Operations Research Interview Questions & Preparation

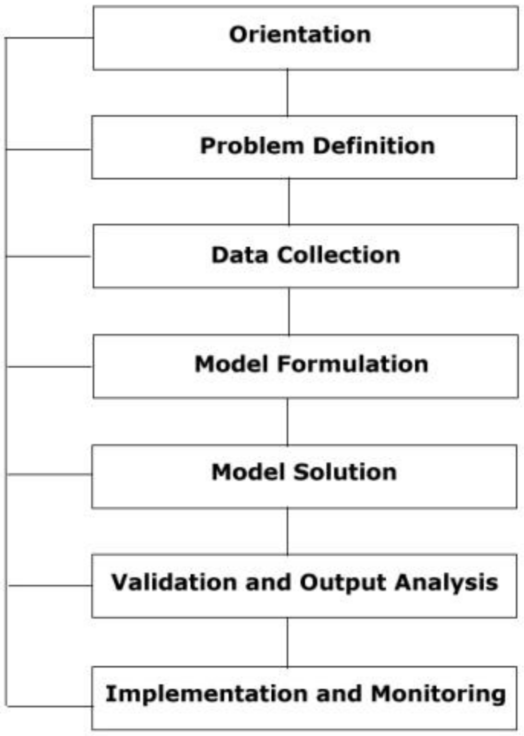
(ongoing)

Compiled by Tanmoy Das

Table of Contents

1 Basic of Operations Research

1. What are the steps of problem solving in OR?



https://sites.pitt.edu/~jrclass/or/or-intro.html

https://web.mit.edu/urban_or_book/www/book/chapter1/1.3.html

2. What do you do for infeasibility analysis?
3. Problem degeneration
4. Convex and concave functions
5. Mathematical programming formula (from verbal to math)
    a. Practise a few+++

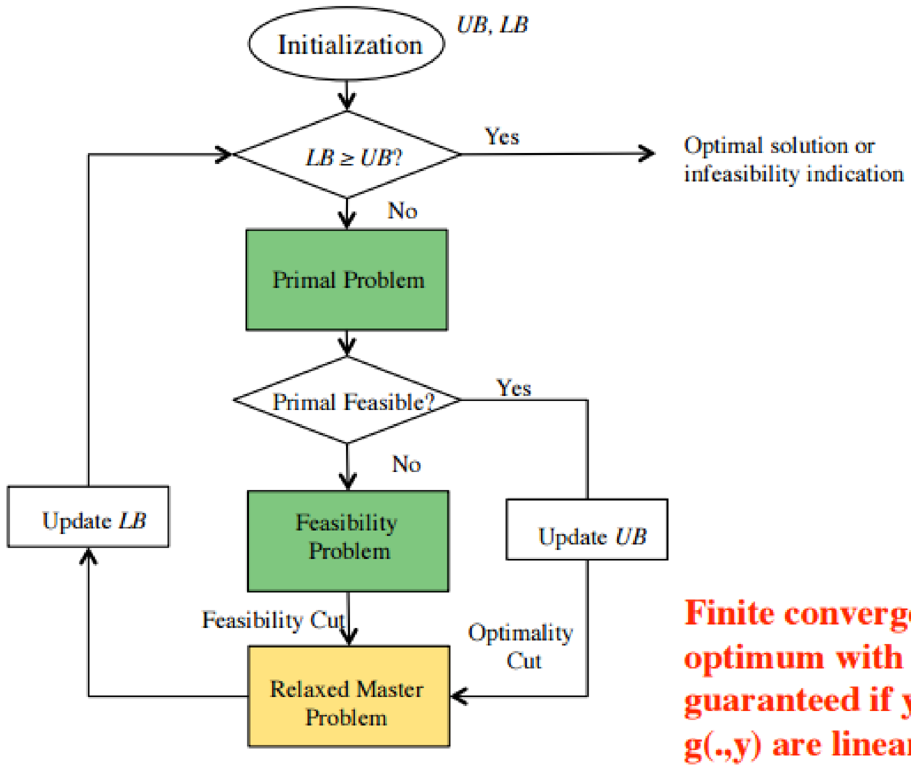
2 Algorithm

2.1 Integer Programming

2.1.1 B&B

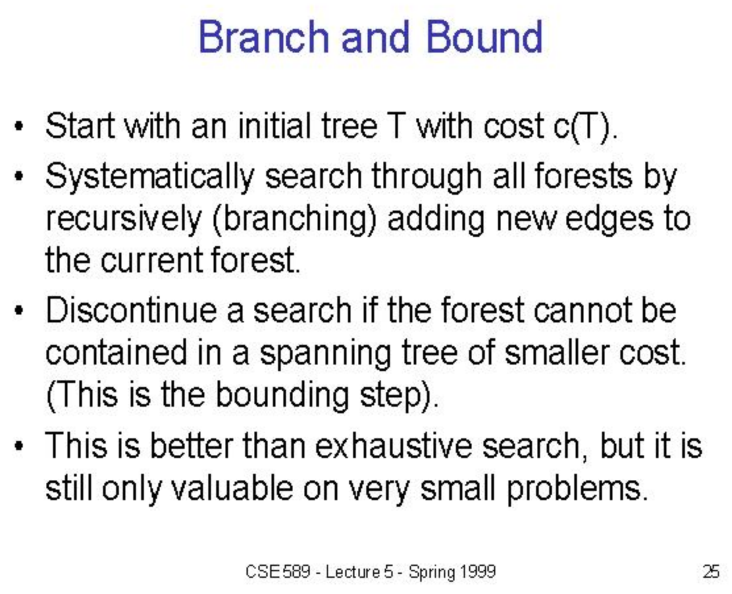
Enumerative (branch and bound, implicit enumeration) methods solve a discrete optimization problem by breaking up its feasible set into successively smaller subsets, calculating bounds on the objective function value over each subset, and using them to discard certain subsets from further consideration. The bounds are obtained by replacing the problem over a given subset with an easier (relaxed) problem, such that the solution value of the latter bounds that of the former. The procedure ends when each subset has either produced a feasible solution, or was shown to contain no better solution than the one already in hand. The best solution found during the procedure is a global optimum.

