14. Apply Huygens principle to derive the relation

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

for a thin lens. (*Mysore 1990*)

15. State and explain Huygens principle of secondary waves. Apply this principle for explaining the simultaneous reflection and refraction of a plane light wave from a plane surface of separation of two optical media. [*Delhi 1984 ; Delhi (Hons.) 1984*]

16. Explain Huygens principle of wave propagation and apply it to prove the laws of reflection of a plane wave at a plane surface. [*Delhi B.Sc.(Hons.) 1991*]

17. State the principle of superposition. Give the mathematical theory of interference between two waves of amplitude a_1 and a_2 with phase difference ϕ. Discuss some typical cases. [*Rajasthan 1985*]

18. Deduce the laws of reflection with the help of Huygens theory of secondary wavelets. (*Rajasthan 1985*)

19. What is Huygens principle ? How would you explain the phenomenon of reflection and refraction of plane waves at plane surfaces on the basis of wave nature of light ? [*Delhi (Sub.) 1986*]

20. State and explain Huygens principle of secondary waves. (*Delhi 1988*)

21. State and explain Huygens principle of secondary waves. [*Delhi ; 1992*]

"The phenomenon of redistribution of light waves or energy due to the superposition of light waves from two or more coherent sources is known as interference".

Superposition of waves: when two or more waves overlap, the resultant displacement at any point and at any instant may be found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.

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INTERFERENCE

8.1 INTRODUCTION

The phenomenon of interference of light has proved the validity of the wave theory of light. Thomas Young successfully demonstrated his experiment on interference of light in 1802. When two or more wave trains act simultaneously on any particle in a medium, the displacement

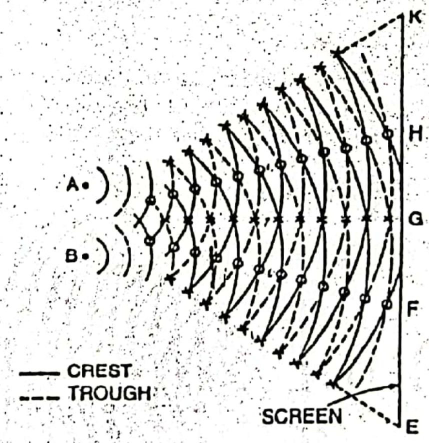

Fig. 8.1

of the particle at any instant is due to the superposition of all the wave trains. Also, after the superposition, at the region of cross over, the wave trains emerge as if they have not interfered at all. Each wave train retains its individual characteristics. Each wave train behaves as if others are absent. This principle was explained by Huygens in 1678.

The phenomenon of interference of light is due to the superposition of two trains within the region of cross over. Let us consider the waves produced on the surface of water. In Fig. 8.1 points A and B are the two sources which produce waves of equal amplitude and constant phase difference. Waves spread out on the surface of water which are circular in shape. At any instant, the particle will be under the action of the displacement due to both the waves. The points shown by circles in the diagram will have minimum displacement because the crest of one wave falls on the trough of the other and the resultant displacement is zero. The points shown by crosses in the diagram will have maximum displacement because, either the crest of one will combine with the crest of the other or the trough of one will combine with the trough of the other. In such a case, the amplitude of the displacement is twice the amplitude of either of the waves. Therefore, at these points the waves **reinforce** with each other. As the intensity (energy) is directly proportional to the square of the amplitude ($I \propto A^2$) the intensity at these points is four times the intensity due to one wave. It should be remembered that **there is no loss of energy due to interference.** The energy is only transferred from the points of minimum displacement to the points of maximum displacement.

8.2 YOUNG'S EXPERIMENT

In the year 1802, Young demonstrated the experiment on the interference of light. He allowed sunlight to fall on a pinhole S and then at some distance away on two pinholes A and B (Fig. 8.2).

