| JEE–MAIN EXAMINATION – JANUARY, 2023 | |
|---|---|
| (Held On Thursday 25th January, 2023) | TIME : 9 : 00 AM to 12 : 00 NOON |

# Physics

## SECTION - A

**1.** Match List I with List II

| List I | List II |
|---|---|
| A. Surface tension | I. $kgm^{-1}\ s^{-1}$ |
| B. Pressure | II. $kgms^{-1}$ |
| C. Viscosity | III. $kgm^{-1}\ s^{-2}$ |
| D. Impulse | IV. $kgs^{-2}$ |

Choose the correct answer from the options given below:

(1) A-II , B-I , C-III , D-IV
(2) A-IV, B-III , C-I , D-II
(3) A-III , B-IV, C-I , D-II
(4) A-IV , B-III , C-II , D-I

**Sol.** 2

Surface tension (S) = $\frac{F}{l} \rightarrow kg\frac{M}{S^2}.\frac{1}{M} \rightarrow Kg\ s^{-2}$

Impulse (J) = $\int Fdt \quad \rightarrow N - S$

$\rightarrow Kg\ ms^{-2}\ .\ S$

$\rightarrow Kg\ ms^{-1}$

Pressure (P) = $\frac{F}{A} \rightarrow Kgms^{-2},\ m^{-2}$

$\rightarrow Kg\ ms^{-1}\ s^{-2}$

Viscocity ( $\eta$ ) = $\frac{F}{6\pi rv}$

$\rightarrow \frac{kg\,ms^{-2}}{m.ms^{-1}}$

$\rightarrow kg\ m^{-1}\ s^{-1}$

**2.** The ratio of the density of oxygen nucleus ( ${}^{16}_{8}O$) and helium nucleus ( ${}^{4}_{2}He$) is

(1) 4:1 (2) 2: 1(3) 1: 1(4) 8: 1

**Sol.** 3

$\rho = \frac{M}{V}$ and $V = \frac{4}{3}\pi r^3$ when r = $R_0A^{1/3}$

$\therefore\ \rho = \frac{M}{\frac{4}{3}\pi R_0^3 A}$

$\therefore\ \rho \propto \frac{M}{A}$

$\frac{\rho_o}{\rho_{He}} = \frac{M_o}{A_o} \times \frac{A_{He}}{M_{He}} = \frac{16}{8} \times \frac{2}{4} = 1$

**3.** The root mean square velocity of molecules of gas is

(1) Inversely proportional to square root of temperature $\left(\sqrt{\frac{1}{T}}\right)$

(2) Proportional to square of temperature $(T^2)$

(3) Proportional to temperature $(T)$

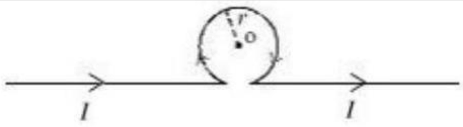
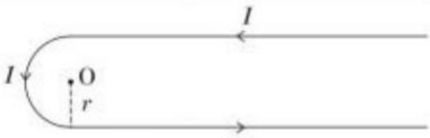
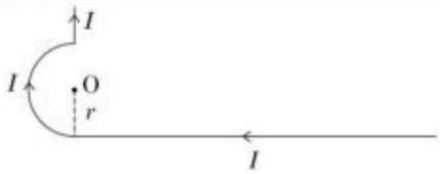
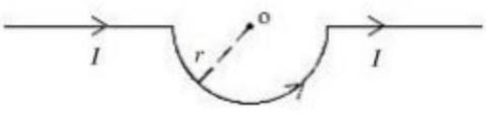
(4) Proportional to square root of temperature $(\sqrt{T})$

**Sol.** 4

$$V_{rms} = \sqrt{\frac{3RT}{M_0}}$$

$$\therefore\ V_{rms} \propto \sqrt{T}$$

4. Match List I with List II

| List I (Current configuration) | List II (Magnitude of Magnetic Field at point O) |
|---|---|
| A.  | I. $B_0 = \frac{\mu_0 I}{4\pi r}[\pi + 2]$ |
| B.  | II. $B_0 = \frac{\mu_0}{4}\frac{I}{r}$ |
| C.  | III. $B_0 = \frac{\mu_0 I}{2\pi r}[\pi - 1]$ |
| D.  | IV. $B_0 = \frac{\mu_0 I}{4\pi r}[\pi + 1]$ |

Choose the correct answer from the options given below :

(1) A-III, B-I, C-IV, D-II

(2) A-I, B-III, C-IV, D-II

(3) A-III, B-IV, C-I, D-II

(4) A-II, B-I, C-IV, D-III

**Sol.** 1

(A) $B = \frac{\mu_0 I}{4\pi r} \times 2 - \frac{\mu_0 I}{2r}$

$$= \frac{\mu I}{2r}\left(\frac{1}{\pi} - 1\right)$$

$$= \frac{\mu I}{2\pi r}(1-\pi) \odot$$

$$= \frac{\mu I}{2\pi r}(\pi-1) \otimes$$

(B) $B = \frac{\mu_0 I}{4\pi r}\times\pi + \frac{\mu_0 I}{4\pi r}\times 2$

$$= \frac{\mu_0 I}{4\pi r}(\pi+2) \odot$$

(C) $B = \frac{\mu_0 I}{4\pi r}.\pi + 0 + \frac{\mu_0 I}{4\pi r}$

$$= \frac{\mu_0 I}{4\pi r}(\pi+1) \otimes$$

(D) $B = \frac{\mu_0 I}{4r} \odot$

**5.** A message signal of frequency 5kHz is used to modulate a carrier signal of frequency 2MHz. The bandwidth for amplitude modulation is:

(1) 20 kHz (2) 5kHz (3) 10kHz (4) 2.5kHz

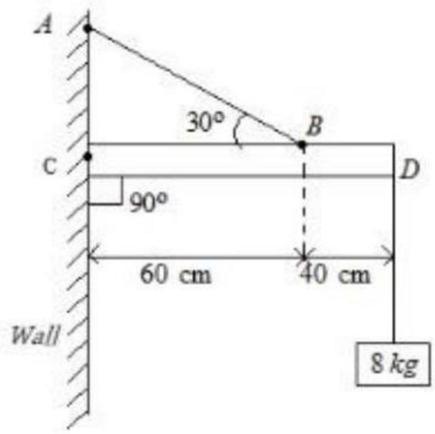
**Sol.** 3

Bandwidth = 2 × highest of base band frequency

= 2 × 5 = 10 kHZ

**6.** An object of mass 8 kg hanging from one end of a uniform rod CD of mass 2 kg and length 1m pivoted at its end C on a vertical walls as shown in figure. It is supported by a cable AB such that the system is in equilibrium. The tension in the cable is:

(Take g = 10 m/s$^2$)




(1) 90 N (2) 30 N (3) 300 N (4) 240 N

**Sol.** 3

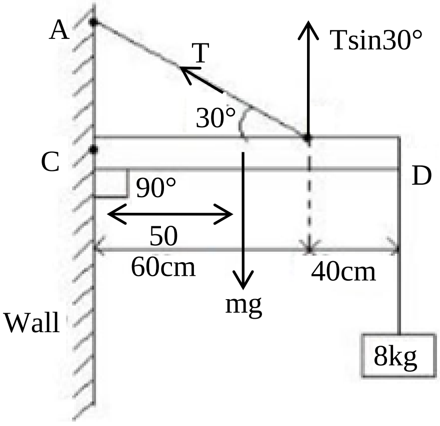
The rod is in equilibrium. So, net torque about any point will be zero.

$\tau_c = 0$

Mg × 50 + 80 × 100 = Tsin30° × 30

$$20 \times 50 + 80 \times 100 = \frac{T}{2} \times 60$$

$$T = \frac{900}{3} = 300\text{ N}$$




**7.** Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R

**Assertion A:** Photodiodes are used in forward bias usually for measuring the light intensity.

**Reason R:** For a p-n junction diode, at applied voltage V the current in the forward bias is more than the current in the reverse bias for $|V_z| > \pm V \geq |V_0|$ where $V_0$ is the threshold voltage and $V_z$ is the breakdown voltage.

In the light of the above statements, choose the correct answer from the options given below

(1) Both $A$ and $R$ are true and $R$ is correct explanation $A$

(2) $A$ is false but $R$ is true

(3) Both A and R are true but R is NOT the correct explanation A

(4) A is true but R is false

**Sol.** 2

Photo diodes are not used in forward bias.

**8.** In an LC oscillator, if values of inductance and capacitance become twice and eight times, respectively, then the resonant frequency of oscillator becomes $x$ times its initial resonant frequency $\omega_0$. The value of $x$ is:

(1) 4 (2) 1/16 (3) 16 (4) ¼

**Sol.** 4

$$\omega = \frac{1}{\sqrt{LC}}$$

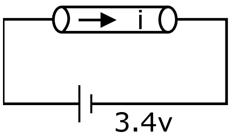
$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{L_2C_2}{L_1C_1}} = \sqrt{\frac{2L.8C}{L.C}} = 4$$

$$\omega_2 = \frac{\omega_1}{4} = \frac{\omega_0}{4}$$

**9.** A uniform metallic wire carries a current 2A, when 3.4 V battery is connected across it. The mass of uniform metallic wires is $8.92 \times 10^{-3}$ kg density is $8.92 \times 10^{3}$ kg/m$^3$ and resistivity is $1.7 \times 10^{-8}$ $\Omega$ – m. The length of wire is:

(1) $l = 10$ m (2) $l = 100$ m (3) $l = 5$ m (4) $l = 6.8$ m

**Sol.** 1




Given, i = 2A

v = 3.4 v

v = iR

$$R = \frac{v}{i} = \frac{3.4}{2} = 1.7\Omega$$

$$\text{volume} = \frac{\text{mass}}{\text{Density}} = \frac{8.92 \times 10^{-3}}{8.92 \times 10^{3}} m^3 = 10^{-6} m^3$$

$\Rightarrow A\ell = 10^{-6}$ m$^3$ ____ (i)

$$R = \frac{\rho\ell}{A}$$

$$\Rightarrow \frac{\rho}{R} = \frac{A}{\ell}$$

$$\frac{1.7 \times 10^{-8}}{1.7} = \frac{A}{\ell}$$

$\frac{A}{\ell} = 10^{-8}$ ____ (ii)

$$\frac{\text{eq(i)}}{\text{eq(ii)}}$$

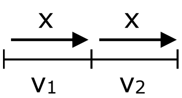
$\ell^2 = 10^2$

$\ell = 10$ m

**10.** A car travels a distance of ' $x$ ' with speed $v_1$ and then same distance ' $x$ ' with speed $v_2$ in the same direction. The average speed of the car is:

(1) $\frac{2v_1v_2}{v_1+v_2}$ (2) $\frac{2x}{v_1+v_2}$ (3) $\frac{v_1v_2}{2(v_1+v_2)}$ (4) $\frac{v_1+v_2}{2}$

**Sol.** 1




$$v_{avg} = \frac{\text{total Distance}}{\text{Total Time}}$$

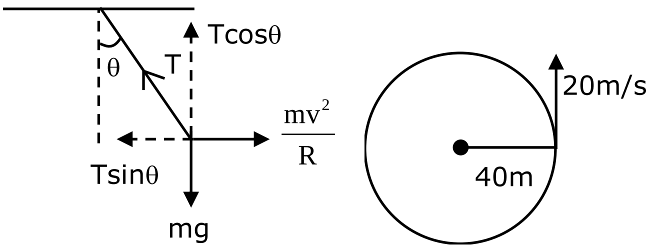
$$= \frac{2x}{\frac{x}{v_1} + \frac{x}{v_2}}$$

$$= \frac{2v_1v_2}{v_1 + v_2}$$

**11.** A car is moving with a constant speed of 20 m/s in a circular horizontal track of radius 40m. A bob is suspended from the roof of the car by a massless string. The angle made by the string with the vertical will be: (Take g = 10 m/s$^2$)

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

**Sol.** 3




$T\cos\theta = mg$ __ (i)

$T\sin\theta = \frac{mv^2}{R}$ __ (ii)

$$\frac{eq(i)}{eq(ii)}; \frac{\cos\theta}{\sin\theta} = \frac{gR}{v^2}$$

$$\tan\theta = \frac{v^2}{Rg} = \frac{400}{40 \times 10} = 1$$

$$\theta = \frac{\pi}{4}$$

**12.** A bowl filled with very hot soup cools from 98°C to 86°C in 2 minutes when the room temperature is 22°C. How long it will take to cool from 75°C to 69°C?

(1) 1 minute (2) 1.4 minutes (3) 0.5 minute (4) 2 minutes

**Sol.** 2

According to NLC,

$$-\frac{d\theta}{dt} = k\theta$$

$$\frac{12}{2} = K\left(\frac{98+86}{2} - 22\right)$$

$\Rightarrow 6 = K(92 - 22) = K \times 70$

$\Rightarrow K = \frac{6}{70}$ …. (i)

Now, $\frac{6}{t_2} = \frac{6}{70}\left(\frac{75+69}{2} - 22\right)$

$= \frac{6}{70} \times (72 - 22)$

$t_2 = \frac{6 \times 70}{6 \times 50}$

$\frac{7}{5} = 1.4 \min$

**13.** A solenoid of 1200 turns is wound uniformly in a single layer on a glass tube 2m long and 0.2m in diameter. The magnetic intensity at the center of the solenoid when a current of 2A flows through it is?

