



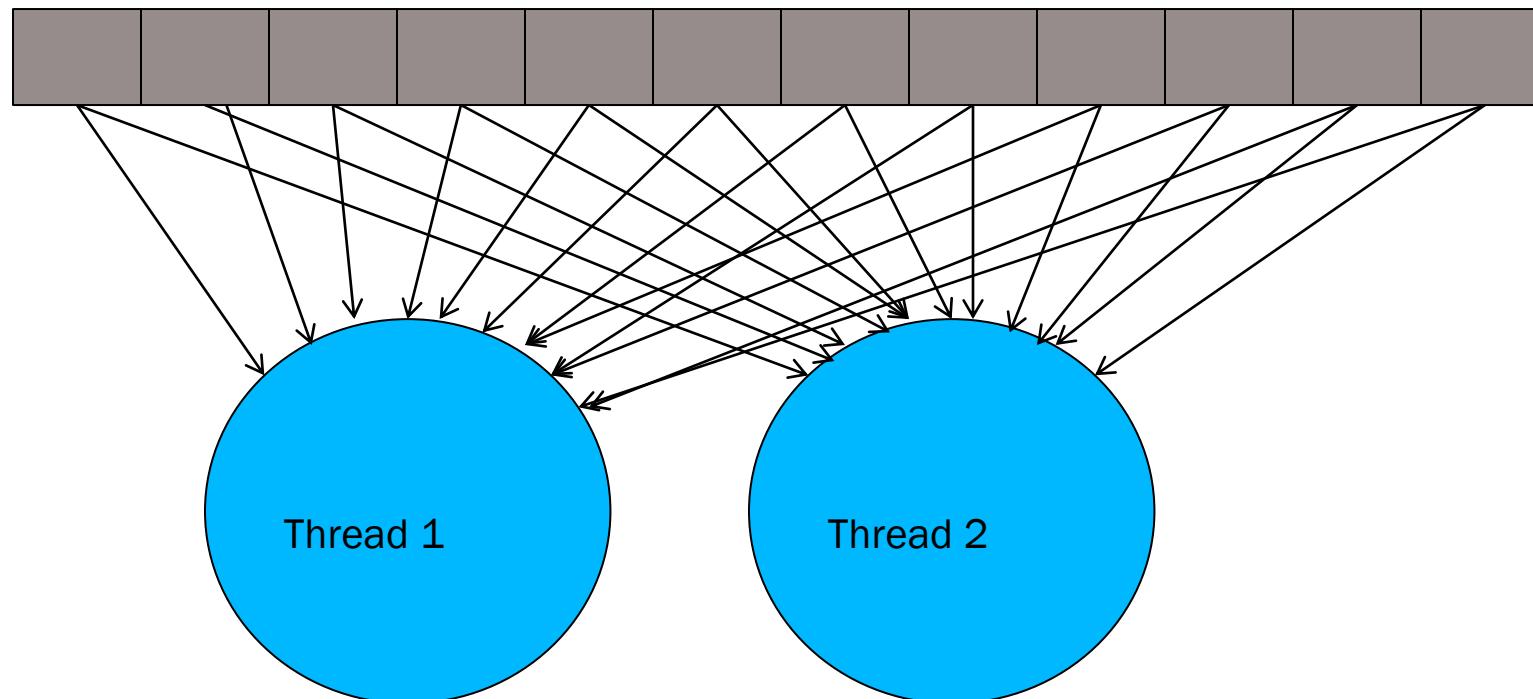
# RECITATION

MATRIX MULTIPLICATION (PARALLEL ARCHITECTURE)

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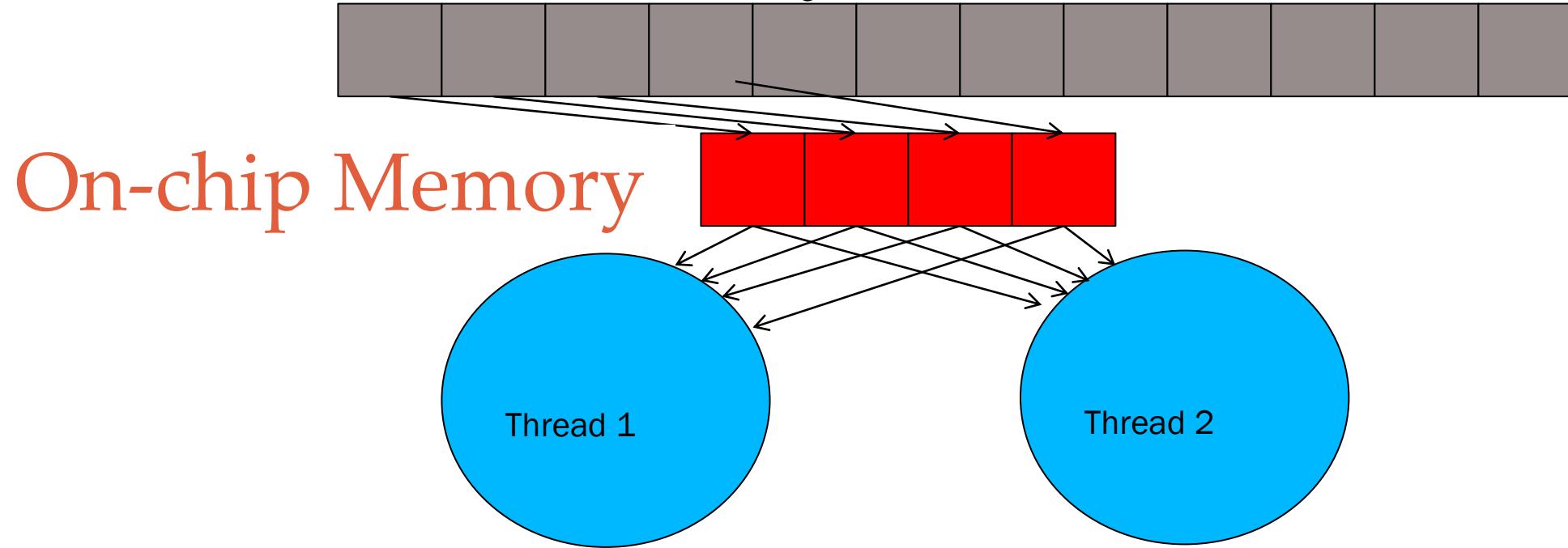
# GLOBAL MEMORY ACCESS PATTERN OF THE BASIC MATRIX MULTIPLICATION KERNEL

