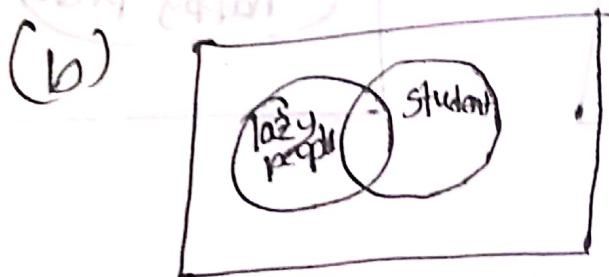
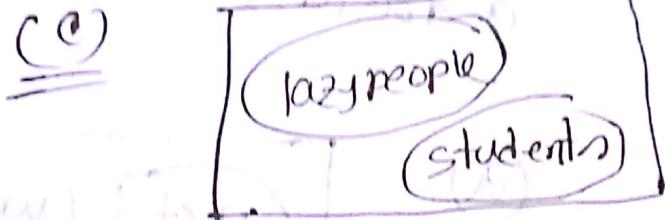
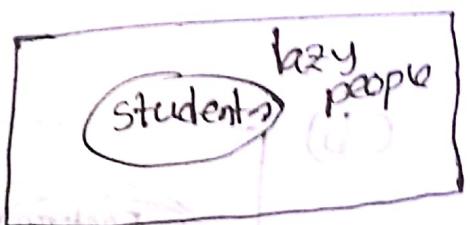


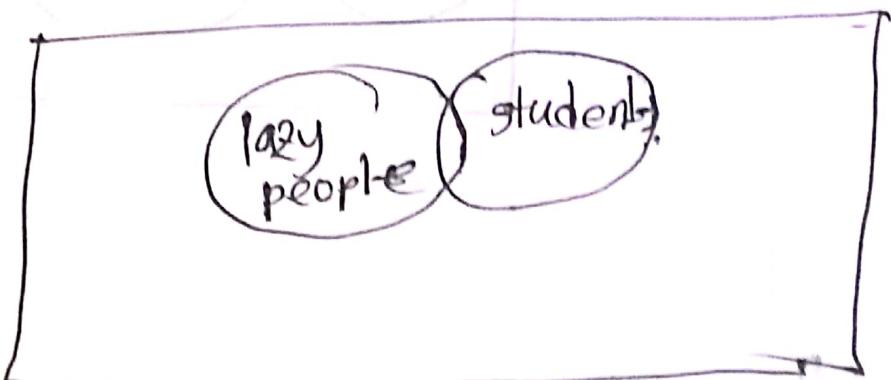
Q Translate each of the following statements intoven diagram.

- (a) All students are lazy.
- (b) Some students are lazy.
- (c) No students are lazy.
- (d) Not all students are lazy.

Ans: (a) The set of students are contain in the set of lazy people.



(d) Here the set of students is not contain in the set of lazy people.



\* Consider the following assumption

S1: poets are happy people.

S2: Every doctor is wealthy.

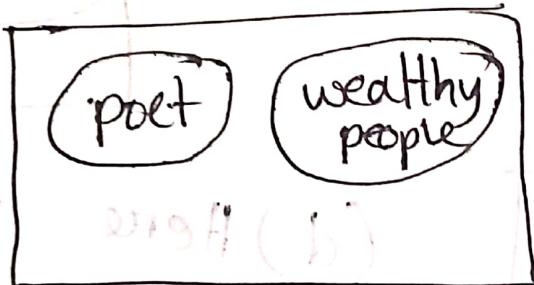
S3: No one who is happy also wealthy determine factors.

(a) No poet is wealthy.

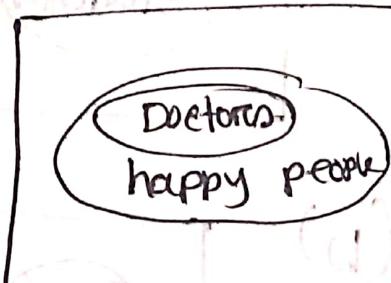
(b) Doctors are happy people.

(c) No one can be both a poet and a doctor.

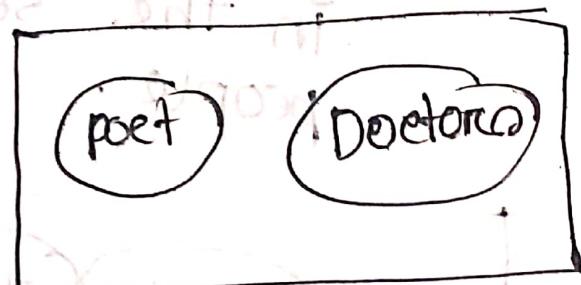
(a)



(b)



(c)



\* (a)  $A = \{x : x \in \mathbb{N}, 3 < x < 9\}$

(a) A consist of all integers between 3 and 9.  $A = \{4, 5, 6, 7, 8\}$ .

