FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Friday 29th July, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

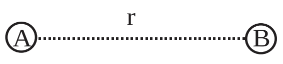
1. Two identical metallic spheres A and B when placed at certain distance in air repel each other with a force of F. Another identical uncharged sphere C is first placed in contact with A and then in contact with B and finally placed at midpoint between spheres A and B. The force experienced by sphere C will be :

(A) 3F/2 (B) 3F/4

(C) F (D) 2F

Official Ans. by NTA (B)

Sol. Let $q_A = q_B = q$

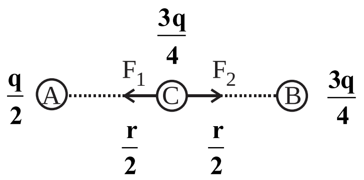


$F = \frac{Kq^2}{r^2}$

When C is placed in contact with A, charge on A & C will be $= \frac{q}{2}$

Now C is placed in contact with B, charge on B & C will be $= \frac{q + \frac{q}{2}}{2} = \frac{3q}{4}$

Now,



$$F' = F_2 - F_1 = \frac{\left(K\frac{3q}{4} - K\frac{q}{2}\right)}{\frac{r^2}{4}} \cdot \frac{3q}{4}$$

$$= \frac{3Kq^2}{4r^2} = \frac{3F}{4} \quad \text{(B)}$$

2. Match List I with List II.

List I		List II	
A.	Torque	I.	Nms^{-1}
B.	Stress	II.	$J\ kg^{-1}$
C.	Latent Heat	III.	Nm
D.	Power	IV.	Nm^{-2}

Choose the correct answer from the options given below:

(A) A-III, B-II, C-I, D-IV

(B) A-III, B-IV, C-II, D-I

(C) A-IV, B-I, C-III, D-II

(D) A-II, B-III, C-I, D-IV

Official Ans. by NTA (B)

Sol. Torque $= F \times r_{\perp}$ Nm

Stress $= \frac{\text{Force}}{\text{Area}}$ N/m^2

Latent heat $= \frac{\text{Energy}}{\text{Mass}}$ $J\ Kg^{-1}$

Power $= \frac{\text{Work}}{\text{Time}}$ $N\ ms^{-1}$

A-III, B-IV, C-II, D-I

3. Two identical thin metal plates has charge q_1 and q_2 respectively such that $q_1 > q_2$. The plates were brought close to each other to form a parallel plate capacitor of capacitance C. The potential difference between them is :

(A) $\frac{(q_1 + q_2)}{C}$ (B) $\frac{(q_1 - q_2)}{C}$

(C) $\frac{(q_1 - q_2)}{2C}$ (D) $\frac{2(q_1 - q_2)}{C}$

Official Ans. by NTA (C)

Sol. Electric field between plates $E = \frac{q_1 - q_2}{2A \epsilon_0}$

$$V = Ed = \frac{q_1 - q_2}{2A \epsilon_0} d$$

$$V = \frac{q_1 - q_2}{2C}$$

4. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Alloys such as constantan and manganin are used in making standard resistance coils.

Reason R: Constantan and manganin have very small value of temperature coefficient of resistance.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Official Ans. by NTA (A)

Sol. Theory based

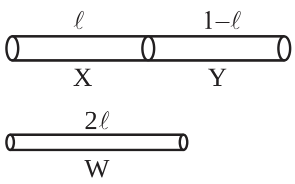
5. A 1 m long wire is broken into two unequal parts X and Y The X part of the wire is streched into another wire W. Length of W is twice the length of X and the resistance of W is twice that of Y. Find the ratio of length of X and Y.

(A) 1 : 4 (B) 1 : 2

(C) 4 :1 (D) 2 : 1

Official Ans. by NTA (B)

Sol.



$$\frac{R_X}{R_Y} = \frac{\ell_X}{\ell_Y}$$

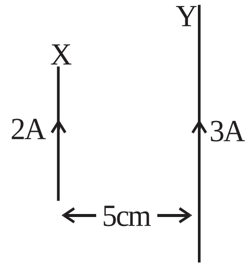
When wire is stretched to double of its length, then resistance becomes 4 times

$R_W = 4R_X = 2R_Y$

$$\frac{R_X}{R_Y} = \frac{1}{2}$$

So. $\frac{\ell_X}{\ell_y} = \frac{1}{2}$

6. A wire X of length 50 cm carrying a current of 2 A is placed parallel to a long wire Y of length 5 m. The wire Y carries a current of 3 A. The distance between two wires is 5 cm and currents flow in the same direction. The force acting on the wire Y is :



(A) 1.2×10^{-5} N directed towards wire X.

(B) 1.2×10^{-4} N directed away from wire X.

(C) 1.2×10^{-4} N directed towards wire X.

(D) 2.4×10^{-5} N directed towards wire X.

Official Ans. by NTA (A)

Sol. Force of interaction = $I_1 \ell_1 B_{12}$

$$= \frac{\mu_0 I_1 I_2}{2\pi r}\ell_1$$

$$= \frac{4\pi\times10^{-7}\times6\times0.5}{2\pi\times5\times10^{-2}}$$

$= 1.2 \times 10^{-5}$ towards X

7. A juggler throws balls vertically upwards with same initial velocity in air. When the first ball reaches its highest position, he throws the next ball. Assuming the juggler throws n balls per second, the maximum height the balls can reach is

(A) g/2n (B) g/n

(C) 2gn (D) $g/2n^2$

Official Ans. by NTA (D)

Sol. Time taken by ball to reach highest point = $\frac{u}{g}$

Frequency of throw = $\frac{g}{u} = n$

$$\Rightarrow u = \frac{g}{n}$$

$$H_{max} = \frac{u^2}{2g} = \frac{\left(\frac{g}{n}\right)^2}{2g}$$

$$\frac{g}{2n^2}$$

8. A circuit element X when connected to an a.c. supply of peak voltage 100 V gives a peak current of 5 A which is in phase with the voltage. A second element Y when connected to the same a.c. supply also gives the same value of peak current which lags behind the voltage by $\frac{\pi}{2}$. If X and Y are connected in series to the same supply, what will be the rms value of the current in ampere ?

(A) $\frac{10}{\sqrt{2}}$ (B) $\frac{5}{\sqrt{2}}$ (C) $5\sqrt{2}$ (D) $\frac{5}{2}$

Official Ans. by NTA (D)

Sol. Element X should be resistive with R = 20Ω

Element Y should be inductive with X_L = 20 Ω

When X and Y are connector in series

$$Z=\sqrt{X_L^2+R^2}=20\sqrt{2}$$

$$I_0=\frac{E_0}{Z}=\frac{100}{20\sqrt{2}}=\frac{5}{\sqrt{2}}A$$

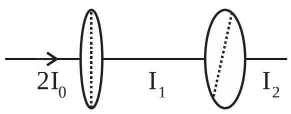
$$I_{rms}=\frac{I_0}{\sqrt{2}}=\frac{5}{2}A$$

9. An unpolarised light beam of intensity $2I_0$ is passed through a polaroid P and then through another polaroid Q which is oriented in such a way that its passing axis makes an angle of 30° relative to that of P. The intensity of the emergent light is

(A) $\frac{I_0}{4}$ (B) $\frac{I_0}{2}$ (C) $\frac{3I_0}{4}$ (D) $\frac{3I_0}{2}$

Official Ans. by NTA (C)

Sol.



$$I_1=\frac{1}{2}(2I_0)=I_0$$

$$I_2=I_1\cos^2 30^\circ$$

$$=I_0.\frac{3}{4}=\frac{3I_0}{4}$$

10. An α particle and a proton are accelerated from rest through the same potential difference. The ratio of linear momenta acquired by above two particals will be :

(A) $\sqrt{2}:1$ (B) $2\sqrt{2}:1$

(C) $4\sqrt{2}:1$ (D) 8 : 1

Official Ans. by NTA (B)

Sol. $p=\sqrt{2mE}=\sqrt{2mqV}$

$$\frac{p_\alpha}{p_p}=\sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}}=\sqrt{\frac{4}{1}\times\frac{2}{1}}$$

$$=\frac{2\sqrt{2}}{1}$$

11. Read the following statements:

(A) Volume of the nucleus is directly proportional to the mass number.

(B) Volume of the nucleus is independent of mass number.

(C) Density of the nucleus is directly proportional to the mass number.

(D) Density of the nucleus is directly proportional to the cube root of the mass number.

(E) Density of the nucleus is independent of the mass number.

Choose the correct option from the following options.

(A) (A) and (D) only. (B) (A) and (E) only.

(C) (B) and (E) only. (D) (A) and (C) only

Official Ans. by NTA (B)

Sol. $R \propto A^{1/3}$

$$V=\frac{4}{3}\pi R^3 \propto A$$

Mass ∝ A

So density is independent of A.

12. An object of mass 1 kg is taken to a height from the surface of earth which is equal to three times the radius of earth. The gain in potential energy of the object will be

[If, $g=10ms^{-2}$ and radius of earth = 6400 km]

(A) 48 MJ (B) 24 MJ

(C) 36 MJ (D) 12 MJ

Official Ans. by NTA (A)

Sol. $U_i = \frac{-GMm}{R}$

$U_f = -\frac{GMm}{4R}$

$\Delta U = U_f - U_i = \frac{3GMm}{4R}$

$= \frac{3}{4} mgR$

$= \frac{3}{4} \times 1 \times 10 \times 64 \times 10^5$

= 48 MJ

13. A ball is released from a height h. If t_1 and t_2 be the time required to complete first half and second half of the distance respectively. Then, choose the correct relation between t_1 and t_2.

