Quantitative Management Final Exam

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The objective of this assignment is to define, formulate and solve a mathematical optimization model of a given problem.

Problem Definition:

An Instructor has been assigned with a task to form 4 groups among 12 students for a group project. The aim is to distribute/ mix the students such a way that each group can have the best chance to succeed. Each of the four groups should have exactly 3 students.

Below are the factors/attributes of the students accountable for mixing them into different groups.

- GPA of the Student (scale of 4.0)
- Relevant work Experience (In years)
- Computer Programming skills level (in a scale of 10)

Let us assume that below are the three factor values for the 12 students.

Students	GPA	Relevant Exp.	Programming Skill level
St1	3.5	10	6
St2	4	2	5
St3	3.7	5	2
St4	3.1	7	7
St5	3.8	9	5
St6	3.2	6	6
St7	3.6	7	7
St8	3	12	6
St9	3.3	7	8
St10	3.7	9	9
St11	4	6	8
St12	3.5	4	4

Above 3 factors have positive impacts on success of the group project.

Factor descriptions:

- **GPA**: Average GPA till date, reflects the sincerity of the student and eventually will be focused on the successful completion of the group project. It also reflects how good are they in the graduate standing. It is the most important factors of the success of the project as it reflects the quality of the student.
- **Relevant Experience**: Having relevant Experience improves the chances of the project success and helps in adding practicality in the project solutions. Also, prior experience adds the attribute of better communication and collaboration within the group work.
- **Programming Skills**: All technological projects need programming skills. Thus, having programming skills adds better chance of success to the project.

Now the question is, how the above factors combine to define success?

As our target is to make all the group successful. Thus, the sum of a factor value in a group should be equal or greater than the average value of the factor of all the students (12). This restriction (constraint) is applicable for all three factors. These three sets of constraints (each factor will have constraints for 4 group of students) will control the evenly distributions of all qualities of students in the four groups and will make sure success of each of the groups.

As each of the factors positively impacts the group's performance, we will be trying to maximize the values of the factors for the success of the group. Out of three factors "Grade point" is the primary factor to lead the group's success. Thus, GPA will be used in the objective function to maximize it values.

Formulation:

The problem statement leads to the formulation of "Assignment Problems" in linear programming methods, as the requirement is to assign people to tasks. The students have to be assigned into four groups. Thus, decision variable values will be either 0 (zero) or 1 (one). That makes the linear programming as an integer programming model.

Below is the list of **decision variables** based on the Group and Student (Let's assume Students are numbered from 1 to 12 as S1 to S12 and Groups from G1 to G4). For Example, G1S1 represents Student 1 presence in Group 1. Rest will be of similar representations. Now, the decision variable values of 1 represents the student and group combination is valid and 0 represents that the student does not belong to that group.

Student	Group1	Group2	Group3	Group4
St1	G1S1	G2S1	G3S1	G4S1
St2	G1S2	G2S2	G3S2	G4S2
St3	G1S3	G2S3	G3S3	G4S3
St4	G1S4	G2S4	G3S4	G4S4
St5	G1S5	G2S5	G3S5	G4S5
St6	G1S6	G2S6	G3S6	G4S6
St7	G1S7	G2S7	G3S7	G4S7
St8	G1S8	G2S8	G3S8	G4S8
St9	G1S9	G2S9	G3S9	G4S9
St10	G1S10	G2S10	G3S10	G4S10
St11	G1S11	G2S11	G3S11	G4S11
St12	G1S12	G2S12	G3S12	G4S12

We are trying to find the assignment of the Students into a group in such a way that it maximizes the probability of having greater GPA of the students in the group.

As there will be certain conditions need to put in place to formulate the linear programming to give the optimum values. Below are the **constraints** for formulating the linear programming.

1> Decision variable values will greater than or equals to zero (>=0). This is the boundary constraint.
2> Each group must have exact 3 students.
3> One student can belong to only one group.
4> As we want to distribute similar mix of students (of their qualities) in each of the groups, average factor values across three factors should be maintained at the minimum average level for all four groups.

Students	GPA	Relevant Exp.	Programming Skill level
St1	3.5	10	6
St2	4	2	5
St3	3.7	5	2
St4	3.1	7	7
St5	3.8	9	5
St6	3.2	6	6
St7	3.6	7	7
St8	3	12	6
St9	3.3	7	8
St10	3.7	9	9
St11	4	6	8
St12	3.5	4	4
Average Value of the Factors	**3.53**	**7.00**	**6.08**

Linear Programming Solution Method:

Below are the LP modeling details.