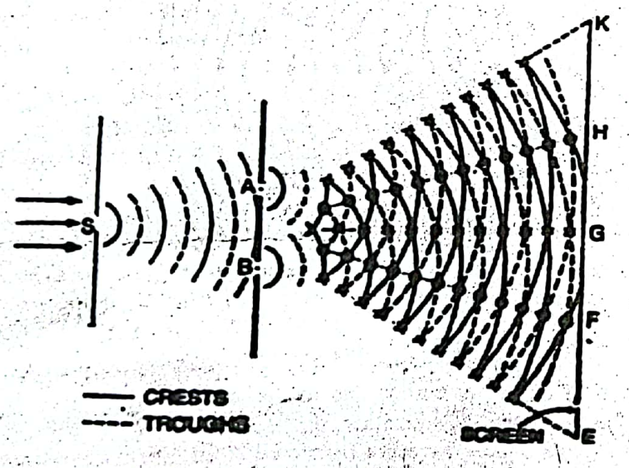

Fig. 8.2

A and B are equidistant from S and are close to each other. Spherical waves spread out from S. Spherical waves also spread out from A and B. These waves are of the same amplitude and wavelength. On the screen interference bands are produced which are alternatively dark and bright. The points such as E are bright because the crest due to one wave coincides with the crest due to the other and therefore they reinforce with each other. The points such as F are dark because the crest of one falls on the trough of the other and they neutralize the effect of each other. Points, similar to E, where the trough of one falls on the trough of the other, are also bright because the two waves reinforce.

It is not possible to show interference due to two independent sources of light, because a large number of difficulties are involved. The two sources may emit light waves of largely different amplitude and wavelength and the phase difference between the two may change with time.

8.3 COHERENT SOURCES

Two sources are said to be coherent if they emit light waves of the same frequency, nearly the same amplitude and are always in phase with each other. It means that the two sources must emit radiations of the same colour (wavelength). In actual practice it is not possible to have two independent sources which are coherent. But for experimental purposes, two virtual sources formed from a single source can act as coherent sources. Methods have been devised where (*i*) interference of light takes place between the waves from the real source and a virtual source (*ii*) interference of light takes place between waves from two sources formed due to a single source. In all such cases, the two sources will act, as if they are perfectly similar in all respects.

Since the wavelength of light waves is extremely small (of the order of 10^{-5} cm), the two sources must be narrow and must also be close to each other. Maximum intensity is observed at a point where the phase difference between the two waves reaching the point is a whole number multiple of 2π or the path difference between the two waves is a whole number multiple of wavelength. For minimum intensity at a point, the phase difference between the two waves reaching the point should be an odd number multiple of π or the path difference between the two waves should be an odd number multiple of half wavelength.

8.4 PHASE DIFFERENCE AND PATH DIFFERENCE

If the path difference between the two waves is λ, the phase difference $= 2\pi$.

Suppose for a path difference x, the phase difference is δ

For a path difference λ, the phase difference $= 2\pi$

$\therefore$ For a path difference x, the phase difference $= \frac{2\pi x}{\lambda}$

Phase difference $\quad \delta = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda} \times$ (pathdifference)

8.5 ANALYTICAL TREATMENT OF INTERFERENCE

Consider a monochromatic source of light S emitting waves of wavelength λ and two narrow pinholes A and B (Fig. 8.3). A and B are equidistant from S and act as two virtual coherent sources. Let a be the amplitude of the waves. The phase difference between the two waves reaching the point P, at any instant, is δ.

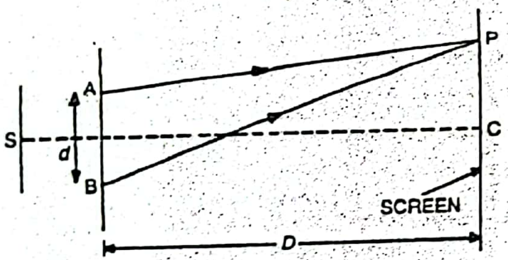

Fig. 8.3

If y_1 and y_2 are the displacements

$$y_1 = a \sin \omega t$$

$$y_2 = a \sin (\omega t + \delta)$$

$$\therefore \quad y = y_1 + y_2 = a \sin \omega t + a \sin (\omega t + \delta)$$

$$y = a \sin \omega t + a \sin \omega t \cos \delta + a \cos \omega t \sin \delta$$

$$= a \sin \omega t (1 + \cos \delta) + a \cos \omega t \sin \delta.$$

Taking $\quad a(1 + \cos \delta) = R \cos \theta \quad$ ...(i)

and $\quad a \sin \delta = R \sin \theta \quad$ ...(ii)

$$y = R \sin \omega t \cos \theta + R \cos \omega t \sin \theta$$

$$y = R \sin (\omega t + \theta) \quad \text{...(iii)}$$

which represents the equation of simple harmonic vibration of amplitude R.

