JEE–MAIN EXAMINATION – JANUARY, 2023	
(Held On Thursday 24th January, 2023)	TIME : 3 : 00 PM to 6 : 00 PM

Physics

SECTION - A

1. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.
Assertion A: A pendulum clock when taken to Mount Everest becomes fast.
Reason: The value of g (acceleration due to gravity) is less at Mount Everest than its value on the surface of earth.
In the light of the above statements, choose the most appropriate answer from the options given below
(1) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**
(2) A is correct but **R** is not correct
(3) Both **A** and **R** are correct and **R** is the correct explanation of **A**
(4) **A** is not correct but **R** is correct

Sol. **4**

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T \propto \frac{1}{\sqrt{g}}$$

on Everest g decreases, so T increases, so moves slow.

2. The frequency (v) of an oscillating liquid drop may depend upon radius (r) of the drop, density (ρ) of liquid and the surface tension (s) of the liquid as : $v = r^a \rho^b s^c$. The values of a, b and c respectively are
(1) $\left(-\frac{3}{2}, \frac{1}{2}, \frac{1}{2}\right)$ (2) $\left(\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$ (3) $\left(-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$ (4) $\left(\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}\right)$

Sol. **3**

$f \; \alpha \; r^a \rho^b s^c$

$T^{-1} = [L]^a [M\,L^{-3}]^b [MT^{-2}]^c$

$T^{-1} = L^{a-3b}, M^{b+c}\; T^{-2c}$

$-2c = -1$ .... (1)

$c = ½$

$b + c = 0$ ....(2)

$b = -½$

$a - 3b = 0$ ....(3)

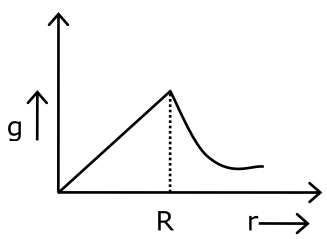
$a = 3b = \frac{-3}{2}$

3. Given below are two statements:
Statement I : Acceleration due to earth's gravity decreases as you go 'up' or 'down' from earth's surface.
Statement II : Acceleration due to earth's gravity is same at a height 'h' and depth 'd' from earth's surface, if h = d.
In the light of above statements, choose the most appropriate answer form the options given below
(1) Both Statement I and Statement II are incorrect
(2) Statement I is incorrect but statement II is correct
(3) Both Statement I and II are correct
(4) Statement I is correct but statement II is incorrect

Sol. **4**

$$g\left(1-\frac{2h}{R}\right) = g\left(1-\frac{d}{R}\right)$$

$$h = \frac{d}{2}$$



4. A long solenoid is formed by winding 70 turns cm^{-1}. If 2.0 A current flows, then the magnetic field produced inside the solenoid is ___________ $(\mu_0 = 4\pi\times10^{-7}\ TmA^{-1})$

(1) 88×10^{-4} T (2) 352×10^{-4} T (3) 176×10^{-4} T (4) 1232×10^{-4} T

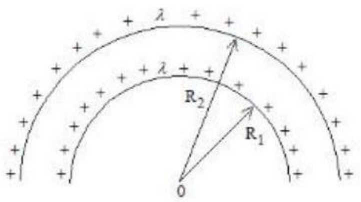
Sol. **3**

$B = \mu_0 ni$

$= 4\times\frac{22}{7}\times10^{-7}\times70\times100\times2$

$= 176 \times 10^{-4}$

5. The electric potential at the centre of two concentric half rings of radii R_1 and R_2, having same linear charge density λ is :



(1) $\frac{\lambda}{2\varepsilon_0}$ (2) $\frac{\lambda}{4\varepsilon_0}$ (3) $\frac{2\lambda}{\varepsilon_0}$ (4) $\frac{\lambda}{\varepsilon_0}$

Sol. **1**

$V_C = \frac{K}{R_1}q_1 + \frac{kq_2}{R_2}$

$= \frac{1}{4\pi\varepsilon_0}\times\frac{\lambda\pi R_1}{R_1} + \frac{1}{4\pi\varepsilon_0}\times\frac{\lambda\pi R_2}{R_2}$

$= \frac{\lambda}{2\varepsilon_0}$

6. If the distance of the earth from Sun is 1.5×10^6 km. Then the distance of an imaginary planet from Sun, if its period of revolution is 2.83 years is :

(1) 6×10^6 km (2) 3×10^6 km (3) 3×10^7 km (4) 6×10^7 km

Sol. **2**

$T^2 \propto R^3$

$\left(\frac{T_E}{T_p}\right)^{\frac{2}{3}} = \left(\frac{R_E}{R_p}\right)$

$\left(\frac{1}{2.83}\right)^{\frac{2}{3}} = \frac{1.5\times10^6}{R}$

$R = 1.5 \times 10^6 \times (2.83)^{\frac{2}{3}}$

$1.5 \times 10^6 \times (1.41\times2)^{\frac{2}{3}}$

$1.5 \times 10^6 \times (2\sqrt{2})^{\frac{2}{3}}$

$1.5 \times 10^6 \times (\sqrt{8})^{\frac{2}{3}}$

3×10^6 KM

7. A photon is emitted in transition from n = 4 to n = 1 level in hydrogen atom. The corresponding wavelength for this transition is (given, $h = 4 \times 10^{-15}$ eVs) :

(1) 99.3 nm (2) 941 nm (3) 974 nm (4) 94.1 nm

Sol. **4**

$\Delta E = E_4 - E_1$

$\frac{hc}{\lambda} = -0.85 - (-13.6)$

$\frac{4\times10^{-15}\times3\times10^{17}}{\lambda_{(nm)}}\frac{nm}{s} = 12.75$

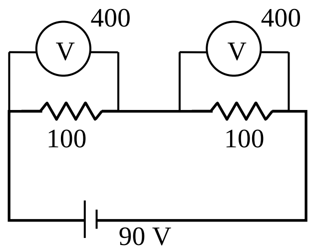
$\lambda = \frac{1200}{12.75}$ nm

= 94.1 nm

8. A cell of emf 90 V is connected across series combination of two resistors each of 100Ω resistance. A voltmeter of resistance 400Ω is used to measure the potential difference across each resistor. The reading of the voltmeter will be:

(1) 90 V (2) 45 V (3) 80 V (4) 40 V

Sol. **2**



as Resistance are same so equal division of potential.

$\therefore \frac{90}{2} = 45$ V

9. If two vectors $\vec{P} = \hat{\imath} + 2m\hat{\jmath} + m\hat{k}$ and $\vec{Q} = 4\hat{\imath} - 2\hat{\jmath} + m\hat{k}$ are perpendicular to each other. Then, the value of m will be:

(1) −1 (2) 3 (3) 2 (4) 1

Sol. **3**

$\vec{P}.\vec{Q} = 0$

$4 \times 1 + 2mx - 2 + m^2 = 0$

$m^2 - 4m + 4 = 0$

$(m - 2)^2 = 0$

$m = 2$

10. The electric field and magnetic field components of an electromagnetic wave going through vacuum is described by

$E_x = E_0 \sin(kz - \omega t)$

$B_y = B_0 \sin(kz - \omega t)$

Then the correct relation between E_o and B_o is given by

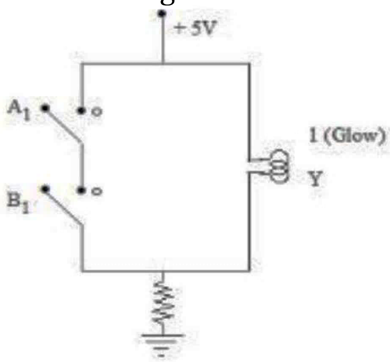
(1) $E_0 B_0 = \omega k$ (2) $E_0 = kB_0$ (3) $kE_o = \omega B_o$ (4) $\omega E_0 = kB_0$

Sol. **3**

by theory of EM wave

$\frac{E_0}{B_0} = v = \frac{\omega}{K}$

11. The logic gate equivalent to the given circuit diagram is :



(1) NAND (2) OR (3) AND (4) NOR

Sol. **1**

by truth table

A_1	B_1	V_1
0	0	1
0	1	1
1	0	1
1	1	0

NAND gate

12. Let γ_1 be the ratio of molar specific heat at constant pressure and molar specific heat at constant volume of a monoatomic gas and γ_2 be the similar ratio of diatomic gas. Considering the diatomic gas molecule as a rigid rotator, the ratio, $\frac{\gamma_1}{\gamma_2}$ is :

(1) $\frac{25}{21}$ (2) $\frac{35}{27}$ (3) $\frac{21}{25}$ (4) $\frac{27}{35}$

Sol. **1**

$$\frac{r_1}{r_2} = \frac{\frac{5}{3}}{\frac{7}{5}} = \frac{25}{21}$$

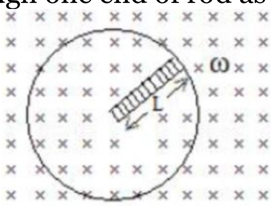
13. When a beam of white light is allowed to pass through convex lens parallel to principal axis, the different colours of light converge at different point on the principle axis after refraction. This is called:

(1) Spherical aberration (2) Polarisation

(3) Chromatic aberration (4) Scattering

Sol. Theory : Colors are due to chromatic aberration.

