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LINEAR POLARIZATION AND PROPER MOTION IN THE AFTERGLOW OF BEAMED GAMMA-RAY BURSTS

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ABSTRACT

We investigate the polarization and proper motion from a beamed gamma-ray burst's ejecta. We find that even if the magnetic field has well-defined orientation relative to the direction of motion of the shock, the polarization is not likely to exceed 20%. Taking into account the dynamics of the beamed ejecta, we find that the polarization rises and decays with peak around the jet break time (when the Lorentz factor of the flow is comparable to the inverse of the initial jet opening angle). Interestingly, we find that when the offset of the observer from the center of the beam is large enough, the polarization as function of time has three peaks and the polarization direction of the middle peak is rotated by 90° relative to the two other peaks. We also show that some proper motion is expected, peaking around the jet break time. Detection of both this proper motion and the direction of polarization can determine which component of the magnetic field is dominant.

Subject headings: gamma rays: bursts — hydrodynamics — polarization — relativity

1. INTRODUCTION

Polarization from an optical afterglow was measured, for the first time, in the case of GRB 990510 (Covino et al. 1999; Wijers et al. 1999). The polarization was relatively small, 1.7%, and was measured about 0.77 days after the burst. Earlier attempts to measure optical polarization gave rise to an upper limit of 2.3% for GRB 990123 (Hjorth et al. 1999), while radio polarization upper limits of 19% (Taylor et al. 1998) and 8% (Frail et al. 1998) were obtained for GRB 980329 and GRB 980703, respectively. A large amount of polarization is usually considered to be the smoking gun of synchrotron emission. Some predictions regarding polarization were put forward by Gruzinov & Waxman (1999) and Medvedev & Loeb (1999). They assumed spherical emission, which by symmetry gives no net polarization except that due to random fluctuations. The polarization from those fluctuations depends on the size of the coherent magnetic field regimes. The smaller the coherent regimes are, the greater the chance of the fluctuations averaging down to zero. The predictions were up to 10% by Gruzinov & Waxman and of order 1% by Medvedev & Loeb.

However, the optical light curve of GRB 990510, for which polarization was measured, showed a strong break into a steeper decline at about 1.5 days (Stanek et al. 1999; Harrison et al. 1999). Although spherical models predict several breaks in the afterglow light curve (see Sari, Piran, & Narayan 1998), these breaks are considerably smaller. The break seen in GRB 990510 is therefore interpreted as a result of beamed ejecta that begins to spread sideways (Rhoads 1999; Panaitescu & Mészáros 1998a; Sari, Piran, & Halpern 1999). Motivated by the measurement of the polarization and the interpretation of a beamed emission, Gruzinov (1999) recalculated the polarization from a beamed ejecta and concluded that if the magnetic field perpendicular to the shock front is significantly different from the magnetic field parallel to the shock front, then the amount of polarization may be as large as the maximal synchrotron polarization, i.e., about 60%. A similar geometric setup (relativistic jet) is also present in BL Lac objects from which there is a typical polarization of the order of 10% (see, e.g., Fan et al. 1997).

Here we reanalyze the expected polarization from a beamed gamma-ray burst (GRB) ejecta. We take an approach similar to that of Gruzinov (1999): we assume that the magnetic field is not completely isotropic behind the shock, with the component parallel to the shock front significantly different from the other components. The discussion here is different from that of Gruzinov in two key points. First, the polarization of the emission of a power-law distribution of electrons, as observed in a given frequency, is estimated (rather than the frequency-integrated polarization which is the energy-weighted average of the polarization). Second, and more important, a more realistic geometric setup for the afterglow emission is considered rather than a pointlike emitter. We take into account the hydrodynamic evolution of the relativistic jet and derive the time-dependent polarization.