Squaring (i) and (ii) and adding,

$$R^2 \sin^2 \theta + R^2 \cos^2 \theta = a^2 \sin^2 \delta + a^2 (1 + \cos \delta)^2$$

or
$$R^2 = a^2 \sin^2 \delta + a^2 (1 + \cos^2 \delta + 2 \cos \delta)$$

$$R^2 = a^2 \sin^2 \delta + a^2 + a^2 \cos^2 \delta + 2a^2 \cos \delta$$

$$= 2a^2 + 2a^2 \cos \delta = 2a^2 (1 + \cos \delta)$$

$$R^2 = 2a^2 . 2 \cos^2 \frac{\delta}{2} = 4a^2 \cos^2 \frac{\delta}{2}$$

The intensity at a point is given by the square of the amplitude

$$\therefore \quad I = R^2$$

or
$$I = 4a^2 \cos^2 \frac{\delta}{2} \quad \text{...(iv)}$$

Special cases : (i) When the phase difference $\delta = 0, 2\pi, 2(2\pi), \ldots n(2\pi)$, or the path difference $x = 0, \lambda, 2\lambda, \ldots n\lambda$;

$$I = 4a^2$$

Intensity is maximum when the phase difference is a whole number multiple of 2π or the path difference is a whole number multiple of wavelength.

(ii) When the phase difference, $\delta = \pi, 3\pi, \ldots (2n + 1)\pi$, or the path difference $x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots (2n + 1)\frac{\lambda}{2}$,

$$I = 0$$

Intensity is minimum when the path difference is an odd number multiple of half wavelength.

(iii) **Energy distribution.** From equation (iv), it is fount that the intensity at bright points is $4a^2$ and at dark points it is zero. According to

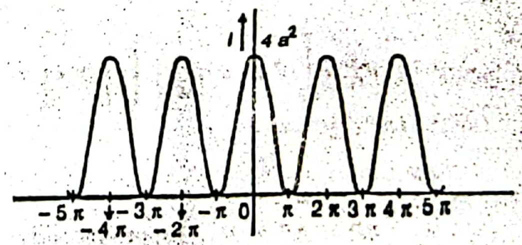

Fig. 8.4

the law of conservation of energy, the energy cannot be destroyed. Here also the energy is not destroyed but only transferred from the points of minimum intensity to the points of maximum intensity. For, at bright points, the intensity due to the two waves should be $2a^2$ but actually it is $4a^2$. As shown in Fig. 8.4 the intensity varies from 0 to $4a^2$, and the average is still $2a^2$. It is equal to the uniform intensity $2a^2$ which will be present in the absence of the interference phenomenon due to the two waves. Therefore, the formation of interference fringes is in accordance with the law of conservation of energy.

8.6 THEORY OF INTERFERENCE FRINGES

Consider a narrow monochromatic source S and two pinholes A and B, equidistant from S. A and B act as two coherent sources separated by a distance d. Let a screen be placed at a distance D from the coherent

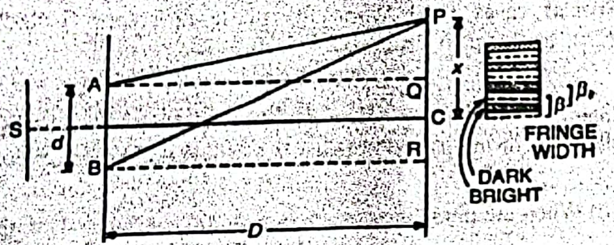

Fig. 8.5

sources. The point C on the screen is equidistant from A and B. Therefore, the path difference between the two waves is zero. Thus, the point C has maximum intensity.

Consider a point P at a distance x from C. The waves reach at the point P from A and B.

Here, $PQ = x - \frac{d}{2}$, $PR = x + \frac{d}{2}$

$$(BP)^2 - (AP)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

$$(BP)^2 - (AP)^2 = 2xd$$

$$BP - AP = \frac{2xd}{BP + AP}$$

But $BP = AP = D$ (approximately)

$$\therefore \text{ Path difference } = BP - AP = \frac{2xd}{2D} = \frac{xd}{D} \quad \text{...(i)}$$

$$\text{Phase difference } = \frac{2\pi}{\lambda}\left(\frac{xd}{D}\right) \quad \text{...(ii)}$$

(i) **Bright fringes.** If the path difference is a whole number multiple of wavelength λ, the point P is bright.

$$\therefore \quad \frac{xd}{D} = n\lambda$$

where $n = 0, 1, 2, 3 \ldots$

$$\text{or} \quad x = \frac{n\lambda D}{d} \quad \text{...(iii)}$$

This equation gives the distances of the bright fringes from the point C. At C, the path difference is zero and a bright fringe is formed.

