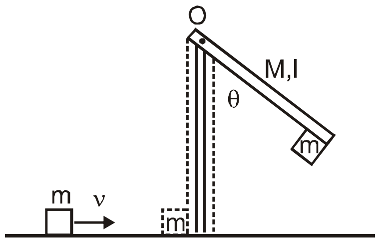
QUESTION PAPER WITH SOLUTION

PHYSICS _ 3 Sep. _ SHIFT - 1

1. A block of mass m = 1 kg slides with velocity ν = 6 m/s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass M = 2 kg, and length l = 1m, the value of θ is approximately :
(take g = 10 m/s^2)



(1) 49°　　(2) 63°　　(3) 69°　　(4) 55°

Sol. **2**

$mv\ell = I\omega$

$\omega = \frac{mv\ell}{I}$

$\frac{1}{2}I\omega^2 = (m+M)\,g\,\ell_{com}(1-\cos\theta)$

$\frac{1}{2}\frac{(mv\ell)^2}{I} = (m+M)\,g\,\ell_{com}(1-\cos\theta)$

$I = \left(\frac{M\ell^2}{3} + m\ell^2\right)$

$I = \left(\frac{2}{3}+1\right) = \frac{5}{3}$

$\frac{36\times 3}{2\times 5} = \frac{3\times 10\times 2}{3}(1-\cos\theta)$

$\frac{27}{50} = (1-\cos\theta)$

$\cos\theta = 1 - \frac{27}{50}$

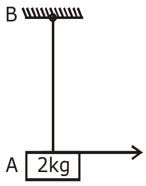
$\cos\theta = \frac{23}{50}$

$\theta = 63°$

2. A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wavetrain of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wavetrain (in cm) when it reaches the top of the rope ?

(1) 12 (2) 3 (3) 9 (4) 6

Sol. **1**



$\ell = 12\text{m}$

$\text{m} = 6\text{kg}$

$$\mu = \frac{6}{12} = \frac{1}{2}\text{kg / m}$$

$$A \to V = \sqrt{\frac{T}{\mu}} = f\lambda$$

$$B \to \sqrt{\frac{T'}{\mu}} = f\lambda'$$

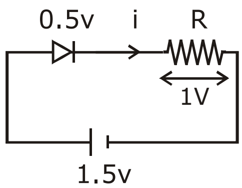
$$\sqrt{\frac{T}{T'}} = \frac{\lambda}{\lambda'}$$

$$\sqrt{\frac{20}{80}} = \frac{6}{\lambda'} = \lambda' = 12$$

3. When a diode is forward biased, it has a voltage drop of 0.5 V. The safe limit of current through the diode is 10 mA. If a battery of emf 1.5 V is used in the circuit, the value of minimum resistance to be connected in series with the diode so that the current does not exceed the safe limit is :

(1) 300 Ω (2) 200 Ω (3) 50 Ω (4) 100 Ω

Sol. **4**



V = i R

$$R = \frac{V}{i} = \frac{1}{10\text{mA}}$$

$$= \frac{1000}{10} = 100\Omega$$

4. Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as :

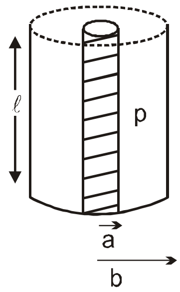
(1) 2.125 cm (2) 2.124 cm (3) 2.123 cm (4) 2.121 cm

Sol. **2**

Least count = $\frac{0.1}{50} = \frac{1}{500} = 0.2 \times 10^{-2} = 0.002$

when we multiply by division no.'s
it must be even because L.C. is 0.002.

5. Model a torch battery of length ℓ to be made up of a thin cylindrical bar of radius 'a' and a concentric thin cylindrical shell of radius 'b' filled in between with an electrolyte of resistivity ρ (see figure). If the battery is connected to a resistance of value R, the maximum joule heating in R will take place for :

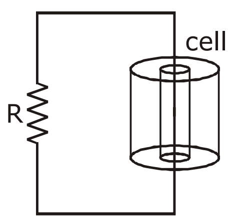


(1) $R = \frac{\rho}{2\pi l}\left(\frac{b}{a}\right)$ (2) $R = \frac{2\rho}{\pi l}\ln\left(\frac{b}{a}\right)$ (3) $R = \frac{\rho}{\pi l}\ln\left(\frac{b}{a}\right)$ (4) $\frac{\rho}{2\pi l}\ln\left(\frac{b}{a}\right)$

Sol. **4**

$$dR = \rho\frac{dr}{2\pi r\ell}$$

$$\int dR = \frac{\rho}{2\pi\ell}\int_a^b \frac{1}{r}dr$$

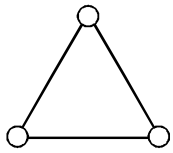


$$R = \frac{\rho}{2\pi\ell}\left[\ln(r)\right]_a^b$$

$$R = \frac{\rho}{2\pi\ell}\ln\left(\frac{b}{a}\right)$$

r = R
For Max heat transfer

6. Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is :



(1) $\frac{3}{2}$ RT (2) 3RT (3) $\frac{5}{2}$RT (4) $\frac{9}{2}$ RT

Sol. **2**

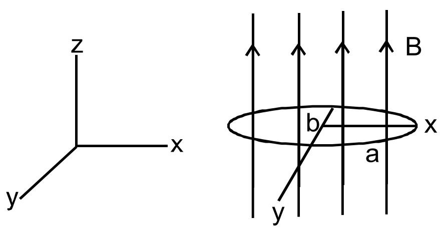
$U = \frac{f}{2}nRT$

$U = \frac{6}{2}nRT$

U = 3nRT n = 1

U = 3RT

7. An elliptical loop having resistance R, of semi major axis a, and semi minor axis b is placed in magnetic field as shown in the figure. If the loop is rotated about the x-axis with angular frequency ω, the average power loss in the loop due to Joule heating is :



(1) $\frac{\pi abB\omega}{R}$ (2) $\frac{\pi^2a^2b^2B^2\omega^2}{R}$ (3) $\frac{\pi^2a^2b^2B^2\omega^2}{2R}$ (4) Zero

Sol. **3**

$\phi = BA\cos\omega t$

$\epsilon = \frac{d\phi}{dt} = BA\omega\sin\omega t$

$\epsilon = B\pi ab\omega \sin\omega t$

$\epsilon_0 = B\pi ab\omega$

$P = \frac{\epsilon^2}{R} = \frac{B^2\pi^2a^2b^2\omega^2}{R}(\sin^2\omega t)$

$P_{av} = \frac{B^2\pi^2a^2b^2\omega^2}{2R}$

8. A balloon filled with helium (32° C and 1.7 atm.) bursts. Immediately afterwards the expansion of helium can be considered as :

(1) reversible isothermal (2) irreversible isothermal

(3) reversible adiabatic (4) irreversible adiabatic

Sol. **4**

irreversible adiabatic $\rightarrow$ Because Energy can not be restored.

9. When the wavelength of radiation falling on a metal is changed from 500 nm to 200 nm, the maximum kinetic energy of the photoelectrons becomes three times larger. The work function of the metal is close to :

(1) 1.02 eV (2) 0.61 eV (3) 0.52 eV (4) 0.81 eV

Sol. **2**

$$KE_{max} = \frac{hc}{\lambda_1} - \phi$$

$$3KE_{max} = \frac{hc}{\lambda_2} - \phi$$

$$3\left(\frac{hc}{\lambda_1} - \phi\right) = \frac{hc}{\lambda_2} - \phi$$

$$\frac{3hc}{\lambda_1} - \frac{hc}{\lambda_2} = 2\phi$$

$$\frac{3 \times 1240}{500} - \frac{1240}{200} = 2\phi$$

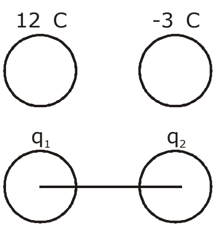
$$\phi = 0.6$$

10. Two isolated conducting spheres S_1 and S_2 of radius $\frac{2}{3}$R and $\frac{1}{3}$R have 12 μC and –3μC charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on S_1 and S_2 are respectively :

(1) 6 μC and 3 μC (2) 4.5 μC on both

(3) + 4.5 μC and –4.5 μC (4) 3 μC and 6 μC

Sol. **1**



$q_1 + q_2 = 9\mu C$ ..(1)

$$\frac{Kq_1}{\frac{2}{3}R} = \frac{Kq_2}{\frac{1}{3}R}$$

$q_1 = 2q_2$

$3q_2 = 9\mu c$

$q_2 = 3\mu c$

$q_1 = 6\mu c$

11. In a radioactive material, fraction of active material remaining after time t is 9/16. The fraction that was remaining after t/2 is :

