JEE Main 31 Jan 2024
(Shift-2)
(Memory Based)

The Actual Paper will be Updated with Solution After the Official Release

PART : PHYSICS

1. By what percent will the illumination (Power) of lamp decreases if the current drops by 20%. if resistance of the lamp is assumed to be constant

(1) 12% (2) 24% (3) 36% (4) 48%

Ans. **(3)**

Sol. $P_1 = I^2R$

$P_2 = (0.8I)^2R = 0.64I^2R$

$P_2 = 0.64\ P_1$

$$\% \text{ drop of Power} = \frac{P_2 - P_1}{P_1} \times 100 = \frac{(0.64 - 1)\,P_1}{P_1} \times 100$$

$$= -0.36 \times 100$$

$$= 36\% \text{ drop}$$

2. Mass of the moon is (1/144) times mass of a planet. Its diameter is 1/16 times diameter of the planet. If the escape velocity from the surface of planet is V, then the escape velocity from surface of moon will be-

(1) $\frac{V}{3}$ (2) $\frac{V}{2}$ (3) $\frac{V}{6}$ (4) $\frac{V}{8}$

Ans. **(1)**

Sol. $V_e\sqrt{\frac{2GM}{R}}$, $\frac{V_{moon}}{V_{planet}} = \frac{\sqrt{\frac{2GM_m}{R_m}}}{\sqrt{\frac{2GM_p}{R_p}}} = \sqrt{\frac{M_m}{M_p}}\sqrt{\frac{R_p}{R_m}}$

$$\Rightarrow \frac{V_{moon}}{V} = \sqrt{\frac{1}{144}}\sqrt{\frac{16}{1}} \Rightarrow V_{moon} = \frac{4}{12}V = \frac{V}{3}$$

3. The speed of sound in oxygen at STP will be approximately? (Given R = 8.3 J/mol K and $\gamma = 1.4$)

(1) 320 (2) 315 (3) 330 (4) 325

Ans. **(2)**

Sol. At STP T = 273 K

$$V = \sqrt{\frac{\gamma RT}{M}} \text{ (Speed of sound)}$$

$$V = \sqrt{\frac{1.4 \times 8.3 \times 273}{32 \times 10^{-3}}} = 314.5 \text{ m/s}$$

4. Force on a 2 kg particle varies with time as $\vec{F} = 6t\hat{i} - 6t^2\hat{j}$. Particle start from rest. Find power delivered by the force as a function of time.

(1) $9t^3 + 4t^5$ (2) $9t^3 + 6t^5$ (3) $6t^2 + 3t^4$ (4) $4t^2 + 6t^3$

Ans. **(2)**

Sol. $\vec{F} = 6t\hat{i} - 6t^2\hat{j}$

$\Rightarrow \vec{a} = \frac{\vec{F}}{m} = 3t\hat{i} - 3t^2\hat{j}$ (m = 2kg)

$\Rightarrow \frac{d\vec{v}}{dt} = 3t\hat{i} - 3t^2\hat{j}$

$\Rightarrow \int_0^v d\vec{v} = \frac{3t^2}{2}\hat{i} - t^3\hat{j}\Big|_0^t$

$\Rightarrow \vec{v} = \frac{3t^2}{2}\hat{i} - t^3\hat{j}$

$P = \vec{F} \cdot \vec{v}$

$\Rightarrow P = ((6t\hat{i} - 6t^2\hat{j}) \cdot \left(\frac{3t^2}{2}\hat{i} - t^3\hat{j}\right)$

$P = 9t^3 + 6t^5$

5. If in the given expression, $E = \frac{b - x^2}{at}$, where E represents energy, x represents length and t represents time Then, find dimension of $\frac{a}{b}$.

(1) $M^{-1}L^{-2}T$ (2) $M^{-1}L^{-2}T^{-1}$ (3) $M^{-1}L^{-2}T^{-2}$ (4) $M^{-1}L^2T$

Ans. **(1)**

Sol. $E = \frac{b}{at} - \frac{x^2}{at} \Rightarrow [E] = \left[\frac{b}{at}\right] = \left[\frac{x^2}{at}\right]$

$[E] = \left[\frac{x^2}{at}\right] \Rightarrow [ML^2T^{-2}] = \frac{[L^2]}{[a][T]}$

$\Rightarrow [a] = \frac{[ML^2T^{-2}][T]}{[L^2]} = [MT^{-1}]$

$\Rightarrow [E] = \frac{[b]}{[MT^{-1}][T]} \Rightarrow [b] = [ML^2T^{-2}][MT^{-1}][T]$

$[b] = [M^2L^2T^{-2}]$

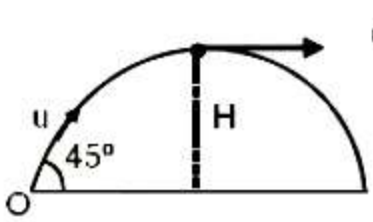
$\left[\frac{a}{b}\right] = \frac{[MT^{-1}]}{[M^2L^2T^{-2}]} = M^{-1}L^{-2}T$

6. A particle is projected at an angle 45° with horizontal with speed u. Find the angular momentum of particle about the point of projection at the time when it reaches maximum height.

(1) $\frac{mu^3}{4g}$ (2) $\frac{mu^3}{4\sqrt{2}g}$ (3) $\frac{mu^3}{2\sqrt{2}g}$ (4) $\frac{\sqrt{2}mu^3}{4g}$

Ans. **(2)**

Sol.



$u\cos 45° = \frac{u}{\sqrt{2}}$

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2}{4g}$$

Angular momentum = mvH $\Rightarrow m\frac{u}{\sqrt{2}}.\frac{u^2}{4g} = \frac{mu^3}{4\sqrt{2}g}$

7. 3 moles of oxygen gas and 2 moles of argon gas are mixed together. If the total internal energy of mixture is xRT. Find the value of x.

(1) $\frac{19}{2}$ (2) 10 (3) 11 (4) $\frac{21}{2}$

Ans. **(4)**

Sol. For oxygen n = 3 mol

f_1 = 5 (D.O.F)

U_1 (internal energy) = $\frac{f_1}{2}nRT = \frac{5}{2}\times nRT = \frac{15}{2}RT$

for argon

n = 2 moles

f_2 = 3 (D.O.F)

$$U_2 = \frac{f_2}{2}nRT = \frac{3}{2}\times 2RT = \frac{6}{2}RT$$

Total internal energy of mixture = $U_1 + U_2$

$$= \frac{15}{2}RT + \frac{6}{2}RT = \frac{21}{2}RT$$

$$x = \frac{21}{2}$$

8. Magnetic flux passing through a loop of resistance 8Ω is given by $\phi = 5t^2 - 3t + 5$. Find current in the loop at t = 2second.

(1) 1.125 A (2) 2.25 A (3) 4.25 A (4) 2.125 A

Ans. **(4)**

Sol. $\varepsilon = -\frac{d\phi}{dt}$

$= -\frac{d}{dt}(5t^2 - 3t + 5)$

$\varepsilon = -(10t - 3)$

at t = 2sec.

$\varepsilon = -(10 \times 2 - 3) = -17$

$i = \frac{\varepsilon}{R} = \frac{17}{8}$ Ampere

i = 2.125 Ampere

9. A nucleus x has mass number 192 and there is a second nucleus y having radius half of radius of x. Find mass number of y nucleus.

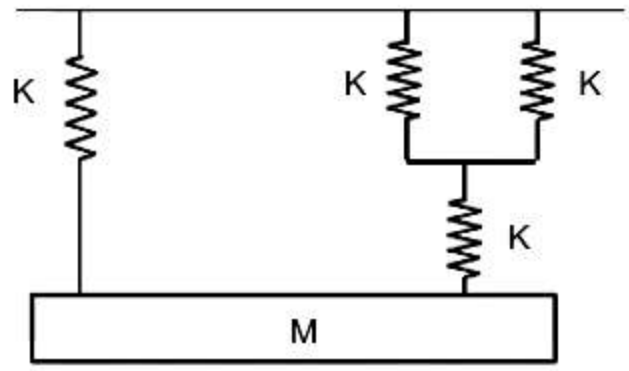
(1)18 (2) 24 (3) 12 (4) 14

Ans. **(2)**

Sol. $\frac{R_x}{R_y} = \frac{R_0(192)^{\frac{1}{3}}}{R_0(A)^{\frac{1}{3}}}$

on solving A = 24

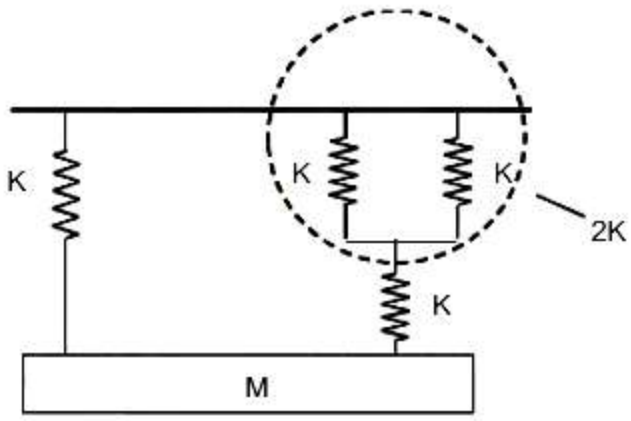
10. The period of oscillation of system shown below is $\pi\sqrt{\frac{am}{5k}}$ then a is ___



(1) 4 (2) 12 (3) 16 (4) 8

Ans. **(2)**

Sol.

