

File name: main_predictive_analytics_and_modeling.ipynb

This file will import data, call the optimization model, provide optimization result.

Outline:

1. Estimating input parameters
2. Modeling
3. Sensitivity analysis

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Date: August 2022

Predictive Analytics

```
In [ ]: # Import essential libraries
# import library
import pandas as pd
import geopandas as gpd
import custom_functions, data_visualization
import model_PAMIP, model_analysis
import numpy as np
from sklearn.cluster import MiniBatchKMeans
import matplotlib.pyplot as plt
%matplotlib inline
```

```
In [2]: # # import data
spill_data_r = pd.read_excel('Inputs/data_PAMIP.xlsx', sheet_name='spills', header=0, index_col=None).copy()
spill_data_200 = spill_data_r.sample(n=200, random_state=42)

station_data = pd.read_excel('Inputs/data_PAMIP.xlsx', sheet_name='stations', header=0).copy()

current_facility = pd.read_excel('Inputs/data_PAMIP.xlsx', sheet_name='current')

# Arctic geomap
ArcticMap = gpd.read_file("Inputs/ArcGIS_data/ArcticShapefile2/ArcticShapefile2.shp")
ArcticMap = ArcticMap.to_crs(epsg=4326) # 3857

# Shipping route
Shipping_route_shp = gpd.read_file('Inputs/ArcGIS_data/Shipping_and_Hydrography/Canadian_Shipping_Routes.shp')
Shipping_route_shp = Shipping_route_shp.to_crs(epsg=4326)

sensitivity_dataR = gpd.read_file('Inputs/ArcGIS_data/Sensitivity_data5.shp').copy()
sensitivity_dataR = sensitivity_dataR.to_crs(epsg=4326)

# Data Scene 2
spill_data_10000 = pd.read_excel('Inputs/Spill_info_4000.xlsx', header=0).copy()
spill_data_scene2 = spill_data_10000.sample(n=2000, random_state=42)
```

```
In [3]:
```

Fig3 Canadian Arctic map

(facilities, shipping route and sensitivity)

```

In [4]: fig, ax3 = plt.subplots(figsize=(8,8))

# Load geometric file for map
ArcticLand = ArcticMap[ArcticMap['Region_EN'].isin(['Arctic'])]
ArcticWater = ArcticMap[ArcticMap['Region_EN'].isin(['Arctic-Water'])]
ArcticLandPlot = ArcticLand.plot(ax=ax3, color="seashell", alpha=.5) # ax=ax,
ArcticWaterPlot = ArcticWater.plot(ax=ax3, color="lightskyblue", alpha=.25)

# Facilities
coordinates_current = custom_functions.extract_coordinate(current_facility)
coordinate_df = pd.DataFrame(coordinates_current) # [0]
coordinate_df.columns = ['Latitude', 'Longitude']
coordinate_gdf = gpd.GeoDataFrame(
    coordinate_df, geometry=gpd.points_from_xy(coordinate_df.Longitude, coordinate_df.Lat
itude))
coordinate_gdf = coordinate_gdf.set_crs(epsg=4326)
st = plt.scatter(data=coordinate_df,
                 x='Longitude', y='Latitude', marker='^', alpha=1, s=250,
                 c='blue')

# Cache
# station_data
coordinates_cache = custom_functions.extract_coordinate(station_data)
coordinate_df_cache = pd.DataFrame(coordinates_cache) # [0]
coordinate_df_cache.columns = ['Latitude', 'Longitude']
coordinate_gdf = gpd.GeoDataFrame(
    coordinate_df_cache, geometry=gpd.points_from_xy(coordinate_df_cache.Longitude, coord
inate_df_cache.Latitude))
coordinate_gdf = coordinate_gdf.set_crs(epsg=4326)
cache = plt.scatter(data=coordinate_df_cache,
                   x='Longitude', y='Latitude', marker='^', alpha=0.5, s=50,
                   c='blue') # ++ why df not gdt in plt.scatter

# Shipping route
Shipping_route_Plot = Shipping_route_shp.plot(ax=ax3, alpha=.5)

# Sensitivities
sensitivity_dataR = sensitivity_dataR.to_crs(epsg=4326)
sensitivity_dataR['Sensitivity_n'] = (sensitivity_dataR['Sensitivit']-min(sensitivity_dat
aR['Sensitivit']))/(max(sensitivity_dataR['Sensitivit'])-min(sensitivity_dataR['Sensitivi
t']))
Sensitivity_Map_Plot = sensitivity_dataR.plot(ax=ax3, color='red', alpha=sensitivity_data
R['Sensitivity_n'])

# fig properties
ax3.grid(False)
ax3.set_xlim([-140, -60])
ax3.set_ylim([50, 80])
plt.axis('off')
#plt.show()
plt.tight_layout()
fig.savefig(f'Outputs/Fig3 gis map.png', transparent=True, bbox_inches='tight')

```

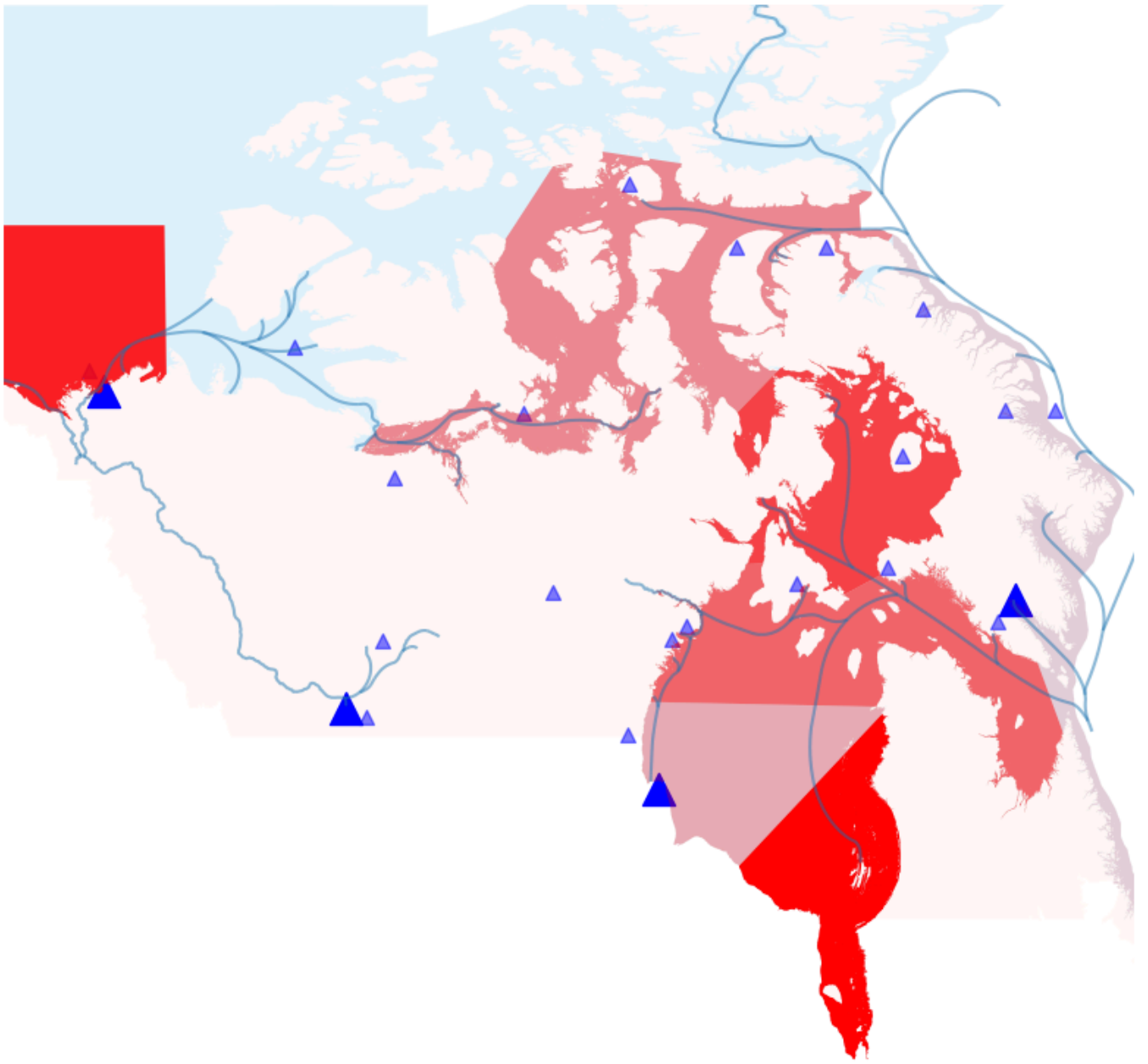
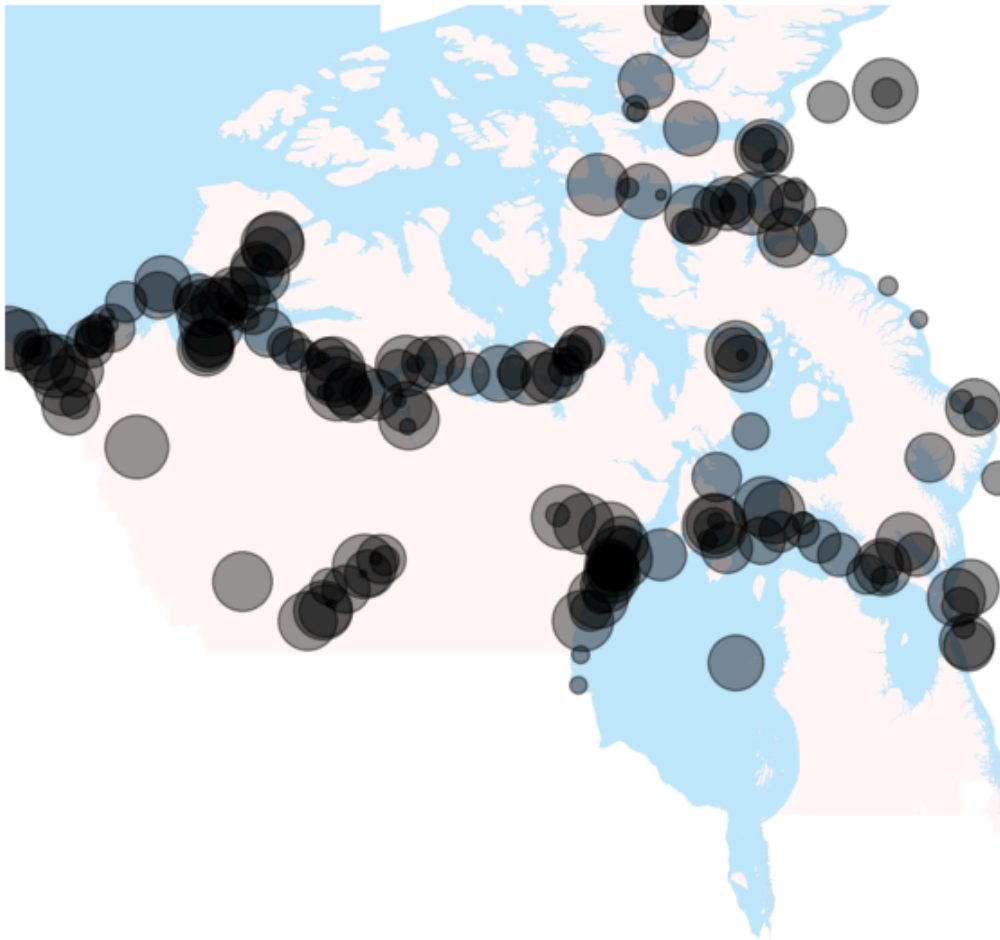