(1) $2.4 \times 10^3$ A $m^{-1}$ (2) $1.2 \times 10^3$ A $m^{-1}$

(3) $2.4 \times 10^{-3}$ A $m^{-1}$ (4) 1 A $m^{-1}$

**Sol.** 2

$B = \mu_0 nI$ and $n = \frac{1200}{2} = 600$

Magnetic field intensity $H = \frac{B}{\mu_0} = nI = 600 \times 2 = 1200 = 1.2 \times 10^3$ A $m^{-1}$

**14.** In Young's double slits experiment, the position of $5^{th}$ bright fringe from the central maximum is 5cm. The distance between slits and screen is 1m and wavelength of used monochromatic light is 600 nm. The separation between the slits is:

(1) $48\mu m$ (2) $36\mu m$ (3) $12\mu m$ (4) $60\mu m$

**Sol.** 4

$5\beta = 5$ cm

$\Rightarrow \beta = 1$cm

$\frac{\lambda D}{d} = 1 \text{ cm} = \frac{1}{100}$ m

$\Rightarrow d = 600 \times 10^{-9} \times 100 \times 1$

$= 60 \times 10^{-6}$ m

$= 60\ \mu m$

**15.** An electromagnetic wave is transporting energy in the negative z direction. At a certain point and certain time the direction of electric field of the wave is along positive y direction. What will be the direction of the magnetic field of the wave at the point and instant?

(1) Negative direction of $y$ (2) Positive direction of $z$

(3) Positive direction of $x$ (4) Negative direction of $x$

**Sol.** 3

$\vec{B} \perp r \vec{E}$ and Direction of propagation is given by

$\vec{E} \times \vec{B}$.

$\boxed{\hat{j} \times \hat{i} = -k}$

**16.** A parallel plate capacitor has plate area 40 $cm^2$ and plates separation 2mm. The space between the plates is filled with a dielectric medium of a thickness 1 mm and dielectric constant 5. The capacitance of the system is:

(1) $24\varepsilon_0$ F (2) $\frac{10}{3}\varepsilon_0$ F (3) $\frac{3}{10}\varepsilon_0$ F (4) $10\varepsilon_0$ F

**Sol.** 2

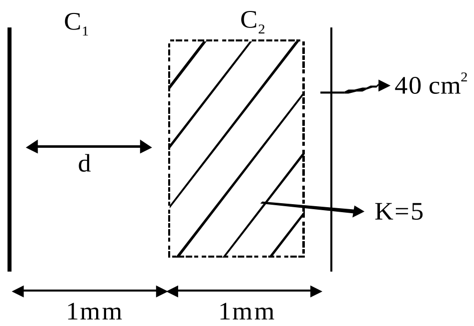
$C_1 = \frac{\varepsilon_0 A}{d} = C_0$

$C_2 = K\frac{\varepsilon_0 A}{d} = K\varepsilon_0$

$Ceq = \frac{C_1C_2}{C_1+C_2} = \frac{C_0 \times KC_0}{(K+1)\varepsilon_0} = \frac{KC_0}{K+1}$

$= \frac{5\times\varepsilon_0\times 40\times 10^{-4}}{1\times 10^{-3}\times 6}$

$= \frac{10}{3}\varepsilon_0 F$




**17.** Assume that the earth is a solid sphere of uniform density and a tunnel is dug along its diameter throughout the earth. It is found that when a particle is released in this tunnel, it executes a simple harmonic motion. The mass of the particle is 100 g. The time period of the motion of the particle will be (approximately)

(Take $g = 10\ m\ s^{-2}$, radius of earth $= 6400$ km )

(1) 12 hours (2) 1 hour 40 minutes

(3) 24 hours (4) 1 hour 24 minutes

**Sol.** 4

Inside earth, force is given by $F = -\frac{GM_e mx}{R_e^3}$

And $g_0 (on\ surface\ of\ earth) = \frac{GM_e}{R_e^2}$

$\therefore F = -\frac{g_0 m}{R_e}x$

$\Rightarrow a = -\frac{g_0}{R_e}x$

$\omega = \sqrt{\frac{g_0}{R_e}}$

$\Rightarrow T = 2\pi\sqrt{\frac{R_e}{g_0}} = 2\pi\sqrt{\frac{6400\times 10^3}{10}} = 2\times 3.13\times 8\times 10^2 sec = 5024\ sec = 1.4\ hr$

$T = 1.4\ hr = 1\ hr\ 24\ minutes$

**18.** Electron beam used in an electron microscope, when accelerated by a voltage of 20kV, has a de – Broglie wavelength of $\lambda_0$. If the voltage is increased to 40kV, then the de-Broglie wavelength associated with the electron beam would be:

(1) $3\lambda_0$ (2) $\frac{\lambda_0}{2}$ (3) $\frac{\lambda_0}{\sqrt{2}}$ (4) $9\lambda_0$

**Sol.** 3

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}}$

$\Rightarrow \lambda\ \alpha \frac{1}{\sqrt{V}}$

$\Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{V_2}{V_1}}$

$\Rightarrow \frac{\lambda_0}{\lambda_2} = \sqrt{\frac{40}{20}} = \sqrt{2}$

$\Rightarrow \lambda_2 = \frac{\lambda_0}{\sqrt{2}}$

**19.** A Carnot engine with efficiency 50% takes heat from a source at 600 K. In order to increase the efficiency to 70%, keeping the temperature of sink same, the new temperature of the source will be :

(1) 300 K (2) 900 K (3) 1000 K (4) 360 K

**Sol.** 3

$\eta = 1 - \frac{T_L}{T_H}$

$0.5 = 1 - \frac{T_L}{600} \Rightarrow T_L = (1 - 0.5) \times 600\,K = 300\,K$

Now $0.7 = 1 - \frac{300}{T_2}$

$\frac{300}{T_2} = 0.3 \Rightarrow T_2 = \frac{300}{0.3} = 1000\,K$

**20.** T is the time period of simple pendulum on the earth's surface. Its time Period becomes x T when taken to a height R (equal to earth's radius) above the earth's surface. Then, the value of x will be:

(1) 4 (2) 2 (3) $\frac{1}{4}$ (4) $\frac{1}{2}$

**Sol.** 2

$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$

And $g_{eff\ above\ earth's\ surface} = \frac{GM}{(R+h)^2} = \frac{GM}{4R^2} = \frac{g_0}{4}$

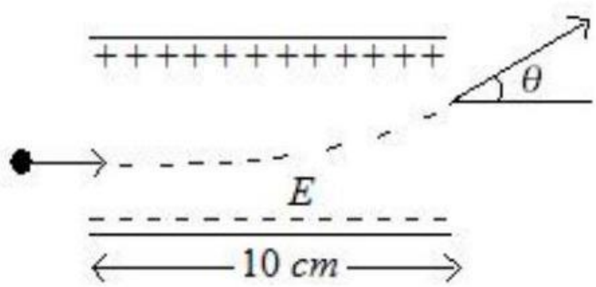
Now $\frac{T_1}{T_2} = \sqrt{\frac{\frac{g_0}{4}}{g_0}} = \frac{1}{2}$

$T_2 = 2T_1$

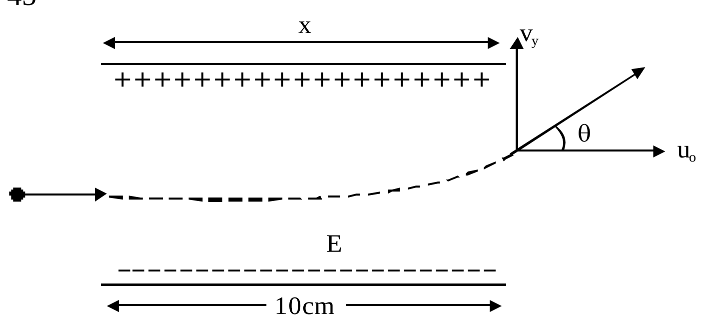
$\therefore x = 2$

## SECTION – B

**21.** A uniform electric field of 10 N/C is created between two parallel charged pates (as shown in figure). An electron enters the field symmetrically between the plates with a kinetic energy 0.5eV. The length of each pate is 10 cm. The angle (θ) of deviation of the path of electron as it comes out of the field is ______ (in degree).




**Sol.** $45^0$



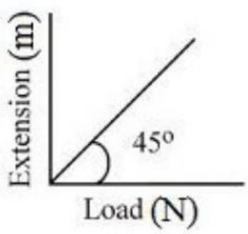

Force due to electric field is given by $\vec{F} = q\vec{E}$

$\therefore F = eE$
$\Rightarrow a = \frac{eE}{m}$
The electron will take a parabolic path i.e., projectile motion.
Here, $s_x = 10cm = 0.1m$
$\therefore t = \frac{0.1}{u_x} ----(i)$
Now $v_y = u_y + a_y t$
$\Rightarrow v_y = 0 + \frac{eE}{m} \times \frac{0.1}{u_x} ----(ii)$
Also $KE = \frac{1}{2} mv^2 = \frac{1}{2} mu_x^2$
$mu_x^2 = 2 \times KE = 2 \times 0.5e = e ----(iii)$
From eq (i),(ii) and (iii), $tan\theta = \frac{v_y}{u_x} = \frac{eE}{m} \times \frac{0.1}{u_x} \times \frac{1}{u_x} = \frac{0.1eE}{mu_x^2} = \frac{0.1eE}{e} = 0.1 \times 10 = 1$
$\Rightarrow tan\theta = 1$
$\therefore \theta = 45^0$

**22.** The wavelength of the radiation emitted is $\lambda_0$ when an electron jumps from the second excited state to the first excited state of hydrogen atom. If the electron jumps from the third excited state to the second orbit of the hydrogen atom, the wavelength of the radiation emitted will be $\frac{20}{x}\lambda_0$. The value of x is _________.

**Sol.** 27
Bohr's energy is given by $E = -13.6 \times \frac{1}{n^2}$ for hydrogen atom.
And $E = \frac{hc}{\lambda}$
For 1st condition, $\frac{hc}{\lambda_0} = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) = 13.6 \times \frac{5}{36} ----- (i)$
For 2nd condition, $\frac{hc}{\lambda} = 13.6\left(\frac{1}{4} - \frac{1}{16}\right) = 13.6 \times \frac{3}{16} ----- (ii)$
Dividing equation (i) by (ii),
$\frac{\lambda}{\lambda_0} = \frac{5}{36} \times \frac{16}{3} = \frac{20}{27}$
$\Rightarrow \lambda = \frac{20}{27}\lambda_0$
$\Rightarrow n = 27$

**23.** As shown in the figure, in an experiment to determine Young's modulus of a wire, the extension-load curve is plotted. The curve is a straight line passing through the origin and makes an angle of 45° with the load axis. The length of wire is 62.8cm and its diameter is 4 mm. The Young's modulus is found to be x × $10^4$ $Nm^{-2}$.
The value of $x$ is __________

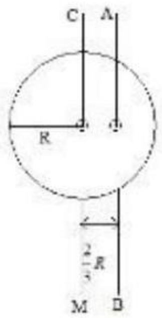



**Sol.** 5
From graph, $tan45^0 = \frac{\Delta l}{F}$
$\Rightarrow \frac{\Delta l}{F} = 1 ----(i)$
Also, Young's modulus is given by $Y = \frac{Fl}{A\Delta l} = \frac{l}{A} \times \frac{F}{\Delta l} = \frac{l}{A} \times 1$
$\therefore Y = \frac{l}{A} = \frac{62.8\times10^{-2}}{\pi\times4\times10^{-6}} = 5 \times 10^4\ Nm^{-2}$
$\therefore x = 5$

**24** $I_{CM}$ is the moment of inertia of a circular disc about an axis (CM)passing through its center and perpendicular. To the plane of disc. $I_{AB}$ is it's moment of inertia about an axis AB perpendicular to plane and parallel to axis $CM$ at a distance $\frac{2}{3}R$ from center.

Where R is the radius of the disc. The ratio of $I_{AB}$ and $I_{CM}$ is $x:9$.

The value of $x$ is ___________




**Sol.** 17

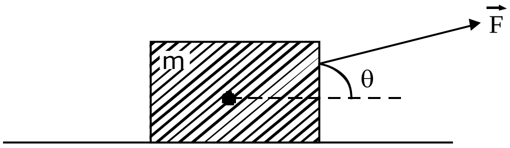
$I_{CM} = \frac{MR^2}{2}$

$I_{AB} = \frac{MR^2}{2} + M\left(\frac{2}{3}R\right)^2 = \frac{MR^2}{2} + \frac{4MR^2}{9} = \frac{17MR^2}{18}$

As per question, $\frac{I_{AB}}{I_{CM}} = \frac{17}{9}$

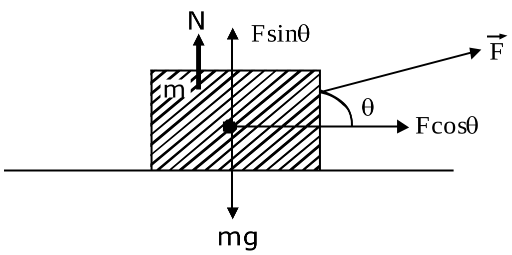
$\therefore x = 17$

**25.** An object of mass 'm' initially at rest on a smooth horizontal plane starts moving under the action of force F = 2N. In the process of its linear motion, the angle θ (as shown in figure) between the direction of force and horizontal varies as θ = kx, where k is constant and x is the distance covered by the object from the initial positon. The expression of kinetic energy of the object will be $E = \frac{n}{k}\sin\theta$, The value of n is _______.