(b) b. consist of all positive integers satisfying the equation  $x^2 + 1 = 10$ .  
Hence  $x^2 = 9$ ,  $x = 3$ .  $3^2 + 1 = 10$ .

Hence  $B = \{3\}$ .  
c. c consist of all positive odd integers between -5 and 5  
hence  $c = \{-1, 3\}$ .

\* (a)  $A \not\subseteq A$  (d)  $A \not\subseteq D$ .  $P = (A)D$  or  $F \supseteq G$ .

(b)  $5 \not\in B$  (e)  $G \subseteq F$  or  $(G)F$

(c)  $A \subseteq C$ .

Q] List all the subsets of A  
there are  $2^5 = 32$  of them.

$$R(A) = [\emptyset, \{\{1\}\}, \{\{2\}\}, \{\{3\}\}, \{\{4\}\}, \{\{5\}\}, \{\{1, 2\}\}, \{\{1, 3\}\}, \{\{1, 4\}\}, \{\{1, 5\}\}, \{\{2, 3\}\}, \{\{2, 4\}\}, \{\{2, 5\}\}, \{\{3, 4\}\}, \{\{3, 5\}\}, \{\{4, 5\}\}, \{\{1, 2, 3\}\}, \{\{1, 2, 4\}\}, \{\{1, 2, 5\}\}, \{\{1, 3, 4\}\}, \{\{1, 3, 5\}\}, \{\{1, 4, 5\}\}, \{\{2, 3, 4\}\}, \{\{2, 3, 5\}\}, \{\{2, 4, 5\}\}, \{\{3, 4, 5\}\}, \{\{1, 2, 3, 4\}\}, \{\{1, 2, 3, 5\}\}, \{\{1, 2, 4, 5\}\}, \{\{2, 3, 4, 5\}\}, A]$$

e.g.)  $P(A) = [\{a, b\}, \{c\}, \{d\}]$ .

$$\therefore P(A) = [\emptyset, \{a, b\}, \{c\}, \{d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, A]$$

$$n(A) = 4$$

$$n(B) = 5$$

$$n(C) = \infty$$
 [as limit because there are no positive integers  $n(C) \geq 0$ .]

... infinite

Given that  $n(F \cup G \cup R) = 120 - 20$

for non-mem.  $n(F) = 65$ , photo  $F = 8 - 20$

$n(G) = 45$ , photo  $G = 8 - 20$

$n(R) = 42$ , photo  $R = 8 - 20$

$n(F \cup G) = 20$

$n(F \cap G) = 25$

From  $n(G \cup R) = 15$ , photo  $F = 8 - 20$

$n(F \cap G \cap R) = 2$

We know  $n(F \cup G \cup R) = n(F) + n(G) + n(R) - n(F \cap G) - n(F \cap R) - n(G \cap R) + n(F \cap G \cap R)$

$n(F \cup G \cup R) = 65 + 45 + 42 - 20 - 25 - 15 + n(F \cap G \cap R)$

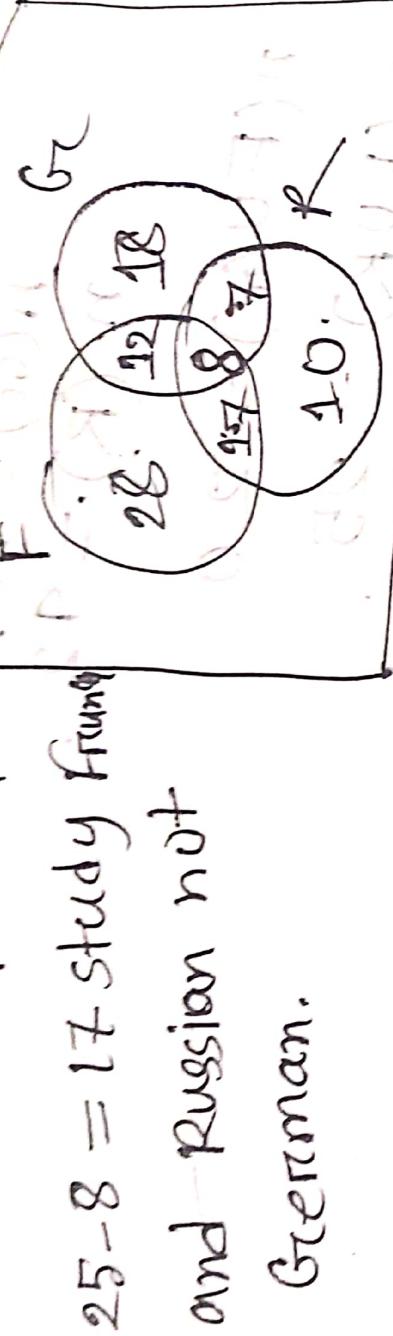
$$\Rightarrow 100 = 65 + 45 + 42 - 20 - 25 - 15 + n(F \cap G \cap R)$$

$$\Rightarrow n(F \cap G \cap R) = 8$$

so, 8 members learn all languages.

that is in (F or Gr) = 100 students.

$20 - 8 = 12$  study French and German not Russian.