1. What are the steps in Benders Decomposition (UPS)



Finite convergence proof see to an optimum with a given tolerance is guaranteed if y is integer or f(.,y), g(.,y) are linear for all y in Y.

https://optimization.cbe.cornell.edu/index.php?title=Generalized_Benders_decomposition_%28GBD%29

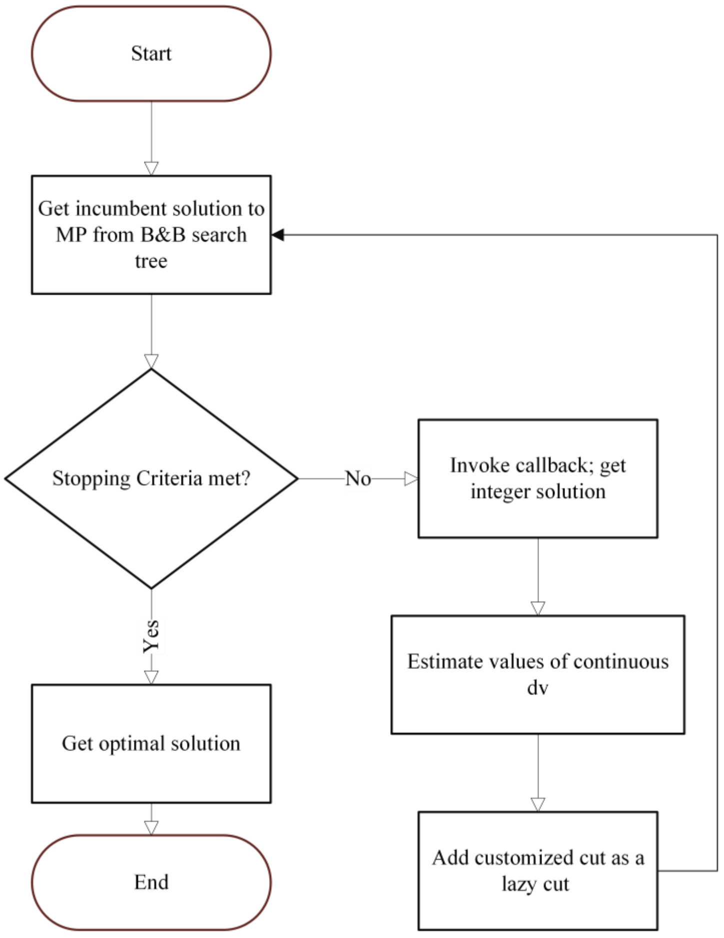


Branch and Bound

- Start with an initial tree T with cost c(T).
- Systematically search through all forests by recursively (branching) adding new edges to the current forest.
- Discontinue a search if the forest cannot be contained in a spanning tree of smaller cost. (This is the bounding step).
- This is better than exhaustive search, but it is still only valuable on very small problems.

CSE 589 - Lecture 5 - Spring 1999 25

2.1.2 Branch & Cut


Start
Get incumbent solution to MP from B&B search tree
Stopping Criteria met?
No
Invoke callback; get integer solution
Estimate values of continuous dv
Add customized cut as a lazy cut
Yes
Get optimal solution
End

What is a cut?

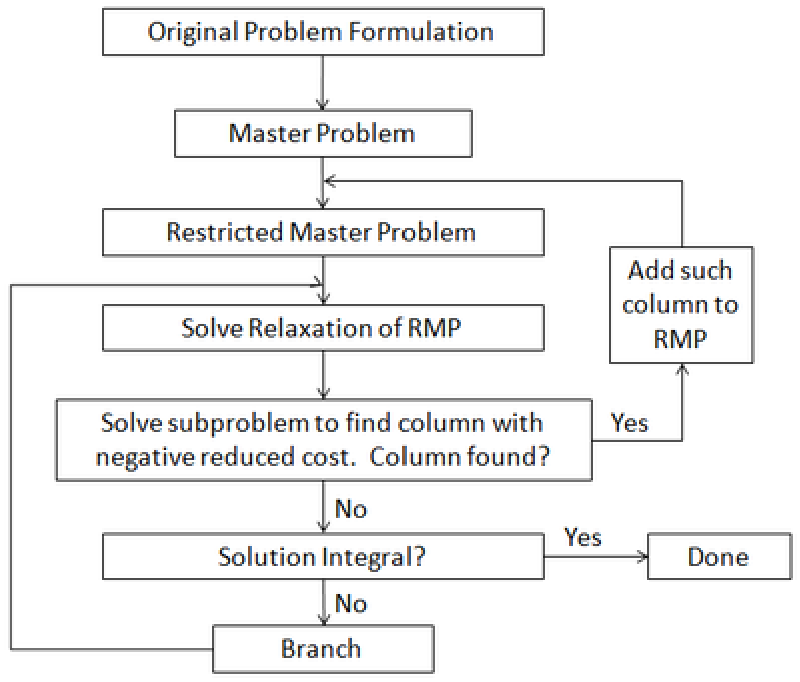
Linear inequality constraints added to the problem

What is incumbent solution?

Best solution found so far

++

2.1.3 B&P


Original Problem Formulation
Master Problem
Restricted Master Problem
Add such column to RMP
Solve Relaxation of RMP
Solve subproblem to find column with negative reduced cost. Column found?
Yes
No
Solution Integral?
Yes
Done
No
Branch

https://www.wikiwand.com/en/Branch_and_price

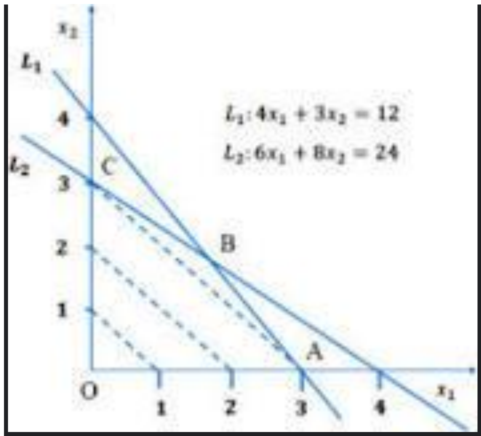
Restricted MP means??