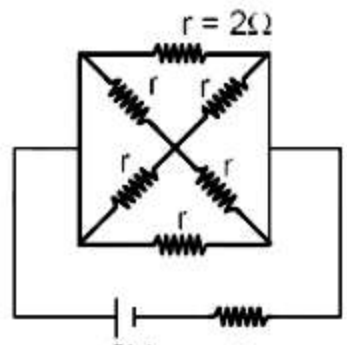


$$K_{eq} = \frac{2k}{3} + k = \frac{5k}{3}$$

$$T(\text{Time period}) = 2\pi\sqrt{\frac{M}{k_{eq}}} = \pi\sqrt{\frac{12M}{5k}}$$

$$a = 12$$

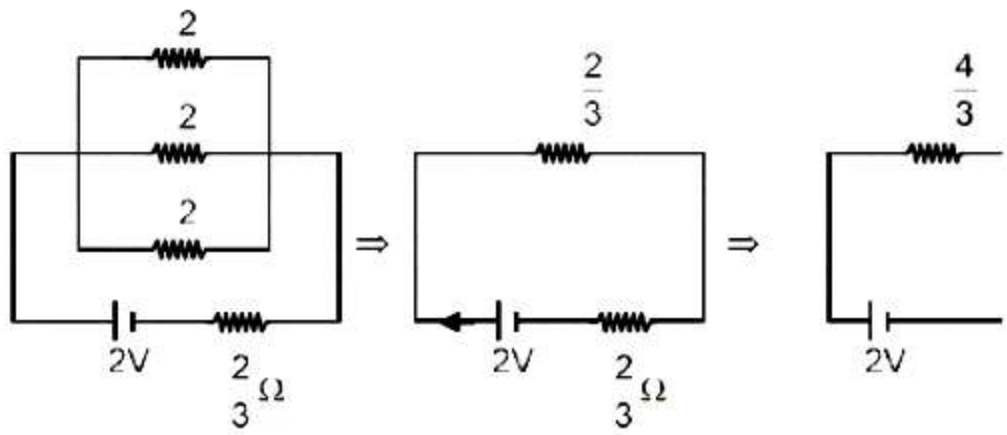
11. In the given figure, find the power delivered by the battery



(1) 1 W (2) 2 W (3) 5 W (4) 3 W

Ans. **(4)**

Sol.



$$P = \frac{V^2}{R} = \frac{(2)^2}{(4/3)} = \frac{4}{4} \times 3$$

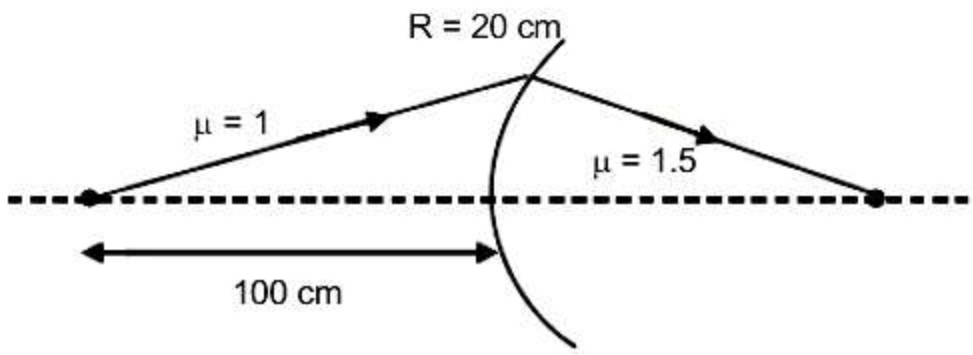
P = 3 watt

12. A point object is placed in air at 100 cm from a convex spherical refractive surface having radius of curvature 20 cm and refractive index on other side is $\mu = 1.5$. Find image distance

(1) 75 cm (2) 100 cm (3) 200 cm (4) 50 cm

Ans. **(2)**

Sol.



$$\frac{\mu_2}{V} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \quad \frac{1.5}{V} - \frac{1}{(-100)} = \frac{1.5-1}{20}$$

$$\Rightarrow \quad \frac{1.5}{V} + \frac{1}{100} = \frac{1}{40} \Rightarrow \frac{1.5}{V} = \frac{1}{40} - \frac{1}{100}$$

$$\Rightarrow \quad \frac{1.5}{V} = \frac{5-2}{200} \Rightarrow \frac{1.5}{V} = \frac{3}{200}$$

$$\Rightarrow \quad V = \frac{200 \times 1.5}{3}$$

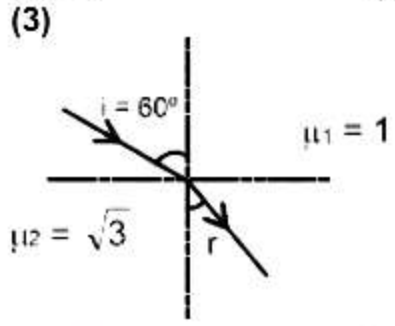
$$\Rightarrow \quad V = 100 \text{ cm}$$

13. Unpolarised light from air is incident on transparent glass at incident angle 60º. If reflected ray is completely polarized, then angle of refraction is -

(1) 15° (2) 60° (3) 30° (4) 45°

Ans. **(3)**

Sol.



$\mu = \tan i$ (Brewster's law)

$\mu_2 = \tan 60° = \sqrt{3}$

$\mu_1 \sin i = \mu_2 \sin r -$ (Snell's law)

$1 \times \sin 60° = \sqrt{3} \sin r$

$$\frac{\sqrt{3}}{2} = \sqrt{3} \sin r = \sin r = \frac{1}{2}$$

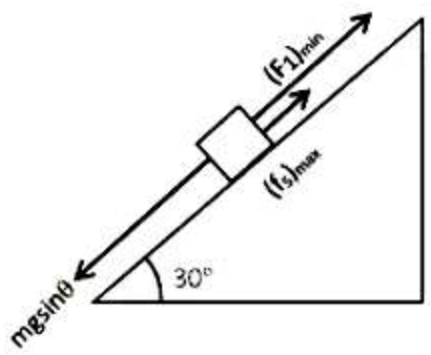
$r = 30°$

14. Find the difference between minimum force required to prevent the block from sliding down and minimum force required to just push it up the plane. The inclined plane is at 30° from horizontal and mass of the block is 5 Kg (use μ= 0.1, g=10m/s^2)

(1) $5\sqrt{3}$ N (2) $2\sqrt{3}$ N (3) 5 N (4) 8 N

Ans. **(1)**

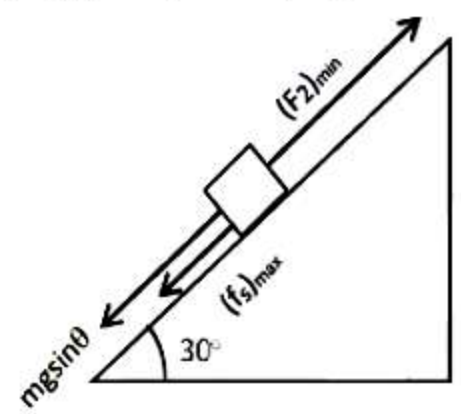
Sol.



$(F_1)_{min} + (f_s)_{max} = mg\sin\theta$

$(F_1)_{min} + \mu mg\cos\theta = mg\sin\theta$

$(F_1)_{min} = mg\sin\theta - \mu mg\cos\theta$



$(F_2)_{min} = (f_s)_{max} + mg\sin\theta$

$(F_2)_{min} = \mu mg\cos\theta + mg\sin\theta$

$\because (F_2)_{min} - (F_1)_{max} = 2\mu mg\cos\theta$

$= 2 \times 0.1 \times 5 \times 10 \times \cos 30°$

$= 5\sqrt{3}$ Newton

15. **Statement-1 :** E.M. waves posses energy.

Statement-2 : When E.M. Waves strike a surface they apply pressure on it.

(1) Both statements are true

(2) Both statements are false

(3) Statement-1 is true and statement-2 is false

(4) Statement-1 is false and statement-2 is true

Ans. **(1)**

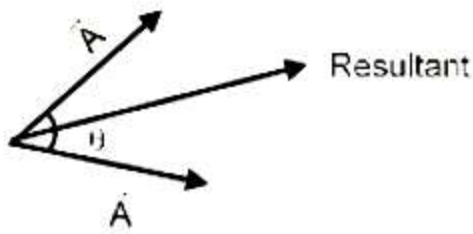
Sol. Theory Based

16. Two vector of equal magnitude A are inclined at an angle θ with each other. Find the magnitude of resultant vector

(1) $2A\cos\left(\frac{\theta}{2}\right)$ (2) $2A\sin\left(\frac{\theta}{2}\right)$ (3) $2A\cos\theta$ (4) $2A\sin\theta$

Ans. **(1)**

Sol.



$$|\text{Resultant}| = \sqrt{A^2 + A^2 + 2A^2\cos\theta}$$

$$= \sqrt{2A^2 + 2A^2\cos\theta}$$

$$= \sqrt{2}A\sqrt{1+\cos\theta}$$

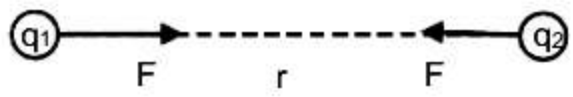
$$= 2A\cos\left(\frac{\theta}{2}\right)$$

17. The force between two charged particle separated by a distance 'r' when placed in air is F. If these charges are immersed in a medium of dielectric constant K = 5. Then find the separation between them to keep the force same.