(A) $t_1 = (\sqrt{2})t_2$ (B) $t_1 = (\sqrt{2}-1)t_2$

(C) $t_2 = (\sqrt{2}+1)t_1$ (D) $t_2 = (\sqrt{2}-1)t_1$

Official Ans. by NTA (D)

Sol. For first $\frac{h}{2}$

$\frac{h}{2} = \frac{1}{2} g t_1^2$

For total height h

$h = \frac{1}{2} g (t_1 + t_2)^2$

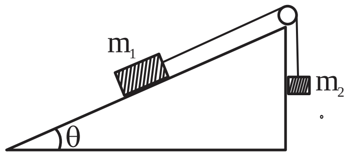
$\frac{1}{\sqrt{2}} = \frac{t_1}{t_1 + t_2}$

$1 + \frac{t_2}{t_1} = \sqrt{2}$

$\frac{t_1}{t_2} = \frac{1}{\sqrt{2}-1}$

$t_2 = (\sqrt{2}-1)t_1$

14. Two bodies of masses m_1 = 5 kg and m_2 = 3 kg are connected by a light string going over a smooth light pulley on a smooth inclined plane as shown in the figure. The system is at rest. The force exerted by the inclined plane on the body of mass m_1 will be :[Take g = 10 ms^{-2}]



(A) 30 N (B) 40 N

(C) 50 N (D) 60 N

Official Ans. by NTA (B)

Sol. For equilibrium $m_2 g = m_1 g \sin\theta$

$\sin\theta = \frac{m_2}{m_1} = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

Normal force on m_1 = 5g cosθ

$= 5 \times 10 \times \frac{4}{5} = 40\,N$

15. If momentum of a body is increased by 20%, then its kinetic energy increases by :

(A) 36% (B) 40%

(C) 44% (D) 48%

Official Ans. by NTA (C)

Sol. $P' = P + \frac{20}{100} P = 1.2\,P$

% change in KE = $\frac{K' - K}{K} \times 100$

$= \left(\frac{\frac{P'^2}{2m} - \frac{P^2}{2m}}{\frac{P^2}{2m}} \right) \times 100$

= $[(1.2)^2 - 1] \times 100$

= 44 %

16. The torque of a force $5\hat{i}+3\hat{j}-7\hat{k}$ about the origin is τ. If the force acts on a particle whose position vector is $2\hat{i}+2\hat{j}+\hat{k}$, then the value of τ will be :

(A) $11\hat{i}+19\hat{j}-4\hat{k}$ (B) $-11\hat{i}+9\hat{j}-16\hat{k}$

(C) $-17\hat{i}+19\hat{j}-4\hat{k}$ (D) $17\hat{i}+9\hat{j}+16\hat{k}$

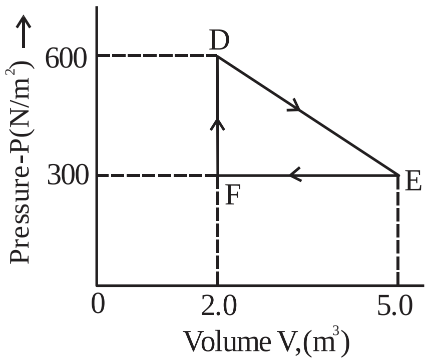
Official Ans. by NTA (C)

Sol. $\vec{\tau}=\vec{r}\times\vec{F}=\begin{vmatrix} i & j & k \\ 2 & 2 & 1 \\ 5 & 3 & -7 \end{vmatrix}$

$=i(-14-3)-j(-14-5)+\hat{k}(6-10)$

$=-17\hat{i}+19\hat{j}-4\hat{k}$

17. A thermodynamic system is taken from an original state D to an intermediate state E by the linear process shown in the figure. Its volume is then reduced to the original volume from E to F by an isobaric process. The total work done by the gas from D to E to F will be



(A) –450 J (B) 450 J

(C) 900 J (D) 1350 J

Official Ans. by NTA (B)

Sol. $W_{DE}=\frac{1}{2}(600+300)3\ J$

$= 1350\ J$

$W_{EF}=-300\times3=-900\ J$

$W_{DEF}=450\ J$

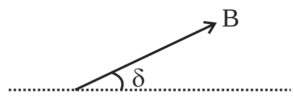
18. The vertical component of the earth's magnetic field is 6×10^{-5} T at any place where the angle of dip is 37°. The earth's resultant magnetic field at that place will be (Given tan 37° = $\frac{3}{4}$)

(A) 8×10^{-5} T (B) 6×10^{-5} T

(C) 5×10^{-4} T (D) 1×10^{-4} T

Official Ans. by NTA (D)

Sol.



($\delta = 37°$)

$B_V = B\sin\delta$

$6\times10^{-5}=B\frac{3}{5}$

$B = 10\times10^{-5}$ T

$= 10^{-4}$ T

19. The root mean square speed of smoke particles of mass 5×10^{-17} kg in their Brownian motion in air at NTP is approximately. [Given $k = 1.38\times10^{-23}\ JK^{-1}$]

(A) 60 mm s^{-1} (B) 12 mm s^{-1}

(C) 15 mm s^{-1} (D) 36 mm s^{-1}

Official Ans. by NTA (C)

Sol. $V_{rms}=\sqrt{\frac{3kT}{m}}=\sqrt{\frac{3\times1.38\times10^{-23}\times293}{5\times10^{-17}}}$

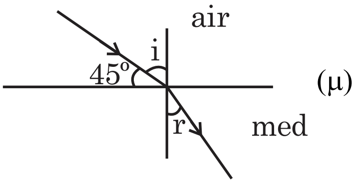
$\approx 15\ mm/s$

20. Light enters from air into a given medium at an angle of 45° with interface of the air-medium surface. After refraction, the light ray is deviated through an angle of 15° from its original direction. The refractive index of the medium is :

(A) 1.732 (B) 1.333

(C) 1.414 (D) 2.732

Official Ans. by NTA (C)

Sol. 

$i = 45°$

$D = i - r$

$15° = 45 - r \Rightarrow r = 30°$

$n_1 \sin i = n_2 \sin r$

$1 \sin 45° = \mu \sin 30°$

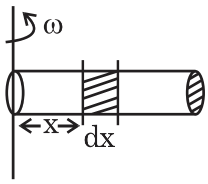
$$\frac{1}{\sqrt{2}} = \mu \frac{1}{2}$$

$$\mu = \sqrt{2} = 1.414$$

SECTION-B

1. A tube of length 50 cm is filled completely with an incompressible liquid of mass 250 g and closed at both ends. The tube is then rotated in horizontal plane about one of its ends with a uniform angular velocity $x\sqrt{F}$ rad s^{-1}. If F be the force exerted by the liquid at the other end then the value of x will be ______.

Official Ans. by NTA (4)



Sol.

$$F = \int (dm)\omega^2 x$$

$$= \int_0^L \left(\frac{m}{L}dx\right)\omega^2 x$$

$$= \frac{m}{L}\omega^2 \frac{L^2}{2}$$

$$= \frac{m\omega^2 L}{2}$$

$$\omega = \sqrt{\frac{2}{mL}}\sqrt{F}$$

$$= \sqrt{\frac{2}{0.25 \times 0.5}}\sqrt{F}$$

$$= \sqrt{16}\sqrt{F}$$

$$= 4\sqrt{F}$$

2. Nearly 10% of the power of a 110 W light bulb is converted to visible radiation. The change in average intensities of visible radiation, at a distance of 1 m from the bulb to a distance of 5 m is $a \times 10^{-2}$ W/m^2. The value of 'a' will be

Official Ans. by NTA (84)

Sol. P' = 10% of 110 W

$$= \frac{10}{100} \times 110\,W$$

$= 11$ W

$$I_1 - I_2 = \frac{P'}{4\pi r_1^2} - \frac{P'}{4\pi r_2^2}$$

$$= \frac{11}{4\pi}\left[\frac{1}{1} - \frac{1}{25}\right]$$

$$= \frac{11}{4\pi} \times \frac{24}{25}$$

$$= \frac{264}{\pi} \times 10^{-2} = 84 \times 10^{-2}\,W/m^2$$

3. A metal wire of length 0.5 m and cross-sectional area 10^{-4} m^2 has breaking stress 5×10^8 Nm^{-2}. A block of 10 kg is attached at one end of the string and is rotating in a horizontal circle. The maximum linear velocity of block will be____ ms^{-1}.

Official Ans. by NTA (50)

Sol. $$T = \frac{mv^2}{\ell} = \frac{10 \times v^2}{0.5} = 20v^2$$

T_{max} = Breaking stress × Area

$= 5 \times 10^8 \times 10^{-4} = 5 \times 10^4$

$20V^2 = 5 \times 10^4$

$$V = \sqrt{\frac{1}{4}10^4} = 50\,m/s$$

4. The velocity of a small ball of mass 0.3g and density 8g/cc when dropped in a container filled with glycerine becomes constant after some time. If the density of glycerine is 1.3 g/cc, then the value of viscous force acting on the ball will be $x \times 10^{-4}$ N, the value of x is____. [use g = 10m/s^2]

Official Ans. by NTA (25)

Sol. $F_V + F_B = mg$ (v = constant)

$F_V = mg - F_B$

$= \rho_B Vg - \rho_L Vg$

$= (\rho_B - \rho_L)Vg$

$$= (8-1.3)\times 10^{+3} \times \frac{0.3\times 10^{-3}}{8\times 10^{3}} \times 10$$

$$= \frac{6.7\times 0.3}{8}\times 10^{-2} \quad (g = 10)$$

$$= \frac{67\times 3}{8}\times 10^{-4} = 25.125\times 10^{-4}$$

Ans. 25.125

5. A modulating signal $2\sin(6.28 \times 10^6)t$ is added to the carrier signal $4\sin(12.56 \times 10^9)t$ for amplitude modulation. The combined signal is passed through a non-linear square law device. The output is then passed through a band pass filter. The bandwidth of the output signal of band pass filter will be___MHz.

Official Ans. by NTA (2)

Sol. Frequencies present in output of square law device

$2f_c,\quad f_c + f_m,\quad f_c,\quad f_c - f_m,\quad 2f_m,\quad f_m$

After passing through band bass filte.

$f_c + f_m, f_c, f_c - f_m$

Band width = $2f_m$

$$= \frac{2\omega_m}{2\pi} = \frac{6.28\times 10^6}{3.14}$$

= 2 MHz

6. The speed of a transverse wave passing through a string of length 50 cm and mass 10 g is 60 ms^{-1}. The area of cross-section of the wire is 2.0 mm^2 and its Young's modulus is 1.2×10^{11} Nm^{-2}. The extension of the wire over its natural length due to its tension will be $x \times 10^{-5}$ m. The value of x is_______.

Official Ans. by NTA (15)

Sol. $V_w = \sqrt{\frac{T}{\mu}}$

$$60 = \sqrt{\frac{T}{10\times 10^{-3}}\times 0.5}$$

$$T = \frac{(60)^2\times 10^{-2}}{0.5} = 72\text{ N}$$

$$\Delta\ell = \frac{F\ell}{AY} = \frac{72\times 0.5}{2\times 10^{-6}\times 1.2\times 10^{11}}$$

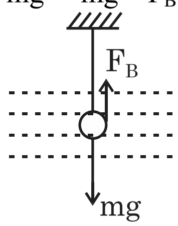
$$= \frac{72\times 5}{24}\times 10^{-5} = 15 \times 10^{-5}$$

Ans. 15

7. The metallic bob of simple pendulum has the relative density 5. The time period of this pendulum is 10 s. If the metallic bob is immersed in water, then the new time period becomes $5\sqrt{x}$ s. The value of x will be _________.