$15 - 8 = 7$  study German and Russian not French.

$$\begin{aligned} & \text{only French} = 28 \\ & 45 - 12 - 8 - 7 = 28 \text{ only German} \\ & 42 - 17 - 8 - 7 = 10 \text{ only Russian.} \end{aligned}$$

$$\begin{aligned} & 12 + 8 + 7 + 5 = 32 \\ & 32 + 28 + 28 + 10 = 90 \text{ total.} \\ & (A \cap B \cap C)^c = 90 \end{aligned}$$

$$x = (A \cap B)^c = 6$$

→ 60% do not choose all three subjects.

100 students were asked what here they have.

Sol: Given that, updating marks on paper

$$n(S) = 45$$

$$n(A) = 38$$

$$n(S \cap A) = 18$$

$$n(S \cap H) = 9$$

$$n(A \cap H) = 4$$

$$n(S \cup H \cup A) = 100 - 23 = 77 \text{ (from a diagram)}$$

$$n(S \cap A \cap H) = ?$$

$$\begin{aligned} n(S \cup H \cup A) &= n(S) + n(H) + n(A) - n(S \cap A) \\ &\quad - n(S \cap H) - n(A \cap H) + n(S \cap A \cap H) \end{aligned}$$

$$\Rightarrow 77 = 45 + 38 + 24 - 18 - 9 - 4 + n(S \cap A \cap H)$$

$$\Rightarrow n(S \cap A \cap H) = 4$$

3 subjects had taken

$18 - 4 = 14$  Sociology and Anthropology  
 $\rightarrow$  not history.

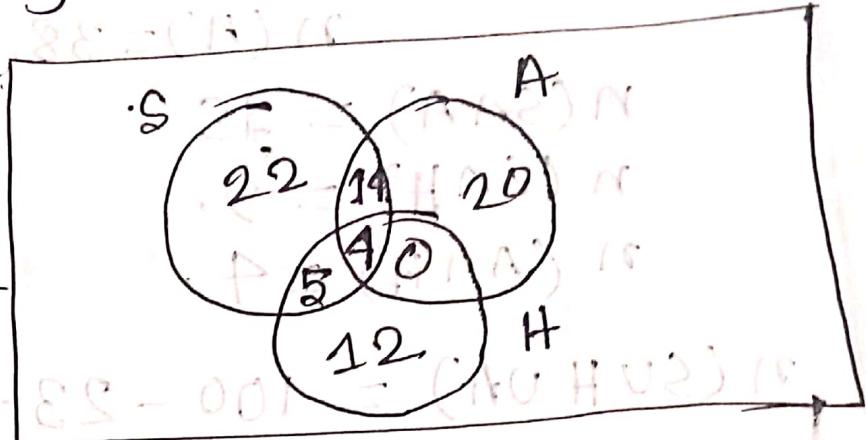
$9 - 4 = 5$  Sociology and History not Anthropology.

$\rightarrow 4-4=0$  Anthropology and History  $\cap$  not History.

only sociology

$$\rightarrow 45 - 14 - 4 - 5 = 22$$

only Anthropology.



$$= 38 - 14 - 10 = 20$$

$$S = (A \cup H) \cap S$$

$$(A \cap S) \cap \text{not History} = 21 - 14 - 5 = 12$$

$$(H \cap S) \cap \text{not Anthropology} = 10 - 5 - 10 = 0$$

$$(A \cap H) \cap \text{not Sociology} = 10 - 5 - 10 = 0$$

$$P = (H \cap A \cap S) \cap \text{not}$$

only Sociology

Eulogopoda has 8 legs  $P(S \cap H \cap A) = P - 8L$

so it contains 6 legs

from part 1 it has 10 legs  $10 - 6 = 4$   $P - 8L = 4$

Eulogopoda

has 4 legs

\* (a) A consists of all the positive integers between 3 and 12

$$A = \{4, 5, 6, 7, 8, 9, 10, 11\}$$

(b) A is infinite since there are four seasons in the year i.e  $n(A) = 4$ , B is finite because there are 50 states in the USA,  $n(B) = 50$ .

(c) There are no positive integers less than 1,  $n(c) = 0$ .

(d) It is infinite,  $n(d) = \infty$ .  
-  $\{1, 3, 4, 21, 2\}$  is difficult to find although it may be difficult to find

(e) Although  $n(\text{population}) = 25$   
~~14~~  $n(T) = 26$  choose 5  
 $n(F) = 26$

$n(N \cap F) = 91$ ,  $n(T \cap F) = 8$ ,  
 $n(N \cap T) = 7$ ,  $n(N \cap T \cap F) = 3$   
and  $T \cup F = 37$ ,  $n(N \cup T \cup F) = 37$

\* (f) Find out the number of people who like both the numbers 5 and 8  
 $n(5 \cap 8) = 8$