14. A metallic rod of length ' L ' is rotated with an angular speed of ' ω ' normal to a uniform magnetic field ' B ' about an axis passing through one end of rod as shown in figure. The induced emf will be:



(1) $\frac{1}{4}\mathrm{BL}^2\omega$ (2) $\frac{1}{2}B^2L^2\omega$ (3) $\frac{1}{4}\ \mathrm{B}^2\ \mathrm{L}\omega$ (4) $\frac{1}{2}\mathrm{BL}^2\omega$

Sol. **4**

$$\in = \int_0^L B\omega x\, dx$$

$$= \frac{1}{2}B\omega L^2$$

15. An a-particle, a proton and an electron have the same kinetic energy. Which one of the following is correct in case of their de-Broglie wavelength:

(1) $\lambda_\alpha < \lambda_p < \lambda_e$ (2) $\lambda_\alpha = \lambda_p = \lambda_e$ (3) $\lambda_\alpha > \lambda_p > \lambda_e$ (4) $\lambda_\alpha > \lambda_p < \lambda_e$

Sol. **1**

$$\lambda = \frac{h}{\sqrt{2mkE}} \propto \frac{1}{\sqrt{m}}$$

$$m_a > m_p > m_e$$

$$\therefore \lambda_\alpha < \lambda_p < \lambda_e$$

16. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason

Assertion A : Steel is used in the construction of buildings and bridges.

Reason R : Steel is more elastic and its elastic limit is high.

In the light of above statements, choose the most appropriate answer from the options given below

(1) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(2) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**

(3) A is correct but **R** is not correct

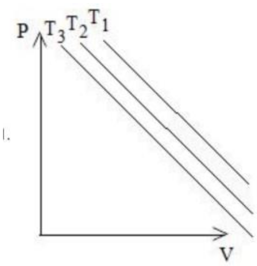
(4) A is not correct but **R** is correct

Sol. **1**

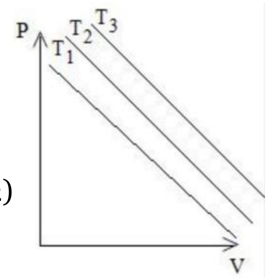
Steel is more elastic.

17. In an Isothermal change, the change in pressure and volume of a gas can be represented for three different temperature; $T_3 > T_2 > T_1$ as:

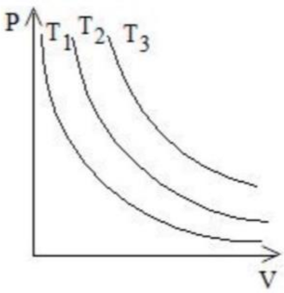
(1)



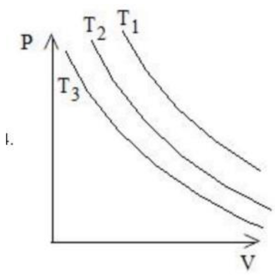
(2)



(3)

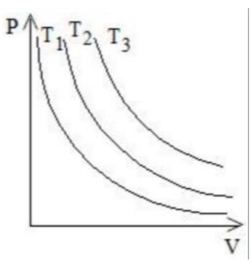


(4)



Sol. **3**

PV = nRT const.



$$P \propto \frac{1}{V}$$

18. Match List I with List II

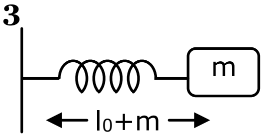
LIST I		LIST II	
A.	AM Broadcast	I.	88 – 108MHz
B.	FM Broadcast	II.	540 – 1600kHz
C.	Television	III	3.7 – 4.2GHz
D.	Satellite Communication	IV.	54MHz – 890MHz

Choose the correct answer from the options given below:
(1) A-II, B-I, C-IV, D-III (2) A-I, B-III, C-II, D-IV
(3) A-IV, B-III, C-I, D-II (4) A-II, B-III, C-I, D-IV

Sol. **1**
by concept of AM & FM freq. range

19. A body of mass 200 g is tied to a spring of spring constant 12.5 N/m, while the other end of spring is fixed at point O. If the body moves about O in a circular path on a smooth horizontal surface with constant angular speed 5rad/s. Then the ratio of extension in the spring to its natural length will be :
(1) 2:5 (2) 1: 1 (3) 2: 3 (4) 1: 2

Sol. **3**



$kx = m\omega^2(\ell_0 + x)$

$\frac{k}{m\omega^2} = \frac{\ell_0}{x} + 1$

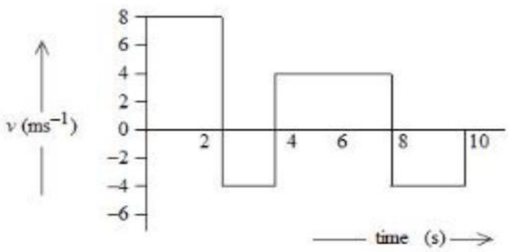
$\frac{12.5}{0.2 \times 25} = \frac{\ell_0}{x} + 1$

$\frac{125}{50} - 1 = \frac{\ell_0}{x}$

$\frac{3}{2} = \frac{\ell_0}{x}$

$\frac{x}{\ell_0} = \frac{2}{3}$

20. The velocity time graph of a body moving in a straight line is shown in figure.



The ratio of displacement to distance travelled by the body in time 0 to 10 s is :
(1) 1: 1 (2) 1: 2 (3) 1: 3 (4) 1: 4

Sol. **3**
disp. = area
= 8 × 2 + (4 × 4) – 2 × 4 – 2 × 4
= 32 – 16
= 16
distance = 32 + 16
= 48

SECTION - B

21. A body of mass 1 kg begins to move under the action of a time dependent force $\vec{F} = (t\hat{i} + 3t^2\hat{j})$N, where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis. The power developed by above force, at the time $t = 2s$, will be __________ W.

Sol. **100**

$\vec{v} = \int_0^2 t\,dt\,\hat{i} + 3\int_0^2 t^2\,dt\,\hat{j}$

$= 2\hat{i} + 8\hat{j}$

$\vec{F} = 2\hat{i} + 12\hat{j}$

$P = \vec{F}.\vec{V}$

= 4 + 96

= 100 w

22. A convex lens of refractive index 1.5 and focal length 18 cm in air is immersed in water. The change in focal length of the lens will be _________ cm

(Given refractive index of water $= \frac{4}{3}$)

Sol. **54**

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{18} = (1.5 - 1)\frac{2}{R}$ .... (1)

$\frac{1}{f} = \left(\frac{1.5}{\frac{4}{3}} - 1\right)\frac{2}{R}$ ..... (2)

Div eq. by eq. 2

$\frac{f}{18} = \frac{0.5 \times 8}{1}$

f = 72 cm

change = 72 – 18

= 54

23. The energy released per fission of nucleus of ^{240}X is 200MeV. The energy released if all the atoms in 120 g of pure ^{240}X undergo fission is _______ $\times 10^{25}$MeV (Given $N_A = 6 \times 10^{23}$)

Sol. **6**

no. of atoms $= \frac{120}{240} \times 6 \times 10^{23}$

$= 3 \times 10^{23}$

Energy rebased $= 200 \times 3 \times 10^{23}$

$= 6 \times 10^{25}$

24. A uniform solid cylinder with radius R and length L has moment of inertia I_1, about the axis of the cylinder. A concentric solid cylinder of radius $R' = \frac{R}{2}$ and length $L' = \frac{L}{2}$ is carved out of the original cylinder. If I_2 is the moment of inertia of the carved out portion of the cylinder then $\frac{I_1}{I_2} =$ ________

(Both I_1 and I_2 are about the axis of the cylinder)

Sol. **32**

$$I_1 = \frac{MR^2}{2}$$

mass = $\rho\pi\frac{R^2}{4}.\frac{L}{2}$

$$m_2 = \frac{M}{8}$$

$$I_2 = \frac{m_2R_2^2}{2} = \frac{MR^2}{8\times4\times2}$$

$$\frac{I_1}{I_2} = 32$$

25. A parallel plate capacitor with air between the plate has a capacitance of 15pF. The separation between the plate becomes twice and the space between them is filled with a medium of dielectric constant 3.5. Then the capacitance becomes $\frac{x}{4}$pF. The value of x is __________

Sol. **105**

$$C = \frac{A\varepsilon_0}{d}$$

$$C = \frac{KA\varepsilon_0}{2d}$$

$$= \frac{KC}{2}$$

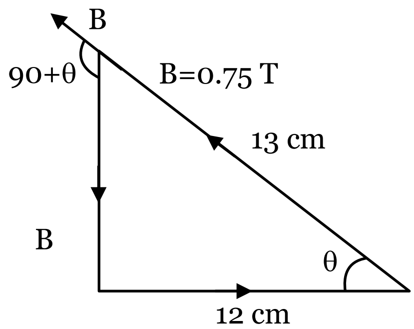
$$= \frac{3.5\times15}{2}$$

$$= \frac{105}{4}$$

= 105

26. A single turn current loop in the shape of a right angle triangle with sides 5 cm, 12 cm, 13 cm is carrying a current of 2 A. The loop is in a uniform magnetic field of magnitude 0.75 T whose direction is parallel to the current in the 13 cm side of the loop. The magnitude of the magnetic force on the 5 cm side will be $\frac{x}{130}$ N. The value of x is ___________

Sol. **9**



Force on 5 cm length = $i\int d\vec{\ell}\times\vec{B}$

$$= i\times\left(\frac{5}{100}\right)\times0.75\times\sin(90+\theta)$$

$$= 2\times\frac{5}{100}\times0.75\times\cos\theta$$

$$= \frac{10}{100}\times0.75\times\frac{12}{13} = \frac{x}{130}$$

$\Rightarrow$ x = 9

27. A mass m attached to free end of a spring executes SHM with a period of 1 s. If the mass is increased by 3 kg the period of oscillation increases by one second, the value of mass m is ______ kg.

Sol. **1**

$2\pi\sqrt{\frac{m}{k}}$ = 1 ………. (1)

$2\pi\sqrt{\frac{m+3}{k}}$ = 2 ……. (2)

(2) ÷ (1)

$\sqrt{\frac{m+3}{m}} = \frac{2}{1}$

$\frac{m+3}{m} = 4$

4 m = m + 3

m = 1 kg.