Official Ans. by NTA (5)

Sol. $mg' = mg - F_B$



$$g' = \frac{mg - F_B}{m}$$

$$= \frac{\rho_B Vg - \rho_w Vg}{\rho_B V}$$

$$= \left(\frac{\rho_B - \rho_w}{\rho_B}\right)g \qquad T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$= \frac{5-1}{5}\times g$$

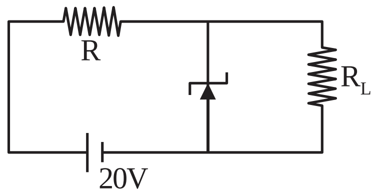
$$= \frac{4}{5}g$$

$$\frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{g}{\frac{4}{5}g}} = \sqrt{\frac{5}{4}}$$

$$T' = T\sqrt{\frac{5}{4}} = \frac{10}{2}\sqrt{5}$$

$$T' = 5\sqrt{5}$$

8. A 8 V Zener diode along with a series resistance R is connected across a 20 V supply (as shown in the figure). If the maximum Zener current is 25 mA, then the minimum value of R will be _____Ω.



Official Ans. by NTA (480)

Sol. $\varepsilon - IR - V_z = 0$

$20 - IR - 6 = 0$

$IR = 12$

$25 \times 10^{-3}\ R = 12$

$$R = \frac{12}{25 \times 10^{-3}} = 480\Omega$$

9. Two radioactive materials A and B have decay constants 25λ and 16λ respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of B to that of A will be "e" after a time $\frac{1}{a\lambda}$. The value of a is_______.

Official Ans. by NTA (9)

Sol. $N = N_0 e^{-\lambda t}$

$$\frac{N_B}{N_A} = \frac{e^{-\lambda_2 t}}{e^{-\lambda_1 t}} = e^{-\lambda_2 t}.e^{\lambda_1 t}$$

$$e^1 = e^{(\lambda_1 - \lambda_2)t}$$

$$(\lambda_1 - \lambda_2)t = 1$$

$$t = \frac{1}{\lambda_1 - \lambda_2} = \frac{1}{25\lambda - 16\lambda} = \frac{1}{9\lambda}$$

10. A capacitor of capacitance 500 μF is charged completely using a dc supply of 100 V. It is now connected to an inductor of inductance 50 mH to form an LC circuit. The maximum current in LC circuit will be ______ A.

Official Ans. by NTA (10)

Sol. Energy stored in capacitor

$$= \frac{1}{2}CV^2 = \frac{1}{2}500 \times 10^{-6} \times 10^4$$

$$= \frac{5}{2}J$$

Current will be maximum when whole energy of capacitor becomes energy of inductor.

$$\frac{1}{2}LI^2 = \frac{5}{2}$$

$$I = \sqrt{\frac{5}{L}} = \sqrt{\frac{5}{50 \times 10^{-3}}} = 10\ A.$$

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(Held On Friday 29th July, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

CHEMISTRY | TEST PAPER WITH SOLUTION

SECTION-A

1. Consider the reaction

$4HNO_3(l) + 3KCl(s) \rightarrow Cl_2(g) + NOCl(g) + 2H_2O(g) + 3KNO_3(s)$

The amount of HNO_3 required to produce 110.0 g of KNO_3 is :

(Given : Atomic masses of H, O, N and K are 1, 16, 14 and 39, respectively.)

(A) 32.2 g (B) 69.4 g

(C) 91.5 g (D) 162.5 g

Official Ans. by NTA (C)

Sol. $4HNO_3(\ell)+3KCl(s)\rightarrow Cl_2(g)+NOCl(g)+2H_2O(g)+3KNO_3(g)$

x gm **110 gm**

$\frac{x}{63}$ Mole $= \frac{110}{101}$

$4 \rightarrow 3$

$1 \rightarrow \frac{3}{4}$

$\frac{x}{63} \rightarrow \frac{3}{4}\times\frac{x}{63} = \frac{110}{101}$

$x = \frac{110\times63\times4}{101\times3} = 91.5\,gm$

2. Given below are the quantum numbers for 4 electrons.

A. $n = 3, l = 2, m_l = 1, m_s = +1/2$

B. $n = 4, l = 1, m_l = 0, m_s = +1/2$

C. $n = 4, l = 2, m_l = -2, m_s = -1/2$

D. $n = 3, l = 1, m_l = -1, m_s = +1/2$

The correct order of increasing energy is :

(A) D < B < A < C (B) D < A < B < C

(C) B < D < A < C (D) B < D < C < A

Official Ans. by NTA (B)

Sol. Energy order of subshell decided by (n+λ) rule.

$A \Rightarrow 3d \Rightarrow n + 1 = 5$

$B \Rightarrow 4p \Rightarrow n + \lambda = 5$

$C \Rightarrow 4d \Rightarrow n + \ell \Rightarrow 6$

$D \Rightarrow 3s \Rightarrow (n+ \ell) = 4$

D < A < B < C

3. $C(s) + O_2(g) \rightarrow CO_2(g) + 400\ kJ$

$C(s) + \frac{1}{2}O_2(g) \rightarrow CO(g) + 100\ kJ$

When coal of purity 60% is allowed to burn in presence of insufficient oxygen, 60% of carbon is converted into 'CO' and the remaining is converted into 'CO_2'.

The heat generated when 0.6 kg of coal is burnt is ______.

(A) 1600 kJ (B) 3200 kJ

(C) 4400 kJ (D) 6600 kJ

Official Ans. by NTA (D)

Sol. $C(S) + O_2(g) \rightarrow CO_2(g) + 400\ kJ$

1 g mole

$C(s)+\frac{1}{2}O_2(g)\rightarrow CO(g)+100kJ$ .... (II)

0.6×1000

= 600 gm

$600\times\frac{60}{100}$ (Pure Carbon)

$= 360gm = \frac{360}{12} = 30mole$ (Pure Carbon)

Carbon converted into $CO_2 = \left(30-30\times\frac{60}{100}\right)$

= 12 mole

and carbon converted in CO = $30\times\frac{60}{100} = 18\,mole$

Energy generated during II equation

= 18 × 100

= 1800 kJ

Energy generated during Ist reaction.

= 12 × 400

= 4800

Total = 1800 + 4800 = 6600 kJ

4. 200 mL of 0.01 M HCl is mixed with 400 mL of 0.01M H_2SO_4. The pH of the mixture is ____.

(A) 1.14 (B) 1.78

(C) 2.34 (D) 3.02

Official Ans. by NTA (B)

Sol. $HCl + H_2SO_4$

$$[H^+] = \frac{(0.01\times200)+(0.01\times2\times400)}{600}$$

$$= \frac{2+8}{600} = \frac{10}{600} = \frac{1}{60}$$

$$pH = -\log\left[\frac{1}{60}\right]$$

= 1.78

5. Given below are the critical temperatures of some of the gases :

Gas	Critical temperature (K)
He	5.2
CH_4	190
CO_2	304.2
NH_3	405.5

The gas showing least adsorption on a definite amount of charcoal is :

(A) He (B) CH_4

(C) CO_2 (D) NH_3

Official Ans. by NTA (A)

Sol. More the critical temp. of gas greater is the ease of liquefaction hence greater is the adsorption.

6. In liquation process used for tin (Sn), the metal :

(A) is reacted with acid

(B) is dissolved in water

(C) is brought to molten form which is made to flow on a slope

(D) is fused with NaOH.

Official Ans. by NTA (C)

Sol. Liquation process is used for metal having low melting point such as tin in which they are heated and brought to molten state and made to flow down the slope while impurities with higher melting point left on the top.

7. Given below are two statements.

Statement I : Stannane is an example of a molecular hydride.

Statement II : Stannane is a planar molecule.

In the light of the above statement, choose the most appropriate answer from the options given below :

(A) Both Statement I and Statement II are true.

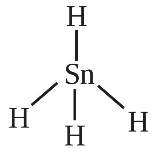
(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II is false.

(D) Statement I is false but Statement II is true.

Official Ans. by NTA (C)

Sol. SnH_4 is non planar molecular hydride



Tetrahedral shape, sp^3 hybridisation

8. Portland cement contains 'X' to enhance the setting time. What is 'X'?

(A) $CaSO_4.\frac{1}{2}H_2O$ (B) $CaSO_4.2H_2O$

(C) $CaSO_4$ (D) $CaCO_3$

Official Ans. by NTA (B)

Sol. Gypsum ($CaSO_4.2H_2O$) is used to enhance setting time in portland cement.

9. When borax is heated with CoO on a platinum loop, blue coloured bead formed is largely due to :

(A) B_2O_3 (B) $Co(BO_2)_2$

(C) CoB_4O_7 (D) $Co[B_4O_5(OH)_4]$

Official Ans. by NTA (B)

Sol. $Na_2B_4O_7 \cdot 10H_2O \xrightarrow{\Delta} Na_2B_4O_7 + 10H_2O$

$Na_2B_4O_7 \xrightarrow{\Delta} 2NaBO_2$(sodium meta borate) $+ B_2O_3$

$B_2O_3 + CoO \rightarrow Co(BO_2)_2$(cobalt (II) meta borate)

Blue Bead

10. Which of the following 3d-metal ion will give the lowest enthalpy of hydration ($\Delta_{hyd}H$) when dissolved in water ?

(A) Cr^{2+} (B) Mn^{2+}

(C) Fe^{2+} (D) Co^{2+}

Official Ans. by NTA (B)

Sol.

Ion	$\Delta H^o_{Hyd.}$ (kJ/mole)
Cr^{2+}	–1925
Mn^{2+}	–1862
Fe^{2+}	–1998
Co^{2+}	–2079

11. Octahedral complexes of copper (II) undergo structural distortion (Jahn-Teller). Which one of the given copper (II) complexes will show the maximum structural distortion ?