Fig4 Spill data map

```
In [5]: coordinates = custom_functions.extract_coordinate(spill_data_200)
coordinate_df = pd.DataFrame(coordinates)
coordinate_df.columns = ['Latitude', 'Longitude']
coordinate_gdf = gpd.GeoDataFrame(
    coordinate_df, geometry=gpd.points_from_xy(coordinate_df.Longitude, coordinate_df.Latitude))
coordinate_gdf.index = spill_data_200.index
coordinate_gdf['Spill size'] = spill_data_200[['Spill size']].copy()/15
coordinate_gdf = coordinate_gdf.set_crs(epsg=4326)

fig, ax4 = plt.subplots(figsize=(8,7))
ArcticLand = ArcticMap[ArcticMap['Region_EN'].isin(['Arctic'])]
ArcticWater = ArcticMap[ArcticMap['Region_EN'].isin(['Arctic-Water'])]
ArcticLandPlot = ArcticLand.plot(ax=ax4, color="seashell", alpha=.5) # ax=ax,
ArcticWaterPlot = ArcticWater.plot(ax=ax4, color="lightskyblue", alpha=.5)
plt.scatter(data=coordinate_gdf,
            x='Longitude', y='Latitude', marker='o', alpha=.4, s='Spill size',
            c='black')
ax4.set_xlim([-140, -60])
ax4.set_ylim([50, 80])
ax4.axis('off')
fig.savefig(f'Outputs/Fig4a Map of spill data.png', transparent=True, bbox_inches='tight')
```



```

In [6]: coordinates = custom_functions.extract_coordinate(spill_data_scene2)
coordinate_df = pd.DataFrame(coordinates)
coordinate_df.columns = ['Latitude', 'Longitude']
coordinate_gdf = gpd.GeoDataFrame(
    coordinate_df, geometry=gpd.points_from_xy(coordinate_df.Longitude, coordinate_df.Lat
itude))
coordinate_gdf.index = spill_data_scene2.index
coordinate_gdf['Spill size'] = spill_data_scene2[['Spill size']].copy()/2
coordinate_gdf = coordinate_gdf.set_crs(epsg=4326)

fig, ax4b = plt.subplots(figsize=(8,7))
ArcticLand = ArcticMap[ArcticMap['Region_EN'].isin(['Arctic'])]
ArcticWater = ArcticMap[ArcticMap['Region_EN'].isin(['Arctic-Water'])]
ArcticLandPlot = ArcticLand.plot(ax=ax4b, color="seashell", alpha=.5) # ax=ax,
ArcticWaterPlot = ArcticWater.plot(ax=ax4b, color="lightskyblue", alpha=.5)
plt.scatter(data=coordinate_gdf,
            x='Longitude', y='Latitude', marker='o', alpha=.2, s='Spill size',
            c='black')
ax4b.set_xlim([-140, -60])
ax4b.set_ylim([50, 80])
ax4b.axis('off')
fig.savefig(f'Outputs/Fig4b Map of spill data.png', transparent=True, bbox_inches='tight')

```



In [6]:

In [6]:

Model

Input parameters

```
In [7]: # %% Input parameters of the model
NumberStMax = 2
DistanceMax = 10 # 5

coordinates_spill = custom_functions.extract_coordinate(spill_data_200)
coordinates_st = custom_functions.extract_coordinate(spill_data_200)
num_customers = len(coordinates_spill)
num_facilities = len(coordinates_st)

# ++ convert 10 into km using google map (for reporting, not related to modeling in this code)
coord1 = (63.31720065616187, -90.65327442130385)
coord2 = (61.99735832040513, -92.36804572739923)
custom_functions.compute_distance(coord1, coord2)

pairings = {(c, f): custom_functions.compute_distance(coordinates_spill[c], coordinates_st[f])
            for c in range(num_customers)
            for f in range(num_facilities)
            if custom_functions.compute_distance(tuple(coordinates_spill[c]), tuple(coordinates_st[f])) < DistanceMax}

print("Number of viable pairings: {}".format(len(pairings.keys())))

# Weights and scaling
# W = [1, 2000, 1]
max_spill_size = max(spill_data_200['Spill size'])
max_sensitivity = max(sensitivity_dataR['Sensitivity'])
max_timeR = pairings[max(pairings, key=pairings.get)]
min_spill_size = min(spill_data_200['Spill size'])
min_sensitivity = min(sensitivity_dataR['Sensitivity'])
min_timeR = pairings[min(pairings, key=pairings.get)]

# x* = (x-x_min)/(x_max - x_min)

#Demand = list(spill_data_200['Resource needed']).copy()

SizeSpill_R = list(spill_data_200['Spill size']).copy()
Sensitivity_R = custom_functions.calculate_sensitivity(coordinates_spill, sensitivity_dataR)
TimeR_R = pairings.copy() # compute_TimeR +++

SizeSpill = [];
Sensitivity = [];
TimeR = []
SizeSpill = [((SizeSpill_R[i] - min_spill_size) / (max_spill_size - min_spill_size)) for i in range(len(SizeSpill_R))]
Sensitivity = [((Sensitivity_R[i] - min_sensitivity) / (max_sensitivity - min_sensitivity)) for i in range(len(Sensitivity_R))]

# TimeR = {((list(TimeR_R.values())[i]-min_timeR)/(max_timeR-min_timeR)) for i in range(len(TimeR_R))}
TimeR_Scaled = [((list(TimeR_R.values())[i] - min_timeR) / (max_timeR - min_timeR)) for i in range(len(TimeR_R))]
keysD = TimeR_R.keys()
TimeR = {}
for i in range(len(keysD)):
    TimeR[list(keysD)[i]] = TimeR_Scaled[i]
```


Number of viable pairings: 6844

Optimization Modeling

Fig5a Tradeoff curve

Predictive Analytics

```

In [8]: # Tradeoff curve for number of stations
# -----
-----
NumberStMax_list = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
W1 = [[0.1, 0.2, 0.7], [0.8, 0.1, 0.1]] # from model configuration table
Tradeoff_output = []
for i in range(len(NumberStMax_list)):
    Wi = W1[i]
    NumberStMax = NumberStMax_list[i]
    m = 'm1' # m2
    model, cover, select, amount, mvars, names, values, \
        cover_1s, select_1s, amountSt_groupby, coverage_percentage, \
        ResponseTimeT, assignment3, spill_df, station_df, \
        sol_y, assignment, assignment2, assignment_name = model_PAMIP.solve(Wi, coordinat
es_st, coordinates_spill,
pairings, Size
Spill, Sensitivity, TimeR,
NumberStMax,
m, spill_data_200)

    Tradeoff_output.append([NumberStMax, coverage_percentage, int(ResponseTimeT * 80) / 1
1])

Tradeoff_Output_df = pd.DataFrame(Tradeoff_output)
Tradeoff_Output_df.columns = ['NumberStMax', 'Coverage %', 'Response time (in hours)']
Tradeoff_Output_df.to_csv('Outputs/Fig5a Tradeoff_Output_df.csv')

NumberStMax_data = pd.read_csv('Outputs/Fig5a Tradeoff_Output_df.csv').copy()
selected = 5
data_visualization.draw_tradeoff_plot(NumberStMax_data, selected)

```

Set parameter Username

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m: m1

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0xb9359ce4

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [3e-03, 2e+01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+00]

Found heuristic solution: objective 310.8444043

Presolve removed 125 rows and 6968 columns

Presolve time: 0.63s

Presolved: 6920 rows, 6920 columns, 20360 nonzeros

Variable types: 0 continuous, 6920 integer (6920 binary)

Root relaxation: objective 5.199648e+02, 7723 iterations, 0.31 seconds (0.12 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	519.9647506	519.96475	0.00%	-	1s

Explored 1 nodes (7723 simplex iterations) in 1.29 seconds (0.32 work units)

Thread count was 8 (of 8 available processors)

Solution count 2: 519.965 310.844

Optimal solution found (tolerance 1.00e-04)

Best objective 5.199647506194e+02, best bound 5.199647506194e+02, gap 0.0000%

m: m1

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0x5db4e314

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [3e-03, 2e+01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 2e+00]

Found heuristic solution: objective 558.3000709

Presolve removed 52 rows and 6895 columns

Presolve time: 0.11s

Presolved: 6993 rows, 6993 columns, 20579 nonzeros

Variable types: 0 continuous, 6993 integer (6993 binary)

Root relaxation: objective 8.169887e+02, 8055 iterations, 0.36 seconds (0.13 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
	0	0	816.98867	0	170	558.30007	816.98867	46.3%	- 0s
H	0	0			809.9178753	816.98867	0.87%	-	0s
*	0	0		0	814.9390179	814.93902	0.00%	-	0s

Cutting planes:

Gomory: 1
Zero half: 1

Explored 1 nodes (8057 simplex iterations) in 0.84 seconds (0.22 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 814.939 809.918 558.3

Optimal solution found (tolerance 1.00e-04)
Best objective 8.149390178791e+02, best bound 8.149390178791e+02, gap 0.00000%
m: m1
Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0x912827cd

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]
Objective range [3e-03, 2e+01]
Bounds range [1e+00, 1e+00]
RHS range [1e+00, 3e+00]

Found heuristic solution: objective 562.1388165

Presolve removed 52 rows and 6895 columns

Presolve time: 0.07s

Presolved: 6993 rows, 6993 columns, 20579 nonzeros

Variable types: 0 continuous, 6993 integer (6993 binary)

Found heuristic solution: objective 562.4123262

Root relaxation: objective 1.101535e+03, 8240 iterations, 0.46 seconds (0.16 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	1101.5352757	1101.53528	0.00%	-	0s

Explored 1 nodes (8240 simplex iterations) in 0.62 seconds (0.22 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 1101.54 562.412 562.139

Optimal solution found (tolerance 1.00e-04)
Best objective 1.101535275653e+03, best bound 1.101535275653e+03, gap 0.00000%
m: m1
Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0x4041212c

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]
Objective range [3e-03, 2e+01]
Bounds range [1e+00, 1e+00]
RHS range [1e+00, 4e+00]

Found heuristic solution: objective 559.4981317

Presolve removed 52 rows and 6895 columns

Presolve time: 0.07s

Presolved: 6993 rows, 6993 columns, 20579 nonzeros

Variable types: 0 continuous, 6993 integer (6993 binary)

Found heuristic solution: objective 566.1200074

Root relaxation: objective 1.267941e+03, 7887 iterations, 0.41 seconds (0.18 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	1267.9405646	1267.94056	0.00%	-	0s

Explored 1 nodes (7887 simplex iterations) in 0.54 seconds (0.24 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 1267.94 566.12 559.498

Optimal solution found (tolerance 1.00e-04)

Best objective 1.267940564558e+03, best bound 1.267940564558e+03, gap 0.00000%

m: m1

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0x8b96955e

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [3e-03, 2e+01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 5e+00]

Found heuristic solution: objective 560.6975938

Presolve removed 52 rows and 6895 columns

Presolve time: 0.10s

Presolved: 6993 rows, 6993 columns, 20579 nonzeros

Variable types: 0 continuous, 6993 integer (6993 binary)

Found heuristic solution: objective 568.8899149

Root relaxation: objective 1.367881e+03, 7102 iterations, 0.38 seconds (0.17 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	1367.8808028	1367.88080	0.00%	-	0s

Explored 1 nodes (7102 simplex iterations) in 0.53 seconds (0.23 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 1367.88 568.89 560.698

Optimal solution found (tolerance 1.00e-04)

Best objective 1.367880802797e+03, best bound 1.367880802797e+03, gap 0.00000%

m: m1

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0xafa97011

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [3e-03, 2e+01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 6e+00]

Found heuristic solution: objective 560.6975938

Presolve removed 52 rows and 6895 columns

Presolve time: 0.07s

Presolved: 6993 rows, 6993 columns, 20579 nonzeros
Variable types: 0 continuous, 6993 integer (6993 binary)
Found heuristic solution: objective 569.1680393

Root relaxation: objective 1.397867e+03, 6448 iterations, 0.40 seconds (0.17 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	1397.8671196	1397.86712	0.00%	-	0s

Explored 1 nodes (6448 simplex iterations) in 0.53 seconds (0.22 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 1397.87 569.168 560.698

Optimal solution found (tolerance 1.00e-04)
Best objective 1.397867119599e+03, best bound 1.397867119599e+03, gap 0.0000%
m: m1
Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros
Model fingerprint: 0xad41a02a
Variable types: 6844 continuous, 7044 integer (7044 binary)
Coefficient statistics:

Matrix range [1e+00, 1e+00]
Objective range [3e-03, 2e+01]
Bounds range [1e+00, 1e+00]
RHS range [1e+00, 7e+00]

Found heuristic solution: objective 703.9959933
Presolve removed 52 rows and 6895 columns
Presolve time: 0.08s
Presolved: 6993 rows, 6993 columns, 20579 nonzeros
Variable types: 0 continuous, 6993 integer (6993 binary)
Found heuristic solution: objective 711.3113333

Root relaxation: objective 1.409621e+03, 6149 iterations, 0.42 seconds (0.16 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	1409.6207930	1409.62079	0.00%	-	0s

Explored 1 nodes (6149 simplex iterations) in 0.57 seconds (0.22 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 1409.62 711.311 703.996

Optimal solution found (tolerance 1.00e-04)
Best objective 1.409620793016e+03, best bound 1.409620793016e+03, gap 0.0000%
m: m1
Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros
Model fingerprint: 0x12391f9e
Variable types: 6844 continuous, 7044 integer (7044 binary)
Coefficient statistics:

Matrix range [1e+00, 1e+00]
Objective range [3e-03, 2e+01]
Bounds range [1e+00, 1e+00]

RHS range [1e+00, 8e+00]
Found heuristic solution: objective 742.7228889
Presolve removed 52 rows and 6895 columns
Presolve time: 0.07s
Presolved: 6993 rows, 6993 columns, 20579 nonzeros
Variable types: 0 continuous, 6993 integer (6993 binary)
Found heuristic solution: objective 754.9757838

Root relaxation: objective 1.419507e+03, 5608 iterations, 0.40 seconds (0.15 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	1419.5073072	1419.50731	0.00%	-	0s

Explored 1 nodes (5608 simplex iterations) in 0.56 seconds (0.20 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 1419.51 754.976 742.723

Optimal solution found (tolerance 1.00e-04)
Best objective 1.419507307183e+03, best bound 1.419507307183e+03, gap 0.00000%
m: m1
Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0xacdba5a7