28. If a copper wire is stretched to increase its length by 20%. The percentage increase in resistance of the wire is __________ %

Sol. **44**

Length becomes = 1.2 times

$\ell' = 1.2\ell$

$R' = n^2R$

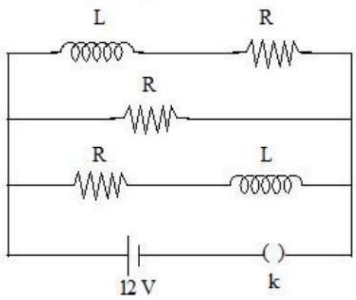
$= (1.2)^2 R$

= 1.44 R

ΔR = 0.44 R

$\frac{\Delta R}{R} \times 100$ % = 44%

29. Three identical resistors with resistance R = 12Ω and two identical inductors with self inductance L = 5mH are connected to an ideal battery with emf of 12 V as shown in figure. The current through the battery long after the switch has been closed will be ________ A.



Sol. **3**

Short all inductor

Req. = $\frac{R}{3} = \frac{12}{3}$ = 4Ω

I = $\frac{12}{4}$ = 3A

30. A Spherical ball of radius 1 mm and density 10.5 g/cc is dropped in glycerine of coefficient of viscosity 9.8 poise and density 1.5 g/cc. Viscous force on the ball when it attains constant velocity is 3696×10^{-x} N. The value of x is (Given, $g = 9.8$ m/s^2 and $\pi = \frac{22}{7}$)

Sol. 7

$$V_T = \frac{2r^2 g(\sigma_s - \rho_\ell)}{a_n}$$

$$\frac{2\times10^{-6}\times9.8\times(10.5-1.5)\times10^3}{9.8\times0.1\times9}$$

$= 2 \times 10^{-2}$ m/s

$F = 6\pi x\, rV_T$

$$= 6\times\frac{22}{7}\times9.8\times0.1\times10^{-3}\times18\times10^{-2}$$

$= 3696 \times 10^{-7}$

$= 7$

Chemistry

SECTION - A

31. Identify the correct statements about alkali metals.

A. The order of standard reduction potential $(M^+ \mid M)$ for alkali metal ions is Na > Rb > Li.

B. CsI is highly soluble in water.

C. Lithium carbonate is highly stable to heat.

D. Potassium dissolved in concentrated liquid ammonia is blue in colour and paramagnetic.

E. All the alkali metal hydrides are ionic solids.

Choose the correct answer from the options given below:

(1) C and E only (2) A, B and E only (3) A, B, D only (4) A and E only

Sol. **4**

(i) These standard potentionals of

Element	Li	Na	Rb
SRP	–3.237	–2.898	–3.079

(ii) All the alkali metal hydrides are ionic solids with high M.P.

32. Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason **R**

Assertion A: Beryllium has less negative value of reduction potential compared to the other alkaline earth metals.

Reason : Beryllium has large hydration energy due to small size of Be^{2+} but relatively large value of atomization enthalpy

In the light of the above statements, choose the most appropriate answer from the options given below

(1) A is not correct but R is correct

(2) A is correct but *R* is not correct
(3) Both A and R are correct and R is the correct explanation of A

(4) Both A and R are correct but R is NOT the correct explanation of A

Sol. **3**

Be has least negative SRP value. Value in alkaline earth metal group as it has high hydration enthalpy and high enthalpy of atomization.

33. A student has studied the decomposition of a gas AB_3 at 25°C. He obtained the following data.

p(mmHg)	50	100	200	400
relative $t_{1/2}$(s)	4	2	1	0.5

The order of the reaction is

(1) 0 (zero) (2) 0.5 (3) 1 (4) 2

Sol. **4**

$t½ \propto (Co)^{1-n}$

$$= \frac{(t½)_1}{(t½)_2} = \left(\frac{P_1}{P_2}\right)^{1-n}$$

$$= \frac{4}{2} = \left(\frac{50}{100}\right)^{1-n} \Rightarrow 2\left(\frac{1}{2}\right)^{1-n}$$

$$2 = (2)^{n-1}$$

n = 2

34. $K_2Cr_2O_7$ paper acidified with dilute H_2SO_4 turns green when exposed to

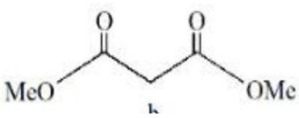
(1) Carbon dioxide (2) Sulphur trioxide (3) Sulphur dioxide (4) Hydrogen sulphide

Sol. **3**

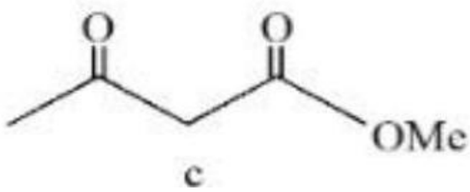
$$K_2Cr_2O_7 + 2H^+ + SO_2 \rightarrow \underset{\text{(green)}}{2Cr^{+3}} + 3SO_4^{-2} + H_2O$$

35. Which will undergo deprotonation most readily in basic medium?

a



b

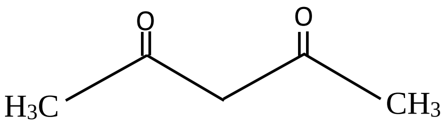


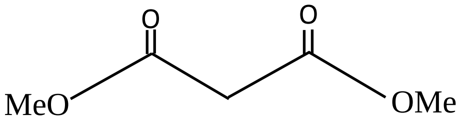
c

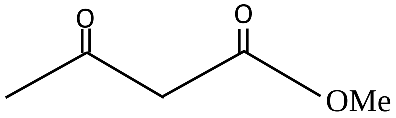
(1) c only (2) only (3) Both a and c (4) b only

Sol. **2**

order of –m effect = $-\overset{\overset{\large O}{||}}{C}-CH_3 > -\overset{\overset{\large O}{||}}{C}-OCH_3$







strong –m effect of both ketone

36. The hybridization and magnetic behaviour of cobalt ion in $[Co(NH_3)_6]^{3+}$ complex, respectively is

(1) d^2sp^3 and paramagnetic(2) $sp^3\ d^2$ and diamagnetic

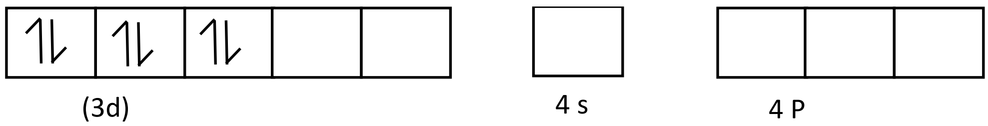
(3) d^2sp^3 and diamagnetic (4) $sp^3\ d^2$ and paramagnetic

Sol. **3**

$[Co(NH_3)_6]^{+3}$

$Co^{+3} \rightarrow [Ar]3d^6 4S^0$

$NH_3 \rightarrow SFL_1$ Pairing of e^-

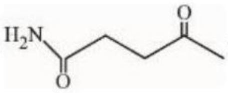


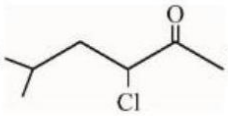
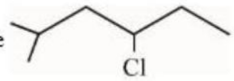
hybridisation d^2sp^3

$\mu = 0$

diamagnetic

37. Given below are two statements:

Statement I :  under Clemmensen reduction conditions will give 

Statement II :  under Wolff-Kishner reduction condition will give 

In the light of the above statements, choose the correct answer from the options given below:

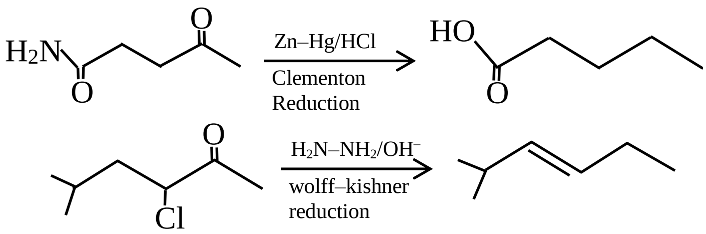
(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Sol. **2**



38. Which of the following cannot be explained by crystal field theory?

(1) The order of spectrochemical series

(2) Stability of metal complexes

(3) Magnetic properties of transition metal complexes

(4) Colour of metal complexes

Sol. **1**

Crystal field theory introduce spectrochemical series based upon the experimental value of Δ but can't explain it's order. While other three points are explained by CFT. Specially when the CFSE increases thermodynamic stability of the comples increases.

39. The number of s-electrons present in an ion with 55 protons in its unipositive state is

(1) 8 (2) 10 (3) 9 (4) 12

Sol. **2**

$$Cs^{+}_{(55)} = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^2, 3d^{10}, 4p^6, 5s^2, 4d^{10}, 5p^6$$

no. of s-electron = 10

40. Which one amongst the following are good oxidizing agents?

(A) Sm^{2+} (B) Ce^{2+} (C) Ce^{4+} (D) Tb^{4+}

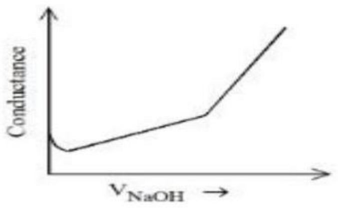
Choose the most appropriate answer from the options given below:

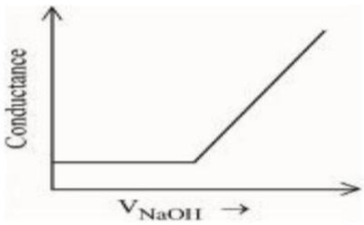
(1) D only (2) C only (3) C and *D* only (4) A and B only

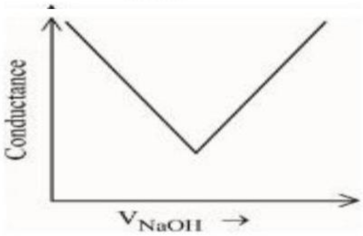
Sol. **3**

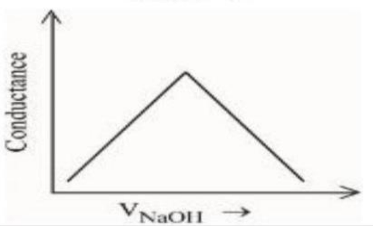
Ce^{+4} & Tb^{+4} are good oxidizing agent.