We denote by Π_0 the polarization from a very small region in which the magnetic field has a given orientation. In § 2, the polarization Π_0 is averaged over the possible orientation of the magnetic field. This is done assuming that the magnetic field is entangled over a pointlike region. Hereafter, the pointlike regime is defined as the region in which the direction toward the observer is approximately constant, i.e., a region of angular size much smaller than $1/\gamma$, where γ is the bulk Lorentz factor. In § 3, we integrate over the entire emitting region and obtain the observed polarization. By producing these steps in that order, one assumes that the magnetic field is randomized on scales that are pointlike, much smaller than the overall size of the emitting region. As pointed out by Gruzinov & Waxman (1999), this is true even if the magnetic field coherent length grows at the speed of light in the local fluid frame. In § 4, we discuss the proper motion associated with a beamed ejecta.

2. POLARIZATION FROM A POINTLIKE EMITTING REGION

At any point in the shock front there is a preferred direction, the radial direction, in which the fluid moves. We call this the parallel direction and choose the z-direction of the local fluid frame coordinate system to be in that direction. The two perpendicular directions (x, y) are assumed equivalent, i.e., the system is isotropic in the plane perpendicular to the direction of motion. We chose $\hat{x}$ to be in the plane that contains $\hat{z}$ and the direction toward the observer $\hat{n}$. Consider now spherical coordinates (θ, φ) where the polar angle θ is measured relative to $\hat{z}$ and the azimuthal angle ϕ is measured relative to the $\hat{x}$ (see insert in Fig. 1). A quite general description of the distribution of the magnetic field in such an anisotropic system would be to allow different values of the magnetic field as a function of θ, $B = B(\theta)$, and define a probability per unit solid angle $f(\theta)$ for the magnetic field to be in the inclination θ.

The relevant component of the magnetic field is that perpendicular to the observer, i.e., $B \sin \delta$, where δ is the angle between the direction of the magnetic field and the observer. This will produce polarization Π_0 in the direction perpendicular both to the observer and to the magnetic field, i.e., in the direction $\hat{n} \times \hat{B}$. However, this polarization should be averaged due to contributions from magnetic fields oriented differently. By our assumption of isotropy in the (x, y) direction, the polarization of radiation emitted from a pointlike region (after averaging on magnetic field orientation) must be in the direction perpendicular to $\hat{z}$ and to the observer, i.e., in the $\hat{y}$ direction. The contribution Π_0 from a single-orientation magnetic field must therefore be multiplied by $\cos 2\eta$, where η is the angle between $\hat{y}$ and $\hat{n} \times \hat{B}$. By doing so, positive total polarization would indicate polarization along the $\hat{y}$ direction, while negative polarization would indicate polarization along the direction perpendicular to $\hat{y}$ and to the observer. Assume now that the emission is proportional to some power of the magnetic field B^{ϵ}. The total polarization from a pointlike region is then

$$\Pi_p = \Pi_0 \frac{\int \cos 2\eta [B(\theta) \sin \delta]^{\epsilon} f(\theta) \sin \theta d\varphi d\theta}{\int [B(\theta) \sin \delta]^{\epsilon} f(\theta) \sin \theta d\varphi d\theta}. \tag{1}$$

For a power-law distribution of electrons, we have $\Pi_0 = (p+1)/(p+7/3)$ and $\epsilon = (p+1)/2$, where p is the electron power-law index, usually in the range of $p = 2$ to $p = 2.5$. Reasonable values are therefore $\Pi_0 \sim 70\%$ and $1.5 < \epsilon < 1.75$. Cooling may increase the effective p by 1/2.

For frequency integrated polarization, the emission is proportional to the square of the magnetic field, $\epsilon = 2$, and the integration can be easily done. We obtain

$$\Pi_p = \Pi_0 \sin^2 \alpha \frac{\langle B_{\parallel}^2 \rangle - \langle B_{\perp}^2 \rangle /2}{\sin^2 \alpha \langle B_{\parallel}^2 \rangle + (1 + \cos^2 \alpha) \langle B_{\perp}^2 \rangle /2}. \tag{2}$$

This is identical to the expression of Gruzinov (1999). As we remarked above, the relevant values of ϵ are probably below 2, and the integration is less simple. The results now depend on higher moments of $B(\theta)$ and $f(\theta)$, rather than simply through $\langle B_{\parallel}^2 \rangle$ and $\langle B_{\perp}^2 \rangle$. One realization of anisotropic magnetic field can be obtained from an isotropic magnetic field in which the component in the parallel direction was multiplied by some factor ξ. In the notation above this translates to

$$B(\theta) \propto (\xi^2 \sin^2 \theta + \cos^2 \theta)^{-1/2}, \quad f(\theta) \propto B^3(\theta). \tag{3}$$