## Global Memory



## TILING/BLOCKING - BASIC IDEA

# Global Memory

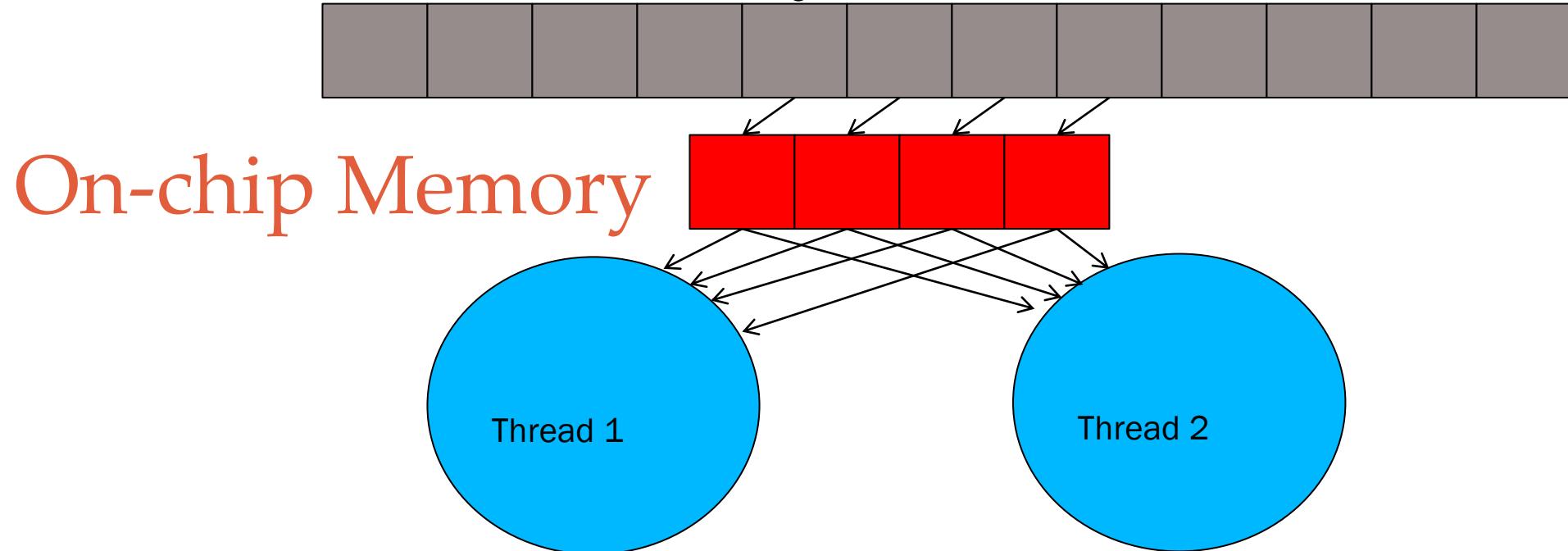


Divide the global memory content into tiles

Focus the computation of threads on one or a small number of tiles at each point in time

## TILING/BLOCKING - BASIC IDEA

# Global Memory



## BASIC CONCEPT OF TILING

- In a congested traffic system, significant reduction of vehicles can greatly improve the delay seen by all vehicles
  - Carpooling for commuters
  - Tiling for global memory accesses
    - drivers = threads accessing their memory data operands
    - cars = memory access requests



## SOME COMPUTATIONS ARE MORE CHALLENGING TO TILE

- Some carpools may be easier than others
  - Car pool participants need to have similar work schedule
  - Some vehicles may be more suitable for carpooling
- Similar challenges exist in tiling



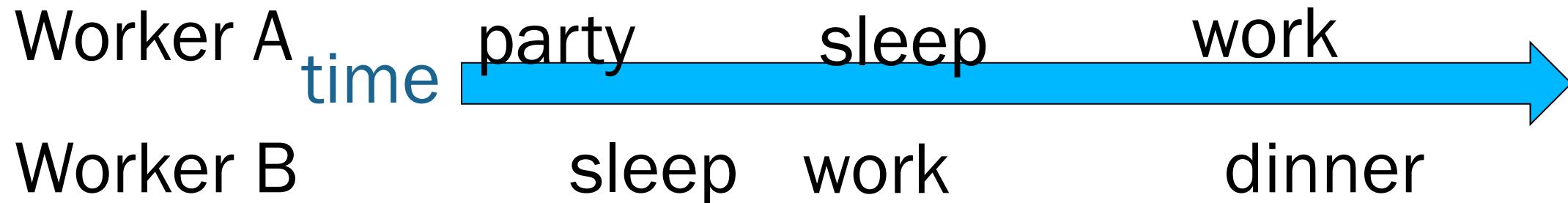
## CARPOOLS NEED SYNCHRONIZATION.