(en–ethylenediamine; $H_2N\text{-}CH_2\text{-}CH_2\text{-}NH_2$)

(A) $[Cu(H_2O)_6]SO_4$ (B) $[Cu(en)(H_2O)_4]SO_4$

(C) cis-$[Cu(en)_2Cl_2]$ (D) trans-$[Cu(en)_2Cl_2]$

Official Ans. by NTA (A)

Sol. There is unsymmetric filling of e_g subset of Cu^{+2} ion, while there is symmetrical distribution in t_{2g} set, if the complex has same ligand there will be equal repulsion which leads to symmetrical bond length along t_{2g} , but due to uneven filling of electron in e_g subset, either octahedral will be elongated or compressed.

12. Dinitrogen is a robust compound, but reacts at high altitude to form oxides. The oxide of nitrogen that can damage plant leaves and retard photosynthesis is :

(A) NO (B) NO_3^-

(C) NO_2 (D) NO_2^-

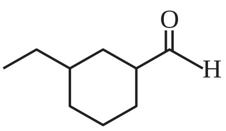
Official Ans. by NTA (C)

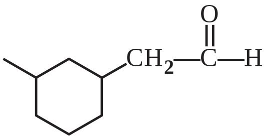
Sol. $N_2(g) + O_2(g) \rightarrow 2NO(g)$

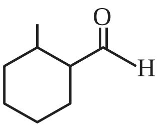
$2NO(g) + O_2(g) \rightarrow 2NO_2(g)$

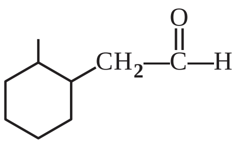
NO_2 damage plant leaves

13. Correct structure of γ-methylcyclohexane carbaldehyde is :

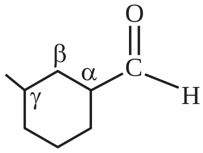
(A) 

(B) 

(C) 

(D) 

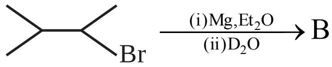
Official Ans. by NTA (A)

Sol. 

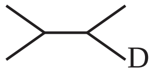
γ-methyl cyclohexane carbaldehyde

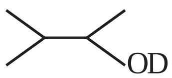
14. Compound 'A' undergoes following sequence of reactions to give compound 'B'. The correct structure and chirality of compound 'B' is:

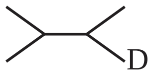
[where Et is $-C_2H_5$]

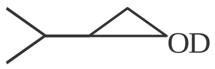


Compound 'A'

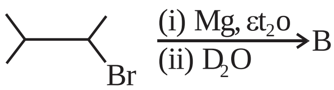
(A)  , Achiral

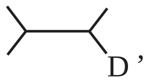
(B)  , Chiral

(C)  , Chiral

(D)  , Achiral

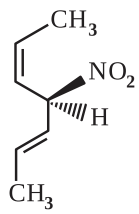
Official Ans. by NTA (C)

Sol. 

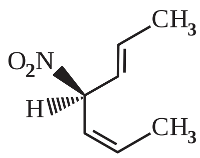


, chiral

15. Given below are two statements.

Statement I : The compound 

is optically active.

Statement II : 

is mirror image of above compound A.

In the light of the above statement, choose the **most appropriate** answer from the options given below.

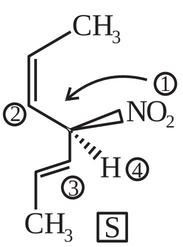
(A) Both Statement I and Statement II are correct

(B) Both Statement I and Statement II are incorrect.

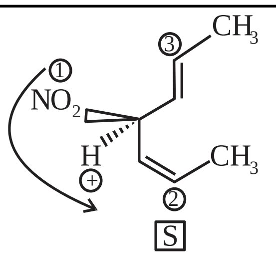
(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (C)

Sol. 

(A)



(B)

Having same configuration.

16. When enthanol is heated with conc. H_2SO_4, a gas is produced. The compound formed, when this gas is treated with cold dilute aqueous solution of Baeyer's reagent, is :

(A) Formaldehyde (B) Formic acid

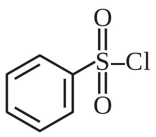
(C) Glycol (D) Ethanoic acid

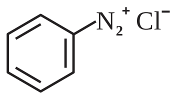
Official Ans. by NTA (C)

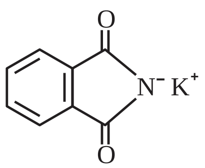
Sol. $$CH_3\text{–}CH_2\text{–}OH \xrightarrow[\Delta]{\text{conc. } H_2SO_4} CH_2{=}CH_2 \xrightarrow{\text{Bayer's Reagent}} \underset{OH}{CH_2} - \underset{OH}{CH_2}$$

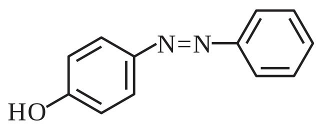
glycol

17. The Hinsberg reagent is :

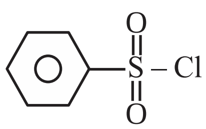
(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (A)

Sol. 

B.S.C (Benzene sulphonyl chloride) is known's Hinsberg Reagent

18. Which of the following is **NOT** a natural polymer?

(A) Protein

(B) Starch

(C) Rubber

(D) Rayon

Official Ans. by NTA (D)

Sol. Rayon is semisynthetic polymer.

19. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Amylose is insoluble in water.

Reason R : Amylose is a long linear molecule with more than 200 glucose units.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both **A** and **R** are correct and **R** is the correct explanation of A.

(B) Both **A** and **R** are correct and **R** is NOT the correct explanation of A.

(C) **A** is correct but **R** is not correct.

(D) **A** is not correct but **R** is correct.

Official Ans. by NTA (D)

Sol. Amylose is water soluble.

20. A compound 'X' is a weak acid and it exhibits colour change at pH close to the equivalence point during neutralization of NaOH with CH_3COOH. Compound 'X' exists in ionized form in basic medium. The compound 'X' is :

(A) methyl orange (B) methyl red

(C) phenolphthalein (D) erichrome Black T

Official Ans. by NTA (C)

Sol. Phenolphthalein is weak acid give colour in basic medium.

SECTION-B

1. 'x' g of molecular oxygen (O_2) is mixed with 200 g of neon (Ne). The total pressure of the non-reactive mixture of O_2 and Ne in the cylinder is 25 bar. The partial pressure of Ne is 20 bar at the same temperature and volume. The value of 'x' is_____.

[Given: Molar mass of O_2 = 32 g mol^{-1}.

Molar mass of Ne = 20 g mol^{-1}]

Official Ans. by NTA (80)

Sol. **O_2 + Ne**

Xgm 200gm

P_{total} = 25 bar ; P_{Ne} = 20

$$P_{O_2} + P_{Ne} = 25$$

$$P_{O_2} = 25 - 20 = 5\,\text{bar}$$

$$5 = \frac{\frac{x}{32}}{\frac{x}{32} + \frac{200}{20}} \times 25$$

$$\frac{1}{5} = \frac{\frac{x}{32}}{\frac{x}{32} + 10}$$

$$\frac{1}{5} = \frac{x \times 32}{32(x + 320)}$$

$5x = x + 320$

$4x = 320$

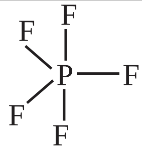
$$x = \frac{320}{4} = 80\,\text{gm}$$

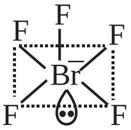
2. Consider, PF_5, BrF_5, PCl_3, SF_6, $[ICl_4]^-$, ClF_3 and IF_5.

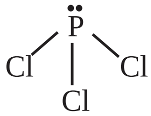
Amongst the above molecule(s)/ion(s), the number of molecule(s)/ion(s) having sp^3d^2 hybridisation is____.

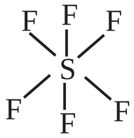
Official Ans. by NTA (4)

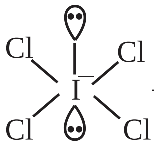
Sol.

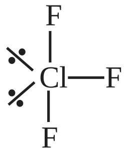

$\rightarrow sp^3d$ Hybridisation

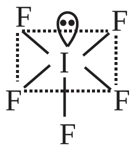

$\rightarrow sp^3d^2$ Hybridisation


$\rightarrow sp^3$ Hybridisation


$\rightarrow sp^3d^2$ Hybridisation


$\rightarrow sp^3d^2$ Hybridisation


$\rightarrow sp^3d$ Hybridisation


$\rightarrow sp^3d^2$ Hybridisation

3. 1.80 g of solute A was dissolved in 62.5 cm^3 of ethanol and freezing point of the solution was found to be 155.1 K. The molar mass of solute A is __g mol^{-1}.

[Given: Freezing point of ethanol is 156.0 K.

Density of ethanol is 0.80 g cm^{-3}.

Freezing point depression constant of ethanol is 2.00 K kg mol^{-1}]

Official Ans. by NTA (80)

Sol. Mass of C_2H_5OH = 62.5 × 0.8 = 50 g

$\Delta T_f = K_f \times m$

$$0.9 = 2 \times \frac{1.8 \times 1000}{M_w \times 50}$$

$$M_w = \frac{2 \times 1.8 \times 1000}{0.9 \times 50} = 80$$

4. For a cell, Cu(s) $|Cu^{2+}(0.001M| |Ag^{+}(0.01M)|$ Ag(s) the cell potential is found to be 0.43 V at 298 K. The magnitude of standard electrode potential for Cu^{2+}/Cu is ___ $\times 10^{-2}$ V.

$$\left[\text{Given}: E^{\ominus}_{Ag^+/Ag} = 0.80\text{V and } \frac{2.303RT}{F} = 0.06\text{V}\right]$$

Official Ans. by NTA (34)

Sol. At anode

$Cu \rightarrow Cu^{2+} + 2e^-$

At cathode

$2Ag^+ + 2e^- \rightarrow 2Ag$

Cell reaction $\rightarrow Cu + 2Ag^+ \rightarrow Cu^{2+} + 2Ag$

$$E_{cell} = E^0_{cell} - \frac{0.06}{2}\log\frac{[Cu^{2+}]}{[Ag^+]^2}$$

$$0.43 = E^0_{cell} - \frac{0.06}{2}\log\frac{(0.001)}{(0.01)^2}$$

$$E^0_{cell} = 0.46$$

$$E^0_{cell} = E^0_{Ag^+/Ag} - E^0_{Cu^{2+}/Cu}$$

$$0.46 = 0.80 - E^0_{Cu^{2+}/Cu}$$

$$E^0_{Cu^{2+}/Cu} = 0.34\,\text{volt}$$

$$E^0_{Cu^{2+}/Cu} = 34 \times 10^{-2}$$

5. Assuming $1\mu g$ of trace radioactive element X with a half life of 30 years is absorbed by a growing tree. The amount of X remaining in the tree after 100 years is___$\times 10^{-1}\mu g$.

