JEE Main 2020 Paper

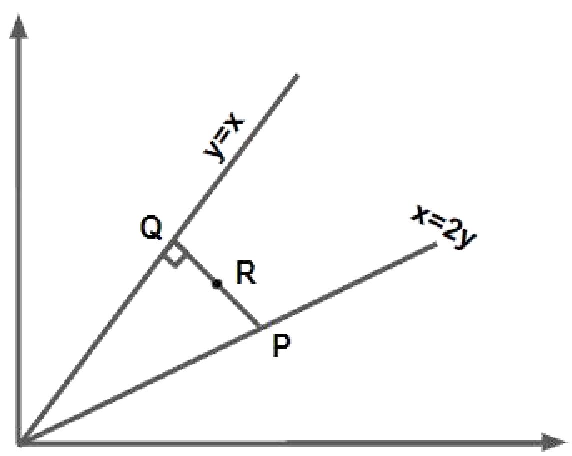
Date**: 7th January 2020 (Shift 2)**
Time**: 2:30 P.M. to 5:30 P.M.**
Subject**: Mathematics**

1. From any point P on the line $x = 2y$, a perpendicular is drawn on $y = x$. Let the foot of perpendicular be Q. Find the locus of mid point of PQ.

a. $5x = 7y$
b. $2x = 3y$
c. $7x = 5y$
d. $3x = 2y$

Answer: (a)

Solution:



Let R be the midpoint of PQ

PQ is perpendicular on line $y = x$

∴ Equation of the line PQ can be written as $y = -x + c$

$y = -x + c$ intersects $y = x$ at Q: $\left(\frac{c}{2}, \frac{c}{2}\right)$

$y = -x + c$ intersects $x = 2y$ at P: $\left(\frac{2c}{3}, \frac{c}{3}\right)$

∴ Midpoint R: $\left(\frac{7c}{12}, \frac{5c}{12}\right)$

Locus of R : $x = \frac{7c}{12}$

$y = \frac{5c}{12}$

$\Rightarrow 5x = 7y$

2. Let θ_1 and θ_2 (where $\theta_1 < \theta_2$) are two solutions of $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0, \theta \in [0,2\pi)$ then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta \, d\theta$ is equal to

a. $\frac{\pi}{9}$

b. $\frac{2\pi}{3}$

c. $\frac{\pi}{3} + \frac{1}{6}$

d. $\frac{\pi}{3}$

Answer: (d)

Solution:

$2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0, \theta \in [0,2\pi)$

$\Rightarrow 2\text{cosec}^2\theta - 2 - 5\text{cosec}\,\theta + 4 = 0$

$\Rightarrow 2\text{cosec}^2\theta - 4\text{cosec}\,\theta - \text{cosec}\,\theta + 2 = 0$

$\Rightarrow \text{cosec}\,\theta = 2$ or $\frac{1}{2}$ (Not possible)

As $\theta \in [0,2\pi)$,

$\theta_1 = \frac{\pi}{6}, \theta_2 = \frac{5\pi}{6}$

$$\Rightarrow \int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{(1+\cos 6\theta)}{2} \, d\theta$$

$$= \frac{1}{2}\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) + \frac{\sin 6\theta}{12}\Bigg|_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$= \frac{\pi}{3}$$

3. Coefficient of x^7 in $(1+x)^{10}+x(1+x)^9+\ x^2(1+x)^8+\cdots+x^{10}$ is

a. 260
b. 210
c. 420
d. 330

Answer: (d)

Solution:

Coefficient of x^7 in $(1+x)^{10}+x(1+x)^9+\ x^2(1+x)^8+\cdots+x^{10}$

Applying sum of terms of G.P. = $\dfrac{(1+x)^{10}\left(1-\left(\frac{x}{1+x}\right)^{11}\right)}{\left(1-\frac{x}{1+x}\right)}$ = $(1+x)^{11}-x^{11}$

Coefficient of $x^7 \Longrightarrow {}^{11}C_7=330$

4. Let α and β are the roots of $x^2-x-1=0$ such that $P_k=\alpha^k+\beta^k, k\geq 1$ then which one is incorrect?

a. $P_5=P_2\times P_3$
b. $P_1+P_2+P_3+P_4+P_5=26$
c. $P_5=11$
d. $P_3=P_5-P_4$

Answer: (a)