41. Choose the correct representation of conductometric titration of benzoic acid vs sodium hydroxide.

1.


2.


3.


4.


Sol. **1**

$$C_6H_5COOH + NaOH \rightarrow C_6H_5COONa + H_2O$$

when weak acid C_6H_5COOH titrated against strong base NaOH in the beginning the conductance Inc. slowly and after equivalent point it increase rapidly.

42. Match List I with List II

LIST I Type		LIST II Name	
A.	Antifertility drug	I.	Norethindrone
B.	Tranquilizer	II.	Meprobomate
C.	Antihistamine	III	Seldane
D.	Antibiotic	IV.	Ampicillin

Choose the correct answer from the options given below:

(1) A-I, B-III, C-II, D-IV

(2) A-IV, B-III, C-II, D-I

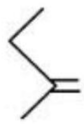
(3) A-I, B-II, C-III, D-IV

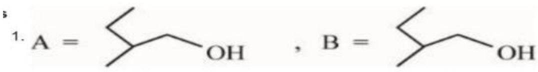
(4)A-II, B-I, C-III, D-IV

Sol. **3**

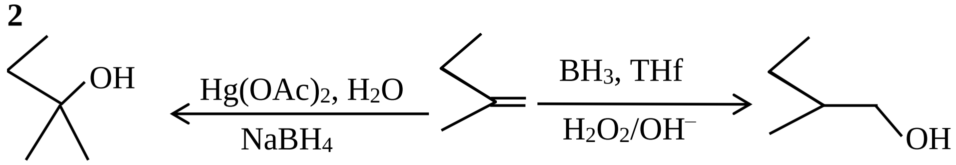
LIST I Type		LIST II Name	
A.	Antifertility drug	I.	Norethindrone
B.	Tranquilizer	II.	Meprobomate
C.	Antihistamine	III	Seldane
D.	Antibiotic	IV.	Ampicillin

43. Find out the major products from the following reaction

$$B \xleftarrow[NaBH_4]{Hg(OAc)_2,\ H_2O}$$  $$\xrightarrow[H_2O_2\,/\,OH^-]{BH_3,\ THF} A$$



Sol. **2**



(A)

44. Given below are two statements, one is labelled as Assertion **A** and the other is labelled as Reason **R**

Assertion : Benzene is more stable than hypothetical cyclohexatriene

Reason : The delocalized π electron cloud is attracted more strongly by nuclei of carbon atoms.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are correct and R is the correct explanation of A

(2) Both A and R are correct but R is NOT the correct explanation of A

(3) A is false but R is true

(4) A is true but R is false

Sol. **1**

Both A and R are correct and R is the correct explanation of A

45. In which of the following reactions the hydrogen peroxide acts as a reducing agent?

(1) $\mathrm{PbS} + 4\mathrm{H_2O_2} \rightarrow \mathrm{PbSO_4} + 4\mathrm{H_2O}$(2) $\mathrm{Mn^{2+}} + \mathrm{H_2O_2} \rightarrow \mathrm{Mn^{4+}} + 2\mathrm{OH^-}$

(3) $\mathrm{HOCl} + \mathrm{H_2O_2} \rightarrow \mathrm{H_3O^+} + \mathrm{Cl^-} + \mathrm{O_2}$(4) $2\mathrm{Fe^{2+}} + \mathrm{H_2O_2} \rightarrow 2\mathrm{Fe^{3+}} + 2\mathrm{OH^-}$

Sol. **3**

$\mathrm{HOCl} + \mathrm{H_2O_2} \rightarrow \mathrm{H_3O^+} + \mathrm{Cl^-} + \mathrm{O_2}$ hydrogen peroxide acts as a reducing agent

46. Given below are two statements:

Statement I : Pure Aniline and other arylamines are usually colourless.

Statement II : Arylamines get coloured on storage due to atmospheric reduction

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are correct

Sol. **3**

47. Correct statement is:

(1) An average human being consumes nearly 15 times more air than food

(2) An average human being consumes 100 times more air than food

(3) An average human being consumes equal amount of food and air

(4) An average human being consumes more food than air

Sol. **1**

An average human being requires. hearly 12 –15 times more air than the food.

48. What is the number of unpaired electron(s) in the highest occupied molecular orbital of the following species : $N_2; N_2^+; O_2; O_2^+$?

(1) 2,1,0,1 (2) 0, 1, 0, 1 (3) 0,1,0,1 (4) 2,1,2,1

Sol. **2**

$N_2 = \sigma_{1s}2, \sigma^*_{1s}2, \sigma_{2s}2, \sigma^*_{2s}2, \pi_{2Px}2 = \pi_{2Py}2, \sigma_{2Pz}2$

no. of e^- present in Homo = 0

$N_2^+ = \sigma_{1s}2, \sigma^*_{1s}2, \sigma_{2s}2, \sigma^*_{2s}2, \pi_{2Px}2 = \pi_{2Py}2, \sigma_{2Pz}1$

no. of unpaired e^- present in HOMO = 1

$O_2 = \sigma_{1s}2, \sigma^*_{1s}2, \sigma_{2s}2, \sigma^*_{2s}2, \sigma_{2Pz}2, \pi_{2Px}2 = \pi_{2Py}2, \pi^*{}_{2Px}1 = \pi^*{}_{2Py}1$

no. of unpaired e^- present in HOMO = 2

$O_2^+ = \sigma_{1s}2, \sigma^*_{1s}2, \sigma_{2s}2, \sigma^*_{2s}2, \sigma_{2Pz}2, \pi_{2Px}2 = \pi_{2Py}2, \pi^*{}_{2Px}1 = \pi^*{}_{2Py}0$

no. of unpaired e^- present in HOMO = 1

49. The metal which is extracted by oxidation and subsequent reduction from its ore is:

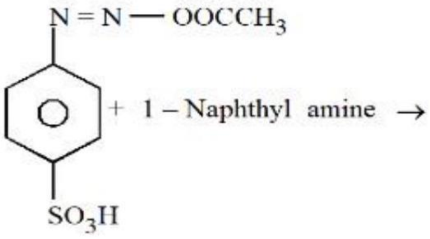
(1) Ag (2) Fe (3) Cu (4) Al

Sol. **1**

$$4Ag(s) + 8CN^-(aq) + 2H_2O(aq) + O_2(g) \rightarrow 4[Ag(CN)_2]^-_{(aq)} + 4OH^-(aq)$$

$$2[Ag(CN)_2]^-_{(aq)} + Zn(s) \rightarrow 2Ag(s) + [Zn(CN)_4]^{-2}_{(aq)}$$

50. Choose the correct colour of the product for the following reaction.



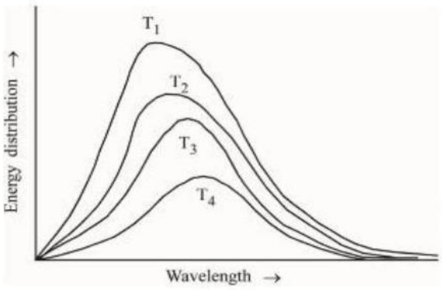
(1) White (2) Red (3) Blue (4) Yellow

Sol. **2**

Red

Section: B

51. Following figure shows spectrum of an ideal black body at four different temperatures. The number of correct statement/s from the following is________________.



A. $T_4 > T_3 > T_2 > T_1$

B. The black body consists of particles performing simple harmonic motion.

C. The peak of the spectrum shifts to shorter wavelength as temperature increases.

D. $\frac{T_1}{v_1} = \frac{T_2}{v_2} = \frac{T_3}{v_3} \neq$ constant

E. The given spectrum could be explained using quantisation of energy.

Sol. **2**

(A) $T_4 > T_3 > T_2 > T_1$

(C) The peak of the spectrum shift to shorter wavelength of temp. Inc.

52. The number of units, which are used to express concentration of solutions from the following is______

Mass percent, Mole, Mole fraction, Molarity, ppm, Molality

Sol. **5**

Conc. Express in → mass percentage

→ mole fraction

→ molarity

→ PPM

→ molality

53. The number of statement/s which are the characteristics of physisorption is______________

A. It is highly specific in nature

B. Enthalpy of adsorption is high

C. It decreases with increase in temperature

D. It results into unimolecular layer

E. No activation energy is needed

Sol. **2**

(C) It decreases with increase in temperature

(E) No activation energy is needed

54. Sum of π - bonds present in peroxodisulphuric acid and pyrosulphuric acid is:

Sol. **8**

Peroxodisulphuric acid ($H_2S_2O_8$)

$$\begin{array}{ccccccc} & O & & & & O & \\ & \| & & & & \| & \\ HO - & S & - O - & O - & & S & - OH \\ & \| & & & & \| & \\ & O & & & & O & \end{array}$$

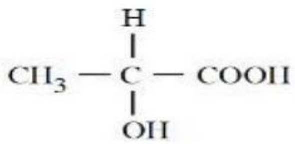
Pyrosulphuric acid ($H_2S_2O_7$)

$$\begin{array}{ccccc} & O & & O & \\ & \| & & \| & \\ HO - & S & - O - & S & - OH \\ & \| & & \| & \\ & O & & O & \end{array}$$

π bond = 4

total π bond = 4 + 4 = 8

55. If the pKa of lactic acid is 5, then the pH of 0.005M calcium lactate solution at 25°C is _________ $\times 10^{-1}$ (Nearest integer)

Lactic acid 

Sol. **85**

$$Ca(Lac)_2 \longrightarrow Ca^{+2} + 2lac^{-}$$

5×10^{-3} $\quad$ 5×10^{-3} $\quad$ 10^{-2} M

Salt of strong base weak acid salt

$$pH = 7 + \frac{1}{2} pka + \frac{1}{2} \log c$$

$$= 7 + \frac{1}{2} \times 5 + \frac{1}{2} \log 10^{-2}$$

$$= 7 + 2.5 - 1 = 8.5$$

$$= 85 \times 10^{-1}$$

56. The total pressure observed by mixing two liquids A and B is 350 mmHg when their mole fractions are 0.7 and 0.3 respectively. The total pressure become 410 mmHg if the mole fractions are changed to 0.2 and 0.8 respectively for A and B. The vapour pressure of pure A is________ mm Hg. (Nearest integer) Consider the liquids and solutions behave ideally.