[Given : ln 10 = 2.303; log2 = 0.30]

Official Ans. by NTA (1)

Sol. $t = \frac{1}{\lambda}\ell n\left(\frac{a}{a-x}\right)$

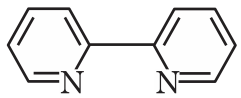
$$100 = \frac{30}{\ell n2}\ell n\left(\frac{1}{w}\right)$$

$$\frac{1}{w} = 10$$

W = 0.1 $\times$ μg

Ans. 1 $\times$ 10^{-1} μg

6. Sum of oxidation state (magnitude) and coordination number of cobalt in $Na[Co(bpy)Cl_4]$ is__.

(Given bpy = )

Official Ans. by NTA (9)

Sol. Coordination no. = 6

Oxidation state = 3

6 + 3 = 9

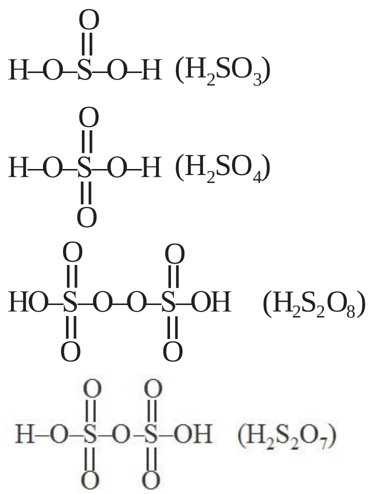
7. Consider the following sulphure based oxoacids.

H_2SO_3, H_2SO_4, $H_2S_2O_8$ and $H_2S_2O_7$.

Amongst these oxoacids, the number of those with peroxo(O-O) bond is______.

Official Ans. by NTA (1)

Sol.



8. A 1.84 mg sample of polyhydric alcoholic compound 'X' of molar mass 92.0 g/mol gave 1.344 mL of H_2 gas at STP. The number of alcoholic hydrogens present in compound 'X' is____.

Official Ans. by NTA (3)

Sol. $R(OH)_x \rightarrow H_2$

PoAC on H –

$$x\left(\frac{1.84\times10^{-3}}{92}\right) = \frac{1.344}{22.4}\times 2$$

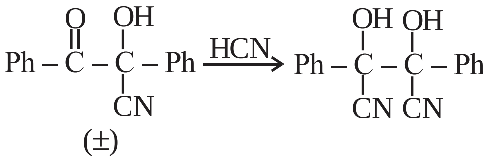
$$x = \frac{1.344\times2\times92\times1000}{1.84\times22400} = 6$$

x = 6

9. The number of stereoisomers formed in a reaction of (±) Ph(C=O) C(OH)(CN)Ph with HCN is_____.

Official Ans. by NTA (3)

Sol.

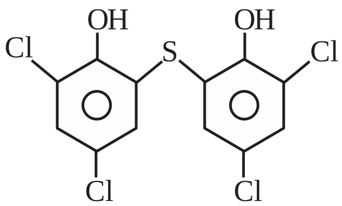


3 stereoisomers

10. The number of chlorine atoms in bithionol is____.

Official Ans. by NTA (4)

Sol. Bithinol



Chlorine atoms = 4

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Friday 29th July, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. If $z \neq 0$ be a complex number such that $|z - \frac{1}{z}| = 2$, then the maximum value of |z| is:

(A) $\sqrt{2}$ (B) 1

(C) $\sqrt{2} - 1$ (D) $\sqrt{2} + 1$

Official Ans. by NTA (D)

Sol. $|z - 1/z| = 2$

$$\left||z| - \frac{1}{|z|}\right| \leq \left|z - \frac{1}{z}\right| \leq |z| + \frac{1}{|z|}$$ Let |z| = r

$$\left|r - \frac{1}{r}\right| \leq 2 \leq r + \frac{1}{r}$$

$$\left|r - \frac{1}{r}\right| \leq 2 \;\&\; r + \frac{1}{r} \geq 2 \text{ always true}$$

$$r - \frac{1}{r} \geq -2 \;\&\; r - \frac{1}{r} \leq 2$$

$r^2 - 1 \leq 2r$

$r^2 - 2r \leq 1$

$(r - 1)^2 \leq 2$

$r - 1 \leq \sqrt{2}$

$\therefore |z|_{max} = 1 + \sqrt{2}$

2. Which of the following matrices can NOT be obtained from the matrix $\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ by a single elementary row operation?

(A) $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 2 \\ -2 & 7 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$

Official Ans. by NTA (C)

Sol. $A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$

(1) $R_1 \to R_1 + R_2; \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ possible

(2) $R_1 \leftrightarrow R_2; \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ possible

(3) Option is not possible

(4) $R_2 \to R_2 + 2R_1; \begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$ possible

3. If the system of equations

$x + y + z = 6$

$2x + 5y + \alpha z = \beta$

$x + 2y + 3z = 14$

has infinitely many solutions, then $\alpha + \beta$ is equal to :

(A) 8 (B) 36

(C) 44 (D) 48

Official Ans. by NTA (C)

Sol. $x + y + z = 6$ ___(1)

$2x + 5y + \alpha z = \beta$ ___(2)

$x + 2y + 3z = 14$ ____(3)

$x + y = 6 - z$

$x + 2y = 14 - 3z$

On solving

$x = z - 2 \Rightarrow y = 8 - 2z$ in (2)

$2(z - 2) + 5(8 - 2z) + \alpha z = \beta$

$(\alpha - 8) z = \beta - 36$ For having infinite solutions

$\alpha - 8 = 0$ & $\beta - 36 = 0$

$\alpha = 8, \beta = 36$ $(\alpha + \beta = 44)$

4. Let the function

$$f(x)=\begin{cases}\dfrac{\log_e(1+5x)-\log_e(1+\alpha x)}{x} & ;\text{if } x\neq 0\\ 10 & ;\text{if } x=0\end{cases}$$

be continuous at x = 0.

The α is equal to :

(A) 10 (B) –10

(C) 5 (D) –5

Official Ans. by NTA (D)

Sol. $$f(x)=\begin{cases}\dfrac{\ln(1+5x)-\ln(1+\alpha x)}{x} & ;x\neq 0\\ 10 & ;x=0\end{cases}$$

$$\lim_{x\to 0}\frac{\ln(1+5x)-\ln(1+\alpha x)}{x}=10$$

Using expension

$$\lim_{x\to 0}\frac{(5x+......)-(\alpha x+......)}{x}=10$$

$5-\alpha=10\Rightarrow\alpha=-5$

5. If [t] denotes the greatest integer $\le$ t, then the value of $\int_0^1\left[2x-|3x^2-5x+2|+1\right]dx$ is:

(A) $\dfrac{\sqrt{37}+\sqrt{13}-4}{6}$ (B) $\dfrac{\sqrt{37}-\sqrt{13}-4}{6}$

(C) $\dfrac{-\sqrt{37}-\sqrt{13}+4}{6}$ (D) $\dfrac{-\sqrt{37}+\sqrt{13}+4}{6}$

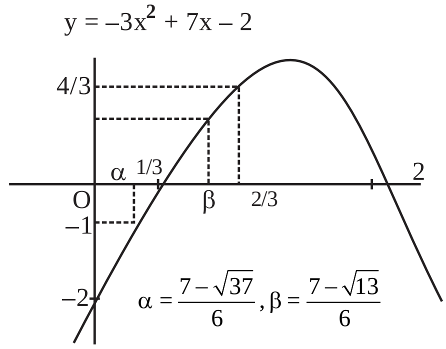
Official Ans. by NTA (A)

Sol. $$I=\int_0^1\left[2x-|3x^2-3x-2x+2|+1\right]dx$$

$$I=\int_0^1\left[2x-|(3x-2)(x-1)|\right]dx+\int_0^1 1\,dx$$

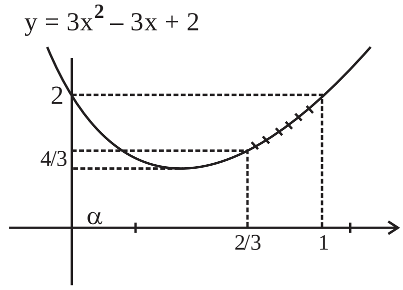
$$I=\int_0^{2/3}\left[\left(2x-(3x^2-5x+2)\right)\right]dx+\int_{2/3}^1\left(2x+(3x^2-5x+2)\right)dx+1$$

$$I=\int_0^{2/3}\left[-3x^2+7x-2\right]dx+\int_{2/3}^1\left(3x^2-3x+2\right)dx+1$$



$$\int_0^{\alpha}(-2)dx+\int_{\alpha}^{1/3}(-1)dx+\int_{1/3}^{\beta}0\,dx+\int_{\beta}^{2/3}1.dx$$

$$=-2\alpha-\left(\frac{1}{3}-\alpha\right)+\frac{2}{3}-\beta=-\alpha-\beta+\frac{1}{3}$$



When $x\in\left(\frac{2}{3},1\right)$

$3x^2-3x+2\in\left(\frac{4}{3},2\right)$

$[3x^2-3x+2]=1$

$$\therefore\int_{2/3}^1\left[3x^2-3x+2\right]dx=1\left(1-\frac{2}{3}\right)=\frac{1}{3}$$

Hence $I=\left(\frac{1}{3}-(\alpha+\beta)\right)+\left(\frac{1}{3}\right)+1$

$$=\frac{5}{3}-\left(\frac{7-\sqrt{37}}{6}+\frac{7-\sqrt{13}}{6}\right)$$

$$=\frac{-2}{3}+\frac{\sqrt{37}+\sqrt{13}}{6}$$

$$=\frac{\sqrt{37}+\sqrt{13}-4}{6}$$

6. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 3a_{n+1} - 2a_n + 1, \forall\ n \ge 0$.