Solution:

Given α, β are the roots of $x^2-x-1=0$

$\Rightarrow \alpha+\beta=1$ & $\alpha\beta=-1$

$\Rightarrow \alpha^2=\alpha+1$ & $\beta^2=\beta+1$

$P_k=\alpha^{k-2}\alpha^2+\beta^{k-2}\beta^2$

$P_k=\alpha^{k-2}(\alpha+1)+\beta^{k-2}(\beta+1)$

$P_k=\alpha^{k-1}+\beta^{k-1}+\alpha^{k-2}+\beta^{k-2}$

$\Rightarrow P_k=P_{k-1}+P_{k-2}$

$\Rightarrow P_3=P_2+P_1=4$

$P_4=P_3+P_2=7$

$P_5 = P_4 + P_3 = 11$

$\therefore P_5 \neq P_2P_3$ & $P_1 + P_2 + P_3 + P_4 + P_5 = 26$

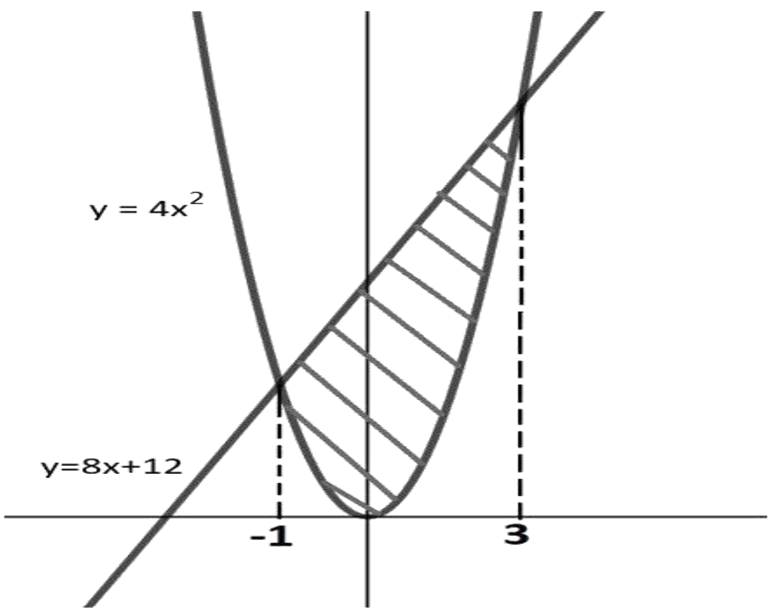
& $P_3 = P_5 - P_4$

5. The area bounded by $4x^2 \leq y \leq 8x + 12$ is

a. $\frac{127}{3}$
b. $\frac{125}{3}$
c. $\frac{128}{3}$
d. $\frac{124}{3}$

Answer: (c)

Solution:



For point of intersection,

$4x^2 = 8x + 12$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x = 3, -1$

Area bounded is given by

$A = \int_{-1}^{3}(8x + 12 - 4x^2)dx$

$$A = \left[\frac{8x^2}{2} + 12x - \frac{4x^3}{3}\right]_{-1}^{3}$$

$$A = (36 + 36 - 36) - \left(4 - 12 + \frac{4}{3}\right)$$

$$A = 44 - \frac{4}{3} = \frac{128}{3}$$

6. Contrapositive of $A \subset B$ and $B \subset C$ then $C \subset D$

a. $C \not\subset D$ or $A \not\subset B$ or $B \not\subset C$
b. $C \subset D$ or $A \not\subset B$ and $B \not\subset C$
c. $C \subset D$ and $A \not\subset B$ or $B \not\subset$C
d. $C \subset D$ or $A \not\subset B$ or $B \not\subset C$

Answer: (d)

Solution:

