

# Interstellar scintillation of compact extragalactic radio sources

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## ABSTRACT

The recent discovery of radio variability of a quasar on short time-scales (hours) prompts us to examine what is expected in respect of the interstellar scintillation of very compact, extragalactic radio sources. We find that large-amplitude, rapid, variability is predicted at commonly observed radio frequencies (1–20 GHz) over the vast majority of the extragalactic sky. As a guide to assist observers in understanding their data, we demonstrate simple techniques for predicting the effects of interstellar scintillation on any extragalactic source.

Key words: ISM: structure – quasars: general – radio continuum: galaxies.

## 1 INTRODUCTION

It is important to understand the short time-scale variations of quasars if we are to make progress in comprehending the nature of their emissions. In particular, intraday variability (IDV) is a well-established phenomenon (see the review by Wagner & Witzel 1995) which, if intrinsic, conflicts squarely with an interpretation in terms of synchrotron emission. However, it is well known (Rickett 1990, and references therein) that compact radio sources can appear to vary as a result of scintillation, particularly in consequence of the small-scale inhomogeneities in the ionized component of the interstellar medium (ISM). It is therefore crucial to understand the role played by interstellar scintillation when interpreting observations of radio galaxies and quasars.

Kedziora-Chudczer et al. (1997) recently discovered a truly extreme example of IDV in the quasar PKS 0405 – 385, in which deep modulations are impressed on a time-scale as short as 2 h. For this source it was concluded that the variations were caused by interstellar scintillation, with the primary requirement of the source being that it contain a component of angular size  $\lesssim 5 \times 10^{-6}$  arcsec. Rickett et al. (1995) have suggested an angular size  $\sim 7 \times 10^{-5}$  arcsec for the quasar 0917 + 624, again making the inference on the basis of a scintillation model. These cases highlight the possibility that some quasars may contain components of very small angular dimension for which, correspondingly, interstellar scintillation will be very important. Where variability is due to scintillation there is presumably no implication that the emission is anything other than synchrotron – certainly this is the case for both PKS 0405 – 385 and 0917 + 624.

In this paper we use a simple model of the ISM (Taylor & Cordes 1993, hereafter TC93) – which is based primarily on observations of pulsars – together with the simplest possible source model – a point source model – to outline the expected scintillation behaviour of compact, extragalactic radio sources. The main purpose is to provide a ready means by which observers may assess the likely effects of scintillation on their own data. Our method and principal results are described in the next section; Section 3 shows how to use these results, including two worked examples, while Section 4 provides some comment on related issues.

## 2 A MODEL FOR INTERSTELLAR SCINTILLATION

The dispersion measure ( $DM = \int n_e ds$ ) is one of the most straightforwardly measured quantities for any pulsar and serves as an approximate gauge of its distance. In order to estimate pulsar distances as accurately as possible, it is necessary to have a good model for the Galactic distribution of ionized ISM; this task is one of the principal functions of the TC93 model. In addition the model accounts, approximately, for the overall distribution of the small-scale inhomogeneities in the ISM. Hence TC93 can be used to predict the scattering of radio waves by the ISM for any line-of-sight, and we have used it for precisely this purpose. Incorporated in the TC93 model are (1) two axisymmetric (disc) components of ionized ISM, with differing scale heights and radial scales, and (2) components corresponding to Galactic spiral arms. For extragalactic point sources there are no free parameters to consider, and the model returns unique predictions for the scattering on any line of sight. Although we

shall use exclusively the TC93 model, we note that in reality the ISM is more complex in its structure than could realistically be reproduced with *any* model. It is therefore essential to regard the results we shall present not as firm predictions, but rather as a guide to the likely circumstance, on any line of sight.

There are several ways in which radio-wave scattering can be characterized. Typically a power-law spectrum of inhomogeneities is assumed to exist everywhere, with the normalization being determined by the structure constant,  $C_n$ . The structure constant measures the local scattering properties of the ISM, while the scattering measure ( $SM = \int C_n^2 ds$ ) describes the cumulative effect of these inhomogeneities along the line of sight. These conventions are followed by TC93. The scattering measure can be readily connected with observable quantities via standard results (e.g. Rickett 1990; Taylor & Cordes 1993). We will also make use of the description set out by Narayan (1992) in which the scintillation behaviour is parametrized in terms of the ‘scattering strength’,  $\xi$ , of the equivalent phase screen.