Sol. **314**

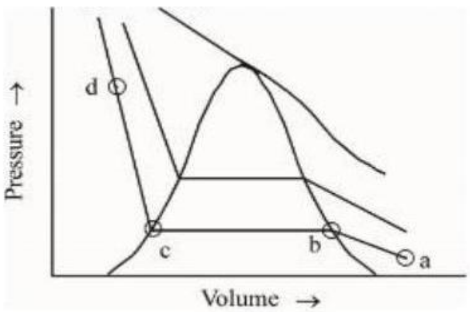
$$XaP_A^o + X_B P_B^o = P_S$$

$$0.7P_A^o + 0.3P_B^o = 350$$

$$0.2P_A^o + 0.8P_B^o = 410$$

$$\therefore P_A^o = 314 \text{torr}$$

57. The number of statement/s, which are correct with respect to the compression of carbon dioxide from point (a) in the Andrews isotherm from the following is_____________



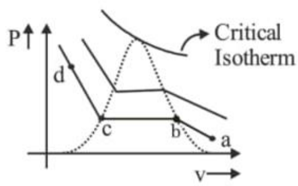
A. Carbon dioxide remains as a gas upto point (b)

B. Liquid carbon dioxide appears at point (c)

C. Liquid and gaseous carbon dioxide coexist between points (b) and (c)

D. As the volume decreases from (b) to (c), the amount of liquid decreases

Sol. **4**



At

(a) $\rightarrow$ CO_2 exist as gas

(b) $\rightarrow$ liquefaction of CO_2 starts

(c) $\rightarrow$ liquefaction ends

(d) $\rightarrow$ CO_2 exist as liquid

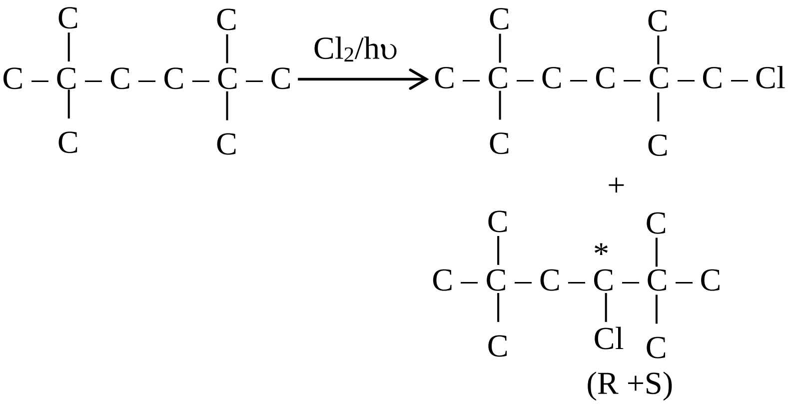
Between (b) & (c) $\rightarrow$ liquid and gaseous CO_2 co-exist.

As volume changes from (b) to (c) gas decreases and liquid increases.

(A), (C) $\rightarrow$ Correct

58. Maximum number of isomeric monochloro derivatives which can be obtained from 2, 2, 5, 5 tetramethylhexane by chlorination is____________

Sol. **3**



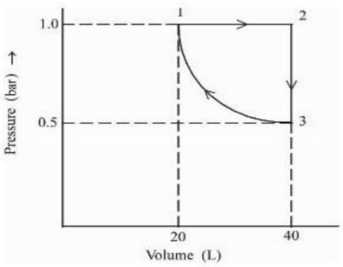
59. Total number of tripeptides possible by mixing of valine and proline is____________

Sol. **8**

(1) P–P–P (2) V–V–V (3) P–V–V (4) V–P–V (5) V–V–P (6) V–P–P

(7) P–V–P (8) P–P–V

60. One mole of an ideal monoatomic gas is subjected to changes as shown in the graph. The magnitude of the work done (by the system or on the system) is_________ J (nearest integer)



Sol. **6**

I→II → Isobaric

II → III → Isochoric

III → I → Isothermal

$W_{I-II} = -1[40-20] = -20\,\text{Lit atm}$

$W_{II-III} = 0$

$$W_{IV-I} = 2.303\,nRt\ \log\frac{V_2}{V_1}$$

$$= 2.303\,PV\log\frac{V_2}{V_1}$$

$$= 2.303(1\times 20)\log 2$$

$$= 2.303\times 20\times 0.3010 = 13.818$$

W total = – 20 + 13.818 = (– 6.182 lit alm) = 6.182 lit alm

Mathematics

SECTION - A

61. If, $f(x)=x^3-x^2f'(1)+xf''(2)-f'''(3), x\in\mathbb{R}$ then

(1) $f(1)+f(2)+f(3)=f(0)$ (2) $2f(0)-f(1)+f(3)=f(2)$

(3) $3f(1)+f(2)=f(3)$ (4) $f(3)-f(2)=f(1)$

Sol. **2**

$f(x) = x^3 - x^2 f'(1) + x f''(2) - f'''(3)$

$f(x) = x^3 - ax^2 + bx - c$

$f'(x) = 3x^2 - 2ax + b$

$f''(x) = 6x - 2a$

$f'''(x) = 6$

$f'''(3) = 6$

$f'(1) = 3 - 2a + b = a \Rightarrow 3a = b + 3$

$f''(2) = 12 - 2a = b \Rightarrow 2a = 12 - b$

$a = 3, b = 6$

$f'''(3) = 6 = c$

$f(x) = x^3 - 3x^2 + 6x - 6$

$f(0) = -6$ $f(2) = 2$

$f(1) = -2$ $f(3) = 12$

62. If the system of equations

$$x+2y+3z=3$$
$$4x+3y-4z=4$$
$$8x+4y-\lambda z=9+\mu$$

has infinitely many solutions, then the ordered pair (λ,μ) is equal to :

(1) $\left(-\frac{72}{5},\frac{21}{5}\right)$ (2) $\left(-\frac{72}{5}\cdot-\frac{21}{5}\right)$ (3) $\left(\frac{72}{5},-\frac{21}{5}\right)$ (4) $\left(\frac{72}{5},\frac{21}{5}\right)$

Sol. **3**

Planes are not parallel

$\therefore (x + 2y + 3z - 3) + a(4x + 3y - 4z - 4)$

$= 8x + 4y - \lambda z - 9 - \mu = 0$

$$\frac{1+4a}{8}=\frac{2+3a}{4}=\frac{3-4a}{-\lambda}=\frac{-3-4a}{-9-\mu}$$

(i) $1 + 4a = 4 + 6a$

$$a=\frac{-3}{2}$$

(ii) $\dfrac{2-\frac{9}{2}}{4}=\dfrac{3+6}{-\lambda}$

$-\lambda = \frac{36}{-5} \times 2$

$\lambda = \frac{72}{5}$

(iii) $\frac{-5}{8} = \frac{-3-4a}{-9-\mu}$

$\frac{5}{8} = \frac{3-6}{-9-\mu}$

$-9-\mu = \frac{-24}{5}$

$\mu = \frac{-45+24}{5}$

$\mu = \frac{-21}{5}$

63. If, then $f(x) = \frac{2^{2x}}{2^{2x}+2}, x \in \mathbb{R}$, then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \cdots + f\left(\frac{2022}{2023}\right)$ is equal to

(1) 1011 (2) 2010 (3) 1010 (4) 2011

Sol. **1**

$f(x) = \frac{4^x}{4^x+2}$

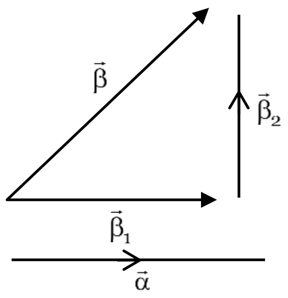
$f(1-x) = \frac{\frac{4}{4^x}}{\frac{4}{4^x}+2} = \frac{4}{4+2.4^x} = \frac{2}{2+4^x}$

f(x) + f(1–x) = 1

64. Let $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k})$ is

(1) 7 (2) 9 (3) 6 (4) 11

Sol. **1**



$\overrightarrow{\beta_1} = \frac{(\vec{\alpha} \cdot \vec{\beta})}{|\vec{\alpha}|}\hat{\alpha}$

$$=\left(\frac{4+6-20}{\sqrt{16+9+25}}\right)\frac{(4,3,5)}{\sqrt{50}}$$

$$=\frac{-10}{50}(4,3,5)$$

$$\vec{\beta}_1=\frac{(-4,-3,-5)}{5}$$

$$\vec{\beta}_1+\vec{\beta}_2-=(1,2,-4)$$

$$\beta_2=\left(1+\frac{4}{5},2+\frac{3}{5},-4+1\right)$$

$$\beta_2=\left(\frac{9}{5},\frac{13}{5},-3\right)$$

$$\therefore 5\beta_2=(9,13,-15)$$

$$\therefore 5\beta_2\cdot(1,1,1)=9+13-15$$

$= 7$

65. Let $y = y(x)$ be the solution of the differential equation $(x^2 - 3y^2)dx + 3xydy = 0, y(1) = 1$. Then $6y^2(e)$ is equal to