When $n = 1$, $x_1 = \frac{\lambda D}{d}$

$n = 2$, $x_2 = \frac{2\lambda D}{d}$

$n = 3$, $x_3 = \frac{3\lambda D}{d}$

$x_n = \frac{n\lambda D}{d}$

Therefore the distance between any two consecutive bright fringes

$$x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d} = \frac{\lambda D}{d} \quad \text{...(iv)}$$

(ii) **Dark fringes.** If the path difference is an odd number multiple of half wavelength, the point P is dark.

$$\frac{xd}{D} = (2n+1)\frac{\lambda}{2} \quad \text{where } n = 0, 1, 2, 3 \ldots$$

$$\text{or} \quad x = \frac{(2n+1)\lambda D}{2d} \quad \text{...(v)}$$

This equation gives the distances of the dark fringes from the point C.

When, $n = 0$, $x_0 = \frac{\lambda D}{2d}$

$n = 1$, $x_1 = \frac{3\lambda D}{2d}$

$n = 2$, $x_2 = \frac{5\lambda D}{2d}$

and $x_n = \frac{(2n+1)\lambda D}{2d}$

The distance between any two consecutive dark fringes,

$$x_2 - x_1' = \frac{5\lambda D}{2d} - \frac{3\lambda D}{2d} = \frac{\lambda D}{d} \quad ...(vi)$$

The distance between any two consecutive bright or dark fringes is known as fringe width. Therefore, alternately bright and dark parallel fringes are formed. The fringes are formed on both sides of C. Moreover, from equations (v) and (vi), it is clear that the width of the bright fringe is equal to the width of the dark fringe. All the fringes are equal in width and are independent of the order of the fringe. The breadth of a bright or a dark fringe is, however, equal to half the fringe width and is equal to $\frac{\lambda D}{2d}$. The fringe width $\beta = \frac{\lambda D}{d}$.

Therefore, (i) the width of the fringe is directly proportional to the wavelength of light, $\beta \propto \lambda$. (ii) The width of the fringe is directly proportional to the distance of the screen from the two sources, $\beta \propto D$. (iii) the width of the fringe is inversely proportional to the distance between the two sources, $\beta \propto \frac{1}{d}$. Thus, the width of the fringe increases (a) with increase in wavelength (b) with increase in the distance D and (c) by bringing the two sources A and B close to each other.

Example 8.1. *Green light of wavelength 5100 Å from a narrow slit is incident on a double slit. If the overall separation of 10 fringes on a screen 200 cm away is 2 cm, find the slit separation.*

[Delhi B.Sc. (Hons.)]

$$\beta = \frac{\lambda D}{d}$$

Here $\lambda = 5100 \times 10^{-8}$ cm, $d = ?$

$D = 200$ cm

$10\,\beta = 2$ cm

or $\beta = 0.2$ cm

$$d = \frac{\lambda D}{\beta}$$

$$d = \frac{5100 \times 10^{-8} \times 200}{0.2}$$

$d = 0.051$ cm

Example 8.2. *Two coherent sources are 0.18 mm apart and the fringes are observed on a screen 80 cm away. It is found that with a certain monochromatic source of light, the fourth bright fringe is situated at a distance of 10.8 mm from the central fringe. Calculate the wavelength of light.*

Here, $D = 80$ cm, $d = 0.18$ mm $= 0.018$ cm

$n = 4$, $x = 10.8$ mm $= 1.08$ cm, $\lambda = ?$

$$x = \frac{n\lambda D}{d}$$

or
$$\lambda = \frac{xd}{nD} = \frac{1.08 \times 0.018}{4 \times 80} = 6075 \times 10^{-8} \text{ cm}$$

$= \mathbf{6075\ Å}$

Example 8.3. *In Young's double slit experiment the separation of the slits is 1.9 mm and the fringe spacing is 0.31 mm at a distance of 1 metre from the slits. Calculate the wavelength of light.*