- Good: when people have similar schedule



## CARPOOLS NEED SYNCHRONIZATION.

- Bad: when people have very different schedule



## SAME WITH TILING

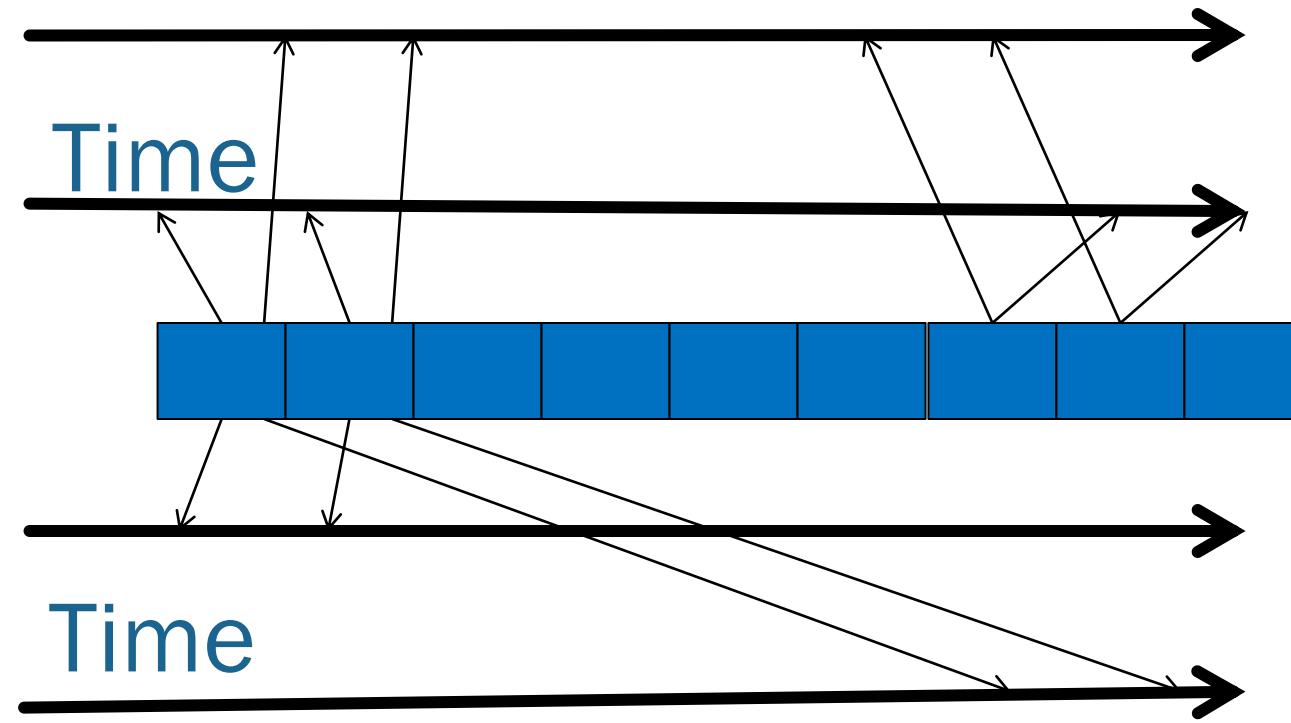
- Good: when threads have similar access timing