Reduced cost: how much objective function value changes with one unit change of RHS

Shadow price: unit change of constraint → obj

2.1.4 Cutting plane

What is the cutting plane method?



The idea of this method is based on developing a sequence of linear inequalities, which is known as cuts. Then these cuts are used to reduce a part of the feasible region and give a feasible region of the ILP problem. The hyperplane boundary of a cut is called the cutting plane.

2.1.5 Column Generation

1. How to use column generation for VRP problem?
   a. Generic: Column generation or delayed column generation is an efficient algorithm for solving large linear programs.
      The overarching idea is that many linear programs are too large to consider all the variables explicitly. The idea is thus to start by solving the considered program with only a subset of its variables. Then iteratively, variables that have the potential to improve the objective function are added to the program.
      https://medium.com/@sean-patrick-kelley/how-to-implement-column-generation-for-vehicle-routing-bdb8027c957f

2.1.6 Other decomposition methods

2.1.6.1 Comparing algorithms

	Advantage	**Disadvantage**
B&B	Whenever the problem is small and the branching can be completed in a reasonable amount of time, the algorithm finds an optimal solution. By using the branch and bound algorithm, the optimal solution is reached in a minimal amount of time. If the problem is simple enough BnB can look at the whole problem at once and provide an optimal solution, if the problem is enormous (as practical problems, opposed to simple ones, usually are) then solving pieces involves later recombining them and reiterating over them; that can lead to	

	small errors, or pieces that were useful in the final solution having been trimmed.	
B&C	Branch-and-cut methods combine branch-and-bound and cutting-plane methods. The cutting-planes are generated throughout the branch-and-bound tree. The underlying idea is to work on getting as **tight as possible bounds in each node** of the tree and thus reducing the number of nodes in the search tree.	Of course, there is an obvious trade-off. If many cuts are added at a node, re-optimization may slow down. In addition, keeping all the information in the tree is more difficult. On the other hand, the size of the search tree may be reduced significantly.
Lagrange relaxation		Difficulty in Solving: Lagrange's method can become computationally complex, especially for problems with **multiple constraints or variables**. Finding the appropriate Lagrange multipliers and solving the resulting system of equations can be challenging
Column generation	Often a problem can be split into a master problem and the subproblem, that is called column generation. Branch-and-price is a hybrid of branch and bound and column generation methods	

LP

How you know its an optimal solution:

Krus-katar's KKT condition.

GAP (0%)

LB known

decomposition methods? Benders, Column Generation, Branch and Bound, Branch and Cut, Branch and price, Dantzig wolf decomposition

2.1.7 Dantzig wolf decomposition

While there are several variations regarding implementation, the Dantzig–Wolfe decomposition algorithm can be briefly described as follows:

1. Starting with a feasible solution to the reduced master program, formulate new objective functions for each subproblem such that the subproblems will offer solutions that improve the current objective of the master program.
2. Subproblems are re-solved given their new objective functions. An optimal value for each subproblem is offered to the master program.
3. The master program incorporates one or all of the new columns generated by the solutions to the subproblems based on those columns' respective ability to improve the original problem's objective.
4. Master program performs *x* iterations of the simplex algorithm, where *x* is the number of columns incorporated.
5. If objective is improved, goto step 1. Else, continue.
6. The master program cannot be further improved by any new columns from the subproblems, thus return.

https://en.wikipedia.org/wiki/Dantzig%E2%80%93Wolfe_decomposition

2.2 Solution Algorithm in Linear Programming

There are basically three types of algorithms for LINEAR PROGRAMMING: the **SIMPLEX** ALGORITHM (see Section 3.2), **interior point** algorithms, and the **ELLIPSOID** METHOD.

4th: **Graphical** Method

Assumption of Linear Programming

1. Linear equation for all obj function and constraints
2. Additivity
   a. (constraint adding will results in increase problem size
3. ++

1. What are the different solution techniques for LPs?
   a. Graphical method, Simplex, interior point algorithms, and the Ellipsoid Method
   b.

2.2.1 Simplex

1. Optimality conditions in Simplex
   a. The entering variable in a maximization (minimization) problem is the non-basic variable having the most negative (positive) coefficient in the Z-row. The optimum is reached at the iteration where all the Z-row coefficient of the non-basic variables are non-negative (non-positive).
   b. The optimality condition is the reduced-cost condition. When reduced costs are positive, you have the optimum

Steps in Simplex

2. 3 rules in converting primal to dual?
3. When to use primal vs dual?

Excel Solver (to solve Simplex Problem)

Steps in SIMPLEX algorithm

I. Standard form
II. Introducing slack variables
III. Creating the tableau
IV. Pivot **variables** (basic, non-basic)
V. Creating a new tableau
VI. Checking for optimality
VII. Identify optimal values

Step 1: Establish a given problem. (i.e.,) write the inequality constraints and objective function.

Step 2: Convert the given inequalities to equations by adding the slack variable to each inequality expression.

Step 3: Create the initial simplex tableau. Write the objective function at the bottom row. Here, each inequality constraint appears in its own row. Now, we can represent the problem in the form of an augmented matrix, which is called the initial simplex tableau.

Step 4: Identify the **greatest negative entry** in the bottom row, which helps to identify the pivot column. The greatest negative entry in the bottom row defines the largest coefficient in the objective function, which will help us to increase the value of the objective function as fastest as possible.

Step 5: Compute the quotients. To calculate the quotient, we need to divide the entries in the far right column by the entries in the first column, excluding the bottom row. The smallest quotient identifies the row. The row identified in this step and the element identified in the step will be taken as the pivot element.

Step 6: Carry out pivoting to make all other entries in column is zero.

Step 7: If there are no negative entries in the bottom row, end the process. Otherwise, start from step 4.

Step 8: Finally, determine the solution associated with the final simplex tableau.

https://byjus.com/maths/linear-programming/#:~:text=The%20linear%20programming%20problem%20can,and%20graphical%20method%20in%20detail.