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]
Objective range [3e-03, 2e+01]
Bounds range [1e+00, 1e+00]
RHS range [1e+00, 9e+00]

Found heuristic solution: objective 831.3381511

Presolve removed 52 rows and 6895 columns

Presolve time: 0.07s

Presolved: 6993 rows, 6993 columns, 20579 nonzeros

Variable types: 0 continuous, 6993 integer (6993 binary)

Found heuristic solution: objective 843.5910460

Root relaxation: objective 1.428018e+03, 5302 iterations, 0.36 seconds (0.14 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	1428.0176897	1428.01769	0.00%	-	0s

Explored 1 nodes (5302 simplex iterations) in 0.49 seconds (0.20 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 1428.02 843.591 831.338

Optimal solution found (tolerance 1.00e-04)
Best objective 1.428017689651e+03, best bound 1.428017689651e+03, gap 0.00000%
m: m1
Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0x774b6428

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [3e-03, 2e+01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 929.2182562

Presolve removed 52 rows and 6895 columns

Presolve time: 0.07s

Presolved: 6993 rows, 6993 columns, 20579 nonzeros

Variable types: 0 continuous, 6993 integer (6993 binary)

Found heuristic solution: objective 943.5607379

Root relaxation: objective 1.434937e+03, 4707 iterations, 0.35 seconds (0.13 work units)

Nodes		Current Node				Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf		Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0		1434.9371765	1434.93718	0.00%	-	0s

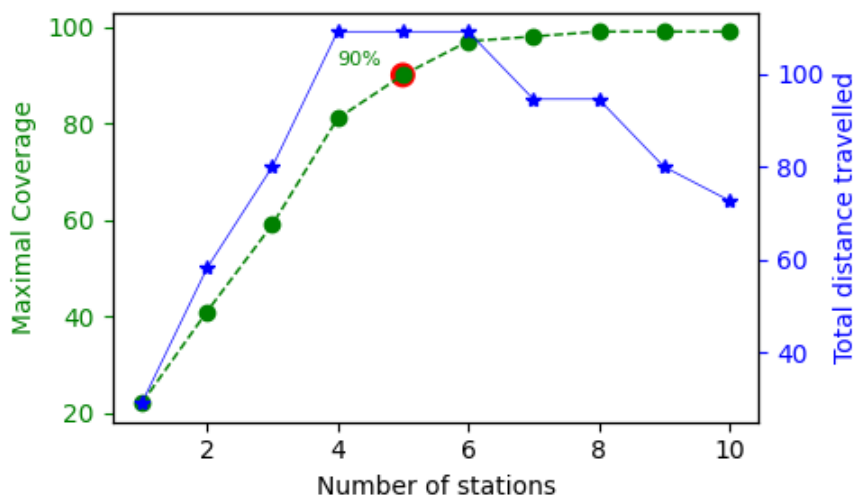
Explored 1 nodes (4707 simplex iterations) in 0.47 seconds (0.18 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 1434.94 943.561 929.218

Optimal solution found (tolerance 1.00e-04)

Best objective 1.434937176486e+03, best bound 1.434937176486e+03, gap 0.0000%



<Figure size 640x480 with 0 Axes>

Table 5 Model configuration

```
In [9]: # Comparing models with different weight vectors
import random

values = [.1, .2, .3, .4, .5, .6, .7, .8]
Wd = []
for i in range(1000):
    w1 = random.choice(values);
    w2 = random.choice(values);
    w3 = random.choice(values)
    if w1 + w2 + w3 == 1.0:
        Wd.append([w1, w2, w3])
# drop duplication values from list W
W_Set = set(tuple(element) for element in Wd)
W0 = [list(t) for t in set(tuple(element) for element in W_Set)]

W = [W0[i] for i in range(10)]

m = 'm2'
model_output = []
# Draw Network Diagram
for i in range(5):
    Wi = W[i]
    model, cover, select, amount, mvars, names, values, \
        cover_1s, select_1s, amountSt_groupby, coverage_percentage, \
        ResponseTimeT, assignment3, spill_df, station_df, \
        sol_y, assignment, assignment2, assignment_name = model_PAMIP.solve(Wi, coordinates_st, coordinates_spill,
                                                                                                     pairings, Size
                                                                                                     Spill, Sensitivity, TimeR,
                                                                                                     NumberStMax,
                                                                                                     m, spill_data_200)

    print(f'coverage_percentage: {coverage_percentage}, i: {i}, W: {Wi}')

    model_output.append([Wi, model.ObjVal, coverage_percentage, int(ResponseTimeT * 80) /
11])
    print('-----')
Model_Output = pd.DataFrame(model_output)
Model_Output.columns = ['Weights', 'Objective Value', 'Coverage %', 'Response time (in hours)']
Model_Output.to_csv('Outputs/Table5 Model_Output.csv')
```

m: m2
Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0x92c68884

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [2e-04, 2e+00]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 112.9800163

Presolve removed 172 rows and 7016 columns

Presolve time: 0.07s

Presolved: 6873 rows, 6872 columns, 20216 nonzeros

Variable types: 0 continuous, 6872 integer (6872 binary)

Found heuristic solution: objective 117.2490937

Root relaxation: objective 1.808593e+02, 4147 iterations, 0.26 seconds (0.10 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	180.8592823	180.85928	0.00%	-	0s

Explored 1 nodes (4147 simplex iterations) in 0.39 seconds (0.14 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 180.859 117.249 112.98

Optimal solution found (tolerance 1.00e-04)

Best objective 1.808592822849e+02, best bound 1.808592822849e+02, gap 0.0000%
coverage_percentage: 97, i: 0, W: [0.6, 0.1, 0.3]

m: m2
Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0x9ce35e29

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [3e-04, 3e+00]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 144.9354570

Presolve removed 828 rows and 7670 columns

Presolve time: 0.08s

Presolved: 6217 rows, 6218 columns, 18254 nonzeros

Variable types: 0 continuous, 6218 integer (6218 binary)

Found heuristic solution: objective 153.9840519

Root relaxation: objective 2.520014e+02, 3085 iterations, 0.22 seconds (0.06 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	252.0014369	252.00144	0.00%	-	0s

Explored 1 nodes (3085 simplex iterations) in 0.35 seconds (0.11 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 252.001 153.984 144.935

Optimal solution found (tolerance 1.00e-04)

Best objective 2.520014368613e+02, best bound 2.520014368613e+02, gap 0.0000%
coverage_percentage: 82, i: 1, W: [0.1, 0.2, 0.7]

m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0xb269c30d

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [4e-04, 2e+00]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 142.8586816

Presolve removed 29 rows and 6873 columns

Presolve time: 0.07s

Presolved: 7016 rows, 7015 columns, 20645 nonzeros

Variable types: 0 continuous, 7015 integer (7015 binary)

Found heuristic solution: objective 144.2929297

Root relaxation: objective 2.134636e+02, 4525 iterations, 0.24 seconds (0.12 work units)

Nodes		Current Node		Objective Bounds			Work	
Expl	Unexpl	Obj	Depth IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0	0	213.4635742	213.46357	0.00%	-	0s

Explored 1 nodes (4525 simplex iterations) in 0.35 seconds (0.17 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 213.464 144.293 142.859

Optimal solution found (tolerance 1.00e-04)

Best objective 2.134635742371e+02, best bound 2.134635742371e+02, gap 0.0000%
coverage_percentage: 100, i: 2, W: [0.8, 0.1, 0.1]

m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0x472d683f

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [5e-04, 5e+00]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 270.9107189

Presolve removed 120 rows and 6963 columns

Presolve time: 0.06s

Presolved: 6925 rows, 6925 columns, 20375 nonzeros

Variable types: 0 continuous, 6925 integer (6925 binary)

Found heuristic solution: objective 276.6028221

Root relaxation: objective 4.241901e+02, 4653 iterations, 0.30 seconds (0.12 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	424.1901116	424.19011	0.00%	-	0s

Explored 1 nodes (4653 simplex iterations) in 0.43 seconds (0.18 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 424.19 276.603 270.911

Optimal solution found (tolerance 1.00e-04)
Best objective 4.241901115605e+02, best bound 4.241901115605e+02, gap 0.0000%
coverage_percentage: 98, i: 3, W: [0.3, 0.3, 0.4]

m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros
Model fingerprint: 0xa847ff83

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [4e-05, 2e+00]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 85.3132628

Presolve removed 624 rows and 7467 columns

Presolve time: 0.08s

Presolved: 6421 rows, 6421 columns, 18863 nonzeros

Variable types: 0 continuous, 6421 integer (6421 binary)

Found heuristic solution: objective 91.4727092

Root relaxation: objective 1.497426e+02, 3961 iterations, 0.23 seconds (0.09 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	149.7425512	149.74255	0.00%	-	0s

Explored 1 nodes (3961 simplex iterations) in 0.36 seconds (0.14 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 149.743 91.4727 85.3133

Optimal solution found (tolerance 1.00e-04)
Best objective 1.497425512126e+02, best bound 1.497425512126e+02, gap 0.0000%
coverage_percentage: 87, i: 4, W: [0.4, 0.1, 0.5]

Data Scene 1

Fig7a. Network Diagram for Data Scene 1

In [9]:

```

In [10]: # Examine model results
# Sensitivity analysis
NumberStMax = 10
W1 = [[0.6, 0.1, 0.3], [0.1, 0.2, 0.7]] # from model configuration table
for i in range(2):
    Wi = W1[i]
    m = 'm2' # m2
    model, cover, select, amount, mvars, names, values, \
        cover_1s, select_1s, amountSt_groupby, coverage_percentage, \
        ResponseTimeT, assignment3, spill_df, station_df, \
        sol_y, assignment, assignment2, assignment_name = model_PAMIP.solve(Wi, coordinates_st, coordinates_spill,
                                                                                                     pairings, Size
Spill, Sensitivity, TimeR,
                                                                                                     NumberStMax,
m, spill_data_200)

    model_analysis.draw_network_diagram(DistanceMax, NumberStMax, spill_df, station_df, ResponseTimeT,
                                                                                                     coverage_percentage,
                                                                                                     assignment3, cover_1s, select_1s, amountSt_groupby, m, Wi, ArcticLand, ArcticWater)

```

m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0x92c68884

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [2e-04, 2e+00]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 112.9800163

Presolve removed 172 rows and 7016 columns

Presolve time: 0.09s

Presolved: 6873 rows, 6872 columns, 20216 nonzeros

Variable types: 0 continuous, 6872 integer (6872 binary)

Found heuristic solution: objective 117.2490937

Root relaxation: objective 1.808593e+02, 4147 iterations, 0.21 seconds (0.10 work units)

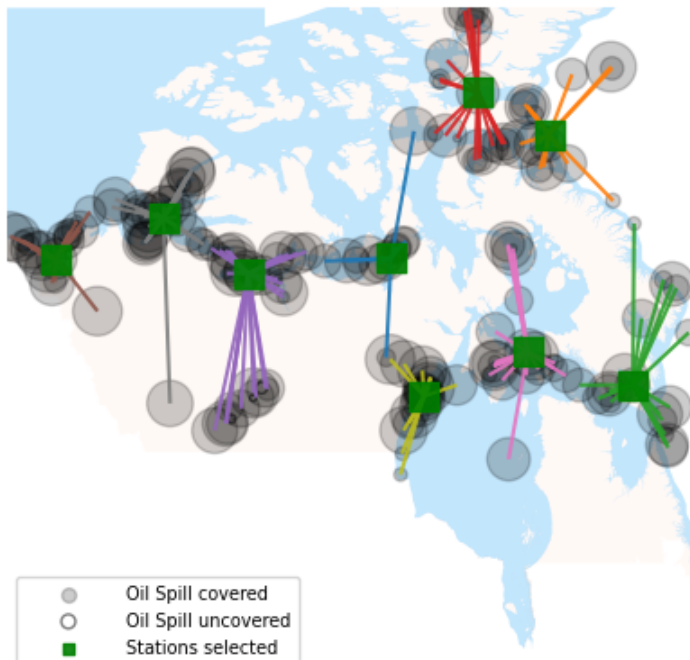
Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	180.8592823	180.85928	0.00%	-	0s

Explored 1 nodes (4147 simplex iterations) in 0.36 seconds (0.14 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 180.859 117.249 112.98

Optimal solution found (tolerance 1.00e-04)

Best objective 1.808592822849e+02, best bound 1.808592822849e+02, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0x9ce35e29

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [3e-04, 3e+00]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 144.9354570

Presolve removed 828 rows and 7670 columns

Presolve time: 0.06s

Presolved: 6217 rows, 6218 columns, 18254 nonzeros

Variable types: 0 continuous, 6218 integer (6218 binary)

Found heuristic solution: objective 153.9840519

Root relaxation: objective 2.520014e+02, 3085 iterations, 0.12 seconds (0.06 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	252.0014369	252.00144	0.00%	-	0s

Explored 1 nodes (3085 simplex iterations) in 0.22 seconds (0.11 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 252.001 153.984 144.935

Optimal solution found (tolerance 1.00e-04)