(1) $2e^2$ (2) $3e^2$ (3) e^2 (4) $\frac{3}{2}e^2$

Sol. **1**

$(x^2 - 3y^2)dx + 3xydy = 0$

$$\frac{dy}{dx}=\frac{3y^2-x^2}{3xy}$$

$y = vx$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$v+x\frac{dv}{dx}=\frac{3v^2x^2-x^2}{3vx^2}$$

$$v+x\frac{dv}{dx}=\frac{3v^2-1}{3v}$$

$$\frac{xdy}{dx}=\frac{3v^2-1}{3v}-v\Rightarrow\frac{-1}{3v}$$

$$3vdv = -\frac{dx}{x}$$

$$\frac{3v^2}{2}=-\ln x+C\ \ [y(1) = 1]$$

$$\frac{3y^2}{2x^2}=-\ln x+C$$

$C=\frac{3}{2}$

$\frac{3y^2}{2x^2}=\frac{3}{2}\ln e=\ln x$

$\therefore 3y^2=3x^2\ln e-2x^2\ln x$

$x = e \quad 3y^2 = 3e^2\ln e - 2e^2\ln e$

$= e^2 \ln e$

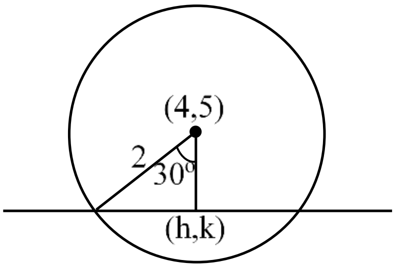
$= e^2$

$6y^2 = 2e^2$

66. The locus of the mid points of the chords of the circle $C_1: (x-4)^2+(y-5)^2=4$ which subtend an angle θ_1 at the centre of the circle C_1, is a circle of radius r_i. If $\theta_1=\frac{\pi}{3}, \theta_3=\frac{2\pi}{3}$ and $r_1^2=r_2^2+r_3^2$, then θ_2 is equal to

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{6}$ (4) $\frac{3\pi}{4}$

Sol. **2**



$r_1 = 2\cos 30^o$

$r_3 = 2\cos 60^o$

$3 = r_2^2 + 1 \Rightarrow r_2 = \sqrt{2}$

$2\cos\theta = \sqrt{2}$, $\theta=\frac{\pi}{4}$

$\therefore \theta_2=\frac{\pi}{2}$

67. The number of real solutions of the equation $3\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)+5=0$, is

(1) 0 (2) 3 (3) 4 (4) 2

Sol. **1**

$3\left[\left(x+\frac{1}{x}\right)^2-2\right]-2\left[x+\frac{1}{x}\right]+5=0$

$3t^2 - 2t - 1 = 0$

$3t^2 - 3t + t - 1 = 0$

$(3t + 1)(t - 1) = 0$

$t = 1, t = \frac{-1}{3}$

68. Let A be a 3×3 matrix such that $|\text{adj}(\text{adj}(\text{adj}\,A))| = 12^4$ Then $|A^{-1}\text{adj}\,A|$ is equal to

(1) $\sqrt{6}$ (2) $2\sqrt{3}$ (3) 12 (4) 1

Sol. **2**

$$|\text{adj}(\text{adj adj}A)| = |A|^{(n-1)^3} = 12^4$$

$$|A|^8 = (12)^4$$

$$|A| = (12)^{\frac{1}{2}}$$

$$\therefore \left|A^{-1} \cdot \text{adj}(A)\right| = \left|A^{-1}\right| \times |\text{adj}A|$$

$$= \frac{1}{|A|} \times |A|^{n-1}$$

$$= \frac{1}{|A|} \times |A|^2 = |A| = \sqrt{12} = 2\sqrt{3}$$

69. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}} dx$ is equal to

(1) 2π (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

Sol. **1**

$$48 \int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{dx}{\sqrt{9-4x^2}} = \frac{48}{2} \sin^{-1}\left\{\frac{2x}{3}\right\}^{\frac{3\sqrt{3}}{4}}$$

$$= 24\left[\sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{\sqrt{2}}\right]^{\frac{3\sqrt{3}}{4}}$$

$$= 24\left[\frac{\pi}{3} - \frac{\pi}{4}\right] = 2\pi$$

70. The number of square matrices of order 5 with entries form the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is

(1) 125 (2) 225 (3) 150 (4) 120

Sol. **4**

$$\begin{bmatrix} \_ & \_ & \_ & \boxed{1} & \_ \\ \_ & \_ & \boxed{1} & \_ & \_ \\ \_ & \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ & \_ \end{bmatrix} \longrightarrow {}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1$$

${}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1$

$= 120$

71. If $\left({}^{30}C_1\right)^2+2\left({}^{30}C_2\right)^2+3\left({}^{30}C_3\right)^2+....+30\left({}^{30}C_{30}\right)^2=\frac{\alpha 60!}{(30!)^2}$ then α is equal to :

(1) 30 (2) 10 (3) 60 (4) 15

Sol. **4**

$C_1^2+2C_2^2+3C_3^2...+30C_{30}^2$

$S=0C_0^2+1C_1^2+...+30C_{30}^2$

$S=30C_{30}^2+29C_{29}^2+...+0C_0^2$

$2S=30\left[C_0^2+C_1^2+...+C_{30}^2\right]$

$S=15\times\ {}^{60}C_{30}=\frac{\alpha\cdot 60!}{(30!)^2}\Rightarrow\alpha=15$

72. Let the plane containing the line of intersection of the planes $P1: x+(\lambda+4)y+z=1$ and $P2: 2x+y+z=2$ pass through the points (0,1,0) and (1,0,1). Then the distance of the point $(2\lambda,\lambda,-\lambda)$ from the plane P2 is

(1) $4\sqrt{6}$ (2) $3\sqrt{6}$ (3) $5\sqrt{6}$ (4) $2\sqrt{6}$

Sol. **2**

$[x+(\lambda+4)y+z-1]+\mu[2x+y+z-2]=0$

(0,1,0)

(i) $(\lambda+4-1)+\mu[-1]=0$

$\lambda-\mu=-3$

(1,0,1) (ii) $1+\mu[1]=0\Rightarrow\mu=-1,\ \lambda=-4$

$\therefore$ point $(-8,-4,4)$; $2x+y+z-2=0$

$$d=\left|\frac{-16-4+4-2}{\sqrt{6}}\right|=\frac{18}{\sqrt{6}}=3\sqrt{6}$$

73. Let f(x) be a funciton such that $f(x+y)=f(x)\cdot f(y)$ for all $x, y\in N$. If f(1) = 3 and $\sum_{k=1}^{n}f(k)=3279$, then the value of n is

(1) 9 (2) 6 (3) 8 (4) 7

Sol. **4**

$f(x+y)=f(x).f(y),\ x,y\in N$

$f(2)=3^2$

$f(3)=3^3$ $\quad\therefore 3\frac{\left[3^n-1\right]}{2}=3279$

$3^n-1=1093\times 2$

$3^n-1=2186$

$3^n=2187$

$n=7$

74. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$, be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to :

(1) 210 (2) 220 (3) 200 (4) 105

Sol. **1**

a + (a + 2d) = 10 $\Rightarrow$ a + d = 5 ...(1)

Mean $\Rightarrow \dfrac{\frac{6}{2}[2a+5d]}{6} = \dfrac{19}{2}$

2a + 5d = 19 ...(2)

from (1) and (2)

3d = 9 $\Rightarrow$ d = 3; a = 2

$\therefore \sigma^2 = \dfrac{\sum x_i^2}{n} - (\bar{x})^2$

$= \dfrac{2^2+5^2+8^2+11^2+14^2+17^2}{6} - \left(\dfrac{19}{2}\right)^2$

$= \dfrac{699}{6} - \dfrac{361}{4}$

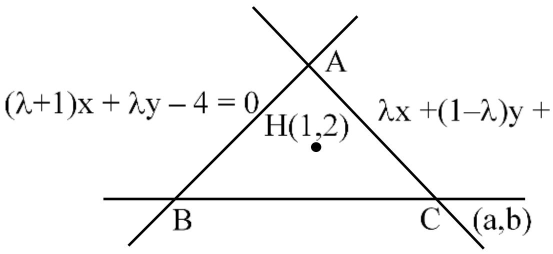
$= \dfrac{233}{2} - \dfrac{361}{4}$

$8\sigma^2 = 932 - 722 = 210$

75. The equations of the sides AB and AC of a triangle ABC are $(\lambda+1)x + \lambda y = 4$ and $\lambda x + (1-\lambda)y + \lambda = 0$ respectively. Its vertex A is on the y - axis and its orthocentre is (1,2). The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is :

(1) 4 (2) 2 (3) $\sqrt{6}$ (4) $2\sqrt{2}$

Sol. **4**



$(\lambda+1)(1-\lambda)x + \lambda(1-\lambda)y = 4(1-\lambda)$

$\lambda^2 x + \lambda(1-\lambda)y = -\lambda^2$

– – +

$(1-\lambda^2-\lambda^2)x = 4 - 4\lambda + \lambda^2$

$(1-2\lambda^2)x = 4 - 4\lambda + \lambda^2$

$x = 0 \Rightarrow \lambda = 2$

$$\left.\begin{array}{l} AB: 3x + 2y = 4 \\ AC: 2x - y + 2 = 0 \end{array}\right\} A(0, 2)$$