1. **Set up the problem.** That is, write the objective function and the inequality constraints.
2. **Convert the inequalities into equations.** This is done by adding one slack variable for each inequality.
3. **Construct the initial simplex tableau.** Write the objective function as the bottom row.
4. **The most negative entry in the bottom row identifies the pivot column.**
5. **Calculate the quotients. The smallest quotient identifies a row. The element in the intersection of the column identified in step 4 and the row identified in this step is identified as the pivot element.** The quotients are computed by dividing the far right column by the identified column in step 4. A quotient that is a zero, or a negative number, or that has a zero in the denominator, is ignored.
6. **Perform pivoting to make all other entries in this column zero.** This is done the same way as we did with the Gauss-Jordan method.
7. **When there are no more negative entries in the bottom row, we are finished; otherwise, we start again from step 4.**
8. **Read off your answers.** Get the variables using the columns with 1 and 0s. All other variables are zero. The maximum value you are looking for appears in the bottom right hand corner.

https://math.libretexts.org/Bookshelves/Applied_Mathematics/Applied_Finite_Mathematics_(Sekhon_and_Bloom)/04%3A_Linear_Programming_The_Simplex_Method/4.02%3A_Maximization_By_The_Simplex_Method

Sensitivity analysis and reduced cost

The opportunity/reduced cost of a given decision variable can be interpreted as the rate at which the value of the objective function (i.e., profit) will deteriorate for each unit change in the optimized value of the decision variable with all other data held fixed.

2.2.2 Dual Simplex

Why we use dual simplex method instead of simplex method?
In the dual simplex, new primal cuts correspond to new dual variables, which are initialized as nonbasic, and thus the previous solution is still dual feasible. Thus, dual simplex does not need to regain feasibility. So in a mixed-integer-programming context, dual simplex will usually outperform primal simplex.
In general it is easier to get dual feasibility than primal feasibility, and dual simplex appears to make more progress in many iterations.

Many commercial solvers also offer Barrier methods to solve LP(-Relaxations). The big drawback of interior point methods is that they can't really be warmstarted, and when solving mixed-integer-programmes using branch & bound are generally given a reasonable warm/hot-start (the solution of the previous LP Relaxation). source

2.2.3 Interior Point for LP

2.3 NLP

1. Contraint optimisation (conditions, Lagrangien relaxation, Quasi-newton, Rampson)

Lagrangian relaxation is a technique well suited for problems where the constraints can be divided into two sets:

"good" constraints, with which the problem is solvable very easily •

"bad" constraints that make it very hard to solve.

The main idea is to relax the problem by removing the "**bad**" constraints and putting them into the objective function, assigned with weights (the Lagrangian multiplier). Each weight represents a penalty which is added to a solution that does not satisfy the particular constraint.

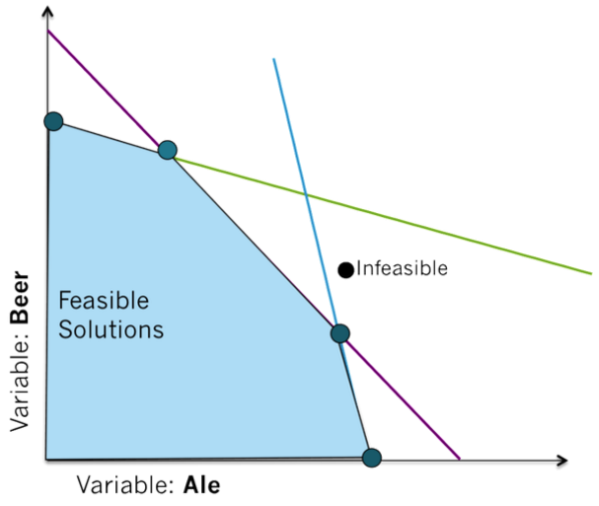
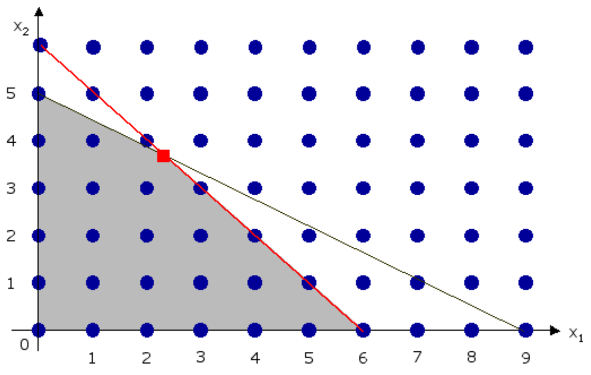
Quasi-Newton methods are methods used to either find zeroes or local maxima and minima of functions, as an alternative to Newton's method. They can be used if the Jacobian or Hessian is unavailable or is too expensive to compute at every iteration. The "full" Newton's method requires the Jacobian in order to search for zeros, or the Hessian for finding extrema. Some iterative methods that reduce to Newton's method, such as SLSQP, may be considered quasi-Newtonian.

https://en.wikipedia.org/wiki/Quasi-Newton_method

2.4 MIP

1. What's the difference between LP & MIP?

LP	MIP
All variables are continuous	Some variables are integer (others are continuous)
In LP, objective function and constraints have to be linear e.g. x1+x2 <=10	

In LP, optimal solution lies in corner points. But, that's not true of IP problem.   Source link	MIP is difficult to solve compared to LP. Why? Because it contains integer variable. But, why integer variable makes it difficult?
	Typically, when we refer to MIP, we meant MILP. When a MIP is

2. What are the different solution techniques for MIPs?
    a. B&B, B&C,
3. ++

Logical transformation

Piecewise linear transformation

2.4.1 Exact Method

2.4.2 Heuristics

For example, a greedy strategy for the travelling salesman problem (which is of high computational complexity) is the following heuristic: "At each step of the journey, visit the nearest unvisited city." This heuristic does not intend to find the best solution, but it terminates in a reasonable number of steps; finding

https://blog.boot.dev/computer-science/examples-of-heuristics-in-computer-science/

Difference between exact method and heuristics?

Heuristic

– Constructive algorithms (greedy algorithm)

– Local search algorithms (hill-climbing...)