Best objective 2.520014368613e+02, best bound 2.520014368613e+02, gap 0.0000%

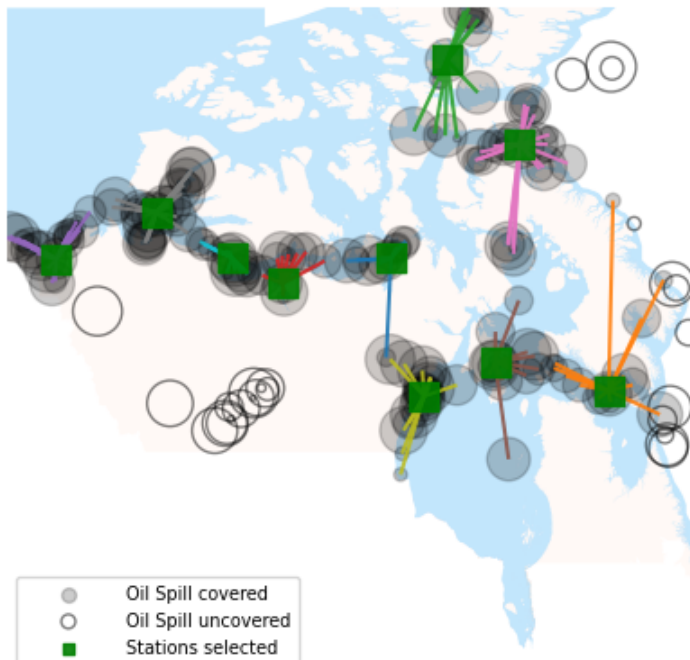


Fig7b

```

In [11]: # Comparing models with different weight vectors
NumberStMax = 5
values = [.1, .2, .3, .4, .5, .6, .7, .8]
Wd = []
for i in range(1000):
    w1 = random.choice(values);
    w2 = random.choice(values);
    w3 = random.choice(values)
    if w1 + w2 + w3 == 1.0:
        Wd.append([w1, w2, w3])
# drop duplication values from list W
W_Set = set(tuple(element) for element in Wd)
W0 = [list(t) for t in set(tuple(element) for element in W_Set)]

W = [W0[i] for i in range(10)]

m = 'm2'
model_output = []
# Draw Network Diagram
for i in range(5):
    Wi = W[i]
    model, cover, select, amount, mvars, names, values, \
        cover_1s, select_1s, amountSt_groupby, coverage_percentage, \
        ResponseTimeT, assignment3, spill_df, station_df, \
        sol_y, assignment, assignment2, assignment_name = model_PAMIP.solve(Wi, coordinates_st, coordinates_spill,
                                                                                                                                                                pairings, Size
                                                                                                                                                                Spill, Sensitivity, TimeR,
                                                                                                                                                                NumberStMax,
                                                                                                                                                                m, spill_data_200)

    print(f'coverage_percentage: {coverage_percentage}, i: {i}, W: {Wi}')
    model_output.append([Wi, model.ObjVal, coverage_percentage, int(ResponseTimeT * 80) /
11])
    print('-----')
Model_Output = pd.DataFrame(model_output)
Model_Output.columns = ['Weights', 'Objective Value', 'Coverage %', 'Response time (in hours)']
Model_Output.to_csv('Outputs/Model_Output.csv')

# Examine model results
# Sensitivity analysis
W1 = [[0.8, 0.1, 0.1], [0.1, 0.2, 0.7]] # from model configuration table
for i in range(2):
    Wi = W1[i]
    m = 'm2' # m2
    model, cover, select, amount, mvars, names, values, \
        cover_1s, select_1s, amountSt_groupby, coverage_percentage, \
        ResponseTimeT, assignment3, spill_df, station_df, \
        sol_y, assignment, assignment2, assignment_name = model_PAMIP.solve(Wi, coordinates_st, coordinates_spill,
                                                                                                                                                                pairings, Size
                                                                                                                                                                Spill, Sensitivity, TimeR,
                                                                                                                                                                NumberStMax,
                                                                                                                                                                m, spill_data_200)

    model_analysis.draw_network_diagram(DistanceMax, NumberStMax, spill_df, station_df, ResponseTimeT,
                                                                                                                                                                coverage_percentage,
                                                                                                                                                                assignment3, cover_1s, select_1s, amountSt_groupby, m, Wi, ArcticLand, ArcticWater)

```


m: m2
Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0xbb5c3e57

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [2e-04, 2e+00]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 5e+00]

Found heuristic solution: objective 65.9433194

Presolve removed 172 rows and 7016 columns

Presolve time: 0.09s

Presolved: 6873 rows, 6872 columns, 20216 nonzeros

Variable types: 0 continuous, 6872 integer (6872 binary)

Found heuristic solution: objective 68.4010157

Root relaxation: objective 1.633502e+02, 7348 iterations, 0.89 seconds (0.19 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	163.3502007	163.35020	0.00%	-	1s

Explored 1 nodes (7348 simplex iterations) in 1.09 seconds (0.24 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 163.35 68.401 65.9433

Optimal solution found (tolerance 1.00e-04)

Best objective 1.633502007105e+02, best bound 1.633502007105e+02, gap 0.0000%
coverage_percentage: 87, i: 0, W: [0.6, 0.1, 0.3]

m: m2
Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0x3b79883d

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [3e-04, 3e+00]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 5e+00]

Found heuristic solution: objective 85.4099661

Presolve removed 828 rows and 7670 columns

Presolve time: 0.07s

Presolved: 6217 rows, 6218 columns, 18254 nonzeros

Variable types: 0 continuous, 6218 integer (6218 binary)

Found heuristic solution: objective 90.0251041

Root relaxation: objective 2.242739e+02, 6353 iterations, 0.42 seconds (0.16 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	224.2738879	224.27389	0.00%	-	0s

Explored 1 nodes (6353 simplex iterations) in 0.55 seconds (0.21 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 224.274 90.0251 85.41

Optimal solution found (tolerance 1.00e-04)

Best objective 2.242738878820e+02, best bound 2.242738878820e+02, gap 0.0000%
coverage_percentage: 78, i: 1, W: [0.1, 0.2, 0.7]

m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0x34e5e30e

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [4e-04, 2e+00]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 5e+00]

Found heuristic solution: objective 83.9033948

Presolve removed 29 rows and 6873 columns

Presolve time: 0.07s

Presolved: 7016 rows, 7015 columns, 20645 nonzeros

Variable types: 0 continuous, 7015 integer (7015 binary)

Found heuristic solution: objective 84.7226269

Root relaxation: objective 2.012354e+02, 7878 iterations, 0.47 seconds (0.20 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
	0	0	201.23542	0	388	84.72263	201.23542	138%	0s
H	0	0				193.1389689	201.23542	4.19%	0s
H	0	0				201.0620166	201.23542	0.09%	0s

Cutting planes:

Gomory: 2

Zero half: 3

Explored 1 nodes (7878 simplex iterations) in 0.94 seconds (0.31 work units)

Thread count was 8 (of 8 available processors)

Solution count 4: 201.062 193.139 84.7226 83.9034

Optimal solution found (tolerance 1.00e-04)

Best objective 2.010620165712e+02, best bound 2.010620167446e+02, gap 0.0000%
coverage_percentage: 92, i: 2, W: [0.8, 0.1, 0.1]

m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0x59ec81ad

Variable types: 6844 continuous, 7044 integer (7044 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [5e-04, 5e+00]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 5e+00]

Found heuristic solution: objective 163.1742325

Presolve removed 120 rows and 6963 columns
Presolve time: 0.10s
Presolved: 6925 rows, 6925 columns, 20375 nonzeros
Variable types: 0 continuous, 6925 integer (6925 binary)
Found heuristic solution: objective 166.4511609

Root relaxation: objective 4.011908e+02, 7148 iterations, 0.53 seconds (0.18 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	401.1908442	401.19084	0.00%	-	0s

Explored 1 nodes (7148 simplex iterations) in 0.70 seconds (0.23 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 401.191 166.451 163.174

Optimal solution found (tolerance 1.00e-04)
Best objective 4.011908442034e+02, best bound 4.011908442034e+02, gap 0.0000%
coverage_percentage: 88, i: 3, W: [0.3, 0.3, 0.4]

m: m2
Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros
Model fingerprint: 0xe24f1506
Variable types: 6844 continuous, 7044 integer (7044 binary)
Coefficient statistics:

Matrix range [1e+00, 1e+00]
Objective range [4e-05, 2e+00]
Bounds range [1e+00, 1e+00]
RHS range [1e+00, 5e+00]

Found heuristic solution: objective 49.6952373
Presolve removed 624 rows and 7467 columns
Presolve time: 0.08s
Presolved: 6421 rows, 6421 columns, 18863 nonzeros
Variable types: 0 continuous, 6421 integer (6421 binary)
Found heuristic solution: objective 52.6907138

Root relaxation: objective 1.292675e+02, 6697 iterations, 0.52 seconds (0.17 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	129.2675292	129.26753	0.00%	-	0s

Explored 1 nodes (6697 simplex iterations) in 0.66 seconds (0.22 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 129.268 52.6907 49.6952

Optimal solution found (tolerance 1.00e-04)
Best objective 1.292675291789e+02, best bound 1.292675291789e+02, gap 0.0000%
coverage_percentage: 80, i: 4, W: [0.4, 0.1, 0.5]

m: m2
Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros

Model fingerprint: 0x34e5e30e
 Variable types: 6844 continuous, 7044 integer (7044 binary)
 Coefficient statistics:
 Matrix range [1e+00, 1e+00]
 Objective range [4e-04, 2e+00]
 Bounds range [1e+00, 1e+00]
 RHS range [1e+00, 5e+00]
 Found heuristic solution: objective 83.9033948
 Presolve removed 29 rows and 6873 columns
 Presolve time: 0.09s
 Presolved: 7016 rows, 7015 columns, 20645 nonzeros
 Variable types: 0 continuous, 7015 integer (7015 binary)
 Found heuristic solution: objective 84.7226269

Root relaxation: objective 2.012354e+02, 7878 iterations, 0.57 seconds (0.20 work units)

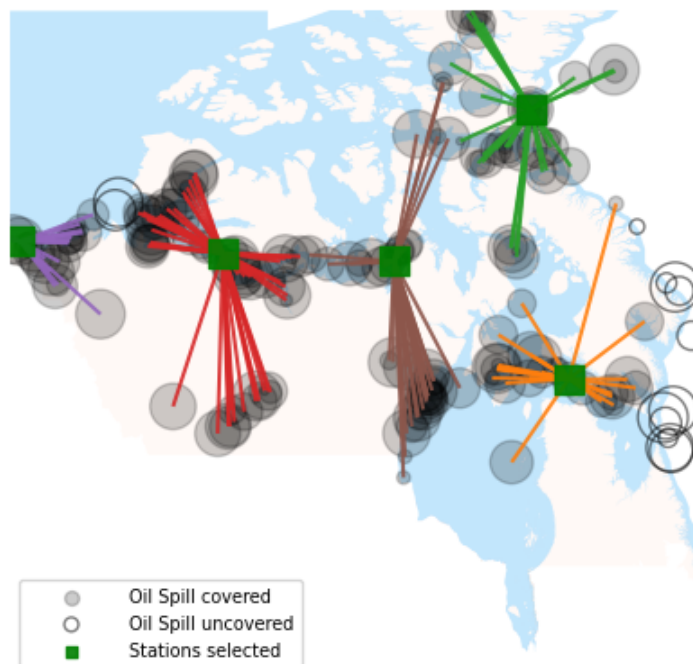
Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
	0	0	201.23542	0	388	84.72263	201.23542	138%	– 0s
H	0	0				193.1389689	201.23542	4.19%	– 0s
H	0	0				201.0620166	201.23542	0.09%	– 1s

Cutting planes:
 Gomory: 2
 Zero half: 3

Explored 1 nodes (7878 simplex iterations) in 1.11 seconds (0.31 work units)
 Thread count was 8 (of 8 available processors)