Smooth horizontal surface

**Sol.** 2




$F_x = 2coskx$

$F_y = 2sinkx - mg$

According to Work Energy Theorem, $\Delta K = \Delta W$

Taking motion only along horizontal direction(X) i.e., linear motion as mentioned in question, $\Delta K = \int_0^x F_x\,dx$

$K_f - K_i = \int_0^x 2coskx\,dx = \frac{2sinkx}{k}$

Hence $K_i = 0, \therefore K_f = \frac{2sinkx}{k}$

$\therefore n = 2$

**26.** An LCR series circuit of capacitance 62.5nF and resistance of 50Ω, is connected to an A.C. source of frequency 2.0kHz. For maximum value of amplitude of current in circuit, the value of inductance is ________ mH. Take $\pi^2 = 10$)

**Sol.** 100

At maximum current, there will be condition of resonance.

So, $\omega = \frac{1}{\sqrt{LC}}$

$\Rightarrow L = \frac{1}{\omega^2 C} = \frac{1}{4\times\pi^2\times4\times10^6\times62.5\times10^{-9}} H = 0.1H = 100mH$

**27.** The distance between two consecutive points with phase difference of $60°$ in a wave of frequency $500$ Hz is $6.0$ m. The velocity with which wave is traveling is _________ km/s

**Sol.** 18

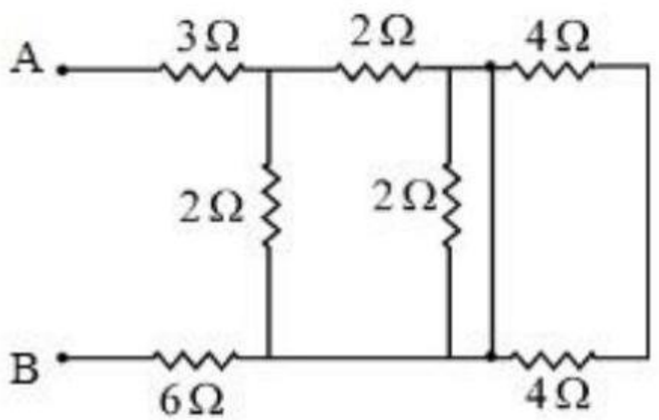
$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$\Rightarrow \frac{\pi}{3} = \frac{2\pi}{\lambda} \times 6$

$\Rightarrow \lambda = 36m$

Now $v = f\lambda = 500 \times 36\ m/s = 18000\ m/s = 18km/s$

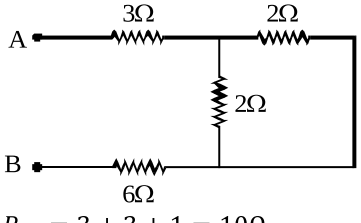
**28.** In the given circuit, the equivalent resistance between the terminal A and B is Ω.




**Sol.** 10

Due to short circuit, 3 resistances get vanished from the circuit.

The circuit is




$R_{eq} = 3 + 3 + 1 = 10\Omega$

**29.** If $\vec{P} = 3\hat{\imath} + \sqrt{3}\hat{\jmath} + 2\hat{k}$ and $\vec{Q} = 4\hat{\imath} + \sqrt{3}\hat{\jmath} + 2.5\hat{k}$ then, The unit vector in the direction of $\vec{P} \times \vec{Q}$ is $\frac{1}{x}(\sqrt{3}i + \hat{\jmath} - 2\sqrt{3}\hat{k})$. The value of $x$ is

**Sol.** 4

Let $\vec{C} = \vec{P} \times \vec{Q} = 3\sqrt{3}\hat{k} - 7.5\hat{\jmath} - 4\sqrt{3}\hat{k} + 2.5\sqrt{3}\hat{\imath} + 8\hat{\jmath} - 2\sqrt{3}\hat{\imath}$

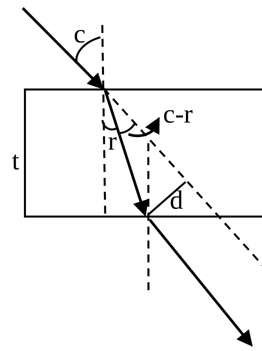
$= \frac{1}{2}(\sqrt{3}\hat{\imath} + \hat{\jmath} - 2\sqrt{3}\hat{k})$

$|\vec{C}| = \frac{1}{2}\sqrt{3+1+12} = \frac{1}{2} \times 4 = 2$

$\therefore \hat{C} = \frac{\vec{C}}{|\vec{C}|} = \frac{1}{4}(\sqrt{3}\hat{\imath} + \hat{\jmath} - 2\sqrt{3}\hat{k})$

**30.** A ray of light is incident from air on a glass plate having thickness $\sqrt{3}$ cm and refractive index $\sqrt{2}$. The angle of incidence of a ray is equal to the critical angle for glass-air interface. The lateral displacement of the ray when it passes through the plate is ________ $\times 10^{-2}$ cm. (given sin 15° = 0.26)

**Sol.** 52




$sinc = \frac{1}{\sqrt{2}} \Rightarrow c = 45^0$

Using Snell's law on 1st surface, $sinc = \sqrt{2}\, sinr$

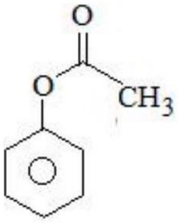
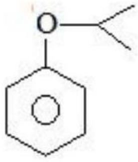
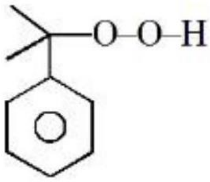
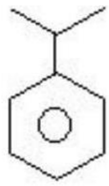
$\Rightarrow sinr = \frac{1}{2} \Rightarrow r = 30^0$

$d = tsecr \times \sin(c - r) = \sqrt{3} \times \frac{2}{\sqrt{3}} \times 0.26 = 0.52cm = 52 \times 10^{-2}cm$

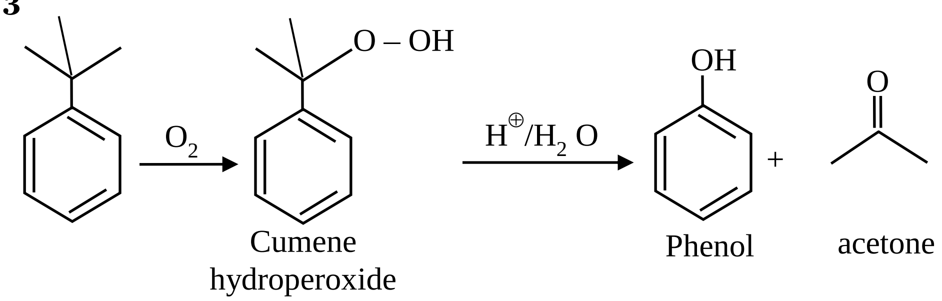
# Chemistry

## SECTION - A

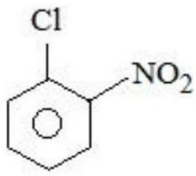
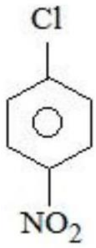
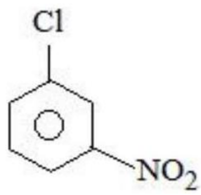
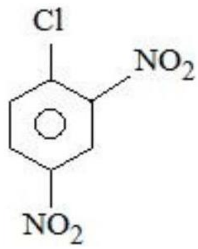
31. In the cumene to phenol preparation in presence of air, the intermediate is

(1)  (2)  (3)  (4) 

Sol. **3**

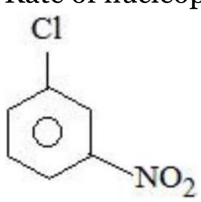



32. The compound which will have the lowest rate towards nucleophilic aromatic substitution on treatment with $OH^-$ is

(1)  (2)  (3)  (4) 

Sol. **3**

Rate of nucleophilic aromatic substitution decrease by $e^-$ withdrawing group



$-NO_2$ of meta shows –I effect which is less dominating than –M

33. Match List I with List II

| LIST I<br>Elements | | LIST II<br>Colour imparted to the flame | |
|---|---|---|---|
| A. | K | I. | Brick Red |
| B. | Ca | II. | Violet |
| C. | Sr | III | Apple Green |
| D. | Ba | IV. | Crimson Red |

Choose the correct answer from the options given below:

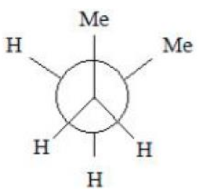
(1) A-II, B-I, C-III, D-IV
(2) A-II, B-I, C-IV, D-III
(3) A-IV, B-III, C-II, D-I
(4) A-II, B-IV, C-I, D-III

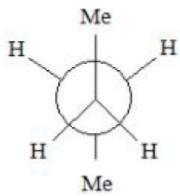
Sol. **2**

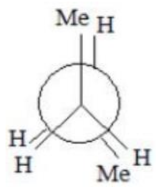
Flame Test.

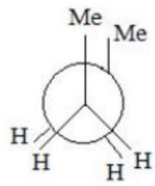
| **Metals** | **Colour of flame test** |
|---|---|
| K | Violet |
| Ca | Brick Red |
| Sr | Crimson Red |
| Ba | Apple Green |

34. Which of the following conformations will be the most stable ?

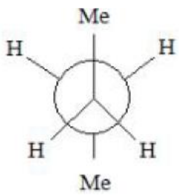
(1) 


(2) 


(3) 

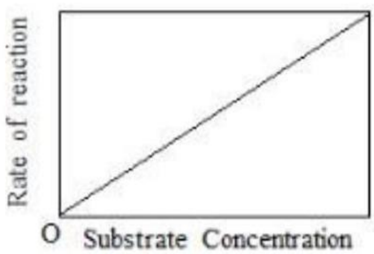

(4) 

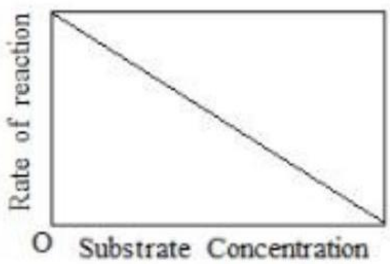

Sol. **2**

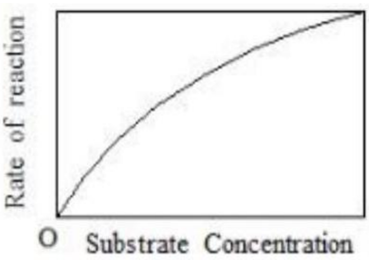


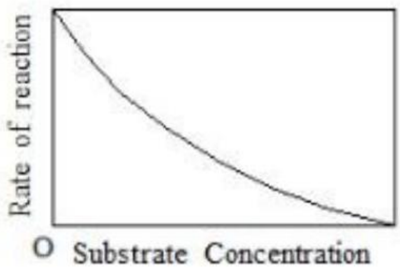

Anti position highly stable (bulky group maximum distance)

35. The variation of the rate of an enzyme catalyzed reaction with substrate concentration is correctly represented by graph

(a) 


(b) 


(c) 


(d) 


(1) (b) (2) (a) (3) (d) (4) (c)

Sol. **4**

Fact base.

36. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason **R** :

**Assertion A :** Acetal / Ketal is stable in basic medium.

**Reason R :** The high leaving tendency of alkoxide ion gives the stability to acetal/ ketal in basic medium.

In the light of the above statements, choose the correct answer from the options given below :

(1) A is true but R is false

(2) A is false but R is true

(3) Both A and R are true but R is NOT the correct explanation of A

(4) Both A and R are true and R is the correct explanation of A

Sol. **1**

Acetal and ketals are basically ether hence they must be stable in basic medium but should break down in acidic medium.

Hence assertion is correct.

Alkoxide ion ($RO^-$) is not considered a good leaving group hence reason must be false.

37. A cubic solid is made up of two elements X and Y. Atoms of X are present on every alternate corner and one at the center of cube. $Y$ is at $\frac{1}{3}$rd of the total faces. The empirical formula of the compound is

(1) $XY_{2.5}$ (2) $X_2Y_{1.5}$ (3) $X_{2.5}Y$ (4) $X_{1.5}Y_2$

Sol. **4**

Number of X-atom per unit cell = $1+4\times\frac{1}{8} = \frac{3}{2}$

Number of Y-atoms per unit cell = $2\times\frac{1}{2}=1$

$\therefore$ Empirical formula of the solid is $X_3Y_2$.