The scattering strength has a simple physical interpretation:  $\xi = 1$  corresponds to the circumstance where the ISM inhomogeneities introduce substantial phase changes, of order half a radian, across the first Fresnel zone. At this point, then, the phase changes due to structure in the ISM are comparable to those attributable to purely geometric (i.e. path-length) variations. In weak scattering there are only small phase changes introduced by the ISM over the first Fresnel zone, and  $\xi < 1$ . By contrast, in strong scattering the wavefront is highly corrugated on scales smaller than the first Fresnel zone, and correspondingly  $\xi > 1$ . These two regimes exhibit qualitatively different behaviour in all measurable quantities.

It is straightforward to merge this description with the quantities utilized by TC93, giving the following numerical result for  $\xi$ :

$$\xi = 2.6 \times 10^3 SM^{0.6} D^{0.5} \nu^{-1.7}. \quad (1)$$

Here  $SM$  takes the unit  $\text{kpc m}^{-20/3}$ , which is appropriate for inhomogeneities arising from a Kolmogorov spectrum of turbulence. The observing frequency,  $\nu$ , is in GHz, and  $D$  is the distance of the equivalent phase screen, in kpc. We have adopted the following definition for  $D$ :

$$D \equiv \frac{1}{SM} \int C_n^2 ds, \quad (2)$$

which is straightforward to calculate, given the TC93 model. Also of interest here is the angular size of the first Fresnel zone, which is given (in microarcsec) by

$$\theta_F = 8/\sqrt{D\nu}, \quad (3)$$

and the corresponding time-scale for traversing this zone (in seconds):

$$t_F = 1.2 \times 10^6 \nu^{-1} \sqrt{D/\nu}, \quad (4)$$

where  $\nu$  ( $\text{km s}^{-1}$ ) is the transverse speed of the phase corrugations, relative to the line of sight to the source. In contrast to the circumstance with pulsars, where the transverse speed of the source is often the primary contribution to  $\nu$ , we should expect this quantity to reflect the intrinsic changes occurring within the ISM when we are observing

extragalactic sources. A sensible value to adopt is  $\nu = 50 \text{ km s}^{-1}$  (Rickett et al. 1995).

For the simple model we are considering – i.e. extragalactic point sources – the three quantities  $\xi$ ,  $\theta_F$  and  $t_F$  suffice to determine all the quantities of interest. Rather than quote  $\xi$  at a particular frequency, however, it makes sense to give the transition frequency,  $\nu_0$ , at which  $\xi = 1$ , as this separates two physically distinct regions of behaviour (i.e. weak and strong scattering). It is then straightforward to infer the expected scintillation behaviour in either strong or weak regimes, as will be seen later. Note that quantities evaluated at the transition (i.e. at  $\xi = 1$ ) will be labelled with the subscript ‘0’, e.g.  $t_{F0}$ .

It is especially valuable to focus on  $\xi = 1$  because it is at this point that the broad-band scintillations are greatest, with rms fractional flux variations being of order unity. This point is often not well appreciated, as the (correct) notion that scattering is stronger at low frequencies often leads to the (incorrect) idea that the flux modulations will necessarily be greatest at low frequencies. Undoubtedly, part of the reason for this common misconception is that there is not a great quantity of (radio astronomical) data which display the peak in modulation index as a function of frequency. In turn, this is mainly because pulsars are almost exclusively studied at low frequencies ( $< 1 \text{ GHz}$ ), where the scintillation is very strong. (Pulsars typically have very steep spectra.) On the other hand, sources which have flat spectra, and so are amenable to study over a broad range of radio frequencies, tend to be much less compact than pulsars, and scintillation is correspondingly less influential in determining their apparent properties.