Solution count 4: 201.062 193.139 84.7226 83.9034

Optimal solution found (tolerance 1.00e-04)
 Best objective 2.010620165712e+02, best bound 2.010620167446e+02, gap 0.0000%



```

m: m2
Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 7045 rows, 13888 columns and 20732 nonzeros
Model fingerprint: 0x3b79883d
Variable types: 6844 continuous, 7044 integer (7044 binary)
Coefficient statistics:
  Matrix range      [1e+00, 1e+00]
  Objective range   [3e-04, 3e+00]
  Bounds range      [1e+00, 1e+00]
  RHS range         [1e+00, 5e+00]
Found heuristic solution: objective 85.4099661
Presolve removed 828 rows and 7670 columns
Presolve time: 0.11s
Presolved: 6217 rows, 6218 columns, 18254 nonzeros
Variable types: 0 continuous, 6218 integer (6218 binary)
Found heuristic solution: objective 90.0251041

Root relaxation: objective 2.242739e+02, 6353 iterations, 0.58 seconds (0.16 work units)

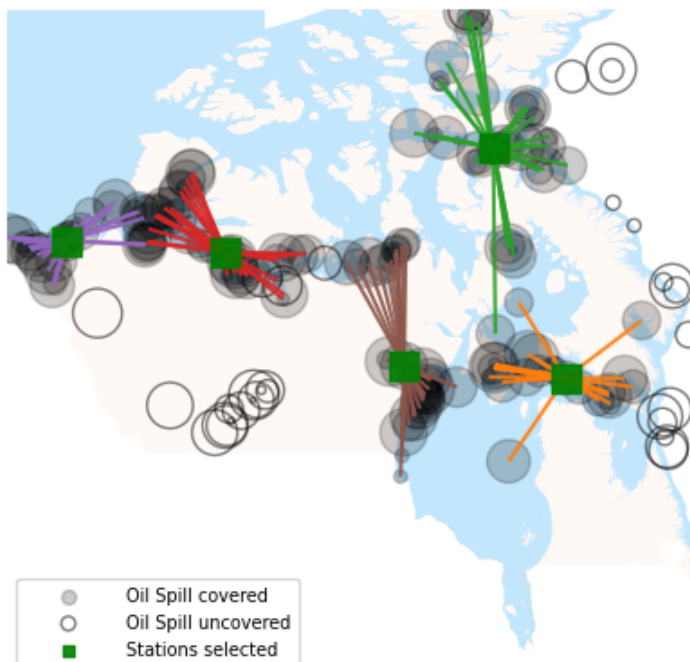
   Nodes      |      Current Node      |      Objective Bounds      |      Work
  Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
*    0       0                0    224.2738879   224.27389   0.00%   -    0s

Explored 1 nodes (6353 simplex iterations) in 0.77 seconds (0.21 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 224.274 90.0251 85.41

Optimal solution found (tolerance 1.00e-04)
Best objective 2.242738878820e+02, best bound 2.242738878820e+02, gap 0.0000%

```



Data Scene 2

In [11]:

Fig5b Kmeans clustering

In [11]:

```

In [12]: # %% Input parameters of the model
NumberStMax = 5
DistanceMax = 10 # 5
num_customers = len(coordinates_spill)
num_facilities = len(coordinates_st)
spill_size = spill_data_scene2[['Spill size']]

coordinates_spill_scene2 = custom_functions.extract_coordinate(spill_data_scene2)
# Cluster them into 50 cluster
num_clusters = 50

kmeans = MiniBatchKMeans(n_clusters=num_clusters, init_size=3 * num_clusters,
                          ).fit(coordinates_spill_scene2)
memberships = list(kmeans.labels_)
centroids = list(kmeans.cluster_centers_) # Center point for each cluster
weights = list(np.histogram(memberships, bins=num_clusters)[0]) # Number of customers in
each cluster
print('First cluster center:', centroids[0])
print('Weights for first 10 clusters:', weights[:10])

# Draw
icon_size_list = []
# Draw the oil spills
for point_spill in range(0, len(coordinates_spill_scene2)):
    icon_size = int((spill_size.iloc[point_spill, 0] / spill_size.max()) * 20)
    icon_size_list.append(icon_size)

size_10times = [item * 10 for item in icon_size_list]
coordinates_spill_df = pd.DataFrame(coordinates_spill_scene2)
coordinates_spill_df.columns = ['Latitude', 'Longitude']

centroids_df = pd.DataFrame(centroids)
centroids_df.columns = ['Latitude', 'Longitude']

data_visualization.draw_cluster(coordinates_spill_df, centroids_df, memberships, ArcticLa
nd, ArcticWater)

```

```
C:\Users\tanmo\OneDrive - Dalhousie University\PhD Tanmoy Das\Implementation of Deterministic Resource Optimization TD 01.2023\deterministic_env\Lib\site-packages\sklearn\cluster\_kmeans.py:870: FutureWarning:
```

The default value of `n_init` will change from 3 to 'auto' in 1.4. Set the value of `n_init` explicitly to suppress the warning

```
First cluster center: [ 73.18462029 -85.27786021]
```

```
Weights for first 10 clusters: [87, 75, 40, 13, 57, 43, 45, 13, 24, 48]
```



In [12]:

In [12]:


```

In [13]: # Input parameters
coordinates_spill_cluster = kmeans.cluster_centers_.tolist()
pairings = custom_functions.compute_pairing(coordinates_spill_cluster, coordinates_st, DistanceMax)
Size_DS1 = list(spill_data_scene2['Spill size']).copy()

cluster_index = {}
for j in range(len(centroids)):
    cluster_index[j] = [i for i, x in enumerate(memberships) if x == j]
SizeSpill_Rc = [sum([e for i, e in enumerate(Size_DS1) if i in cluster_index[ii]]) for ii in range(len(cluster_index))]
Sensitivity_Rc = custom_functions.calculate_sensitivity(coordinates_spill_cluster, sensitivity_dataR)
TimeRc = pairings.copy()

max_spill_size = max(SizeSpill_Rc)
min_spill_size = min(SizeSpill_Rc)

max_sensitivity = max(Sensitivity_Rc)
min_sensitivity = min(Sensitivity_Rc)

max_timeR = pairings[max(pairings, key=pairings.get)]
min_timeR = pairings[min(pairings, key=pairings.get)]

SizeSpill = [];
Sensitivity = [];
TimeR = [];
SizeSpill = [((SizeSpill_Rc[i] - min_spill_size) / (max_spill_size - min_spill_size)) for i in range(len(SizeSpill_Rc))]
Sensitivity = [((Sensitivity_Rc[i] - min_sensitivity) / (max_sensitivity - min_sensitivity)) for i in range(len(Sensitivity_Rc))]

# TimeR = {((list(TimeR_R.values())[i]-min_timeR)/(max_timeR-min_timeR)) for i in range(len(TimeR_R))}
TimeR_Scaled = [((list(TimeRc.values())[i] - min_timeR) / (max_timeR - min_timeR)) for i in range(len(TimeRc))]
keysD = TimeRc.keys()
TimeR = {}
for i in range(len(keysD)):
    TimeR[list(keysD)[i]] = TimeR_Scaled[i]

```

Fig8a Network Diagram for data scene 2

```
In [14]: m = 'm2'
NumberStMax = 10

# Solve the model
W1 = W #[[0.1, 0.2, 0.7], [0.2, 0.7, 0.1]] # from model configuration table
for i in range(10):
    Wi = W1[i]
    m = 'm2' # m2
    model, cover, select, amount, mvars, names, values, \
        cover_1s, select_1s, amountSt_groupby, coverage_percentage, \
        ResponseTimeT, assignment3, spill_df, station_df, \
        sol_y, assignment, assignment2, assignment_name = model_PAMIP.solve(Wi, coordinates_st, coordinates_spill_cluster,
                                                                                                                                                                pairings, Size
Spill, Sensitivity, TimeR,
                                                                                                                                                                NumberStMax,
m, spill_data_scene2)

    model_analysis.draw_network_diagram(DistanceMax, NumberStMax, spill_df, station_df, ResponseTimeT,
                                                                                                                                                                coverage_percentage,
                                                                                                                                                                assignment3, cover_1s, select_1s, amountSt_groupby,
y, m, Wi, ArcticLand, ArcticWater)
```

m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0x2aad51c6

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [1e-04, 6e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 0.8316521

Presolve removed 1145 rows and 2716 columns

Presolve time: 0.02s

Presolved: 488 rows, 648 columns, 1556 nonzeros

Variable types: 0 continuous, 648 integer (647 binary)

Found heuristic solution: objective 1.0601163

Root relaxation: objective 2.924985e+00, 310 iterations, 0.01 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	2.9249847	2.92498	0.00%	-	0s

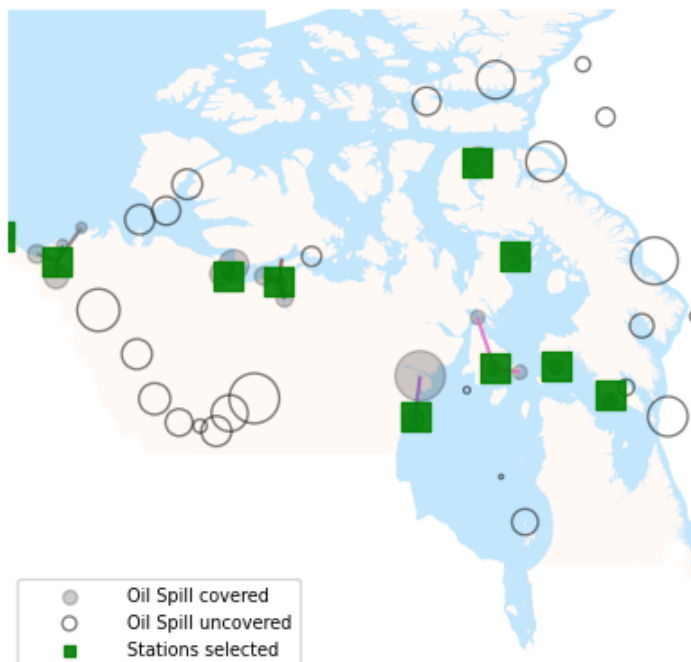
Explored 1 nodes (310 simplex iterations) in 0.16 seconds (0.01 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 2.92498 1.06012 0.831652

Optimal solution found (tolerance 1.00e-04)

Best objective 2.924984740910e+00, best bound 2.924984740910e+00, gap 0.0000%



m: m2
Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0x9c4683d1

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [3e-04, 7e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 0.3406055

Presolve removed 1534 rows and 3221 columns

Presolve time: 0.02s

Presolved: 99 rows, 143 columns, 317 nonzeros

Found heuristic solution: objective 1.4936694

Variable types: 0 continuous, 143 integer (143 binary)

Found heuristic solution: objective 1.6457332

Root relaxation: objective 1.793407e+00, 74 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	1.7934066	1.79341	0.00%	-	0s

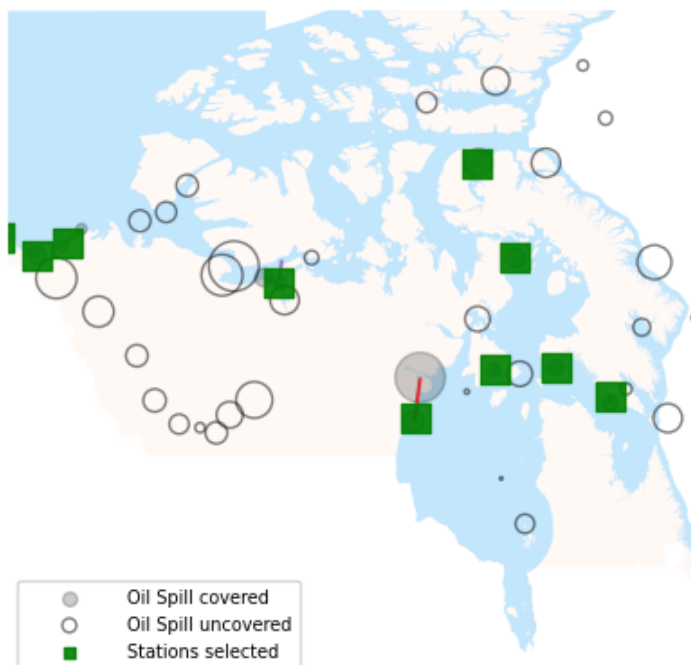
Explored 1 nodes (74 simplex iterations) in 0.06 seconds (0.00 work units)

Thread count was 8 (of 8 available processors)

Solution count 4: 1.79341 1.64573 1.49367 0.340605

Optimal solution found (tolerance 1.00e-04)

Best objective 1.793406555093e+00, best bound 1.793406555093e+00, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0xdc364994

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [3e-04, 8e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 2.5000747

Presolve removed 380 rows and 1958 columns

Presolve time: 0.02s

Presolved: 1253 rows, 1406 columns, 3818 nonzeros

Variable types: 0 continuous, 1406 integer (1406 binary)

Found heuristic solution: objective 2.7980470

Root relaxation: objective 4.969606e+00, 700 iterations, 0.03 seconds (0.01 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	4.9696056	4.96961	0.00%	-	0s

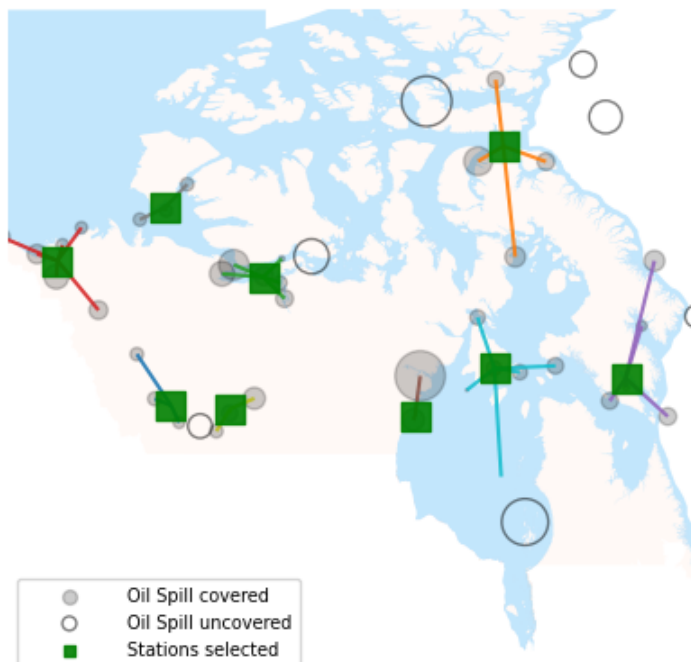
Explored 1 nodes (700 simplex iterations) in 0.10 seconds (0.02 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 4.96961 2.79805 2.50007