(1) $\frac{3}{4}$ (2) $\frac{7}{8}$ (3) $\frac{4}{5}$ (4) $\frac{3}{5}$

Sol. **1**

$N = N_0\ e^{-\lambda t}$

$\left(\frac{N}{N_0}\right) = e^{-\lambda t}$

$\frac{9}{16} = e^{-\lambda t}$

$\frac{N}{N_0} = e^{-\lambda t/2}$

$\frac{N}{N_0} = \left(\frac{9}{16}\right)^{\frac{1}{2}} = \frac{3}{4}$

12. Moment of inertia of a cylinder of mass M, length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of a material, the ratio L /R for it to have minimum possible I is :

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\sqrt{\frac{2}{3}}$ (4) $\sqrt{\frac{3}{2}}$

Sol. **4**

M = d.V

$M = d\pi R^2 L$

$I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)$

$I = M\left(\frac{M}{4d\pi L} + \frac{L^2}{12}\right)$

$$I = \left(\frac{M^2}{4d\pi L} + \frac{ML^2}{12}\right)$$

$$\frac{dI}{dL} = \frac{M^2}{4d\pi}\left(\frac{-1}{L^2}\right) + \frac{2LM}{12} = 0$$

$$\frac{M^2}{4d\pi L^2} = \frac{2LM}{12}$$

$$\frac{d\pi R^2 L}{4d\pi L^2} = \frac{L}{6}$$

$$\frac{R^2}{L^2} = \frac{2}{3}$$

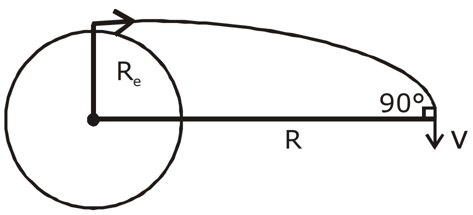
$$\frac{R}{L} = \sqrt{\frac{2}{3}} \Rightarrow \frac{L}{R} = \sqrt{\frac{3}{2}}$$

13. A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e. By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that it become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R. Value of R is :

(1) $2R_e$ (2) $3R_e$ (3) $4R_e$ (4) $2.5\ R_e$

Sol. **2**

$$V_0 = \sqrt{\frac{GM}{R_e}} \times \sqrt{\frac{3}{2}}$$



$$mV_0 R_e = mVR$$

$$\sqrt{\frac{3}{2}}\sqrt{\frac{GM}{R_e}}\ R_e = VR$$

$$-\frac{GMm}{R_e} + \frac{1}{2}mv_0^2 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$-\frac{GMm}{R_e} + \frac{1}{2}m\frac{3}{2}\frac{GM}{R_e} = -\frac{GMm}{R} + \frac{1}{2}m\frac{3}{2}\frac{GM}{R_e}\frac{R_e^2}{R^2}$$

$$-\frac{1}{R_e}+\frac{3}{4R_e}=-\frac{1}{R}+\frac{3}{4}\frac{R_e}{R^2}$$

$$-\frac{1}{4R_e}=-\frac{1}{R}+\frac{3}{4}\frac{R_e}{R^2}$$

By further calculating

$R = 3R_e$

14. Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is :

(1) 4 : 1 (2) 2 : 1 (3) 0.8 : 1 (4) 8 : 1

Sol. **4**

$$P_{in} = P_0 + \frac{4T}{R_1}$$

$$1.01 = 1+\frac{4T}{R_1}$$

$$0.01 = \frac{4T}{R_1}$$

$$0.02 = \frac{4T}{R_2}$$

$$\frac{1}{2}=\frac{R_2}{R_1}$$

$$\frac{V_1}{V_2}=\frac{8}{1}$$

15. In a Young's double slit experiment, light of 500 nm is used to produce an interference pattern. When the distance between the slits is 0.05 mm, the angular width (in degree) of the fringes formed on the distance screen is close to :

(1) 0.17° (2) 0.07° (3) 0.57° (4) 1.7°

Sol. **3**

$$B = \frac{\lambda D}{d}$$

$$\theta = \frac{\beta}{D} = \left(\frac{\lambda}{d}\right)$$

$$\theta = \frac{500\times 10^{-9}}{(0.05\times 10^{-3})}$$

$$\theta = \frac{5\times 10^{-2}}{5} = \frac{5}{100\times 5}$$

$$\theta^\circ = \frac{5}{100}\times\frac{180}{5\times\pi}$$

$$\theta^\circ = 0.57^\circ$$

16. A 750 Hz, 20 V (rms) source is connected to a resistance of 100 Ω, an inductance of 0.1803 H and a capacitance of 10 μF all in series. The time in which the resistance (heat capacity 2 J/°C) will get heated by 10°C. (assume no loss of heat to the surroundings) is close to :

(1) 245 s (2) 365 s (3) 418 s (4) 348 s

Sol. **4**

f = 750Hz, V_{rms} = 20V, R = 100Ω, L = 0.1803H

C = 10μf

$S = 2J / °C$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fc}\right)^2}$$

$$|z| = \sqrt{(100)^2 + (2 \times 3.14 \times 750 \times 0.1803 - \frac{1}{2 \times 3.14 \times 750 \times 10^{-5}})^2}$$

|Z| = 834Ω

In AC

$$P = V_{rms} i_{rms} \cos\phi$$

$$P = \left(V_{rms} \cdot \frac{V_{rms}}{|Z|} \cdot \frac{R}{|Z|}\right)$$

$$P = \left(\frac{V_{rms}}{|Z|}\right)^2 R$$

$$P = \left(\frac{20}{834}\right)^2 \times 100 = 0.0575 J / S$$

P×t = S$\Delta\theta$

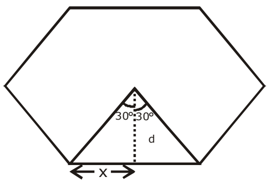
$$t = \frac{2(10)}{0.0575}$$

t = 348 sec

17. Magnitude of magnetic field (in SI units) at the centre of a hexagonal shape coil of side 10 cm, 50 turns and carrying current I (Ampere) in units of $\frac{\mu_0 I}{\pi}$ is :

(1) $250\sqrt{3}$ (2) $50\sqrt{3}$ (3) $500\sqrt{3}$ (4) $5\sqrt{3}$

Sol. **3**



$$\tan 30 = \frac{x}{d}$$

$$d = \frac{x}{\tan 30}$$

$$d = \frac{5 \times 10^{-2}}{\frac{1}{\sqrt{3}}}$$

$$d = 5\sqrt{3} \times 10^{-2}$$

$$B = \frac{6 \times \mu_0 IN}{4\pi d}\left(\sin\theta_1 + \sin\theta_2\right)$$

$$B = \frac{\mu_o I}{4\pi} \times \frac{50 \times 6}{5\sqrt{3} \times 10^{-2}}\left(\sin 30 + \sin 30\right)$$

$$B = \frac{10 \times 6 \times 100}{\sqrt{3} \times 4}\left(\frac{\mu_0 I}{\pi}\right)$$

$$B = 500\sqrt{3}$$

18. The magnetic field of a plane electromagnetic wave is

$\vec{B} = 3 \times 10^{-8} \sin[200\pi (y + ct)]\,\hat{i}$ T

where $c = 3 \times 10^8$ ms^{-1} is the speed of light.