Thread 1

Thread 2

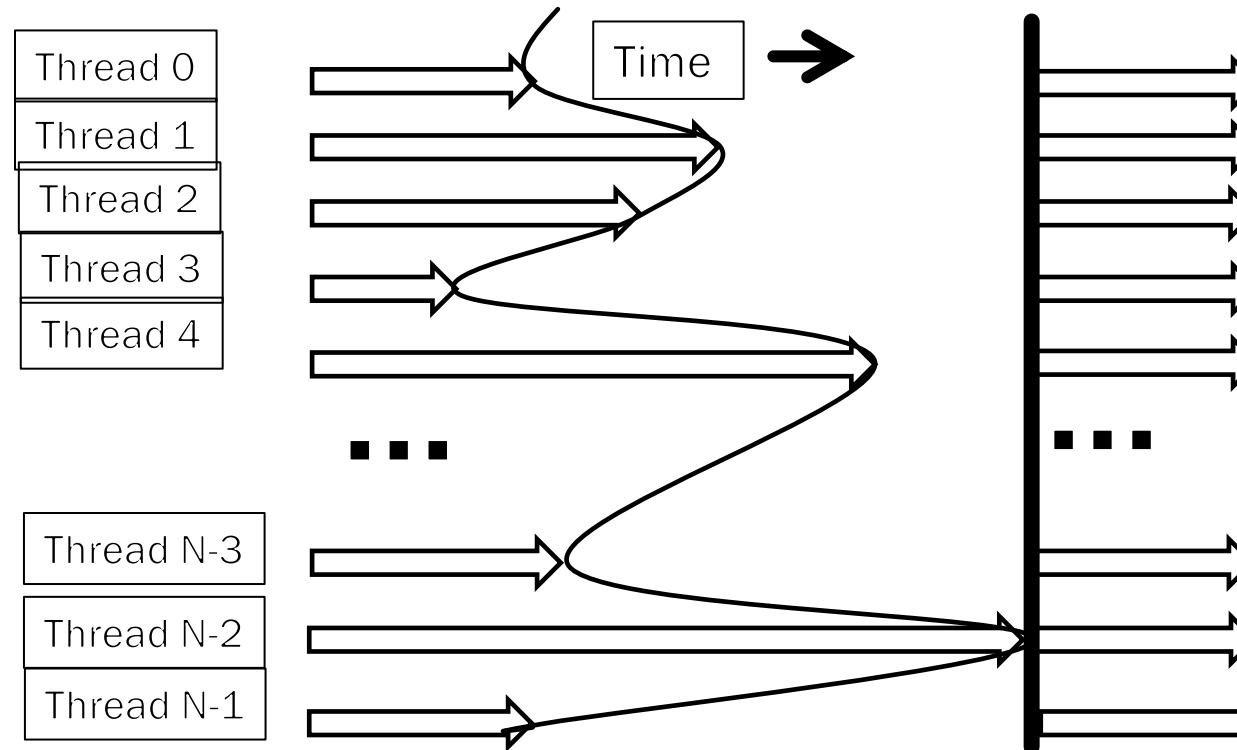
Thread 1

Thread 2



- Bad: when threads have very different timing

# BARRIER SYNCHRONIZATION FOR TILING



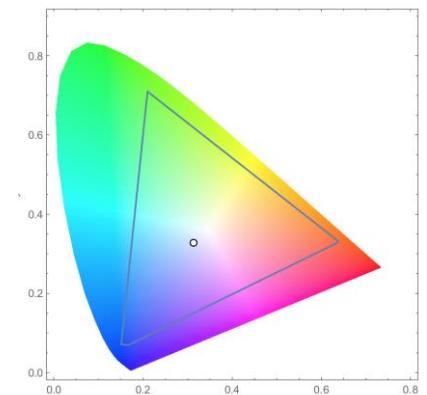
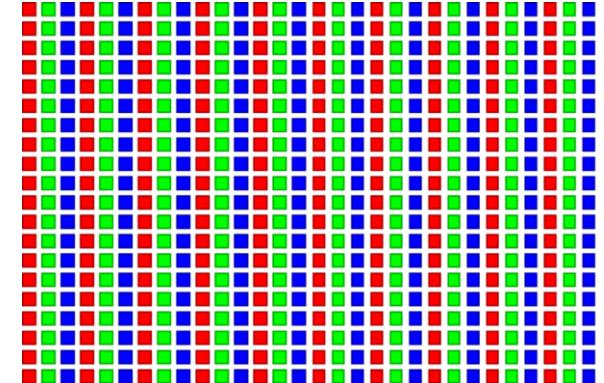
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## OUTLINE OF TILING TECHNIQUE

- Identify a tile of global memory contents that are accessed by multiple threads
- Load the tile from global memory into on-chip memory
- Use barrier synchronization to make sure that all threads are ready to start the phase
- Have the multiple threads to access their data from the on-chip memory
- Use barrier synchronization to make sure that all threads have completed the current phase
- Move on to the next tile

# RGB COLOR IMAGE REPRESENTATION

- Each pixel in an image is an RGB value
- The format of an image's row is  
 $(r\ g\ b)\ (r\ g\ b)\ \dots\ (r\ g\ b)$
- RGB ranges are not distributed uniformly
- Many different color spaces, here we show the constants to convert to AdobeRGB color space
  - The vertical axis (y value) and horizontal axis (x value) show the fraction of the pixel intensity that should be allocated to G and B. The remaining fraction ( $1-y-x$ ) of the pixel intensity that should be assigned to R
  - The triangle contains all the representable colors in this color space



## RGB TO GRayscale CONVERSION



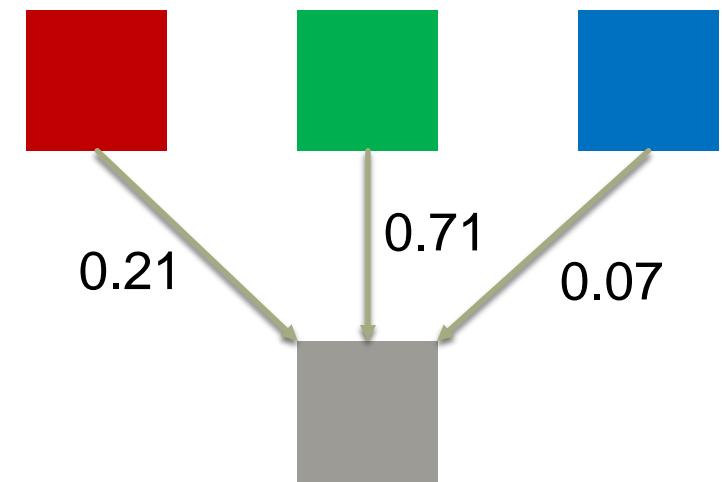
A grayscale digital image is an image in which the value of each pixel carries only intensity information.

## COLOR CALCULATING FORMULA

- For each pixel (r g b) at (I, J) do:

$$\text{grayPixel}[I,J] = 0.21*r + 0.71*g + 0.07*b$$

- This is just a dot product  
 $\langle [r,g,b], [0.21, 0.71, 0.07] \rangle$  with the constants being specific to input RGB space



## RGB TO GRayscale CONVERSION CODE

```
// we have 3 channels corresponding to RGB
// The input image is encoded as unsigned characters [0, 255]
__global__ void colorConvert(unsigned char * grayImage,
                             unsigned char * rgblImage,
                             int width, int height) {
    int x = threadIdx.x + blockIdx.x * blockDim.x;
    int y = threadIdx.y + blockIdx.y * blockDim.y;

    if (x < width && y < height) {

    }
}
```

# RGB TO GRayscale CONVERSION CODE

```
#define CHANNELS 3 // we have 3 channels corresponding to RGB
// The input image is encoded as unsigned characters [0, 255]
__global__ void colorConvert(unsigned char * grayImage,
                             unsigned char * rgbImage,
                             int width, int height) {
    int x = threadIdx.x + blockIdx.x * blockDim.x;
    int y = threadIdx.y + blockIdx.y * blockDim.y;

    if (x < width && y < height) {
        // get 1D coordinate for the grayscale image
        int grayOffset = y*width + x;
        // one can think of the RGB image having
        // CHANNEL times columns than the gray scale image
        int rgbOffset = grayOffset*CHANNELS;
        unsigned char r = rgbImage[rgbOffset]; // red value for pixel
        unsigned char g = rgbImage[rgbOffset + 1]; // green value for pixel
        unsigned char b = rgbImage[rgbOffset + 2]; // blue value for pixel
    }
}
```

# RGB TO GRayscale CONVERSION CODE

```
// we have 3 channels corresponding to RGB
// The input image is encoded as unsigned characters [0, 255]
__global__ void colorConvert(unsigned char * grayImage,
                             unsigned char * rgbImage,
                             int width, int height) {
    int x = threadIdx.x + blockIdx.x * blockDim.x;
    int y = threadIdx.y + blockIdx.y * blockDim.y;

    if (x < width && y < height) {
        // get 1D coordinate for the grayscale image
        int grayOffset = y*width + x;
        // one can think of the RGB image having
        // CHANNEL times columns than the gray scale image
        int rgbOffset = grayOffset*CHANNELS;
        unsigned char r = rgbImage[rgbOffset]; // red value for pixel
        unsigned char g = rgbImage[rgbOffset + 2]; // green value for pixel
        unsigned char b = rgbImage[rgbOffset + 3]; // blue value for pixel
        // perform the rescaling and store it
        // We multiply by floating point constants
        grayImage[grayOffset] = 0.21f*r + 0.71f*g + 0.07f*b;
    }
}
```

