. **(4)**



$A \rightarrow B$ = isotheraml process

$B \rightarrow C$ = isobaric process

$C \rightarrow A$ = isochoric process

also, $V_2 = 2V_1$

work done by gas in the complete cycle ABCA is –

$\Rightarrow w = w_{AB} + w_{BC} + w_{CA}$ ....(1)

$\Rightarrow w_{CA} = 0$, as isochoric process

$\Rightarrow w_{AB} = 2P_1V_1 \ln\left(\frac{V_2}{V_1}\right) = 2\ nRT \ln(2)$

$\Rightarrow w_{BC} = P_2(V_1 - V_2) = P_2(V_1 - 2V_1) = -P_2V_1 = -nRT$

$\Rightarrow$Now put the value of w_{AB}, w_{BC} and w_{CA} in equation, we get

$\Rightarrow w = 2nRT \ln(2) - nRT + 0$

$\Rightarrow w = nRT\ [2\ln(2) - 1]$

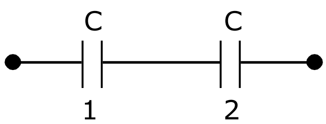
$\Rightarrow w = nRT\ [\ln(2) - \frac{1}{2}]$

4. Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities capacities in the two cases will be -

(1)2 : 1 (2)1 : 4 (3)4 : 1 (4) 1 : 2

Sol. **(2)**

Given that first connection



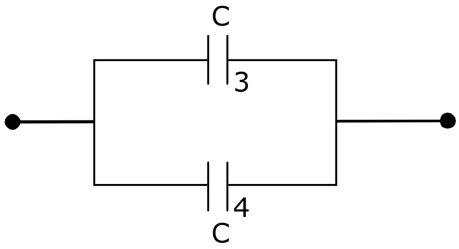
$\Rightarrow \frac{1}{C_{12}} = \frac{1}{C} + \frac{1}{C} \Rightarrow C_{12} = \frac{C}{2}$

Second connection

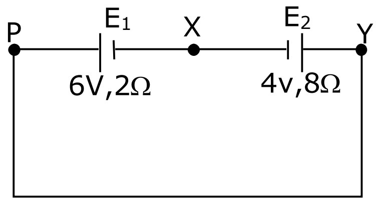
$C_{34} = C + C = 2\ C$

Now, the ratio of equivalent capacities in the two cases will be –

$$\Rightarrow \frac{C_{12}}{C_{34}} = \frac{C/2}{2C} \Rightarrow \frac{C_{12}}{C_{34}} = \frac{1}{4}$$

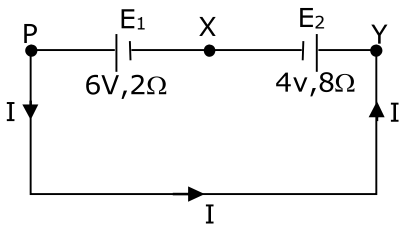


5. A cell E_1 of emf 6V and internal resistance 2Ω is connected with another cell E_2 of emf 4V and internal resistance 8Ω (as shown in the figure). The potential difference across points X and Y is –



(1)3.6V (2)10.0V (3)5.6V (4) 2.0V

Sol. **(3)**



emf of E_1 = 6v

$r_1 = 2\ \Omega$

emf of $E_2 = 4\ \Omega$

$r_2 = 8\Omega$

$|v_x - v_y|$ = potential difference across points x and y

$E_{eff} = 6 - 4 = 2$ V

$R_{eq} = 2 + 8 = 10\ \Omega$

So, current in the circuit will be

$$\Rightarrow I = \frac{E_{eff}}{R_{eq}} \Rightarrow I = \frac{2}{10} = 0.2 \text{ A}$$

Now, potential difference across points X and Y is

$|v_x - v_y| = E + iR$

$\Rightarrow |v_x - v_y| = 4 + 0.2 \times 8 = 5.6$ V

$\Rightarrow |v_x - v_y| = 5.6$ v

6. If Y,K and η are the values of Young's modulus, bulk modulus and modulus of rigidity of any material respectively. Choose the correct relation for these parameters.

(1) $K = \dfrac{Y\eta}{9\eta - 3Y}$ N/m^2 (2) $\eta = \dfrac{3YK}{9K + Y}$ N/m^2

(3) $Y = \dfrac{9K\eta}{3K - \eta}$ N/m^2 (4) $Y = \dfrac{9K\eta}{2\eta + 3K}$ N/m^2

Sol. **(1)**

$\Rightarrow y = 3k\,(1 - 2\sigma)$

$\Rightarrow \sigma = \dfrac{1}{2}\left(1 - \dfrac{y}{3k}\right)$ ....(1)

$\Rightarrow y = 2\eta\,(1 + \sigma)$

$\Rightarrow \sigma = \dfrac{y}{2\eta} - 1$ ....(2)

by comparing equation (1) and (2), we get

$\Rightarrow \dfrac{y}{2\eta} - 1 = \dfrac{1}{2}\left(1 - \dfrac{y}{3k}\right)$

$\Rightarrow \dfrac{y}{\eta} - 2 = 1 - \dfrac{y}{3k}$

$\Rightarrow \dfrac{y}{\eta} = 1 + 2 - \dfrac{y}{3k} \Rightarrow \dfrac{y}{\eta} = 3 - \dfrac{y}{3k}$

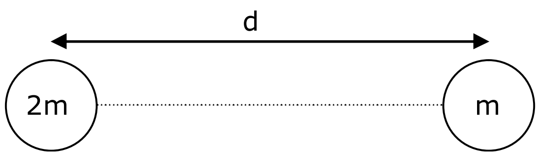
$\Rightarrow \dfrac{y}{3k} = 3 - \dfrac{y}{\eta} \Rightarrow \dfrac{y}{3k} = \dfrac{3\eta - y}{\eta}$

$\Rightarrow k = \dfrac{\eta y}{9\eta - 3y}$

7. Two stars of masses m and 2m at a distance d rotate about their common centre of mass in free space. The period of revolution is -

(1) $2\pi\sqrt{\dfrac{d^3}{3Gm}}$ (2) $\dfrac{1}{2\pi}\sqrt{\dfrac{3Gm}{d^3}}$ (3) $\dfrac{1}{2\pi}\sqrt{\dfrac{d^3}{3Gm}}$ (4) $2\pi\sqrt{\dfrac{3Gm}{d^3}}$

Sol. **(1)**



$\Rightarrow \dfrac{G(m)(2m)}{d^2} = m\omega^2 \times \dfrac{2d}{3}$

$\Rightarrow \dfrac{2Gm}{d^2} = \omega^2 \times \dfrac{2d}{3}$

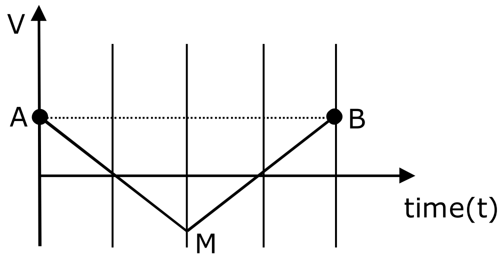
$$\Rightarrow \omega^2 = \frac{3Gm}{d^3}$$

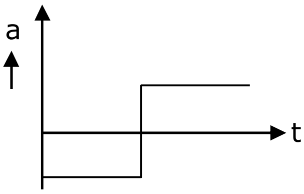
$$\Rightarrow \omega = \sqrt{\frac{3Gm}{d^3}}$$

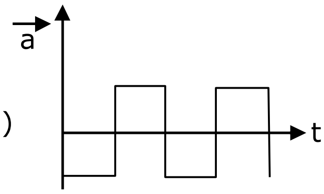
we know that, $\omega = \frac{2\pi}{T}$ so $T = \frac{2\pi}{\omega}$

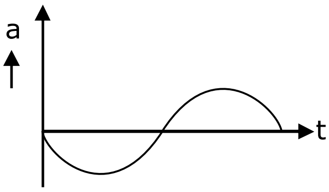
$$\Rightarrow T = \frac{2\pi}{\sqrt{\frac{3Gm}{d^3}}} \Rightarrow T = 2\pi\sqrt{\frac{d^3}{3Gm}}$$

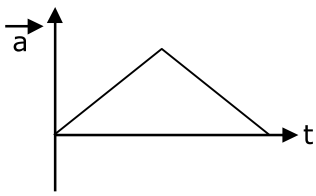
8. If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph ?



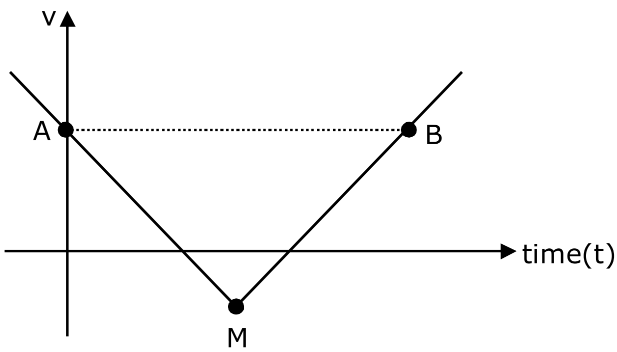
(1)


(2)


(3)


(4)


Sol. **(1)**



$a = \frac{dv}{dt}$ = slope of (v – t) curve

If m = +ve, then equation of straight line is

$y = mx + c \Rightarrow v = mt + c$ (for MB)

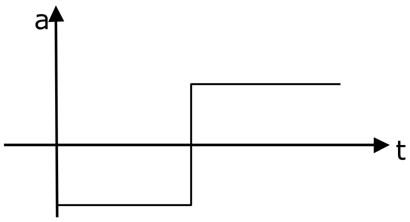
If m =-ve, then equation of straight line is

$y = -mx + c \Rightarrow v = -mt + c$ (for AM)

If we differentiate equation (1) and (2), we get

a_{MB} = +ve = m

a_{AM} =-ve = -m, so graph of (a-t) will be



9. Given below are two statements :

Statement – I: Two photons having equal linear momenta have equal wavelengths.

Statement-II: If the wavelength of photon is decreased, then the momentum and energy of a photon will also decrease.

In the light of the above statements, choose the correct answer from the options given below.

(1)Statemnet-I is false but Statement-II is true

(2)Both Statement-I and Statement-II are true

(3)Both Statement-I and Statement-II are false

(4) Statement-I is true but Statement-II is false

Sol. **(4)**

By theory

10. A current through a wire depends on time as $i = \alpha_0 t + \beta t^2$

where α_0 = 20 A/s and β = 8 As^{-2}. Find the charge crossed through a section of the wire in 15 s.

(1)2100 C (2)260 C (3)2250 C (4) 11250 C

Sol. **(4)**

given :$i = \alpha_0 t + \beta t^2$

α = 20 A/s and β = 8As^{-2}

t = 15 sec

we know that, $i = \frac{dq}{dt} \Rightarrow \int_0^t i dt = \int_0^Q dq$

$$\Rightarrow \int_0^{15} (\alpha_0 t + \beta t^2) dt = \int_0^Q dq$$

$$\Rightarrow Q = \left[\frac{\alpha_0 t^2}{2} + \frac{\beta t^3}{3}\right]_0^{15}$$

$$\Rightarrow Q = \frac{20 \times 15 \times 15}{2} + \frac{8 \times 15 \times 15 \times 15}{3} - 0$$

$\Rightarrow Q = 11250$ C

11. match List I with List II

List-I	List-II
(a) Isothermal	(i) Pressure constant
(b) Isochoric	(ii) Temperature constant
(c) Adiabatic	(iii) Volume constant
(d) Isobaric	(iv) Heat content is constant

Choose the correct answer from the options given below –

(1)(a) – (ii), (b) – (iv), (c) – (iii), (d) – (i)

(2)(a) – (ii), (b) – (iii), (c) – (iv), (d) – (i)

(3)(a) – (i), (b) – (iii), (c) – (ii), (d) – (iv)

(4) (a) – (iii), (b) – (ii), (c) – (i), (d) – (iv)

Sol. (**2**)

(a)$\rightarrow$(ii), (b)$\rightarrow$(iii), (c)$\rightarrow$(iv), (d)$\rightarrow$(i),

By theory

In isothermal process, temperature is constant.