Then $a_{25}\,a_{23} - 2\,a_{25}\,a_{22} - 2\,a_{23}\,a_{24} + 4\,a_{22}\,a_{24}$ is equal to:

(A) 483 (B) 528 (C) 575 (D) 624

Official Ans. by NTA (B)

Sol. $a_0 = 0,\ a_1 = 0$

$a_{n+2} = 3\,a_{n+1} - 2\,a_{n+1} : n \ge 0$

$a_{n+2} - a_{n+1} = 2\,(a_{n+1} - a_n) + 1$

$n = 0 \qquad a_2 - a_1 = 2\,(a_1 - a_0) + 1$

$n = 1 \qquad a_3 - a_2 = 2\,(a_2 - a_1) + 1$

$n = 2 \qquad a_4 - a_3 = 2\,(a_3 - a_2) + 1$

$n = n \qquad a_{n+2} - a_{n+1} = 2\,(a_{n+1} - a_n) + 1$

$(a_{n+2} - a_1) - 2\,(a_{n+1} - a_0) - (n + 1) = 0$

$a_{n+2} = 2a_{n+1} + (n + 1)$

$n \to n - 2$

$a_n - 2a_{n-1} = n - 1$

Now $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$

$= (a_{25} - 2a_{24})\,(a_{23} - 2a_{22}) = (24)\,(22) = 528$

7. $\sum_{r=1}^{20}\left(r^2+1\right)\left(r!\right)$ is equal to:

(A) 22! – 21! (B) 22! – 2 (21!)

(C) 21! – 2 (20!) (D) 21! – 20!

Official Ans. by NTA (B)

Sol. $\sum_{x=1}^{20}\left(r^2+1\right)r!$

$\sum_{x=1}^{20}\left(\left(r+1\right)^2-2r\right)r!$

$\sum_{x=1}^{20}\left(\left(r+1\right)\left(r+1\right)!-r.r!\right)-\sum_{r=1}^{20}r.r!$

$\sum_{x=1}^{20}\left(\left(r+1\right)\left(r+1\right)!-r.r!\right)-\sum_{r=1}^{20}\left(\left(r+1\right)!-r!\right)$

$=\left(21.\underline{|21}-1\right)-\left(\underline{|21}-1\right)$

$= 20.21! = 22! - 2.21!$

8. For $I(x)=\int\frac{\sec^2 x-2022}{\sin^{2022}x}dx$, if $I\left(\frac{\pi}{4}\right)=2^{1011}$, then

(A) $3^{1010}I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$

(B) $3^{1010}I\left(\frac{\pi}{6}\right)-I\left(\frac{\pi}{3}\right)=0$

(C) $3^{1011}I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$

(D) $3^{1011}I\left(\frac{\pi}{6}\right)-I\left(\frac{\pi}{3}\right)=0$

Official Ans. by NTA (A)

Sol. $I(x)=\int\underset{\text{II}}{\sec^2 x}.\underset{\text{I}}{\sin^{-2022}x}\,dx-2022\int\sin^{-2022}x\,dx$

$=\tan x.\left(\sin x\right)^{-2022}+\int\left(2022\right)\tan x.\left(\sin x\right)^{-2023}\cos x\,dx$

$-2022\int\left(\sin x\right)^{-2022}dx$

$I(x) = (\tan x)\,(\sin x)^{-2022} + C$

At $X = \pi/4$, $2^{1011}=\left(\frac{1}{\sqrt{2}}\right)^{-2022}+C \therefore C=0$

Hence $I\left(x\right)=\frac{\tan x}{\left(\sin x\right)^{2022}}$

$I\left(\pi/6\right)=\frac{1}{\sqrt{3}\left(\frac{1}{2}\right)^{2022}}=\frac{2^{2022}}{\sqrt{3}}$

$I\left(\pi/3\right)=\frac{\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)^{2022}}=\frac{2^{2022}}{\left(\sqrt{3}\right)^{2021}}=\frac{1}{3^{1010}}I\left(\frac{\pi}{6}\right)$

$3^{1010}I\left(\pi/3\right)=I\left(\pi/6\right)$

9. If the solution curve of the differential equation $\frac{dy}{dx}=\frac{x+y-2}{x-y}$ passes through the point (2,1) and (k + 1,2), k > 0, then

(A) $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(k^2+1\right)$

(B) $\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(k^2+1\right)$

(C) $2\tan^{-1}\left(\frac{1}{k+1}\right)=\log_e\left(k^2+2k+2\right)$

(D) $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(\frac{k^2+1}{k^2}\right)$

Official Ans. by NTA (A)

Sol. $\frac{dy}{dx}=\frac{x+y-2}{x-y}=\frac{(x-1)+(y-1)}{(x-1)-(y-1)}$

$x-1=X,\ y-1=Y$

$\frac{dY}{dX}=\frac{X+Y}{X-Y}$

$Y=VX \qquad \frac{dY}{dX}=V+X\frac{dV}{dX}$

$V+X\frac{dV}{dX}=\frac{1+V}{1-V} \qquad X\frac{dV}{dX}=\frac{V^2+1}{1-V}$

$\int\frac{1-V}{1+V^2}dV=\int\frac{dX}{X}$

$\int\frac{dV}{1+V^2}-\frac{1}{2}\int\frac{2VdV}{1+V^2}=\int\frac{dX}{X}$

$\tan^{-1}V-\frac{1}{2}\ln\left(1+V^2\right)=\ln X+c$

$\tan^{-1}\left(\frac{Y}{X}\right)-\frac{1}{2}\ln\left(1+\frac{Y^2}{X^2}\right)=\ln(X)+c$

$\tan^{-1}\left(\frac{y-1}{x-1}\right)-\frac{1}{2}\ln\left(1+\frac{(y-1)^2}{(x-1)^2}\right)=\ln(x-1)+c$

Passes through (2,1)

$0-\frac{1}{2}\ \ln 1=\ln 1+c\ \therefore\ c=0$

Passes through (k + 1, 2)

$\therefore\ \tan^{-1}\left(\frac{1}{k}\right)-\frac{1}{2}\ln\left(1+\frac{1}{k^2}\right)=\ln k$

$2\tan^{-1}\left(\frac{1}{k}\right)=\ln\left(\frac{1+k^2}{k^2}\right)+2\ln k$

$2\tan^{-1}\left(\frac{1}{k}\right)=\ln(1+k^2)$

10. Let y = y (x) be the solution curve of the differential equation $\frac{dy}{dx}+\left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y=\frac{(x+3)}{x+1}$, $x>-1$, which passes through the point (0,1). Then y (1) is equal to:

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{7}{2}$

Official Ans. by NTA (B)

Sol. $\frac{dy}{dx}+\left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y=\frac{x+3}{x+1}$

$\int p(x)dx \qquad \text{I.F.}=e^{\int p(x)dx}$

$\int p(x)dx=\int\frac{(2x^2+11x+13)dx}{(x+1)(x+2)(x+3)}$

Using partial fraction

$\frac{2x^2+11x+13}{(x+1)(x+2)(x+3)}=\frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{x+3}$

$A=\frac{4}{2}=2$

$B=1$

$C=-1$

$\because\int p(x)dx=A\ln(x+1)+B\ln(x+2)+c\ln(x+3)$

$=\ln\left(\frac{(x+1)^2(x+2)}{x+3}\right)$

$\text{I.F.}=e^{\int p(x)dx}=\frac{(x+1)^2(x+2)}{(x+3)}$

Solution $y(\text{IF})=\int Q.(\text{IF})dx$

$y\left(\frac{(x+1)^2(x+2)}{x+3}\right)=\int\left(\frac{\cancel{x+3}}{\cancel{x+1}}\right)\frac{(x+1)^{\cancel{2}}(x+2)}{(\cancel{x+3})}dx$

$y\left(\frac{(x+1)^2(x+2)}{x+3}\right)=\frac{x^3}{3}+\frac{3x^2}{2}+2x+c$

Passes through (0, 1) $C=\frac{2}{3}$

Now put x = 1

$\Rightarrow y(1)=\frac{3}{2}$

11. Let m_1, m_2 be the slopes of two adjacent sides of a square of side a such that $a^2+11a+3\left(m_2^2+m_2^2\right)=220$. If one vertex of the square is $(10(\cos\alpha-\sin\alpha), 10(\sin\alpha+\cos\alpha))$, where $\alpha\in\left(0,\frac{\pi}{2}\right)$ and the equation of one diagonal is $(\cos\alpha-\sin\alpha)x+(\sin\alpha+\cos\alpha)y=10$, then $72(\sin^4\alpha+\cos^4\alpha)+a^2-3a+13$ is equal to:

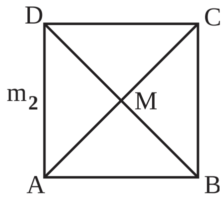
(A) 119 (B) 128

(C) 145 (D) 155

Official Ans. by NTA (B)

Sol. $m_1m_2 = -1$

$$a^2 + 11a + 3\left(m_1^2 + \frac{1}{m_1^2}\right) = 220$$



Eq. of AC

AC = (cosα – sinα) + (sinα + cosα) y = 10

BD = (sinα – cosα) x + (sinα – cosα) y = 0

(10 (cosα – sinα), 10 (sinα – cosα))

$$\text{Slope of AC} = \left(\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}\right) = \tan\theta = M$$

Eq. of line making an angle π_4 with AC

$$m_1, m_2 = \frac{m \pm \tan\frac{\pi}{4}}{1 \pm m\tan\frac{\pi}{4}}$$

$$= \frac{m+1}{1-m} \text{ or } \frac{m-1}{1+m}$$

$$\frac{\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha} + 1}{1 - \left(\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}\right)}, \frac{\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha} - 1}{1 + \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}}$$

m_1, m_2 = tanα , cotα

mid point of AC & BD

= M (5(cosα–sinα), 5(cosα+sinα))

B (10(cosα–sinα), 10(cosα+sinα))

$$a = AB = \sqrt{2}\,BM = \sqrt{2}\left(5\sqrt{2}\right) = 10$$

a = 10

$$\because a^2 + 11a + 3\left(m_1^2 + \frac{1}{m_1 2}\right) = 220$$

100 + 110 + 3 ($\tan^2\alpha + \cot^2\alpha$) = 220

Hence $\tan^2\alpha = 3$, $\tan^2\alpha = \frac{1}{3} \Rightarrow \alpha = \frac{\pi}{3} \text{ or } \frac{\pi}{6}$

Now 72 ($\sin^4\alpha + \cos^4\alpha$) + a^2 – 3a + 13

$$= 72\left(\frac{9}{16} + \frac{1}{16}\right) + 100 - 30 + 13$$

$$= 72\left(\frac{5}{8}\right) + 83 = 45 + 83 = 128$$

12. The number of elements in the set

$$S = \left\{x \in \mathbb{R} : 2\cos\left(\frac{x^2 + x}{6}\right) = 4^x + 4^{-x}\right\} \text{ is:}$$

(A) 1 (B) 3

(C) 0 (D) infinite

Official Ans. by NTA (A)

Sol. $2\cos\left(\frac{x^2 + x}{6}\right) = 4^x + 4^{-x}$

L.H.S ≤ 2. & R.H.S. ≥ 2

Hence L.H.S = 2 & R.H.S = 2

$2\cos\left(\frac{x^2 + x}{6}\right) = 2 \quad 4^x + 4^{-x} = 2$

Check x = 0 Possible hence only one solution.