Figure 1 shows contours of the polarization obtained from a pointlike region after averaging over all possible directions of the magnetic field as a function of the inclination angle α and the anisotropy ξ, both in the case of $\epsilon = 1.5$ and in the analytic case $\epsilon = 2$. The $\epsilon = 2$ case gives higher polarization than lower values of ϵ. However, it is evident that the differences in polarization for the two values of ϵ are not large and are mostly less than 10%. Given the much higher uncertainties, such as the anisotropy ξ and the uncertainty in the geometry when averaging over the emitting regions (see the next section), one

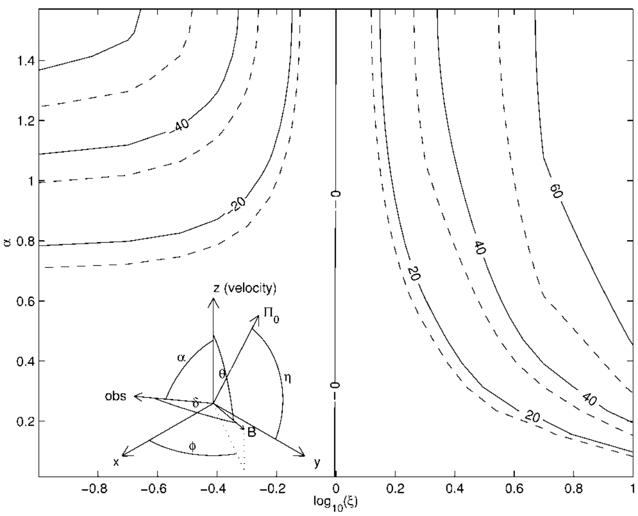

FIG. 1.—Percentage of polarization Π_p from a pointlike emitting region, after averaging over the possible orientation of the magnetic field, as function of the inclination of the observer relative to the preferred direction α and the ration between the two components of the magnetic field ξ. The insert shows the geometry of the calculation, which averages over θ and ϕ for a given α.

can use the analytic result even though it uses $\epsilon = 2 > 1.75$. The following properties seem to be general: if $\xi \gg 1$ ($B_{\parallel} \gg B_{\perp}$) the polarization is $\Pi_p \cong \Pi_0$ quite independent of the exact value of α and ϵ. If, on the other hand, $\xi \ll 1$ ($B_{\parallel} \ll B_{\perp}$) as suggested by Medvedev & Loeb (1999), then the polarization is small for small values of $\sin^2 \alpha$. In the following section we shall assume the favorite conditions in which the polarization from a pointlike emitter is $\Pi_p \cong \Pi_0 \cong 70\%$.

3. POLARIZATION FROM A BEAMED RELATIVISTIC EJECTA

The calculation of the expected polarization requires the knowledge of the hydrodynamic evolution of a beamed ejecta. There is no detailed description of that for now; however, several key features are understood. At first, the ejecta behaves like a spherical one, since it has no time to spread laterally. This stage lasts as long as $\gamma \gg \theta_0^{-1}$, where γ is the bulk Lorentz factor of the jet and θ_0 is its initial opening angle. During this stage the emission also looks spherical, since the observer is only able to see a small fraction of the jet surface, of the order of $\gamma^{-1} \ll \theta_0$. The emission in this stage is mostly from a ring at high frequencies (above the peak synchrotron frequency) and from an almost uniform disk below the peak frequency and especially uniform below the self-absorption frequency (Waxman 1998; Panaitescu & Mészáros 1998b; Sari 1998; Granot, Piran, & Sari 1999a, 1999b). Since the emitting region has spherical symmetry around the observer, the net expected polarization is zero, except perhaps for fluctuations in the manner discussed by Gruzinov & Waxman and Medvedev & Loeb. These fluctuations will also cause small deviations of the direction of the polarization from the prediction presented below. We ignore this kind of fluctuation in the rest of this Letter and focus on the average net polarization.