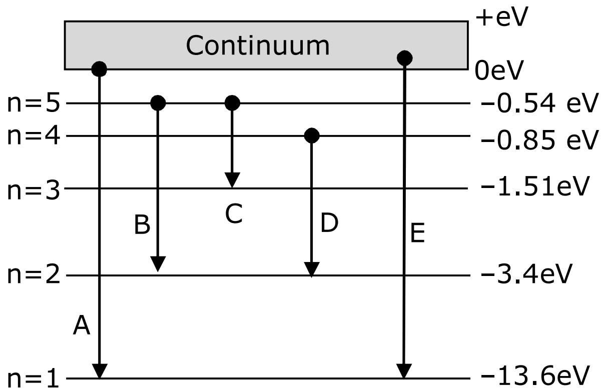
In isochoric process, volume is constant.

In adiabatic process, heat content is constant.

In isobaric process, pressure is constant.

12. In the given figure, the energy levels of hydrogen atom have been shown along with some transitions marked A,B,C,D and E.

The transitions A,B and C respectively represents –



(1)The series limit of Lyman series, third member of balmer series and second member of paschen series

(2)The first member of the Lyman series, third member of Balmer series and second member of paschen series

(3)The ionization potential of hydrogen, second member of Balmer series and third member of Paschen series

(4) The series limit of Lyman series, second memebr of Balmer series and second member of Paschen series.

Sol. **(1)**

A $\rightarrow$ series limit of lyman.

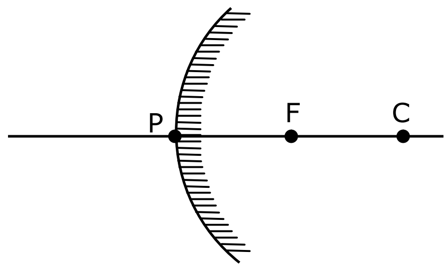
B $\rightarrow$ 3^{rd} member of Balmer series.

C $\rightarrow$ 2^{nd} member of Paschen series.

13. The focal length f is related to the radius of curvature r of the spherical convex mirror by -

(1) $f = r$ (2) $f = -\frac{1}{2}r$ (3) $f = +\frac{1}{2}r$ (4) $f = -r$

Sol. **(3)**



So, $\frac{R}{2} = f$

$F = +\frac{1}{2}R$

14. Moment of inertia (M.I.) of four bodies, having same mass and radius, are reported as –

I_1 = M.I. of thin circular ring about its diameter,

I_2 = M.I. of circular disc about an axis perpendicular to disc and going through the centre,

I_3 = M.I. of solid cylinder about its axis and

I_4 = M.I. of solid sphere about its diameter.

Then :-

(1)$I_1 = I_2 = I_3 < I_4$ (2)$I_1 + I_2 = I_3 + \frac{5}{2} I_4$

(3)$I_1 + I_3 < I_2 + I_4$ (4) $I_1 = I_2 = I_3 > I_4$

Sol. **(4)**

Given$\Rightarrow$ I_1 = M.I. of thin circular ring about its diameter

I_2 = M.I. circular disc about an axis perpendicular to disc and going through the centre.

I_3 = M.I. of solid cylinder about its axis

I_4 = M.I. of solid sphere about its diameter

we know that,

$I_1 = \frac{MR^2}{2}$, $I_2 = \frac{MR^2}{2}$, $I_3 = \frac{MR^2}{2}$

$I_4 = \frac{2}{5} MR^2$

So, $I_1 = I_2 = I_3 > I_4$

15. The workdone by a gas molecule in an isolated system is given by, $W = \alpha\beta^2 e^{-\frac{x^2}{\alpha kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. α and β are constants. Then the dimensions of β will be -

(1)$[M^0LT^0]$ (2)$[M^2LT^2]$ (3)$[MLT^{-2}]$ (4) $[ML^2T^{-2}]$

Sol. **(3)**

given : work $= \alpha.\beta^2.e^{-\frac{x^2}{\alpha.k.T}}$

k = boltzmann constant

T = temperature

x = displacement

we know that, $\frac{x^2}{\alpha.k.T}$ = dimensionless

$\left[\frac{x^2}{\alpha.k.T}\right] = [M^0L^0T^0]$

$[\alpha] = \left[\frac{L^2}{K.T}\right]$

$\Rightarrow [K] = [M^1L^2T^{-2}K^{-1}]$

$[T] = [K]$

$\Rightarrow [\alpha] = \left[\frac{L^2}{M^1L^2T^{-2}K^{-1} \times K}\right] \Rightarrow [\alpha] = [M^{-1}T^2]$

$\Rightarrow \omega = \alpha.\beta^2$

$\Rightarrow \frac{[M^1L^1T^{-2}][L^{-1}]}{[M^{-1}T^2]} = [\beta^2] = [M^2L^2T^{-4}]$

$[\beta] = [MLT^{-2}]$

16. If an emitter current is changed by 4mA, the collector current changes by 3.5 mA.The value of β will be -

(1)7 (2)0.875 (3)0.5 (4) 3.5

Sol. **(1)**

Given :

$\Delta I_E = 4$ mA

$\Delta I_C = 3.5$ mA

we know that, $\alpha = \frac{\Delta I_C}{\Delta I_E}$

$\Rightarrow \alpha = \frac{3.5}{4} = \frac{7}{8}$

Also, $\beta = \frac{\alpha}{1-\alpha}$, so

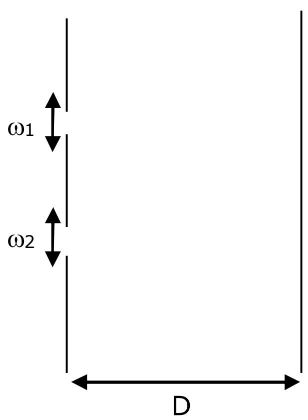
$\beta = \frac{\frac{7}{8}}{1-\frac{7}{8}} = \frac{7}{1}$

$\beta = 7$

17. In a Young's double slit experiment, the width ofthe one of the slit is three times the other slit. The amplitude of the light coming from a slit is proportional to the slit-width. Find the ratio of the maximum to the minimum intensity in the interference pattern.

(1)4 : 1 (2)2 : 1 (3)3 : 1 (4) 1 : 4

Sol. **(1)**



given : $\omega_2 = 3\omega_1$

also, $A \propto \omega$

$\frac{\omega_1}{\omega_2} = \frac{1}{3}$ ...(1)

Assume $\omega_1 = x$, $\omega_2 = 3x$

we know that

$I_{max} = (A_1 + A_2)^2$, and

$I_{min} = (A_1 - A_2)^2$

$$\frac{A_1}{A_2} = \frac{\omega_1}{\omega_2} \quad \text{....(2)}$$

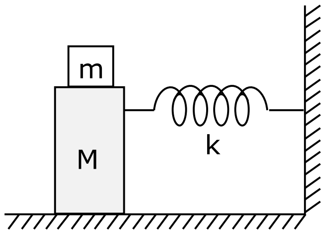
from equation (2) we can say that

$A_1 = A$ and $A_2 = 3A$

Now, $\frac{I_{max}}{I_{min}} = \frac{(A+3A)^2}{(A-3A)^2} = \frac{16A^2}{4A^2} = \frac{4}{1}$

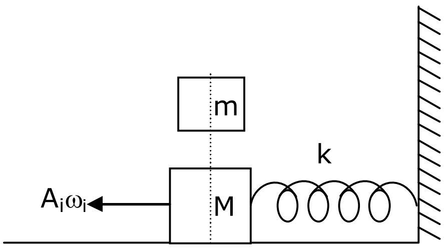
$\Rightarrow \frac{I_{max}}{I_{min}} = \frac{4}{1}$

18. In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position, as shown in the figure, another mass m is gently fixed upon it. The new amplitude of oscillation will be -

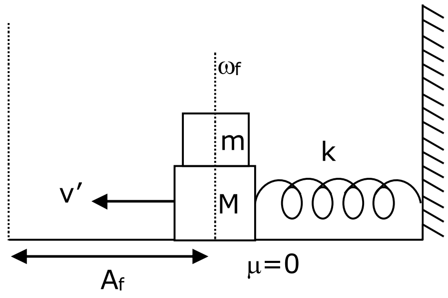


(1) $A\sqrt{\frac{M}{M+m}}$ (2) $A\sqrt{\frac{M}{M-m}}$ (3) $A\sqrt{\frac{M-m}{M}}$ (4) $A\sqrt{\frac{M+m}{M}}$

Sol. **(1)**



Before placing



After placing

We know that $\omega = \sqrt{\frac{k}{m}}$ and $\omega_i = \sqrt{\frac{k}{M}}$ $\quad A_i = A$

Also, momentum is conserved just before and just after the block of mass (m) is placed because there is no implusive force. So -

$MA_i\omega_i = (M + m)\, v'$

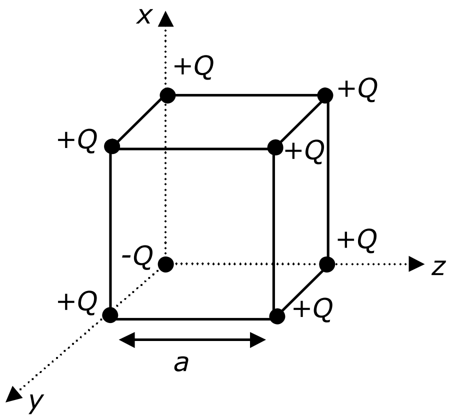
$v' = \frac{MA_i\omega_i}{(M+m)} \Rightarrow v' = A_f\omega_f$

$\frac{MA\omega_i}{(M+m)} = A_f\sqrt{\frac{K}{(M+m)}}$

$\Rightarrow \frac{MA\sqrt{\frac{K}{M}}}{M+m} \times \sqrt{\frac{M+m}{K}} = A_f$

$\Rightarrow A_f = A\sqrt{\frac{M}{(M+m)}}$

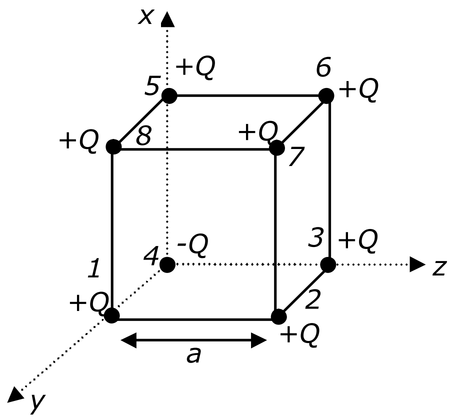
19. A cube of side 'a' has point charges +Q located at each of its vertices except at the origin where the charge is –Q. The electric field at the centre of cube is :



(1) $\frac{2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$ (2) $\frac{Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$

(3) $\frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$ (4) $\frac{-Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$

Sol. **(3)**



If only +Q charges are placedat the corners of cube of side a then electric field at the centre of the cube will be zero.

But in the given condition one (-Q) is placed at one corner of cube so here

$E_1 = E_6$, $E_2 = E_5$ and $E_3 = E_8$ (So it will cancel out each other so electric field at centre is due to Q_4 and Q_7.