13. Let A (α, –2), B (α, 6) and $C\left(\frac{\alpha}{4}, -2\right)$ be vertices of a ΔABC. If $\left(5, \frac{\alpha}{4}\right)$ is the circumcentre of ΔABC, then which of the following is NOT correct about ΔABC:

(A) ares is 24 (B) perimeter is 25

(C) circumradius is 5 (D) inradius is 2

Official Ans. by NTA (B)

Sol. A (α, –2) : B (α, 6) : $C\left(\frac{\alpha}{4}, -2\right)$

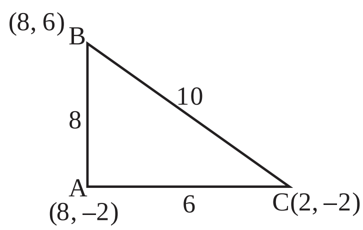
since AC is perpendicular to AB.

So, ΔABC is right angled at A.

Circumcentre = mid point of BC. = $\left(\frac{5\alpha}{8}, 2\right)$

$$\therefore \frac{5\alpha}{8} = 5 \;\&\; \frac{\alpha}{4} = 2$$

α = 8



Area $= \frac{1}{2}(6)(8) = 24$

Perimeter = 24

Circumradius = 5

Inradius $= \frac{\Delta}{s} = \frac{24}{12} = 2$

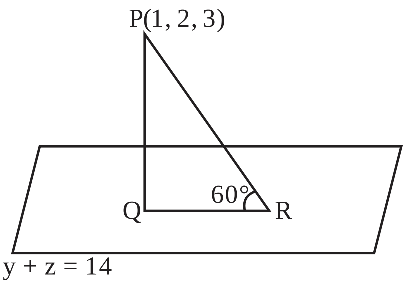
14. Let Q be the foot of perpendicular drawn from the point P (1, 2, 3) to the plane x + 2y + z = 14. If R is a point on the plane such that $\angle PRQ = 60°$, then the area of ΔPQR is equal to:

(A) $\frac{\sqrt{3}}{2}$ (B) $\sqrt{3}$

(C) $2\sqrt{3}$ (D) 3

Official Ans. by NTA (B)

Sol.



Length of perpendicular

$$PQ = \left|\frac{1+4+3-14}{\sqrt{6}}\right| = \sqrt{6}$$

$$QR = (PQ)\cot 60° = \sqrt{2}$$

$$\therefore \text{ Area of } \Delta PQR = \frac{1}{2}(PQ)(QR) = \sqrt{3}$$

15. If (2, 3, 9), (5, 2, 1), (1, λ, 8) and (λ, 2, 3) are coplanar, then the product of all possible values of λ is:

(A) $\frac{21}{2}$ (B) $\frac{59}{8}$

(C) $\frac{57}{8}$ (D) $\frac{95}{8}$

Official Ans. by NTA (D)

Sol. A(2, 3, 9) ; B(5, 2, 1) ; C(1, λ, 8) ; D(λ, 2, 3)

$$\left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right] = 0$$

$$\begin{vmatrix} 3 & -1 & -8 \\ -1 & \lambda-3 & -1 \\ \lambda-2 & -1 & -6 \end{vmatrix} = 0$$

$\Rightarrow [-6(\lambda - 3) - 1] - 8(1 - (\lambda - 3)(\lambda - 2)) + (6 + (\lambda - 2) = 0$

$3(-6\lambda + 17) - 8(-\lambda^2 + 5\lambda - 5) + (\lambda + 4) = 8$

$8\lambda^2 - 57\lambda + 95 = 0$

$$\lambda_1 \lambda_2 = \frac{95}{8}$$

16. Bag I contains 3 red, 4 black and 3 white balls and Bag II contains 2 red, 5 black and 2 white balls. One ball is transferred from Bag I to Bag II and then a ball is draw from Bag II. The ball so drawn is found to be black in colour. Then the probability, that the transferred ball is red, is:

(A) $\frac{4}{9}$ (B) $\frac{5}{18}$ (C) $\frac{1}{6}$ (D) $\frac{3}{10}$

Official Ans. by NTA (B)

Sol.

3R	2R
4B	5B
3W	2W

A : Drown ball from boy II is black

B : Red ball transferred

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{3}{9} \times \frac{5}{10}}{\frac{3}{9} \times \frac{5}{10} + \frac{4}{9} \times \frac{6}{10} + \frac{3}{9} \times \frac{5}{10}}$$

$$= \frac{15}{15+24+15} = \frac{15}{54} = \frac{5}{18}$$

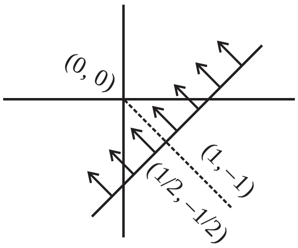
17. Let $S = \{z = x + iy : |z - 1 + i| \geq |z|, |z| < 2, |z + i| = |z - 1|\}$. Then the set of all values of x, for which $w = 2x + iy \in S$ for some $y \in \mathbb{R}$, is

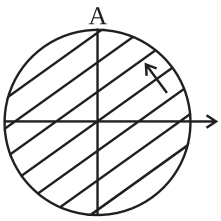
(A) $\left(-\sqrt{2}, \frac{1}{2\sqrt{2}}\right]$ (B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right]$

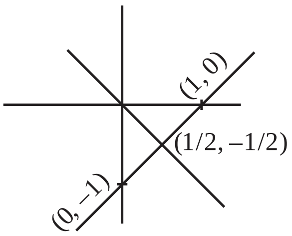
(C) $\left(-\sqrt{2}, \frac{1}{2}\right]$ (D) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$

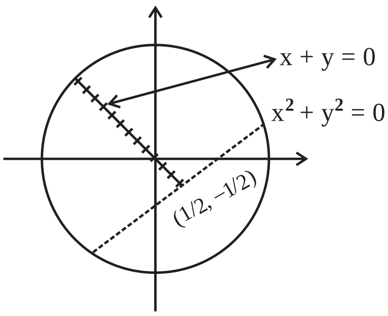
Official Ans. by NTA (B)

Sol. $|z - 1 + i| \geq |z|$; $|z| < 2$; $|z + i| = |z - 1|$









Hence

$w = 2x + iy \in S$

$2x \leq \frac{1}{2} \quad \therefore x \leq \frac{1}{4}$

Now

$(2x)^2 + (2x)^2 < 4$

$x^2 < \frac{1}{2} \quad \Rightarrow x \in \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\therefore x \in \left(\frac{-1}{\sqrt{2}}, \frac{1}{4}\right]$

18. Let $\vec{a}, \vec{b}, \vec{c}$ be three coplanar concurrent vectors such that angles between any two of them is same. If the product of their magnitudes is 14 and $(\vec{a} \times \vec{b}).(\vec{b} \times \vec{c}) + (\vec{b} \times \vec{c}).(\vec{c} \times \vec{a}) + (\vec{c} \times \vec{a}).(\vec{a} \times \vec{b}) = 168$ then $|\vec{a}| + |\vec{b}| + |\vec{c}|$ is equal to:

(A) 10 (B) 14

(C) 16 (D) 18

Official Ans. by NTA (C)

Sol. $|\vec{a}||\vec{b}||\vec{c}| = 14$

$\vec{a} \wedge \vec{b} = \vec{b} \wedge \vec{c} = \vec{c} \wedge \vec{a} = \theta = \frac{2\pi}{3}$

So, $\vec{a}.\vec{b} = -\frac{1}{2}ab,\ \vec{b}.\vec{c} = -\frac{1}{2}bc,\ \vec{a}.\vec{c}. = -\frac{1}{2}ac$

(let)

$(\vec{a} \times \vec{b}).(\vec{b} \times \vec{c}) = (\vec{a}.\vec{b})(\vec{b}.\vec{c}) - (\vec{a}.\vec{c})(\vec{b}.\vec{b})$

$= \frac{1}{4}ab^2c + \frac{1}{2}ab^2c = \frac{3}{4}ab^2c$

Similarly

$(\vec{b} \times \vec{c}).(\vec{c} \times \vec{a}) = \frac{3}{4}abc^2$

$(\vec{c} \times \vec{a}).(\vec{a} \times \vec{b}) = \frac{3}{4}a^2bc$

$168 = \frac{3}{4}abc(a + b + c)$

So, $(a + b + c) = 16$

19. The domain of the function $f(x) = \sin^{-1}\left(\frac{x^2 - 3x + 2}{x^2 + 2x + 7}\right)$ is :

(A) $[1, \infty)$ (B) $(-1, 2]$

(C) $[-1, \infty)$ (D) $(-\infty, 2]$

Official Ans. by NTA (C)

Sol. $f(x) = \sin^{-1}\left(\frac{x^2 - 3x + 2}{x^2 + 2x + 7}\right)$ Domain

$\frac{x^2 - 3x + 2}{x^2 + 2x + 7} \geq -1$ and $\frac{x^2 - 3x + 2}{x^2 + 2x + 7} \leq 1$

$2x^2 - x + 9 \geq 0$ and $5x \geq -5 \Rightarrow x \geq -1$

$x \in R$

Hence Domain $x \in [-1, \infty)$

20. The statement $(p \Rightarrow q) \vee (p \Rightarrow r)$ is NOT equivalent to:

(A) $(p \wedge (\sim r)) \Rightarrow q$ (B) $(\sim q) \Rightarrow ((\sim r) \vee p)$

(C) $p \Rightarrow (q \vee r)$ (D) $(p \wedge (\sim q)) \Rightarrow r$

Official Ans. by NTA (B)

Sol. $(p \to q) \vee (p \to r)$

$(\sim p \vee q) \vee (\sim p \vee r)$

$= \sim p \vee (q \vee r)$

$= p \to (q \vee r) \equiv (3)$ is true.