The transition frequency,  $\nu_0$ , is displayed in Fig. 1 in the form of a contour plot in galactic coordinates. At the Galactic poles its value is close to 4 GHz, increasing monotonically towards the Galactic plane. The bulging of contours around the Galactic Centre and antiCentre is attributable to the proximity of spiral arms in these directions (cf. fig. 4 of TC93). It can be seen from Fig. 1 that *for a very large portion of the extragalactic sky the transition occurs in or near one of*

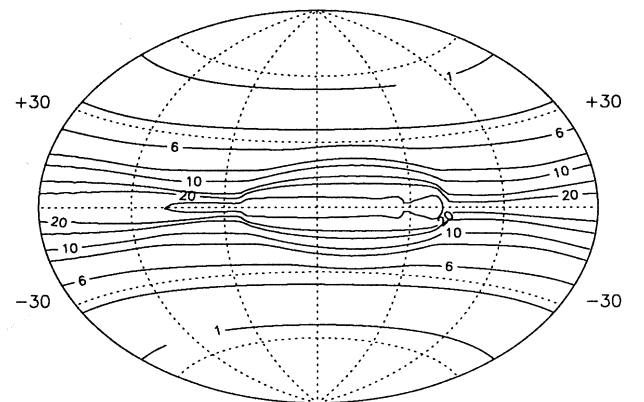


Figure 1. The transition frequency,  $\nu_0$  – i.e., the frequency at which the scattering strength is unity – for extragalactic sources, plotted in galactic coordinates. The dashed lines mark intervals of  $30^\circ/60^\circ$  in latitude/longitude, respectively, while the solid lines show contours of constant transition frequency, with levels 4, 5, 6, 8, 10, 15, 20 and 40 GHz.

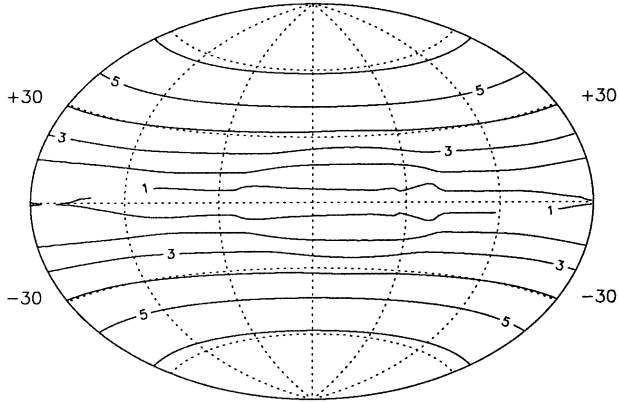


Figure 2. Angular size limits,  $\theta_{F0}$ , for extragalactic sources, at an observing frequency equal to the transition frequency, plotted in galactic coordinates. Sources (or components thereof) smaller than the appropriate limit can be approximated as point sources, and will show deep modulations in their received flux. Contours are plotted a 1-microarcsec intervals.

the commonly observed bands at 5, 8 or 15 GHz. In consequence of this fact, together with the recognition that quasars can contain extremely compact components, it is important to be aware of the possible role which scattering might play when interpreting observations of extragalactic sources – especially at high radio frequencies.

In Fig. 2 we plot the angular size of the first Fresnel zone at an observing frequency equal to the transition frequency. The largest Fresnel zones correspond to high galactic latitudes (6.1 microarcsec near the poles), with submicroarcsec values obtaining near the Galactic plane. These dimensions form approximate upper limits on source size for which deep modulations can arise from interstellar scintillation. (Rough scaling laws for larger sources are given by Narayan 1992, and are reproduced in Section 3 of this paper.) These limits are very small in comparison with the  $\sim$ milliarc-second dimensions one usually imagines for quasar cores: the discovery of rapid scintillation of quasars will, in the longer term, allow us to refine our concept of the structure of their emission regions (see the discussion in Section 4).

We have also computed the characteristic time-scale of the scintillations at the transition frequency. These values are not reproduced here as there is very little variation of this quantity over the sky, being everywhere in the range 1–3 h. To understand this we need only recognize that the transition frequency and screen distance vary in opposite senses, so their ratio (equation 4) varies relatively little. In fact, we can quite trivially solve equation (1) for the transition frequency and insert this into equation (4), in order to reveal the dependencies more explicitly; taking  $v = 50 \text{ km s}^{-1}$ , one finds

$$t_{F0} = 2.4 \times 10^3 D^{6/17} / \text{SM}^{3/17}. \quad (5)$$

If  $C_n^2$  is reasonably uniform in the ISM – which is a fair approximation locally for the TC93 model, and therefore good for lines of sight away from the Galactic plane – then  $D \propto \text{SM}$ , and we recognize a very weak functional dependence of  $t_{F0}$  on the actual line of sight considered. We will henceforth make use of the approximation  $t_{F0} \sim 2 \text{ h}$  for all

lines of sight to the extragalactic sky.

