Homogeneous Differential Equations:

Homogeneous की equation power: every term same.

$$\frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)}$$

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Ex(1) $(x^2+y^2)\,dx + 2xy\,dy = 0$

$$\Rightarrow (x^2+y^2)\,dx = -2xy\,dy$$

$$\Rightarrow -\frac{x^2+y^2}{2xy} = \frac{dy}{dx}$$

$$\Rightarrow y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$-\frac{x^2+y^2}{2xy} = v + x\frac{dv}{dx}$$

$$\Rightarrow -\frac{x^2+v^2x^2}{2x\cdot vx} = v + x\frac{dv}{dx}$$

$$\Rightarrow \frac{-x^2(1+v^2)}{2x^2v} = v + x\frac{dv}{dx}$$

$$\Rightarrow -\frac{1+v^2}{2v} = v + x\frac{dv}{dx}$$

$$\Rightarrow -x\frac{dv}{dx} = v + \frac{1+v^2}{2v}$$

$$\Rightarrow -x\frac{dv}{dx} = \frac{1+3v^2}{2v}$$

$$\Rightarrow -\left(\frac{2v}{1+3v^2}\right)dv = \frac{1}{x}dx$$

$$\Rightarrow -\int \frac{2v}{1+3v^2}dv = \int \frac{1}{x}dx$$

$$\Rightarrow -\frac{1}{3}\log|1+3v^2| + \log c = \log x$$

$$\Rightarrow \log x + \frac{1}{3}\log\left|1+3\frac{y^2}{x^2}\right| = \log c$$

$$\Rightarrow \log x\left(1+\frac{3y^2}{x^2}\right)^{1/3} = \log c$$

$\Rightarrow x\left(1+\frac{3y^2}{x^2}\right)^{\frac{1}{3}} = c$

Ans.

Ex 2 $x^2y\,dx - (x^3+y^3)\,dy = 0$

$\Rightarrow x^2y\,dx = (x^3+y^3)\,dy$

$\Rightarrow \frac{x^2y}{x^3+y^3} = \frac{dy}{dx}$

$\Rightarrow y = vx$

$\Rightarrow \frac{dy}{dx} = 1$

$y = vx$

$\frac{dy}{dv} = v\frac{dx}{dv} + x$

(Ex 2) $x^2y\,dx - (x^3+y^3)dy = 0$

$\Rightarrow x^2y\,dx = (x^3+y^3)dy$

$\Rightarrow \frac{x^2y}{x^3+y^3} = \frac{dy}{dx}$

$\Rightarrow y = vx$

x এর সাপেক্ষে,

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\frac{x^2y}{x^3+y^3} = v + x\frac{dv}{dx}$

$\Rightarrow \frac{x^2v.x}{x^3+v^3x^3} = v + x\frac{dv}{dx}$

$\Rightarrow \frac{v}{1+v^3} = v + x\frac{dv}{dx}$

$\Rightarrow \frac{v - v(1+v^3)}{1+v^3} = x\frac{dv}{dx}$

$\Rightarrow \frac{v - v - v^4}{1+v^3} = x\frac{dv}{dx}$

$$\Rightarrow \frac{-v^4}{1+v^3} = x \cdot \frac{dv}{dx}$$

$$\Rightarrow \frac{1}{x}dx = -\frac{1+v^3}{v^4}dv$$

$$\Rightarrow \int \frac{1}{x}dx = -\int \frac{1+v^3}{v^4}dv$$

$$\Rightarrow \log x = -\left[\int \frac{1}{v^4}dv + \int \frac{1}{v}dv\right]$$

$$\Rightarrow \log x = \frac{1}{3}\frac{1}{v^3} + \log v + \log c$$

$$\Rightarrow \log vx = \frac{1}{3v^3} + \log c$$

$$\Rightarrow \log y = \frac{x^3}{3y^3} + \log c$$

(Ans)

(Ex 4) $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$

$\Rightarrow y^2 = (xy - x^2)\frac{dy}{dx}$

$\Rightarrow \frac{y^2}{xy - x^2} = \frac{dy}{dx}$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\frac{y^2}{xy - x^2} = v + x\frac{dv}{dx}$

$\Rightarrow \frac{v^2x^2}{x^2v - x^2} = v + x\frac{dv}{dx}$

$\Rightarrow \frac{v^2}{v - 1} = v + x\frac{dv}{dx}$

$\Rightarrow \frac{v^2 - v^2 + v}{v - 1} = x\frac{dv}{dx}$

$\Rightarrow \frac{v}{v - 1} = x\frac{dv}{dx}$

$\Rightarrow \frac{1}{x}dx = \frac{v - 1}{v}dv$

$\Rightarrow \frac{1}{x}dx = 1\,dv - \frac{1}{v}dv$

$\Rightarrow \int \frac{1}{x}dx = \int 1\,dv - \int \frac{1}{v}dv$

$\Rightarrow \log x = v - \log v + \log c$

$\Rightarrow \log xv = v + \log c$

$\Rightarrow \log y = \frac{y}{x} + \log c$ (Ans.