38. Match the List-I with List-II

| List-I | List-II |
|---|---|
| Cations | Group reagents |
| A → $Pb^{2+}, Cu^{2+}$ | i) $H_2$ S gas in presence of dilute HCl |
| B → $Al^{3+}, Fe^{3+}$ | ii) $(NH_4)_2CO_3$ in presence of $NH_4OH$ |
| C → $Co^{2+}, Ni^{2+}$ | iii) $NH_4OH$ in presence of $NH_4Cl$ |
| D → $Ba^{2+}, Ca^{2+}$ | iv) $H_2$ S in presence of $NH_4OH$ |

Correct match is -

(1) A → iii, B → i, C → iv, D → ii

(2) A → i, B → iii, C → ii, D → iv

(3) A → iv, B → ii, C → iii, D → i

(4) A → i, B → iii, C → iv, D → ii

Sol. **4**

| Cations | Group No. | Group reagents |
|---|---|---|
| $Pb^{2+}, Cu^{2+}$ | II | $H_2S + HCl$ |
| $Al^{3+}, Fe^{3+}$ | III | $NH_4Cl + NH_4OH$ |
| $Co^{2+}, Ni^{2+}$ | IV | $NH_4OH + H_2S$ |
| $Ba^{2+}, Ca^{2+}$ | V | $NH_4OH$ , $Na_2CO_3$ |

39. Which of the following statements is incorrect for antibiotics?
(1) An antibiotic must be a product of metabolism.
(2) An antibiotic should promote the growth or survival of microorganisms.
(3) An antibiotic is a synthetic substance produced as a structural analogue of naturally occurring antibiotic.
(4) An antibiotic should be effective in low concentrations.

Sol. **2**
Antibiotic kill or inhibit the growth of microorganism

40. The correct order in aqueous medium of basic strength in case of methyl substituted amines is :
(1) $Me_3\,N > Me_2NH > MeNH_2 > NH_3$
(2) $Me_2NH > MeNH_2 > Me_3\,N > NH_3$
(3) $Me_2NH > Me_3\,N > MeNH_2 > NH_3$
(4) $NH_3 > Me_3\,N > MeNH_2 > Me_2NH$

Sol. **2**
In aqueous medium basic strength is dependent on electron density on nitrogen as well as solvation of cation formed after accepting $H^+$.After considering all these factors overall basic strength order is $Me_2NH > MeNH_2 > Me_3\,N > NH_3$

41. '25 volume' hydrogen peroxide means
(1) 1 L marketed solution contains 25 g of $H_2O_2$.
(2) 1 L marketed solution contains 75 g of $H_2O_2$.
(3) 1 L marketed solution contains 250 g of $H_2O_2$.
(4) 100 mL marketed solution contains 25 g of $H_2O_2$.

Sol. **2**
$25\,V\,H_2O_2$ means : 1 lit of $H_2O_2$ on decomposition give 25 lit of $O_2(g)$ at STP.

$$2H_2O_2(\ell) \rightarrow 2H_2O(\ell) + O_2(g)$$
$$2\left[\frac{25}{22.4}\right]\text{mole} \qquad \left[\frac{25}{22.4}\right]\text{mole}$$

Mass of $H_2O_2 = \frac{2\times 25}{22.4}\times 34 = 75.89\,\text{gram}$.

42. The radius of the $2^{nd}$ orbit of $Li^{2+}$ is $x$. The expected radius of the $3^{rd}$ orbit of $Be^{3+}$ is
(1) $\frac{27}{16}x$ (2) $\frac{4}{9}x$ (3) $\frac{9}{4}x$ (4) $\frac{16}{27}x$

Sol. **1**

$$R = 0.529\times\frac{n^2}{Z}$$

$$r_{Li^{2+}\ n-2} = 0.529\times\frac{(2)^2}{3} = x$$

$$r_{Be^{3+}\ n-3} = 0.529\times\frac{(3)^2}{4}$$

$$\frac{r_{Li^{2+}\ n-2}}{r_{Be^{3+}\ n-3}} = \frac{\frac{r_0\times(2)^2}{3}}{\frac{r_0\times(3)^2}{4}}$$

$$\frac{x}{r_{Be^{3+}\ n-3}} = \frac{16}{27}$$

$$\therefore \quad (r_{Be^{3+}})_{n=3} = \frac{27x}{16}$$

43. Reaction of thionyl chloride with white phosphorus forms a compound [A], which on hydrolysis gives [B], a dibasic acid. [A] and [B] are respectively
(1) $P_4O_6$ and $H_3PO_3$ (2) $PCl_5$ and $H_3PO_4$ (3) $POCl_3$ and $H_3PO_4$ (4) $PCl_3$ and $H_3PO_3$

Sol. **4**

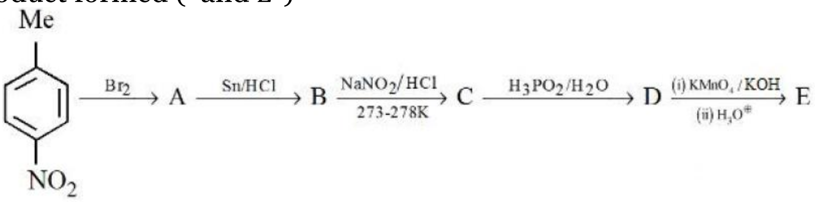
$$P_4 + 8SOCl_2 \longrightarrow \underset{(A)}{4PCl_3} + 4SO_2 + 2S_2Cl_2$$

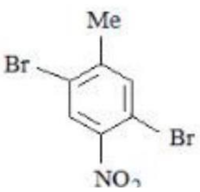
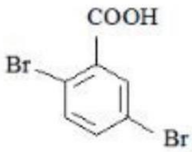
$$PCl_3 + 3H_2O \longrightarrow \underset{(B)}{H_3PO_3} + 3HCl$$

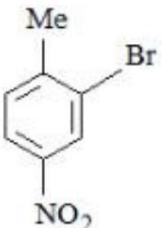
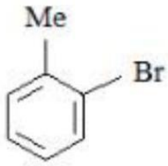
44. Inert gases have positive electron gain enthalpy. Its correct order is
(1) He < Kr < Xe < Ne (2) He < Xe < Kr < Ne
(3) He < Ne < Kr < Xe (4) Xe < Kr < Ne < He

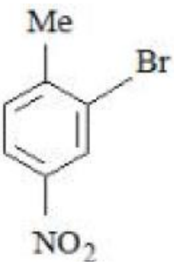
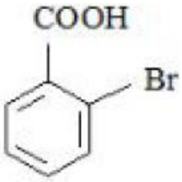
Sol. **2**
Positive electron gain enthalpy. of inert gas is in order of
Ne > Ar = Kr > Xe > He

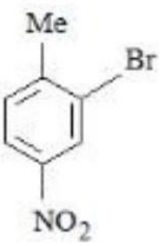
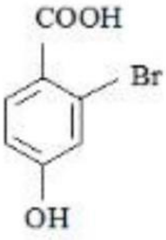
45. Identify the product formed ( and $E$ )



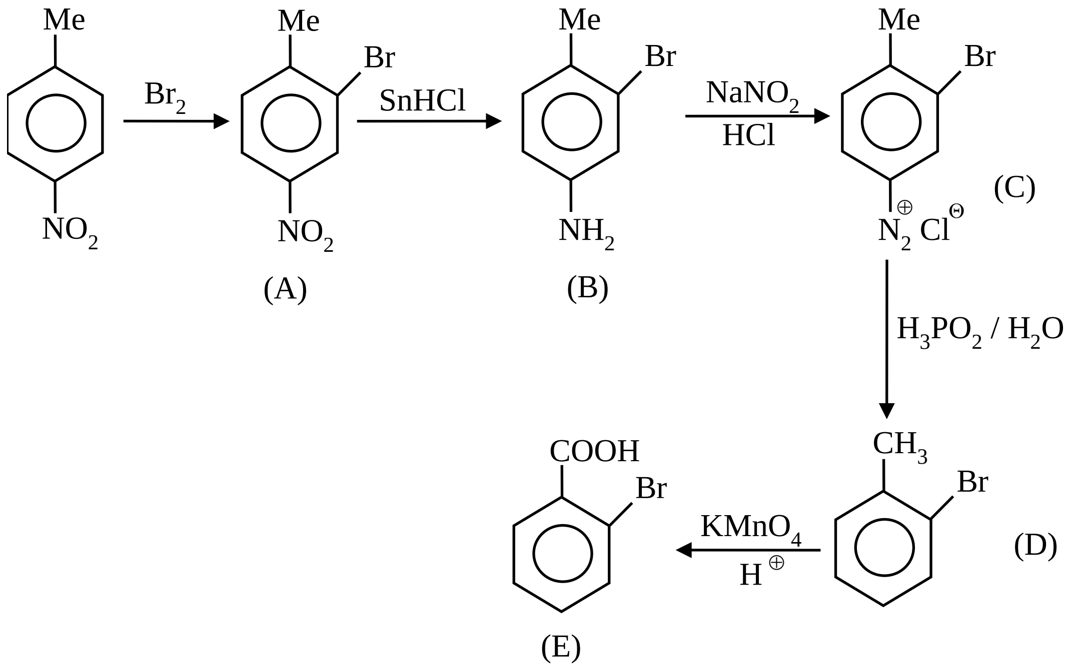
(1) A =  , E = 

(2)  , E = 

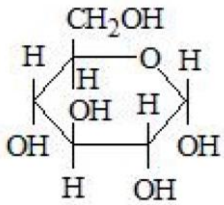
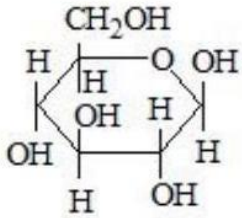
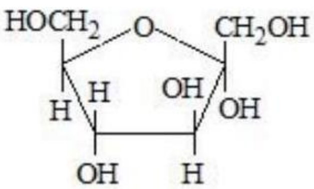
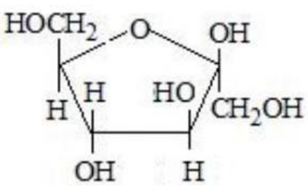
(3)  , E = 

(4)  , E = 

Sol. **3**



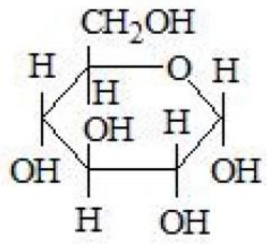

46. Match items of Row I with those of Row II.

| Row I | Row II |
|---|---|
| (A)  | (i) $\alpha-\mathrm{D}-(-)$-Fructofuranose, |
| (B)  | (ii) $\beta-\mathrm{D}-(-)-$ Fructofuranose |
| (C)  | (iii) $\alpha-D-(-)$ Glucopyranose, |
| (D)  | (iv) $\beta-D-(-)$-Glucopyranose |

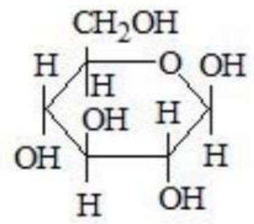
Correct match is

(1) $A \rightarrow i, B \rightarrow ii, C \rightarrow ii, D \rightarrow iv$ (2) $A \rightarrow iv, B \rightarrow iii, C \rightarrow i, D \rightarrow ii$
(3) $A \rightarrow iii, B \rightarrow iv, C \rightarrow ii, D \rightarrow I$ (4) $A \rightarrow iii, B \rightarrow iV, C \rightarrow i, D \rightarrow ii$

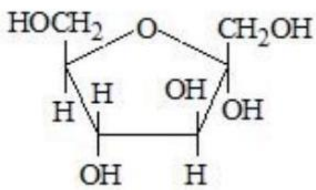
Sol. **4**



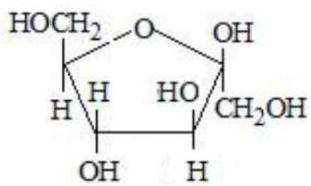
$\alpha - D - (-)$ Glucopyranose



$\beta - D - (-)$-Glucopyranose



$\alpha$ – D – (–)-Fructofuranose



$\beta$ – D – (–) – Fructofuranose

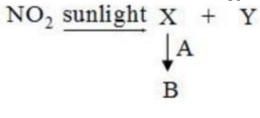
47. Which one of the following reactions does not occur during extraction of copper ?
(1) $2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$ (2) $FeO + SiO_2 \rightarrow FeSiO_3$
(3) $2FeS + 3O_2 \rightarrow 2FeO + 2SO_2$ (4) $CaO + SiO_2 \rightarrow CaSiO_3$

Sol. **4**

$\underset{\text{Impmily}}{CaO} + \underset{\text{Flux}}{SiO_2} \rightarrow \underset{\text{Slog}}{CaSiO_3}$

In metallurgy iron will occur not in metallurgy of Cu.