### 3 SCALING AND USAGE OF RESULTS

In practice, one needs to know how to connect the results presented in Section 2 with data on quasars and radio galaxies taken at various different frequencies. For a thorough exposition we refer the reader to Narayan (1992), but for convenience we give a summary in this section; practical examples are also given. Readers should be aware that the results we shall quote here are asymptotic results and therefore are strictly correct only in the limits  $\xi \gg 1$  (strong scattering) or  $\xi \ll 1$  (weak scattering). The frequency dependence of the scattering strength is given by equation (1), whence  $\xi = (v_0/v)^{17/10}$ .

#### 3.1 Weak scattering

In weak scattering we expect the following dependencies to hold for a point source:

$$m_p = \xi^{5/6} = \left( \frac{v_0}{v} \right)^{17/12} \quad (6)$$

is the modulation index (i.e., the rms fractional flux variation), while the size of the first Fresnel zone is just

$$\theta_F = \theta_{F0} \sqrt{v_0/v}, \quad (7)$$

and the corresponding time-scale is given in hours by

$$t_F \sim 2 \sqrt{v_0/v}. \quad (8)$$

Evidently, if the source of interest contains only a fraction of its flux in a compact component then, while equations (7) and (8) remain valid, the observed modulation index will be reduced by just this fraction. (Similar remarks pertain to scintillations in the strong-scattering regime.)

Because extragalactic sources often emit their radiation over a logarithmically broad range of length-scales, the point source approximation is a description which will often fail in practice. In this case the observed modulation index is always smaller than for a point source, while the time-scale for variability is, of course, always longer. (These remarks hold also for strong scattering.) For a source of size  $\theta_s$  the results are:

$$m = m_p \left( \frac{\theta_F}{\theta_s} \right)^{7/6}, \quad (9)$$

for the modulation index, and

$$t = t_F \frac{\theta_s}{\theta_F} \quad (10)$$

for the variability time-scale.

#### 3.2 Strong scattering

For  $\xi \gg 1$  two types of variability are expected for point sources – slow, broad-band changes, and fast, narrow-band variations. The former effects can be understood in terms of ray-optics and correspond to lens-like phenomena, hence the term ‘refractive scintillation’. The narrow-band pheno-

mena, in contrast, are interference effects and are commonly referred to as ‘diffractive scintillation’.

### 3.2.1 Refractive scintillation

For refractive scintillation the point source variations have the following properties:

$$m_p = \xi^{-1/3} = \left(\frac{v}{v_0}\right)^{17/30} \quad (11)$$

is the modulation index; the scattering disc is much larger than the first Fresnel zone, being

$$\theta_r = \theta_F \xi = \theta_{F0} \left(\frac{v_0}{v}\right)^{11/5}; \quad (12)$$

and the refractive time-scale is correspondingly longer, being given (in hours) by

$$t_r \sim 2 \left(\frac{v_0}{v}\right)^{11/5}. \quad (13)$$

Again, where the point source approximation fails (for  $\theta_s > \theta_r$  in this case) the modulation index is reduced by a factor  $(\theta_r/\theta_s)^{7/6}$ , while the time-scale for variability simply increases as  $\theta_s/\theta_r$ .

### 3.2.2 Diffractive scintillation

Because diffractive scintillation is an interference phenomenon, and the phase changes are much greater than 1 rad (in the strong-scattering regime), the modulation index is unity for a point source:

$$m_p = 1. \quad (14)$$

It must be remembered, however, that this modulation is *not* broad-band: it manifests itself in the form of interference fringes having a characteristic frequency scale  $\Delta\nu$ , such that

$$\frac{\Delta\nu}{\nu} = \xi^{-2} = \left(\frac{v}{v_0}\right)^{17/5}. \quad (15)$$

It is therefore necessary to observe with frequency resolution of  $\Delta\nu$ , or better, in order to be sensitive to diffractive scintillations.