The corresponding electric field is :

(1) $\vec{E} = -9 \sin[200\pi (y + ct)]\,\hat{k}$ V/m

(2) $\vec{E} = 9 \sin[200\pi (y + ct)]\,\hat{k}$ V/m

(3) $\vec{E} = -10^{-6} \sin[200\pi (y + ct)]\,\hat{k}$ V/m

(4) $\vec{E} = 3 \times 10^{-8} \sin[200\pi (y + ct)]\,\hat{k}$ V/m

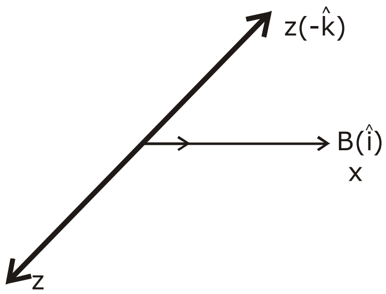
Sol. **1**

$E = BC$

$E = 3 \times 10^{-8} \times 3 \times 10^{8}$

$E = 9$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{r}$$



$$E = 9\sin\left[200\pi(y + ct)(-\hat{k})\right]$$

19. A charged particle carrying charge 1 μC is moving with velocity $(2\hat{i} + 3\hat{j} + 4\hat{k})$ ms^{-1}. If an external magnetic field of $(5\hat{i} + 3\hat{j} - 6\hat{k}) \times 10^{-3}$ T exists in the region where the particle is moving then the force on the particle is $\vec{F} \times 10^{-9}$ N. The vector $\vec{F}$ is :

(1) $-0.30\hat{i} + 0.32\hat{j} - 0.09\hat{k}$

(2) $-3.0\hat{i} + 3.2\hat{j} - 0.9\hat{k}$

(3) $-30\hat{i} + 32\hat{j} - 9\hat{k}$

(4) $-300\hat{i} + 320\hat{j} - 90\hat{k}$

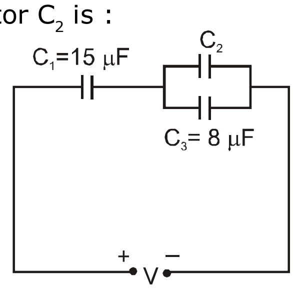
Sol. **3**

$\vec{F} = q(\vec{V} \times \vec{B})$

$\vec{F} = 10^{-6}(2\hat{i} + 3\hat{j} + 4\hat{k}) \times (5\hat{i} + 3\hat{j} - 6\hat{k}) \times 10^{-3}$

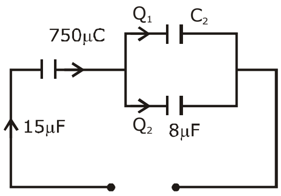
$\vec{F} = (-30\hat{i} + 32\hat{j} - 9\hat{k}) \times 10^{-9}$

20. In the circuit shown in the figure, the total charge is 750 μC and the voltage across capacitor C_2 is 20 V. Then the charge on capacitor C_2 is :



(1) 650 μC (2) 450 μC (3) 590 μC (4) 160 μC

Sol. **3**



$Q_2 = CV = 8\mu f 20 = 160\mu C$

$Q_1 = 750 - 160 = 590\mu C$

21. A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis at 5 revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre ______.

Sol. **9**

$I_1\omega_1 = \omega_2 I_2$

$$\left(\frac{MR^2}{2} + mR^2\right)\omega_1 = \omega_2 \frac{MR^2}{2}$$

$$\left(1 + \frac{2mR^2}{MR^2}\right)\omega_1 = \omega_2$$

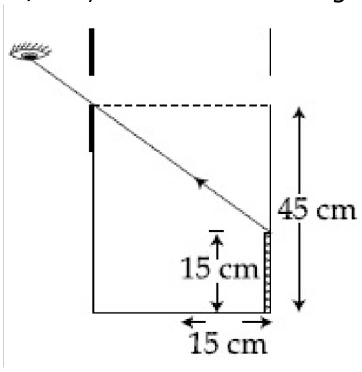
$$\left(1 + \frac{2 \times 80}{200}\right)\omega_1 = \omega_2$$

$\omega_2 = \omega_1 1.8$

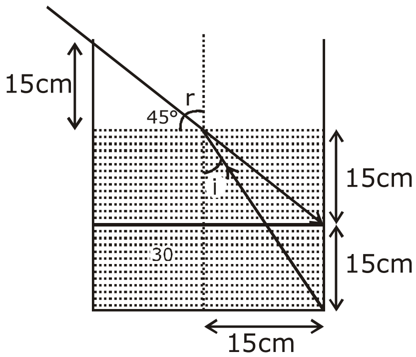
$2\pi f_2 = 2\pi f_1 \times 1.8$

$f_2 = 5 \times 1.8 = 9$

22. An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15 cm from the bottom (see figure). The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid is N/100, where N is an integer, the value of N is _____.



Sol. **1.58**



$\mu \sin i = 1 \sin 45$

$\mu \frac{15}{\sqrt{1125}} = \frac{1}{\sqrt{2}}$

$\mu = \frac{\sqrt{1125}}{15\sqrt{2}}$

$\mu = \sqrt{\frac{1125}{450}}$

$\mu = 1.58$

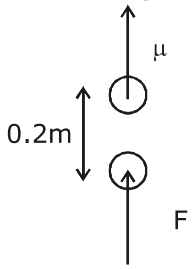
$\mu = \frac{N}{100} = 1.58$

$\Rightarrow N = 158$

23. A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizotally a distance of 0.2 m while launching the ball, the value of F(in N) is ($g = 10\ ms^{-2}$)_____.

Sol. **150**

m = 0.15 kg

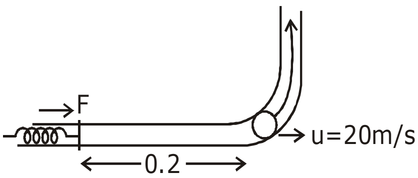


$$\mu = \sqrt{2gh}$$
$$\mu = \sqrt{2 \times 10 \times 20}$$
$$\mu = 20m / sec$$

$$Fd = \frac{1}{2}mv^2$$

$$F \times 0.2 = \frac{1}{2} \times \frac{0.15}{100} \times 400$$

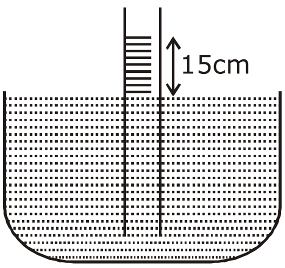
$$F = \frac{15 \times 200 \times 10}{100 \times 0.2} = 150$$



24. When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it. If the contact angle between the liquid and glass to close to 0°, the surface tension of the liquid, in milliNewton m^{-1}, is
[$\rho_{(liquid)} = 900\ kgm^{-3}$, $g = 10\ ms^{-2}$] (Give answer in closest integer)____.

Sol. **101**

r = 0.015



$$h = \frac{2T}{\rho g r}$$

$$T = \frac{h\rho g r}{2}$$

$$T = \frac{15 \times 10^{-2} \times 900 \times 10 \times 0.015 \times 10^{-2}}{2 \times 10^{3}}$$

T = 101 milli N/m

25. A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with V_m volume (at 30°C) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(beaker)} = 6 \times 10^{-6}$ °C^{-1} and $\gamma_{(mercury)} = 1.5 \times 10^{-4}$ °C^{-1}, where γ is the coefficient of volume expansion, then V_m (in cc) is close to ____.

Sol. **20**

$\Delta V = v\gamma\Delta T$

$v_1\gamma_1 = v_2\gamma_2$

$500cc \times 6 \times 10^{-6} = V_m \times 1.5 \times 10^{-4}$

$$V_m = \frac{500 \times 6 \times 10^{-6}}{1.5 \times 10^{-4}} = \frac{30}{1.5}$$

$V_m = 20cc$

QUESTION PAPER WITH SOLUTION

CHEMISTRY _ 3 Sep. _ SHIFT - 1

1. It is true that :

(1) A second order reaction is always a multistep reaction

(2) A first order reaction is always a single step reaction

(3) A zero order reaction is a multistep reaction

(4) A zero order reaction is a single step reaction

Sol. **3**

Factual

2. An acidic buffer is obtained on mixing :

(1) 100 mL of 0.1 M HCl and 200 mL of 0.1 M CH_3COONa

(2) 100 mL of 0.1 M HCl and 200 mL of 0.1 M NaCl

(3) 100 mL of 0.1 M CH_3 COOH and 100 mL of 0.1 M NaOH

(4) 100 mL of 0.1 M CH_3COOH and 200 mL of 0.1 M NaOH

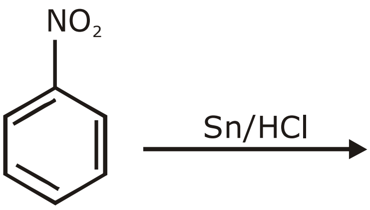
Sol. **1**

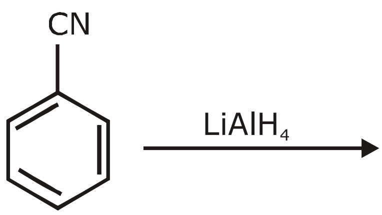
$2HCl + CH_3COO^- \longrightarrow CH_3COOH + OH^-$