48. Some reactions of $NO_2$ relevant to photochemical smog formation are

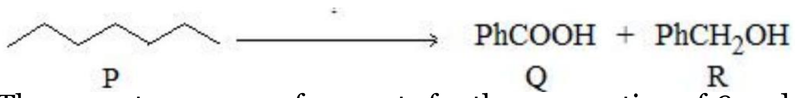


Identify A, B, X and Y
(1) $X = \frac{1}{2}O_2, Y = NO_2, A = O_3, B = O_2$ (2) $X = [O], Y = NO, A = O_2, B = O_3$
(3) $X = N_2O, Y = [O], A = O_3, B = NO$ (4) $X = NO, Y = [O], A = O_2, B = N_2O_3$

Sol. **2**

$$NO_2 \xrightarrow{hv} \underset{(x)}{O} + \underset{(y)}{NO}$$

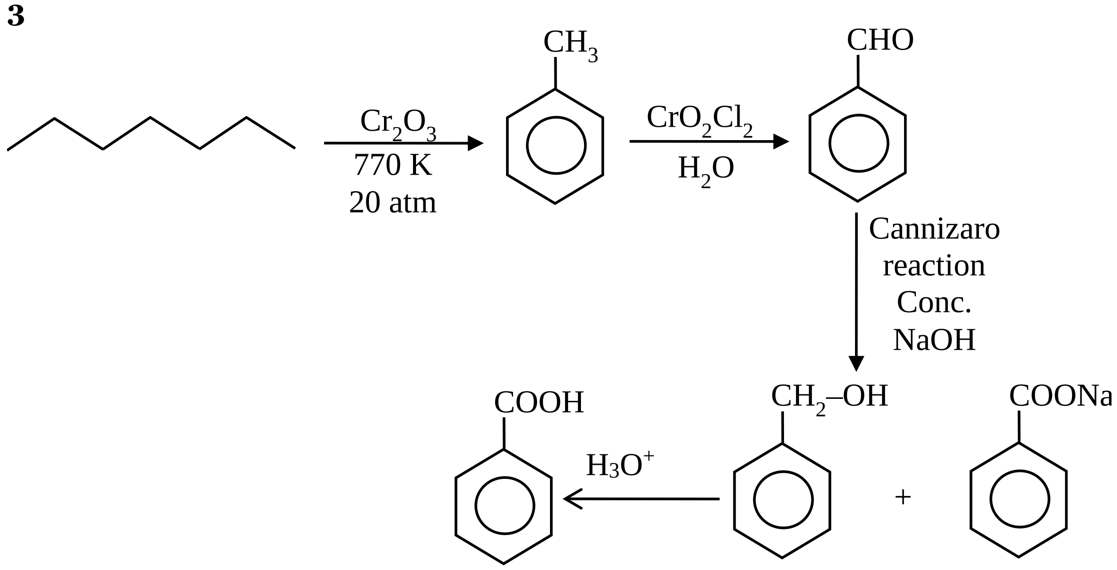
$$O \xrightarrow{O_2\,(A)} O_3\,(B)$$

49. 


The correct sequence of reagents for the preparation of $Q$ and $R$ is :

(1) (i) $CrO_2Cl_2, H_3O^+$; (ii) $Cr_2O_3, 770$ K, 20 atm; (iii) NaOH; (iv) $H_3O^+$
(2) (i) $KMnO_4, OH^-$; (ii) $Mo_2O_3, \Delta$; (iii) NaOH; (iv) $H_3O^+$
(3) (i) $Cr_2O_3, 770$ K, 20 atm; (ii) $CrO_2Cl_2, H_3O^+$; (iii) NaOH; (iv) $H_3O^+$
(4) (i) $Mo_2O_3, \Delta$; (ii) $CrO_2Cl_2, H_3O^+$; (iii) NaOH; (iv) $H_3O^+$

Sol. **3**




50. Compound A reacts with $NH_4Cl$ and forms a compound B. Compound B reacts with $H_2O$ and excess of $CO_2$ to form compound C which on passing through or reaction with saturated NaCl solution forms sodium hydrogen carbonate.
Compound A, B and C, are respectively.

(1) $CaCl_2, NH_3, NH_4HCO_3$
(2) $Ca(OH)_2, NH_4^{\oplus}, (NH_4)_2CO_3$
(3) $CaCl_2, NH_4^{\oplus}, (NH_4)_2CO_3$
(4) $Ca(OH)_2, NH_3, NH_4HCO_3$

Sol. **4**

$$\underset{(A)}{Ca(OH)_2} + 2NH_4Cl \xrightarrow{\Delta} \underset{(B)}{2NH_3} + CaCl_2 + 2H_2O$$

$$\xrightarrow[\text{Excess}]{CO_2 + H_2O} \underset{(C)}{NH_4HCO_3} + NaCl \longrightarrow NH_4Cl + NaHCO_3$$

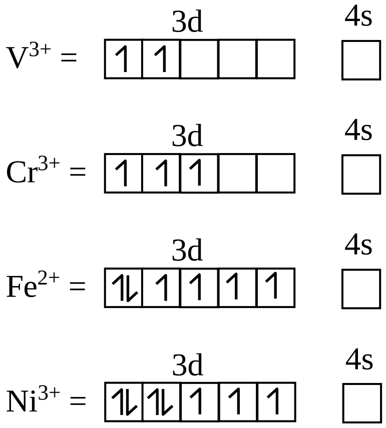
# SECTION - B

51. For the first order reaction $A \rightarrow B$, the half life is 30 min. The time taken for 75% completion of the reaction is_____min. (Nearest integer)
Given : $\log 2 = 0.3010$
$\log 3 = 0.4771$
$\log 5 = 0.6989$

Sol. **60**
$t_{75\%} = 2t_{1/2}$ [For 1st order reaction]
$t_{75\%} = 2 \times 30 = 60\,\text{min}$.

52. How many of the following metal ions have similar value of spin only magnetic moment in gaseous state?
(Given: Atomic number : $V, 23; Cr, 24; Fe, 26; Ni, 28$ )
$V^{3+}, Cr^{3+}, Fe^{2+}, Ni^{3+}$

Sol. **2 ($Cr^{+3}$ & $Ni^{+3}$)**




53. In sulphur estimation, 0.471 g of an organic compound gave 1.4439 g of barium sulphate.
The percentage of sulphur in the compound is_______(Nearest Integer)
(Given: Atomic mass Ba: $137u, S: 32u, O: 16u$ )

Sol. **42**
Organic compound $\longrightarrow$ $BaSO_4$
Weight = 0.417 g    Weight = 1.44 g

Moles $BaSO_4 = \frac{1.44}{233}$ = moles of Sulphur

Weight Sulphur = $\frac{1.44}{233} \times 32\,\text{gram}$

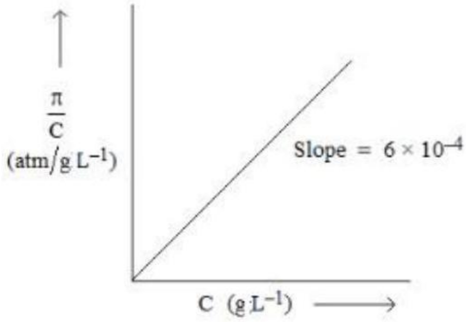
$\% S = \frac{\text{weight of sulphur}}{\text{weight of organic}} \times 100$

$\Rightarrow \frac{1.44 \times 32}{233 \times 0.471} \times 100$

$\Rightarrow \frac{46.08}{109.743} \times 100$

$\Rightarrow 41.98 \simeq 42$

54. The osmotic pressure of solutions of PVC in cyclohexanone at 300 K are plotted on the graph. The molar mass of PVC is_____$\text{g mol}^{-1}$ (Nearest integer)




(Given : $R = 0.083 \text{ L atm K}^{-1} \text{ mol}^{-1}$ )

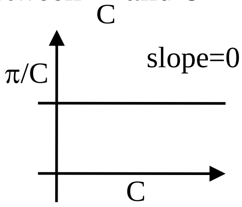
Sol. 41500

$$\pi = M'RT = \left(\frac{W/M}{V}\right)RT$$

$$\Rightarrow \pi = \left(\frac{W}{V}\right)\left(\frac{1}{M}\right)RT = C\left(\frac{RT}{M}\right)$$

$$\Rightarrow \frac{\pi}{C} = \frac{RT}{M} \neq f(c)$$

If we assume graph between $\frac{\pi}{C}$ and C




Assuming π vs C graph

$$\text{Slope} = \frac{RT}{M} = \frac{0.083 \times 300}{M} = 6 \times 10^{-4}$$

$$\therefore M = \frac{0.083 \times 300}{6 \times 10^{-4}} = \frac{830 \times 300}{6}$$

= 41,500

55. The density of a monobasic strong acid (Molar mass 24.2 g/mol ) is 1.21 kg/L. The volume of its solution required for the complete neutralization of 25 mL of 0.24MNaOH is____$\times 10^{-2}$ mL (Nearest integer)

Sol. **12**

Molarity of acid = $\frac{1.2 \times 10^3}{24.2} = \frac{1000}{20} = 50\,M$

Neutralization reaction :

$$HA + NaOH \rightarrow NaA + H_2O$$

$$M_1V_1 = M_2V_2$$

$$[50] \times V = [0.24 \times 25]$$

V = 00. 12 ml

56. An athlete is given 100 g of glucose ($C_6H_{12}O_6$) for energy. This is equivalent to 1800 kJ of energy. The 50% of this energy gained is utilized by the athlete for sports activities at the event. In order to avoid storage of energy, the weight of extra water he would need to perspire is_____ *g* (Nearest integer)
Assume that there is no other way of consuming stored energy.
Given : The enthalpy of evaporation of water is 45 kJ $mol^{-1}$
Molar mass of C, H&O are 12,1 and 16 g $mol^{-1}$

Sol. **360**

$$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O(\ell)$$

$$n = \frac{100}{180}$$

Energy needed to perspire water = $1800 \times \frac{1}{2}$

Moles of water evaporated = $\frac{900}{45} = 20\,\text{moles}$

Weight of water evaporated $\Rightarrow 20 \times 18$

$\Rightarrow$ 360 gram

57. The number of paramagnetic species from the following is

$[Ni(CN)_4]^{2-}$, $[Ni(CO)_4]$, $[NiCl_4]^{2-}$
$[Fe(CN)_6]^{4-}$, $[Cu(NH_3)_4]^{2+}$
$[Fe(CN)_6]^{3-}$ and $[Fe(H_2O)_6]^{2+}$

Sol. **4**

$(NiCl_4)^{-2} \rightarrow Ni^{+2} \rightarrow 3d^8$
$Cl^{+} \rightarrow$ weak field layered

$\boxed{\parallel}\boxed{\parallel}\boxed{\parallel}\boxed{\mid}\boxed{\mid}$ paramagnetic (n-e=2)

$(Cu(NH_3)_4)^{+2} \rightarrow Cu^{+2} \rightarrow 3d^9$

$\boxed{\parallel}\boxed{\parallel}\boxed{\parallel}\boxed{\parallel}\boxed{\mid}$ u-e = 1 paramagnetic

$(Fe(CN)_6)^{-3} \rightarrow Fe^{+3} \rightarrow 3d^5$

$\boxed{\parallel}\boxed{\parallel}\boxed{\mid}\boxed{\phantom{\mid}}\boxed{\phantom{\mid}}$

$CN^{-}$ is strong field ligand so u-e=1
so paramagnetic

$(Fe(H_2O)_6)^{+2} \rightarrow Fe^{+2} \rightarrow 3d^6$
$H_2O$ is weak field ligand

$\boxed{\parallel}\boxed{\mid}\boxed{\mid}\boxed{\mid}\boxed{\mid}$ u-e=4 so paramagnetic

58. Consider the cell
Pt(s) | $H_2$(g) (1 atm) | $H^+$(aq, $[H^+] = 1$) || $Fe^{3+}$(aq), $Fe^{2+}$(aq) | Pt(s)
Given $E^{o}_{Fe^{3+}/Fe^{2+}} = 0.771$ V and $E^{o}_{H/1/2H_2} = 0$ V, T= 298 K
If the potential of the cell is 0.712 V, the ratio of concentration of $Fe^{2+}$ to $Fe^{3+}$ is (Nearest integer)

Sol. **10**

$$\text{Anode} \Rightarrow \frac{1}{2}H_2(g) \rightarrow H^+(aq) + e^-$$

$$\underline{\text{Cathode} \Rightarrow Fe^{3+} + e^- \rightarrow Fe^{2+}}$$

$$\text{Overall} \frac{1}{2}H_2 + Fe^{3+} \xrightarrow{n-1} H^+ + Fe^{2+}$$

$$E_{cell} = E^o_{cell} - \frac{0.059}{1}\log\frac{[Fe^{2+}]}{[Fe^{3+}]} \times \frac{[H^+]}{[P_{H_2}]^{\frac{1}{2}}}$$

$$0.712 = 0.771 - 0.059\log\frac{[Fe^{2+}]}{[Fe^{3+}]}$$

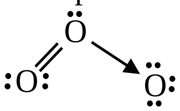
$$\log\frac{[Fe^{2+}]}{[Fe^{3+}]} = 1$$

$$\text{So } \frac{[Fe^{2+}]}{[Fe^{3+}]} = 10$$

59. The total number of lone pairs of electrons on oxygen atoms of ozone is

Sol. **6**

Not l.p. $e^-$ in $O_3$ is = 6



60. A litre of buffer solution contains 0.1 mole of each of $NH_3$ and $NH_4Cl$. On the addition of 0.02 mole of HCl by dissolving gaseous HCl, the pH of the solution is found to be_____$\times 10^{-3}$ (Nearest integer)
[Given : $pK_b(NH_3) = 4.745$
$\log 2 = 0.301$
$\log 3 = 0.477$
$T = 298$ K ]