The angular size of relevance here is the dimension on which phase changes of order 1 rad are introduced into the wavefront – i.e., the scale of the wavefront corrugations – this is

$$\theta_d = \theta_F \xi^{-1} = \theta_{F0} \left(\frac{v}{v_0}\right)^{6/5}. \quad (16)$$

(Notice that  $\theta_d = \theta_r \xi^{-2}$ , so that the diffractive and refractive scales diverge rapidly – as  $v^{22/5}$  – below the transition frequency.) Correspondingly, the diffractive time-scale is typically very short:

$$t_d = t_F \xi^{-1} \sim 2 \left(\frac{v}{v_0}\right)^{6/5} \text{ h}. \quad (17)$$

For a source with  $\theta_d < \theta_s$  the modulation index is reduced

to  $(\theta_d/\theta_s)$ , while the time-scale for variations increases by a factor  $\theta_s/\theta_d$ . Interestingly, there are no recorded examples of diffraction scintillation of extragalactic sources, and this is presumably a result of the circumstance  $\theta_d < \theta_s$  (see Section 4).

### 3.3 Two examples

Suppose we make observations at 15 GHz of a quasar located at  $l=180^\circ$ ,  $b=24^\circ$ , with the result that variations of 20 per cent rms are seen on a time-scale  $\sim 1$  h; can these variations be explained by scintillation? To proceed, we first determine the transition frequency – using Fig. 1 we see that  $v_0=6$  GHz; evidently this quasar is in the weak-scattering regime at 15 GHz. In this case we compute the modulation index from equation (6):  $m_p=0.27$ , so the depth of the modulation can indeed be explained, provided that essentially the whole source scintillates. In turn, this requires that all of the flux at 15 GHz originates from a source of dimension less than the first Fresnel zone at this frequency. Fig. 2 gives the size of the first Fresnel zone at the transition frequency as roughly 4 microarcsec, whence equation (7) yields a source size limit of 2.5 microarcsec at 15 GHz. Similarly equation (8) predicts a variability time-scale of around 75 min. In this way we see that the overall properties of the variations observed are broadly consistent with interstellar scintillation of a source of dimension  $\theta_s < 2.5$  microarcsec.

As a second example, consider a 5 per cent variation at 1.4 GHz occurring on a time-scale of 10 h, for a quasar at  $l=180^\circ$ ,  $b=5^\circ$ ; could this be scintillation? In this case the transition frequency is roughly 20 GHz, so at 1.4 GHz the scattering is very strong. In this case  $m_p=0.22$  (refractive variations; equation 11), which is easily sufficient to explain the observed modulation depth. However, the variability time-scale is proportional to  $v^{-11/5}$  in refractive scintillation (equation 13), so we should expect to see variations having a time-scale of 30 d, not 10 h. By contrast, the diffractive time-scale is short – too short, in fact, as we expect  $t_d \sim 5$  min (equation 17). For an extended source the diffractive time-scale would be longer, but if we increase the source dimension to  $\theta_s \approx \theta_d \times (10 \text{ h}/5 \text{ min})$ , in order to match the observed time-scale, then the implied modulation index would fall from unity to  $m \sim 8 \times 10^{-3}$ . Moreover, although the observing bandwidth is not given in this example, most configurations for continuum observations would fail to resolve the scintillation bandwidth (equation 15) of  $\Delta\nu \approx 160$  kHz, thereby further reducing the observable diffraction modulation. Hence the variations cannot be explained as interstellar scintillation in this case.

When assessing the role of scintillation from the results we have given, and in connection with real data, it is important to admit a broad envelope for what is held to be consistent between predictions and data. This is primarily because the tremendous complexity of the ISM is necessarily rendered simplistically in the TC93 model.