Here electric field at centre = 2 $(E.f.)_4$

As, $|E_4| = |E_7|$

$$(E.F)_C = \frac{2kQ}{\left(\frac{\sqrt{3}a}{2}\right)^2} = \frac{8KQ}{3a^2} \qquad \left\{\because K = \frac{1}{4\pi\varepsilon_0}\right\}$$

$$(E.F)_C = \frac{2Q}{3a^2\pi\varepsilon_0}$$

In vector form $\Rightarrow \vec{E} = \frac{-2Q}{3a^2\pi\varepsilon_0} \times \left(\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}\right)$

20. Each side of a box made of metal sheet in cubic shape is 'a' at room temperature 'T', the coefficient of linear expansion of the metal sheet is 'α'. The metal sheet is heated uniformly, by a small temperature ΔT, so that its new temeprature is $T+\Delta T$. Calculate theincrease in the volume of the metal box-

(1) $\frac{4}{3}\pi a^3\alpha\Delta T$ (2) $4\pi a^3\alpha\Delta T$ (3) $3a^3\alpha\Delta T$ (4) $4a^3\alpha\Delta T$

Sol. **(3)**

volume expansion $\gamma = 3\alpha$

$$\frac{\Delta V}{V} = \gamma\Delta T$$

$\Delta V = V.\gamma\Delta T$

$\Delta V = a^3.\ 3\alpha\Delta T$

SECTION-B

1. A resonance circuit having inductance and resistance 2×10^{-4} H and 6.28 Ω respectively oscillates at 10 MHz frequency. The value of quality factor of this resonator is________.
[π = 3.14]

Sol. **2000**

Given : R = 6.28 Ω

f = 10 MHz

L = 2×10^{-4} Henry

we know that quality factor Q is given by

$\Rightarrow Q = \frac{X_L}{R} = \frac{\omega L}{R}$

also, $\omega = 2\pi f$, so

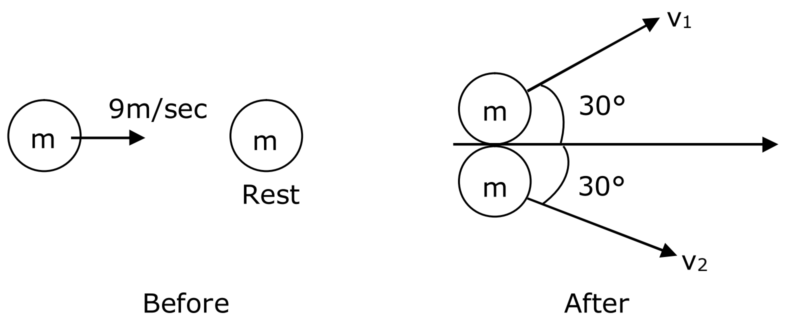
$\Rightarrow Q = \frac{2\pi f L}{R}$

$\Rightarrow Q = \frac{2\pi \times 10 \times 10^6 \times 2 \times 10^{-4}}{6.28} = 2000$

Q = 2000

2. A ball with a speed of 9 m/s collides with another identical ball at rest. After thecollision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is x : y, where x is _________.

Sol. **1**



Momentum is conserved just before and just after the collision in both x-y direction.

In y-direction

$p_i = 0$

$P_f = m \times \frac{1}{2} v_1 - m \times \frac{1}{2} v_2$

$p_i = p_f$, so

$= \frac{mv_1}{2} - \frac{mv_2}{2} = 0$

$\Rightarrow \frac{mv_1}{2} = \frac{mv_2}{2} \Rightarrow v_1 = v_2$

$\frac{v_1}{v_2} = 1$

3. An audio signal $\upsilon_m = 20\sin 2\pi(1500t)$ amplitude modulates a carrier $\upsilon_c = 80 \sin 2\pi$ (100,000t). The value of percent modulation is ________.

Sol. **25**

Given :$\upsilon_m = 20 \sin \left[100\pi t + \frac{\pi}{4}\right]$

$\upsilon_c = 80 \sin \left[10^4 \pi t + \frac{\pi}{6}\right]$

we know that, modulation index = $\frac{A_m}{A_c}$

from given equations, $A_m = 20$ and $A_c = 80$

percentage modulation index = $\frac{A_m}{A_c} \times 100$

$\Rightarrow \frac{20}{80} \times 100 = 25\%$

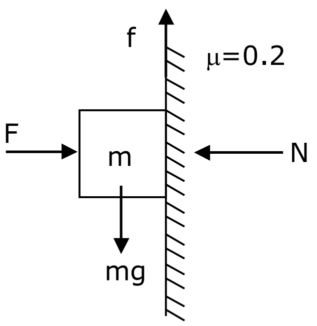
The value of percentage modulation index is

=25

4. The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be _______ N.

[$g = 10\ ms^{-2}$]

Sol. **25**



Given : $\mu_s = 0.2$

m = 0.5 kg

g = 10 m/s^2

we know that

$f_s = \mu N$ and ....(1)

To keep the block adhere to the wall

here N = F ...(2)

$f_s = mg$ ....(3)

from equation (1), (2), and (3), we get

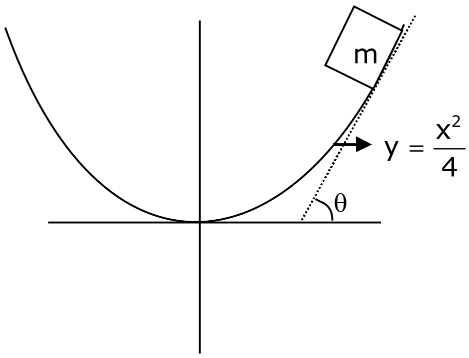
$\Rightarrow mg = \mu F$

$\Rightarrow F = \dfrac{mg}{\mu} \Rightarrow F = \dfrac{0.5 \times 10}{0.2}$

F = 25 N

5. An inclined plane is bent in such a way that the vertical cross-section is given by $y = \dfrac{x^2}{4}$ where y is in vertical and x in horizontal direction. If the upper surface of this curved plane is rough with coefficient of friction $\mu = 0.5$, the maximum height in cm at which a stationary block will not slip downward is ______ cm.

Sol. **25**



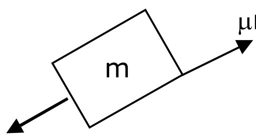
given

$y = \dfrac{x^2}{4}$

$\mu = 0.5$

condition for block will not slip downward

$mg \sin \theta = \mu mg \cos \theta$



mgsinθ

$\Rightarrow \tan\theta = \mu$

and we know that

$\Rightarrow \tan\theta = \dfrac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx} = \mu \Rightarrow \dfrac{x}{2} = 0.5$ $\qquad \left[\begin{array}{l} y = \dfrac{x^2}{4} \\ \dfrac{dy}{dx} = \dfrac{x}{2} \end{array}\right]$

$\Rightarrow x = 1,$

put $x = 1$ in equation $y = x^2/4$

$\Rightarrow y = \dfrac{(1)^2}{4} \Rightarrow y = \dfrac{1}{4} \Rightarrow y = 0.25$

$y = 25$ cm

6. An electromagnetic wave of frequency 5 GHz, is travelling in a medium whose relative electric permittivity and relative magnetic permeability bothare 2. Its velocity in this medium is _______ $\times 10^7$ m/s.

Sol. 15

Given : $f = 5$ GHz

$\varepsilon_r = 2$

$\mu_r = 2$

velocity of wave $\Rightarrow v = \dfrac{c}{n}$ ....(1)

where, $n = \sqrt{\mu_r \varepsilon_r}$ and c = speed of light $= 3 \times 10^8$ m/s

$n = \sqrt{2 \times 2} = 2$

put the value of n in we get

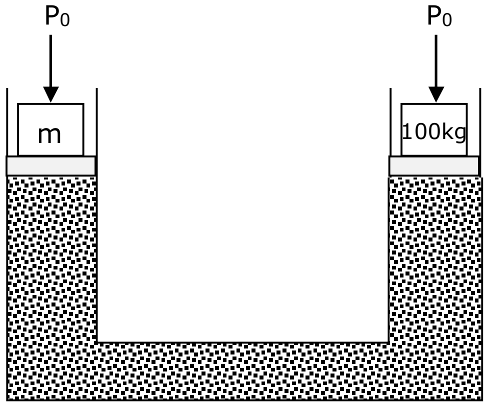
$\Rightarrow v = \dfrac{3 \times 10^8}{2} = 15 \times 10^7$ m/s

$\Rightarrow X \times 10^7 = 15 \times 10^7$

$X = 15$

7. A hydraulic press can lift 100 kg when a mass 'm' is placed on the smaller piston. It can lift _______ kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass 'm' on the smaller piston.

Sol. **25600**



Atmospheric pressure P_0 will be acting on both the limbs of hydraulic lift.

Applying pascal's law for same liquid level

$$\Rightarrow P_0 + \frac{mg}{A_1} = P_0 + \frac{(100)g}{A_2}$$

$$\Rightarrow \frac{Mg}{A_1} = \frac{(100)g}{A_2} \Rightarrow \frac{m}{100} = \frac{A_1}{A_2} \qquad ...(1)$$

Diameter of piston on side of 100 kg is increased by 4 times so new area = $16A_2$

Diameter of piston on side of (m) kg is decreasing

$$A_1 = \frac{A_1}{16}$$

(In order to increasing weight lifting capacity, diameter of smaller piston must be reduced)

Again, $\frac{mg}{\left(\frac{A_1}{16}\right)} = \frac{M'g}{16A_2} \Rightarrow \frac{256m}{M'} = \frac{A_1}{A_2}$

From equation (1) = $\frac{256m}{M'} = \frac{m}{100} \Rightarrow \therefore M' = 25600$ kg

8. A common transistor radio set requires 12 V (D.C.) for its operation. The D.C. source is constructed by using a transformer and a rectifier circuit, which are operated at 220 V (A.C.) on standard domestic A.C. supply. The number of turns of secondary coil are 24, then the number of turns of primary are _______.

Sol. **440**

Given

Primary voltage, V_p = 220 V

Secondary voltage, v_s = 12 V

No. of turns in secondary coil is N_s = 24

no. of turns in primary coil, N_p = ?

We know that for a transformer

$$\Rightarrow \frac{N_p}{N_s} = \frac{V_p}{V_s}$$

$$\Rightarrow N_p = \frac{V_p \times N_S}{V_s} = \frac{220 \times 24}{12}$$

$$\Rightarrow N_p = 440$$

9. An unpolarized light beam is incident on the polarizer of a polarization experiment and the intensity of light beam emerging fromthe analyzer is measured as 100 Lumens. Now, if the analyzer is rotated around the horizontal axis (direction of light) by 30° in clockwise direction, the intensity of emerging light will be________ Lumens.

Sol. **75**

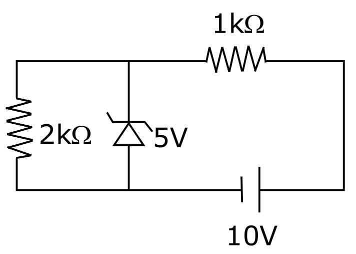
Given :I_0 = 100 lumens, $\theta = 30^{\circ}$

$I_{net} = I_0 \cos^2\theta$

$$I_{net} = 100 \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{100 \times 3}{4}$$

I_{net} = 75 lumens

10. In connection with the circuit drawn below, the value of current flowing through 2kΩresistor is ________ × 10^{-4} A.



Sol. **25**

In zener diode there will be o change in current after 5V
zener diode breakdown

$$\Rightarrow i = \frac{5}{2 \times 10^3}$$

$\Rightarrow i = 2.5 \times 10^{-3}$ A

$\Rightarrow i = 25 \times 10^{-4}$ A

24th Feb. 2021 | Shift - 1
CHEMISTRY

SECTION - A

1. The gas released during anaerobic degradation of vegetation may lead to:
(1) Global warming and cancer
(2) Acid rain
(3) Corrosion of metals
(4) Ozone hole

Ans. **(1)**

Sol. Biogas is the mixtrue of gases produced by the breakdown of organic matter in the absence of oxygen (anaerobically), primary consisting of methane and carbondioxide. Biogas can be produced from raw material such as agricultural waste, manure, municiple waste, plant material, sewage, green waste or good waste. Due to release of CH_4 gas during anaerobic vegetative degradstion which caueses globle warming and cancer.

2. Out of the following, which type of interaction is responsible for the stabilisation α–helix structure of proteins ?
(1) Ionic bonding
(2) Hydrogen bonding
(3) vander Waals forces
(4) Covalent bonding

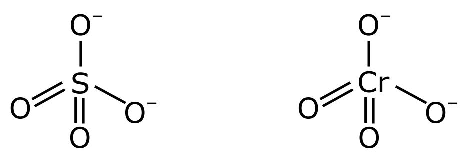
Ans. **(2)**

Sol. The α-helix is stabilized by hydrogen bond between the NH and CO group of the main chain.