Meta-heuristic

– Trajectory methods: Describe a trajectory in the search space during the search process

- Variable Neighbourhood Search
- Iterated Local Search
- Simulated Annealing
- Tabu Search

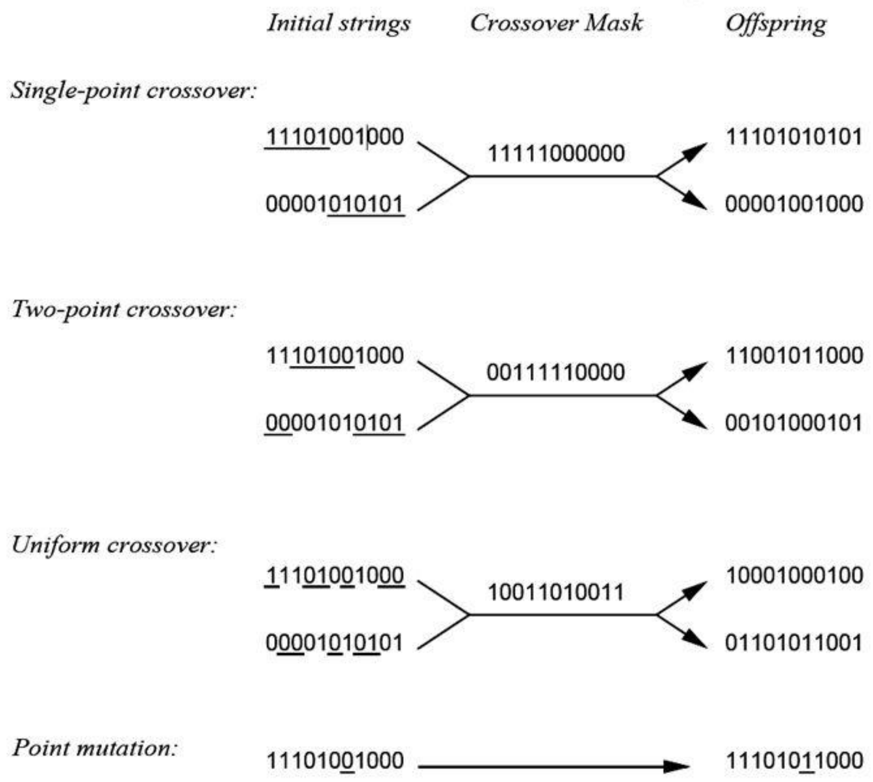
– Population-based: Perform search processes which describe the evolution of a set of points in the search space e.g. Evolutionary computation (Genetic Algorithms)

Source: https://mat.uab.cat/~alseda/MasterOpt/IntroHO.pdf

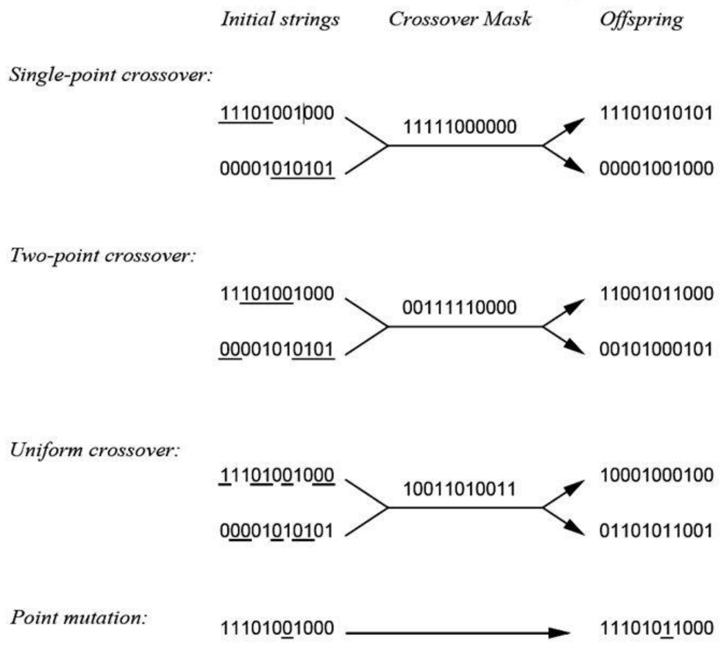
2.4.3 Meta-heuristics

2.4.3.1 GA

Operators for Genetic Algorithms



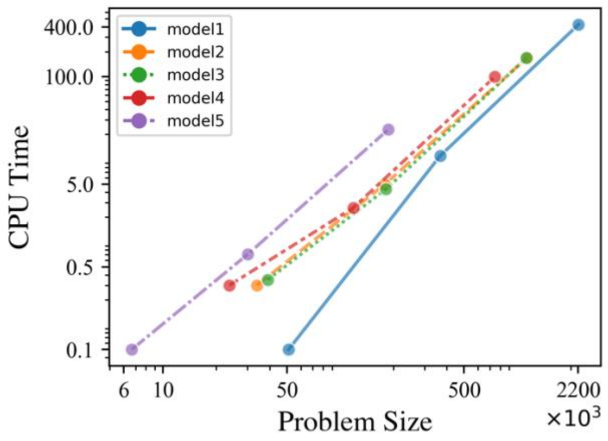
Operators for Genetic Algorithms



Source

Regarding your question on complexity and runtime, I am trying to summary one of my most-recent facility location-allocation problem.

The size of the problem is the sum of the number of continuous and discrete variables, number of linear and nonlinear constraints. The figure shows the CPU time, which grows exponentially with problem size; for instance, the CPU time rises from 0.03 seconds to 424 seconds – shown by the blue line – when dealing with 100 and 3100 number of oil spills (the problem size is raising from 50000 to 2200000) with 12 scenarios by Model 1.



We were able to reduce the runtime approximately by 10 times using meta-heuristics (Genetic algorithm) rather than exact solution algorithm used before. More details can be found in Section 4.3 of the attached paper.

2.4.3.2 PSO

3 Modeling

3.1 Knap-sack problem

3.2 Set Covering

$$\begin{aligned} \max \quad & \sum_j a_j y_j \\ \text{s.t} \quad & \sum_{i \in C_j} x_i \geq y_j \quad \forall j \\ & \sum_i x_i = k \\ & x_i, y_j \in \{0,1\} \end{aligned}$$

https://optimization.cbe.cornell.edu/index.php?title=Set_covering_problem

https://en.wikipedia.org/wiki/Set_cover_problem

3.3 P-median

The "p-median" problem is a distance based optimization problem in which **p facilities** need to be located and assigned to the demand points such that each demand point is mapped to a single facility, and the sum of the weighted distance between all demand points and corresponding facilities is minimized.