It is likely that the observer is not directed exactly at the center of the jet. Therefore, once the viewing angle becomes large enough, the observer "feels" the asymmetry: most of the emission comes from the direction toward the center of the jet and some net polarization is expected (see also Wijers et al. 1999). However, one should not expect the maximal linear polarization Π_p, since the emitting regions have angular extent $\sim 1/\gamma$ and therefore a considerable averaging will take place.

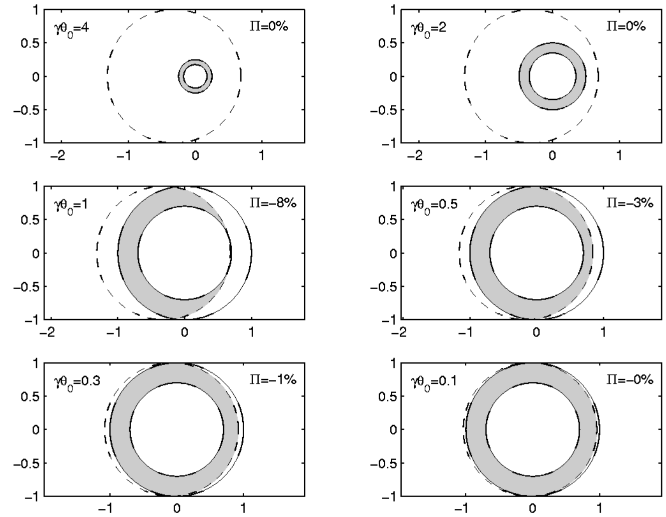

FIG. 2.—Emitting region at several times in the toy model, the case of a relatively small offset of $q = 0.32$. The dashed line marks the physical extent of the jet, while solid lines give the viewable region. The shaded region is where the radiation is coming from. On each frame, the percentage of linear polarization is given on the top right and the initial size of the jet relative to $1/\gamma$ is given on the left. The frames are scaled so that the size of the jet is unity. Since the offset is small, more than half of the ring is seen at all times and the polarization is, therefore, always at the same direction.

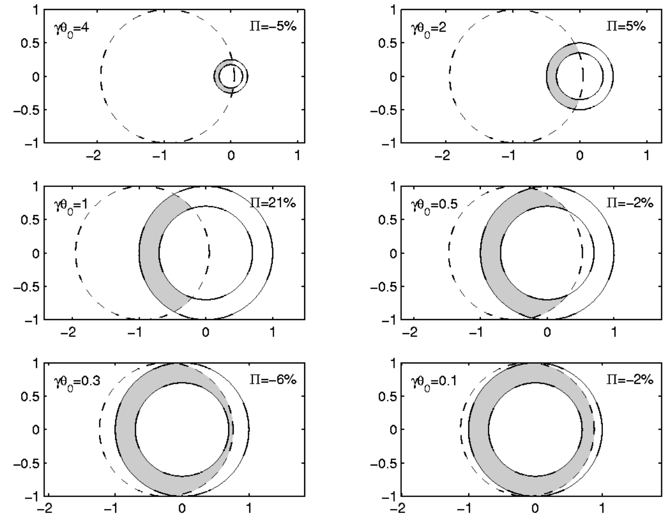

FIG. 3.—Same as Fig. 2 but for a relatively large offset of $q = 0.95$. For this high value of q, when $\gamma\theta_0 = 2$ or $\gamma\theta_0 = 1$, more than half of the ring is missing and therefore the polarization direction is rotated by 90° relative to its initial and final directions. This corresponds to the middle polarization peak in Fig. 4 for $q = 0.95$. A similar situation is obtained for $q = 0.71$.

The time when the edge effects become visible is comparable to the time when the jet begins to spread. Later in time the angular extent of the jet increases, while the offset of the observer from the center of the jet is, of course, fixed in time. The observer therefore becomes more and more in the center of the ejecta and the system, once again, approaches cylindrical symmetry. The amount of polarization is expected to fade.