Given statements: $A \subset B$ and $B \subset C$

Let $A \subset B$ be p

$B \subset C$ be q

$C \subset D$ be r

Modified statement: $(p \wedge q) \Rightarrow r$

Contrapositive: $\sim r \Rightarrow \sim (p \wedge q)$

$\therefore r \vee (\sim p \vee \sim q)$

$\Rightarrow$ $C \subset D$ or $A \not\subset B$ or $B \not\subset$C

7. Let $3 + 4 + 8 + 9 + 13 + 14 + 18 + \ldots \ldots \ldots .40$ terms $= S$. If $S = (102)m$ then $m =$

a. 5
b. 10
c. 25
d. 20

Answer: (d)

Solution:

$S = \underline{3 + 4} + \underline{8 + 9} + 13 + 14 + \ldots \ldots$ 40 terms

$S = 7 + 17 + 27 + 37 + \cdots \ldots \ldots .20$ terms

$$S=\frac{20}{2}[14+(19)10]=20\times 102$$

$\therefore m=20$

8. $({}^{36}C_{r+1})\times(k^2-3)={}^{35}C_r\times 6$, then the number of ordered pairs (r,k), where $k\ \in \mathbf{I}$, are

a. 2
b. 6
c. 3
d. 4

Answer: (d)

Solution:

using ${}^{36}C_{r+1}=\frac{36}{r+1}\times{}^{35}C_r$

$\frac{36}{r+1}\times{}^{35}C_r\times(k^2-3)={}^{35}C_r\times 6$

$$k^2-3=\frac{r+1}{6}$$

$$k^2=\frac{r+1}{6}+3$$

$k\in\mathbf{I}$

$r\rightarrow$ Non-negative integer $0\le r\le 35$

$r=5\Rightarrow k=\pm 2$

$r=35\Rightarrow k=\pm 3$

No. of ordered pairs $(r,k)=4$

9. Let $f(x)$ be a five-degree polynomial which has critical points $x=\pm 1$ and $\lim_{x\to 0}\left(2+\frac{f(x)}{x^3}\right)=4$ then which one is incorrect.

a. $f(x)$ has minima at $x\ =\ 1$ and maxima at $x\ =-1$
b. $f(1)-4f(-1)\ =\ 4$
c. $f(x)$ has maxima at $x\ =\ 1$ and minima at $x\ =-1$
d. $f(x)$ is odd

Answer: (a)

Solution:

Given $\lim_{x \to 0}\left(2+\frac{f(x)}{x^3}\right)=4$

$\lim_{x \to 0}\frac{f(x)}{x^3}=2$

$\lim_{x \to 0}\frac{f(x)}{x^3}$ Limit exists and it is finite

$\therefore f(x)=ax^5+bx^4+cx^3$

$\Longrightarrow \lim_{x \to 0}(ax^2+bx+c)=2$

$c=2$

Also $f'(x)=5ax^4+4bx^3+6x^2$

$f'(1)=5a+4b+6=0$

$f'(-1)=5a-4b+6=0$

$b=0, \quad a=-\frac{6}{5}$

$f(x)=-\frac{6}{5}x^5+2x^3 \quad \Rightarrow f(x)$ is odd

$f'(x)=-6x^4+6x^2$

$f''(x)=-24x^3+12x \qquad (f''(1)<0, \quad f''(-1)>0)$

At $x=-1$ local minima at $x=1$ local maxima

And $f(1)-4f(-1)=4$

10. If LMVT is applicable on $f(x)=x^3-4x^2+8x+11$ in $[0,1]$, the value of c is

a. $\frac{4+\sqrt{5}}{3}$

b. $\frac{4+\sqrt{7}}{3}$

c. $\frac{4-\sqrt{7}}{3}$

d. $\frac{4-\sqrt{5}}{3}$

Answer: (c)

Solution:

LMVT is applicable on $f(x)$ in $[0,1]$, therefore it is continuous and differentiable in $[0,1]$

Now, $f(0) = 11, f(1) = 16$

$f'(x) = 3x^2 - 8x + 8$

$\therefore f'(c) = \frac{f(1)-f(0)}{1-0} = \frac{16-11}{1}$

$\Rightarrow 3c^2 - 8c + 8 = 5$

$\Rightarrow 3c^2 - 8c + 3 = 0$

$\Rightarrow c = \frac{8\pm2\sqrt{7}}{6} = \frac{4\pm\sqrt{7}}{3}$

As $c \in (0,1)$

We get, $c = \frac{4-\sqrt{7}}{3}$

11. Consider there are 5 machines. Probability of a machine being faulty is $\frac{1}{4}$. Probability of at most two machines being faulty is $\left(\frac{3}{4}\right)^3 k$, then the value of k is

a. $\frac{17}{4}$

b. $\frac{17}{8}$

c. $\frac{17}{2}$

d. 4

Answer: (b)