## 4 DISCUSSION

As remarked in Section 3.1, the point source model is unlikely to offer a good description of the interstellar scintillation of quasars. We emphasize in particular that real

(extragalactic) sources will often contain structure on scales *spanning* the characteristic angular scales of the phase screen, and furthermore the source structure may be very different indeed in different observing bands. Despite these complexities the point source model retains its relevance as a limiting case – the actual modulation index cannot be larger, and the time-scale cannot be shorter than would be the case for a point-like source. In this context the results we have presented offer a simple point of reference, which we believe will be of practical use. Readers interested in developing more accurate models of the scintillation of real extragalactic sources are referred to Rickett et al. (1995), which is the most sophisticated example of such modelling to date.

Perhaps the most important points to bear in mind, in connection with non-point-like sources, are the following: the radio ‘core’ flux, as measured with an interferometer, may well contain a substantial contribution from structures which are sufficiently large that they do not scintillate, in addition to a small, scintillating component – in this case the apparent modulation depth will be reduced relative to the point source model, even if the compact component on its own satisfies the point source requirement. In general, the presence of source structure changes the frequency dependence of all observable quantities and, for example, the observed peak in modulation index might occur at a frequency quite different from the transition frequency.

In the absence of any direct imaging data on the angular scales of interest here, it would be useful to pursue some theoretical models of the structure of compact radio cores. More specifically, models with very few free parameters are essential in practice. A profitable line of investigation may be to model the core as a power-law extrapolation (interpolation?) of the jets seen on larger scales in many sources. We note, in particular, that at least one well-studied source can be modelled as scale-free over the whole observed range (3C 120; Walker, Benson & Unwin 1987). Obviously, the source cannot remain scale-free on dimensions comparable with the origin of the jets. It is, however, likely that the scale-free assumption will fail at much larger radii, set by the transition between optically thin and optically thick regimes for the source – the relevant opacity being synchrotron self-absorption.

At this point it behoves us to examine what we believe about the dimensions of radio quasar cores. Typically there is an unresolved component even in VLBI images of radio sources, so this core component is sub  $\sim$ milliarcsec in dimension. It is well known (see, e.g., Kellermann & Owen 1988) that static synchrotron sources are limited to brightness temperatures  $T_b \leq 10^{12}$  K as a consequence of Compton scattering within the source – the ‘Compton catastrophe’. For a source of flux density  $S$  (Jy), at  $\nu$  GHz, the angular size is thereby limited to values greater than  $0.6\nu^{-1}\sqrt{S}$  milliarcsec. Thus we expect that bright ( $S \sim 1$  Jy) sources observed at  $\nu \sim 1$  GHz will have dimensions  $\sim$ milliarcsec. However, relativistic motion (with Lorentz factors up to  $\gamma \sim 10$ ) is quite common amongst radio quasars, and if this motion is directed towards us the perceived brightness tem-

perature can be higher by a factor of  $\sim \gamma$ . Thus we should admit the possibility of there being reasonably bright sources which are only a few microarcsec in size, at frequencies of a few GHz. Evidently, such sources will exhibit strong interstellar scintillation, but they will constitute only a small fraction, of order  $\gamma^{-2} \sim 10^{-2}$ , of the population – a consequence of the favourable orientation required for an observer to perceive a high brightness temperature.

Following on from these remarks it is worth emphasizing the difficulty of observing *diffractive* scintillation, as opposed to weak or refractive scintillation, primarily because of the very severe requirement on source dimension. As a simple numerical example, consider the first hypothetical source of Section 3.3. If we were to look for interference phenomena at 1.4 GHz, then equation (16) indicates that the relevant source-size limit becomes a mere 1.4 microarcsec. More critically, for synchrotron emission, we note that the observation of diffractive scintillation would place an extreme requirement on the brightness temperature of the source. If the requisite brightness temperature is  $T_{b0}$  for scintillation at the transition frequency, then diffractive scintillation at a frequency  $\nu$  ( $< \nu_0$ ) demands  $T_b > T_{b0}(\nu_0/\nu)^{22/5}$ , for a source flux independent of frequency. Hence even an octave below the transition frequency the brightness temperature requirement is up by a factor of 20. It is not surprising, then, that searches for the interference effects characteristic of diffractive scintillation have produced null results for extragalactic sources (Dennison & Condon 1981).