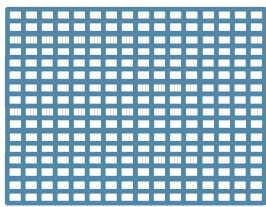
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## IMAGE BLURRING

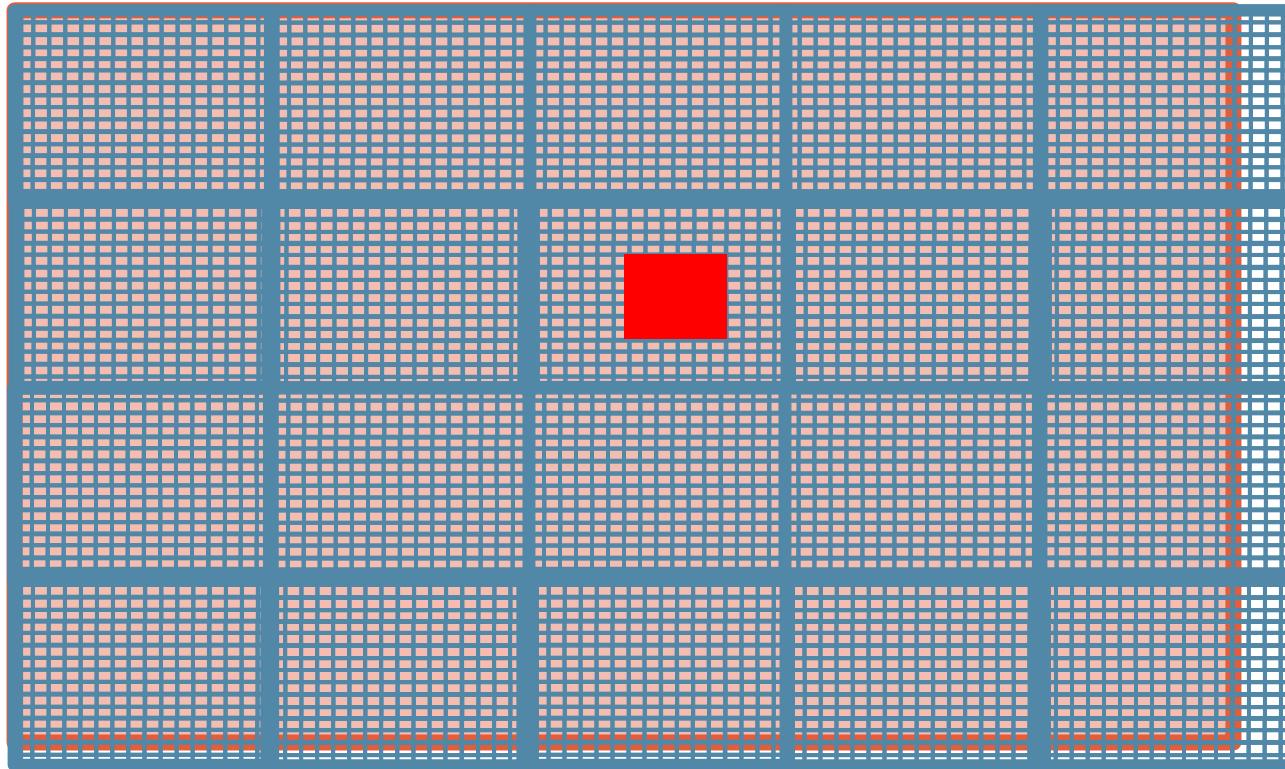
---



## BLURRING BOX



Pixels  
processed by  
a thread  
block



## IMAGE BLUR AS A 2D KERNEL

```
__global__
void blurKernel(unsigned char * in, unsigned char * out, int w, int h)
{
    int Col = blockIdx.x * blockDim.x + threadIdx.x;
    int Row = blockIdx.y * blockDim.y + threadIdx.y;

    if (Col < w && Row < h) {
        ... // Rest of our kernel
    }
}
```

## \_\_global\_\_

```
void blurKernel(unsigned char * in, unsigned char * out, int w, int h) {
    int Col = blockIdx.x * blockDim.x + threadIdx.x;
    int Row = blockIdx.y * blockDim.y + threadIdx.y;

    if (Col < w && Row < h) {
        int pixVal = 0;
        int pixels = 0;

        // Get the average of the surrounding 2xBLUR_SIZE x 2xBLUR_SIZE box
        for(int blurRow = -BLUR_SIZE; blurRow < BLUR_SIZE+1; ++blurRow) {
            for(int blurCol = -BLUR_SIZE; blurCol < BLUR_SIZE+1; ++blurCol) {

                int curRow = Row + blurRow;
                int curCol = Col + blurCol;
                // Verify we have a valid image pixel
                if(curRow > -1 && curRow < h && curCol > -1 && curCol < w) {
                    pixVal += in[curRow * w + curCol];
                    pixels++; // Keep track of number of pixels in the accumulated total
                }
            }
        }

        // Write our new pixel value out
        out[Row * w + Col] = (unsigned char)(pixVal / pixels);
    }
}
```