Solution:

$P(\text{machine being faulty}) = p = \frac{1}{4}$

$\therefore q = \frac{3}{4}$

$P(\text{at most two machines being faulty}) = P(\text{zero machine being faulty})$

$+P(\text{one machine being faulty}) + P(\text{two machines being faulty})$

$= {}^5C_0 p^0 q^5 + {}^5C_1 p^1 q^4 + {}^5C_2 p^2 q^3$

$= q^5 + 5pq^4 + 10p^2q^3$

$= \left(\frac{3}{4}\right)^5 + 5 \times \frac{1}{4}\left(\frac{3}{4}\right)^4 + 10\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$

$$= \left(\frac{3}{4}\right)^3 \left[\frac{9}{16} + \frac{15}{16} + \frac{10}{16}\right]$$

$$= \left(\frac{3}{4}\right)^3 \times \frac{34}{16} = \left(\frac{3}{4}\right)^3 \times \frac{17}{8}$$

$\therefore k = \frac{17}{8}$

12. $a_1, a_2, a_3, \ldots, a_9$ are in geometric progression where $a_1 < 0$ and $a_1 + a_2 = 4, a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to

a. 171

b. -513

c. $-\frac{511}{3}$

d. -171

Answer: (d)

Solution:

$a_1 + a_2 = 4 \Rightarrow a + ar = 4 \Rightarrow a(1 + r) = 4$

$a_3 + a_4 = 16 \Rightarrow ar^2 + ar^3 = 16 \Rightarrow ar^2(1 + r) = 16 \Rightarrow 4r^2 = 16$

$\Rightarrow r = \pm 2$

If $r = 2, a = \frac{4}{3}$ which is not possible as $a_1 < 0$

If $r = -2, a = -4$

$\sum_{i=1}^{9} a_i = \frac{a(r^9 - 1)}{r - 1} = \frac{(-4)[(-2)^9 - 1]}{-3} = \frac{4}{3}(-512 - 1) = 4(-171)$

$\lambda = -171$

13. If $y\sqrt{1 - x^2} = k - x\sqrt{1 - y^2}$ and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x = \frac{1}{2}$ is

a. $\frac{2}{\sqrt{5}}$

b. $-\frac{\sqrt{5}}{2}$

c. $-\frac{\sqrt{5}}{4}$

d. $\frac{\sqrt{5}}{2}$

Answer: (b)

Solution:

$y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$

Differentiating w.r.t. x on both the sides, we get:

$y'\sqrt{1-x^2} + y \times \frac{1}{2\sqrt{1-x^2}} \times (-2x) = -\sqrt{1-y^2} - x \times \frac{1}{2\sqrt{1-y^2}} \times (-2y)y'$

$\Rightarrow y'\sqrt{1-x^2} - \frac{xy}{\sqrt{1-x^2}} + \sqrt{1-y^2} - \frac{xy}{\sqrt{1-y^2}}y' = 0$

$\Rightarrow y'\left[\sqrt{1-x^2} - \frac{xy}{\sqrt{1-y^2}}\right] = \frac{xy}{\sqrt{1-x^2}} - \sqrt{1-y^2}$

Putting $x = \frac{1}{2}, y = -\frac{1}{4}$

$\Rightarrow y'\left[\frac{\sqrt{3}}{2} + \frac{\frac{1}{8}}{\frac{\sqrt{15}}{4}}\right] = -\frac{\frac{1}{8}}{\frac{\sqrt{3}}{2}} - \frac{\sqrt{15}}{4}$

$\Rightarrow y'\left[\frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{15}}\right] = -\frac{1}{4\sqrt{3}} - \frac{\sqrt{15}}{4}$