3.4 MCLP

Problem Name	**Aim**	**Objective**	**Formulation**
Set covering problem	locate the minimum number of facilities required to "cover" all of the demand nodes.	minimizes the number of facilities located	$\text{Minimize} \quad \sum_{j \in J} x_j$ subject to: $\sum_{j \in N_i} x_j \geq 1 \ \forall i \in I$; $x_j \in \{0,1\} \ \forall j \in J$
MCLP	Budget constraint, so build p number of facilities	Maximize demand	$\text{Maximize} \quad \sum_{i \in I} h_i z_i$ subject to: $\sum_{j \in N_i} x_j - z_i \geq 0 \ \forall i \in I$; $\sum_{j \in J} x_j = p$; $x_j \in \{0,1\} \quad \forall j \in J$; $z_i \in \{0,1\} \quad \forall i \in I$

p-center	minimizing the maximum distance that demand is from its closet facility given that we are siting a pre-determined number of facilities	Minimize max distance	Maximize W subject to: $\sum_{j\in J} x_j = p$ $\sum_{j\in J} y_{ij} = 1 \quad \forall i \in I$ $y_{ij} - x_j \leq 0 \quad \forall i \in I, j \in J$ $W - \sum_{j\in J} h_i d_{ij} y_{ij} \geq 0 \; \forall i \in I$ $x_j \in \{0,1\} \quad \forall j \in J$ $y_{ij} \in \{0,1\} \quad \forall i \in I, j \in J$
p-median	Locations of p facilities to minimize the demand-weighted total distance between demand nodes and the facilities	minimize the demand-weighted total distance	Minimize $\sum_{i\in I}\sum_{j\in J} h_i d_{ij} y_{ij}$ subject to: $\sum_{j\in J} x_j = p$ $\sum_{j\in J} y_{ij} = 1 \quad \forall i \in I$ $y_{ij} - x_j \leq 0 \quad \forall i \in I, j \in J$ $x_j \in \{0,1\} \quad \forall j \in J$ $y_{ij} \in \{0,1\} \quad \forall i \in I, j \in J$

Maximal Covering Location Problem

$$\max \sum_{i\in I} h_i y_i$$

$s.t.$

$$\sum_{j\in N_i} x_j \geq y_i, \qquad i \in I$$

$$\sum_{j\in J} x_j = p,$$

$$x_j \in \{0,1\}, \qquad j \in J$$

$$y_i \in \{0,1\}, \qquad i \in I$$

https://en.wikipedia.org/wiki/Maximum_coverage_problem

3.5 Location Allocation

1. Can you write a simple facility allocation model? When we use location-allocation modeling?

Verbal:

Objective: Min cost, $FixedCost_f \, .open_f$

St. number of facility <= 10

Allocation_f,c <= availability_f

3.6 Network Optimization

3.6.1 VRP formulation

3.6.2 TSP formulation

$$\begin{aligned}
\min \sum_{i=1}^{n} \sum_{j\neq i, j=1}^{n} c_{ij} x_{ij} &\colon \\
x_{ij} &\in \{0,1\} && i,j = 1,\ldots,n; \\
u_i &\in \mathbf{Z} && i = 2,\ldots,n; \\
\sum_{i=1, i\neq j}^{n} x_{ij} &= 1 && j = 1,\ldots,n; \\
\sum_{j=1, j\neq i}^{n} x_{ij} &= 1 && i = 1,\ldots,n; \\
u_i - u_j + (n-1)x_{ij} &\le n-2 && 2 \le i \neq j \le n; \\
2 \le u_i &\le n && 2 \le i \le n.
\end{aligned}$$

3.6.3 Minimum Spanning Tree

3.6.3.1 Kruskal

Learning source: link.

4 Computational Complexity

1. Please describe one **linear optimization problem/algorithm** and state its **computational complexity**.

Linear programming:

Linear programming is **an optimization technique for a system of linear constraints and a linear objective function**. An objective function defines the quantity to be optimized, and the goal of linear programming is to find the values of the variables that maximize or minimize the objective function.

Complexity

The complexity of an optimization algorithm depends on the following factors:

1- Number of iterations, 2- Number of individuals in the population, 3-complexity of the objective function, 4- If you sort the individuals, the complexity of sorting should be added

Example O(N*M*P*Q) where N is #1, M is #2, P is #3, Q is #4

The better estimation for #1 and #2 is computing the NFE(number of function evaluations) source: https://www.researchgate.net/post/How-can-I-calculate-the-computational-complexity-of-any-optimization-algorithm

The computational complexity, in general, depends on the optimization algorithm and the technique that you use. In some algorithms, the complexity can be measured by the time that the CPU needs to run the algorithm, others consider the computational complexity as the number of nested loops (for loops and others) per run and can be written as O(x), where x is your nested loops.

5 Uncertainty

5.1 Robust optimization

Why we always use “worst-case”

https://math.stackexchange.com/questions/646380/reasons-for-the-worst-case-scenario-in-robust-optimization

less worst case, then not much robust, right?

If we add worst situation, and train model accordingly, it will be robust for any other less-worse conditions, as well.