# L8 - PARALLEL ARCHITECTURE

BMCS3003  
DISTRIBUTED  
SYSTEMS AND  
PARALLEL  
COMPUTING

## CONTENTS



Handler's Classification



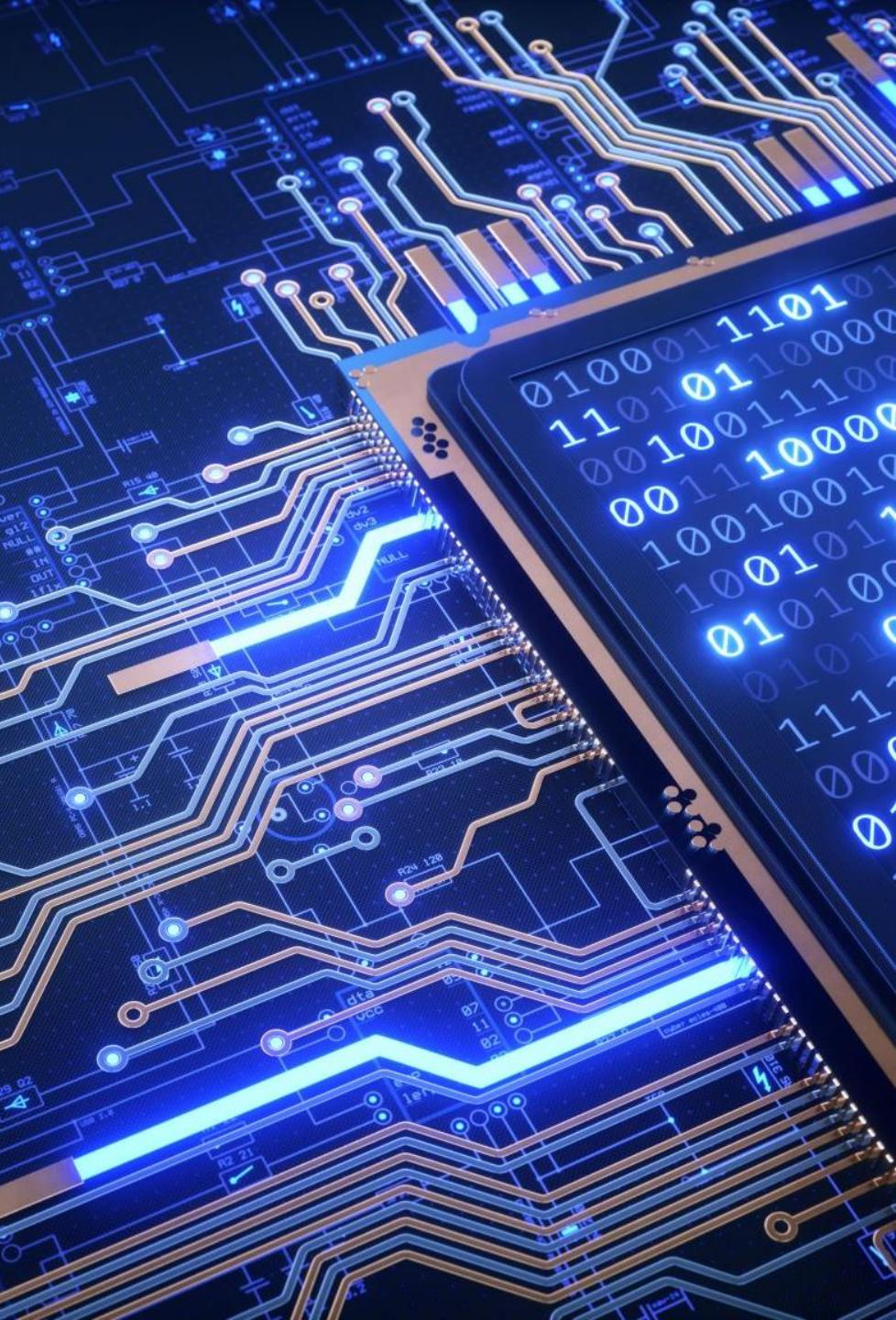
Dependency and its types



Bernstein Conditions for Detecting Parallelism

## HANDLER'S CLASSIFICATION

- Wolfgang Handler has proposed a classification scheme for identifying the parallelism degree and pipelining degree built into the hardware structure of a computer system. He considers at three subsystem levels:
  - Processor Control Unit (PCU)
  - Arithmetic Logic Unit (ALU)
  - Bit Level Circuit (BLC)
- Each PCU corresponds to one processor or one CPU. The ALU is equivalent to Processor Element (PE). The BLC corresponds to combinational logic circuitry needed to perform 1 bit operations in the ALU.



# LOOP DEPENDENCY



Consumer



Dependencies



Container

Statement 2:  $b=a+2;$

"5"

Statement 1:  $a=5;$

- Find dependencies within iterations of a loop
- Goal of determining different relationships between statements.
- To allow multiple processors to work on different portions of the loop in parallel
- First analyze the dependencies within individual loops.
- It help determine which statements in the loop need to be completed before other statements can start.
- Two general categories of dependencies: Data and Control dependency

# NEED FOR DEPENDENCE ANALYSIS

- Reordering transformations.
  - As the name suggests, these transformations change the order of computations in a program. Reordering transformations are a very powerful class of transformations and can be used at source-level as well as at the intermediate-language level.
  - Consider a simple implementation of matrix-multiply.

```
for i = 1:N {  
    for j = 1:N {  
        C(i,j) = 0.0;  
        for k = 1:N {  
            C(i,j) += A(i,k) * B(k,j);  
        }  
    }  
}
```

Works very well in a scalar machine that executes one instruction at a time

## REORDERING TRANSFORMATION

However, suppose that we had a machine that had a 64-word vector floating point unit. Clearly the loop in this form does not take advantage of the vector unit. We could rewrite the code.

```
for i = 1:N {  
    for j = 1:64:N {  
        C(i,j:j+63) = 0.0;  
        for k = 1:N {  
            C(i,j:j+63) += A(i,k) * B(k,j:j+63)  
        }  
    }  
}
```

## BERNSTEIN'S CONDITIONS

- Since computationally intensive programs are likely to spend most of their time in loops it makes sense to pay attention to improving the performance of loops by parallelizing them. In a 1966 paper, Bernstein established that two iterations  $I_1$  and  $I_2$  can be safely execute in parallel if
  1. iteration  $I_1$  does not write into a location that is read by  $I_2$ ,
  2. iteration  $I_2$  does not write into a location that is read by  $I_1$ , and
  3. iteration  $I_1$  does not write into a location that is also written into by iteration  $I_2$ .
- This forms the basis for all loop parallelization algorithms.

```
function Dep(a,b)
c:=a.b
d:=2.c
end function
```