$\Rightarrow y'\left[\frac{\sqrt{45}+1}{2\sqrt{15}}\right] = -\frac{1+\sqrt{45}}{4\sqrt{3}}$

$\Rightarrow y' = -\frac{\sqrt{5}}{2}$

14. Let $A = [a_{ij}], B = [b_{ij}]$ are two 3×3 matrices such that $b_{ij} = \lambda^{i+j-2}a_{ij}$ and $|B| = 81$. Find $|A|$ if $\lambda = 3$

a. $\frac{1}{81}$

b. $\frac{1}{27}$

c. $\frac{1}{9}$

d. 3

Answer: (c)

Solution:

$b_{ij} = \lambda^{i+j-2}a_{ij}$, $\lambda = 3$

$$B = \begin{bmatrix} 3^0a_{11} & 3a_{12} & 3^2a_{13} \\ 3a_{21} & 3^2a_{22} & 3^3a_{23} \\ 3^2a_{31} & 3^3a_{32} & 3^4a_{33} \end{bmatrix}$$

$$|B| = \begin{vmatrix} 3^0 a_{11} & 3a_{12} & 3^2 a_{13} \\ 3a_{21} & 3^2 a_{22} & 3^3 a_{23} \\ 3^2 a_{31} & 3^3 a_{32} & 3^4 a_{33} \end{vmatrix}$$

Taking 3^2 Common each from C_3 & R_3

$$|B| = 81 \begin{vmatrix} a_{11} & 3a_{12} & a_{13} \\ 3a_{21} & 3^2 a_{22} & 3a_{23} \\ a_{31} & 3a_{32} & a_{33} \end{vmatrix}$$

Taking 3 common each from C_2 & R_2

$$|B| = 81(9) \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Given $|B| = 81$

$\Rightarrow 81 = 81(9)|A|$

$\Rightarrow |A| = \frac{1}{9}$

15. Pair of tangents are drawn from the origin to the circle $x^2 + y^2 - 8x - 4y + 16 = 0$, then the square of length of chord of contact is

a. $\frac{8}{5}$

b. $\frac{8}{13}$

c. $\frac{24}{5}$

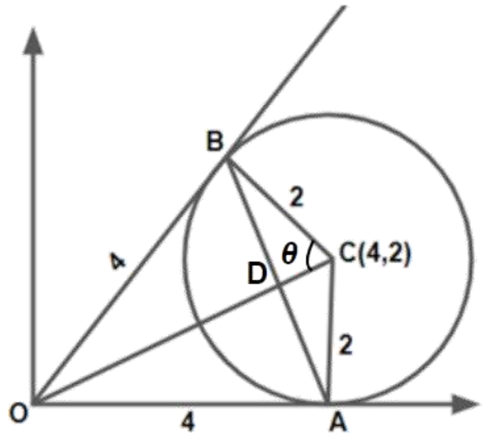
d. $\frac{64}{5}$

Answer: (d)

Solution:

$x^2 + y^2 - 8x - 4y + 16 = 0$

$(x - 4)^2 + (y - 2)^2 = 4 \Rightarrow$ Centre (4,2) , radius (2)



$OA = 4 = OB$

In ΔOBC

$\tan\theta = \frac{4}{2} = 2 \Rightarrow \sin\theta = \frac{2}{\sqrt{5}}$

In ΔBDC

$\sin\theta = \frac{BD}{2} \Rightarrow BD = \frac{4}{\sqrt{5}}$

Length of chord of contact $(AB) = \frac{8}{\sqrt{5}}$

<u>Alternative</u>

(l) length of tangent = 4

(r) radius =2

$\Rightarrow$Length of chord of contact = $\frac{2lr}{\sqrt{(l^2+r^2)}}$

Square of length of chord of contact = $\frac{64}{5}$

16. Let $y(x)$ is the solution of differential equation $(y^2 - x)\frac{dy}{dx} = 1$ and $y(0) = 1$, then find the value of x where the curve cuts the $x - axis$.

a. $2 - e$
b. 2
c. $2 + e$
d. e

Answer: (a)