Optimal solution found (tolerance 1.00e-04)

Best objective 4.969605632768e+00, best bound 4.969605632768e+00, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0xbb4d26bc

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [1e-05, 4e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 1.1005805

Presolve removed 1117 rows and 2718 columns

Presolve time: 0.02s

Presolved: 516 rows, 646 columns, 1624 nonzeros

Variable types: 0 continuous, 646 integer (646 binary)

Found heuristic solution: objective 1.5576830

Root relaxation: objective 4.322181e+00, 288 iterations, 0.01 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	4.3221811	4.32218	0.00%	-	0s

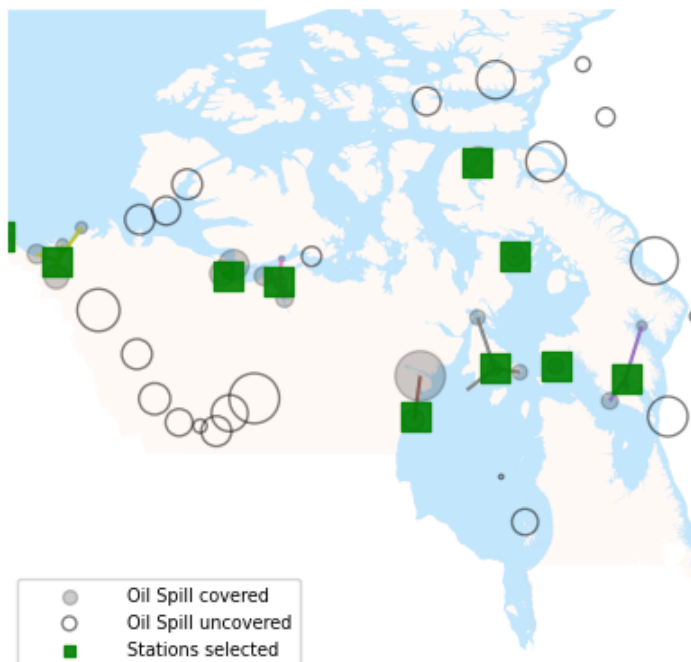
Explored 1 nodes (288 simplex iterations) in 0.07 seconds (0.01 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 4.32218 1.55768 1.10058

Optimal solution found (tolerance 1.00e-04)

Best objective 4.322181126895e+00, best bound 4.322181126895e+00, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0x8ef2d5e4

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [8e-05, 5e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 0.3846419

Presolve removed 1491 rows and 3160 columns

Presolve time: 0.02s

Presolved: 142 rows, 204 columns, 462 nonzeros

Found heuristic solution: objective 1.2728967

Variable types: 0 continuous, 204 integer (204 binary)

Found heuristic solution: objective 1.4468650

Root relaxation: objective 1.837952e+00, 123 iterations, 0.01 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	1.8379521	1.83795	0.00%	-	0s

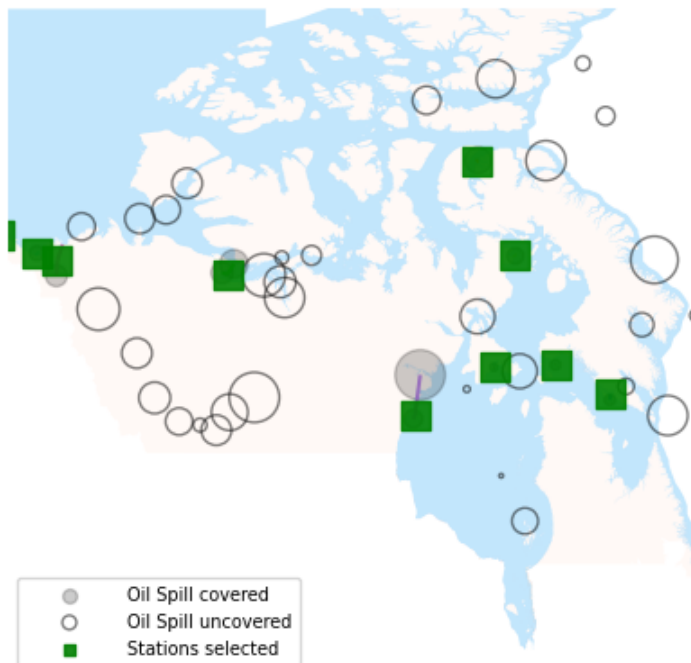
Explored 1 nodes (123 simplex iterations) in 0.05 seconds (0.01 work units)

Thread count was 8 (of 8 available processors)

Solution count 4: 1.83795 1.44687 1.2729 0.384642

Optimal solution found (tolerance 1.00e-04)

Best objective 1.837952051280e+00, best bound 1.837952051280e+00, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0xf19dbbfd

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [2e-04, 8e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 9.1458755

Presolve removed 186 rows and 1766 columns

Presolve time: 0.02s

Presolved: 1447 rows, 1598 columns, 4394 nonzeros

Variable types: 0 continuous, 1598 integer (1598 binary)

Found heuristic solution: objective 9.4471768

Root relaxation: objective 1.384226e+01, 894 iterations, 0.02 seconds (0.01 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	13.8422636	13.84226	0.00%	-	0s

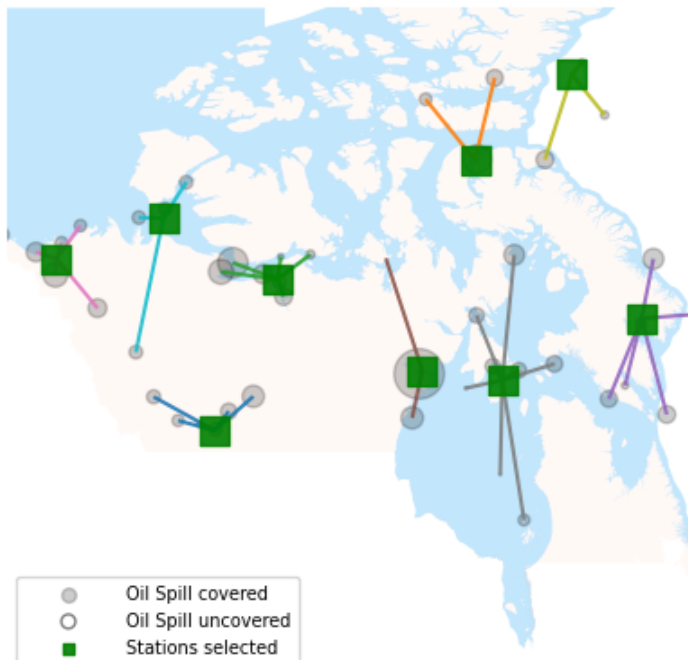
Explored 1 nodes (894 simplex iterations) in 0.08 seconds (0.02 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 13.8423 9.44718 9.14588

Optimal solution found (tolerance 1.00e-04)

Best objective 1.384226359926e+01, best bound 1.384226359926e+01, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0x0bda0ce9

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [2e-05, 6e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 6.2913468

Presolve removed 182 rows and 1762 columns

Presolve time: 0.02s

Presolved: 1451 rows, 1602 columns, 4406 nonzeros

Variable types: 0 continuous, 1602 integer (1602 binary)

Found heuristic solution: objective 6.5898192

Root relaxation: objective 9.953644e+00, 848 iterations, 0.03 seconds (0.01 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	9.9536445	9.95364	0.00%	-	0s

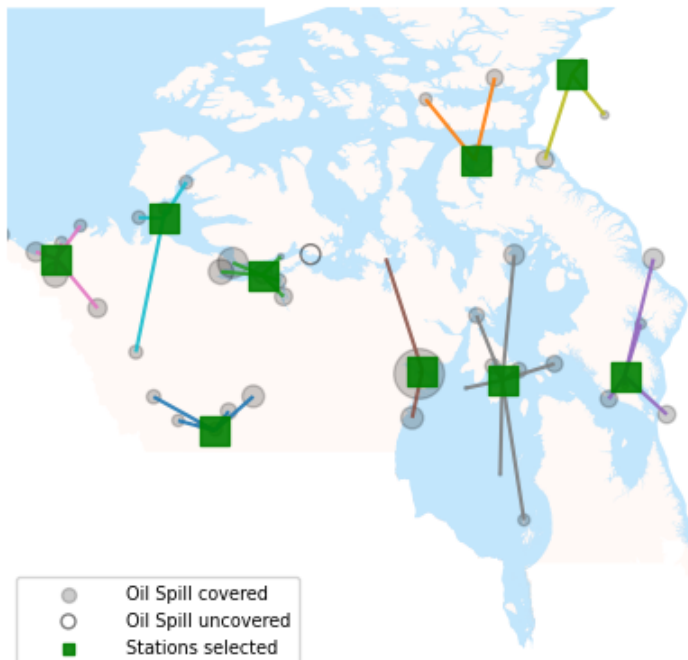
Explored 1 nodes (848 simplex iterations) in 0.13 seconds (0.02 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 9.95364 6.58982 6.29135

Optimal solution found (tolerance 1.00e-04)

Best objective 9.953644463120e+00, best bound 9.953644463120e+00, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0xfe6ea3a6

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [5e-05, 5e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 4.6491131

Presolve removed 524 rows and 2099 columns

Presolve time: 0.01s

Presolved: 1109 rows, 1265 columns, 3395 nonzeros

Variable types: 0 continuous, 1265 integer (1265 binary)

Found heuristic solution: objective 5.0826553

Root relaxation: objective 8.405966e+00, 538 iterations, 0.01 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	8.4059658	8.40597	0.00%	-	0s

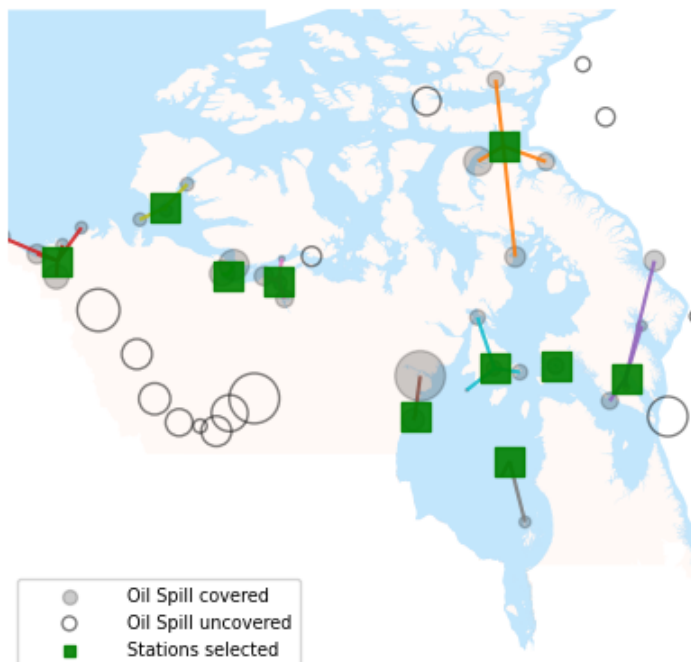
Explored 1 nodes (538 simplex iterations) in 0.06 seconds (0.01 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 8.40597 5.08266 4.64911

Optimal solution found (tolerance 1.00e-04)

Best objective 8.405965756269e+00, best bound 8.405965756269e+00, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0x2f08aa00

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [2e-04, 7e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 1.3633573

Presolve removed 818 rows and 2388 columns

Presolve time: 0.02s

Presolved: 815 rows, 976 columns, 2530 nonzeros

Variable types: 0 continuous, 976 integer (975 binary)

Found heuristic solution: objective 1.5054000

Root relaxation: objective 3.657158e+00, 471 iterations, 0.01 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	3.6571582	3.65716	0.00%	-	0s