/* Objective function */

max: +3.5 G1S1 +4 G1S2 +3.7 G1S3 +3.1 G1S4 +3.8 G1S5 +3.2 G1S6 +3.6 G1S7 +3 G1S8 +3.3 G1S9 +3.7 G1S10 +4 G1S11

+3.5 G1S12 +3.5 G2S1 +4 G2S2 +3.7 G2S3 +3.1 G2S4 +3.8 G2S5 +3.2 G2S6 +3.6 G2S7 +3 G2S8 +3.3 G2S9 +3.7 G2S10

+4 G2S11 +3.5 G2S12 +3.5 G3S1 +4 G3S2 +3.7 G3S3 +3.1 G3S4 +3.8 G3S5 +3.2 G3S6 +3.6 G3S7 +3 G3S8 +3.3 G3S9

+3.7 G3S10 +4 G3S11 +3.5 G3S12 +3.5 G4S1 +4 G4S2 +3.7 G4S3 +3.1 G4S4 +3.8 G4S5 +3.2 G4S6 +3.6 G4S7 +3 G4S8

+3.3 G4S9 +3.7 G4S10 +4 G4S11 +3.5 G4S12;

/* Constraints */

Group1Students: +G1S1 +G1S2 +G1S3 +G1S4 +G1S5 +G1S6 +G1S7 +G1S8 +G1S9 +G1S10 +G1S11 +G1S12 = 3;

Group2Students: +G2S1 +G2S2 +G2S3 +G2S4 +G2S5 +G2S6 +G2S7 +G2S8 +G2S9 +G2S10 +G2S11 +G2S12 = 3;

Group3Students: +G3S1 +G3S2 +G3S3 +G3S4 +G3S5 +G3S6 +G3S7 +G3S8 +G3S9 +G3S10 +G3S11 +G3S12 = 3;

Group4Students: +G4S1 +G4S2 +G4S3 +G4S4 +G4S5 +G4S6 +G4S7 +G4S8 +G4S9 +G4S10 +G4S11 +G4S12 = 3;

Student1: +G1S1 +G2S1 +G3S1 +G4S1 = 1;

Student2: +G1S2 +G2S2 +G3S2 +G4S2 = 1;

Student3: +G1S3 +G2S3 +G3S3 +G4S3 = 1;

Student4: +G1S4 +G2S4 +G3S4 +G4S4 = 1;

Student5: +G1S5 +G2S5 +G3S5 +G4S5 = 1;

Student6: +G1S6 +G2S6 +G3S6 +G4S6 = 1;

Student7: +G1S7 +G2S7 +G3S7 +G4S7 = 1;

Student8: +G1S8 +G2S8 +G3S8 +G4S8 = 1;

Student9: +G1S9 +G2S9 +G3S9 +G4S9 = 1;

Student10: +G1S10 +G2S8 +G3S8 +G4S10 = 1;

Student11: +G1S11 +G2S11 +G3S11 +G4S11 = 1;

Student12: +G1S12 +G2S12 +G3S12 +G4S12 = 1;

Group1Factor1: +3.5 G1S1 +4 G1S2 +3.7 G1S3 +3.1 G1S4 +3.8 G1S5 +3.2 G1S6 +3.6 G1S7 +3 G1S8 +3.3 G1S9 +3.7 G1S10 +4 G1S11

+3.5 G1S12 >= 3.53;

Group2Factor1: +3.5 G2S1 +4 G2S2 +3.7 G2S3 +3.1 G2S4 +3.8 G2S5 +3.2 G2S6 +3.6 G2S7 +3 G2S8 +3.3 G2S9 +3.7 G2S10 +4 G2S11

+3.5 G2S12 >= 3.53;

Group3Factor1: +3.5 G3S1 +4 G3S2 +3.7 G3S3 +3.1 G3S4 +3.8 G3S5 +3.2 G3S6 +3.6 G3S7 +3 G3S8 +3.3 G3S9 +3.7 G3S10 +4 G3S11

+3.5 G3S12 >= 3.53;

Group4Factor1: +3.5 G4S1 +4 G4S2 +3.7 G4S3 +3.1 G4S4 +3.8 G4S5 +3.2 G4S6 +3.6 G4S7 +3 G4S8 +3.3 G4S9 +3.7 G4S10 +4 G4S11

+3.5 G4S12 >= 3.53;

Group1Factor2: +10 G1S1 +2 G1S2 +5 G1S3 +7 G1S4 +9 G1S5 +6 G1S6 +7 G1S7 +12 G1S8 +7 G1S9 +9 G1S10 +6 G1S11 +4 G1S12 >= 7;