Solution:

$$(y^2 - x)\frac{dy}{dx} = 1$$

$$\frac{dx}{dy} + x = y^2$$

$$xe^y = \int y^2 e^y dy$$

$$x = y^2 - 2y + 2 + ce^{-y}$$

Given $y(0) = 1$

$\Rightarrow c = -e$

$\therefore$ Solution is $x = y^2 - 2y + 2 - e^{-y+1}$

$\therefore$ The value of x where the curve cuts the $x - axis$ will be at $x = 2 - e$

17. Let $4\alpha \int_{-1}^{2} e^{-\alpha|x|}\, dx = 5$ then $\alpha =$

a. $\ln\sqrt{2}$

b. $\ln\frac{3}{4}$

c. $\ln 2$

d. $\ln\frac{4}{3}$

Answer: (c)

Solution:

$$4\alpha \int_{-1}^{2} e^{-\alpha|x|} dx = 5$$

$$4\alpha\left[\int_{-1}^{0} e^{-\alpha|x|} dx + \int_{0}^{2} e^{-\alpha|x|} dx\right] = 5$$

$$= 4\alpha\left[\int_{-1}^{0} e^{\alpha x} dx + \int_{0}^{2} e^{-\alpha x} dx\right] = 5$$

$$= 4\alpha\left[\left(\frac{1-e^{-\alpha}}{\alpha}\right)+\left(\frac{e^{-2\alpha}-1}{-\alpha}\right)\right] = 5$$

$$= 4[1-e^{-2\alpha}-e^{-\alpha}+1] = 5$$

Let $e^{-\alpha} = t$

$$\Rightarrow\ -4t^2 - 4t + 3 = 0$$

$$\Rightarrow\ t = \frac{1}{2} = e^{-\alpha} \Rightarrow \alpha = \ln 2$$

18. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $\lambda = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ and $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ then $(\lambda, d) =$

a. $\left(\frac{3}{2}, 3\vec{a} \times \vec{b}\right)$

b. $\left(-\frac{3}{2}, 3\vec{a} \times \vec{c}\right)$

c. $\left(-\frac{3}{2}, 3\vec{a} \times \vec{b}\right)$

d. $\left(\frac{3}{2}, 3\vec{a} \times \vec{c}\right)$

Answer: (c)

Solution:

Given $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ $\quad \lambda = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$

$$\Rightarrow \left|\vec{a} + \vec{b} + \vec{c}\right|^2 = |\vec{0}|^2$$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$$

$$\lambda = -\frac{3}{2}$$

Also $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$

$$\Rightarrow \vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \left(-\vec{a} - \vec{b}\right) + \left(-\vec{a} - \vec{b}\right) \times \vec{a}$$

$$\Rightarrow \vec{d} = \vec{a} \times \vec{b} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b}$$

$$\Rightarrow \vec{d} = 3\left(\vec{a} \times \vec{b}\right) = 3\vec{a} \times \vec{b}$$

19. $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$, then the distance between foci of ellipse is

a. $2\sqrt{5}$
b. $2\sqrt{7}$
c. 4
d. $2\sqrt{3}$

Answer: (b)

Solution:

$3x + 4y = 12\sqrt{2}$ is tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$

Equation of tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ is $y = mx + \sqrt{a^2m^2 + 9}$

Now, $3x + 4y = 12\sqrt{2} \Rightarrow y = -\frac{3}{4}x + 3\sqrt{2}$

$\Rightarrow m = -\frac{3}{4}$ and $\sqrt{a^2m^2 + 9} = 3\sqrt{2}$

$$\Rightarrow a^2\left(-\frac{3}{4}\right)^2 + 9 = 18$$

$$\Rightarrow a^2 \times \frac{9}{16} = 9$$

$$\Rightarrow a^2 = 16 \Rightarrow a = 4$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$$

Distance between foci is $2ae = 2 \times 4 \times \frac{\sqrt{7}}{4} = 2\sqrt{7}$

20. If mean and variance of $2, 3, 16, 20, 13, 7, x, y$ are 10 and 25 respectively then xy is equal to _______.

Answer: (124)

Solution:

Mean $= 10 \Rightarrow \frac{61+x+y}{8} = 10$

$\Rightarrow x + y = 19$

Variance $= \frac{\sum x_i^2}{n} - (\bar{x}\ )^2$

$\Rightarrow 25 = \frac{2^2+3^2+16^2+20^2+13^2+7^2+x^2+y^2}{8} - 100$

$\Rightarrow 1000 = 887 + x^2 + y^2$

$\Rightarrow x^2 + y^2 = 113$

$\Rightarrow (x + y)^2 - 2xy = 113$

$\Rightarrow 361 - 2xy = 113$

So, $xy = 124$

21. If $Q \equiv \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ is foot of perpendicular drawn from $P(1, 0, 3)$ onto a line L and line L is passing through $(\alpha, 7,1)$, then value of α is _________.