3. Which of the following are isostructural pairs ?
(A) SO_4^{2-} and CrO_4^{2-}
(B) $SiCl_4$ and $TiCl_4$
(c) NH_3 and NO_3^-
(D) BCl_3 and $BrCl_3$
1. A and C only
2. A and B only
3. B and C only
4. C and D only

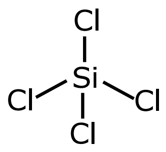
Ans. **(2)**

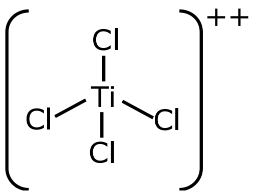
Sol. (1) SO_4^{-2} and CrO_4^{2-} both have tetrahedral structure.



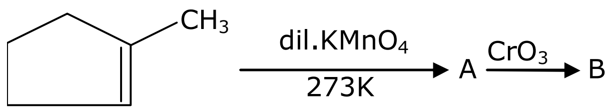
Tetrahedral Tetrahedral

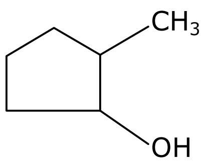
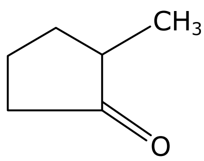
(2) $SiCl_4$ and $TiCl_4$ both have Tetrahedral structure also.

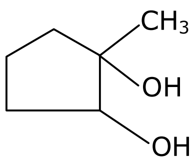
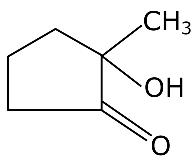




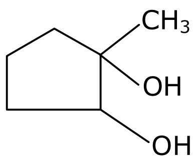
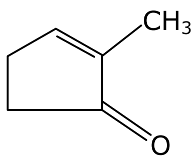
4. Identify products A and B.



(1) A :  B : 

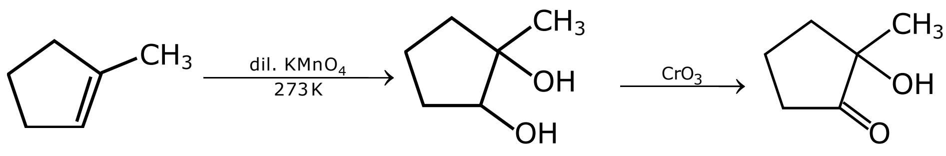
(2) A :  B : 

(3) A : $OHC-CH_2CH_2CH_2-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$ B : $HOOC-CH_2CH_2CH_2-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$

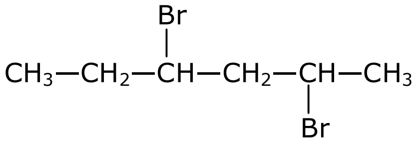
(4) A :  B : 

Ans. (2)

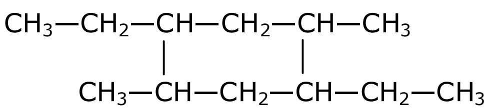
Sol.

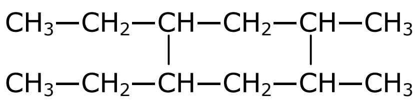


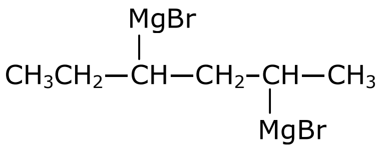
5. The product formed in the first step of the reaction of

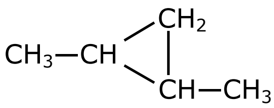


with excess $Mg/Et_2O(Et = C_2H_5)$ is :

(1) 

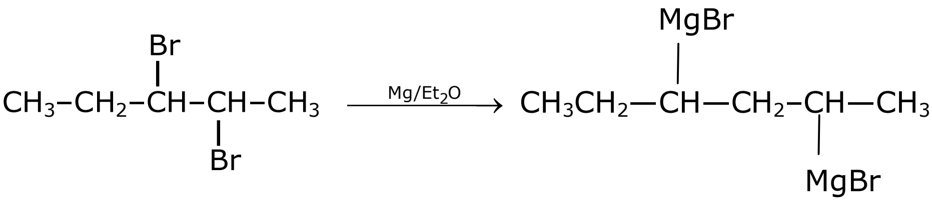
(2) 

(3) 

(4) 

Ans. **(3)**

Sol.



6. The electrode potential of M^{2+}/M of 3d– series elements shows positive value for:

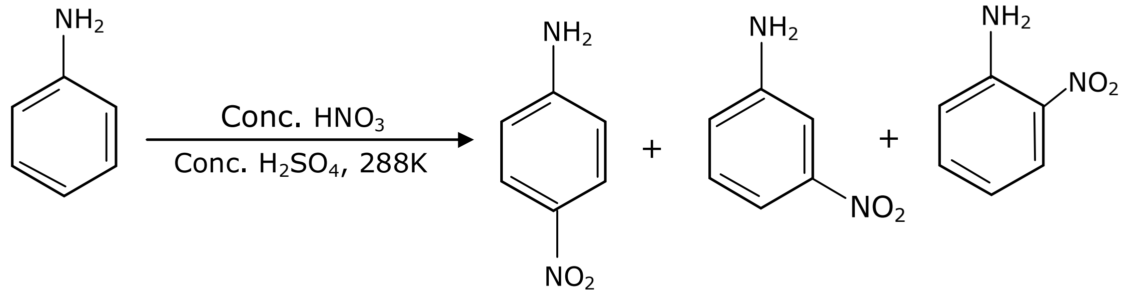
(1) Zn (2) Co (3) Fe (4) Cu

Ans. **(4)**

Sol.

(A) Zn	–0.76
(B) CO	–0.28
(C) Fe	–0.44
(D) Cu	+0.34

7. In the following reaction the reason why meta–nitro product also formed is:



[A] 51% [B] 47% [C] 2%

(1) Formation of anilinium ion

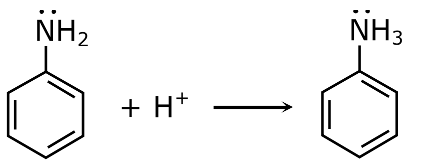
(2) $-NO_2$ substitution always takes place at meta–position

(3) low temperature

(4) $-NH_2$ group is highly meta–directive

Ans. **(1)**

Sol.



anilinium ion

In acidic medium the $-NH_2$ group in aniline converts into anilinium ion which is meta directing.

8. (A) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

(B) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

Choose the correct option.

(1) H_2O_2 act as oxidizing and reducing agent respectively in equations (A) and (B).

(2) H_2O_2 acts as oxidizing agent in equations (A) and (B).

(3) H_2O_2 acts as reducing agent in equations (A) and (B).

(4) H_2O_2 acts as reducing and oxidising agent respectively in equation (A) and (B).

Ans. **(3)**

Sol. When H_2O_2 acts a reducing agent it liberates the O_2.

$H_2O_2 \rightleftharpoons 2H^+ + O_2 + 2e^-$

9. Which of the following ore is concentrated using group 1 cyanide salt ?

(1) Sphalerite (2) Siderite

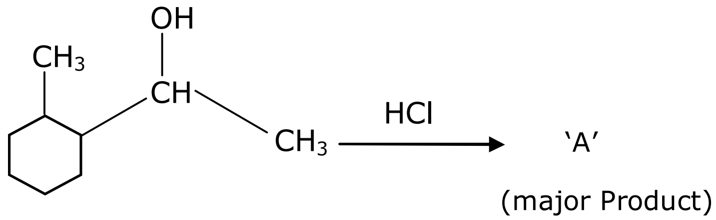
(3) Malachite (4) Calamine

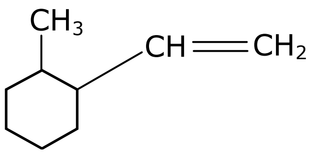
Ans. (1)

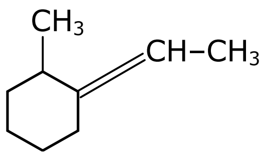
Sol. Conc. of sphalerite, first by cyanide salt as a depressant to remove the impurity of galena

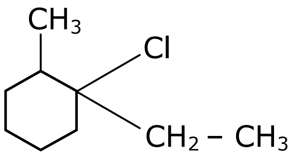
$$\underset{}{Zns} + Pbs + \underset{\text{excess}}{NaCN} \longrightarrow \underset{\text{solution}}{Na_2[Zn(CN)_4]} + PbS\uparrow$$

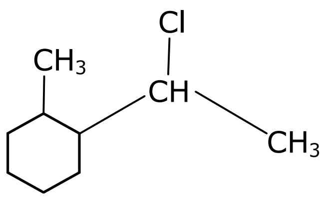
10. Which is the final product (major) 'A' in the given reaction?



(1) 

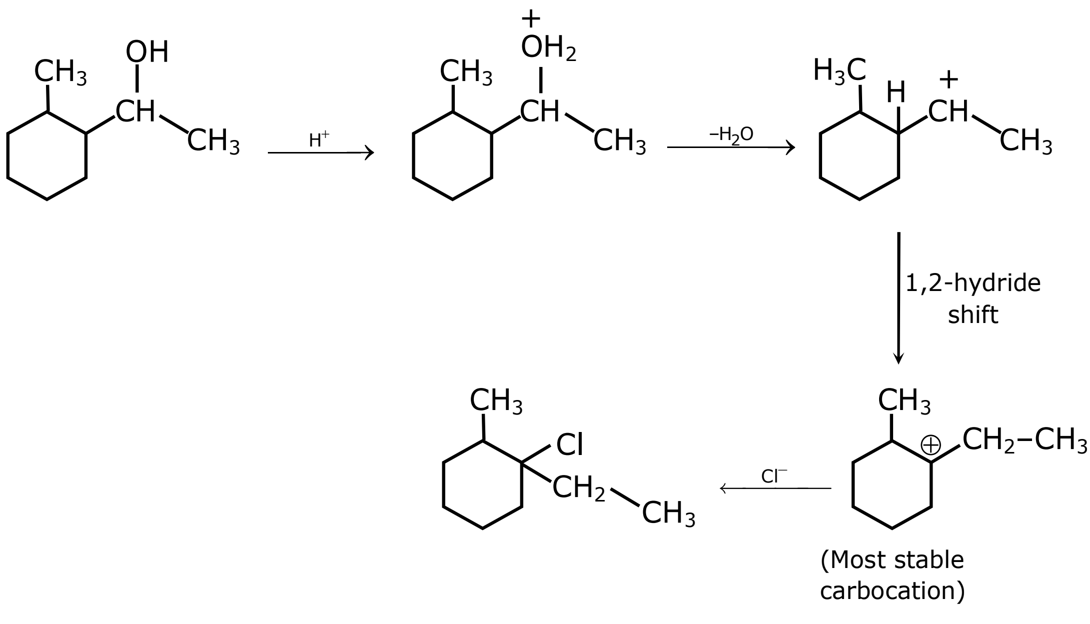
(2) 

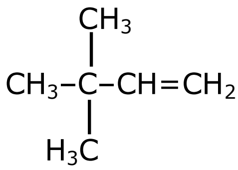
(3) 

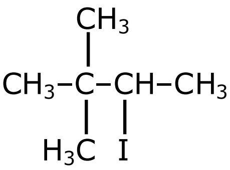
(4) 

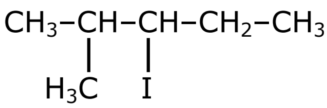
Ans. (3)

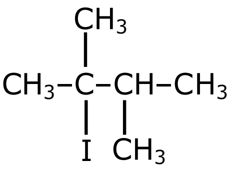
Sol.