**(flow dependency)**

```
function Nodep(a,b)
c:=a.b
d:=2.b
e:=a+b
end function
```

**(no dependency)**

# DATA DEPENDENCY

| TYPE                   | NOTATION                | DESCRIPTION   |
|------------------------|-------------------------|---|
| True (Flow) Dependence | $S_1 \rightarrow_T S_2$ | A true dependence between $S_1$ and $S_2$ means that $S_1$ writes to a location later read from by $S_2$              |
| Anti Dependence        | $S_1 \rightarrow_A S_2$ | An anti-dependence between $S_1$ and $S_2$ means that $S_1$ reads from a location later written to by $S_2$ .(before) |
| Output Dependence      | $S_1 \rightarrow_I S_2$ | An input dependence between $S_1$ and $S_2$ means that $S_1$ and $S_2$ read from the same location.                   |

## **True dependence**

S0: int a, b;

S1: a = 2;

S2: b = a + 40;

S1  $\rightarrow T$  S2, meaning that S1 has a true dependence on S2 because S1 writes to the variable a, which S2 reads from.

## **Anti-dependence**

S0: int a, b = 40;

S1: a = b - 38;

S2: b = -1;

S1  $\rightarrow A$  S2, meaning that S1 has an anti-dependence on S2 because S1 reads from the variable b before S2 writes to it.

## **Output-dependence**

S0: int a, b = 40;

S1: a = b - 38;

S2: a = 2;

S1  $\rightarrow O$  S2, meaning that S1 has an output dependence on S2 because both write to the variable a.

# ANTI DEPENDENCES

- WAR - Write-after-read

```
Sum = a+1;  
  
First_term = sum*scale1;  
  
Sum = b+1;  
  
Second_term = sum*scale2;
```

- This is **false dependence**
- Rewrite with variable renaming

```
Sum1=a+1;  
  
First_term=sum1*scale1;  
  
Sum2 = b+1;  
  
Second_term = sum2*scale2;
```

# OUTPUT DEPENDENCES

- WAW - Write-after-write

```
Sum = a+1;  
  
First_term = sum*scale1;  
  
Sum = b+1;  
  
Second_term = sum*scale2;
```

- This is **false dependence**
- Rewrite with variable renaming

```
Sum1=a+1;  
  
First_term=sum1*scale1;  
  
Sum2 = b+1;  
  
Second_term = sum2*scale2;
```

# CONTROL DEPENDENCY

Control flow gives rise to control dependences. Consider the following code.

```
for ( . . . ) {  
S1      if (d == 0) continue;  
S2      x = x / d;  
      . . .  
}
```

Here, the statement  $S_1$  must be executed before  $S_2$ , since the execution of  $S_2$  is conditional upon the execution of the branch in  $S_1$ . Executing  $S_2$  before  $S_1$  could generate a divide-by-zero error that would be impossible in the original code.

---

# DEPENDENCY IN LOOP

Loops can have two types of dependence:

- Loop-carried dependency
- Loop-independent dependency

# LOOP CARRIED DEPENDENCY

- In loop-carried dependence, statements in an iteration of a loop depend on statements in another iteration of the loop.

```
for(i=0;i<4;i++)
{
    S1: b[i]=8;
    S2: a[i]=b[i-1] + 10;
}
```

# LOOP INDEPENDENT DEPENDENCY

- In loop-independent dependence, loops have inter-iteration dependence, but do not have dependence between iterations.
- Each iteration may be treated as a block and performed in parallel without other synchronization efforts.

```
for (i=0;i<4;i++)  
{  
S1: b[i] = 8;  
S2: a[i] =b[i] + 10;  
}
```

---

## SIMPLE RULE OF THUMB

- A loop that matches the following criteria has no dependences and can be parallelized:
  - All assignments (to shared data) are to arrays
  - Each element is assigned by at most one iteration
  - No iteration reads elements assigned by any other iteration

## ARE THESE LOOPS PARALLEL?

1. `for (i=1; i<N; i+=2)`

`a[i] = a[i]+a[i-1];`

2. `for (i=0; i<N/2; i++)`

`a[i] = a[i]+a[i+N/2];`

3. `for (i=0; i<=N/2; i++)`

`a[i] = a[i]+a[i+N/2];`

4. `for (i=0; i<N; i++)`

`a[idx[i]] = a[idx[i]]+b[idx[i]];`

# LOOP PARALLELIZATION

- Extraction parallel tasks from loops
- Data is stored in random access data structures
- A program exploiting loop-level parallelism will use multiple threads or processes which operate on same time
- It provides speedup
- Amdhal's law