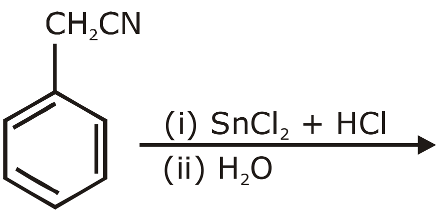
10	20	
X	10	10

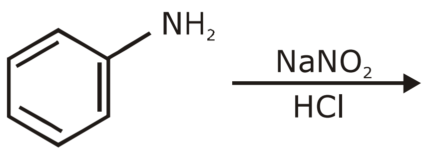
Acidic buffer

3. The Kjeldahl method of Nitrogen estimation fails for which of the following reaction products?

(a) 

(b) 

(c) 

(d) 

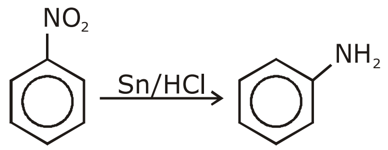
(1) (a), (c) and (d)

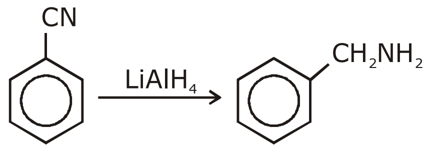
(2) (b) and (c)

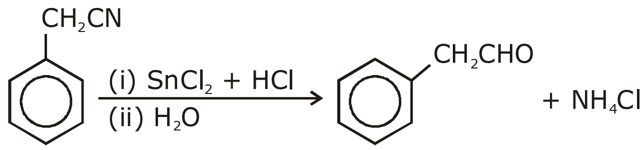
(3) (c) and (d)

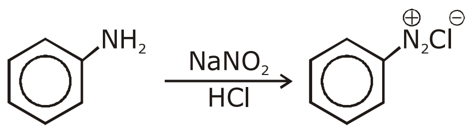
(4) (a) and (d)

Sol. **3**

(A) 

(B) 

(C) 

(D) 

Diazo compound and inorganic nitrogen can't be estimeted by kjeldal method.

4. If the boiling point of H_2O is 373 K, the boiling point of H_2S will be :

(1) greater than 300 K but less than 373 K

(2) equal to 373 K

(3) more than 373 K

(4) less than 300 K

Sol. **4**

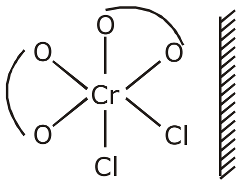
Less than 300 K (factual)

5. The complex that can show optical activity is :

(1) cis $-\left[CrCl_2(ox)_2\right]^{3-}$ (ox = oxalate) (2) trans $-\left[Fe(NH_3)_2(CN)_4\right]^{-}$

(3) trans $-\left[Cr(Cl_2)(ox)_2\right]^{3-}$ (4) cis $-\left[Fe(NH_3)_2(CN)_4\right]^{-}$

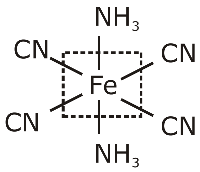
Sol. **1**



Nonsuper imposable M.I.

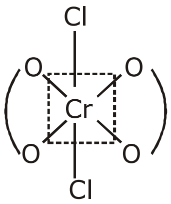
optically active

$cis-\left[CrCl_2(ox)_2\right]^{3-}$ $(ox = oxalate)$



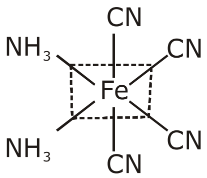
POS optically inactive

$trans-\left[Fe(NH_3)_2(CN)_4\right]^{-}$



$\rightarrow$ POS optically inactive

$trans-\left[Cr(Cl_2)(ox)_2\right]^{3-}$

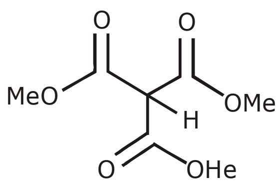


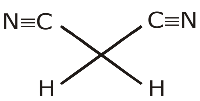
$\rightarrow$ POS opticaly inactive

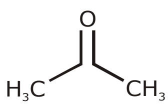
$cis-\left[Fe(NH_3)_2(CN)_4\right]^{-}$

6. Which one of the following compounds possesses the most acidic hydrogen?

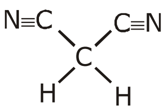
(1) $H_3C-C\equiv C-H$

(2) 

(3) 

(4) 

Sol. **3**



has most acidic hydrogen among given compound , this is due to strong –M effect of –CN group which stabilize –ve charge significantly.

7. Aqua regia is used for dissolving noble metals (Au, Pt, etc.). The gas evolved in this process is :

(1) N_2O_3 (2) N_2 (3) N_2O_5 (4) NO

Sol. **4**

$Au + HNO_3 + HCl \rightarrow HAuCl_4 + NO + H_2O$

$Pt + \underbrace{HNO_3 + HCl}_{\text{aqua regia}} \rightarrow H_2PtCl_6 + NO + H_2O$

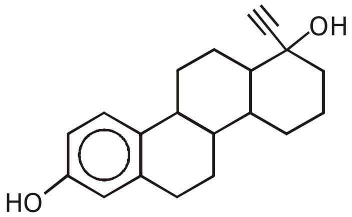
8. The antifertilituy drug "Novestrol" can react with :

(1) Br_2/water; $ZnCl_2$/HCl; $FeCl_3$ (2) Br_2/water; $ZnCl_2$/HCl; NaOCl

(3) Alcoholic HCN; NaOCl; $ZnCl_2$/HCl (4) $ZnCl_2$/HCl; $FeCl_3$; Alcoholic HCN

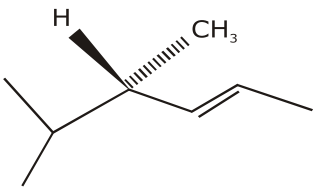
Sol. **1**

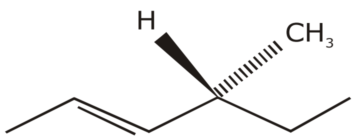
Novestrol

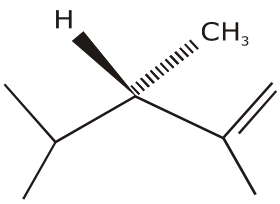


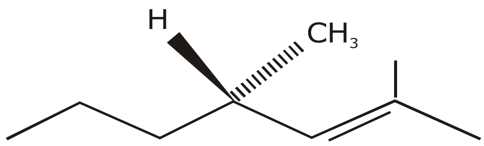
It can reacts with Br_2/water due to presence of unsaturation, with $ZnCl_2$/HCl due to –OH group and with $FeCl_3$ due to phenol.

9. Which of the following compounds produces an optically inactive compound on hydrogenation?

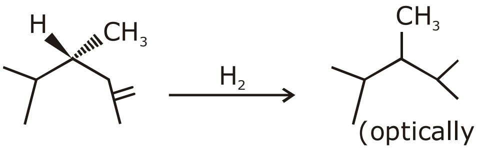
(1) 

(2) 

(3) 

(4) 

Sol. **3**



(optically inactive)

10. Of the species, NO, NO^+, NO^{2+} and NO^-, the one with minimum bond strength is :

(1) NO^- (2) NO^+ (3) NO^{2+} (4) NO

Sol. **1**

B.O. $NO^{-} = 2$

BO $NO^{+} = 3$

BO $NO^{2+} = 2.5$

BO NO = 2.5

B.O $\alpha \frac{1}{B.L}$

11. Glycerol is separated in soap industries by :

(1) Fractional distillation
(2) Distillation under reduced pressure
(3) Differential extraction
(4) Steam distillation

Sol. **2**

conceptual

Glycerol is separated in soap industries by distillation under reduced pressure

12. Thermal power plants can lead to :

(1) Ozone layer depletion
(2) Blue baby syndrome
(3) Eutrophication
(4) Acid rain

Sol. **4**

Refer enviornmental chemistry

It emits CO_2 that combine with mositure of atmosphere and forms H_2CO_3 (carbonic acid)

13. Henry's constant (in kbar) for four gases α, β, γ and δ in water at 298 K is given below :

	α	β	γ	δ
K_H	50	2	2×10^{-5}	0.5

(density of water = 10^3 kg m^{-3} at 298 K)

This table implies that :

(1) solubility of γ at 308 K is lower than at 298 K

(2) The pressure of a 55.5 molal solution of δ is 250 bar

(3) α has the highest solubility in water at a given pressure

(4) The pressure of a 55.5 molal solutio of γ is 1 bar

Sol. **1**

p = K_HX mol fraction of gas in liquid.