Now (1) $(p \wedge \sim r) \to q$

$\sim(p \wedge \sim r) \vee q = (\sim p \vee r) \vee q$

$= \sim p \vee (r \vee q) = p \to (q \vee r)$

(4) $(p \wedge \sim q) \to r = p \to (q \vee r)$

SECTION-B

1. The sum and product of the mean and variance of a binomial distribution are 82.5 and 1350 respectively. They the number of trials in the binomial distribution is:

Official Ans. by NTA (96)

Sol. Let, mean = m = np

& variance = v = npq, p + q = 1

Sum = m + v = $\frac{165}{2}$

Product = mv = 1350

On solving,

$m = np = 60$ & $v = npq = \frac{45}{2}$ $\therefore q = \frac{3}{8}$ $\therefore P = \frac{5}{8}$

Hence n = 96

2. Let α, β $(\alpha > \beta)$ be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n, n \in \mathbb{N}$, then

$\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ is equal to ______.

Official Ans. by NTA (16)

Sol. $Pn = \alpha^n - \beta^n$ $\qquad x^2 - x - 4 = 0$

$\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ __(1)

As $P_n - P_{n-1} = (\alpha^n - \beta^n) - (\alpha^{n-1} - \beta^{n-1})$

$= \alpha^{n-2}(\alpha^2 - \alpha) - \beta^{n-2}(\beta^2 - \beta)$

$= 4(\alpha^{n-2} - \beta^{n-2})$

$P_n - P_{n-1} = 4P_{n-2}$

Hence Expression (1)

$\frac{P_{16}(P_{15} - P_{14}) - P_{15}(P_{15} - P_{14})}{P_{13}P_{14}}$

$= \frac{(P_{15} - P_{14})(P_{16} - P_{15})}{P_{13}P_{14}} = \frac{(4P_{13})(4P_{14})}{P_{13}P_{14}} = 16$

3. Let $x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$. For $k \in \mathbb{N}$, if

$X' A^k X = 33$, then k is equal to:

Official Ans. by NTA (10)

Sol. $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$; $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$

$X^T A^K X = 33$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}^k \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

$$\text{As } A^2 = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^8 = \begin{bmatrix} 1 & 0 & 24 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{10}=\begin{bmatrix}1&0&6\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&24\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}1&0&30\\0&1&0\\0&0&1\end{bmatrix}$$

$$\text{for K}\to\text{Even } A^{K}=\begin{bmatrix}1&0&3K\\0&1&0\\0&0&1\end{bmatrix}$$

$X^TA^KX = 33$ (This is not correct)

$$[1\ 1\ 1]\begin{bmatrix}1&0&3K\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$=[1\ 1\ 3K+1]\begin{bmatrix}1\\1\\1\end{bmatrix}=[3K+3]$$

$\therefore$ 3K + 3 = 33 $\therefore$ K = 10

But it should be dropped as 33 is not matrix

If K is odd

$X^TA^KX = 33$

$X^TAA^{K-1}X = 33$

$$\begin{bmatrix}1&1&1\end{bmatrix}\begin{bmatrix}-1&2&3\\0&1&6\\0&0&-1\end{bmatrix}\begin{bmatrix}1&0&3k-3\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}=33$$

$$\begin{bmatrix}-1&3&8\end{bmatrix}\begin{bmatrix}3k-2\\1\\1\end{bmatrix}=[33]$$

[–3k + 13] = [33]

k = 20/3 (not possible)

4. The number of natural numbers lying between 1012 and 23421 that can be formed using the digits 2, 3, 4, 5, 6 (repetition of digits is not allowed) and divisible by 55 is____,

Official Ans. by NTA (6)

Sol. 4 digit numbers

For divisibility by 55, no. should be

div. by 5 and 11 both

Also, for divisibility by 11

			5
a	b	c	d

a + c = b + 5

for b = 1   a = 2, c = 4

a = 4, c = 2

for b = 2   a = 3, c = 4

a = 4, c = 3

for b = 3   a = 6, c = 2

a = 2, c = 6

$\therefore$ 6 possible four digit no.s are div. by 55

(II) 5 digit number is not possible

(Not possible)

5. If $\sum_{k=1}^{10} K^2\left(10_{C_K}\right)^2 = 22000L$, then L is equal to ____.

Official Ans. by NTA (221)

Sol. $\sum_{K=1}^{10} K^2\left({}^{10}C_K\right)^2$

$$\sum_{K=1}^{10}\left(K.{}^{10}C_K\right)^2=\sum_{K=1}^{10}\left(10.{}^{9}C_{K-1}\right)^2$$

$$=100\sum_{K=1}^{10}{}^{9}C_{K-1}.{}^{9}C_{10-K}$$

$$=100\left({}^{18}C_9\right)=100\left(\frac{18!}{9!9!}\right)$$

$\Rightarrow$ 4862000 = 22000L

Hence L = 221

6. If [t] denotes the greatest integer $\le$ t, then number of points, at which the function f(x) = 4 | 2x + 3| + $9\left[x+\frac{1}{2}\right]$–12 [x + 20] is not differentiable in the open interval (–20, 20), is____.

Official Ans. by NTA (79)

Sol. $f(x) = 4|2x+3| + 9\left[x+\frac{1}{2}\right] - 12[x+20]$

$x \in (-20, 20)$

f(x) is not Diff. at $x = I \in \{-19, -18, ...0,...19\} = 39$

at $x = I + \frac{1}{2}$, f(x) Non diff. at 39 points

Check at $x = \frac{-3}{2}$ Discount at $x = \frac{-3}{2}$ $\therefore$ N. R(1)

No. of point of non-differentiabilty

= 39 + 39 + 1 = 79

7. If the tangent to the curve $y = x^3 - x^2 + x$ at the point (a , b) is also tangent to the curve $y = 5x^2 + 2x - 25$ at the point (2, –1), then |2a + 9b| is equal to ______.

Official Ans. by NTA (195)

Sol. $y = 5x^2 + 2x - 25$ P(2, –1)

$y' = 10x + 2$

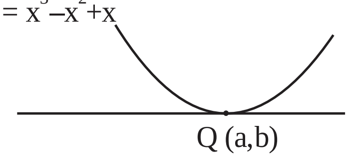
$y'_P = 22$

$\therefore$ tangent to curve at P

$y + 1 = 22(x - 2)$

$y = 22x - 45$

$y = x^3 - x^2 + x$



$$\left.\frac{dy}{dx}\right|_{C_2} = 3x^2 - 2x + 1$$

$$\left.\frac{dy}{dx}\right|_{Q} = 3a^2 - 2a + 1$$

Hence $3a^2 - 2a + 1 = 22$

$\therefore\ 3a^2 - 2a - 21 = 0$

$3a^2 - 9a + 7a - 21 = 0$

$(3a + 7)(a - 3) = 0$ $\begin{cases} a = 3 \\ a = -7/3 \end{cases}$

from curve $b = a^3 - a^2 + a$

$a = 3$

$b = 21$ $|2a + 9b| = 195$

at $a = -7/3$ tangent will be parallel

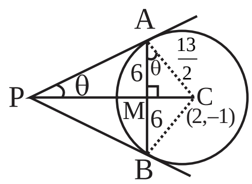
Hence it is rejected

8. Let AB be a chord of length 12 of the circle $(x-2)^2 + (y+1)^2 = \frac{169}{4}$.

If tangents drawn to the circle at points A and B intersect at the point P, then five times the distance of point P from chord AB is equal to ___.

Official Ans. by NTA (72)

Sol.



$$\cos\theta = \frac{6}{\frac{13}{2}} = \frac{12}{13}$$

$$\sin\theta = \frac{5}{13}$$

PM = AM cotθ

$$PM = 6\left(\frac{12}{5}\right) \quad \therefore\ 5(PM) = 72$$

9. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2, \vec{a}.\vec{b} = 3$ and $|\vec{a}\times\vec{b}|^2 = 75$. Then $|\vec{a}|^2$ is equal to____.

Official Ans. by NTA (14)

Sol. $\left|\vec{a}+\vec{b}\right|^2 = \left|\vec{a}\right|^2 + 2\left|\vec{b}\right|^2$; $\vec{a}.\vec{b} = 3$

As $\cancel{|\vec{a}|^2} + |\vec{b}|^2 + 2\vec{a}.\vec{b} = \cancel{|\vec{a}|^2} + 2|\vec{b}|^2$

$|\vec{b}|^2 = 2\vec{a}.\vec{b} = 6$

$|\vec{a}\times\vec{b}|^2 = 75$

$|\vec{a}|^2|\vec{b}|^2 - \left(\vec{a}.\vec{b}\right)^2 = 75$

$6|\vec{a}|^2 - 9 = 75 \Rightarrow |\vec{a}|^2 = 14$

10. Let

$S=\left\{(x,y)\in\mathbb{N}\times\mathbb{N}:9(x-3)^2+16(y-4)^2\le 144\right\}$

and $T=\left\{(x,y)\in\mathbb{R}\times\mathbb{R}:(x-7)^2+(y-4)^2\le 36\right\}$.

The $n(S\cap T)$ is equal to_____.

Official Ans. by NTA (27)

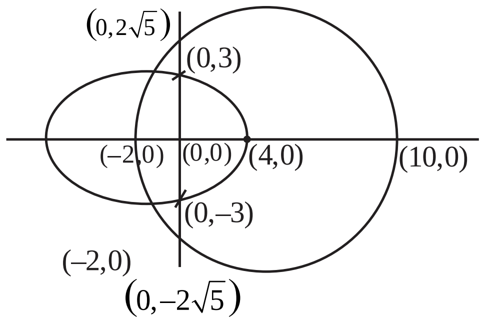
Sol. $S:\dfrac{(x-3)^2}{16}+\dfrac{(y-4)^2}{9}\le 1$; x, y ∈ {1, 2, 3, ...}

$T:(x-7)^2+(y-4)^2\le 36$ x, y ∈ R

Let x – 3 = x : y – 4 = y

$S:\dfrac{x^2}{16}+\dfrac{y^2}{9}\le 1$; x ∈ {–2, –1, 0, 1, ...}

$T:(x-4)^2+y^2\le 36$; y ∈ {–3, –2, –1, 0, ...}



S ∩ T = (–2, 0), (–1, 0), ...(4, 0) → (7)

(–1, 1), (0, 1), ...(3, 1) → (5)

(–1, –1), (0, –1), ...(3, –1) → (5)

(–1, 2), (0, 2), (1, 2), (2, 2) → (4)

(–1, –2), (0, –2), (1, –2), (2, –2) → (4)

(0, 3) (0, –3) → (2)