$CH \perp AB$

$$\left(\frac{b-2}{a-1}\right) \times \left(\frac{-3}{2}\right) = -1$$

$3b - 6 = 2a - 2$ Also $2a - b + 2 = 0$

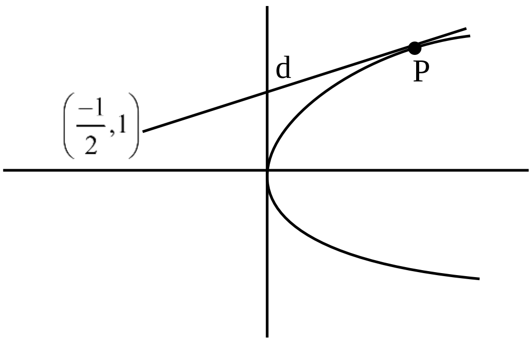
$3b - 2a = 4$ $b = 2a + 2$

$6a + 6 = 2a + 4$ $C\left(-\frac{1}{2}, 1\right)$

$4a = -2$

$a = \frac{-1}{2}, b = 1$

$\therefore y^2 = 6x$



$ty = x + \frac{3}{2}t^2$

$t = -\frac{1}{2} + \frac{3}{2}t^2$

$3t^2 - 1 = 2t$

$3t^2 - 2t - 1 = 0$

$3t^2 - 3t + t - 1 = 0$

$(3t + 1)(t - 1) = 0$

$t = 1$

$P\left(\frac{3}{2}, 3\right)$ $\therefore d = \sqrt{\left(\frac{3}{2} + \frac{1}{2}\right)^2 + (3-1)^2}$

$= \sqrt{4+4} = 2\sqrt{2}$

76. Let p and q be two statements. Then $\sim (p \wedge (p \Rightarrow \sim q))$ is equivalent to

(1) $p \vee (p \wedge q)$ (2) $p \vee (p \wedge (\sim q))$ (3) $(\sim p) \vee q$ (4) $p \vee ((\sim p) \wedge q)$

Sol. **3**

$P \wedge (P \Rightarrow \sim q)$ $\qquad$ $P \rightarrow q$

$P \wedge (\sim P \vee \sim q)$

$\therefore$ Its negation will be

$\sim P \vee [P \wedge q]$

$= [\sim P \vee P] \wedge [\sim P \vee q]$

$= \sim P \vee q$

77. The set of all values of a for which $\lim_{x \to a}([x-5]-[2x+2]) = 0$, where $[\propto]$ denotes the greatest integer less than or equal to α is equal to

(1) $[-7.5, -6.5)$ (2) $[-7.5, -6.5]$ (3) $(-7.5, -6.5]$ (4) $(-7.5, -6.5)$

Sol. **4**

$\lim_{x \to \alpha}([x]-5-[2x]-2) = 0$

$\lim_{x \to \alpha}([x]-[2x]) = 7$

$[\alpha]-[2\alpha] = 7$

If $\alpha = -7.5$ $\quad$ $[-7.5] = -8$

$[-15] = -15$ $\quad$ $\therefore -8 + 15 = 7$

If $\alpha = -6.5$ $\quad$ $[-6.5] = -7$ $\quad$ $-7 + 13 = 6$

$[-13] = -13$

$\therefore \quad \alpha \in (-7.5, -6.5)$

78. If the foot of the perpendicular drawn from (1,9,7) to the line passing through the point (3,2,1) and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

(1) 3 (2) 1 (3) -1 (4) 5

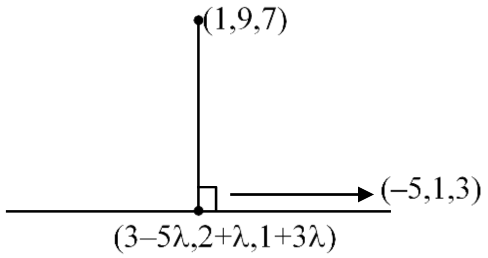
Sol. **4**

$\bar{n}_1 = (1,2,1) \bar{n}_2 = (0,3,-1)$

$$\bar{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix} = \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3)$$

$= (-5, 1, 3)$

$\therefore \text{line}: \frac{x-3}{-5} = \frac{y-2}{1} = \frac{z-1}{3} = \lambda$



$(2–5\lambda).–5+(\lambda–7).1+(3\lambda–6).3=0$

$25\lambda–10+\lambda–7+9\lambda–18=0$

$35\lambda=35$

$\lambda=1$

$\therefore$ Point is $(–2,3,4)$ $\quad\alpha+\beta+\gamma=5$

79. The number of integers, greater than 7000 that can be formed, using the digits 3,5,6,7,8 without repetition, is

(1) 168 (2) 220 (3) 120 (4) 48

Sol. **1**

C-1 □ □ □ □
$2\times4\times3\times2=48$

C-2 5!(5 digit nos) = $\frac{120}{168}$

80. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is

(1) $-\frac{1}{2}(\sqrt{3}-i)$ (2) $-\frac{1}{2}(1-i\sqrt{3})$ (3) $\frac{1}{2}(1-i\sqrt{3})$ (4) $\frac{1}{2}(\sqrt{3}+i)$

Sol. **1**

$$\frac{\pi}{2}-\frac{2\pi}{9}$$

$$=\frac{95-4\pi}{18}=\frac{5\pi}{18}$$

$$\Rightarrow\frac{1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18}-i\sin\frac{5\pi}{18}}$$

$$=\frac{2\cos^2\frac{5\pi}{36}+2i\sin\frac{5\pi}{36}.\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36}-2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}}\Rightarrow\left(\frac{e^{i\frac{5\pi}{36}}}{e^{-i\frac{5\pi}{36}}}\right)^3$$

$$=e^{i\left(\frac{5\pi}{18}\right)^3}\quad=e^{i\left(\frac{5\pi}{6}\right)}$$

$$\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)=-\frac{\sqrt{3}}{2}+\frac{i}{2}$$

SECTION B

81. If the shortest distance between the lines $\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4}$ and $\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{z+2\sqrt{6}}{5}$ is 6 , then the square of sum of all possible values of λ is

Sol. **384**

$P\left(-\sqrt{6},\sqrt{6},\sqrt{6}\right)$ $\qquad Q\left(\lambda,2\sqrt{6},-2\sqrt{6}\right)$

$\bar{n}_1=(2,3,4)$ $\qquad \bar{n}_2=(3,4,5)$

$$\bar{n}_1\times\bar{n}_2\Rightarrow\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&4\\3&4&5\end{vmatrix}=\hat{i}(-1)-\hat{j}(-2)+\hat{k}(-1)$$

= (–1, 2, –1)

$$\therefore S_d\left|\frac{\overline{PQ}\cdot(-1,2,-1)}{\sqrt{6}}\right|=\frac{\left(\lambda+\sqrt{6},\sqrt{6},-3\sqrt{6}\right)\cdot(-1,2,-1)}{\sqrt{6}}$$

$$=\left|\frac{-\lambda-\sqrt{6}+2\sqrt{6}+3\sqrt{6}}{\sqrt{6}}\right|=6$$

$$\Rightarrow\left|-\lambda+4\sqrt{6}\right|=6\sqrt{6}$$

(+) $\quad -\lambda+4\sqrt{6}=6\sqrt{6}$ $\qquad$ (–) $\quad \lambda-4\sqrt{6}=6\sqrt{6}$

$\lambda=-2\sqrt{6}$ $\qquad \lambda=10\sqrt{6}$

$$\therefore\left(8\sqrt{6}\right)^2=384$$

82. Three urns A, B and C contain 4 red, 6 black; 5 red, 5 black; and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2=\lambda x$ with one vertex at the vertex of the parabola, is

Sol. 432

A	B	C
4R	5R	4R
6B	5B	4B

$$\text{P(Red from C)}=\frac{\frac{1}{3}\times\frac{\lambda}{\lambda+4}}{\frac{1}{3}\cdot\frac{\lambda}{\lambda+4}+\frac{1}{3}\cdot\frac{4}{10}+\frac{1}{3}\cdot\frac{5}{10}}$$

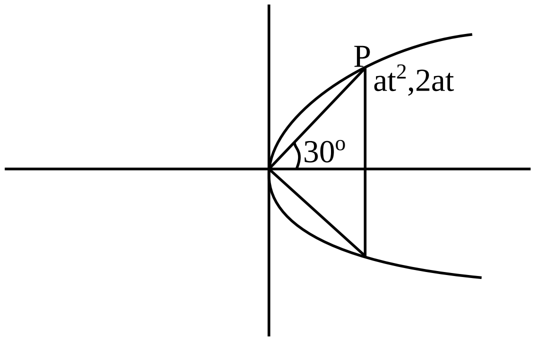
$$=\frac{\frac{\lambda}{\lambda+4}}{\frac{\lambda}{\lambda+4}+\frac{9}{10}}$$

$$\Rightarrow \frac{10\lambda}{10\lambda + 9(\lambda+4)} = \frac{4}{10}$$

$$\Rightarrow 100\lambda = 40\lambda + 36\lambda + 144$$

$24\lambda = 144$

$\lambda = 6$



$m = \frac{2}{t} = \frac{1}{\sqrt{3}}$

$t = 2\sqrt{3}$

$P\left(12a, 4\sqrt{3}a\right)$

$(\text{Side})^2 = 144a^2 + 48a^2$

$= 192 \times \frac{9}{4} = 432$

83. Let $S = \{\theta \in [0,2\pi) : \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$.

Then $\sum_{\theta \in s} \sin^2\left(\theta + \frac{\pi}{4}\right)$ is equal to

Sol. **2**

$\tan(\pi\cos\theta) = \tan[-\pi\sin\theta]$

$\pi\cos\theta = n\pi - \pi\sin\theta \qquad (n \in I)$

$\cos\theta + \sin\theta = n$

$n \in \left[-\sqrt{2}, \sqrt{2}\right] \qquad n \in \{-1, 0, 1\}$

$\cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}, \qquad \cos\left(\theta - \frac{\pi}{4}\right) = 0, \qquad \cos\left(\theta - \frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

$\theta - \frac{\pi}{4} = 2m\pi \pm \frac{\pi}{4}, \ \theta - \frac{\pi}{4} = 2m\pi + \frac{\pi}{2}, \qquad \theta - \frac{\pi}{4} = 2m\pi \pm \frac{3\pi}{4}$

$\theta = 2m\pi + \frac{\pi}{2}, \qquad \theta = 2m\pi + \frac{3\pi}{4}, \qquad \theta = 2m\pi + \pi$

$\theta = 2m\pi, \qquad \theta = 2m\pi - \frac{\pi}{2}$

$\theta = \left\{\frac{\pi}{2}, 0, \cancel{\frac{\pi}{4}}, \pi, \frac{3\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

$$\sin\left(\theta+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}},\frac{-1}{\sqrt{2}}$$

$$\therefore \sum\sin^2\left(\theta+\frac{\pi}{4}\right)=\frac{1}{2}+\frac{1}{2}+\cancel{1}+\frac{1}{2}+\frac{1}{2}=2$$ Ans.