Answer: (4)

Solution:

Direction ratios of line L: $\left(\alpha - \frac{5}{3}, 7 - \frac{7}{3}, 1 - \frac{17}{3}\right)$

$= \left(\frac{3\alpha - 5}{3}, \frac{14}{3}, -\frac{14}{3}\right)$

Direction ratios of PQ: $\left(-\frac{2}{3}, -\frac{7}{3}, -\frac{8}{3}\right)$

As line L is perpendicular to PQ

So, $\left(\frac{3\alpha-5}{3}\right)\left(-\frac{2}{3}\right) + \left(\frac{14}{3}\right)(-\frac{7}{3}) + \left(-\frac{14}{3}\right)\left(-\frac{8}{3}\right) = 0$

$\Rightarrow -6\alpha + 10 - 98 + 112 = 0$

$\Rightarrow 6\alpha = 24\ \Rightarrow \alpha = 4$

22. If system of equations $x + y + z = 6, x + 2y + 3z = 10, 3x + 2y + \lambda z = \mu$ has more than 2 solutions, then $(\mu - \lambda^2)$ is _______.

Answer: (13)

Solution:

The system of equations has more than 2 solutions

$\therefore\ D = D_3 = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix} = 0 \ \Rightarrow 2\lambda - 6 - \lambda + 9 + 2 - 6 = 0$$

$\Rightarrow \lambda = 1$

$$\begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 3 & 2 & \mu \end{vmatrix} = 0 \Rightarrow 2\mu - 20 - \mu + 30 - 24 = 0$$

$\Rightarrow \mu = 14$

So, $\mu - \lambda^2 = 13$

23. If $f(x)$ is defined in $x \in \left(-\frac{1}{3}, \frac{1}{3}\right)$ &

$$f(x) = \begin{cases} \left(\frac{1}{x}\right) \log_e \left(\frac{1+3x}{1-2x}\right) & x \neq 0 \\ k & x = 0 \end{cases}$$

The value of k such that $f(x)$ is continuous is _______.

Answer: (5)

Solution:

As $f(x)$ is continuous

$$\Rightarrow \lim_{x \to 0} f(x) = f(0) = k$$

$$\Rightarrow \lim_{x \to 0} \left(\frac{1}{x}\right) \log_e \left(\frac{1+3x}{1-2x}\right) = k$$

$$\Rightarrow \lim_{x \to 0} \frac{3 \log(1+3x)}{3x} - \lim_{x \to 0} \frac{(-2) \log(1-2x)}{(-2x)} = k$$

$\Rightarrow 3+2=k \ \Rightarrow k=5$

24. Let $X=\{x: 1 \leq x \leq 50, x \in \mathbf{N}\}$, $A=\{x: x$ is a multiple of $2\}$, $B=\{x: x$ is a multiple of $7\}$. Then the number of elements in the smallest subset of X which contain elements of both A and B is ______.

Answer: (29)

Solution:

$A = \{x: x \text{ is multiple of } 2\} = \{2,4,6,8, \ldots\}$

$B = \{x: x \text{ is multiple of } 7\} = \{7,14,21, \ldots\}$

$X = \{x: 1 \leq x \leq 50, x \in \mathbf{N}\}$

Smallest subset of X which contains elements of both A and B is a set with multiples of 2 or 7 less than 50.

$P=\{x: x$ is a multiple of 2 less than or equal to $50\}$

$Q=\{x: x$ is a multiple of 7 less than or equal to $50\}$

$n(P \cup Q)=n(P)+n(Q)-n(P \cap Q)$

$=25+7-3$

$=29$