## REMOVING DEPENDENCES

```
for (i=0; i<N; i++) {  
    x=(b[i]+c[i])/2;  
    a[i]=a[i+1]+x;  
}
```

**What kind of dependences are there?**

## SAME LOOP MADE PARALLEL

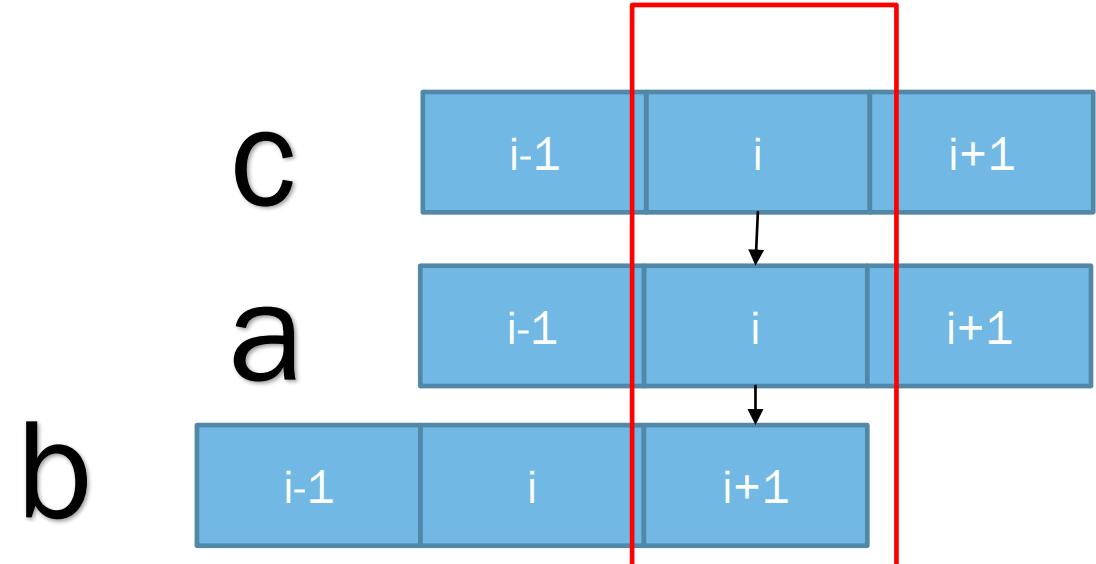
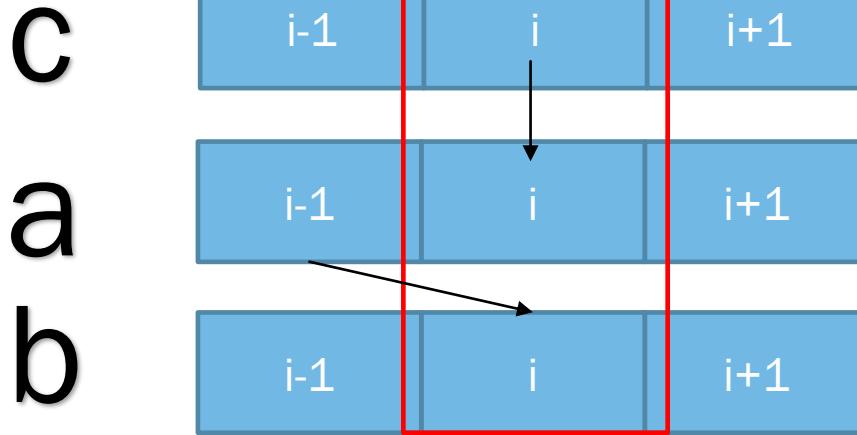
```
#pragma omp parallel for
for (i=0;i<N;i++) {
    a2[i]=a[i+1];
}

#pragma omp parallel for private(x)
for (i=0;i<N;i++) {
    x=(b[i]+c[i])/2;
    a[i]=a2[i]+x;
}
```

- Space and extra data movement is traded for parallelism

# WHAT ARE THE DEPENDENCES?

```
for(i=1;i<N;i++){  
    b[i]+=a[i-1];  
    a[i]+=c[i];  
}
```



---

## USE SOFTWARE PIPELINING TO MAKE IT PARALLEL

```
b[1] += a[0];  
#pragma omp parallel for  
for (i=1; i<N-1; i++) {  
    a[i] += c[i];  
    b[i+1] += a[i];  
}  
a[N] += c[N];
```

# BERNSTEIN CONDITIONS FOR DETECTION OF PARALLELISM,

# BERNSTEIN CONDITIONS FOR DETECTION OF PARALLELISM

- For implementation of instructions or block of instructions in parallel, it should be guaranteed that the instructions are independent of each other. These instructions can be / control dependent / data dependent resource dependent on each other. We are considering only data dependency between the statements for taking decisions of parallel implementation. A.J. Bernstein has implicated the work of data dependency and resultant some conditions based on which we can settle on the parallelism of processes or instructions.
- Bernstein conditions are rely on the subsequent two sets of variables:
  - i) The input set or Read set  $R_I$  that consists of memory locations examine by the statement of instruction  $I_1$ .
  - ii) The output set or Write set  $W_I$  that consists of memory locations printed into by instruction  $I_1$ .
- The sets  $W_I$  and  $R_I$  are not disjoint as the similar locations are used for writing and reading by  $S_I$ .

---

## BERNSTEIN PARALLELISM CONDITIONS WHICH ARE DETERMINE TO CONCLUDE WHETHER THE STATEMENTS ARE PARALLEL OR NOT:

1. Position in  $R_1$  from which  $S_1$  reads and the Position  $W_2$  onto which  $S_2$  writes must be commonly exclusive. It means  $S_1$  doesn't read from any memory location onto which  $S_2$  writes. It can be indicated as:

$$R_1 \cap W_2 = \emptyset$$

2.  $R_1$  Likewise, locations in  $R_2$  from which  $S_1$  writes and the locations  $W_1$  onto which  $S_2$  reads must be commonly exclusive. It means  $S_2$  does not read from some memory location onto which  $S_1$  writes. It can be indicated as:

$$R_2 \cap W_1 = \emptyset$$

3. The memory locations  $W_1$  and  $W_2$  onto which  $S_1$  and  $S_2$  write, should not be examined by  $S_1$  and  $S_2$ . It means  $R_1$  and  $R_2$  should be free of  $W_1$  and  $W_2$ . It can be indicated as :

$$W_1 \cap W_2 = \emptyset$$

---

---

LET CONSIDER THE FOLLOWING INSTRUCTIONS OF SEQUENTIAL PROGRAM:

$$I_1 : x = (a + b) / (a * b)$$

$$I_2 : y = (b + c) * d$$

$$I_3 : z = x^2 + (a * e)$$

Now, the read set and write set of  $I_1$ ,  $I_2$  and  $I_3$  are as given:

$$R_1 = \{a,b\} \quad W_1 = \{x\}$$

$$R_2 = \{b,c,d\} \quad W_2 = \{y\}$$

$$R_3 = \{x,a,e\} \quad W_3 = \{z\}$$