Sol. **9**

$$\begin{array}{lll} NH_3 & + & NH_4Cl \\ 0.1\text{mole} & & 0.1\text{mole} \end{array}$$

$$\begin{array}{lllll} NH_3 & + & HCl & \rightarrow & NH_4Cl \\ 0.1\text{mole} & & 0.02 & & 0.1 \\ 0.08\,\text{mole} & & - & & 0.12\,\text{mole} \end{array}$$

$$P_{OH} \Rightarrow P_{Kb} + \log\frac{[NH_4Cl]}{[NH_3]}$$

$$\Rightarrow 4.745 + \log\left(\frac{0.12}{0.08}\right)$$

$$\Rightarrow 4.745 + \log\left(\frac{3}{2}\right)$$

$\Rightarrow 4.745 + (0.477 - 0.301)$
$\Rightarrow 4.745 + 0.176$
$\Rightarrow 4.569$
$pH \Rightarrow 14 - 4.569$
$\Rightarrow 9.431 \simeq 9$

# Mathematics

## Section A

**61.** The points of intersection of the line $ax + by = 0, (a \neq b)$ and the circle $x^2 + y^2 - 2x = 0$ are $A(\alpha, 0)$ and $B(1, \beta)$. The image of the circle with AB as a diameter in the line $x + y + 2 = 0$ is :

(1) $x^2 + y^2 + 3x + 3y + 4 = 0$
(2) $x^2 + y^2 + 3x + 5y + 8 = 0$
(3) $x^2 + y^2 - 5x - 5y + 12 = 0$
(4) $x^2 + y^2 + 5x + 5y + 12 = 0$

**Sol.** **4**

Only possibilities is $\alpha = 0, \beta = 1$

Equation of circle

$(x - 0)(x - 1) + (y - 0)(y - 1) = 0$

$x^2 + y^2 - x - y = 0$

Image of circle in live $x + y + 2 = 0$

$x^2 + y^2 + 5x + 5y + 12 = 0$

**62.** The distance of the point $(6, -2\sqrt{2})$ from the common tangent $y = mx + c, m > 0$, of the curves $x = 2y^2$ and $x = 1 + y^2$ is :

(1) $\frac{14}{3}$
(2) $5\sqrt{3}$
(3) $\frac{1}{3}$
(4) 5

**Sol.** **4**

$$\left.\begin{array}{l} y^2 = \frac{x}{2} \\ y^2 = x - 1 \end{array}\right]$$

Tangent to $y^2 = \frac{x}{2}$ is $y = mx + \frac{1}{8m}$ ...(1)

$y^2 = x-1$ is $y = m(x-1) + \frac{1}{4m}$

$y = mx - m + \frac{1}{4m}$ ...(2)

(1) & (2)

$$\frac{1}{8m} = -m + \frac{1}{4m}$$

$$m = \frac{1}{4m} - \frac{1}{8m}$$

$$m = \frac{1}{8m} \Rightarrow m^2 = \frac{1}{8} \Rightarrow m = \frac{1}{2\sqrt{2}} (m > 0)$$

From (1)

$$y = \frac{1}{2\sqrt{2}} x + \frac{1}{2\sqrt{2}}$$

distance from $(6, -2\sqrt{2})$

$$\frac{\left|\frac{3}{\sqrt{2}} + 2\sqrt{2} + \frac{1}{2\sqrt{2}}\right|}{\sqrt{1 + \frac{1}{8}}} = \frac{6+8+1}{3} = \frac{15}{3} = 5$$

**63.** Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non zero vectors such that $\vec{b} \cdot \vec{c} = 0$ and $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b}-\vec{c}}{2}$.
If $\vec{d}$ be a vector such that $\vec{b} \cdot \vec{d} = \vec{a} \cdot \vec{b}$, then $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$ is equal to

(1) $-\frac{1}{4}$ (2) $\frac{1}{4}$ (3) $\frac{3}{4}$ (4) $\frac{1}{2}$

**Sol.** **2**

$$(\bar{a}.\bar{c})\bar{b} - (\bar{a}.\bar{b})\bar{c} = \frac{\bar{b}}{2} - \frac{\bar{c}}{2}$$

$$\bar{a}.\bar{c} = \frac{1}{2}, \bar{a}.\bar{b} = \frac{1}{2}$$

$$\bar{b}.\bar{d} = \frac{1}{2}$$

$$(\bar{a} \times \bar{b}).(\bar{c} \times \bar{d}) = \bar{a}.\left[\bar{b} \times (\bar{c} \times \bar{d})\right]$$

$$= \bar{a}.\left[(\bar{b}.\bar{d})\bar{c} - (\bar{b}.\bar{c})\bar{d}\right]$$

$$= \bar{a}.\left[\bar{c}/2\right]$$

$$= \frac{1}{2}(\bar{a}.\bar{c})$$

$$= \frac{1}{4}$$

**64.** The vector $\vec{a} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ is rotated through a right angle, passing through the y-axis in its way and the resulting vector is $\vec{b}$. Then the projection of $3\vec{a} + \sqrt{2}\vec{b}$ on $\vec{c} = 5\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$ is :

(1) $2\sqrt{3}$ (2) 1 (3) $3\sqrt{2}$ (4) $\sqrt{6}$

**Sol.** **3**

$$\bar{b} = \lambda\bar{a} + \mu\hat{J}$$

$$= \lambda(-\hat{i} + 2\hat{j} + \hat{k}) + \mu\hat{j}$$

$$\bar{b} = -\lambda\hat{i} + (2\lambda + \mu)\hat{j} + \lambda\hat{k}$$

$$|\bar{a}| = |\bar{b}|$$

$$|\bar{a}|^2 = |\bar{b}|^2 \quad \Rightarrow 6 = \lambda^2 + (2\lambda + \mu)^2 + \lambda^2 \quad \ldots\ldots(1)$$

$$\because \bar{a}.\bar{b} = 0 \quad \Rightarrow \lambda + 2(2\lambda + \mu) + (1)(\lambda) = 0$$

$$\Rightarrow 6\lambda + 2\mu = 0$$

$$\Rightarrow \mu = -3\lambda \ \ldots\ldots..(2)$$

from (1) & (2)

$$3\lambda^2 = 6$$

$$\lambda^2 = 2 \quad \Rightarrow \lambda = \pm\sqrt{2}$$

$$\Rightarrow \mu = \pm 3\sqrt{2}$$

Projection of $3\bar{a} + 2\bar{b}$ on $\bar{c}$ is $= \dfrac{(3\bar{a} + \sqrt{2}\bar{b}).\bar{c}}{|\bar{c}|}$

$$= \frac{3\bar{a}.\bar{c} + \sqrt{2}\bar{b}.\bar{c}}{|\bar{c}|}$$

$$= \frac{18+\sqrt{2}\left(-6\sqrt{2}\right)}{\sqrt{50}}$$

$$= \frac{6}{\sqrt{50}} = \frac{6}{5\sqrt{2}} = \frac{3\sqrt{2}}{5}$$

**Case I :**

$$\left(\bar{a}.\bar{c} = -5+8+3=6\right) \qquad \frac{18+\sqrt{2}\left(-6\sqrt{2}\right)}{\sqrt{50}}$$

$\lambda = \sqrt{2}$ $\qquad \bar{b} = -\sqrt{2}\hat{i} + 12\sqrt{2} - 3\sqrt{2}\hat{j} + \sqrt{2}\hat{k}$

$$\bar{b} = -\sqrt{2}\hat{i} - \sqrt{2}\hat{j} + \sqrt{2}\hat{k}$$

$$\bar{b}.\bar{c} = -5\sqrt{2} - 4\sqrt{2} + 3\sqrt{2}$$

$$= -6\sqrt{2}$$

**Case II :** $\left.\begin{matrix}\lambda = -\sqrt{2}\\ \mu = 3\sqrt{2}\end{matrix}\right]$ $\qquad \bar{b} = \sqrt{2}\hat{i} + \left(\sqrt{2}\right)\hat{j} + \left(-\sqrt{2}\right)\hat{k}$

$$= \frac{18+\sqrt{2}\left(6\sqrt{2}\right)}{\sqrt{50}}$$

$$= \frac{30}{\sqrt{50}} = \frac{30}{5\sqrt{2}} = \frac{6}{\sqrt{2}} = 3\sqrt{2} \text{ Ans.}$$

**Complex Number, Easy**

**65.** Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$. The set $S = \{z \in C: |z - z_1|^2 - |z - z_2|^2 = |z_1 - z_2|^2\}$ represents a

(1) hyperbola with the length of the transverse axis 7

(2) hyperbola with eccentricity 2

(3) straight line with the sum of its intercepts on the coordinate axes equals $-18$

(4) straight line with the sum of its intercepts on the coordinate axes equals 14

**Sol.** **4**

Let $z = x + iy$

$z - z_1 = (x-2) + i(y-3)$

$|z - z_1|^2 = (x-2)^2 + (y-3)^2$

$z - z_2 = (x-3) + i(y-4)$

$|z - z_2|^2 = (x-3)^2 + (y-4)^2$

$((x-2)^2 + (y-3)^2) - ((x-3)^2 + (y-4)^2) = 2$

$\Rightarrow 2x + 2y = 14$

$= x + y = 7$

straight line with sum of intercept on C.A = 14

**66.** The mean and variance of the marks obtained by the students in a test are 10 and 4 respectively. Later, the marks of one of the students is increased from 8 to 12 . If the new mean of the marks is 10.2, then their new variance is equal to :

(1) 3.96 (2) 4.08 (3) 4.04 (4) 3.92

**Sol.** **3.96**

Let Number of observations is = n

$$\frac{\sum x_i}{n} = 10 \qquad \frac{\sum x_i - 8 + 12}{n} = 10.2$$

$$\sum x_i = (10.2)n - 4 \_\_\_(2)$$

$$\sum x_i = 10n \_\_\_\_\_(1)$$

10n = (10.2)n–4

$\Rightarrow$ (.2)n = 4 $\Rightarrow$ $\boxed{n = 20}$

Given $\frac{\sum x_i^2}{20} - (10)^2 = 4 \qquad \Rightarrow \sum x_i^2 = 2080$

After Change

$$\sum x_i^2 = 2080 - 8^2 + (12)^2$$

$$= 2160$$

New vanance $= \frac{\sum x_i^2}{20} - (\bar{x})^2$

$= \frac{2160}{20} - (10.2)^2$

$= 108 - (10.2)^2$

$= 3.96$

**67.** Let $S_1$ and $S_2$ be respectively the sets of all $a \in \mathbb{R} - \{0\}$ for which the system of linear equations

$$ax + 2ay - 3az = 1$$

$$(2a+1)x + (2a+3)y + (a+1)z = 2$$

$$(3a+5)x + (a+5)y + (a+2)z = 3$$

has unique solution and infinitely many solutions. Then

(1) $S_1$ is an infinite set and $n(S_2) = 2$ (2) $S_1 = \Phi$ and $S_2 = \mathbb{R} - \{0\}$

(3) $n(S_1) = 2$ and $S_2$ is an infinite set (4) $S_1 = \mathbb{R} - \{0\}$ and $S_2 = \Phi$

**Sol.** **4**

$$\Delta = \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$$

$\Delta = a(15a^2 + 31a + 36) = 0$

$a = 0$

$\Delta \neq 0$ for all $a \in \mathbb{R} - \{0\}$

$\therefore S_1 = \mathbb{R} - \{0\}, S_2 = \Phi$

**68.** The value of $\lim_{n\to\infty} \frac{1+2-3+4+5-6+\cdots\ldots+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3-\sqrt{n^4+5n+4}}}$ is :

(1) $\frac{3}{2}(\sqrt{2}+1)$ (2) $\frac{3}{2\sqrt{2}}$ (3) $\frac{\sqrt{2}+1}{2}$ (4) $3(\sqrt{2}+1)$

**Sol.** **1**

$$\lim_{n\to\infty} \frac{\left(1+2+4+5+\ldots+(3n-2)+(3n-1)-3+6+\ldots+3n\right)}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

Let $N^r = \sum_{n=1}^{\infty}(3n-2)+(3n-1)-3n$

$$= \sum_{n=1}^{\infty}(3n-3)$$

$$= \frac{3n(n+1)}{2} - 3n = \frac{3}{2}(n^2 - n)$$

$$\frac{3}{2}\lim_{n\to\infty}\frac{n^2\left(1-\frac{1}{n}\right)}{n^2\left(\sqrt{2+\frac{4}{n^3}+\frac{3}{n^4}}-\sqrt{1+\frac{5}{n^3}+\frac{4}{n^4}}\right)}$$

$$= \frac{3}{2(\sqrt{2}-1)} \text{ or } \frac{3}{2}(\sqrt{2}+1) \text{ Ans.}$$

**69.** The statement $(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$ is

(1) a tautology (2) a contradiction (3) equivalent to $p \vee q$ (4) equivalent to $(\sim p) \vee (\sim q)$

**Sol.** **1**

$(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$

| P | q | $\sim q$ | $p \wedge \sim q$ | $p \Rightarrow \sim q$ | $(p \wedge \sim q) \Rightarrow (p \Rightarrow \sim q)$ |
|---|---|---|---|---|---|
| T | T | F | F | F | T |
| T | F | T | T | T | T |
| F | T | F | F | T | T |
| F | F | T | F | T | T |

**Tautology**

**70.** Consider the lines $L_1$ and $L_2$ given by

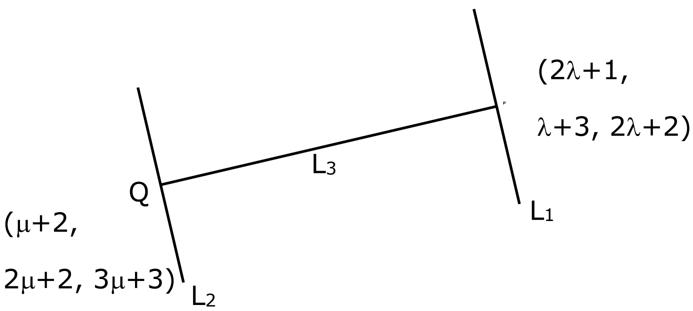
$L_1: \dfrac{x-1}{2} = \dfrac{y-3}{1} = \dfrac{z-2}{2}$

$L_2: \frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{3}$.