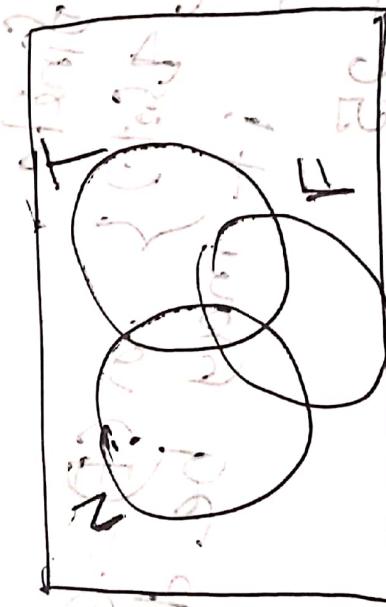
(g) Find out the number of people who like either 5 or 8 or both  
 $n(5 \cup 8) = 15$

read at least one of the three magazines.

$$\rightarrow (a) n(T \cup F \cup N) = ?$$
$$n(N \cap T \cap F) = n(N) + n(T) + n(F)$$

$$= n(N \cap T) + n(N \cap F) - n(T \cap F)$$
$$= n(N \cap T) - n(N \cap T \cap F)$$

$$= 25 + 26 + 26 - 9 - 11 - 8 + 3$$
$$= 52$$



$$\rightarrow (b) n \rightarrow$$
$$n(T \cup F \cup N) = ?$$
$$n(T \cap F \cap N) = n(T) + n(F) + n(N)$$

3 reads all three magazines.

$$= 9 + 3 + 7$$

6 fortune not week  
fortune week  
 $n(T \cap F \cap N) = 8$  reads  
 $\rightarrow 113 - 8 = 105$   
 $\rightarrow 105 - 3 = 102$  readers  
newspaper not newspaper  
newspaper newspaper

$$\begin{array}{r} \text{252} \\ \times 3 \\ \hline \text{756} \end{array}$$

$$\begin{array}{r} 26 - 8 - 3 - 5 \\ \hline = 10 \text{ read only} \\ \hline \rightarrow 26 - 8 - 5 - 3 - 6 = \\ \hline 60 - 52 = 8 \text{ read} \\ \hline (c) 8 + 10 + 12 = 30 \end{array}$$

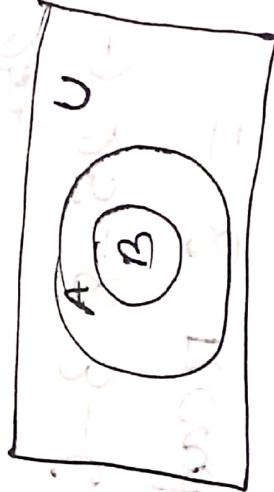
magazine. I had no  
idea + it was  
a surprise.

→ 4 UBC (BIO) 1000



Shade the set

$$\text{if } (A \cap B) \subseteq B \text{ i.e. } (A \cap B) \subseteq A \cup B \text{ i.e. } (A \cap B) \subseteq A$$



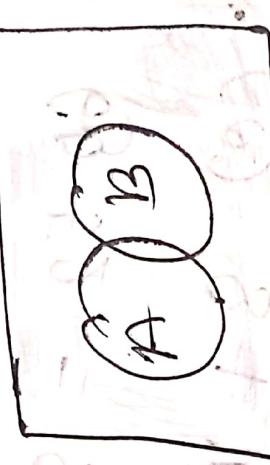
$$(c) A \cap B = A \cap C \text{ and } A \cap B = A \cap C.$$



$$= 2 - 3 + 2 + 8 - 3 + 6$$

area one shaded

$$\Rightarrow (A \cup B) \cap C \text{ is shaded}$$



$\rightarrow A \cap B \cap C$  is shaded.