Here $\beta = 0.31$ mm $= 0.031$ cm

$d = 1.9$ mm $= 0.19$ cm

$D = 1$ m $= 100$ cm

$$\beta = \frac{\lambda D}{d}$$

or
$$\lambda = \frac{\beta d}{D}$$

$$\lambda = \frac{0.031 \times 0.19}{100}$$

$\lambda = 5890 \times 10^{-8}$ cm $= \mathbf{5890\ Å}$

Example 8.4. *Two straight and narrow parallel slits 1 mm apart are illuminated by monochromatic light. Fringes formed on the screen held at a distance of 100 cm from the slits are 0.50 mm apart. What is the wavelength of light ?*

[Delhi 1977]

Here $\beta = 0.50$ mm $= 0.05$ cm

$d = 1$ mm $= 0.1$ cm

$D = 100$ cm

$$\beta = \frac{\lambda D}{d}$$

or
$$\lambda = \frac{\beta d}{D}$$

$$\lambda = \frac{0.05 \times 0.1}{100}$$

$\lambda = 5 \times 10^{-5}$ cm

$\lambda = \mathbf{5000\ Å}$

Example 8.5. *A Young's double slit experiment is arranged such that the distance between the centers of the two slits is d and the source slit, emitting light of wavelength λ, is placed at a distance x from the double slit. If now the source slit is gradually opened up, for what width will the fringes first disappear?* [Delhi (Hons) 1992]

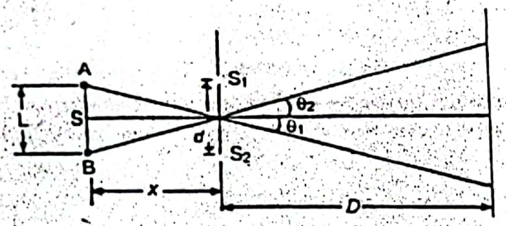

Fig. 8.6

A and B are two extreme points of the source S separated by distance L.

Here $\theta_1 = -\left(\frac{L}{2x}\right)$ when $x >> L$

$$\theta_2 = \left(\frac{L}{2x}\right)$$

The fringe pattern first disappears when the central maximum of one pattern overlaps on the first minimum of the second pattern. The first minimum occurs at a distance given by

$$y = \pm \frac{\lambda D}{2d}$$

Also $\frac{y}{D} = \theta = \pm \frac{\lambda}{2d}$

For source A, these minima occur at an angle

$$\theta_1 \pm \frac{\lambda}{2d}$$

The fringe width is very large when d is very small. As d increases, the first minimum of S_1, moves towards the zeroeth maximum of S_2. These two meet when $d = d_0$.

Here $\theta_2 = \theta_1 + \frac{\lambda}{2d_0}$

or $\frac{L}{2x} = -\frac{L}{2x} + \frac{\lambda}{2d_0}$

to the slit S_1 [Fig. 10.1(b)].

$$d_0 = \left(\frac{\lambda x}{2L}\right)$$

$$\therefore \quad L = \left[\frac{\lambda x}{2d_0}\right]$$

Example 8.6. *A light source emits light of two wavelength* $\lambda_1 = 4300$ Å *and* $\lambda_2 = 5100$ Å. *The source is used in a double slit interference experiment. The distance between the sources and the screen is 1.5 m and the distance between the slits is 0.025 mm. Calculate the separation between the third order bright fringes due to these two wavelengths.*

Here
$$D = 1.5\,\text{m}$$
$$d = 0.025\ \text{mm} = 25 \times 10^{-6}\ \text{m}$$
$$\lambda_1 = 4300\ \text{Å} = 4.3 \times 10^{-7}\ \text{m}$$
$$\lambda_2 = 5100\ \text{Å} = 5.1 \times 10^{-7}\ \text{m}$$
$$n = 3$$
$$x_1 = \frac{n\lambda_1 D}{d}$$
$$x_2 = \frac{n\lambda_2 D}{d}$$
$$x_2 - x_1 = \left(\frac{n\lambda_2 D}{d}\right) - \left(\frac{n\lambda_1 D}{d}\right)$$
$$= \frac{nD}{d}\left[\lambda_2 - \lambda_1\right]$$
$$= \left(\frac{3 \times 1.5}{25 \times 10^{-6}}\right)\left[5.1 \times 10^{-7} - 4.3 \times 10^{-7}\right]$$
$$= \mathbf{0.0144\ m}$$
$$= \mathbf{1.44\ cm}$$

Hence, the separation between the two fringes is 1.44 cm.