A line $L_3$ having direction ratios $1, -1, -2$, intersects $L_1$ and $L_2$ at the points $P$ and $Q$ respectively. Then the length of line segment $PQ$ is

(1) $3\sqrt{2}$ (2) $4\sqrt{3}$ (3) 4 (4) $2\sqrt{6}$

**Sol.** **4**




D.R's of PQ are = $(2\lambda-\mu-1, \lambda-2\mu+1, 2\lambda-3\mu-1)$

given D.R's are = $(1, -1, -2)$

$$\frac{2\lambda-\mu-1}{1} = \frac{\lambda-2\mu+1}{-1} = \frac{2\lambda-3\mu-1}{-2}$$

$\lambda = \mu = 3$

P = (7, 6, 8)

Q = (5, 8, 12)

PQ = $2\sqrt{6}$

**71.** Let $f(x)=\int \frac{2x}{(x^2+1)(x^2+3)}dx$. If $f(3)=\frac{1}{2}(\log_e 5-\log_e 6)$, then $f(4)$ is equal to

(1) $\log_e 19-\log_e 20$ (2) $\log_e 17-\log_e 18$

(3) $\frac{1}{2}(\log_e 19-\log_e 17)$ (4) $\frac{1}{2}(\log_e 17-\log_e 19)$

**Sol.** **4**

Let $x^2 = t$

$2xdx = dt$

$$f(x)=\int\frac{dt}{(t+1)(t+3)}$$

$$=\frac{1}{2}\int\left(\frac{1}{t+1}-\frac{1}{t+3}\right)dt$$

$$=\frac{1}{2}\ln\left|\frac{t+1}{t+3}\right|+C$$

$$f(x)=\frac{1}{2}\ln\left|\frac{x^2+1}{x^2+3}\right|+C$$

$x = 3$

$$\frac{1}{2}\ln\left(\frac{5}{6}\right)=\frac{1}{2}\ln\left(\frac{5}{6}\right)+C\Rightarrow C=0$$

$$f(x)=\frac{1}{2}\ln\left(\frac{x^2+1}{x^2+3}\right)$$

$$f(x)=\frac{1}{2}\ln\left(\frac{17}{19}\right)$$

$$=\frac{1}{2}\ [\ln 17-\ln 19]$$

**72.** The minimum value of the function $f(x)=\int_0^2 e^{|x-t|}dt$ is :

(1) $e(e-1)$ (2) $2(e-1)$ (3) 2 (4) $2e-1$

**Sol.** **2**

**Case I** $x < 0$

$$f(x)=\int_0^2 e^{-(x-t)}dt$$

$$=e^{-x}\int_0^2 e^{t}dt=e^{-x}\left(e^2-1\right)$$

**Case II**

$(0 < x < 2)$ $$f(x)=\int_0^x e^{x-t}dt+\int_x^2 e^{-(x-t)}dt$$

$$=e^{x}\left(-e^{-t}\right)_0^x+e^{-x}[e^t]_x^2$$

$$=e^{x}\left[-e^{-x}+1\right]+e^{-x}\left[e^2-e^x\right]$$

$$=-1+e^x+e^{2-x}-1$$

**Case III**

$x\ge 2$ $$f(x)=\int_0^2 e^{(x-t)}dt$$

$$=e^{x}\left[-e^{-t}\right]_0^2$$

$$=e^{x}\left[-e^{-2}+1\right]$$

$$=e^{x}\left(1-e^{-2}\right)$$

$$f(x)=\begin{bmatrix} e^{-x}\left(e^2-1\right), & x\le 0\rightarrow & \left(e^2-1\right) \\ e^x+e^{2-x}-2, & 0\le x\le 2\rightarrow & 2(e-1) \\ ex\left(1-e^{-2}\right), & x\ge 2\rightarrow & \left(e^2-1\right) \end{bmatrix}$$

Minimum value = 2(e–1)

**73.** Let $M$ be the maximum value of the product of two positive integers when their sum is 66. Let the sample space $S=\left\{x\in\mathbb{Z}: x(66-x)\ge \frac{5}{9}M\right\}$ and the event $A=\{x\in S: x \text{ is a multiple of } 3\}$. Then P(A) is equal to

(1) $\frac{7}{22}$ (2) $\frac{1}{5}$ (3) $\frac{15}{44}$ (4) $\frac{1}{3}$

**Sol.** **4**

Let a, b $\rightarrow$ 2 positive number

$$\frac{a+b}{2}\ge\sqrt{ab}$$

$$\sqrt{ab}\le 33$$

$$\boxed{ab\le(33)^2} \qquad M=(33)^2$$

$$x(66-x)\ge\frac{5}{9}(33)^2$$

$66x-x^2\ge 605$

$0\ge x^2-66x+605$

$(x-11)(x-55)\le 0$

$x\in[11,55]$

A = {12, 15, 18...54}

Total number in A=15

$P(A)=\frac{15}{45}=\frac{1}{3}$ Ans.

**74.** Let $x=2$ be a local minima of the function $f(x)=2x^4-18x^2+8x+12$, $x\in(-4,4)$. If M is local maximum value of the function $f$ in $(-4,4)$, then M =

(1) $18\sqrt{6}-\frac{31}{2}$ (2) $18\sqrt{6}-\frac{33}{2}$ (3) $12\sqrt{6}-\frac{33}{2}$ (4) $12\sqrt{6}-\frac{31}{2}$

**Sol.** **3**

$f'(x)=8x^3-36x+8$

$=4[2x^3-9x+2]$

$=4[(x-2)(2x^2+4x-1)]$

$$=4\left[(x-2)\left(x-\left(-\frac{2-\sqrt{6}}{2}\right)\right)\left(x-\left(\frac{-2+\sqrt{6}}{2}\right)\right)\right]$$

| – | + | – | + |
|---|---|---|---|

$\frac{-2-\sqrt{6}}{2}$ $\quad$ $\frac{-2+\sqrt{6}}{2}$ $\quad$ 2

$\frac{-2+\sqrt{6}}{2}\searrow$ maximum

$$M=2\left(\frac{-2+\sqrt{6}}{2}\right)^4-18\left(\frac{-2+\sqrt{6}}{2}\right)^2+8\left(\frac{-2+\sqrt{6}}{2}\right)+12$$

$$=12\sqrt{6}-\frac{33}{2}$$

**75.** Let $f:(0,1)\to\mathbb{R}$ be a function defined by $f(x)=\frac{1}{1-e^{-x}}$, and $g(x)=(f(-x)-f(x))$. Consider two statements

(I) $g$ is an increasing function in $(0,1)$

(II) g is one-one in $(0,1)$

Then,

(1) Both (I) and (II) are true
(2) Neither (I) nor (II) is true
(3) Only (I) is true
(4) Only (II) is true

**Sol.** **1**

$$f(x)=\frac{1}{1-e^{-x}}$$

g(x) = (f(–x) – f(x))

$$=\frac{1}{1-e^{x}}-\frac{1}{1-e^{-x}}$$

$$=\frac{1}{1-e^{x}}-\frac{e^{x}}{e^{x}-1}$$

$$g(x)=\frac{1+e^{x}}{1-e^{x}}$$

$$g'(x)=\frac{(1-e^{x})(e^{x})-(1+e^{x})(-e^{x})}{(1-e^{x})^{2}}$$

$$=\frac{e^{x}-e^{2x}+e^{x}+e^{2x}}{(1-e^{x})^{2}}$$

$$g'(x)=\frac{2e^{x}}{(1-e^{x})^{2}}$$

$$g'(x)>0\Rightarrow g(x)\uparrow$$

g(x) is one-one

**76.** Let $y(x)=(1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$. Then $y'-y''$ at $x=-1$ is equal to :

(1) 976
(2) 944
(3) 464
(4) 496

**Sol.** **4**

$$f(x)=y=\frac{(1-x)(1+x)(1+x^{2})(1+x^{4})(1+x^{8})(1+x^{16})}{(1-x)}$$

$$f(x)=y=\frac{(1-x^{32})}{1-x}\Rightarrow f(-1)=0$$

$(1-x)y = 1-x^{32}$

differentiate both side

$(1-x)y' + y(-1) = -32x^{31}$ $\boxed{x=-1\Rightarrow y'=16}$

differentiate both side

$(1-x)y' + y'(-1)-y' = -(32)(31)\,x30$

Put $x=-1$

$2y'' - 2y' = -(32)(31)$

$y'' - y' = -(16)(31)$

$\boxed{y'-y''=496}$

**77.** The distance of the point $P(4,6,-2)$ from the line passing through the point $(-3,2,3)$ and parallel to a line with direction ratios $3,3,-1$ is equal to :

(1) $\sqrt{14}$ (2) 3 (3) $\sqrt{6}$ (4) $2\sqrt{3}$

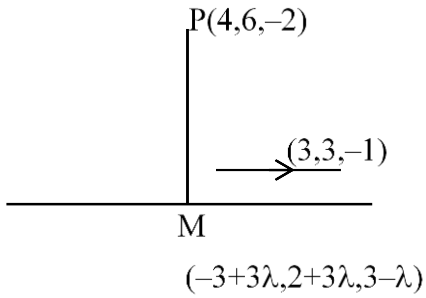
**Sol.** **1**

equation of line

$$\bar{r} = (-3,2,3) + \lambda(3,3,-1)$$

$\overline{PM}.(3,3,-1) = 0$

$(3\lambda - 7, 3\lambda - 4, 5 - \lambda) . (3,3,-1) = 0$




$\Rightarrow 3(3\lambda - 7) + 3(3\lambda - 4) - 1(15 - \lambda) = 0$

$\Rightarrow 19\lambda = 38 \Rightarrow \lambda = 2$

$M = (3, 8, 1)$

$PM = \sqrt{1+4+9} = \sqrt{14}$

**78.** Let $x, y, z > 1$ and $A = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{bmatrix}$. Then $|\operatorname{adj}(\operatorname{adj} A^2)|$ is equal to

(1) $2^8$ (2) $4^8$ (3) $6^4$ (4) $2^4$

**Sol.** **1**

$|\operatorname{adj}(\operatorname{adj}A^2| = \left|A^2\right|^{(3-1)^2} = |A|^8$

$$|A| = \begin{vmatrix} 1 & \frac{\ln y}{\ln} & \frac{\ln z}{\ln x} \\ \frac{\ln x}{\ln y} & 2 & \frac{\ln z}{\ln y} \\ \frac{\ln x}{\ln z} & \frac{\ln y}{\ln z} & 3 \end{vmatrix}$$

$$= \frac{1}{\ln x \ln y \ln z}\begin{vmatrix} \ln x & \ln y & \ln z \\ \ln x & 2\ln y & \ln z \\ \ln x & \ln y & 3\ln z \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{vmatrix}$$

$|A| = 2$

$\therefore |\operatorname{adj}(\operatorname{adj}A)| = 2^8$ Ans.

**79.** If $a_r$ is the coefficient of $x^{10-r}$ in the Binomial expansion of $(1+x)^{10}$, then $\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2$ is equal to

(1) 5445 (2) 3025 (3) 4895 (4) 1210

**Sol.** **4**

$$\sum_{r=1}^{10} r^3 \left[\frac{a_r}{a_{r-1}}\right]^2$$

$\because \frac{a_r}{a_{r-1}} = \frac{10-r+1}{r} = \frac{11-r}{r}$

$\because (1+x)^{10} \Rightarrow {}^{10}C_r n^r$

$\Rightarrow {}^{10}C_{10-r}\, n^{10-r}$

$= {}^{10}C_{10-r}$ or $= {}^{10}C_r\, x^{10-r}$

$a_r = {}^{10}C_r$

$$\sum_{r=1}^{10} r^3\left[\frac{11-r}{r}\right]^2$$

$$\sum_{r=1}^{10} r(11-r)^2$$

$$\sum_{r=1}^{10}\left[r^3-22r^2+121r\right]$$

$$=\left(\frac{(10)(11)}{2}\right)^2-22\left(\frac{(10)(11)(21)}{6}\right)+\left(\frac{(10)(11)}{2}\right)(121)$$

= 1210 Ans.