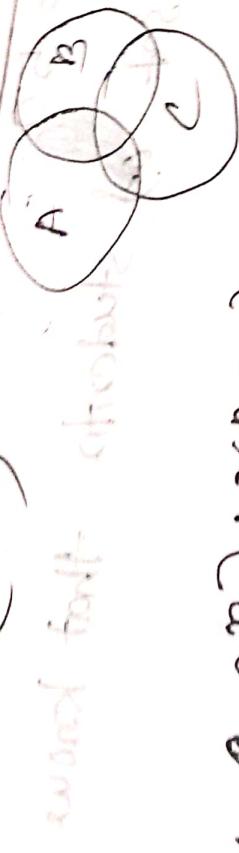


$\rightarrow A \cap B \cap C$  are shaded



Probability for all students who have got A grade

$$A \cap (B \cup C)$$



Probability for all students who have got A grade

and got full marks

$$A \cap (B \cap C)$$

Probability for all students who got full marks

$$A = 100 \quad B = 100 \quad C = 100$$

Probability of A and B Union

more

than 70%

$$P(A \cap B) = \frac{1}{2} \times 100 = 50$$

Probability of A and C Union

$$P(A \cap C) = \frac{1}{2} \times 100 = 50$$

$$P(B \cap C) = \frac{1}{2} \times 100 = 50$$

$$P(A \cap B \cap C) = \frac{1}{2} \times 100 = 50$$

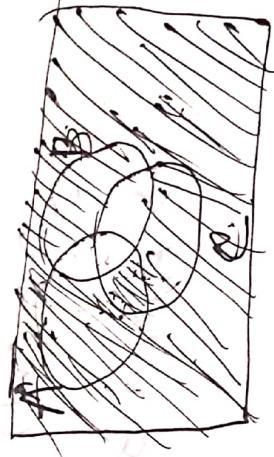
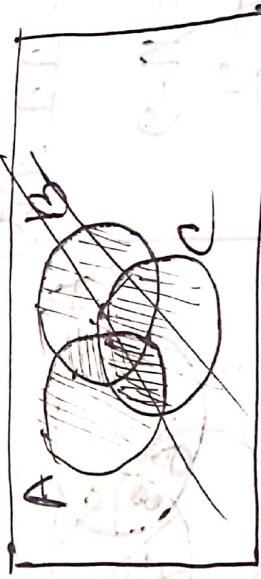
$$P(A \cap (B \cup C)) = 100 - 50 = 50$$

$$P(A \cap (B \cup C)) = 100 - 50 - 50 + 50 = 50$$

$$P(A \cap (B \cup C)) = 100 - 50 - 50 + 50 = 50$$

Probability for all students who have got A grade

(Q-3) shade the set  $(A \cup B) \cap (A \cap C)$



$$\frac{\text{mid}}{(Q-4)}$$

$$n(A) = 15, \quad n(A \cap W) = 5.$$

$$n(B) = 12, \quad n(A \cap B) = 9$$

$$n(W) = 25 - 2 \times 9 = 7, \quad n(B \cap W) = 4.$$

$$n(A \cap B \cap W) = 3.$$

$$n(A \cup B \cup W) = 21.$$

$$n(A \cap \bar{B} \cap \bar{W}) = 2 \text{ (shaded)}.$$

$$n(A \cap B \cup W) = 2.$$

$$n(A \cup B \cup W) = 25 - 2 = 23.$$

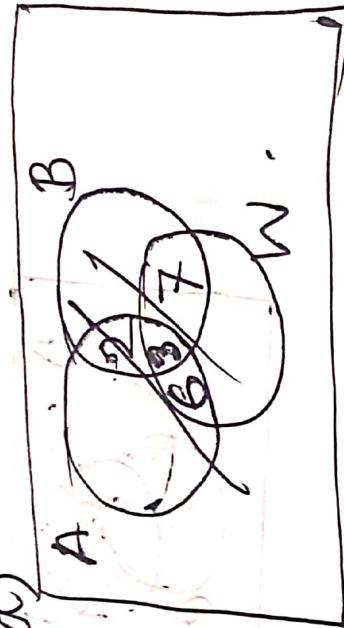
$$\Rightarrow n(A \cup B \cup W) = n(A) + n(B) + n(W) - n(A \cap B) - n(A \cap W) - n(B \cap W) + n(A \cap B \cap W).$$

$$\therefore n(A \cup B \cup W) = 15 + 12 + 7 - 9 - 3 - 4 + 3 = 23.$$

$$\therefore 23 = 15 + 12 + 7 - 3.$$

$\Rightarrow 2(3) - 12 = 6$  (in) with whole (8-10)

$\Rightarrow n(W) = 11 \cdot (3)$   
 $\Rightarrow A \text{ and } B = 11 \cdot 9$



$\rightarrow \text{Bond } W = 11 \cdot 4$

$\Rightarrow 11 \cdot 4 = 44$

$A \text{ and } B = 11 \cdot 5 = 55 = (10) \text{ n}$

$\Rightarrow 11 \cdot 5 = 55 = 15 - 2 - 3 - 6 = (4) \text{ n}$

$\Rightarrow \text{only } B = 12 - 2 - 3 - 7 = \overline{\text{Wg. } g(A)} / 16$   
 $\Rightarrow \text{only window at } 1, 2, \dots, 16$

$\Rightarrow \text{only } B = 12 - 2 - 3 - 7 = \overline{\text{Wg. } g(A)} / 16$

$\Rightarrow A \text{ and } B = 9 - (3) \text{ n}$

$\Rightarrow A \text{ and } B = 6 \text{ n}$   
 $\Rightarrow A \text{ and } B = 6 \text{ n}$

$\Rightarrow A \text{ and } B = 3 \text{ (Wg. } g(A)) \text{ n}$

$\Rightarrow B \text{ and } W = 4 - 3$

$\Rightarrow B \text{ and } W = 1 \text{ n} = 6$

only A =  $15 - 6 - 3 - 2 = 4$ .

only B =  $12 - 6 - 3 - 1 = 2$

only W =  $11 - 2 - 3 - 1 = 5$ .

only power window = 5.

(a) only air  $\rightarrow n(A) = 4$ .

(b) only radio = 2.

only power window not  
radio and power window not  
air condition.

(c) Radio and air condition = 1.

(d) Air and radio = 6.