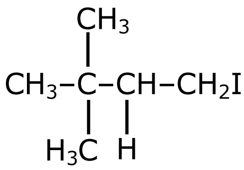


11. What is the major product formed by HI on reaction with  ?

(1) 

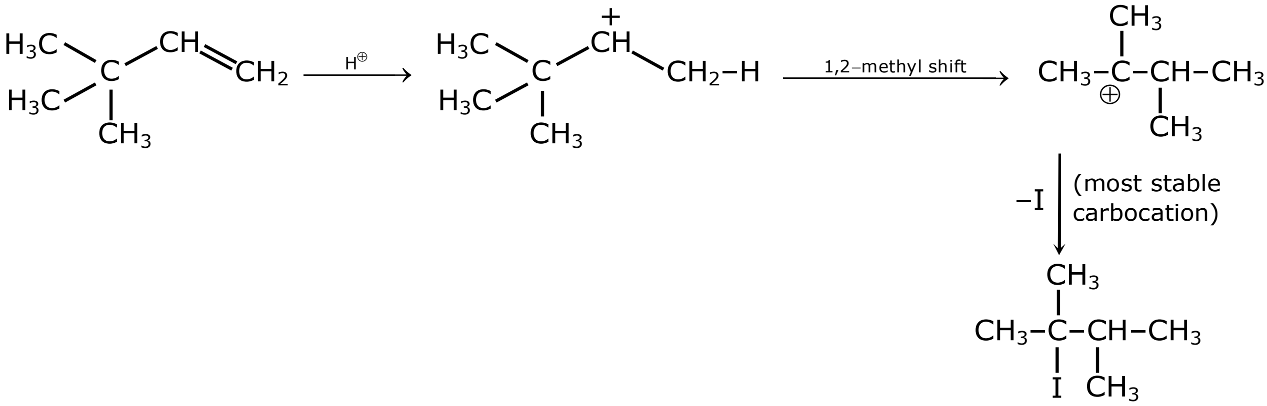
(2) 

(3) 

(4) 

Ans. (3)

Sol.



12. Which of the following reagent is used for the following reaction ?

$CH_3CH_2CH_3 \xrightarrow{?} CH_3CH_2CHO$

(1) Potassium permanganate

(2) Molybdenum oxide

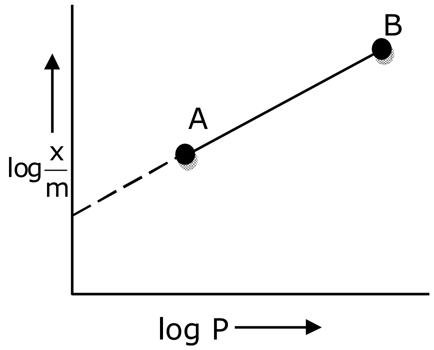
(3) Copper at high temperature and pressure

(4) Manganese acetate

Ans. (2)

Sol. $CH_3{-}CH_2{-}CH_3 \xrightarrow{MO_2O_3} CH_3{-}CH_2{-}CH{=}0$

13. In Freundlich adsorption isotherm, slope of AB line is :



(1) $\frac{1}{n}$ with $\left(\frac{1}{n} = 0 \text{ to } 1\right)$

(2) log $\frac{1}{n}$ with (n<1)

(3) log n with (n>1)

(4) n with (n, 0.1 to 0.5)

Ans. **(1)**

Sol. Freundlich adsorption isotherm is :

$\frac{x}{m} = kp^{1/n}$

x = mass of adsorbate

m = mass of adsorbent

P = eq. pressure

$k_1 n = \frac{1}{n}$ log p + log k

y = mx + c

compairing

m = $\frac{1}{n}$ = slope $[\frac{1}{n} = 0 \text{ to } 1]$

n > 1

14. The major components in "Gun Metal" are:

(1) Al, Cu, Mg and Mn

(2) Cu, Sn and Zn

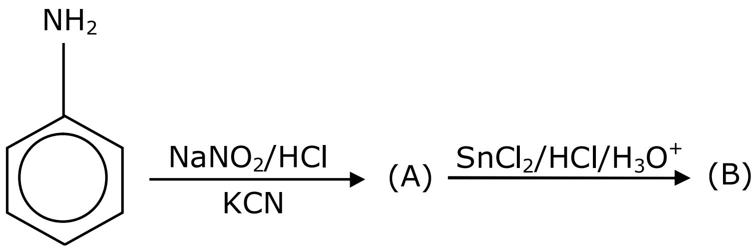
(3) Cu, Zn and Ni

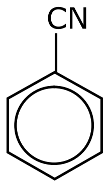
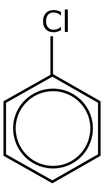
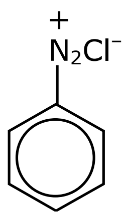
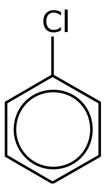
(4) Cu, Ni and Fe

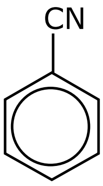
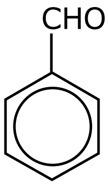
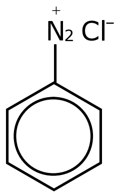
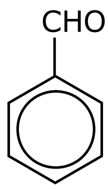
Ans. **(2)**

Sol. "Gun metal" is alloy of copper with tin and zinc.

15. 'A' and 'B' in the following reactions are :

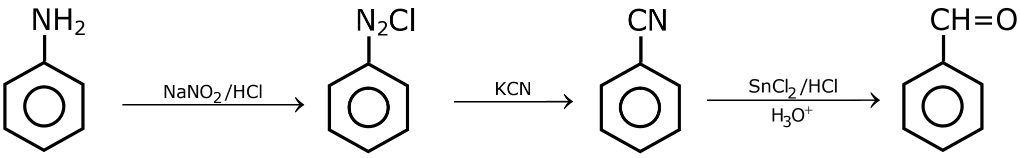


(1) (A) :  (B) :  (2) (A) :  (B) : 

(3) (A) :  (B) :  (4) (A) :  (B) : 

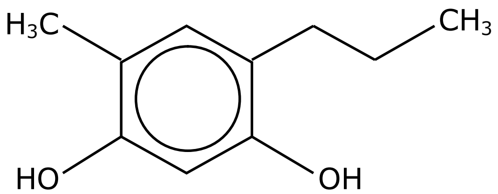
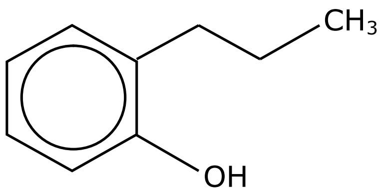
Ans. (3)

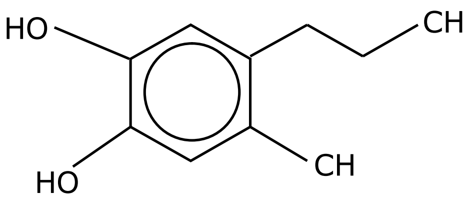
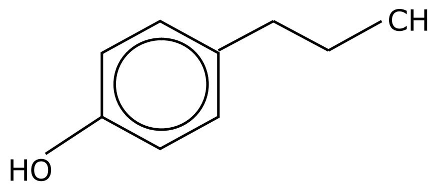
Sol.



(A) (Stephen's rexn) (B)

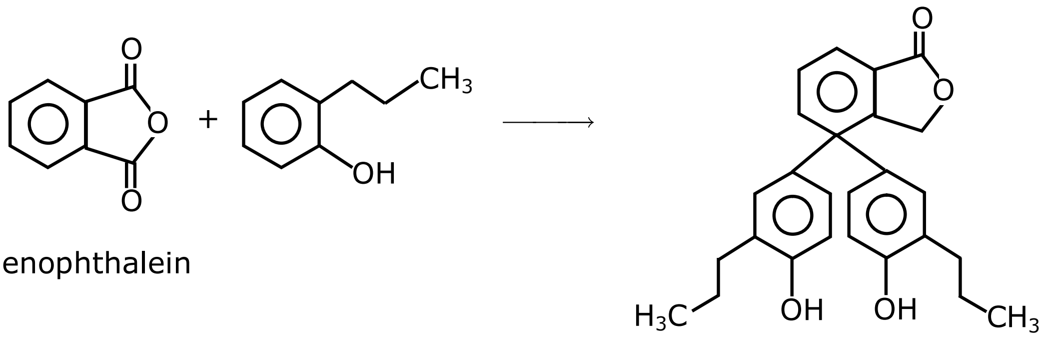
16. Which of the following compound gives pink colour on reaction with phthalic anhydride in conc.H_2SO_4 followed by treatment with NaOH ?

(1)  (2) 

(3)  (4) 

Ans. (2)

Sol.



Phenophthalein

(Pink colour)

17. Consider the elements Mg, Al, S, P and Si, the correct increasing order of their first ionization enthalpy is:

(1) Al < Mg < Si < S < P
(2) Al < Mg < S < Si < P
(3) Mg < Al < Si < S < P
(4) Mg < Al < Si < P < S

Ans. (1)

Sol. Order of IE, in 3rd period is

Na < Mg > Al < Si < P > S < Cl < Ar

Na < Al < Mg < Si < S < P < Cl < Ar

due to stable full filed 3s-orbital and more penetrating power

due to half filed $3p^3$-orbital of phosphorous

18. Given below are two statements :

Statement I : Colourless cupric metaborate is reduced to cuprous metaborate in a luminous flame.

Statement II : Cuprous metaborate is obtained by heating boric anhydride and copper sulphate in a non-luminous flame.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Statement I is false but statement II is true.
(2) Statement I is true but Statement II is false.
(3) Both Statement I and Statement II are true.
(4) Both Statement I and Statement II are false.

Ans. (4)

Sol. Both are False

(1) Copper sulphate form copper meta boric with beric an hydride

$CuSO_4 \longrightarrow CuO + SO_3$

$CuO + B_2O_3 \longrightarrow Cu(BO_2)_2$

blue in cold oxidising flame (non luminous flame)

(2) Blue coloured metal borate is reduced to copper in a luminous flame.

19. Al_2O_3 was leached with alkali to get X. The solution of X on passing of gas Y, forms Z. X, Y and Z respectively are :

(1) X = $Na[Al(OH)_4]$, Y=CO_2, Z = $Al_2O_3.xH_2O$

(2) X=$Na[Al(OH)_4]$, Y=SO_2, Z = Al_2O_3

(3) X=$Al(OH)_3$, Y=SO_2, Z = $Al_2O_3.xH_2O$

(4) X = $Al(OH)_3$, Y=CO_2, Z= Al_2O_3

Ans. (1)

Sol. (1) $Al_2O_3 + NaOH \longrightarrow \underset{\text{"X"}}{Na[Al(OH)_4]}$

(2) $Na[Al(OH)_4] \xrightarrow[\text{"Y"}]{CO_2} \underset{\text{"Z"}}{Al(OH)_3 \text{ or } Al_2O_3 \cdot xH_2O}$

20. Match List I with List II.

List I	**List II**
(Monomer Unit)	(Polymer)
(a) Caprolactum	(i) Natural rubber
(b) 2-Chloro-1,3-butadiene	(ii) Buna-N
(c) Isoprene	(iii) Nylon 6
(d) Acrylonitrile	(iv) Neoprene

Choose the correct answer from the options given below :

(1) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (iv), (c) $\rightarrow$ (i), (d) $\rightarrow$ (ii)

(2) (a) $\rightarrow$ (i), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (iii), (d) $\rightarrow$ (iv)

(3) (a) $\rightarrow$ (ii), (b) $\rightarrow$ (i), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (iii)

(4) (a) $\rightarrow$ (iv), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (ii), (d) $\rightarrow$ (i)

Ans. (1)

Sol. (1) Polymer of caprolactum is nylon-6

(2) Polymer of 2-chloro-1,3-butadiene is neoprene.