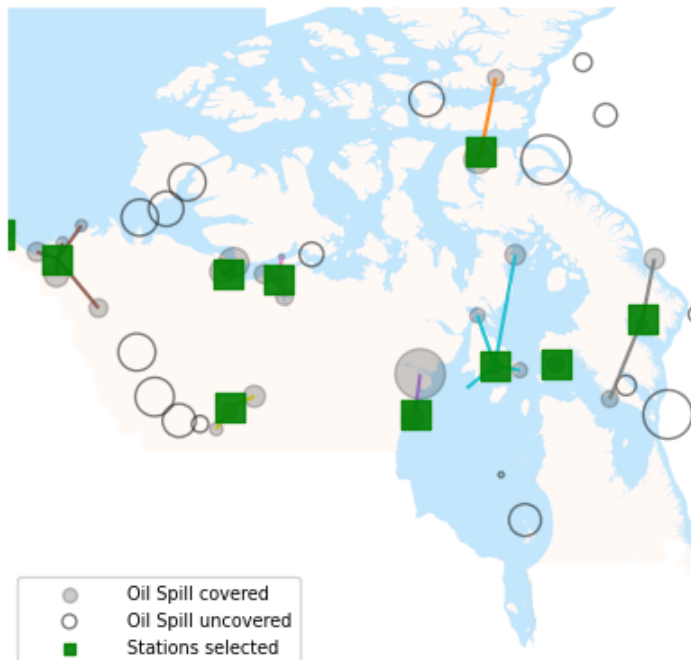
Explored 1 nodes (471 simplex iterations) in 0.08 seconds (0.01 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 3.65716 1.5054 1.36336

Optimal solution found (tolerance 1.00e-04)

Best objective 3.657158240696e+00, best bound 3.657158240696e+00, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0xeab7e787

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [8e-06, 6e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 1e+01]

Found heuristic solution: objective 0.4570986

Presolve removed 1465 rows and 3131 columns

Presolve time: 0.02s

Presolved: 168 rows, 233 columns, 536 nonzeros

Found heuristic solution: objective 1.6472684

Variable types: 0 continuous, 233 integer (233 binary)

Root relaxation: objective 2.186747e+00, 128 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	2.1867466	2.18675	0.00%	-	0s

Explored 1 nodes (128 simplex iterations) in 0.05 seconds (0.01 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 2.18675 1.64727 0.457099

Optimal solution found (tolerance 1.00e-04)

Best objective 2.186746604230e+00, best bound 2.186746604230e+00, gap 0.0000%

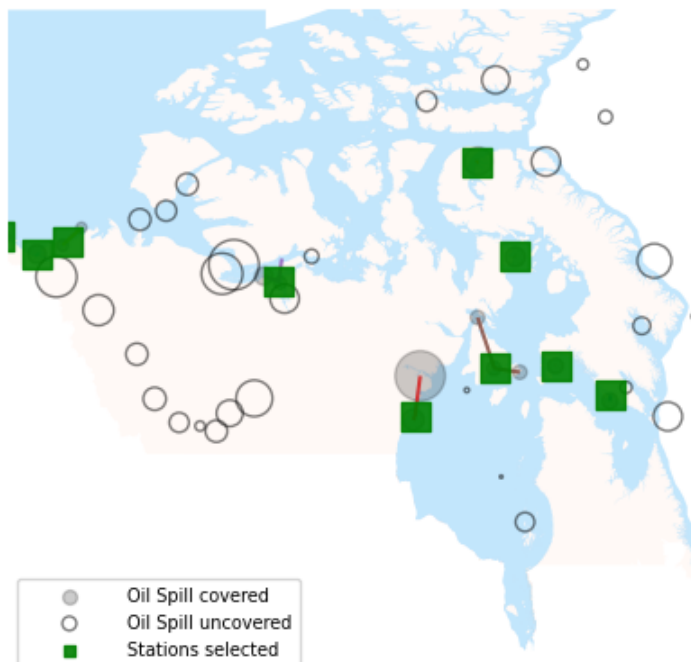


Fig8b

```
In [15]: m = 'm2'
NumberStMax = 5

# Solve the model
W1 = W #[[0.1, 0.2, 0.7], [0.2, 0.7, 0.1]] # from model configuration table
for i in range(10):
    Wi = W1[i]
    m = 'm2' # m2
    model, cover, select, amount, mvars, names, values, \
        cover_1s, select_1s, amountSt_groupby, coverage_percentage, \
        ResponseTimeT, assignment3, spill_df, station_df, \
        sol_y, assignment, assignment2, assignment_name = model_PAMIP.solve(Wi, coordinates_st, coordinates_spill_cluster,
                                                                                                                                                                pairings, Size
Spill, Sensitivity, TimeR,
                                                                                                                                                                NumberStMax,
m, spill_data_scene2)

    model_analysis.draw_network_diagram(DistanceMax, NumberStMax, spill_df, station_df, ResponseTimeT,
                                                                                                                                                                coverage_percentage,
                                                                                                                                                                assignment3, cover_1s, select_1s, amountSt_groupby,
y, m, Wi, ArcticLand, ArcticWater)
```

m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0xe2748484

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [1e-04, 6e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 5e+00]

Found heuristic solution: objective 0.1874289

Presolve removed 1201 rows and 2814 columns

Presolve time: 0.02s

Presolved: 432 rows, 550 columns, 1360 nonzeros

Found heuristic solution: objective 0.9154055

Variable types: 0 continuous, 550 integer (550 binary)

Found heuristic solution: objective 1.0155796

Root relaxation: objective 2.090409e+00, 360 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	2.0904092	2.09041	0.00%	-	0s

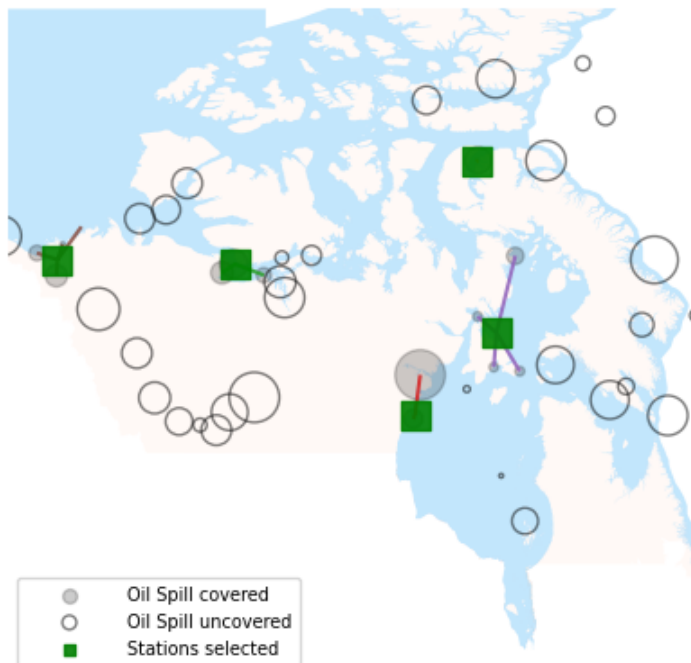
Explored 1 nodes (360 simplex iterations) in 0.06 seconds (0.01 work units)

Thread count was 8 (of 8 available processors)

Solution count 4: 2.09041 1.01558 0.915406 0.187429

Optimal solution found (tolerance 1.00e-04)

Best objective 2.090409207139e+00, best bound 2.090409207139e+00, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0x9274900d

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [3e-04, 7e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 5e+00]

Found heuristic solution: objective 0.0601409

Presolve removed 1534 rows and 3226 columns

Presolve time: 0.01s

Presolved: 99 rows, 138 columns, 312 nonzeros

Found heuristic solution: objective 0.9251680

Variable types: 0 continuous, 138 integer (138 binary)

Found heuristic solution: objective 0.9331133

Root relaxation: objective 9.778047e-01, 83 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	0.9778047	0.97780	0.00%	-	0s

Explored 1 nodes (83 simplex iterations) in 0.05 seconds (0.00 work units)

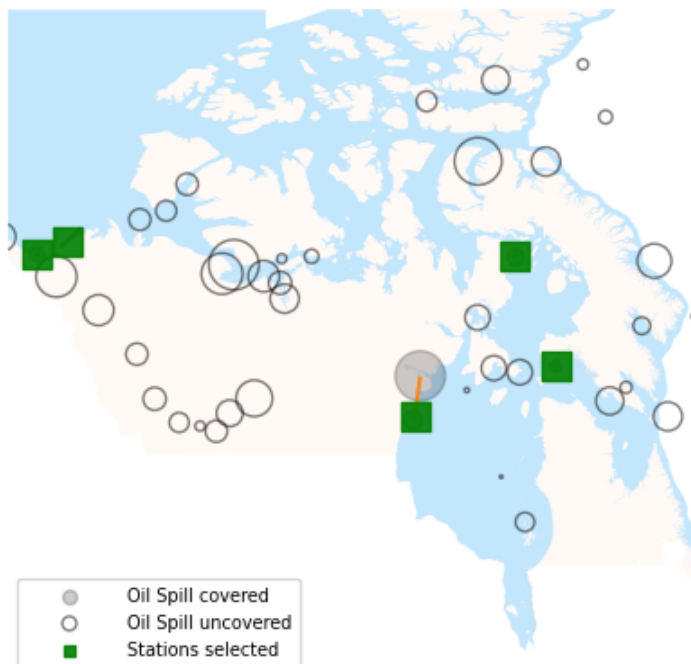
Thread count was 8 (of 8 available processors)

Solution count 4: 0.977805 0.933113 0.925168 0.0601409

No other solutions better than 0.977805

Optimal solution found (tolerance 1.00e-04)

Best objective 9.778046684917e-01, best bound 9.778046684917e-01, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0xfb1db6af

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [3e-04, 8e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 5e+00]

Found heuristic solution: objective 1.2767653

Presolve removed 380 rows and 1958 columns

Presolve time: 0.02s

Presolved: 1253 rows, 1406 columns, 3818 nonzeros

Variable types: 0 continuous, 1406 integer (1406 binary)

Found heuristic solution: objective 1.3998416

Root relaxation: objective 4.235669e+00, 1017 iterations, 0.03 seconds (0.01 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	4.2356686	4.23567	0.00%	-	0s

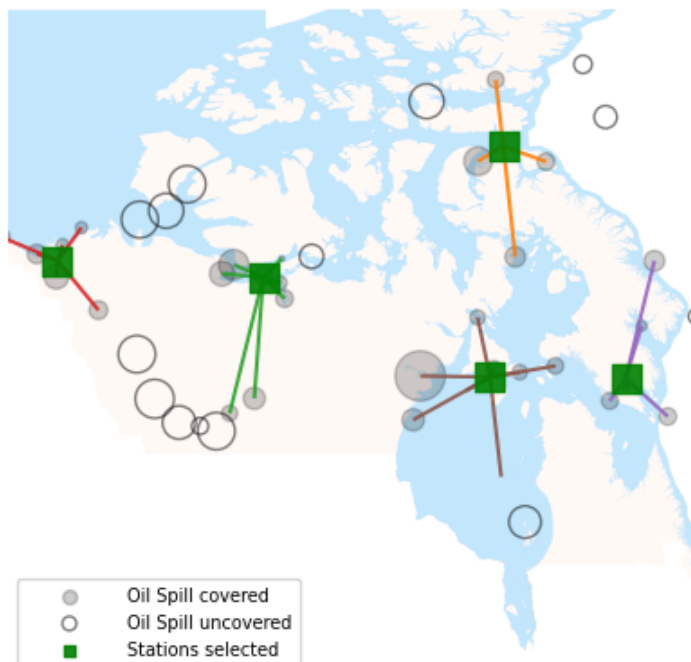
Explored 1 nodes (1017 simplex iterations) in 0.08 seconds (0.02 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 4.23567 1.39984 1.27677

Optimal solution found (tolerance 1.00e-04)

Best objective 4.235668644968e+00, best bound 4.235668644968e+00, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0x2e06af83

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [1e-05, 4e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 5e+00]

Found heuristic solution: objective 0.3106614

Presolve removed 1117 rows and 2718 columns

Presolve time: 0.03s

Presolved: 516 rows, 646 columns, 1624 nonzeros

Variable types: 0 continuous, 646 integer (646 binary)

Found heuristic solution: objective 0.6361293

Root relaxation: objective 3.000222e+00, 404 iterations, 0.01 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	3.0002224	3.00022	0.00%	-	0s

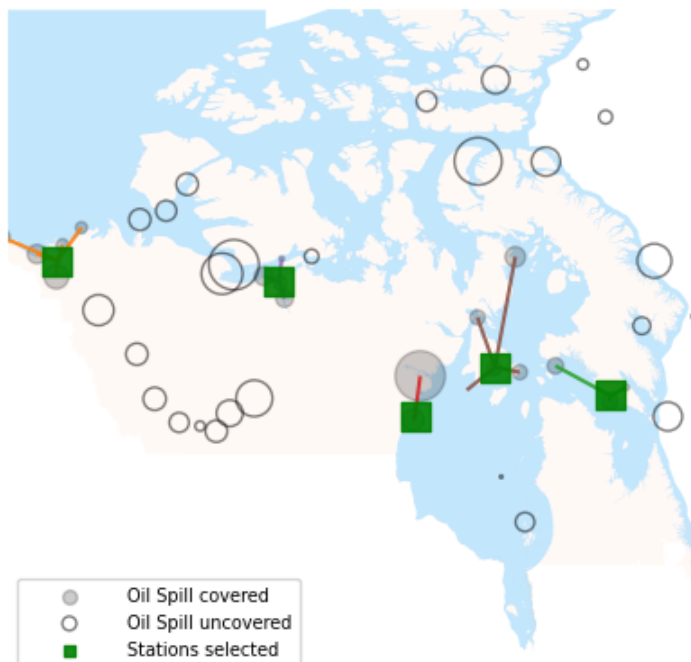
Explored 1 nodes (404 simplex iterations) in 0.10 seconds (0.01 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 3.00022 0.636129 0.310661