When the emission is from a ring centered around the observer, and assuming that the dominant component of the magnetic field is perpendicular to the shock, then the north and south quarters of the ring produce polarization in the north-south direction while the west and east quarters give rise to west-east polarization (the opposite is true if the parallel magnetic field is the dominant one). The total effect is a zero net polarization. If part of the ring is missing (due to the finite extent of the jet), say, a small part on the east direction, then the net polarization would be in the south-north direction. If it is a big fraction of the ring that is missing, say the east part as well as the north and south parts, then the polarization would be east-west. As discussed above, the part of the ring that is missing is initially growing, reaching a maximum around the time when the jet begins to spread, and then decreases again. If, at the maximum, a large part (more than half) of the ring is missing (or radiates less efficiently), then the direction of polarization is expected to change by 90°! This is a unique signature of the geometric setup of beamed GRBs. The detection of such a signature would therefore give very strong support both to the synchrotron radiation as well as the geometric structure of the jet and its evolution. Some possible examples of this behavior are given in the toy model below.

We suggest a toy model in order to explore the possible observable effects and roughly estimate the maximal polarization: (1) The line of sight to the observer is always crossing the jet, i.e., the angular offset between the observer and the center of the jet is smaller than the initial angular extent of the jet θ_0. (2) The viewable region is a thin ring of radius γ^{-1} centered around the line of sight to the observer. The width of the ring is taken to be 30% of its radius. (3) The jet spans an angular size given by its initial size θ_0 as long as $\gamma \geq \theta_0^{-1}$, and after that expands with angular size of γ^{-1}, i.e., $\theta(t) = \max[\gamma^{-1}, \theta_0]$. (4) The portion of the viewable region (defined in item 2) that overlaps the jet radiates uniformly, while the portion of the viewable region that is outside of the jet is not emitting. (5) The Lorentz factor of the fluid is related to the observed time T by $\gamma \propto T^{-3/8}$ as long as $\gamma \geq \theta_0^{-1}$ and by $\gamma \propto T^{-1/2}$ after that. Under these assumptions, the evolution of the polarization as a function of time depends only on the initial offset between the line of sight to the observer and center of the jet measured in units of the jet's initial angular size $q \equiv \theta_{\rm offset}/\theta_0$. Jets with $q = 0$ are centered exactly at the observer, while with $q = 1$ the observer is located exactly at the edge of the jet. Values of $q > 1$ are excluded by our first assumption. This reflects the fact that in such cases the GRB itself will be hardly seen.

Figures 2 and 3 display the emission geometry for the two radical values $q = \sqrt{0.1} \cong 0.32$ and $q = \sqrt{0.9} \cong 0.95$, where 80% of the cases are. Figure 4 summarizes the polarization evolution for these two extreme values of q as well as the median value $q = 0.71$. Figure 4 also includes the observational detections and upper limits obtained so far. Although one would imagine that the low values of polarization that were measured would imply an almost isotropic magnetic field, the sparse data does not allow us to constrain any of the free parameters of the model. For example, GRB 990510 is completely consistent with the maximal value of $\Pi_p = 70\%$ (obtained for extremely anisotropic magnetic field) and $q = 0.71$. Similarly, GRB 990123 is consistent with any value of $q < 0.37$ even for the maximal $\Pi_p = 70\%$. This demonstrates the need for semi-continuous polarization data in order to test and constrain the geometry of the ejecta and of the magnetic field.

4. PROPER MOTION

A straightforward consequence of the discussion and the toy model of the previous section is proper motion of the source that peaks around the time of the jet spreading. Since initially the source is centered around the observer due to relativistic