(3) Polymer of isoprene is natureal rubber

(4) Polymer of acrylonitrile and 1,3-butadiene is buna-N

SECTION – B

1. The stepwise formation of $[Cu(NH_3)]^{2+}$ is given below:

$$Cu^{2+} + NH_3 \xrightleftharpoons{K_1} \left[Cu(NH_3)_4\right]^{2+}$$

$$\left[Cu(NH_3)_2\right]^{2+} + NH_3 \xrightleftharpoons{K_2} \left[Cu(NH_3)_2\right]^{2+}$$

$$\left[Cu(NH_3)_2\right]^{2+} + NH_3 \xrightleftharpoons{K_3} \left[Cu(NH_3)_3\right]^{2+}$$

$$\left[Cu(NH_3)_3\right]^{2+} + NH_3 \xrightleftharpoons{K_4} \left[Cu(NH_3)_4\right]^{2+}$$

The value of stability constants K_1, K_2, K_3 and K_4 are 10^4, 1.58×10^2 , 5×10^2 and 10^2 respectively. The overall equilibrium constants for dissociation of $[Cu(NH_3)_4]^{2+}$ is $x \times 10^{-12}$. The value of x is ______. (Rounded off to the nearest integer)

Ans. **(1)**

Sol. $\left[Cu(NH_3)_4\right]^{+2} \xrightleftharpoons{k} cu^{+2} + 4NH_3$ .....(A)

For this :

$$Cu^{+2} + NH_3 \xrightleftharpoons{k1} \left[Cu(NH_3)\right]^{+2} ......(1)$$

$$\left[Cu(NH_3)\right]^{+2} + NH_3 \xrightleftharpoons{k_2} \left[cu(NH_3)_2\right]^{+2} .....(2)$$

$$\left[Cu(NH_3)\right]^{+2} + NH_3 \xrightleftharpoons{k_3} \left[cu(NH_3)_3\right]^{+2} ....(3)$$

$$\left[Cu(NH_3)_3\right]^{+2} + NH_3 \xrightleftharpoons{k_4} \left[Cu(NH_3)_4\right]^{+2} ....(4)$$

(1) + (2) + (3) + (4)

$$Cu^{+2} + 4NH_3 \xrightleftharpoons{k_1.k_2.k_3.k_4} \left[C_4(NH_3)_4\right]^{+4} ....(B)$$

So for (A)

$$K = \frac{1}{k_1.K_2.K_3.K_4}$$

Putting the value of k_1,k_2,k_3 and k_4.

$$K = \frac{1}{(10)^4 .(1.58 \times 10^3)(5 \times 10^2)(10)^2} = 1.26 \times 10^{-12}$$

x = 1.

2. At 1990 K and 1 atrm pressure, there are equal number of Cl_2 molecules and Cl atoms in the reaction mixture. The value of K_p for the reaction $Cl_{2(g)} \rightleftharpoons 2Cl_{(g)}$ under the above conditions is x × 10^{-1}. The value of x is _______. (Rounded off to the nearest integer)

Ans. **(5)**

Sol.

$$Cl_2 \rightleftharpoons 2Cl^-$$

Lets mole of eq. $\quad x \qquad x$

P.P. at eq. $\quad \frac{x}{2x}\times 1 \qquad \frac{x}{2x}\times 1$

$\qquad \frac{1}{2} \qquad \frac{1}{2}$

$$K_p = \frac{[P_{c1}]^2}{[P_{Cl_2}]} = \frac{\left[\frac{1}{2}\right]^2}{\frac{1}{2}} = \frac{1}{2} = 0.5 = 5\times 10^{-1}$$

X = 5.

3. 4.5 g of compound A (MW = 90) was used to make 250 mL of its aqueous solution. The molarity of the solution in M is x × 10^{-1}. The value of x is _______. (Rounded off to the nearest integer)

Ans. **(2)**

Sol. Moles of A = $\frac{\text{Weight}}{\text{M.w}}$

$= \frac{4.5}{90} = \frac{1}{20} = 0.05$

Volume (Lit) = $= \frac{250}{1000} = 0.250$ lit lit

Moles of A = $\frac{\text{Weight}}{\text{M.w}}$

$= \frac{4.5}{90} = \frac{1}{20} = 0.05$

Volume (Lit) = $= \frac{250}{1000} = 0.250$ lit lit

Molarity (M) = $\frac{\text{Mole}}{\text{(Lit)volume}} = \frac{0.05}{0.250} = 0.2$

$= 2\times 10^{-1}\frac{\text{mol}}{\text{Lit}}$

x = 2

Molarity (M) = $\frac{\text{Mole}}{\text{(Lit)volume}} = \frac{0.05}{0.250} = 0.2$

$= 2\times 10^{-1}\frac{\text{mol}}{\text{Lit}}$ x = 2

4. The coordination number of an atom in a body-centered cubic structure is ________________.
[Assume that the lattice is made up of atoms]

Ans. **(4)**

Sol. Fact

5. Number of amphoteric compounds among the following is ________.

(A) BeO (B) BaO (C) $Be(OH)_2$ (D) $Sr(OH)_2$

Ans. **(2)**

Sol. BeO and $Be(OH)_2$ are amphoteric in nature

6. When 9.45g of $ClCH_2COOH$ is added to 500 mL of water, its freezing point drops by 0.5°C. The dissociation constant of $ClCH_2COOH$ is x × 10^{-3}. The value of x is _______. (Rounded off to the nearest integer)

$[K_{f(H_2O)} = 1.86\,K\,kg\,mol^{-1}]$

Ans. **(35)**

$$ClCH_2COOH \longrightarrow ClCH_2CO\overset{\ominus}{O} + \overset{\oplus}{H}$$

t = 0	C	O	O
t = t	C−cα	cα	cα

Sol.

Total no. of moles = c+ cα = c(1+ α)

$$i = \frac{\text{observed}}{\text{calculate}} = \frac{c(1+\alpha)}{c} = (1+\alpha)$$

M.W. = 94.5

$\Delta T_f = i \times k_f \times m$

$\Delta T_f = 0.5°C$

$i = 1 + \alpha$

$$0.5 = (1+\alpha) \times 1.86 \times \frac{\frac{9.45}{94.5}}{\frac{500}{1000}}$$

$$m = \frac{\text{mole}}{\text{k.g(Solvent)}}$$

$k_t = 1.86$k kg/mol

$$(1+\alpha) = \frac{2.5}{1.86}$$

$$\alpha = \frac{0.64}{1.86} = \frac{32}{93}$$

$$K_a = \frac{C\alpha^2}{1-\alpha} = \frac{0.2 \times 1024}{93 \times 93 \times \frac{61}{93}}$$

$K_a = 0.0351 = 35.1 \times 10^{-3}$

7. A proton and a Li^{3+} nucleus are accelerated by the same potential. If λ_{Li} and λ_p denote the de Broglie wavelengths of Li^{3+} and proton respectively, then the value of $\frac{\lambda_{Li}}{\lambda_p}$ is $x \times 10^{-1}$. The value of x is _______. [Rounded off to the nearest integer]

[Mass of Li^{3+} = 8.3 mass of proton]

Ans. (2)

Sol. De Brogir Davelength

$$\lambda = \frac{h}{\sqrt{2m\ k.E.}}$$

$$\frac{\lambda_{Li^{+3}}}{\lambda_p} = \sqrt{\frac{m_p \times (e^- v)_p}{m_{Li^{+3}} \times 3e_p v}}$$

$m_{Li^{+3}}$ = 8.3 mp

$$\frac{\lambda_{Li^{+3}}}{\lambda_p} = \sqrt{\frac{m_p}{3 \times 8.3 m_p}} = \sqrt{\frac{1}{25}}$$

$$= \frac{1}{5} = 0.2 = 2 \times 10^{-1}$$

x = 2.

8. Gaseous cyclobutene isomerizes to butadiene in a first order process which has a 'k' value of $3.3 \times 10^{-4}\ s^{-1}$ at 153°C. The time in minutes it takes for the isomerization to proceed 40% to completion at this temperature is ______. (Rounded off to the nearest integer)

Ans. (26)

Sol. For firdst order Rxn :-

$$t = \frac{2.303}{k} \log\left[\frac{100}{100 - x}\right]$$

X = 40, k = 3.3×10^{-4}

$$t = \frac{2.303}{3.3 \times 10^{-4}} \log\left[\frac{100}{60}\right]$$

For firdst order Rxn :-

$$t = \frac{2.303}{k} \log\left[\frac{100}{100 - x}\right]$$

X = 40, k = 3.3×10^{-4}

$$t = \frac{2.303}{3.3 \times 10^{-4}} \log\left[\frac{100}{60}\right]$$

$$t = \frac{2.303}{3.3 \times 10^{-4}} \times 0.22$$

t = $0.1535.3 \times 10^4$

t = 1535 sec.

t = $0.1535.3 \times 10^4$

t = 1535 sec = 25.6 Min.

9. For the reaction $A_{(g)} \rightarrow B_{(g)}$, the value of the equilibrium constant at 300 K and 1 atm is equal to 100.0. The value of $\Delta_r G$ for the reaction at 300 K and 1 atm in J mol^{-1} is $-xR$, where x is ______. (Rounded off to the nearest integer)
[R = 8.31 J $mol^{-1}K^{-1}$ and ln 10 = 2.3]

Ans. **(1380)**

Sol. $\Delta G° = -RT \ln Keq.$
$= -R \times 300 \times \ln(10^2)$
$= 300 \times 2 \times 2.3 \times (-R)$
$= -1380R$
x = 1380 ans.

10. The reaction of sulphur in alkaline medium is given below:

$$S_{8(s)} + a\, OH^-_{(aq)} \longrightarrow b\, S^{2-}_{(aq)} + c\, S_2O_{3\,(aq)}^{2-} + d\, H_2O_{(\ell)}$$

The values of 'a' is __________. (Integer answer)

Ans. **(12)**

Sol. $S_8 + aOH^- \longrightarrow bs^{-2} + CdS_2O_3^{-2} + dH_2O$
$S_8 + bOH^- \longrightarrow 4S^{-2} + 2S_2O_3^{-2} + dH_2O$
$S_8 + 12OH^- \longrightarrow 4S^{-2} + 2S_2O_3^{-2} + 6H_2O$
a = 12

24^{th} Feb. 2021 | Shift - 1
MATHEMATICS

1. The locus of the mid-point of the line segment joining the focus of the parabola $y^2=4ax$ to a moving point of the parabola, is another parabola whose directrix is:.

(1) $x = a$ (2) $x = 0$ (3) $x = -\frac{a}{2}$ (4) $x = \frac{a}{2}$

Ans. (2)

Sol. $h = \frac{at^2 + a}{2}$, $k = \frac{2at + 0}{2}$

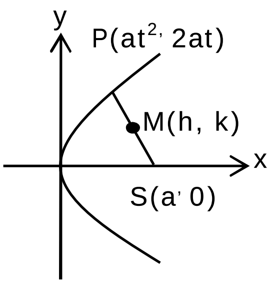
$\Rightarrow t^2 = \frac{2h - a}{a}$ and $t = \frac{k}{a}$

$\Rightarrow \frac{k^2}{a^2} = \frac{2h - a}{a}$

$\Rightarrow$ Locus of (h, k) is $y^2 = a(2x - a)$

$\Rightarrow y^2 = 2a\left(x - \frac{a}{2}\right)$

Its directrix is $x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$



2. A scientific committee is to formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed is:

(1) 560 (2) 1050 (3) 1625 (4) 575

Ans. (3)

Sol. (2I, 4F) + (3I, 6F) + (4I, 8F)

$= {}^6C_2 {}^8C_4 + {}^6C_3 {}^8C_6 + {}^6C_4 {}^8C_8$

$= 15 \times 70 + 20 \times 28 + 15 \times 1$

$= 1050 + 560 + 15 = 1625$

3. The equation of the plane passing through the point (1, 2, –3) and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is:

(1) $3x - 10y - 2z + 11 = 0$
(2) $6x - 5y - 2z - 2 = 0$
(3) $11x + y + 17z + 38 = 0$
(4) $6x - 5y + 2z + 10 = 0$

Ans. (3)

Sol. Normal vector of required plane is $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} - 17\hat{k}$

$\therefore$ 11 (x – 1) + (y – 2) + 17 (z + 3) = 0

$11x + y + 17z + 38 = 0$

4. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man?