Optimal solution found (tolerance 1.00e-04)

Best objective 3.000222439523e+00, best bound 3.000222439523e+00, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0x55b381e9

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [8e-05, 5e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 5e+00]

Found heuristic solution: objective 0.0835744

Presolve removed 1491 rows and 3165 columns

Presolve time: 0.01s

Presolved: 142 rows, 199 columns, 457 nonzeros

Found heuristic solution: objective 0.8132200

Variable types: 0 continuous, 199 integer (199 binary)

Found heuristic solution: objective 0.8831827

Root relaxation: objective 1.225426e+00, 123 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	1.2254261	1.22543	0.00%	-	0s

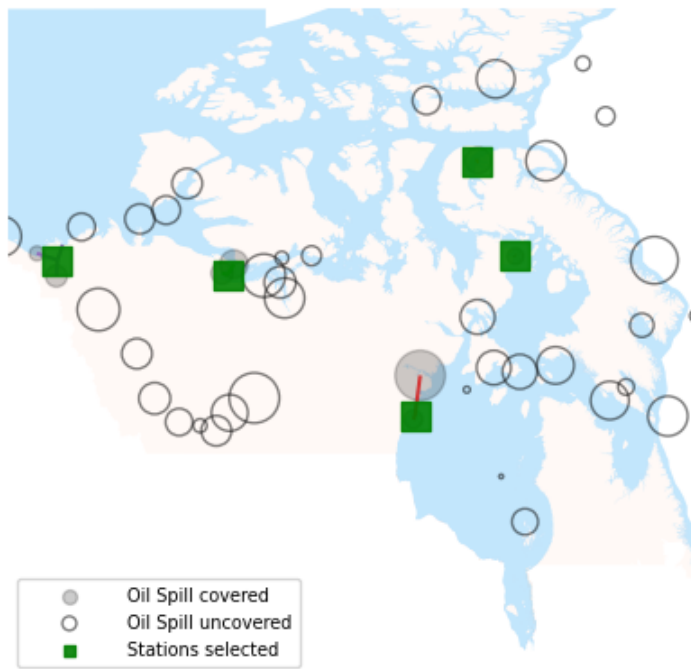
Explored 1 nodes (123 simplex iterations) in 0.05 seconds (0.00 work units)

Thread count was 8 (of 8 available processors)

Solution count 4: 1.22543 0.883183 0.81322 0.0835744

Optimal solution found (tolerance 1.00e-04)

Best objective 1.225426148272e+00, best bound 1.225426148272e+00, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0x8c2f7785

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [2e-04, 8e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 5e+00]

Found heuristic solution: objective 5.0580245

Presolve removed 186 rows and 1766 columns

Presolve time: 0.02s

Presolved: 1447 rows, 1598 columns, 4394 nonzeros

Variable types: 0 continuous, 1598 integer (1598 binary)

Found heuristic solution: objective 5.1811009

Root relaxation: objective 1.276075e+01, 1492 iterations, 0.05 seconds (0.02 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	12.7607535	12.76075	0.00%	-	0s

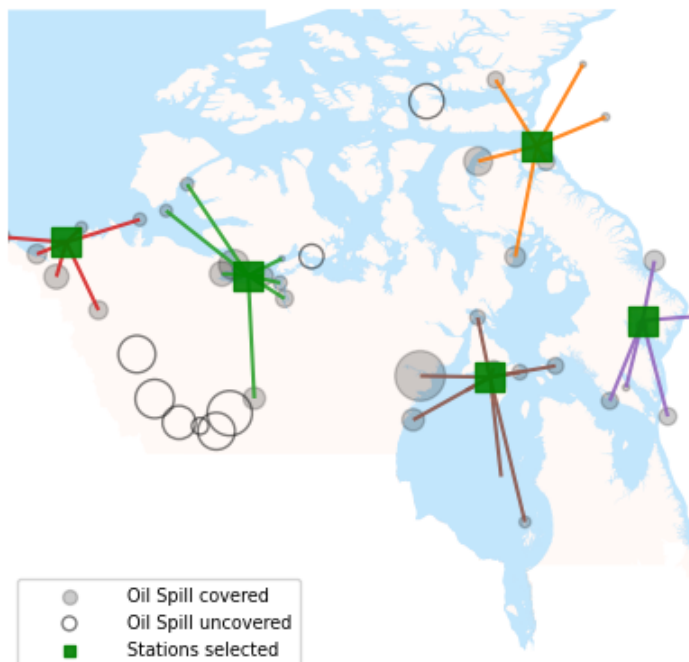
Explored 1 nodes (1492 simplex iterations) in 0.14 seconds (0.02 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 12.7608 5.1811 5.05802

Optimal solution found (tolerance 1.00e-04)

Best objective 1.276075348261e+01, best bound 1.276075348261e+01, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0xf3b01677

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [2e-05, 6e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 5e+00]

Found heuristic solution: objective 3.4350165

Presolve removed 182 rows and 1762 columns

Presolve time: 0.02s

Presolved: 1451 rows, 1602 columns, 4406 nonzeros

Variable types: 0 continuous, 1602 integer (1602 binary)

Found heuristic solution: objective 3.5580929

Root relaxation: objective 9.035758e+00, 1346 iterations, 0.04 seconds (0.01 work units)

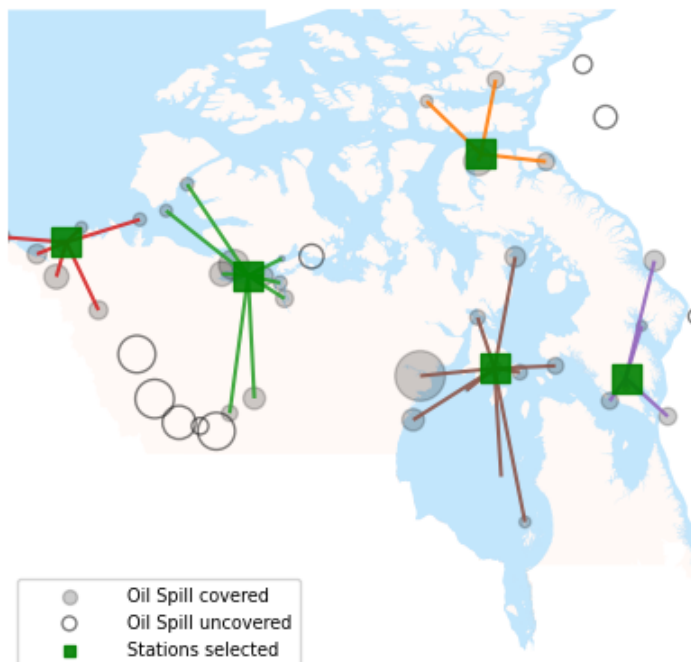
Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	9.0357577	9.03576	0.00%	-	0s

Explored 1 nodes (1346 simplex iterations) in 0.10 seconds (0.02 work units)
Thread count was 8 (of 8 available processors)

Solution count 3: 9.03576 3.55809 3.43502

Optimal solution found (tolerance 1.00e-04)

Best objective 9.035757686020e+00, best bound 9.035757686020e+00, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0x665b6b00

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [5e-05, 5e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 5e+00]

Found heuristic solution: objective 2.4099470

Presolve removed 524 rows and 2099 columns

Presolve time: 0.01s

Presolved: 1109 rows, 1265 columns, 3395 nonzeros

Variable types: 0 continuous, 1265 integer (1265 binary)

Found heuristic solution: objective 2.5587951

Root relaxation: objective 7.357897e+00, 865 iterations, 0.01 seconds (0.01 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	7.3578971	7.35790	0.00%	-	0s

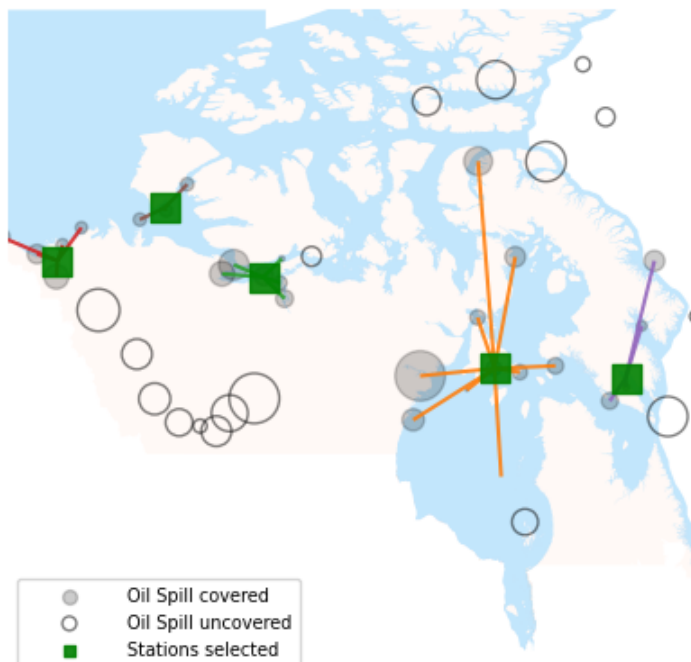
Explored 1 nodes (865 simplex iterations) in 0.07 seconds (0.02 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 7.3579 2.5588 2.40995

Optimal solution found (tolerance 1.00e-04)

Best objective 7.357897074182e+00, best bound 7.357897074182e+00, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0x9cdc1158

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [2e-04, 7e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 5e+00]

Found heuristic solution: objective 0.4480215

Presolve removed 818 rows and 2388 columns

Presolve time: 0.01s

Presolved: 815 rows, 976 columns, 2530 nonzeros

Variable types: 0 continuous, 976 integer (975 binary)

Found heuristic solution: objective 0.5595553

Root relaxation: objective 2.862374e+00, 629 iterations, 0.02 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	2.8623742	2.86237	0.00%	-	0s

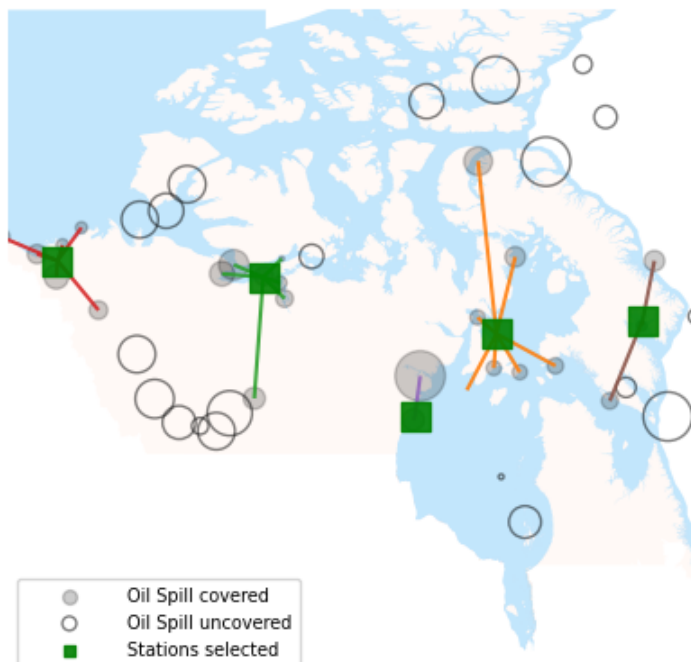
Explored 1 nodes (629 simplex iterations) in 0.06 seconds (0.01 work units)

Thread count was 8 (of 8 available processors)

Solution count 3: 2.86237 0.559555 0.448021

Optimal solution found (tolerance 1.00e-04)

Best objective 2.862374190842e+00, best bound 2.862374190842e+00, gap 0.0000%



m: m2

Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64)

CPU model: Intel(R) Core(TM) i5-8265U CPU @ 1.60GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 1633 rows, 3364 columns and 4946 nonzeros

Model fingerprint: 0xce346049

Variable types: 1582 continuous, 1782 integer (1782 binary)

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [8e-06, 6e-01]

Bounds range [1e+00, 1e+00]

RHS range [1e+00, 5e+00]

Found heuristic solution: objective 0.0874160

Presolve removed 1465 rows and 3136 columns

Presolve time: 0.01s

Presolved: 168 rows, 228 columns, 531 nonzeros

Found heuristic solution: objective 0.9601647

Variable types: 0 continuous, 228 integer (228 binary)

Found heuristic solution: objective 0.9837613

Root relaxation: objective 1.239571e+00, 150 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
*	0	0		0	1.2395713	1.23957	0.00%	-	0s

Explored 1 nodes (150 simplex iterations) in 0.04 seconds (0.01 work units)

Thread count was 8 (of 8 available processors)

Solution count 4: 1.23957 0.983761 0.960165 0.087416

Optimal solution found (tolerance 1.00e-04)

Best objective 1.239571348778e+00, best bound 1.239571348778e+00, gap 0.0000%