(1) $\frac{r}{\sqrt{3}}$ (2) $\sqrt{5}r$ (3) $\sqrt{3}r$ (4) $\frac{r}{\sqrt{5}}$

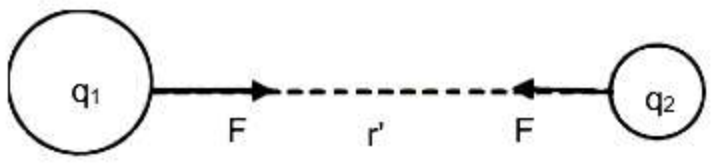
Ans. **(4)**

Sol. Case – I : In air



$$F = \frac{q_1q_2}{4\pi\varepsilon_0 r^2} \text{ (Electrostatic force)}$$

Case -2 : In medium (K = 5)



$$F = \frac{q_1q_2}{4\pi\varepsilon_0 K \times r'^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 \times 5r'^2}$$

$$5r'^2 = r^2$$

$$r' = \frac{r}{\sqrt{5}}$$

18. Length of a pendulum is 20 cm and error in its measurement is 2mm. If it completes 50 oscillations in 40 sec. and time was measured by a watch of resolution 1 sec. Find % error in calculation of acceleration due to gravity

(1) 8 % (2) 4 % (3) 2 % (4) 6 %

Ans. **(4)**

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$ $\left[T = \frac{40}{50} = \frac{4}{5}s\right]$

$$g = \frac{4\pi^2\ell}{T^2}$$

$$\frac{\Delta g}{g} \times 100\% = \frac{\Delta \ell}{\ell} \times 100\% + 2\frac{\Delta T}{T} \times 100\%$$

$$\frac{\Delta g}{g} \times 100\% = \left(\frac{0.2}{20}\right) \times 100 + 2 \times \frac{1}{40} \times 100$$

$$\frac{\Delta g}{g} \times 100\% = 6\%$$

19. A nucleus has mass number A_1 and volume V_1. Another nucleus has mass number A_2 and volume V_2 If relation between mass number is $A_2 = 4A_1$ then find V_2

(1) $2V_1$ (2) $8V_1$ (3) $4V_1$ (4) V_1

Ans. **(3)**

Sol. $r = r_0(A)^{\frac{1}{3}}$

$$V_1 = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(r_0(A_1)^{\frac{1}{3}}\right)^3 = \frac{4}{3}\pi(r_0)^3 A_1$$

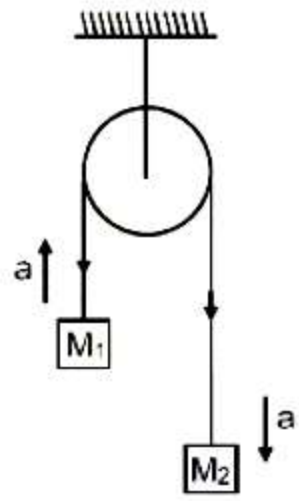
$$V_2 = \frac{4}{3}\pi\left(r_0(A_2)^{\frac{1}{3}}\right)^3 = \frac{4}{3}\pi r_0^3 A_2$$

$A_2 = 4A_1$ (given)

$$V_2 = \frac{4}{3}\pi r_0^3(4A_1) = 4V_1$$

$V_2 = 4V_1$

20. In the given pulley block system



given a = g/8

find the ratio of $\frac{M_1}{M_2} = ?$

(1) $\frac{7}{8}$ (2) $\frac{5}{7}$ (3) $\frac{9}{7}$ (4) $\frac{7}{9}$

Ans. **(4)**

Sol. $a = \frac{(M_2 - M_1)}{M_1 + M_2} g$

$\frac{g}{8} = \frac{(M_2 - M_1)}{M_1 + M_2} \times g$

$M_1 + M_2 = 8M_2 - 8M_1$

$\frac{M_1}{M_2} = \frac{7}{9}$

21. The frequency of incident light is equal to threshold frequency for the metal surface υ_{th}. When frequency is halved and intensity is doubled then the number of photo electrons will be

(1) Doubled

(2) halved

(3) Will remain same

(4) Photo electrons will not be emitted

Ans. **(4)**

Sol. electrons will not release below the reshold frequency $\upsilon < \upsilon_{th}$

Here, $\upsilon = \frac{\upsilon_{th}}{2}$

22. Find average power in electric circuit if source voltage V=20 sin (100 t) & current in the circuit is i = 2 sin(100t+π/3)

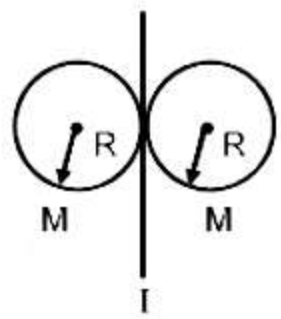
(1) 5 W (2) 10 W (3) $5\sqrt{3}$ W (4) $10\sqrt{3}$ W

Ans. **(2)**

Sol. $P_{avg}= \frac{i_m V_m}{2}\cos\phi = \frac{20\times 2}{2}\times\cos\pi/3$

$= 20 \times \frac{1}{2} = 10$ W

23. Two solid spheres each of mass 2 kg and radius 75 cm are arranged as shown. Find moment of inertia of the system about the gives axis shown.



(1) $\frac{63}{20}$kg.m^2 (2) $\frac{126}{30}$kg.m^2 (3) $\frac{7}{5}$kg.m^2 (4) $\frac{9}{7}$kg.m^2

Ans. **(1)**

Sol. $I = 2\left(MR^2\right)+2\left(\frac{2}{5}MR^2\right)$

$I = \frac{14MR^2}{5}$

$I = \frac{14}{5}\times 2\times\left(\frac{3}{4}\right)^2$

$I = \frac{28}{5}\times\frac{9}{16}$

$I = \frac{63}{20}$kg.m^2

24. A resistor of 25Ω is in left side with resistor x on the right side. Resistance per unit length is r and now the resistance per unit length is changed by 2r. Find the change in balance position, If the earlier balanced position was 40 cm from the left side.

(1) Does not shift

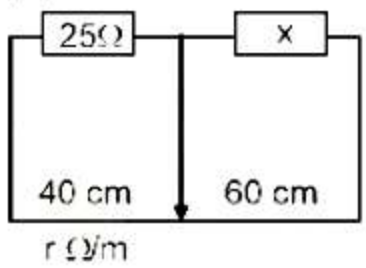
(2) Shift by 10 cm right

(3) Shift by 20 cm right

(4) Shift by 10 cm left

Ans. **(1)**

Sol.



Given bridge balanced.

when r is change by 2r balance condition does not change became resistance ratio for balanced bridge will match.

JEE Main 31 Jan 2024 (Shift-2) (Memory Based)

PART : CHEMISTRY

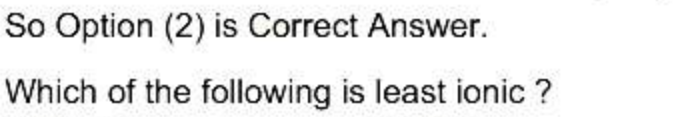
1. Find the correct set of quantum number for the last electron of potassium.

(1) $n = 4 \quad \ell = 2 \quad m = +2 \quad s = +\frac{1}{2}$ (2) $n = 2 \quad \ell = 0 \quad m = 0 \quad s = +\frac{1}{2}$

(3) $n = 3 \quad \ell = 0 \quad m = 0 \quad s = +\frac{1}{2}$ (4) $n = 4 \quad \ell = 0 \quad m = 0 \quad s = +\frac{1}{2}$

Ans. **(4)**

Sol. K = 19



$\Rightarrow n = 4 , \ell = 0 , m = 0, s = +\frac{1}{2}$

2. Which of the following is correct ?

(1) $[NiCl_4]^{2-}$ diamagnetic, $[Ni(CO)_4]$ diamagnetic

(2) $[Ni(CO)_4]$ diamagnetic, $[Ni(Cl)_4]^{2-}$ Paramagnetic

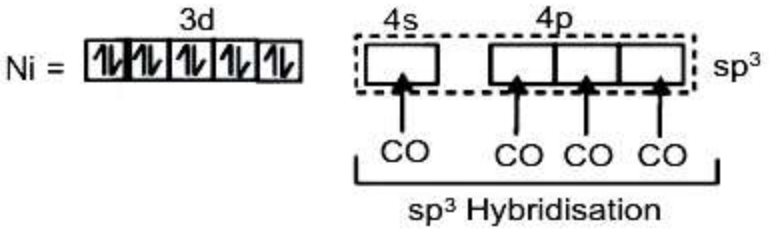
(3) $[NCl_4]^{2-}$ paramagnetic, $[Ni(CO)_4]$ Paramagnetic

(4) $[NiCl_4]^{2-}$ diamagnetic $[Ni(CO)_4]$ Paramagnetic

Ans. **(2)**

Sol. $Ni(CO)_4 \longrightarrow Ni(0) \longrightarrow$ In Presence of strong field ligand CO $\longrightarrow$ more interaction and Δ value high so paring will takes place.

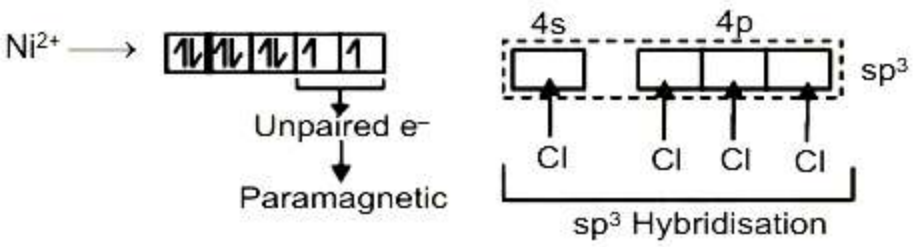
In presence of SFL CO



No unpaired electron - So diamagnetic compound.