(1) B only
(2) A only
(3) All the three
(4) C only

Ans. (1)

Sol. $\frac{x}{a} + \frac{y}{b} = 1$

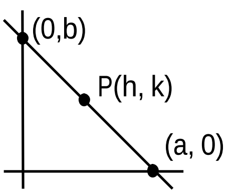
$\frac{h}{a} + \frac{k}{b} = 1$ ....(1)

$\frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{1}{4}$

$\therefore \frac{1}{a} + \frac{1}{b} = \frac{1}{2}$ ....(ii)

$\therefore$ Line passes through fixed point B(2, 2)

(from (1) and (2))



5. The statement among the following that is a tautology is:

(1) $A \wedge (A \vee B)$ (2) $B \rightarrow [A \wedge (A \rightarrow B)]$ (3) $A \vee (A \wedge B)$ (4) $[A \wedge (A \rightarrow B)] \rightarrow B$

Ans. (4)

Sol. $A \wedge (\sim A \vee B) \rightarrow B$

$= [(A \wedge \sim A) \vee (A \wedge B)] \rightarrow B$

$= (A \wedge B) \rightarrow B$

$= \sim A \vee \sim B \vee B$

$= t$

6. Let $f : R \rightarrow R$ be defined as $f(x) = 2x-1$ and $g : R - \{1\} \rightarrow R$ be defined as $g(x) = \dfrac{x - \frac{1}{2}}{x-1}$.

Then the composition function $f(g(x))$ is :

(1) both one-one and onto (2) onto but not one-one

(3) neither one-one nor onto (4) one-one but not onto

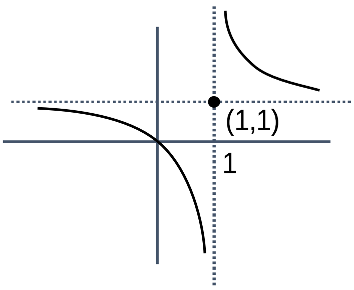
Ans. (4)

Sol. $f(g(x)) = 2g(x) - 1$

$= 2\dfrac{\left(x - \frac{1}{2}\right)}{x-1} = \dfrac{x}{x-1}$

$f(g(x)) = 1 + \dfrac{1}{x-1}$

one-one, into



7. If f:R $\rightarrow$ R is a function defined by f(x) = [x–1] $\cos\left(\frac{2x-1}{2}\right)\pi$, where [.] denotes the greatest integer function, then f is :

(1) discontinuous only at x = 1

(2) discontinuous at all integral values of x except at x = 1

(3) continuous only at x = 1

(4) continuous for every real x

Ans. (4)

Sol. Doubtful points are x = n, n $\in$ I

L.H.L = $\lim\limits_{x\to n^-}\left[x-1\right]\cos\left(\frac{2x-1}{2}\right)\pi$ = (n – 2)cos $\left(\frac{2n-1}{2}\right)\pi$ =0

R.H.L = $\lim\limits_{x\to n^+}\left[x-1\right]\cos\left(\frac{2x-1}{2}\right)\pi$ = (n – 1)cos $\left(\frac{2n-1}{2}\right)\pi$ =0

f(n) = 0

Hence continuous.

8. The function $f(x)=\frac{4x^3-3x^2}{6}-2\sin x+(2x-1)\cos x$:

(1) increases in $\left[\frac{1}{2},\infty\right)$ (2) decreases $\left(-\infty,\frac{1}{2}\right]$

(3) increases in $\left(-\infty,\frac{1}{2}\right]$ (4) decreases $\left[\frac{1}{2},\infty\right)$

Ans. (1)

Sol. f'(x) = (2x – 1) (x –sinx)

$\Rightarrow$ f'(x) $\geq$ 0 in x $\in\left[\frac{1}{2},\infty\right)$

and f'(x) $\leq$ 0 in x $\in\left(-\infty,\frac{1}{2}\right]$

9. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$ and the plane x + y + z = 17 is:

(1) $\sqrt{38}$ (2) $19\sqrt{2}$ (3) $2\sqrt{19}$ (4) 38

Ans. **(1)**

Sol. $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}=\lambda$

$\Rightarrow$ x = λ+3, y = 2λ+4, z = 2λ+5

Which lines on given plane hence

$\Rightarrow \lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 17$

$\Rightarrow \lambda=\frac{5}{5}=1$

Hence, point of intersection is Q (4, 6, 7)

$\therefore$ Required distance = PQ

$= \sqrt{9+25+4}$

$= \sqrt{38}$

10. $\lim_{x\to 0}\frac{\int_0^{x^2}\left(\sin\sqrt{t}\right)dt}{x^3}$ is equal to :

(1) $\frac{2}{3}$ (2) 0 (3) $\frac{1}{15}$ (4) $\frac{3}{2}$

Ans. **(1)**

Sol. $\lim_{x\to 0}\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^3}=\lim_{x\to 0}\frac{\left(\sin|x|\right)2x}{3x^2}=\lim_{x\to 0}\left(\frac{\sin x}{x}\right)\times\frac{2}{3}=\frac{2}{3}$

11. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is:

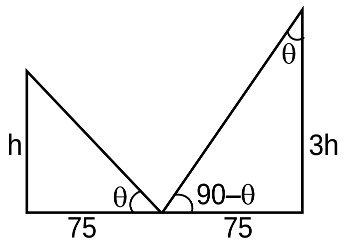
(1) 25 (2) $20\sqrt{3}$ (3) 30 (4) $25\sqrt{3}$

Ans. **(4)**

Sol. $\tan\theta = \frac{h}{75}=\frac{75}{3h}$

$\Rightarrow h^2 = \frac{(75)^2}{3}$

$h = 25\sqrt{3}$m



12. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is :

(1) $-2t^3$ (2) $-t^3$ (3) 0 (4) $2t^3$

Ans. **(1)**

Sol. Equation of tangent at $P(t, t^3)$

$(y - t^3) = 3t^2(x - t)$ ....(1)

Now solve the above equation with

$y = x^3$ ....(2)

By (1) & (2)

$x^3 - t^3 = 3t^2 (x - t)$

$x^2 + xt + t^2 = 3t^2$

$x^2 + xt - 2t^2 = 0$

$(x - t)(x + 2t) = 0$

$\Rightarrow x = -2t \Rightarrow Q(-2t, -8t^3)$

Ordinate of required point = $\dfrac{2t^3 + \left(-8t^3\right)}{3} = -2t^3$

13. The area (in sq. units) of the part of the circle $x^2+y^2=36$, which is outside the parabola $y^2=9x$, is :

(1) $24\pi + 3\sqrt{3}$

(2) $12\pi + 3\sqrt{3}$

(3) $12\pi - 3\sqrt{3}$

(4) $24\pi - 3\sqrt{3}$

Ans. (4)

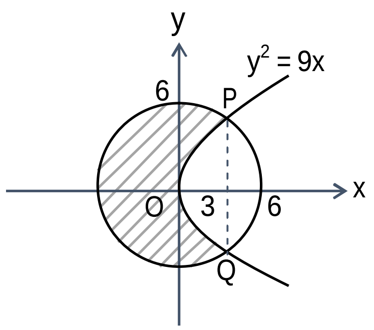
Sol. The curves intersect at point $(3, \pm 3\sqrt{3})$

Required area

$$= \pi r^2 - 2\left[\int_0^3 \sqrt{9x}\,dx + \int_3^6 \sqrt{36 - x^2}\,dx\right]$$

$$= 36\pi - 12\sqrt{3} - 2\left(\frac{x}{2}\sqrt{36 - x^2} + 18\sin^{-1}\left(\frac{x}{6}\right)\right)_3^6$$

$$= 36\pi - 12\sqrt{3} - 2\left(9 - \left(\frac{9\sqrt{3}}{2} + 3\pi\right)\right) = 24\pi - 3\sqrt{3}$$



14. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}}\,dx = a\sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to :

(1) (1, –3) (2) (1, 3) (3) (–1, 3) (4) (3, 1)

Ans. (2)

Sol. put sin x + cos x = t $\Rightarrow$ 1 + sin 2x = t^2

$\Rightarrow$ (cos x – sin x) dx = dt

$$\therefore\ I = \int \frac{dt}{\sqrt{8 - (t^2 - 1)}} = \int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1}\left(\frac{t}{3}\right) + C = \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + C$$

$\Rightarrow$ a = 1 and b = 3

15. The population P = P(t) at time 't' of a certain species follows the differential equation $\frac{dP}{dt}$ = 0.5P – 450. If P(0) = 850, then the time at which population becomes zero is :

(1) $\frac{1}{2}\log_e 18$ (2) $2\log_e 18$ (3) $\log_e 9$ (4) $\log_e 18$

Ans. **(2)**

Sol. $\frac{dp}{dt} = \frac{p-900}{2}$

$\int_{850}^{0} \frac{dp}{p-900} = \int_{0}^{t} \frac{dt}{2}$

$\ell n\left|P-900\right|\Big|_{850}^{0} = \frac{t}{2}$

$\ell n|900| - \ell n|50| = \frac{t}{2}$

$\frac{t}{2} = \ell n|18|$

$\Rightarrow t = 2\ell n 18$

16. The value of

$-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + ... -15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + ... + {}^{14}C_{11}$ is:

(1) 2^{14} (2) $2^{13} - 13$ (3) $2^{16} - 1$ (4) $2^{13} - 14$

Ans. **(4)**

Sol. $S_1 = -{}^{15}C_1 + 2.{}^{15}C_2 - .... - 15\ {}^{15}C_{15}$

$= \sum_{r=1}^{15}(-1)^r .r.{}^{15}C_r = 15\sum_{r=1}^{15}(-1)^r\ {}^{14}C_{r-1}$

$= 15\ (-{}^{14}C_0 + {}^{14}C_1 - ....-{}^{14}C_{14}) = 15\ (0) = 0$

$S_2 = {}^{14}C_1 + {}^{14}C_3 + ...+{}^{14}C_{11}$

$= ({}^{14}C_1 + {}^{14}C_3 + ...+{}^{14}C_{11} + {}^{14}C_{13}) - {}^{14}C_{13}$

$= 2^{13} - 14$

$= S_1 + S_2 = 2^{13} - 14$

17. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

(1) $\frac{3}{16}$ (2) $\frac{1}{2}$ (3) $\frac{5}{16}$ (4) $\frac{1}{32}$

Ans. (2)

Sol. P(odd no. twice) = P(even no. thrice)

$\Rightarrow {}^nC_2\left(\frac{1}{2}\right)^n = {}^nC_3\left(\frac{1}{2}\right)^n \Rightarrow n = 5$

Success is getting an odd number then P(odd successes) = P(1) + P(3) + P(5)

$= {}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$

$= \frac{16}{2^5} = \frac{1}{2}$

18. Let p and q be two positive number such that p + q = 2 and $p^4 + q^4 = 272$. Then p and q are roots of the equation :

(1) $x^2 - 2x + 2 = 0$ (2) $x^2 - 2x + 8 = 0$

(3) $x^2 - 2x + 136 = 0$ (4) $x^2 - 2x + 16 = 0$

Ans. (4)

Sol. $(p^2 + q^2)^2 - 2p^2q^2 = 272$

$((p + q)^2 - 2pq)^2 - 2p^2q^2 = 272$

$16 + 16pq + 2p^2q^2 = 272$

$(pq)^2 - 8pq - 128 = 0$

$pq = \frac{8 \pm 24}{2} = 16, -8$

$pq = 16$

Now

$x^2 - (p + q)x + pq = 0$

$x^2 - 2x + 16 = 0$

19. If $e^{\left(\cos^2 x+\cos^4 x+\cos^6 x+...\infty\right)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2\sin x}{\sin x+\sqrt{3}\cos x}\left(0<x<\frac{\pi}{2}\right)$ is :

(1) $\frac{3}{2}$ (2) $2\sqrt{3}$ (3) $\frac{1}{2}$ (4) $\sqrt{3}$

Ans. **(3)**

Sol. $e^{\left(\cos^2 x+\cos^4 x+......\infty\right)\ell n 2} = 2^{\cos^2 x+\cos^4 x+......\infty}$