On increasing tamp, 'K'_H increases

Hence solubility $\downarrow$

therefore, option 1

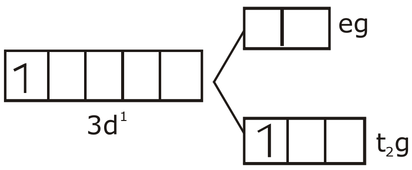
14. The electronic spectrum of $[Ti(H_2O)_6]^{3+}$ shows a single broad peak with a maximum at 20,300 cm^{-1}. The crystal field stabillization energy (CFSE) of the complex ion, in kJ mol^{-1}, is :

(1 kJ mol-1 = 83.7 cm^{-1})

(1) 83.7 (2) 242.5 (3) 145.5 (4) 97

Sol. **4**

$[Ti(H_2O)_6]^{3+}$ Ti^{3+} $3d^1$ in octahedral field of ligend



CFSE = $-0.4\ \Delta_0$

$$\text{CFSE} = \frac{-0.4 \times 20300}{83.7}$$

= 97 kJ mol

15. The atomic number of the element unnilennium is :

(1) 109 (2) 102 (3) 119 (4) 108

Sol. **1**

Unnilennium 109

16. An organic compound [A], molecular formula $C_{10}H_{20}O_2$ was hydrolyzed with dilute sulphuric acid to give a carboxylic acid [B] and an alcohol [C]. Oxidation of [C] with CrO_3 - H_2SO_4 produced [B]. Which of the following strucutres are not possible for [A]?

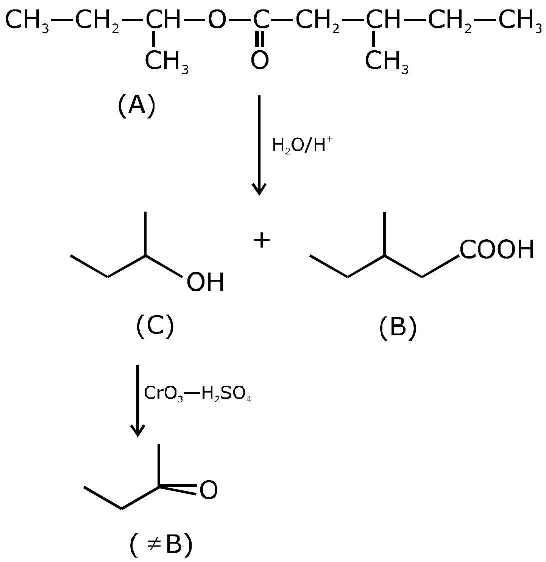
(1) $(CH_3)_3$-C-$COOCH_2C(CH_3)_3$

(2) $CH_3-CH_2-CH(CH_3)-OCOCH_2CH(CH_3)-CH_2CH_3$

(3) $CH_3CH_2CH_2COOCH_2CH_2CH_2CH_3$

(4) $CH_3-CH_2-CH(CH_3)-COOCH_2-CH(CH_3)-CH_2CH_3$

Sol. **2**



17. The mechanism of S_N1 reaction is given as :

$$R-X \to \underset{\text{Ion pair}}{R^{\oplus}X^{\ominus}} \to R^{\oplus}\Vert X^{\ominus} \xrightarrow{Y^{\ominus}} R-Y+X^{\ominus}$$

Solvent Separated ion pair

A student writes general characteristics based on the given mechanism as :
(a) The reaction is favoured by weak nucleophiles.
(b) $R^{\oplus}$ would be easily formed if the substituents are bulky.
(c) The reaction is accompanied by racemization.
(d) The reaction is favoured by non-polar solvents.
Which observations are correct?
(1) (a) and (b)
(2) (a), (b) and (c)
(3) (a) and (c)
(4) (b) and (d)

Sol. **2**
Statement (a), (b) & (c) are correct for $S_N{}^1$ reaction mechanism.

18. Tyndall effect is observed when:
(1) The diameter of dispersed particles is much smaller than the wavelength of light used.
(2) The diameter of dispersed particles is much larger than the wavelength of light used.
(3) The refractive index of dispersed phase is greater than that of the dispersion medium.
(4) The diameter of dispersed particles is similar to the wavelenght of light used.

Sol. **4**
Diameter of dispersed particles should not be much smaller than wavelength of light used.
Refer topic surface chemistry

19. Let C_{NaCl} and C_{BaSO_4} be the conductances (in S) measured for saturated aqueous solutions of NaCl and BaSO4, respectively, at a temperature T.
Which of the following is false?

(1) $C_{NaCl}\ (T_2) > C_{NaCl}\ (T_1)$ for $T_2 > T_1$

(2) $C_{BaSO_4}\ (T_2) > C_{BaSO_4}\ (T_1)$ for $T_2 > T_1$

(3) Ionic mobilities of ions from both salts increase with T.

(4) $C_{NaCl} >> C_{BaSO_4}$ at a given T

Sol. **4**

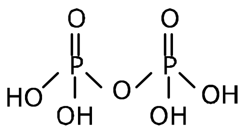
Ionic

$C_{NaCl} >> C_{BaSO_4}$ at temp 'T'

20. In a molecule of pyrophosphoric acid, the number of P-OH, P = O and P - O - P bonds/moiety(ies) respectively are :

(1) 3, 3 and 3 (2) 4, 2 and 1 (3) 2, 4 and 1 (4) 4, 2 and 0

Sol. **2**



P – OH bonds = 4
P = O bonds = 2
P – O – P linkage = 1
Ans. 4, 2, 1

21. The mole fraction of glucose ($C_6H_{12}O_6$) in an aqueous binary solution is 0.1. The mass percentage of water in it, to the nearest integer, is ________.

Sol. **47 %**

$x_{Glucose} = 0.1$

mass% of glucose $= \dfrac{0.1 \times 180}{0.1 \times 180 + 0.9 \times 18} \times 100$

$= \dfrac{1800}{18 + 16.2}$

$= \dfrac{1800}{34.2}\%$

$= 52.63\%$

$= 53\%$

$\therefore$ mass % of H_2O = 47%

22. The volume strength of 8.9 M H_2O_2 solution calculated at 273 K and 1 atm is ______. (R = 0.0821 L atm K^{-1} mol^{-1}) (rounded off ot the nearest integer)

Sol. **100**

Vol. strength $= \frac{8.9}{2} \times \frac{0.821 \times 273}{1}$

$= 99.73$

$= 100$

23. An element with molar mass 2.7×10^{-2} kg mol^{-1} forms a cubic unit cell with edge length 405 pm. If its density is 2.7×10^{3} kg m^{-3}, the radius of the element is approximately ______ $\times$ 10^{-12} m (to the nearest integer).

Sol. **143**

Density $= \frac{Z \times GMM}{N_A \times a^3}$

$2.7 \times 10^3 = \frac{Z \times 2.7 \times 10^{-2}}{6.023 \times 10^{23} \times (405 \times 10^{-12})^3}$

$Z = 6.023 \times 405 \times 405 \times 405 \times 10^{23-36+3+2}$

$Z = 6.023 \times 405 \times 405 \times 405 \times 10^{-8}$

$Z = 4$

FCC

$4R = \sqrt{2} \times a$

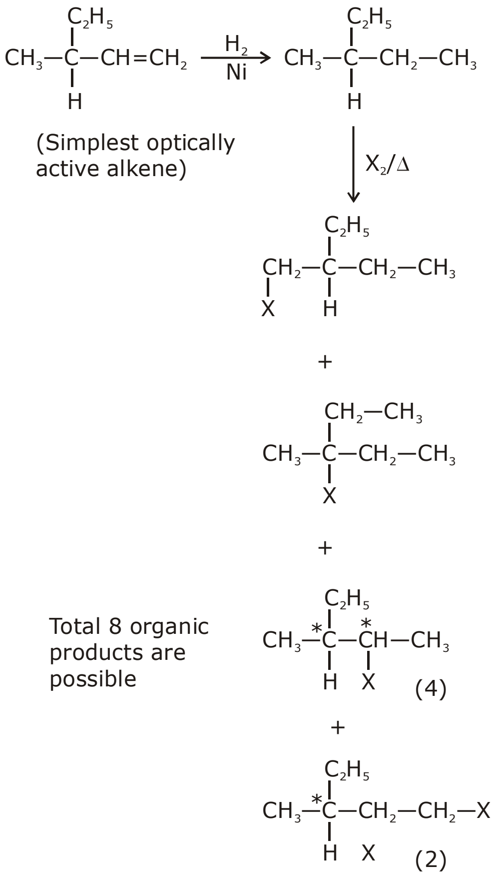
$R = \frac{405}{2\sqrt{2}} \times 10^{-12} = 143.21 \times 10^{-12}$m

= 143 ans

24. The total number of monohalogenated organic products in the following (including stereoisomers) reaction is ______.

A (Simplest optically active alkene) $\xrightarrow[\text{(ii) } X_2/\Delta]{\text{(i) } H_2/Ni/\Delta}$

Sol. **8**



25. The photoelectric current from Na (Work function, w_0 = 2.3 eV) is stopped by the output voltage of the cell Pt(s) H_2(g, 1 Bar) HCl (aq. pH =1)$\left|AgCl(s)\right|$ Ag(s).