But in $[NiCl_4]^{2-} \longrightarrow Ni^{+2}$

$Cl^- \longrightarrow$ WFL $\longrightarrow \Delta_0$ Value low $\longrightarrow$ high spin complex



So Option (2) is Correct Answer.

3. Which of the following is least ionic ?

(1) $BaCl_2$ (2) KCl (3) AgCl (4) $CoCl_2$

Ans. **(3)**

Sol. Order of ionic character = $BaCl_2 > KCl > CoCl_2 > AgCl$.

$Ag^{\oplus}$ due to pseudo inert gas configuration have high polarizing power.

4 **Statement-I** : 13th group element hydrolyse due to covalent nature.

Statement-II : On hydrolysis Al give $[Al(H_2O)_6]^{3+}$

(1) Both statement I and II are correct

(2) Statement I is correct and statement II is incorrect

(3) Statement I is incorrect and statement II is correct

(4) Both statement I and II are incorrect

Ans. **(1)**

Sol. 13th group element hydrolyse due to covalent character and on hydrolysis Al give $[Al(H_2O)_6]^{3+}$.

5.

	List-I		**List-II**
	Complex		**Electronic configuration**
(i)	$[Fe(H_2O)_6]^{3+}$	**(P)**	$t_{2g}^{2,2,2}, e_g^{1,1}$
(ii)	$[Ni(H_2O)_6]^{2+}$	**(Q)**	$t_{2g}^{1,1,1}, e_g^{1,1}$
(iii)	$[Cr(H_2O)_6]^{3+}$	**(R)**	$t_{2g}^{1,1,0}, e_g^{0,0}$
(iv)	$[V(H_2O)_6]^{3+}$	**(S)**	$t_{2g}^{1,1,1}, e_g^{0,0}$

Identify correct match:

(1) i-Q, ii-P, iii-S, iv-R (2) i-P, ii-Q, iii-R, iv-S (3) i-Q, ii-P, iii-R, iv-S (4) i-P, ii-R, iii-Q, iv-S

Ans. **(1)**

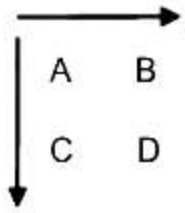
Sol. (i) $_{26}Fe^{3+}$ = $3d^5$ = $t_{2g}^{1,1,1}, e_g^{1,1}$

(ii) $_{28}Ni^{2+}$ = $3d^8$ = $t_{2g}^{2,2,2}, e_g^{1,1}$

(iii) $_{24}Cr^{3+}$ = $3d^3$ = $t_{2g}^{1,1,1}, e_g^{0,0}$

(iv) $_{23}V^{3+}$ = $3d^2$ = $t_{2g}^{1,1,0}, e_g^{0,0}$

6.



Where A, B, C and D are elements in periodic table

Which of the following order is correct ?

(1) Atomic radius: A < B < C < D (2) Metallic radius: A < B < D < C

(3) Ionic radius: $B^+ < A^+ < D^+ < C^+$ (4) None of these.

Ans. **(3)**

Sol. (i) On moving left to right atomic radius decrease. So atomic radius order is B < A.

(ii) Metallic radius is also decrease on moving left to right so order $\Rightarrow$ B < A.

(iii) Order of ionic radius $B^+ < A^+ < D^+ < C^+$.

7. (A) Mn_2O_7 is an oil at room temperature (B) V_2O_5 reacts with acid to give VO_2^+

(C) CrO is a basic oxide (D) V_2O_5 does not react with acids

Choose the correct answer

(1) A, B and C (2) B, C and D only (3) A only (4) B and C only

Ans. **(1)**

Sol. **Ref. NCERT 4.4.1**

- Mn_2O_7 is a covalent green oil. So (A) is correct.
- V_2O_5 is amphoteric, though mainly acidic, it gives VO_4^{3-} as well as VO_2^{+} salts. So (B) is correct.
- CrO is basic because Cr in CrO is +2 oxidation state. So Cr in +2 oxidation state shows basic character. So (C) is correct.

8. **Statement-I :** In 15^{th} group hydrides reducing character decreases from NH_3 to BiH_3.
Statement-II : E_2O_5 is more acidic than E_2O_3 (where E is the 15^{th} group elements)
(1) Statement I is incorrect and statement II is correct
(2) Statement I is correct and statement II is incorrect
(3) Both statement I and II are correct
(4) Both statement I and II are incorrect

Ans. **(1)**

Sol. (1) Reducing character of 15^{th} group hydrides increases from NH_3 to BiH_3.
(2) The oxide in the higher oxidation state of element is more acidic than that of lower oxidation state.

9. **Statement-I :** In the reduction of permanganate ion to magnate ion, one e^- is involved.
Statement-II : $CrO_4^{2-} \xrightarrow{H^+}$ Product
In product Oxidation number of Cr is 6.
(1) Statement I is incorrect and statement II is correct
(2) Statement I is correct and statement II is incorrect
(3) Both statement I and II are correct
(4) Both statement I and II are incorrect

Ans. **(1)**

Sol. $MnO_4^- + e^- \longrightarrow MnO_4^{2-}$
$2CrO_4^{2-} + 2H^+ \longrightarrow Cr_2O_7^{2-} + H_2O$
In product $Cr_2O_7^{2-}$ Oxidation number of Cr is 6.

10. **Statement-I :** S_8 disproportionate into S^{2-} and $S_2O_3^{2-}$ in alkaline medium
Statement-II : ClO_4^- undergoes disproportionate in acidic medium.
(1) Statement I is correct and statement II is incorrect
(2) Statement I is incorrect and statement II is correct
(3) Both statement I and statement II are correct
(4) Both statement I and statement II are incorrect

Ans. **(1)**

Sol. (i) $S_8 + OH^- \longrightarrow S^{2-} + S_2O_3^{2-}$
So S_8 disproportionate in alkaline medium.
(ii) ClO_4^- do not show disproportionate reaction in any medium.

11. Nessler's reagent is used for identification of following cation:
(1) Na^+ (2) K^+ (3) NH_4^+ (4) Pb^{+2}

Ans. **(3)**

Sol. $NH_4^+ + \underbrace{2[HgI_4]^{2-} + 4OH^-}_{\text{Nessler's reagent}} \rightarrow \underbrace{HgO.\ Hg(NH_2)I \downarrow}_{\text{brown ppt (iodide of millon's base)}} + I^- + H_2O$

12. For an equilibrium reaction $A(g) \rightleftharpoons B(g) + \frac{C}{2}(g)$ the relation between equilibrium constant (Kp). degree of dissociation (α) and total equilibrium pressure (P) is :

(1) $Kp = \frac{\alpha^{3/2} \cdot p^{1/2}}{(2+\alpha)^{1/2}(1-\alpha)}$ (2) $Kp = \frac{\alpha^{1/2} \cdot p^{1/2}}{\left(1+\frac{\alpha}{2}\right)^{1/2}(1-\alpha)}$

(3) $Kp = \frac{\alpha^{3/2} \cdot p^{1/2}}{\left(1+\frac{\alpha}{2}\right)(1-\alpha)}$ (4) $Kp = \frac{\alpha^{1/2} \cdot p}{\left(1+\frac{\alpha}{2}\right)(-\alpha)}$

Ans. **(1)**

Sol.

	$A(g) \rightleftharpoons$	$B(g)$	$+ \frac{C}{2}(g)$
$t = 0$	a	0	0
$t = t_{eq}$	a – x	x	x/2

For a mole, x moles are dissociated

For 1 mole, $\frac{x}{a}$ moles = α are dissociated

$x = a\alpha$

	$A(g) \rightleftharpoons$	$B(g)$	$+ \frac{C}{2}(g)$
At $t = t_{eq}$	$a - a\alpha$	$a\alpha$	$\frac{a\alpha}{2}$

Total no. of moles at equilibrium = $a + \frac{a\alpha}{2} = a\left(1+\frac{\alpha}{2}\right)$

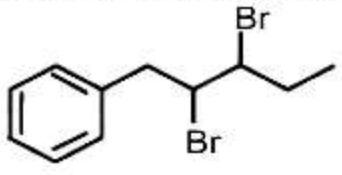
$P_{A(g)} = \frac{a(1-\alpha)P}{a\left(1+\frac{\alpha}{2}\right)}$ $P_{B(g)} = \frac{a\alpha \cdot P}{a\left(1+\frac{\alpha}{2}\right)}$ $P_{C(g)} = \frac{(a\alpha/2)\cdot P}{a\left(1+\frac{\alpha}{2}\right)}$

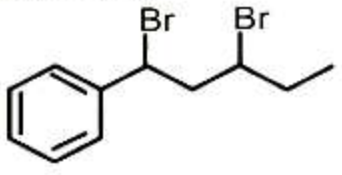
$= \frac{(1-\alpha)P}{1+\frac{\alpha}{2}}$ $= \frac{\alpha P}{1+\frac{\alpha}{2}}$ $= \frac{(\alpha/2)\cdot P}{1+\frac{\alpha}{2}}$