84. If $\frac{1^3+2^3+3^3+\cdots \text{ up to n terms}}{1\cdot3+2\cdot5+3\cdot7+\cdots \text{ up to n terms}}=\frac{9}{5}$, then the value of n is

Sol. **5**

$$\frac{\left(\frac{n(n+1)}{2}\right)^2}{\sum r(2r+1)}$$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{\frac{2n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}}$$

$$\Rightarrow \frac{\frac{n(n+1)}{4}}{\frac{2n+1}{3}+\frac{1}{2}} \Rightarrow \frac{\frac{n(n+1)}{4}}{\frac{(4n+5)}{6}}=\frac{9}{5}$$

$$\Rightarrow \frac{3(n+1)n}{2(4n+5)}=\frac{9}{5}$$

$\Rightarrow 5n^2 + 5n = 24n + 30$

$\Rightarrow 5n^2 - 19n - 30 = 0$

$5n^2 - 25n + 6n - 30 = 0$

$(5n + 6)(n - 5) = 0$

$n = 5$

85. Let the sum of the coefficients of the first three terms in the expansion of $\left(x-\frac{3}{x^2}\right)^n, x \neq 0. n \in \mathbb{N}$, be 376. Then the coefficient of x^4 is

Sol. **405**

${}^nC_0 - {}^nC_1 \cdot 3 + {}^nC_2 \cdot 3^2$

$1 - 3n + \frac{9n(n-1)}{2} = 376$

$2 - 6n + 9n^2 - 9n = 752$

$9n^2 - 15n - 750 = 0$

$3n^2 - 5n - 250 = 0$

$3n^2 - 30n + 25n - 250 = 0$

$(3n + 25)(n - 10) = 0$

$n = 10$

$\therefore T_{r+1} = {}^{10}C_r (x)^{10-r} \left(\frac{-3}{x^2}\right)^r$

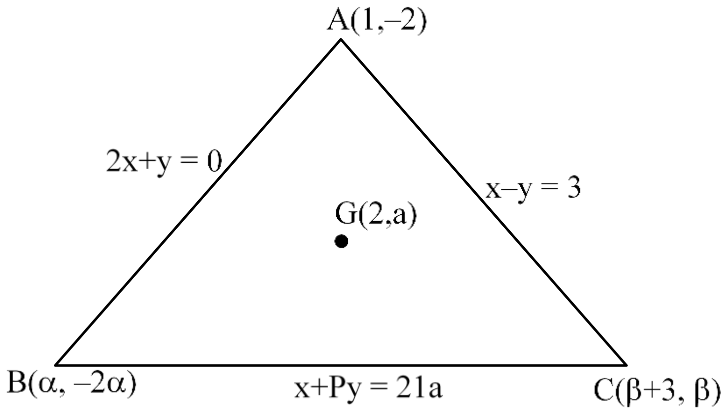
$x^{10-3r} = x^4 \Rightarrow 3r = 6$

$r = 2$

$\therefore T_3 = {}^{10}C_2 \times 3^2 \Rightarrow \frac{10\times 9}{2}\times 9 = 405$

86. The equations of the sides AB, BC and CA of a triangle ABC are : $2x + y = 0, x + py = 21a, (a \neq 0)$ and $x - y = 3$ respectively. Let $P(2, a)$ be the centroid of $\triangle$ ABC. Then $(BC)^2$ is equal to

Sol. **122**



$\frac{\alpha+\beta+4}{3} = 2 \qquad \frac{-2\alpha-2+\beta}{3} = a$

$\alpha + \beta = 2 \qquad -2\alpha + \beta = 3a + 2$

$-2\alpha + 2 - \alpha = 3a + 2$

$\alpha = -a$

put 'B' in BC

$\alpha - 2p\alpha = 21a$

$\alpha.(1 - 2p) = 21a \qquad 2p - 1 = 21$

$p = 11$

put 'C' in BC

$\beta + 3 + 11\beta = 21a$

$21\alpha + 12\beta + 3 = 0$

also $\beta = 2 - \alpha \qquad$ Solving $\alpha = -3, \beta = 5$

$\therefore BC = \sqrt{122}$

$BC^2 = 122$

87. Let $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}, \vec{a}\cdot\vec{c} = 7, 2\vec{b}\cdot\vec{c} + 43 = 0, \vec{a}\times\vec{c} = \vec{b}\times\vec{c}$. Then $|\vec{a}\cdot\vec{b}|$ is equal to

Sol. **8**

$\bar{a} = (1, 2, \lambda) \qquad \bar{b} = (3, -5, -\lambda)$

$\bar{a}\cdot\bar{c}=7$, $\quad \bar{b}\cdot\bar{c}=\frac{-43}{2}$,

$(\bar{a}-\bar{b})\times\bar{c}=0$

$c=x[-2,7,2\lambda]=(-2x,7x,2\lambda x)$

now, $\quad \bar{a}.\bar{c}=-2x+14x+2\lambda^2x=7$

$$2\lambda^2x+12x=7 \qquad ...(1)$$

$\bar{b}\cdot\bar{c}=-6x-35x-2\lambda^2x=\frac{-43}{2}$

$$-41x-2\lambda^2x=\frac{-43}{2} \qquad ...(2)$$

by adding (1) + (2)

$$-29x=\frac{-29}{2}\Rightarrow x=\frac{1}{2}$$

$\therefore \lambda^2+6=7 \quad \Rightarrow \lambda^2=1$

$|\bar{a}\cdot\bar{b}|=|3-10-\lambda^2|=|-8|$

88. The minimum number of elements that must be added to the relation $R=\{(a,b),(b,c),(b,d)\}$ on the set $\{a,b,c,d\}$ so that it is an equivalence relation, is

Sol. **13**

	a	b	c	d
a	1	✓	8	9
b	5	2	✓	✓
c	10	6	3	11
d	12	7	13	4

1, 2, 3, 4 $\rightarrow$ for reflexive

5, 6, 7 $\rightarrow$ for symmetric

8, 9, 10, 11, 12, 13 $\rightarrow$ for transitive

89. If the area of the region bounded by the curves $y^2-2y=-x, x+y=0$ is A, then 8 A is equal to

Sol. **36**

$y^2-2y+1=-x+1$ $\qquad$ $x+y=0$

$(y-1)^2=-(x-1)$ $\qquad$ $x+1+y+1=0$

$y^2=-4Ax$ $\qquad$ $x+y+2=0$

$y=y-1$

$x=x-1$

$y^2=-x$

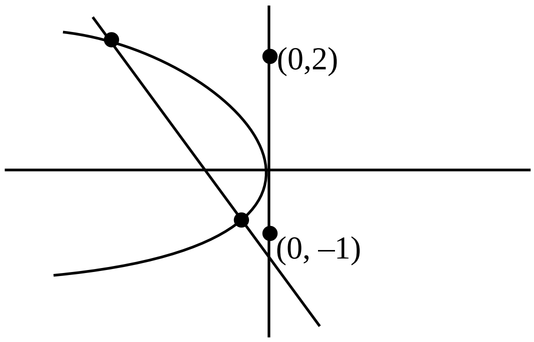
$x+y=-2$

$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2, y = -1$



Area $= \int_{-1}^{2} \left(-y^2\right) - (-2 - y)\,dy$

$= \left(\frac{-y^3}{3} + 2y + \frac{y^2}{2}\right)_{-1}^{2}$

$= \left(\frac{-8}{3} + 4 + 2\right) - \left(\frac{1}{3} - 2 + \frac{1}{2}\right)$

$= \frac{-8}{3} + 6 - \frac{1}{3} + 2 - \frac{1}{2}$

$A = -3 + 8 - \frac{1}{2} \Rightarrow \frac{9}{2}$

$\therefore$ 8A = 36

90. Let f be a differentiable function defined on $\left[0, \frac{\pi}{2}\right]$ such that $f(x) > 0$ and $f(x) + \int_0^x f(t)\sqrt{1 - (\log_e f(t))^2}\,dt = e, \forall x \in \left[0, \frac{\pi}{2}\right]$. Then $\left(6\log_e f\left(\frac{\pi}{6}\right)\right)^2$ is equal to

Sol. $f'(x) + f(x) \cdot \sqrt{1 - \log^2 \cdot f(x)} = 0$

$\frac{dy}{dx} = -y\sqrt{1 - \log^2 y}$

$f(0) = e$

$\frac{dy}{y\sqrt{1 - \log^2 y}} = -dx$

$\log y = t$

$\frac{1}{y}dy = dt$

$\frac{dt}{\sqrt{1 - t^2}} = -dx$

$\sin^{-1}(t) = -x + C$

$\sin^{-1}[\log y] = -x + C$

$x = 0 \quad \mathrm{Sin}^{-1}(1) = C \Rightarrow \frac{\pi}{2}$

$\log(y) = \mathrm{Sin}\left(\frac{\pi}{2} - x\right)$

$x = \frac{\pi}{6} \quad \log_e\left[f\left(\frac{\pi}{6}\right)\right] = \mathrm{Sin}\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\therefore \left(6 \times \frac{\sqrt{3}}{2}\right)^2 = 27$