Other perspective: lightly robust

- An optimal (feasible) solution is robust if it stays optimal (feasible) under any realization of the data

5.2 Stochastics Optimization

6 Large Scale Optimization

1. Tell me about large-scale optimization
2. How do you solve them?
3. What heuristics you will apply for large-scale?

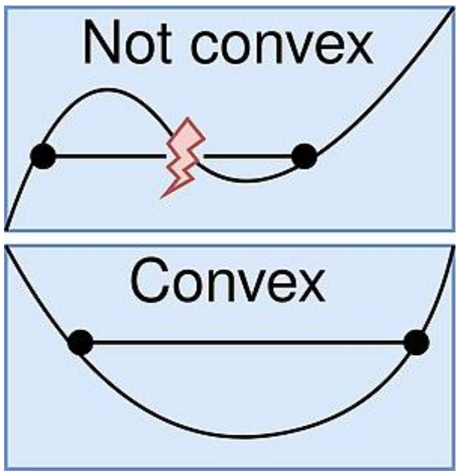
7 Gurobi

1. How you set parameters of B&B
2. How to handle infeasibility?
    a. Model.computeIIS (Irreducible Inconsistent Subsystem)
        i. This set is still infeasible
        ii. feasible if single constraint or bound removed
    b. How to manually check infeasibility?
3. Warm start
    a. How to use pretrained optimization model as initial sol

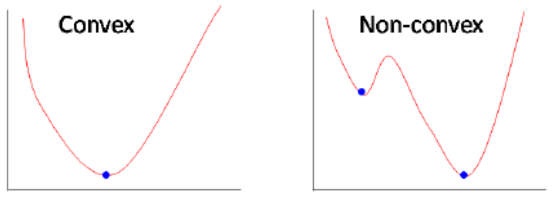
IIS

8 Miscellaneous

8.1 Convex function



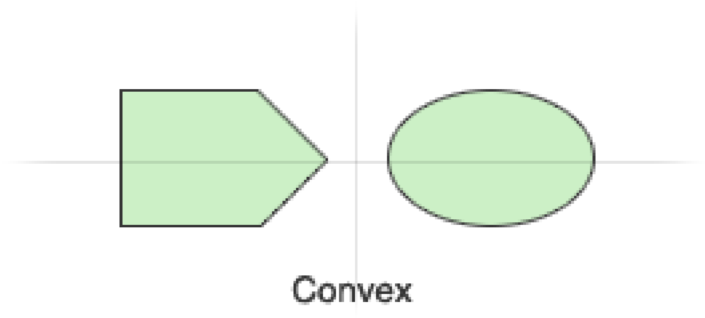
A line drawn from any two points of the function falls within the surface of the function (concave if outside)



8.1.1.1 *Convex Optimization Problems*

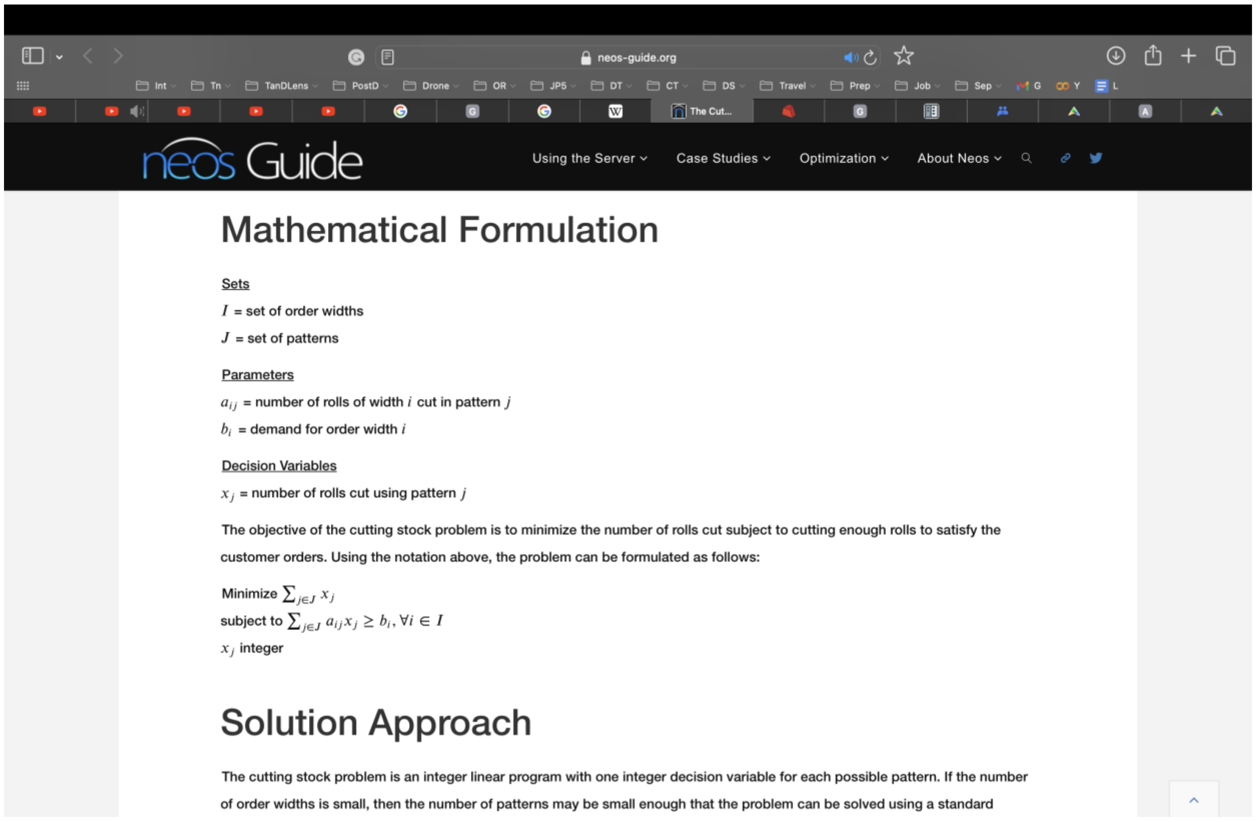
A *convex optimization problem* is a problem where all of the constraints are convex functions, and the objective is a convex function if minimizing, or a concave function if maximizing.

Linear functions are convex, so linear programming problems are convex problems. Conic optimization problems -- the natural extension of linear programming problems -- are also convex problems. In a convex optimization problem, the feasible region -- the intersection of convex constraint functions -- is a convex region, as pictured below.