(e) one of the option.  
one only

(f)

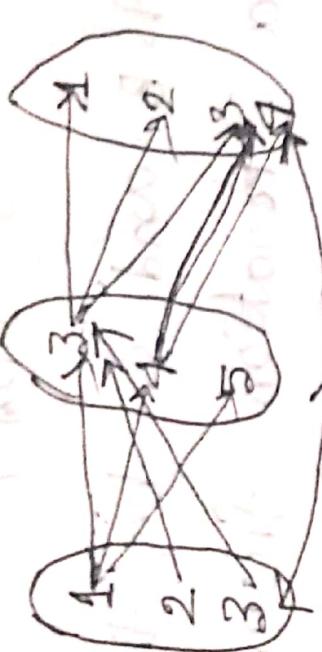
### composition of Relation

Let  $A$ ,  $B$ , and  $C$  be three sets, suppose  $R$  is a relation from  $A$  to  $B$  and  $S$  is a relation from  $B$  to  $C$ .

The ~~(Ans)~~ composite of  $R$  and  $S$ , denoted by  $S \circ R$ , is a binary relation from  $A$  to  $C$  consisting of ordered pairs  $(a, c)$  where  $a \in A$  and  $c \in C$ . Also, prove such that  $(a, b) \in R$  and  $(b, c) \in S \circ R$  such that  $(a, c) \in S$ .

Example: What is the composition of the relations  $R$  and  $S$ , where  $R$  is the relation from set  $A = \{1, 2, 3\}$  to set  $B = \{3, 4, 5\}$  with  $R = \{(1, 3), (1, 4), (1, 5)\}$  and  $S = \{(3, 1), (3, 2), (3, 3), (4, 3), (4, 4)\}$

Solution:

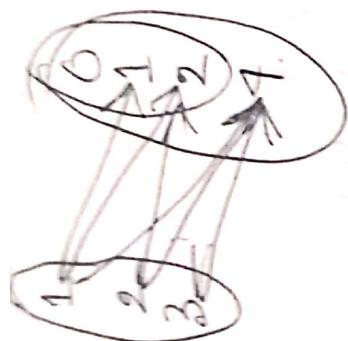


$$S \circ R = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}.$$

Note: If  $R$  and  $S$  are relations from  $A$  to  $B$  and  $A$  to  $C$  respectively then  $S \circ R$  is not defined unless  $B = C$ .

If  $R$  is a relation from  $A$  to  $B$  and  $S$  is a relation from  $B$  to  $C$  then  $S \circ R$  is a relation from  $A$  to  $C$ .  
 $(A, B)$   $\cap (C, D) = \emptyset$  if and only if  $(A, C) \cap (B, D) = \emptyset$

$$\{ (E, E), (E, G), (G, E), (G, G) \}^2 = ?$$



2.1) The ordered elements of the elements in  $(a, b)$  does make difference, here a is designated as the first element and b as second element. Thus  $(a, b) \neq (b, a)$  unless  $a = b$ . On the other hand  $\{a, b\}$  and  $\{b, a\}$  represent the same set.

2.2) If and only if the corresponding elements  $a_1 = b_1, a_2 = b_2 \dots a_n = b_n$

$$2.6(a) A = \{1, 2, 3\} \quad B = \{a, b\}$$

$$A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$$

$$(b) \quad B \times A = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$$

19

$$n(A) = 4 \quad (A) \quad AXB.$$

$$n(B) = 3 \quad m(AXB)$$

$$= 3 \times 2 = 6.$$

(b)