Group2Factor2: +10 G2S1 +2 G2S2 +5 G2S3 +7 G2S4 +9 G2S5 +6 G2S6 +7 G2S7 +12 G2S8 +7 G2S9 +9 G2S10 +6 G2S11 +4 G2S12 >= 7;

Group3Factor2: +10 G3S1 +2 G3S2 +5 G3S3 +7 G3S4 +9 G3S5 +6 G3S6 +7 G3S7 +12 G3S8 +7 G3S9 +9 G3S10 +6 G3S11 +4 G3S12 >= 7;

Group4Factor2: +10 G4S1 +2 G4S2 +5 G4S3 +7 G4S4 +9 G4S5 +6 G4S6 +7 G4S7 +12 G4S8 +7 G4S9 +9 G4S10 +6 G4S11 +4 G4S12 >= 7;

Group1Factor3: +6 G1S1 +5 G1S2 +2 G1S3 +7 G1S4 +5 G1S5 +6 G1S6 +7 G1S7 +6 G1S8 +8 G1S9 +9 G1S10 +8 G1S11 +4 G1S12 >= 6.08;

Group2Factor3: +6 G2S1 +5 G2S2 +2 G2S3 +7 G2S4 +5 G2S5 +6 G2S6 +7 G2S7 +6 G2S8 +8 G2S9 +9 G2S10 +8 G2S11 +4 G2S12 >= 6.08;

Group3Factor3: +6 G3S1 +5 G3S2 +2 G3S3 +7 G3S4 +5 G3S5 +6 G3S6 +7 G3S7 +6 G3S8 +8 G3S9 +9 G3S10 +8 G3S11 +4 G3S12 >= 6.08;

Group4Factor3: +6 G4S1 +5 G4S2 +2 G4S3 +7 G4S4 +5 G4S5 +6 G4S6 +7 G4S7 +6 G4S8 +8 G4S9 +9 G4S10 +8 G4S11 +4 G4S12 >= 6.08;

/* Integer definitions */

int
G1S1,G1S2,G1S3,G1S4,G1S5,G1S6,G1S7,G1S8,G1S9,G1S10,G1S11,G1S12,G2S1,G2S2,G2S3,G2S4,G2S5,G2S6,G2S7,G2S8,G2S9,G2S10,G2S11,G2S12,G3S1,G3S2,G3S3,G3S4,G3S5,G3S6,G3S7,G3S8,G3S9,G3S10,G3S11,G3S12,G4S1,G4S2,G4S3,G4S4,G4S5,G4S6,G4S7,G4S8,G4S9,G4S10,G4S11,G4S12;

Optimal solution:

Based on the factor values of the students the Linear programming model provides solutions as

Maximized Objective function value: 42.4 This is the sum of GPA values of all the students.

Decision variable values: Below are the values of 48 decision variables which are the combinations of Groups and the Students.

[1] 1 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 1 0 0 0 1 0 1 0 0 0 0 1 0 0 1 0 0 0 0 1 0 1 1 0 0 1 0 0 0 0 0 0

Based the Linear Programming solution the optimal distribution of the students will be:

Groups	Students
Group-1	Student-1
	Student-8
	Student-10
Group-2	Student-5
	Student-9
	Student-11
Group-3	Student-4
	Student-7
	Student-12
Group-4	Student-2
	Student-3
	Student-6

Constraints: The Constraints values are

[1] 3.0 3.0 3.0 3.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 10.2 11.1 10.2 10.9 31.0 22.0 18.0 13.0 21.0 21.0 18.0 13.0

- The first 4 values represent the total number of students in each four groups. Value 3 represents that all group has exact 3 students.
- Values from 4 to 16 represents the Students assigned in number of groups. Value 1 represents that each student is assigned to only one group.
- Values from 17 to 28 represents total values of each factors corresponding to the groups.

Conclusion:

For any assignment problem where assignees are being assigned to perform tasks, can be solved using linear programming. Above problem is one of the examples of the assignment problem. The mathematical model for assignment problem uses the decision variables as Xij = 1 (if assignee i performs task j) or 0 (if not). Thus, each Xij is a binary variable (it has value 0 or 1). This eventually makes it a binary integer programming. Binary integer programming (BIP) is widely used to aid in these decisions. Investment Analysis, Site selection, Dispatching shipments, scheduling Interrelated activities, Airline applications are one of the areas where BIP is used for Optimization.

Above stated student assignment problem has been dealt using Binary Integer Programing. The solution explains that the number of decision variables will increase if the students or the group counts gets increased. All combinations of students and groups needs to feed in the objective functions to get the optimal combination. For the twelve students and four groups, BIP is able to successfully derive the student assignments to the appropriate groups to make the optimum distribution of students into the groups.