The pH of aq. HCl required to stop the photoelectric current form K(w_0 = 2.25 eV), all other conditions remaining the same, is _______ $\times$ 10^{-2} (to the nearest integer).

Given,

$$2.303\frac{RT}{F} = 0.06\ V; E^0_{AgCl|Ag|Cl^-} = 0.22\ V$$

Sol. **58**

Energy of photon = 2.3 – E_{cell} {for Na}

Energy of photon = 2.25 – E_{cell} {for K}

E_{cell} {for 'Na'} + 0.05 = E_{cell} {for 'K'}

$0.22 + 0.06 \log [H^+][Cl^-] + 0.05 = 0.22 + 0.06 \log [H^+][Cl^-]$

$6 \log (10^{-2}) + 5 = 6 \log [H^+][Cl^-]$

$\log (10^{-12}) + \log (10^5) = \log \{[H^+][Cl^-]\}^6$

$\{[H^+][Cl^-]\}^6 = 10^{-7}$

$[H^+]^{12} = 10^{-7}$

$$pH = \frac{7}{12} = 0.58$$

$= 58 \times 10^{-2}$ = 58 Ans

QUESTION PAPER WITH SOLUTION

MATHEMATICS _ 3 Sep. _ SHIFT - 1

Q.1 The value of ($2.{}^1P_0 - 3.{}^2P_1 + 4.{}^3P_2 - \ldots$ up to 51^{th} term) +($1! - 2! + 3! - \ldots$ up to 51^{th} term) is equal to:

(1) 1-51(51)! (2) 1+(52)! (3) 1 (4) 1+ (51)!

Sol. **2**

$2.\,{}^1P_0 = \underline{|2}$

$3.\,{}^2P_1 = \underline{|3}$

$4.\,{}^3P_2 = \underline{|4}$

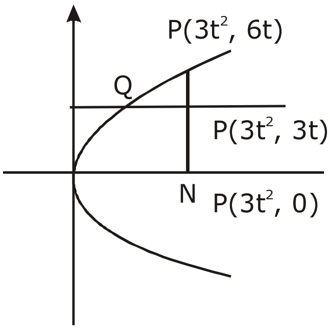
$(\underline{|2} - \underline{|3} + \underline{|4} - \underline{|5} + \ldots\ldots\underline{|52}) + (\underline{|1} - \underline{|2} + \underline{|3} - \underline{|4} \ldots\ldots + \underline{|51})$

$= \underline{|52} + 1$

Q.2 Let P be a point on the parabola, $y^2=12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then:

(1) PN=4 (2) MQ=$\frac{1}{3}$ (3) PN=3 (4) MQ=$\frac{1}{4}$

Sol. **4**



Q (h, 3t) lie on
Parabola
$9t^2 = 12\,h$

$h = \frac{3t^2}{4}$

$Q = \left(\frac{3t^2}{4}, 3t\right)$

Equation of NQ

$$y = \frac{3t}{\left(\frac{3t^2}{4} - 3t^2\right)} \quad (x - 3t^2)$$

$$y = \frac{-4t}{3t^2}\left(x - 3t^2\right)$$

put x = 0

$$y = \frac{-4}{3t}\left(-3t^2\right) = 4t$$

$$4t = \frac{4}{3} \Rightarrow t = \frac{1}{3}$$

$$PN = 6t = 6.\frac{1}{3} = 2$$

$$M = \left[\frac{1}{3}, 1\right], Q\left[\frac{1}{12}, 1\right]$$

$$MQ = \frac{1}{3} - \frac{1}{12} = \frac{1}{4}$$

Q.3 If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3+Bx^2+Cx+D$, then B+C is equal to:

(1) 1 (2) -1 (3) -3 (4) 9

Sol. **3**

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3+Bx^2+Cx+D$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & -x+2 & -2x+3 \\ 1 & -x+1 & x-5 \end{vmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & -x+2 & -2x+3 \\ 0 & -1 & 3x-8 \end{vmatrix} = Ax^3+Bx^2+Cx+D$$

$\Rightarrow -1[(3 - 2x)(x - 2) - (3x - 4)] + (3x - 8)[(x - 2)(-x + 2) - (2x - 3)] = Ax^3+Bx^2+Cx+D$

$\Rightarrow 3x - 2x^2 - 6 + 4x - 3x + 4 + (3x - 8)[-x^2 + 4x - 4 - 2x + 3] = Ax^3+Bx^2+Cx+D$

A = -3, B = 12, C = -15

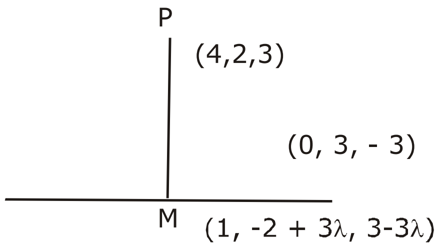
B + C = −3

Q.4 The foot of the perpendicular drawn form the point (4,2,3) to the line joining the points (1,-2,3) and (1,1,0) lies on the plane:

(1) x-y-2z=1 (2) x-2y+z=1 (3)2x+y-z=1 (4) x+2y-z=1

Sol. **3**

$\vec{r} = (1,-2,3) + \lambda(0,3,-3)$



$\overrightarrow{pm} \perp \vec{b}$

$\overrightarrow{pm} . \vec{b} = 0$

$(-3, 3\lambda - 4, -3\lambda).(0, 3, -3) = 0$

$\Rightarrow 0 + 9\lambda - 12 + 9\lambda = 0 \Rightarrow \lambda = \frac{12}{18} = \frac{2}{3}$

m = (1, 0, 1) are on 2x + y - z = 1

Q.5 If $y^2+\log_e(\cos^2x)=y$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then

(1) $|y'(0)|+|y''(0)|=1$ (2) $y''(0)=0$

(3) $|y'(0)|+|y''(0)|=3$ (4) $|y''(0)|=2$

Sol. **4**

$2yy' + 2(-\tan x) = y'$ ....(1)

diff. w.r.t.x

$2yy'' + 2(y')^2 - 2\sec^2 x = y''$ ....(2)

Put x = 0 in given equation we get y = 0, 1

from (1) x = 0, y = 0 $\Rightarrow$ y′(0) = 0

x = 0, y = 1, $\Rightarrow$ y′(0) = 0

from (2) $x = 0, y = 0, y'(0) = 0 \Rightarrow y''(0) = -2$
$x = 0, y = 1, y'(0) = 0 \Rightarrow y''(0) = 2$
$|y''(0)| = 2$

Q.6 $2\pi - \left(\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{25}\right)$ is equal to:

(1) $\frac{5\pi}{4}$ (2) $\frac{3\pi}{2}$ (3) $\frac{7\pi}{4}$ (4) $\frac{\pi}{2}$

Sol. **2**

$$2\pi - \left[\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\frac{16}{63}\right]$$

$$2\pi - \tan^{-1}\left(\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\cdot\frac{5}{12}}\right) - \tan^{-1}\left(\frac{16}{63}\right)$$

$$\Rightarrow 2\pi - \tan^{-1}\left(\frac{48+15}{36-20}\right) - \tan^{-1}\left(\frac{16}{63}\right)$$

$$\Rightarrow 2\pi - \left[\tan^{-1}\left(\frac{63}{16}\right) + \cot^{-1}\left(\frac{63}{16}\right)\right]$$

$$\Rightarrow 2\pi - \frac{\pi}{2}$$

$$= \frac{3\pi}{2}$$

Q.7 A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2+4y^2=12$, then this hyperbola does not pass through which of the following points ?