$$n(B \times A) = 12.$$

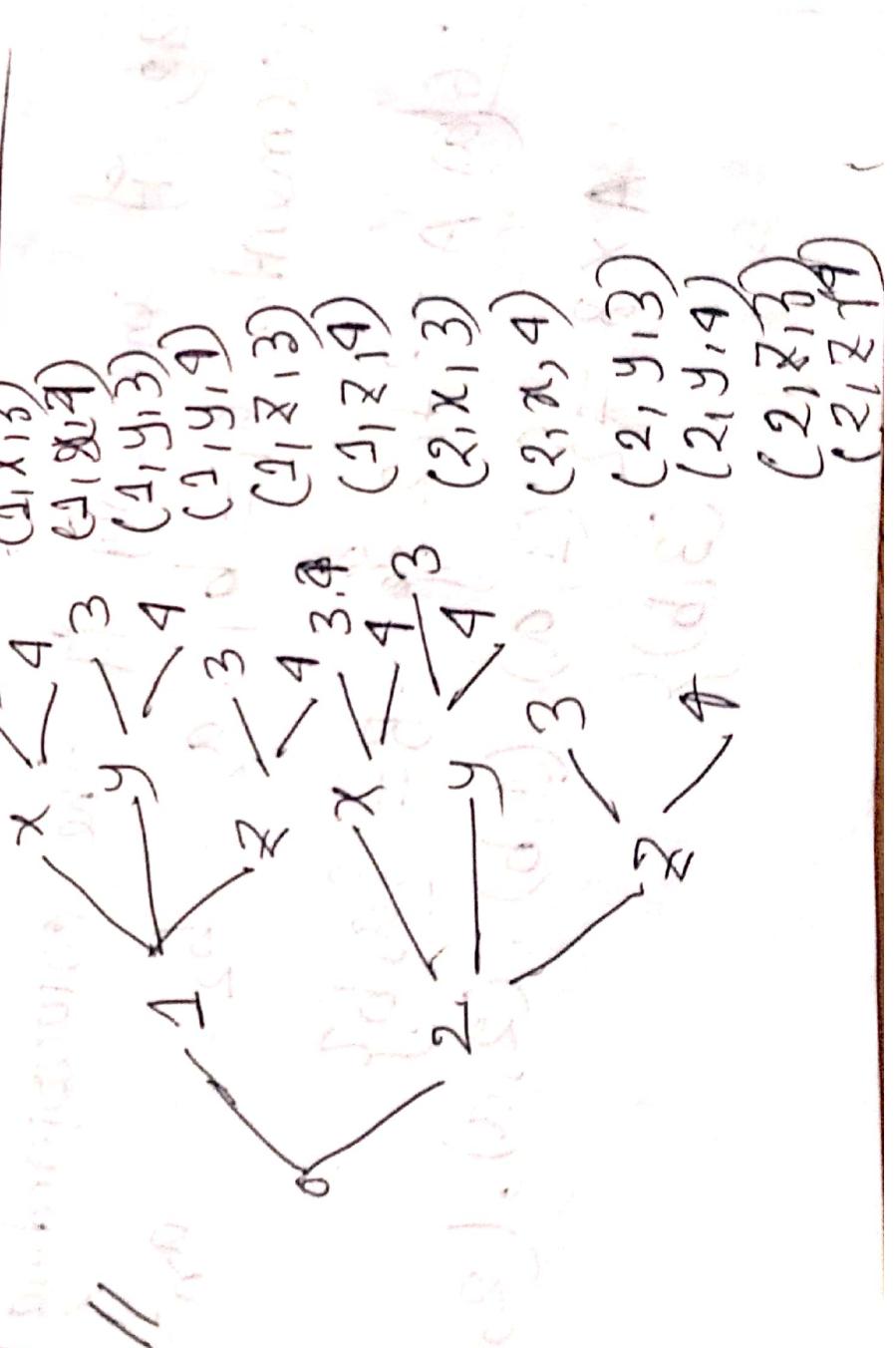
$$(c) \quad n(A^r) = 4, 1 = 16.$$

$$(d) \quad n(B^4) = n(B^4) = 3^4 = 81.$$

$$(e) \quad n(AXAXB) = 4 \cdot 4 \cdot 3 = 48.$$

(f)

$$n(B \times AXB) = 3 \times 4 \times 3 = 36$$



$A \times B \times C$  consists of all ordered triples  $(a, b, c)$ , where  $a \in A$ ,  $b \in B$ ,  $c \in C$ . These elements of  $A \times B \times C$  can be systematically obtained by a so called tree diagram.

$$n(A) = 2, n(B) = 3, n(C) = 2.$$

$$n(A \times B \times C) = 2 \cdot 3 \cdot 2 = 12$$

$$\begin{aligned} & \text{# } (A \times B) \cap (A \times C) : (x, y) \in (A \times B) \text{ and } (x, y) \in (A \times C) \\ & \Rightarrow \{(x, y) : x \in A, y \in B \text{ and } x \in A, y \in C\} \\ & = \{(x, y) : x \in A, y \in B \text{ or } x \in A, y \in C\} \\ & = \{(x, y) : x \in A, y \in B \text{ or } x \in A, y \in C\} \\ & = A \times (B \cup C) \end{aligned}$$

$$\begin{aligned} & \text{# } (A \times B) \cap (A \times C) = A \times (B \cap C) \text{ since } B \cap C \\ & = F \cdot \{(x, y) : (x, y) \in B \cap C\} \\ & = F \cdot \{(x, y) : (x, y) \in B \text{ and } (x, y) \in C\} \\ & = F \cdot \{(x, y) : x \in A, y \in B \text{ and } x \in A, y \in C\} \\ & = F \cdot \{(x, y) : x \in A, y \in B \text{ or } x \in A, y \in C\} \\ & = A \times (B \cup C) \end{aligned}$$

$\{(x, y) : x \in A \text{ and } y \in B \text{ only if}\}$

$= \{(x, y) : x \in A, y \in B \cup e\}$

$= (A \times (B \cup e))$

$$32] \quad 27] \quad A = \{a, b\} \text{ of } , B = \{1, 2\};$$

2) R They are all subset of  $A \times B$ .

$$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$$

3) a) The domain of R consists of

the first elements of the ordered pair . and the range of R. consists of second elements of the ordered pair.

ans. (a)  $A \times B$

b)  $R^{-1}$  is obtained by reversing

the ordered pairs in  $R$ .  
 $R = \{(1, y), (1, z), (3, y), (4, y), (4, z)\}$   
 $R^{-1} = \{(y, 1), (z, 1), (y, 3), (z, 3), (y, 4), (z, 4)\}$

31) Let,  $A = \{1, 2, 3\}$ ,  $R = \{(1, 1), (2, 1), (3, 2), (1, 3)\}$ .