Example 8.7. *Two coherent sources of monochromatic light of wave length 6000 Å produce an interference pattern on a screen kept at a distance of 1 m from them. The distance between two consecutive bright fringes on the screen is 0.5 mm. Find the distance between the two coherent sources.* [IAS

Here
$$\lambda = 6000\ \text{Å} = 6 \times 10^{-7}\ \text{m}$$
$$D = 1\ \text{m}$$
$$\beta = 0.5\ \text{mm} = 5 \times 10^{-4}\ \text{m}$$

$$d = ?$$

$$\beta = \frac{\lambda D}{d}$$

$$d = \frac{\lambda D}{\beta}$$

$$d = \frac{6 \times 10^{-7} \times 1}{5 \times 10^{-4}}$$

$$d = 1.2 \times 10^{-3} \text{ m}$$

$$d = 1.2 \text{ mm}$$

Example 8.8. *Light of wavelength 5500 Å from a narrow slit is incident on a double slit. The overall separation of 5 fringes on a screen 200 cm away is 1 cm, calculate (a) the slit separation and (b) the fringe width.*

Here $x = \frac{n \lambda D}{d}$

$$n = 5$$

$$D = 200 \text{ cm} = 2 \text{ m}$$

$$\lambda = 5500 \text{ Å} = 5.5 \times 10^{-7} \text{ m}$$

$$x = 1 \text{ cm} = 10^{-2} \text{ m}$$

$$d = ?$$

(a) $d = \frac{n \lambda D}{x}$

$$d = \frac{5 \times 5.5 \times 10^{-7} \times 2}{10^{-2}}$$

$$d = 5.5 \times 10^{-4} \text{ m}$$

$$d = 0.055 \text{ cm}$$

(b) $\beta = \frac{x}{n}$

$$\beta = \frac{1}{5} \text{ cm}$$

$$\beta = 0.2 \text{ cm}$$

8.7 FRESNEL'S MIRRORS

Fresnel produced the interference fringes by using two plane mirrors M_1 and M_2 arranged at an angle of nearly 180° so that their surfaces are nearly (not exactly) coplanar (Fig. 8.7).

A monochromatic source of light S is used. The pencil of light from S incident on the two mirrors, after reflection appears to come from two virtual sources A and B at some distance d apart. Therefore, A and B act

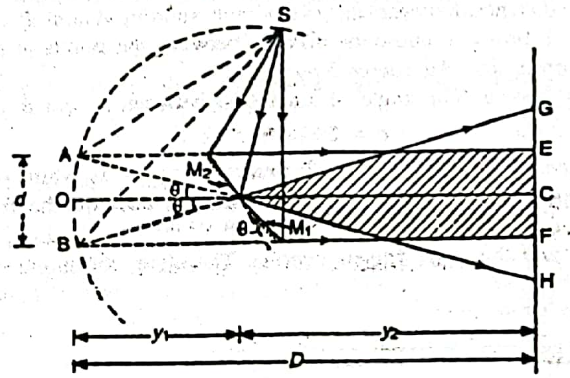

Fig. 8.7

as two virtual coherent sources and interference fringes are obtained on the screen. These fringes are of equal width and are alternately dark and bright.

Theory. A and B are two coherent sources at a distance d apart. The screen is at a distance D from the virtual sources. The two reflected beams from the mirrors M_1 and M_2 overlap between E and F (shown as shaded in the diagram) and interference fringes are formed.

(For complete theory read Article 8.6)

Here, $D = Y_1 + Y_2$

Fringe width $\beta = \frac{\lambda D}{d}$

A point on the screen will be at the centre of a bright fringe, if its distance from C is $\frac{n \lambda D}{d}$ where n = 0, 1, 2, 3 *etc*, and it will be at the centre of a dark fringe, if its distance from C is

$$\frac{(2n+1) \lambda D}{2d}$$

where n = 0, 1, 2, 3, . . . *etc*.

For the fringes to be formed, the following conditions must be satisfied. The two mirrors M_1 and M_2 should be made from optically flat glass and silvered on the front surfaces. No reflection should take place from