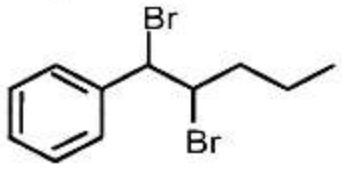
$K_P = \frac{P_B \cdot (Pc)^{1/2}}{P_A} = \frac{\left(\frac{\alpha}{1+\frac{\alpha}{2}}P\right)\left(\frac{\frac{\alpha}{2}P}{1+\frac{\alpha}{2}}\right)^{1/2}}{\frac{(1-\alpha)P}{\left(1+\frac{\alpha}{2}\right)}}$

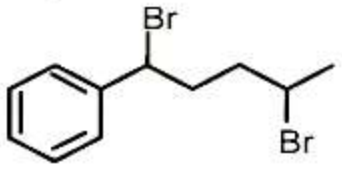
$K_P = \frac{\alpha \cdot \alpha^{1/2} \cdot P^{1/2}}{(2+\alpha)^{1/2}(1-\alpha)} = \frac{\alpha^{3/2} \cdot P^{1/2}}{(2+\alpha)^{1/2}(1-\alpha)}$

13. Find correct structure of 2,3-Dibromo-1-phenypentane

(1) 

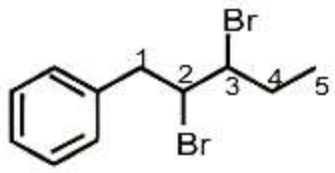
(2) 

(3) 

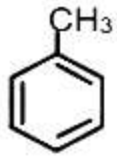
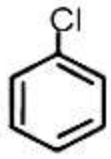
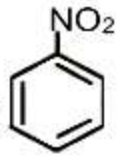
(4) 

Ans. **(1)**

Sol.



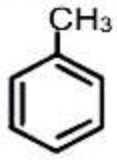
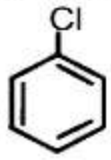
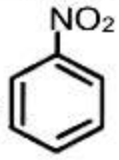
14. The correct order of reactivity towards electrophilic aromatic substitution reaction is.

(A)  (B)  (C)  (D) 

(1) A < B < C < D (2) C < B < D < A (3) D < C < A < B (4) D < C < B < A

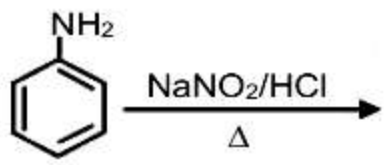
Ans. **(3)**

Sol. Greater the electron density of Aromatic ring, greater will be the rate of electrophilic Aromatic substitution.

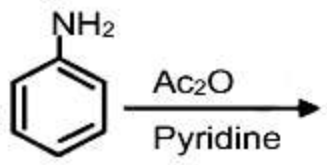
(A)  (B)  (C)  (D) 

D < C < A < B

Answer is (3).

15. **Statement-I :** 

formed product give Libermann nitroso test.

Statement-II : 

formed product is highly deactivating due to protonation of nitrogen.

(1) Both Statement-I & Statement-II are correct.

(2) Both Statement-I & Statement-II are incorrect.

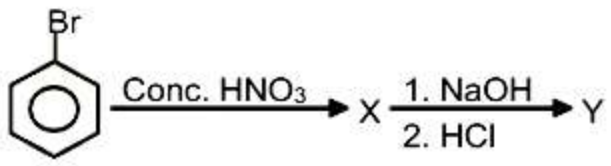
(3) Statement-I is correct whereas Statement-II is incorrect.

(4) Only Statement-II is correct.

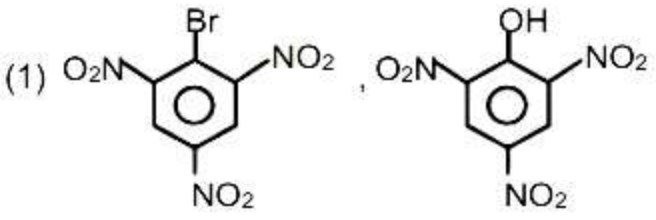
Ans. **(3)**

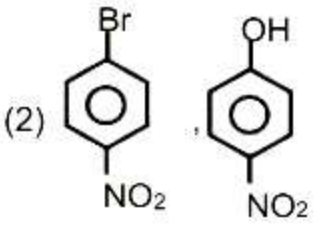
Sol. In statement-I : Phenol is formed which gives Libermann nitroso test.

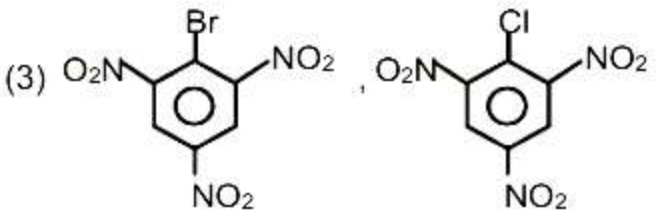
In statement-II : The product benzanilide is weakly acitivating.

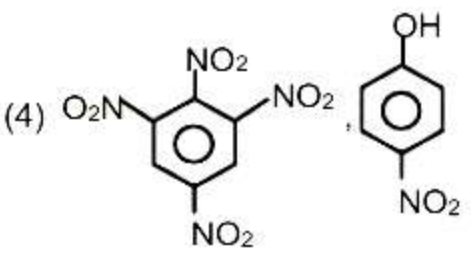
16. 

X and Y are respectively.

(1) 

(2) 

(3) 

(4) 

Ans. **(2)**

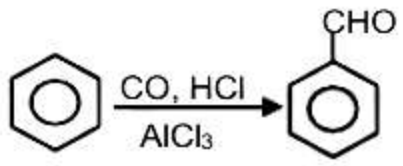
17. How many of the vitamins among A, B_1, B_2, B_{12}, C, D and K, can be stored in human body.

(1) 2 (2) 3 (3) 4 (4) 5

Ans. **(2)**

Sol. Only water insoluble and fat soluble vitamins A, D and K can be stored in human body.

18.



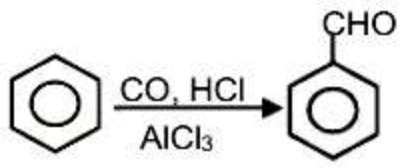
Above reaction is known as

(1) Etard reaction. (2) Gatterman Koch reaction

(3) Stephen reaction (4) Rosenmund reaction

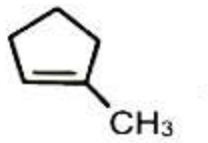
Ans. **(2)**

Sol.

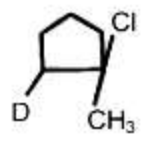


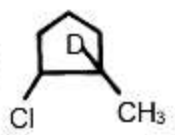
is called Gatterman Koch reaction

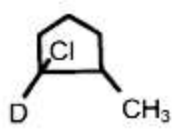
19.

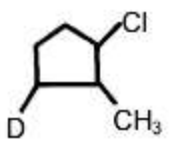


$\xrightarrow{DCl}$ P, Product "P" is

(1) 

(2) 

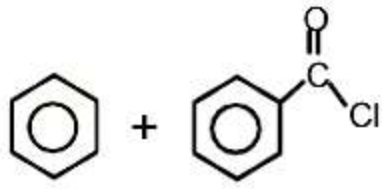
(3) 

(4) 

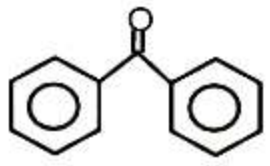
Ans. **(1)**

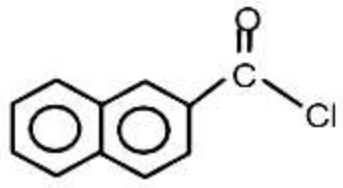
Sol. It is example of electrophilic addition reaction

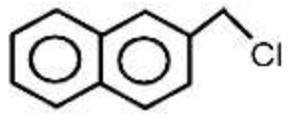
20.

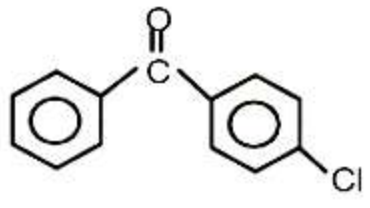


$\xrightarrow{\text{Anhydrous } AlCl_3}$ Major Product

(1) 

(2) 

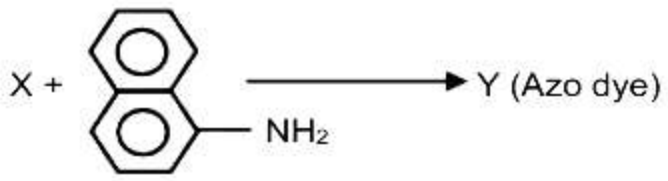
(3) 

(4) 

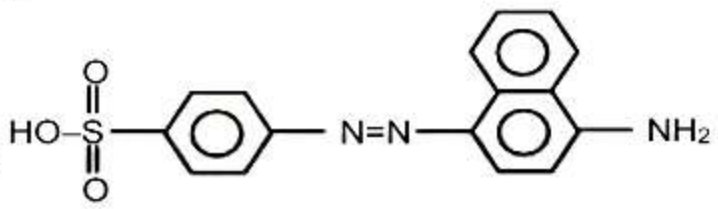
Ans. **(1)**

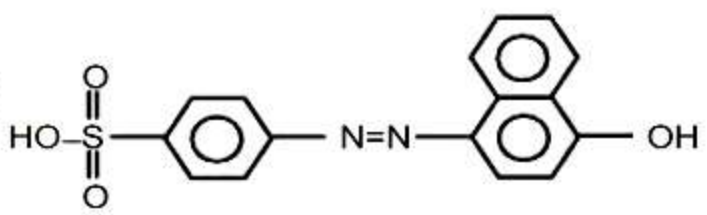
Sol. It is example of Friedel Craft acylation.