**80.** Let $y = y(x)$ be the solution curve of the differential equation

$$\frac{dy}{dx}=\frac{y}{x}\left(1+xy^2(1+\log_e x)\right), x>0, y(1)=3. \text{ Then } \frac{y^2(x)}{9} \text{ is equal to :}$$

(1) $\frac{x^2}{2x^3(2+\log_e x^3)-3}$

(2) $\frac{x^2}{3x^3(1+\log_e x^2)-2}$

(3) $\frac{x^2}{7-3x^3(2+\log_e x^2)}$

(4) $\frac{x^2}{5-2x^3(2+\log_e x^3)}$

**Sol.** **(4)**

$$\frac{dy}{dx}=\frac{y}{x}\left[1+xy^2(1+\ln x)\right]$$

$$\frac{dy}{dx}-\frac{y}{x}=y^3(1+\ln x)$$

$$\frac{1}{y^3}\frac{dy}{dx}-\frac{1}{x}\cdot\frac{1}{y^2}=1+\ln x \qquad ...(1)$$

$$-\frac{1}{y^2}=t \Rightarrow \frac{2}{y^3}\frac{dy}{dx}=\frac{dt}{dx}$$

From (1)

$$\frac{1}{2}\frac{dy}{dx}+t\left(\frac{1}{x}\right)=1+\ln x$$

$$\frac{dy}{dx}+t\left(\frac{2}{x}\right)=2(1+\ln x)$$

$$\text{I.F} = e^{\int\frac{2}{x}dx}=e^{2\ln x}=x^2$$

$$t\left(x^2\right)=\int\left[2(1+\ln x)\cdot x^2\right]dx$$

$$\Rightarrow t\cdot x^2=\frac{2x^3}{3}+2\int x^2\ln x\,dx$$

$$\Rightarrow \frac{-x^2}{y^2}-\frac{2x^3}{3}+2\left[\ln x\cdot\frac{x^3}{3}-\frac{x^2}{9}\right]+C$$

$$x = 1,\ y = 3 \qquad \Rightarrow C=\frac{-5}{9}$$

$$\frac{-x^2}{y^2}=\frac{2x^3}{3}+2\left[\ln x\cdot\frac{x^3}{3}-\frac{x^3}{9}\right]\frac{-5}{9}$$

# Section B

**81.** The constant term in the expansion of $\left(2x+\frac{1}{x^7}+3x^2\right)^5$ is

**Sol.** **1080**

General term $=\frac{5!}{r_1!r_2!r_3!}(2x)^{r_1}\left(1/x\right)^{r_2}\left(3x^2\right)^{r_3}$

$=\frac{5!}{r_1!r_2!r_3!}2^{r_1}.3^{r_3}\left[x^{r_1-7r_2+2r_3}\right]$

$$\left.\begin{array}{r} r_1-7r_2+2r_3=0 \\ r_1+r_2+r_3=5 \end{array}\right]$$

$$r_1=1, r_2=1, r_3=3$$

Constant term = 1080

**82.** For some $a,b,c\in\mathbb{N}$, let $f(x)=ax-3$ and $g(x)=x^b+c, x\in\mathbb{R}$. If $(f\circ g)^{-1}(x)=\left(\frac{x-7}{2}\right)^{1/3}$, then $(f\circ g)(ac)+(g\circ f)(b)$ is equal to

**Sol.** **2039**

Let f(g(x)) = h(x)

$f(g(x)) = 2x^3 + 7$

$a(x^b + c) - 3 = 2x^3 + 7$

$$\left.\begin{array}{r} a=2, b=3, ac=10 \\ c=5 \end{array}\right]$$

$$\left.\begin{array}{r} g(f(x))(3)=32 \\ f(g(10))=2007 \end{array}\right]$$

Sum = 2039

**83.** Let $S=\{1,2,3,5,7,10,11\}$. The number of non-empty subsets of $S$ that have the sum of all elements a multiple of 3 , is

**Sol.** **43**

No. of element 1 = {3}

No. of element 2 = {(3K + 1), (3k + 2)}

(3) (3) = 9

No. of element 3 = {3k, 3k+1, 3K + 2} = (1) (3) (3) = 9

= {(3k + 1), (3k + 1), (3k + 1)} = 1

= {(3K + 2), (3k + 2), (3k + 2)} = $\frac{1}{11}$

No. of element 4 = {3k, 3k+1, 3k+1, 3k+1} $\rightarrow$ 1

= {3k, 3k + 2, 3k+2, 3k+2} $\rightarrow$ 1

= (3k+1, 3k+2, 3k+2, 3k+1} $\rightarrow$ ${}^3C_2\times{}^3C_2 = 9$

No. of element 5 = 9, no. of element 6 = 1, no. of element 7 = 1

Total = 43.

**84.** Let the equation of the plane passing through the line $x-2y-z-5=0=x+y+3z-5$ and parallel to the line $x+y+2z-7=0=2x+3y+z-2$ be $ax+by+cz=65$. Then the distance of the point $(a,b,c)$ from the plane $2x+2y-z+16=0$ is

**Sol.** Equation of plane is

$(x - 2y - z - 5) + b(x + y + 3z - 5) = 0$

$$\begin{vmatrix} 1+b & -2+b & -1+3b \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$\Rightarrow b = 12$

Plane is $\quad 13x + 10y + 35z = 65$

Distance From given point is = 9

**85.** If the sum of all the solutions of $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}, -1 < x < 1, x \neq 0$, is $\alpha - \frac{4}{\sqrt{3}}$, then $\alpha$ is equal to

**Sol.** $\boldsymbol{\alpha = 2}$

$x \in (-1, 1) \qquad \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$

$x \in (0, 1) \qquad \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$

$x \in (-1, 0) \qquad \cot^{-1}\left(\frac{1-x^2}{2x}\right) = IT + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = IT + 2\tan^{-1}x$

$x \in (0, 1) \qquad 2\tan^{-1}x + 2\tan^{-1}x = \pi/3$

$\tan^{-1}x = \frac{\pi}{12}$

$\boxed{x = 2 - \sqrt{3}}$

$x \in (-1, 0) \qquad 2\tan^{-1}x + IT + 2\tan^{-1}x = \pi/3$

$4\tan^{-1}x = \frac{-2\pi}{3}$

$\tan^{-1}x = -\pi/6$

$\boxed{x = -1/\sqrt{3}}$

$(2 - \sqrt{3}) + \left(-\frac{1}{\sqrt{3}}\right) = \alpha - \frac{4}{\sqrt{3}}$

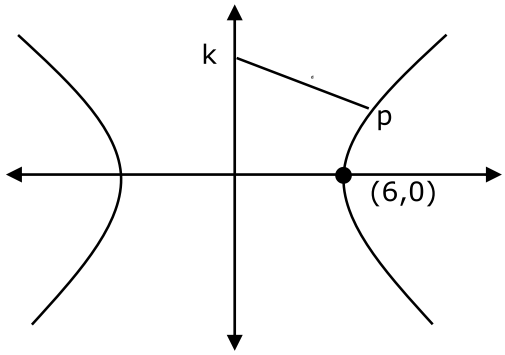
$2 - 4/\sqrt{3} = \alpha - 4/\sqrt{3}$

$\boxed{\alpha = 2}$

**86.** The vertices of a hyperbola H are $(\pm 6, 0)$ and its eccentricity is $\frac{\sqrt{5}}{2}$. Let N be the normal to H at a point in the first quadrant and parallel to the line $\sqrt{2}x + y = 2\sqrt{2}$.

If $d$ is the length of the line segment of $N$ between $H$ and the $y$-axis then $d^2$ is equal to

**Sol.** **216**




H : $\frac{x^2}{36}-\frac{y^2}{9}=1$

Equation of normal is $6x\cos\theta + 3y\cot\theta = 45$

$M = -2\sin\theta = -\sqrt{2}$

$\theta = \pi/4$

Equation of normal is $\sqrt{2}x + y = 15$

$P(a\sec\theta, b\tan\theta)$

$P\left(6\sqrt{2},3\right)$, $k(0,15)$

$d^2 = 216$

**87.** Let $x$ and $y$ be distinct integers where $1 \le x \le 25$ and $1 \le y \le 25$. Then, the number of ways of choosing $x$ and $y$, such that $x + y$ is divisible by 5 , is

**Sol.**

$x + y = 5\lambda$

| X | y | No. of ways |
|---|---|---|
| $5\lambda$ | $5\lambda$ | 20 |
| $5\lambda + 1$ | $5\lambda + 4$ | 25 |
| $5\lambda + 2$ | $5\lambda + 3$ | 25 |
| $5\lambda + 3$ | $5\lambda + 2$ | 25 |
| $5\lambda + 4$ | $5\lambda + 1$ | 25 |
| | | 1200 |

Total Ways = 120

**88.** Let $S = \left\{\alpha: \log_2(9^{2\alpha-4} + 13) - \log_2\left(\frac{5}{2}\cdot 3^{2\alpha-4} + 1\right) = 2\right\}$. Then the maximum value of $\beta$ for which the equation $x^2 - 2\left(\sum_{\alpha\in s}\alpha\right)^2 x + \sum_{\alpha\in s}(\alpha+1)^2\beta = 0$ has real roots, is

**Sol.** **25**

$$\log_2\left[\frac{9^{2\alpha-4}+13}{3^{2\alpha-4}.\frac{5}{2}+1}\right] = 2$$

$$= \frac{9^{2\alpha-4}+13}{3^{2\alpha-4}.\frac{5}{2}+1} = 4$$

$= 9^{2\alpha-4} + 13 = 10 . 3^{2\alpha-4} + 4$

$t^2 - 10\,t + 9 = 0$

$t = 1, 9$

$3^{2\alpha-4} = 3^0, 3^2$

$2\alpha - 4 = 0, 2$

$= \alpha = 2, 3$

$x^2 - 2(25)\,x + 25\beta = 0$

$D \geq 0$

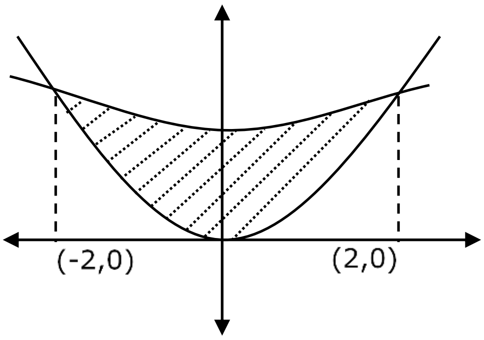
$(2)^2\,(25)^2 - 4\,(25)\,(\beta) \geq 0$

$\beta \leq 25$

$\beta_{max} = 25$

**89.** It the area enclosed by the parabolas $P_1: 2y = 5x^2$ and $P_2: x^2 - y + 6 = 0$ is equal to the area enclosed by $P_1$ and $y = \alpha x, \alpha > 0$, then $\alpha^3$ is equal to

**Sol. 600**




$y = \frac{5x^2}{2}, \qquad y = x^2 + 6$

$\frac{5x^2}{2} = x^2 + 6$

$3x^2 = 12 \Rightarrow x^2 = 4$

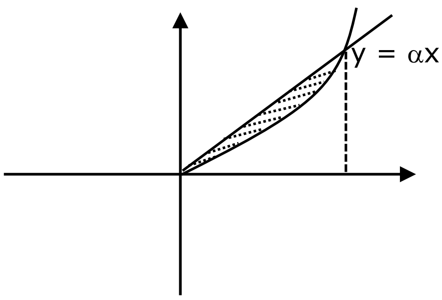
$\boxed{x = \pm 2}$

$A_1 = 2\int_0^2 \left( x^2 + 6 - \frac{5}{2}x^2 \right) dx$

$= 2\int_0^2 \left( 6 - \frac{3x^2}{2} \right) dx$

$= 2\left[ 6x - {x^3}/{2} \right]_0^2 = 2[12 - 4]$

$= 16$



$y=\frac{5}{2}x^2, y=\alpha x(\alpha>0)$

area $=\frac{8}{3}\left[a^2m^3\right]$

$=\frac{8}{3}\left[\frac{1}{10}\right]^2.\alpha^3$

$=\frac{8}{300}-\alpha^3=\frac{2}{75}\alpha^3$

$\because \frac{2}{75}-\alpha^3=16 \qquad \Rightarrow \alpha^3=8\times 75$

$\boxed{\alpha^3=600}$

**90.** Let $A_1,\ A_2,\ A_3$ be the three A.P. with the same common difference d and having their first terms as A, A +1, A + 2, respectively. Let $a, b, c$ be the $7^{th}$ , $9^{th}$ , $17^{th}$ terms of $A_1,\ A_2,\ A_3$, respectively such that

$$\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix}+70=0.$$

If $a=29$, then the sum of first 20 terms of an AP whose first term is $c-a-b$ and common difference is $\frac{d}{12}$, is equal to

**Sol.** $$\begin{vmatrix} A+6d & 7 & 1 \\ 21(A+1+8d) & 17 & 1 \\ A+2+16d & 17 & 1 \end{vmatrix}+70=0$$

A = –7, d = 6

$\therefore$ c – a – b = 20

$\therefore$ $5_{20}$ = 495