## 5 CONCLUSIONS

Contrary to the usual notion that scintillation is not important at ‘high’ frequencies, we have explicitly demonstrated that this phenomenon may be *especially* important, for some extragalactic sources, at frequencies of several GHz. Large-amplitude variations are possible on time-scales as short as an hour. The role played by interstellar scintillation is, however, strongly influenced by the angular size of the source, and large-amplitude scintillations at high frequencies are contingent on source dimensions much less than a milliarcsecond. In turn, this implies high brightness temperatures and, typically, some degree of relativistic beaming towards the observer is necessary.

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# Erratum: Interstellar scintillation of compact extragalactic radio sources

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**Key words:** errata, addenda – ISM: structure – quasars: general – radio continuum: galaxies.

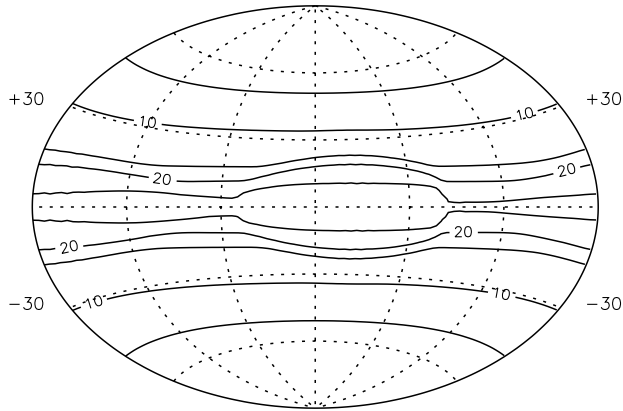
The paper ‘Interstellar scintillation of compact extragalactic radio sources’ was published in Mon. Not. R. Astron. Soc. **294**, 307–311 (1998). The numerical factor in equation (1) of that paper is in error by a factor of  $\pi$ ; the correct formulation is

$$\xi = 7.9 \times 10^3 \text{ SM}^{0.6} D^{0.5} \nu^{-1.7}. \quad (1)$$

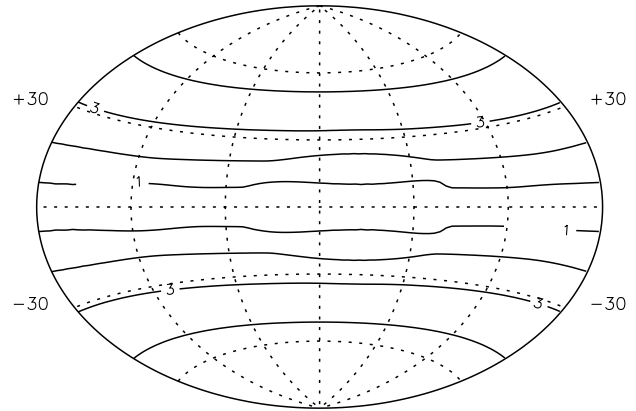
This error affects the numerical estimates, given in Section 2, of transition frequency between weak and strong scattering –  $\nu_0$ , which should be increased by a factor of 1.9 – and the angular

Fresnel radius at the transition frequency –  $\theta_{F0}$ , which should be decreased by a factor of 0.7. Figs 1 and 2 should therefore appear as shown here. The scintillation time-scale at the transition frequency (equation 5) is  $t_{F0} \propto \theta_{F0}$ , and should also be decreased by a factor of 0.7. The error does not affect the scaling relations given in Section 3, nor the discussion and conclusions of the paper (Sections 4 and 5, respectively).

Thanks go to Leon Koopmans for drawing my attention to the error.



**Figure 1.** The transition frequency,  $\nu_0$  – i.e. the frequency at which the scattering strength is unity – for extragalactic sources, plotted in galactic coordinates. The dashed lines mark intervals of  $30^\circ/60^\circ$  in latitude/longitude, respectively, while the solid lines show contours of constant transition frequency, with levels 8, 10, 15, 20 and 40 GHz.



**Figure 2.** Angular size limits,  $\theta_{F0}$ , for extragalactic sources, at an observing frequency equal to the transition frequency, plotted in galactic coordinates. Sources (or components thereof) smaller than the appropriate limit can be approximated as point sources, and will show deep modulations in their received flux. Contours are plotted at one-microarcsecond intervals.

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