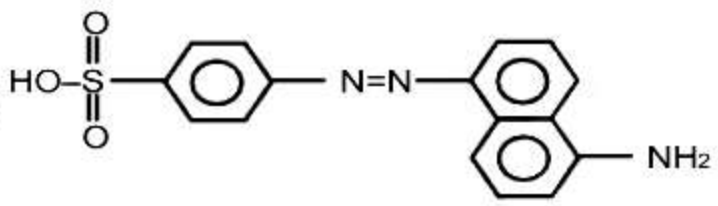
21. Sulphanlic acid + $NaNO_2$ + $CH_3COOH \longrightarrow X$

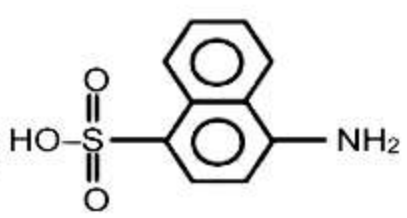


Y is :

(1) 

(2) 

(3) 

(4) 

Ans. **(1)**

Sol. Y is red violet Azo dye.

22. Given rate law for a reaction r = k[A]
If reaction complete 50% in 120 min then determine in how many minute reaction gets completed 90%?

Ans. **(400)**

Sol. For first order,

$t_{90\%} = 3.33 \times t_{½}$

$= 3.33 \times 120 \approx 400$ min

23. $KMnO_4$ oxidise $C_2O_4^{2-}$ to CO_2 during this reaction no of mole of H^+ ions used with 1 mole of MnO_4^- is _____.

Ans. **(8)**

Sol. $[MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O] \times 2$

$[C_2O_4^{2-} \rightarrow 2CO_2 + 2e^-] \times 5$

$2MnO_4^- + 16H^+ + 5C_2O_4^{2-} \rightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$

24. 1 mole of an ideal gas expands from 10 lit to 100 lit isothermally and reversibly at 300 K, then magnitude of work done is _____ (in kJ) [Nearest integer]

[Given R = 8.314 $\frac{J}{Mole \times K}$]

Ans. **(6)**

Sol. For **isothermal reversible process**

$$W = -nRT \ln\left(\frac{V_2}{V_1}\right)$$

$$= -[1\times8.314\times300]\, 2.303 \log\left(\frac{100}{10}\right)$$

$$= -2.303\times8.314\times300$$

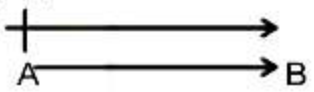
$$= -5744 \text{ J}$$

$$= -5.744 \text{ kJ}$$

25. In compound AB dipole moment of A–B bond and bond distance are 1 Å and 1.2 D respectively, then magnitude of fraction of charge on A atom is _____ $\times 10^{-2}$ [Nearest integer]

Ans. **(25)**

Sol.



$\mu = \delta \times d = 1.2$

$\delta \times 1 \times 10^{-10} = 1.2 \times 3.33 \times 10^{-30}$ C×meter

$\delta = 1.2 \times 3.33 \times 10^{-20}$

$= 3.996 \times 10^{-20}$

$= 0.3996 \times 10^{-19}$C

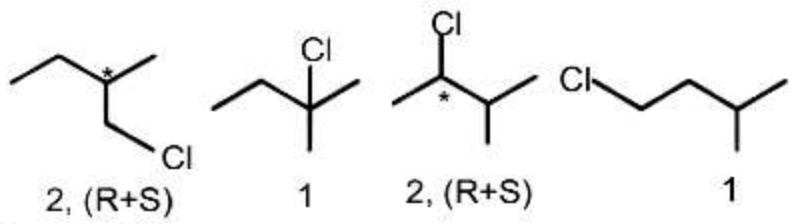
$$\text{Fraction of charge} = \left(\frac{\delta}{e}\right) = \left(\frac{0.3996\times10^{-19}}{1.6\times10^{-19}}\right)$$

$$= 0.24975 = 24.975 \times 10^{-2}$$

26. Number of monochlorination product of 2-Methylbutane formed in presence of sunlight is.

Ans. **(6)**

Sol.



Answer (6)

27. Compound X with molar mass 108 g mol^{-1} undergoes acetylation to give product with molar mass 192 g mol^{-1}. Total number of NH_2 group in benzoid molecule X is.

Ans. **(2)**

Sol. Let the benzoid molecule has x no of NH_2 group, therefore on acetylation it will form $R–(NH–COCH_3)_x$,

$$R-(NH_2)_x \xrightarrow{CH_3-COCl} R-(NHCOCH_3)_x$$

$R + 16x = 108$ $\qquad$ $R + 58x = 192$

$42x = 84$

$x = 2.$

JEE Main 31 Jan 2024 (Shift-2) (Memory Based)

PART : MATHEMATICS

1. The number of solutions of the equation $e^{\sin x} - 2e^{-\sin x} = 2$ is

(1) 1 (2) 0 (3) infinite (4) 2

Ans. **(2)**

Sol. Let $e^{\sin x} = t$

$\Rightarrow t - \frac{2}{t} = 2 \Rightarrow t^2 - 2t - 2 = 0$

$(t-1)^2 = 3$

$t - 1 = \pm\sqrt{3}$

$t = 1 \pm \sqrt{3}$ but $t > 0$

so, $e^{\sin x} = 1 + \sqrt{3}$

Now, $-1 \le \sin x \le 1$

$e^{-1} \le e^{\sin x} \le e$

$e \cong 2.72$ but $1+\sqrt{3} \cong 2.73$

$\Rightarrow$ No. of solutions = 0

2. If $a = \sin^{-1}(\sin 5)$ and $b = \cos^{-1}(\cos 5)$ then value of $a^2 + b^2$ is

(1) $(2\pi - 5)^2$ (2) $(3\pi - 7)^2$ (3) $2(2\pi - 5)^2$ (4) $2(3\pi - 7)^2$

Ans. **(3)**

Sol. $a = \sin^{-1}(\sin 5)$, $b = \cos^{-1}(\cos 5)$

$a = 5 - 2\pi$, $b = 2\pi - 5$

$a^2 + b^2 = (5 - 2\pi)^2 + (2\pi - 5)^2$

$= 2(2\pi - 5)^2$

3. If 2nd, 8th, 44th term of an non-constant arithmetic progression is same as 1st, 2nd & 3rd term of Geometric progression respectively and first term of arithmetic progression is 1, then sum of first 20 terms of that arithmetic progression is

Ans. **(970)**

Sol. Let common difference = d

$t_2 = 1 + d$, $t_8 = 1 + 7d$, $t_{44} = 1 + 43d$

but these are in G.P.

$\Rightarrow (1 + 7d)^2 = (1 + d)(1 + 43d)$

$\Rightarrow 6d^2 - 30d = 0 \Rightarrow d = 5$ ($\because d \neq 0$)

Now, sum of first 20 terms of an AP $= \frac{20}{2}(2a + (20-1)d) = 10(2 + 95) = 970$

4. If $f : R \to (0, \infty)$ is an increasing function such that $\lim\limits_{x\to\infty} \frac{f(7x)}{f(x)} = 1$, then the value of $\lim\limits_{x\to\infty}\left[\frac{f(5x)}{f(x)} - 1\right]$ (where [•] denote the greatest integer function) is

(1) 1 (2) 0 (3) 2 (4) 3

Ans. **(2)**

Sol. $f(x) \le f(5x) \le f(7x)$, $\forall\ x > 0$

$1 \le \frac{f(5x)}{f(x)} \le \frac{f(7x)}{f(x)}$

As $x \to \infty$ $\frac{f(5x)}{f(x)} \to 1^+$

$\lim\limits_{x\to\infty}\left[\frac{f(5x)}{f(x)} - 1\right] = 0$

5. The area bounded by the curves $3y = (x-4)^2$ and $y = 4x - x^2$ is

(1) 10 (2) 6 (3) 14 (4) 27

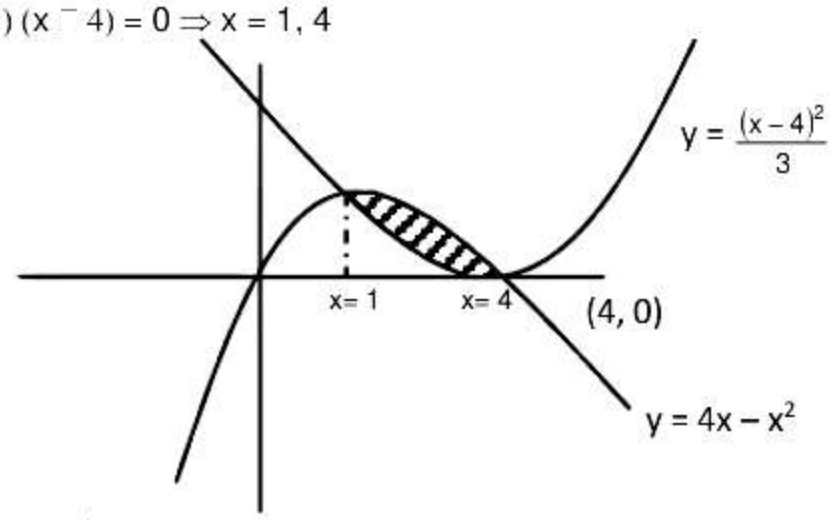
Ans. **(2)**

Sol. Solving $3y = (x-4)^2$ and $y = 4x - x^2$

$12x - 3x^2 = x^2 - 8x + 16$

$\Rightarrow 4x^2 - 20x + 16 = 0$

$(x - 1)(x - 4) = 0 \Rightarrow x = 1, 4$



$$\text{Area} = \int_1^4 \left((4x - x^2) - \frac{(x-4)^2}{3} \right) dx$$

$$= \left(2x^2 - \frac{x^3}{3} - \frac{(x-4)^3}{9} \right)_1^4$$

$$= 2(16 - 1) - \frac{1}{3}(3)(21) - \frac{1}{9}(0 + 27)$$

$$= 30 - 21 - 3 = 6$$

6. The number of ways in which 21 identical apples to be distributed into 3 children in such a way that each children get at least 2 apples is