(1) $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$ (2) $\left(1, -\frac{1}{\sqrt{2}}\right)$ (3) $\left(\frac{1}{\sqrt{2}}, 0\right)$ (4) $\left(-\sqrt{\frac{3}{2}}, 1\right)$

Sol. **1**

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$b_1^2 = a_1^2\left(1 - e_1^2\right)$$

$3 = 4(1 - e_1^2)$

$e_1 = \frac{1}{2}$

focus = $(\pm a_1e_1, 0)$
$= (\pm 1, 0)$

Length of transverse axis $2a_2 = \sqrt{2} \rightarrow a_2 = \frac{1}{\sqrt{2}}$

$a_2e_2 = 1$
$= e_2 = \sqrt{2}$
$b_2^2 = a_2^2 (e_2^2 - 1)$

$b_2^2 = \frac{1}{2}(2-1) = \frac{1}{2}$

equation of Hyperbola

$x^2 - y^2 = \frac{1}{2}$

Q.8 For the frequency distribution:

Variate(x):	x_1	x_2	$x_3....x_{15}$
Frequency(f):	f_1	f_2	$f_3.....f_{15}$

where $0<x_1<x_2<x_3<....<x_{15}=10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be:

(1) 1 (2) 4 (3) 6 (4) 2

Sol. **3**

$\sigma^2 \le \frac{1}{4}(M - m)^2$

(M = upper bound of value of any random variable,
m = Lower bound of value of any random variable)

$\sigma^2 \le \frac{1}{4}(10-0)^2$

$\sigma^2 < 25$
$-5 < \sigma < 5$
$\sigma \ne 6$

Q.9 A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is:

(1) $\frac{1}{3}$ (2) $\frac{1}{4}$ (3) $\frac{1}{8}$ (4) $\frac{1}{9}$

Sol. **4**

Total Possibilities = (1, 3), (3, 1), (2, 2),
(2, 6), (6, 2) (4, 4)
(3, 5), (5, 3) (6, 6)
fav. = 1 = (4, 4)

prob. = $\frac{1}{9}$

Q.10 If the number of integral terms in the expansion of $\left(3^{\frac{1}{2}}+5^{\frac{1}{8}}\right)^{n}$ is exactly 33, then the least value of n is:

(1) 128 (2) 248 (3) 256 (4) 264

Sol. **3**

$T_{r+1} = {}^{n}C_{r}\left(3^{\frac{1}{2}}\right)^{n-r}\left(5^{\frac{1}{8}}\right)^{r}$

$\frac{n-r}{2} \rightarrow$ n - r = 0, 2, 4, 6, 8, ...

$\frac{r}{8} \rightarrow r = 0, 8, 16, 24$................

common r = 0, 8, 16, 24..........
no. of integral term = 33.
L = 0 + (33 - 1) × 8 → L = 32 × 8
= 256

Q.11 $\int_{-\pi}^{\pi} |\pi - |x|| \, dx$ is equal to:

(1) π^2 (2) $\frac{\pi^2}{2}$ (3) $\sqrt{2}\pi^2$ (4) $2\pi^2$

Sol. **1**

$\int_{-\pi}^{\pi} |\pi - |x|| \, dx$

even function

$2\int_{0}^{\pi} |\pi - x| \, dx$

$= 2\int_{0}^{\pi} (\pi - x) \, dx \Rightarrow 2\left[\pi x - \frac{x^2}{2}\right]_0^{\pi}$

$= 2\left[\frac{\pi^2}{2}\right] = \pi^2$

Q.12 Consider the two sets:
A={m $\in$ R : both the roots of $x^2-(m+1)x+m+4=0$ are real} and B=[-3,5).
Which of the following is not true ?

(1) $A-B=(-\infty,-3)\cup(5,\infty)$
(2) $A\cap B=\{-3\}$
(3) B-A=(-3,5)
(4) $A\cup B=R$

Sol. **1**

$D \geq 0$
$(m+1)^2-4(m+4)\geq 0$
$\Rightarrow m^2-2m-15\geq 0$
$(m-5)(m+3)\geq 0$
$m\in(-\infty,-3]\cup[5,\infty)$
$A=(-\infty,-3]\cup[5,\infty)$
$B=[-3,5)$
$A-B=(-\infty,-3)\cup[5,\infty)$
$A\cup B=R$

Q.13 The proposition $p\rightarrow\sim(p\wedge\sim q)$ is equivalent to :

(1) $(\sim p)\vee(\sim q)$
(2) $(\sim p)\wedge q$
(3) q
(4) $(\sim p)\vee q$

Sol. **4**

$\sim(p\wedge\sim q)\rightarrow\sim p\vee q$
$p\rightarrow(\sim p\vee q)$
$\Rightarrow\sim p\vee(\sim p\vee q)$
$\Rightarrow\sim p\vee q$

Q.14 The function, $f(x)=(3x-7)x^{2/3}$, $x\in R$ is increasing for all x lying in:

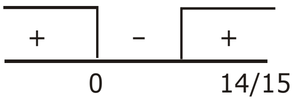
(1) $\left(-\infty,-\frac{14}{15}\right)\cup(0,\infty)$
(2) $\left(-\infty,\frac{14}{15}\right)$
(3) $(-\infty,0)\cup\left(\frac{14}{15},\infty\right)$
(4) $(-\infty,0)\cup\left(\frac{3}{7},\infty\right)$

Sol. **3**

$$f(x)=(3x-7).\frac{2}{3x^{\frac{1}{3}}}+x^{\frac{2}{3}}.3$$

$$=\frac{6x-14+9x}{3x^{\frac{1}{3}}}$$

$$= \frac{15x - 14}{3x^{\frac{1}{3}}}$$



$f(x) > 0 \uparrow \Rightarrow x \in (-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

Q.15 If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is:

(1) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{7}$

Sol. **1**

a = 3

$$\frac{25}{2}\left[2a + 24d\right] = \frac{15}{2}\left[2 \times (a + 25d) + 14d\right]$$

$\Rightarrow 50a + 600d = 15\,[2a + 50d + 14d]$

$\Rightarrow 20a + 600d = 960d$

$\Rightarrow 60 = 360d$

$d = \frac{1}{6}$

Q.16 The solution curve of the differential equation, $(1 + e^{-x})(1 + y^2)\frac{dy}{dx} = y^2$, which passes through the point (0,1), is:

(1) $y^2 = 1 + y\log_e\left(\frac{1 + e^{-x}}{2}\right)$ (2) $y^2 + 1 = y\left(\log_e\left(\frac{1 + e^{-x}}{2}\right) + 2\right)$

(3) $y^2 + 1 = y\left(\log_e\left(\frac{1 + e^{x}}{2}\right) + 2\right)$ (4) $y^2 = 1 + y\left(\log_e\left(\frac{1 + e^{x}}{2}\right)\right)$

Sol. **4**

$$\int\left(\frac{1 + y^2}{y^2}\right)dy = \int\left(\frac{1}{1 + e^{-x}}\right)dx$$

$$\int\frac{1}{y^2}dy + \int dy = \int\left(\frac{e^x}{e^x + 1}\right)dx$$

$$\Rightarrow \frac{-1}{y} + y = \ln\left|e^x + 1\right| + c$$

x = 0, y = 1

$\Rightarrow$ -1 + 1 = ln2 + c $\Rightarrow$ c = -ln2

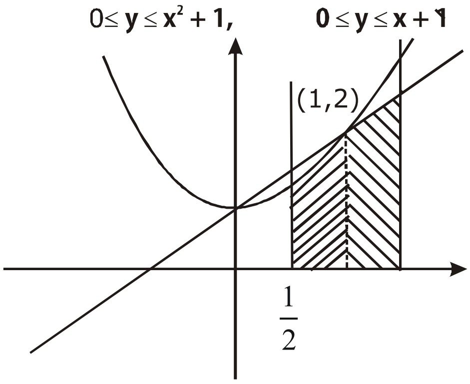
$$\Rightarrow \frac{-1}{y} + y = \ln\left|e^x + 1\right| - \ln 2$$

$$\Rightarrow y^2 = 1 + y\left[\ln\left(\frac{e^x + 1}{2}\right)\right]$$

Q.17 The area (in sq. units) of the region $\left\{(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, \frac{1}{2} \le x \le 2\right\}$ is

(1) $\frac{23}{16}$ (2) $\frac{79}{16}$ (3) $\frac{23}{6}$ (4) $\frac{79}{24}$

Sol. **4**



$$A = \int_{\frac{1}{2}}^{1} (x^2 + 1)dx + \int_{1}^{2} (x + 1)\,dx$$

$$\left(\frac{x^3}{3} + x\right)_{\frac{1}{2}}^{1} + \left(\frac{x^2}{2} + x\right)_{1}^{2}$$

$$= \left(\frac{1}{3} + 1\right) - \left(\frac{1}{24} + \frac{1}{2}\right) + \left((2 + 2) - \left(\frac{3}{2}\right)\right)$$

$$= \left(\frac{4}{3} - \frac{13}{24}\right) + \left(\frac{5}{2}\right)$$

$$= \left(\frac{32-13}{24}\right) + \left(\frac{5}{2}\right) = \frac{19+60}{24} = \frac{79}{24}$$