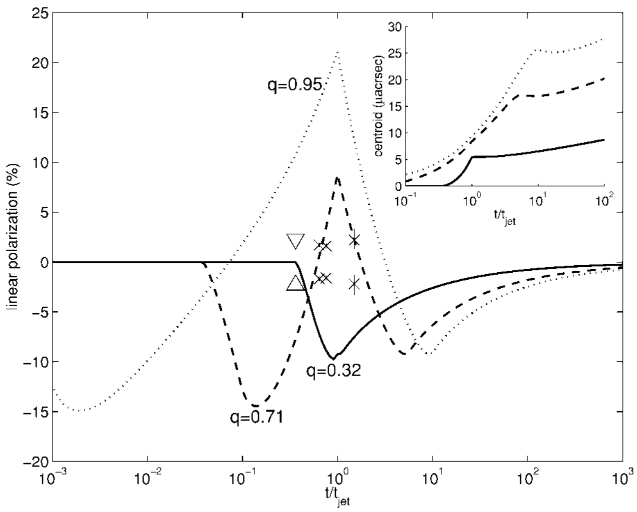

FIG. 4.—Polarization as function of time for three values of $q = 0.32, 0.71, 0.95$. The polarization scales linearly with the pointlike polarization Π_p, and we assumed here $\Pi_p = 70\%$. Observational data for GRB 990510 is marked by *x*, assuming $t_{\rm jet} = 1.2$ days. The upper limit for GRB 990123 is given by a triangle, assuming $t_{\rm jet} = 2.1$ days. The insert shows the emission centroid position on the sky for the same values of *q*, assuming $\theta_0 = 0.2$, $R_{\rm jet} = 10^{18}$ cm, and $D = 2.2 \times 10^{27}$ cm.

beaming, the centroid of the emitting ring is fixed in time. Once the Lorentz factor becomes comparable to one over the opening angle of the jet, the symmetry breaks and more emission comes from the side directed toward the center of the jet. This results in proper motion.

The most extreme change in the angular position can be estimated by $q\theta_0 R/D$, where R is the emission radius at the time of the observation and D is the (angular) distance to the observer. R has a very weak dependence on observed time ($t^{1/4}$ before the jet break and logarithmic dependence after that). Since the proper motion peaks at the jet break time, R can be approximated by $R_{\rm jet}$, the radius at the jet break time. Even with favorite parameters of $\theta_0 = 0.2$, $q = 1$, $R_{\rm jet} = 10^{18}$ cm, and a distance of $D \sim 2 \times 10^{27}$ cm ($z \cong 0.2$), we get angular displacement of order of $\sim 10^{-10} \cong 20$ μas, which is about the current observational capabilities with VLBI. A more detailed time-dependent calculation could be done by finding the centroid of the shaded regions in Figures 2 and 3. The proper position as a function of time is displayed in the insert for Figure 4. For $t \gg t_{\rm jet}$ the centroid of the emitting region is the center of the jet, and further proper motion is only due to the logarithmic growth of the radius as function of time.

Since the proper motion is toward the center of the jet, it must be either perpendicular or parallel to that of the polarization. If the direction of the motion is parallel (perpendicular) to the direction of the polarization (during the first peak in case that the polarization changes its direction), then the dominant component of the magnetic field is the parallel (perpendicular) component. Combination of detection of polarization and proper motion enables us to determine the orientation of the magnetic field behind the shock.

5. DISCUSSION

High polarization is expected if the magnetic field is significantly different in the parallel and perpendicular directions. Averaging over the whole emission site has a dramatic effect on the total polarization. It completely destroys the polarization at early and late times, and polarization is expected only around the jet break time. Even under the extreme conditions of our toy model, the polarization is unlikely to get to the maximal synchrotron value of ∼70%. The maximal value we get is around 20%.

A striking and quite unique signature of our model is that the polarization changes direction by 90° around the jet spreading time for cases in which the observer is not very close to the center of the jet. It rises once in a given direction, decays to zero and rises again in a direction rotated by 90°, vanishes again, and finally rises in the original direction and slowly decays. Within our toy model, most beamed ejecta are expected to change the direction of their polarization in the above manner.

Beamed GRBs are subject to proper motion of the centroid of the emission region, mostly around the jet spreading time. The centroid position change, of order of a few microarcseconds, is marginally detectable by current instruments. If proper motion is detected, it would be toward the center of the physical jet. This is either perpendicular or parallel to the polarization direction. It will tell us which component of the magnetic field is larger. Ghisellini & Lazzati (1999) have simultaneously and independently completed similar work and found two peaks for the polarization in the limit of $\xi = 0$ and a nonspreading jet. The jet-spreading effect, which is taken into account here, brings back the symmetry at late times and results in a third polarization peak in the same direction as the first peak. The spreading also destroys the second peak if the offset is small enough.

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