(1) 133 (2) 134 (3) 135 (4) 136

Ans. **(4)**

Sol. Let 1st student get x

2nd student get y

3rd student get z

$\Rightarrow x + y + z = 21, \quad x, y, z \geq 2$

Let $x = 2 + t_1, \quad y = 2 + t_2, z = 2 + t_3$

$\Rightarrow t_1 + t_2 + t_3 = 15, \; t_1, t_2, t_3 \geq 0$

number of ways $= {}^{17}C_2 = \frac{17 \times 16}{2} = 17 \times 8 = 136$

7. If $z_1 + z_2 = 5$ & $z_1^3 + z_2^3 = 20 + 15i$ then $\left| z_1^4 + z_2^4 \right|$ is equal to

Ans. **(75)**

Sol. $z_1^3 + z_2^3 = (z_1 + z_1)((z_1 + z_2)^2 - 3z_1 z_2) \Rightarrow 20 + 15i = 5\,(25 - 3z_1 z_2) \Rightarrow z_1 z_2 = 7 - i$

Now $z_1^2 + z_2^2 + 2z_1 z_1 = 25 \Rightarrow z_1^2 + z_2^2 = 25 - 2(7 - i) \Rightarrow z_1^2 + z_2^2 = 11 + 2i$

Now $z_1^4 + z_2^4 + 2(7 - i)^2 = 121 - 4 + 44i \Rightarrow z_1^4 + z_2^4 = 21 + 72i$

So $\left| z_1^4 + z_2^4 \right| = \sqrt{441 + (72)^2} = \sqrt{5625} = 75$

8. If A is a matrix of order 3 × 3 and det A = 2 and n = det $\underbrace{(adj(adj.....(adjA))}_{2024 times}$, the remainder when n is divided by 9, is

(1) 2 (2) 4 (3) 6 (4) 7

Ans. **(4)**

Sol. $|adjA| = |A|^2$

$|adj.adj.....adj(A)| = |A|^{2^{2024}} = 2^{2^{2024}}$ ..... (1)

$\because 2^{2024} = 4^{1012} = (3+1)^{1012} = 3k+1$, where k is odd

$\Rightarrow |adj(adj).....adj(A)| = 2^{3k+1} = 2.8^k$

$= 2(9-1)^k = 9m - 2$

$\Rightarrow \quad = 9P + 7$

Remainder = 7

9. A biased coin in which probability of getting head is twice to that of tail. If coin is tossed 3 times then the probability of getting two tails and one head is

(1) 1/9 (2) 2/9 (3) 2/27 (4) 1/27

Ans. **(2)**

Sol. $P(H) = p \quad P(T) = q \quad \Rightarrow p = 2q$ and

$p + q = 1 \Rightarrow \quad q = 1/3 \quad \Rightarrow p = 2/3$

$P(2T, 1H) = {}^3C_2\, q^2 p$

$= {}^3C_2 . \frac{1}{9} . \frac{2}{3} = \frac{2}{9}$

10. The value of $\frac{120}{\pi^3}\left|\int_0^{\pi} \frac{x^2 \sin x \cos x}{\sin^4 x + \cos^4 x} dx\right|$ is

Ans. **(15)**

Sol. Let $I = \int_0^{\pi} \frac{x^2 \sin x \cos x}{\sin^4 x + \cos^4 x} dx$ .....(1)

$I = \int_0^{\pi} -\frac{(\pi - x)^2 \sin x \cos x}{\sin^4 x + \cos^4 x} dx$, by king property .....(2)

adding equation (1) and (2)

$2I = \int_0^{\pi} \frac{\pi(2x - \pi)\sin x \cos x}{\sin^4 x + \cos^4 x} dx$

$I = \pi \int_0^{\pi/2} \frac{(2x - \pi)\sin x \cos x}{\sin^4 x + \cos^4 x} dx$ .....(3)

$I = \pi \int_0^{\pi/2} \frac{-2x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$ .....(4)

adding equation (3) and (4)

$2I = -\pi^2 \int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$

$I = -\frac{\pi^2}{2 \times 2} \int_0^{\pi/2} \frac{2 \sin x \cos x dx}{\sin^4 x + \cos^4 x}$

$$I = -\frac{\pi^2}{4}\int_0^{\pi/2}\frac{2\tan x\sec^2 x}{\tan^4 x+1}dx$$

$$I = -\frac{\pi^2}{2\times 2}\tan^{-1}\left(\tan^2 x\right)_0^{\pi/2}$$

$$I = -\frac{\pi^3}{8}$$

$$\text{Now } \frac{120}{\pi^3}\left|\int_0^{\pi}\frac{x^2\sin x\cos x}{\sin^4+\cos^4 x}dx\right|$$

$$= \frac{120}{\pi^3}\times\left(\frac{\pi^3}{8}\right) = 15$$

11. If the mean and variance of 6 observations a, b, 68, 44, 48, 60 are 55 and 194 respectively and a > b then a + 3b is

(1) 190 (2) 180 (3) 200 (4) 210

Ans. **(2)**

Sol. $\text{Mean} = \frac{a+b+68+44+48+60}{6} = 55 \Rightarrow a+b = 110$ ..... (1)

$$\text{Variance} = \frac{\sum(x_i-\bar{x})^2}{n} = \frac{(55-a)^2+(55-b)^2+(13)^2+(11)^2+7^2+5^2}{6} = 194$$

$\Rightarrow a^2 + b^2 - 110(a + b) = -5250 \Rightarrow a^2 + b^2 = 6850.$ ..... (2)

after solving equation (1) and (2), we get a = 75 & b = 35 $\Rightarrow$ a + 3b = 75 + 105 = 180

12. If $\lim\limits_{x\to 0}\frac{ax^2e^x - b\log_e(1+x) + cxe^{-x}}{x^2\sin x} = 1$ then the value of $16(a^2 + b^2 + c^2)$ is

Ans. **(81)**

Sol. $$\lim_{x\to 0}\frac{ax^2\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.....\right)-b\left(x-\frac{x^2}{2}+\frac{x^3}{3}-.....\right)+cx\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+.....\right)}{x^3\left(\frac{\sin x}{x}\right)} = 1$$

$$\Rightarrow \lim_{x\to 0}\frac{(c-b)x+\left(\frac{b}{2}-c+a\right)x^2+\left(a-\frac{b}{3}+\frac{c}{2}\right)x^3.....}{x^3} = 1$$

$\Rightarrow c - b = 0, \quad \frac{b}{2} - c + a = 0$

and $a - \frac{b}{3} + \frac{c}{2} = 1$

$a = \frac{3}{4}, \quad b = c = \frac{3}{2}$

$a^2 + b^2 + c^2 = \left(\frac{9}{16}+\frac{9}{4}+\frac{9}{4}\right)$

$16(a^2 + b^2 + c^2) = 9 + 36 + 36 = 81$

13. A line of negative slope passing through the centre of circle $x^2 + y^2 - 16x - 4y = 0$ intersects +ve x and y-axis at A and B respectively then the minimum value of OA + OB (O is origin) is

Ans. **(18)**

Sol. Let slope of line is – m

$\therefore$ Equation of straight line $(y - 2) = -m(x - 8) \Rightarrow mx + y = 8m + 2$

$A \equiv \left(8+\frac{2}{m},0\right)$ $B \equiv (0, 2 + 8m)$

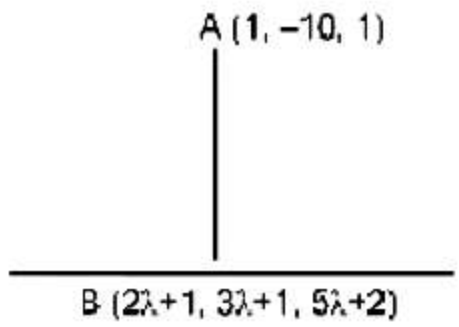
$OA + OB = 10 + \frac{2}{m} + 8m \geq 18$

$(OA + OB)_{min} = 18$

14. If reflection of A(1, –10, 1) about the line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-2}{5}$ is (α, β, γ) then the value of $|2\alpha + 3\beta + 5\gamma|$ is.

Ans. **(23)**

Sol.