(a)  $1 R 1$ . (b)  $1 \not R 2$  (c)  $2 R 3$  (d)  $2 \not R 4$ ,

(e)  $3 R 2$  (f)  $3 \not R 4$ .

(a) True, Since  $(1, 1) \in R$ .

(b) False, Since  $(1, 2) \notin R$ .

(c) False,

(d) False,  $(2, 1) \in R$ .

(e) True,  $(3, 2) \in R$ .

(f) True.

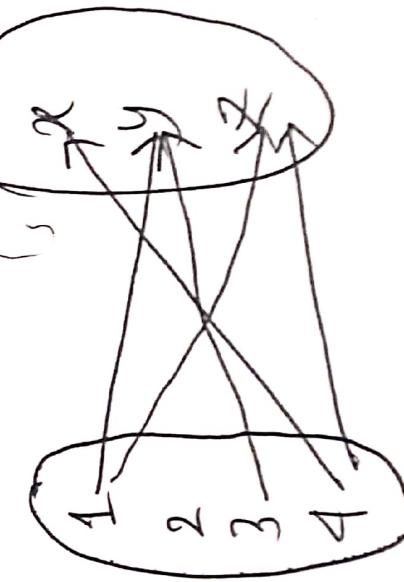
(g)

$B = \{x, y, z\}$ , set  $B = \{x, y, z\}$

$A = \{1, 2, 3, 4\}$ ,  $(4, x), (4, y), (4, z)$

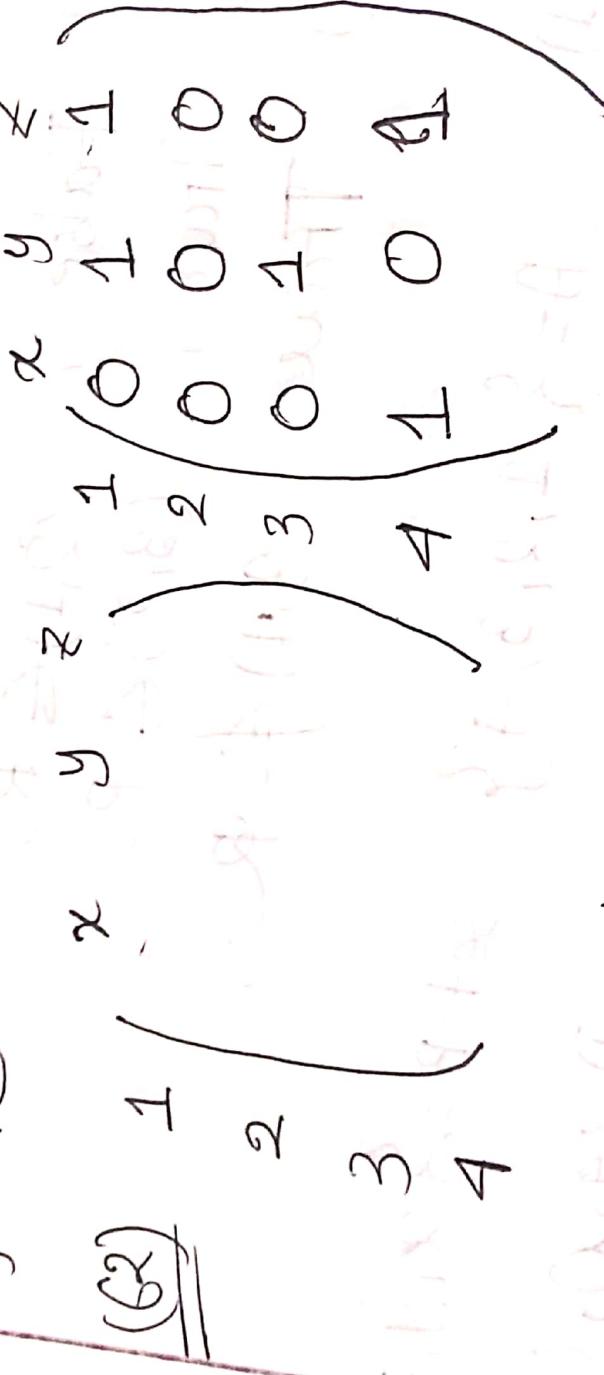
(6A)

$R = \{(1, y), (1, z), (3, y), (3, z)\}$



Write down the elements of A and the elements of B in two columns within two disjoint disks, and then draw an arrow from  $a \in A$  to  $b \in B$  whenever  $a$  is related to  $b$ .  
 $((a, b) \in R)$ , as shown in Fig 2.

The figure is called the arrow diagram of  $R$ .



From a rectangle  $(A, B)$  we get