$\Rightarrow$

Ex 5 $(x^2+y^2)dy = xy\,dx$

$$\frac{dy}{dx} = \frac{xy}{x^2+y^2}$$

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\frac{xy}{x^2+y^2} = v + x\frac{dv}{dx}$$

$$\frac{vx^2}{x^2+v^2x^2} = v + x\frac{dv}{dx}$$

$$\frac{v}{1+v^2} = v + x\frac{dv}{dx}$$

$$\frac{v - v(1+v^2)}{1+v^2} = x\frac{dv}{dx}$$

$$\frac{-v^3}{1+v^2} = x\frac{dv}{dx}$$

$$\frac{1}{x}dx = \frac{1+v^2}{-v^3}dv$$

$$\int \frac{1}{x}dx = -\int \frac{1}{v^3}dv - \int \frac{1}{v}dv$$

$\Rightarrow \log x = -\frac{1}{2} \cdot \frac{1}{v^2} - \log v + \log c$

$\Rightarrow \log xv = -\frac{1}{2v^2} + \log c$

$\Rightarrow \log y = -\frac{1}{2v^2} + \log c$

$\Rightarrow$ (Ans)

(vi) $\sin^{-1}\left(\frac{dy}{dx}\right) = x+y$

$x+y = z$

$1 + \frac{dy}{dx} = \frac{dz}{dx}$; $\frac{dy}{dx} = \frac{dz}{dx} - 1$

$\sin z = \frac{dz}{dx} - 1$

$\therefore \sin z + 1 = \frac{dz}{dx}$

$\Rightarrow \frac{1}{\sin z + 1} \, dz = dx$

$\Rightarrow \int \frac{1 - \sin^2 z}{1 - \sin^2 z} \, dz = \int dx$

$\Rightarrow \int \frac{1 - \sin z}{\cos^2 z} \, dz = \int dx$

$\Rightarrow \int \sec^2 z \, dz - \int \sec z \cdot \tan z \, dz = \int dx$

$= \tan z - \sec z = x + c$

$= \tan(x+y) - \sec(x+y) = x + c$

(xiv) $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$

$\Rightarrow y^2 = (xy - x^2) \frac{dy}{dx}$

$\Rightarrow \frac{y^2}{xy - x^2} = \frac{dy}{dx}$

$\Rightarrow y = vx$

$\frac{dy}{dx} = v + x \frac{dv}{dx}$

$v + x \frac{dv}{dx} = \frac{v^2 x^2}{vx^2 - x^2}$

$\Rightarrow v + x \frac{dv}{dx} = \frac{v^2}{v-1}$

$\Rightarrow x \frac{dv}{dx} = \frac{v^2}{v-1} - v$

$\Rightarrow x \frac{dv}{dx} = \frac{v^2 - v^2 + v}{v-1}$

$\Rightarrow x \frac{dv}{dx} = \frac{v}{v-1}$

$\Rightarrow \frac{v-1}{v} \quad \frac{v}{v-1}$

$\Rightarrow \frac{v-1}{v} dv = \frac{1}{x} dx$

$$\int 1\,dv - \int \frac{1}{v}\,dv = \int \frac{1}{x}\,dx$$

$$\Rightarrow v - \ln v = \ln x + c$$

$$\Rightarrow v - c = \ln x + \ln v$$

$$\Rightarrow v - c = \ln y$$

$$\Rightarrow \frac{y}{x} - c = \ln y$$

7. $(e^y + 1)\cos x\,dx + e^y \sin x\,dy = 0$

$$(e^y + 1)\cos x\,dx = -e^y \sin x\,dy$$

$$\Rightarrow \frac{e^y + 1}{e^y}\,dy = -\frac{\sin x}{\cos x}\,dy$$

$$\Rightarrow \int \frac{e^y}{e^y + 1}\,dy = -\int \cot x\,dx$$

$$\ln|e^y + 1| = -\ln|\sin x| + c$$

$$\sin x\,(e^y + 1) = c$$

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

$$\frac{dy}{dx} = e^x \cdot e^{-y} + x^2 \cdot e^{-y}$$

$$\frac{dy}{dx} = e^{-y}(e^x + x^2)$$

$$\Rightarrow \int e^y \, dy = \int (e^x + x^2) \, dx$$

$$\Rightarrow e^y = e^x + \frac{x^3}{3} + c$$