Let co-ordinate of foot is

B(2λ +1, 3λ + 1, 5λ +2)

Direction ratio of AB is 2λ, 3λ + 11, 5λ + 1

AB is perpendicular to the given line so $\Rightarrow 2\lambda(2) + (3\lambda+11)3 + (5\lambda+1)5 = 0 \Rightarrow \lambda = -1$

So, foot is B(–1, –2, –3)

Now image of A (1, –10, 1) about the line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-2}{5}$ is (α, β, γ)

so, $\frac{\alpha+1}{2} = -1, \frac{\beta-10}{2} = -2, \frac{\gamma+1}{2} = -3 \Rightarrow \alpha = -3, \beta = 6, \gamma = -7$

$\Rightarrow |2\alpha + 3\beta + 5\gamma| = |-6 + 18 - 35| = 23$

15. If ${}^6C_m + 2\left({}^6C_{m+1}\right) + {}^6C_{m+2} = {}^8C_3$ (where $m \neq 1$) & $\frac{{}^{n-1}P_3}{{}^nP_4} = \frac{1}{8}$, then value of ${}^{n+1}C_m + {}^nP_m$ is

Ans. **(420)**

Sol. ${}^6C_m + {}^6C_{m+1} + {}^6C_{m+1} + {}^6C_{m+2} = {}^8C_3$

$\Rightarrow {}^7C_{m+1} + {}^7C_{m+2} = {}^8C_3$

$\Rightarrow {}^8C_{m+2} = {}^8C_3 = {}^8C_5$

$\Rightarrow m + 2 = 5 \ (\because m \neq 1)$

$\Rightarrow m = 3$

$\frac{{}^{(n-1)}P_3}{{}^nP_4} = \frac{1}{8} \Rightarrow \frac{1}{n} = \frac{1}{8} \Rightarrow n = 8$

Now ${}^{n+1}C_m + {}^nP_m = {}^9C_3 + {}^8P_3 = 84 + 336 = 420$

16. If $\frac{dT}{dt} = -k(T-85)$ and $T = 160$ at $t = 0$ then the value of T at $t = 45$, is

(1) $85 + 75\, e^{45k}$ (2) $85 + 75\, e^{-45k}$ (3) $75 + 85\, e^{45k}$ (4) $75 + 85\, e^{-45k}$

Ans. **(2)**

Sol. $\int \frac{dt}{T-85} = -\int k dt$

$\ln|T-85| = -kt + c$

at $t = 0$, $T = 160$

$\ln 75 = c$

$\ln\left|\frac{T-85}{75}\right| = -kt$

$\frac{T-85}{75} = + e^{-kt}$ (–rejected because $T = 160$ at $t = 0$)

$T = 85 + 75\, e^{-kt}$

at $t = 45$, $T = 85 + 75e^{-45k}$

17. If $f(x) = e^{-|\ln x|}$; $x \in (0, \infty)$ is discontinuous at m points and non-differentiable at n points then the value of $m + n$ is

Ans. **(1)**

Sol. Since $|\ln x|$ is continuous in $(0, \infty)$

$\Rightarrow f(x) = e^{-|\ln x|}$ is continuous is $(0, \infty)$

$\Rightarrow$ So number of points where f(x) is discontinuous, $m = 0$

$$f(x) = \begin{cases} e^{\ln x}; & 0 < x < 1 \\ e^{-\ln x}; & x \geq 1 \end{cases}$$

$f'(1^-) = 1$, $f'(1^+) = -1 \Rightarrow$ So number of points where f(x) is non-differentiable, $n = 1$

$m + n = 0 + 1 = 1$

18. A is a square matrix of order 3 and v_1, v_2, v_3 are 3 column matrices such that

$Av_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $Av_2 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$ $Av_3 = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$ where $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ then the value of $|A|$ is

(1) 9 (2) $\frac{9}{2}$ (3) $\frac{9}{4}$ (4) $\frac{9}{8}$

Ans. **(3)**

Sol. $A\begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & -1 \\ 3 & 2 & 2 \end{bmatrix}$

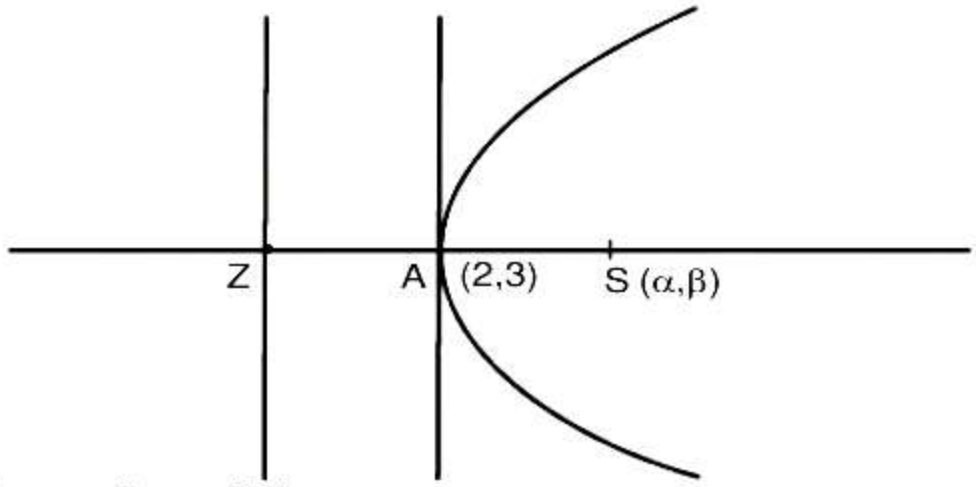
$|A|\begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & 3 & -1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 0 \\ 2 & 0 & -1 \\ 3 & 2 & 2 \end{vmatrix} \Rightarrow |A| = \frac{9}{4}$

19. An ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ whose eccentricity is $\frac{1}{\sqrt{2}}$ and passes through the focus of the parabola whose vertex is (2,3) and directrix is $2x + y - 6 = 0$ then the length of the latus rectum of ellipse is

(1) $\sqrt{656}$ (2) $\frac{\sqrt{656}}{5}$ (3) $\frac{\sqrt{656}}{10}$ (4) $\frac{\sqrt{656}}{2}$

Ans. **(2)**

Sol. Let foot of perpendicular from vertex of parabola on the directix is point Z



So, coordinate of Z is

$$\frac{x-2}{2}=\frac{y-3}{1}=-\frac{(4+3)-6}{4+1} \Rightarrow z\equiv\left(\frac{8}{5},\frac{14}{5}\right)$$

$$\Rightarrow \text{focus } S\equiv\left(\frac{12}{5},\frac{16}{5}\right)$$

Now eccentricity equal to $\frac{1}{\sqrt{2}} \Rightarrow b^2=\frac{a^2}{2}$

$$\therefore \frac{144}{25a^2}+\frac{256}{25\times\frac{a^2}{2}}=1$$

$$a^2=\frac{656}{25},\ b^2=\frac{328}{25}$$

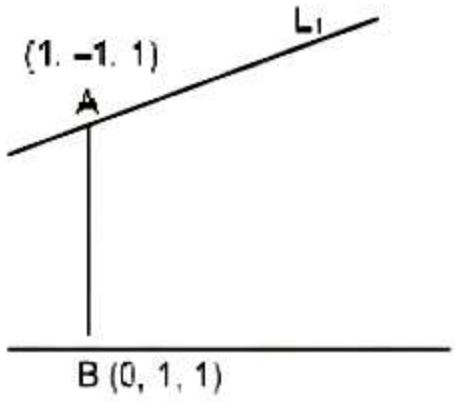
Now length of latus rectum $=\frac{2b^2}{a}=\frac{\sqrt{656}}{5}$

20. The shortest distance between the line $L_1=\left(\hat{i}-\hat{j}+\hat{k}\right)+\lambda\left(2\hat{i}-14\hat{j}+5\hat{k}\right)$ and $L_2=\left(\hat{j}+\hat{k}\right)+\mu\left(-2\hat{i}-4\hat{j}+7\hat{k}\right)$ then L_1 and L_2 is

(1) $\frac{5}{\sqrt{221}}$ (2) $\frac{10}{\sqrt{221}}$ (3) $\frac{2}{\sqrt{221}}$ (4) $\frac{5}{11}$

Ans. **(1)**

Sol.



Let $\vec{p}=2\hat{i}-14\hat{j}+5\hat{k}$

and $\vec{q}=-2\hat{i}-4\hat{j}+7\hat{k}$

$$\vec{p}\times\vec{q}=\begin{vmatrix} i & j & k \\ 2 & -14 & 5 \\ -2 & -4 & 7 \end{vmatrix}$$

$$= \hat{i}(-98+20) - \hat{j}(14+10) + \hat{k}(-8-28)$$

$$= -78\hat{i} - 24\hat{j} - 36\hat{k}$$

$$\overrightarrow{AB} = -\hat{i} + 2\hat{j}$$

$$\text{S.D.} = \frac{|\overrightarrow{AB}.(\bar{p}\times\bar{q})|}{|\bar{p}\times\bar{q}|} = \frac{|78-48|}{\sqrt{(78)^2+(24)^2+(36)^2}} = \frac{30}{\sqrt{7956}}$$

$$= \frac{30}{6\sqrt{221}} = \frac{5}{\sqrt{221}}$$

21. If $F : (-\infty, -1] \to (a, b]$ is defined as $f(x) = e^{x^3-3x+1}$ such that F is both one–one and onto then the distance from a point $P(2a + 4, b + 2)$ to curve $x + ye^{-3} - 4 = 0$ is

(1) $\sqrt{e^3+2}$ (2) $\frac{e^3+2}{\sqrt{e^3+1}}$ (3) $\frac{e^3+2}{\sqrt{e^6+1}}$ (4) e

Ans. **(3)**

Sol. $f'(x) = e^{(x^3-3x+1)}.3(x-1)(x+1)$

$$a = \lim_{x\to-\infty} e^{x^3-3x+1} = 0$$

$$b = f(-1) = e^{-1+3+1} = e^3$$

$P(2a + 4, b + 2) \equiv (4, 2 + e^3)$

$$\text{Distance} = \frac{|4+(2+e^3)e^{-3}-4|}{\sqrt{1+e^{-6}}} = \frac{e^3+2}{\sqrt{1+e^6}}$$