Q.18 If α and β are the roots of the equation $x^2+px+2=0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2+2qx+1=0$, then $\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$ is equal to :

(1) $\frac{9}{4}\left(9+p^2\right)$ (2) $\frac{9}{4}\left(9+q^2\right)$ (3) $\frac{9}{4}\left(9-p^2\right)$ (4) $\frac{9}{4}\left(9-q^2\right)$

Sol. **3**

$\alpha + \beta = -p$, $\alpha\beta = 2$

$\frac{1}{\alpha}+\frac{1}{\beta} = -q$, $\frac{1}{\alpha\beta} = \frac{1}{2}$

$\frac{\alpha+\beta}{\alpha\beta} = -q \Rightarrow \frac{-p}{2} = -q$

$\Rightarrow p = 2q$

$$\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right) = \alpha\beta + \frac{1}{\alpha\beta} + 2$$

$$= 2 + \frac{1}{2} + 2 = \frac{9}{2}$$

$$\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right) = \alpha\beta + \frac{1}{\alpha\beta} - \frac{\alpha}{\beta} - \frac{\beta}{\alpha}$$

$$= 2 + \frac{1}{2} - \left[\frac{\alpha^2+\beta^2}{\alpha\beta}\right]$$

$$= \frac{5}{2} - \left[\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}\right]$$

$$= \frac{5}{2} - \left[\frac{p^2-4}{2}\right]$$

$$= \frac{9-p^2}{2}$$

$$\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)=\left(\frac{9-p^2}{2}\right)\left(\frac{9}{2}\right)$$

$$= \frac{9}{4}\left(9-p^2\right)$$

Q.19 The lines $\vec{r}=(\hat{i}-\hat{j})+l(2\hat{i}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j})+m(\hat{i}+\hat{j}-\hat{k})$

(1) do not intersect for any values of l and m
(2) intersect when l=1 and m=2
(3) intersect when l=2 and m=$\frac{1}{2}$
(4) intersect for all values of l and m

Sol. **1**

$\frac{2}{1}\neq\frac{0}{1}\neq\frac{1}{-1}\rightarrow$ lines are intersecting

$\vec{r}=(1+2l)\hat{i}-\hat{j}+l\hat{k}$ ...(1)

$\vec{r}=(2+m)\hat{i}+(m-1)\hat{j}-m\hat{k}$ ....(2)

compare coff. of $\hat{i},\hat{j},\hat{k}$

$1+2l=2+m$ $\left|\begin{array}{c}-1=m-1\\ m=0\end{array}\right|$ $l=0$

Lines do not intersect

Q.20 Let [t] denote the greatest integer $\leq$t. if for some $\lambda\in$ $R-\{0, 1\}$

$\lim_{x\to0}\left|\frac{1-x+|x|}{\lambda-x+[x]}\right|=L$, then L is equal to:

(1) 0 (2) 2 (3) $\frac{1}{2}$ (4) 1

Sol. **2**

$$\lim_{x\to 0}\left|\frac{1-x+|x|}{\lambda-x+[x]}\right| = L$$

$$\lim_{h\to 0}\left|\frac{1-h+h}{\lambda-h+[h]}\right|$$

$$\lim_{h\to 0}\left|\frac{1}{\lambda-h+0}\right| = \left|\frac{1}{\lambda}\right| \quad [x] = 0$$

$$\lim_{h\to 0}\left|\frac{1+h+h}{\lambda+h+[-h]}\right|$$

$$= \left|\frac{1}{\lambda-1}\right| \qquad [-h] = -1$$

$\because$ $|\lambda| = |\lambda - 1|$

$\lambda^2 = \lambda^2 - 2\lambda + 1 \Rightarrow \lambda = \dfrac{1}{2}$

L = 2

Q.21 If $\lim_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\} = 2^{-k}$, then the vlaue of k is ....

Sol. **8**

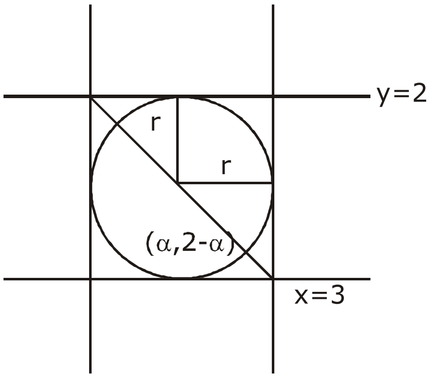
$$\lim_{x\to 0}\frac{\left(1-\cos\frac{x^2}{2}\right)}{\left(\frac{x^2}{2}\right)^2}\frac{\left(1-\cos\frac{x^2}{4}\right)}{\left(\frac{x^2}{4}\right)^2}.\frac{\left(\frac{x^2}{2}\right).\left(\frac{x^2}{4}\right)^2}{x^8}$$

$$\lim_{x\to 0}\frac{1}{4}.\frac{1}{4}.\frac{1}{16} \Rightarrow \frac{1}{256} = 2^{-k}$$

$2^{-8} = 2^{-k} \Rightarrow k = 8$

Q.22 The diameter of the circle, whose centre lies on the line x + y = 2 in the first quadrant and which touches both the lines x=3 and y=2, is ....

Sol. **2**



p = r

for y = 2

r = $\left|\dfrac{2-\alpha-2}{1}\right| = |\alpha|$

for x = 3

r = $\left|\dfrac{\alpha-3}{1}\right| = |\alpha-3|$

$|\alpha| = |\alpha - 3|$

$\Rightarrow \alpha^2 + \alpha^2 - 6\alpha + 9 \Rightarrow \alpha = \dfrac{3}{2}$

$2\alpha = 3 = 2r$

Q.23 The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^2}+..........\text{to } \infty\right)}$ is equal to..............

Sol. **4**

$$\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+........\infty = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$$

$$\log_{2.5}\left(\frac{1}{2}\right) \Rightarrow \log_{\frac{5}{2}}\frac{1}{2}$$

$$.16 = \frac{16}{100} = \frac{4}{25} = \left(\frac{2}{5}\right)^2$$

$$\Rightarrow \left(\frac{2}{5}\right)^{2\log_{\frac{5}{2}}\frac{1}{2}} = \left(\frac{5}{2}\right)^{-2\log_{\frac{5}{2}}\frac{1}{2}}$$

$$\Rightarrow \left(\frac{5}{2}\right)^{\log_{\frac{5}{2}}\left(\frac{1}{2}\right)^{-2}}$$

$= 4$

Q.24 Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}, x \in R$ and $A^4 = \left[a_{ij}\right]$. If $a_{11}=109$, then a_{22} is equal to ....

Sol. **10**

$$A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} x^3+x+x & x^2+1 \\ x^2+1 & x \end{bmatrix}$$

$$A^4 = \begin{bmatrix} x^3+2x & x^2+1 \\ x^2+1 & x \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x^4+2x^2+x^2+1 & x^3+2x \\ x^3+x+x & x^2+1 \end{bmatrix}$$

$a_{11} \Rightarrow x^4 + 3x^2 + 1 = 109$

$x^4 + 3x^2 - 108 = 0$

$\Rightarrow (x^2 + 12)(x^2 - 9) = 0$

$x = \pm 3$

$a_{11} = x^2 + 1 = 10$

Q.25 If $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}} = \left(\frac{1+i}{i-1}\right)^{\frac{n}{3}} = 1$, (m, n $\in$ N) then the greatest common divisor of the least values of m and n is ....

Sol. **4**

$$\left[\frac{(1+i)(1+i)}{(1+i)(1-i)}\right]^{\frac{m}{2}} = \left[\left(\frac{1+i}{-1+i}\right)\left(\frac{-1-i}{-1-i}\right)\right]^{\frac{n}{3}} = 1$$

$$= \left(\frac{2i}{2}\right)^{\frac{m}{2}} = 1 \quad \Bigg| \quad \left(\frac{-1-i-i+1}{1+1}\right)^{\frac{n}{3}} = 1$$

m = 8

$(-i)^{n/3} = 1$

n = 12

greatest common divisor of m & n is 4