$= 2^{\cot^2 x}$

$t^2 - 9t + 8 = 0 \Rightarrow t = 1, 8$

$\Rightarrow 2^{\cot^2 x} = 1, 8 \Rightarrow \cot^2 x = 0, 3$

$0 < x < \frac{\pi}{2} \Rightarrow \cot x = \sqrt{3}$

$\Rightarrow \frac{2\sin x}{\sin x+\sqrt{3}\cos x} = \frac{2}{1+\sqrt{3}\cot x} = \frac{2}{4} = \frac{1}{2}$

20. The system of linear equations

$3x - 2y - kz = 10$

$2x - 4y - 2z = 6$

$x + 2y - z = 5m$

is inconsistent if :

(1) $k = 3,\ m = \frac{4}{5}$ (2) $k \neq 3, m \in R$

(3) $k \neq 3,\ m \neq \frac{4}{5}$ (4) $k = 3,\ m \neq \frac{4}{5}$

Ans. **(4)**

Sol. $\Delta = \begin{vmatrix} 3 & -2 & -k \\ 1 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$

$3(4 + 4) + 2(-2 + 2) - k(4 + 4) = 0$

$\Rightarrow k = 3$

$\Delta_x = \begin{vmatrix} 10 & -2 & -3 \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix} \neq 0$

$10(4 + 4) + 2(-6 + 10m) - 3(12 + 20m) \neq 0$

$80 - 12 + 20m - 36 - 60m \neq 0$

$40m \neq 32 \Rightarrow m \neq \frac{4}{5}$

$$\Delta_y = \begin{vmatrix} 3 & 10 & -3 \\ 2 & 6 & -2 \\ 1 & 5m & -1 \end{vmatrix} \neq 0$$

$3(-6 + 10m) - 10(-2 + 2) - 3(10m - 6) \neq 0$

$-18 + 30m - 30m + 18 \neq 0 \Rightarrow 0$

$$\Delta_z = \begin{vmatrix} 3 & -2 & 10 \\ 2 & -4 & 6 \\ 1 & 2 & 5m \end{vmatrix} \neq 0$$

$3(-20m - 12) + 2(10m - 6) + 10(4 + 4) - 40m + 32 \neq 0 \Rightarrow m \neq \frac{4}{5}$

Section – B

1. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to ______

Ans. **17**

Sol. As $PQ = KI \Rightarrow Q = kP^{-1}I$

now $Q = \frac{k}{|P|}(adjP)\,I \Rightarrow Q = \frac{k}{(20+12\alpha)}\begin{bmatrix} - & - & - \\ - & - & (-3\alpha - 4) \\ - & - & - \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\because q_{23} = \frac{-k}{8} \Rightarrow \frac{k}{(20+12\alpha)}(-3\alpha - 4) = \frac{-k}{8} \Rightarrow 2(3\alpha + 4) = 5 + 3\alpha$

$3\alpha = -3 \Rightarrow \alpha = -1$

also $|Q| = \frac{k^3|I|}{|P|} \Rightarrow \frac{k^2}{2} = \frac{k^3}{(20+12\alpha)}$

$(20+12\alpha) = 2k \Rightarrow 8 = 2k \Rightarrow k = 4$

2. Let $B_i(i=1, 2, 3)$ be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha-2\beta)p=\alpha\beta$ and $(\beta-3\gamma)p=2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to ____

Ans. **6**

Sol. Let x, y, z be probability of B_1, B_2, B_3 respectively

$\Rightarrow x(1-y)(1-z) = \alpha$

$\Rightarrow y(1-x)(1-z) = \beta$

$\Rightarrow z(1-x)(1-y) = \gamma$

$\Rightarrow (1-x)(1-y)(1-z) = p$

$(\alpha - 2\beta)p = \alpha\beta$

$(x(1-y)(1-z)-2y(1-x)(1-z))(1-x)(1-y)(1-z) = xy(1-x)(1-y)(1-z)$

$x - xy - 2y + 2xy = xy$

$x = 2y \qquad ...(1)$

Similarly $(\beta-3r)p = 2\beta r$

$\Rightarrow y = 3z \qquad ...(2)$

From (1) & (2)

$x = 6z$

Now

$\frac{x}{z} = 6$

3. The minimum value of α for which the equation $\frac{4}{\sin x}+\frac{1}{1-\sin x}=\alpha$ has at least one solution in $\left(0,\frac{\pi}{2}\right)$ is _____

Ans. **9**

Sol. $f(x)=\frac{4}{\sin x}+\frac{1}{1-\sin x}$

Let $\sin x = t \quad \because x \in \left(0,\frac{\pi}{2}\right) \quad \Rightarrow 0 < t < 1$

$$f(t) = \frac{4}{t} + \frac{1}{1-t}$$

$$f'(t) = \frac{-4}{t^2} + \frac{1}{(1-t)^2}$$

$$= \frac{t^2 - 4(1-t)^2}{t^2(1-t)^2}$$

$$= \frac{(t-2(1-t))(t+2(1-t))}{t^2(1-t)^2}$$

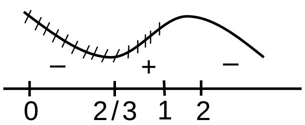
$$= \frac{(3t-2)(2-t)}{t^2(1-t)^2}$$

f_{min} at $t = \frac{2}{3}$

$$\alpha_{min} = f\left(\frac{2}{3}\right) = \frac{4}{\frac{2}{3}} + \frac{1}{1-\frac{2}{3}}$$

= 6 + 3

= 9



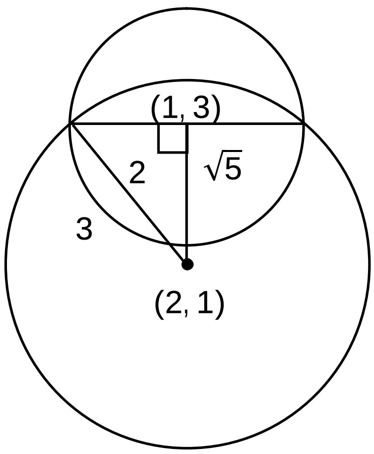
4. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C' whose center is at (2,1), then its radius is ______

Ans. **3**

distance between (1, 3) and (2, 1) is $\sqrt{5}$

$\therefore \left(\sqrt{5}\right)^2 + (2)^2 = r^2$

$\Rightarrow$ r = 3



5. $\lim\limits_{x \to \infty} \tan\left\{\sum\limits_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ is equal to ______

Ans. **1**

Sol. $\tan\left(\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\left[\tan^{-1}(r+1)-\tan^{-1}(r)\right]\right)$

$= \tan\left(\lim\limits_{n\to\infty}\left(\tan^{-1}(n+1)-\frac{\pi}{4}\right)\right)$

$= \tan\left(\frac{\pi}{4}\right) = 1$

6. If $\int\limits_{-a}^{a}(|x|+|x-2|)\,dx = 22$,(a > 2) and [x] denotes the greatest integer $\le$ x, then $\int\limits_{a}^{-a}(x+[x])\,dx$ is equal to _____

Ans. **3**

Sol. $\int\limits_{-a}^{0}(-2x+2)\,dx + \int\limits_{0}^{2}(x+2-x)\,dx + \int\limits_{2}^{a}(2x-2)\,dx = 22$

$x^2 - 2x\,|_0^{-a} + 2x\,|_0^2 + x^2 - 2x\,|_2^a = 22$

$a^2 + 2a + 4 + a^2 - 2a - (4 - 4) = 22$

$2a^2 = 18 \Rightarrow a = 3$

$\int\limits_{3}^{-3}(x+[x])\,dx = -\left(\int\limits_{-3}^{3}(x+[x])\,dx\right) = -\left(\int\limits_{-3}^{3}[x]\,dx\right)$

$= -(-3 - 2 - 1 + 0 + 1 + 2) = 3$

7. Let three vectors $\vec{a},\vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a}.\vec{c} = 7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a} = -\hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = 2\hat{i}+\hat{k}$, then the value of $2\left|\vec{a}+\vec{b}+\vec{c}\right|^2$ is _______

Ans. **75**

Sol. $\vec{c} = \lambda\left(\vec{b}\times\left(\vec{a}\times\vec{b}\right)\right)$

$= \lambda\left(\left(\vec{b}.\vec{b}\right)\vec{b} - \left(\vec{b}.\vec{a}\right)\vec{b}\right)$

$= \lambda\left(5\left(-\hat{i}+\hat{j}+\hat{k}\right)+2\hat{i}+\hat{k}\right)$

$= \lambda\left(-3\hat{i} + 5\hat{j} + 6\hat{k}\right)$

$\vec{c}.\vec{a} = 7 \Rightarrow 3\lambda + 5\lambda + 6\lambda = 7$

$\lambda = \frac{1}{2}$

$\therefore\ 2\left|\left(\frac{-3}{2} - 1 + 2\right)\hat{i} + \left(\frac{5}{2} + 1\right)\hat{j} + (3 + 1 + 1)\hat{k}\right|^2$

$= 2\left(\frac{1}{4} + \frac{49}{4} + 25\right) = 25 + 50 = 75$

8. Let $A = \{n \in N : n \text{ is a 3-digit number}\}$

$B = \{9k + 2 : k \in N\}$

and $C : \{9k + \ell : k \in N\}$ for some ℓ $(0 < \ell < 9)$

If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then ℓ is equal to __

Ans. 5

Sol. 3 digit number of the form 9K + 2 are {101, 109, ...992}

$\Rightarrow$ Sum equal to $\frac{100}{2}$ (1093) = s_1 = 54650

$274 \times 400 = s_1 + s_2$

$274 \times 400 = \frac{100}{2}[101 + 992] + s_2$

$274 \times 400 = 50 \times 1093 + s_2$

$s_2 = 109600 - 54650$

$s_2 = 54950$

$s_2 = 54950 = \frac{100}{2}\left[(99 + \ell) + (990 + \ell)\right]$

$1099 = 2\ell + 1089$

$\ell = 5$

9. If the least and the largest real values of α, for which the equation $z + \alpha|z-1| + 2i = 0$ $\left(z \in C \text{ and } i = \sqrt{-1}\right)$ has a solution, are p and q respectively; then $4(p^2 + q^2)$ is equal to ___

Ans. **10**

Sol. $x + iy + \alpha\sqrt{(x-1)^2 + y^2} + 2i = 0$

$\therefore y + 2 = 0$ and $x + \alpha\sqrt{(x-1)^2 + y^2} = 0$

$y = -2$ & $x^2 = \alpha^2(x^2 - 2x + 1 + 4)$

$\alpha^2 = \dfrac{x^2}{x^2 - 2x + 5} \Rightarrow x^2(\alpha^2-1) - 2x\alpha^2 + 5\alpha^2 = 0$

$x \in R \Rightarrow D \geq 0$

$4\alpha^4 - 4(\alpha^2 - 1)5\alpha^2 \geq 0$

$\alpha^2\left[4\alpha^2 - 2\alpha^2 + 20\right] \geq 0$

$\alpha^2\left[-16\alpha^2 + 20\right] \geq 0$

$\alpha^2\left[\alpha^2 - \dfrac{5}{4}\right] \leq 0$

$0 \leq \alpha^2 \leq \dfrac{5}{4}$

$\therefore \alpha^2 \in \left[0, \dfrac{5}{4}\right]$

$\therefore \alpha \in \left[-\dfrac{\sqrt{5}}{2}, \dfrac{\sqrt{5}}{2}\right]$

then $4[(q)^2 + (p)^2] = 4\left[\dfrac{5}{4} + \dfrac{5}{4}\right] = 10$

10. Let M be any 3×3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of M^TM is seven, is __

Ans. **540**

Sol. $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case I : Seven (1's) and two (0's)

$^9C_2 = 36$

Case II : One (2) and three (1's) and five (0's)

$$\frac{9!}{5!3!} = 504$$

$\therefore$ Total = 540