8.2 Cutting stock problem

https://neos-guide.org/case-studies/sc/mfg/the-cutting-stock-problem/



Mathematical Formulation

Sets

I = set of order widths

J = set of patterns

Parameters

a_{ij} = number of rolls of width i cut in pattern j

b_i = demand for order width i

Decision Variables

x_j = number of rolls cut using pattern j

The objective of the cutting stock problem is to minimize the number of rolls cut subject to cutting enough rolls to satisfy the customer orders. Using the notation above, the problem can be formulated as follows:

Minimize $\sum_{j \in J} x_j$

subject to $\sum_{j \in J} a_{ij} x_j \geq b_i, \forall i \in I$

x_j integer

Solution Approach

The cutting stock problem is an integer linear program with one integer decision variable for each possible pattern. If the number of order widths is small, then the number of patterns may be small enough that the problem can be solved using a standard

9 MCQs

1. If the feasible set of an optimization problem is unbounded, which of the following is true? (C3AI-T)
    a. No finite optimum point exists
    b. It has an infinite number of feasible points
    c. The existence of finite optimum points can not be assured
    d. None of the above.

Nash, "an **equilibrium** point is an n-tuple such that each player's mixed strategy maximizes his payoff if the strategies of the others are held

How many routes possible in the traveling salesman problem with n cities?

$(n-1)!$

10 Applied questions

Common optimization problems in production planning and optimization challenges

10.1 Optym Interview Questions

Q1: Buy and Sell stock

The cost of one unit of a stock on each day is given in an array of size n (# of days). The decision problem is to find days on which stocks should be bought and sold to maximize the profit by buying and selling of stocks. For example, if the given array is {100, 180, 260, 310, 40, 535, 695}, the maximum **profit** can be earned by buying on day 1, selling on day 4; buying on day 5 and selling on day 7. **Provide a pseudocode of your algorithm for above problem.** Also provide the running time of your algorithm.

Brainstorm

Profit = Sell – Buy

Maximize profit

Clarifying question:

One-time transaction

For each day, track

1. Min_buy
2. Max_profit

# Data

Cost = {100, 180, 260, 310, 40, 535, 695}

Pseudocode of the algorithm:

```
# Initialize
    - Maximum profit, max_p = 0 If we don't sell
    - Minimum buy, min_b = cost[0]
# Iterate
    - for i in Cost():
        Max_p = max (max_p, cost[i]-min_b)
        min_b = min(min_b, cost[i])
```

Runtime:

Since I am calculating everything in one pass, Runtime = O(n)

Q2: Workforce scheduling

Set & Index

$\boldsymbol{i} \in \boldsymbol{I}$ I is a set of day shifts, indexed by i

Day 1= Monday

Day 7 = Sunday

$e \in E$ E is a set of Employees, indexed by e

Decision Variables

X_{ei} 1 if employee e is hired for day i

0 otherwise

A non-negative integer? Or binary

Parameters

$C_i = c_1, c_2, \dots, c_7$ Cost of each shift

$\boldsymbol{D_i}$ Demand for # of people, where $0 \le i \le 7$

R_i Rate/pay per day

Objective function

Min wage = Daily rate x # of day for each employee x Number of Employee?++

$$\sum_e R_i . X_{ei}$$

Constraints

1. working at max 5 consecutive days for each employee

$$\sum_e \sum_i X_{ei} \le 5$$

2. Two days weekend

3. **Workload Demand** of people in day I

$$\sum_e X_{ei} \ge D_i \quad \forall i$$

https://medium.com/walmartglobaltech/automating-shift-scheduling-with-linear-programming-fe1720f13620

Q3: Handshake

Handshaking lemma: the sum of the degrees (the numbers of times each vertex is touched) equals twice the number of edges in the graph. https://en.wikipedia.org/wiki/Handshaking_lemma

1. group even
2. group odd

++ Sajeeb

Q4: Euler Tour

https://www.gatevidyalay.com/euler-graph-euler-trail-euler-circuit/

Graph theory – Network optimization

Q5: Dynamic Programming – Print neatly

https://github.com/samuelklam/print-neatly/blob/master/print-neatly.py

DP pseudocode https://www.albany.edu/~ravi/pdfs_csi604/handout_1.1.pdf

Q7: Cutting stock

The production of paper or cloth often uses the following process. We first produce the paper on long rolls which we then cut to meet specific demand requirements. The cutting stock problem is to meet the specified demand by using the minimum number of rolls. Suppose that the rolls all have length L and that we have a demand of di for smaller rolls of size Li • There might be many ways to cut any roll into smaller rolls. For example, if L = 50 and the Li have lengths 10, 15, 25, 30, we could cut the rolls into several, different patterns: for example, (1) 5 rolls of size 10; (2) 3 rolls of size 15; (3) 2 rolls of size 10 and 1 roll of size 25. Note that in the first pattern, we use the entire roll of size L, but in the other two patterns we incur a waste of 5. Let P denote the collection of all patterns, and for any pattern p ∈ P, let ni,p denote the number of small rolls of size Li in pattern p.

(a) Letting xp be a nonnegative integer variable indicating how many rolls we cut into pattern p, show how to formulate the cutting stock problem of finding the fewest number of large rolls to meet the specified demands as an optimization model with a variable for each cutting pattern.

(b) Suppose that we wish to cut a single roll of size L into sub-rolls and that we receive a profit of πi for every sub-roll of size Li that we use in the solution. Show how to formulate this problem as an integer knapsack problem.

(c) Show how to use the column generation technique to solve the linear programming relaxation of the cutting stock problem in part (a), using knapsack problems of the form stated in part (b) to price out columns.

(a) Cutting Stock Formulation

Set & Index

$p \in P$	Pattern of stock
$o \in O$	Set of Order width

Decision Variable

X_p= Number of rolls cut into pattern

D_o= Demand of o

Parameters

c_p= cost of pattern p

Obj

$$\min c_p X_p$$

Constraints

1. Enough roll to satisfy demand

$$\sum_p a_{po} X_p \geq D_o \quad \forall o$$

(b) Knapsack

Basic cutting plane

https://en.wikipedia.org/wiki/Cutting_stock_problem

https://neos-guide.org/case-studies/sc/mfg/the-cutting-stock-problem/

Two pointer

Card

5 card hand → from 52 cards (4*13)

At least one card from each suite.

13^4

n_c_r = $\frac{n!}{r!(n-r)!}$

$1 - \frac{4}{13}$ *

American Airline (Sr. OR Scientist)