

# Gamma-ray burst afterglows: effects of radiative corrections and non-uniformity of the surrounding medium

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## ABSTRACT

The afterglow of a gamma-ray burst (GRB) is commonly thought to be the result of continuous deceleration of a relativistically expanding fireball in the surrounding medium. Assuming that the expansion of the fireball is adiabatic and that the density of the medium is a power-law function of shock radius, i.e.  $n_{\text{ext}} \propto R^{-k}$ , we study the effects of the first-order radiative correction and the non-uniformity of the medium on a GRB afterglow analytically. We first derive a new relation among the observed time, the shock radius and the Lorentz factor of the fireball:  $t_{\oplus} = R/4(4 - k)\gamma^2 c$ , and also derive a new relation among the comoving time, the shock radius and the Lorentz factor of the fireball:  $t_{\text{co}} = 2R/(5 - k)\gamma c$ . We next study the evolution of the fireball by using the analytic solution of Blandford & McKee. The radiation losses may not significantly influence this evolution. We further derive new scaling laws both between the X-ray flux and observed time and between the optical flux and observed time. We use these scaling laws to discuss the afterglows of GRB 970228 and GRB 970616, and find that if the spectral index of the electron distribution is  $p = 2.5$ , implied from the spectra of GRBs, the X-ray afterglow of GRB 970616 is well fitted by assuming  $k = 2$ .

**Key words:** radiation mechanisms: non-thermal – gamma-rays: bursts.

## 1 INTRODUCTION

The popular theoretical explanation for cosmological gamma-ray bursts (GRBs) is based on the fireball model. In this model, a GRB is thought to result from the dissipation of the kinetic energy of a relativistically expanding fireball. This dissipation can be the result either of internal shocks formed during the collision between the shells with different Lorentz factors in the fireball (Rees & Mészáros 1994; Paczyński & Xu 1994; Sari & Piran 1997), or of external shocks formed by the fireball colliding with the surrounding medium (Rees & Mészáros 1992; Mészáros & Rees 1993; Katz 1994; Sari, Narayan & Piran 1996). After the main GRB event, the expanding fireball is predicted to produce delayed emission at longer wavelengths (Paczynski & Rhoads 1993; Katz 1994; Mészáros & Rees 1997; Wijers, Rees & Mészáros 1997; Reichart 1997; Waxman 1997a,b; Vietri 1997a,b; Tavani 1997; Sari 1997). It is easily understood that such an X-ray, optical and/or radio afterglow is in fact a result of continuous deceleration of the expanding fireball.

Fortunately, the afterglows of GRBs have been detected recently in the error boxes at the sites of seven GRBs, e.g. GRB 970228, 970402, 970508, 970616, 970815, 970828 and 971214. The current fireball model used to explain these afterglows has in fact been divided into two sub-models. In the first sub-model, the fireball

expansion following a GRB is thought to be adiabatic (Wijers et al. 1997; Reichart 1997; Waxman 1997a,b). This is a reasonable assumption if the time-scale for the cooling of the accelerated electrons behind the shocks rapidly becomes longer than the expansion time-scale of the fireball. The adiabatic expansion model has given a scaling relation between the expansion Lorentz factor and observed time:  $\gamma \propto t_{\oplus}^{-3/8}$ , provided that the surrounding medium is uniform and the expansion is ultra-relativistic. The studies of Wijers et al. (1997), Reichart (1997) and Waxman (1997a,b) showed that the adiabatic expansion model may satisfactorily explain the long-term behaviour of the afterglows of GRB 970228 and GRB 970508. The more detailed numerical calculations by Huang et al. (1998) are consistent with these studies. An alternative sub-model that may account for several properties of the afterglows of these two bursts has been presented by Vietri (1997b), who postulated that the expansion is highly radiative. This requires that the accelerated electrons always cool more rapidly than the fireball expands.

The purpose of the present work is to study analytically the effects of radiative corrections and non-uniformity of the surrounding matter on a GRB afterglow based on the adiabatic expansion model. Our study is stimulated by two motivations. First, Sari (1997) recently found that radiation losses may significantly influence the hydrodynamical evolution of the fireball in the uniform medium. Secondly, it is possible that the sources of GRBs are merging neutron star binaries (Narayan et al. 1992; Vietri 1996),

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failed supernovae (Woosley 1993), accretion-induced phase transitions of neutron stars (Cheng & Dai 1996), or hypernovae (Paczynski 1998). If so, one may not assume that the surrounding matter is uniform. A massive star as a progenitor of a GRB may have produced a stellar wind, or a supernova remnant may exist around a neutron star which is a source of the GRB. The stellar wind and/or the supernova remnant constitute the non-uniform surrounding medium of the GRB. It can be expected that the non-uniformity of the medium can shorten or prolong the relativistic expansion of the fireball. Such a behaviour should be compared with observations of afterglows of GRBs.

In order to achieve the purpose of the present work, in the next section we first derive a new relation among the observed time, the shock radius and the Lorentz factor of the fireball, assuming that the density of the surrounding medium is a power-law function of shock radius. We also obtain a new relation among the comoving time, the shock radius and the Lorentz factor of the fireball. We show that, in the case of a uniform medium, the former relation turns out to be consistent with the expression derived by Sari (1997), but the latter relation is different from the usual expression by a factor of 5/2. We next study the evolution of the relativistic fireball in the non-uniform medium by using the analytic solution found by Blandford & McKee (1976), and investigate the effect of the first-order radiative correction on the evolution. We find this effect may be insignificant. In Section 3, we derive the X-ray flux from the fireball as a function of observed time, and give a scaling relation between the optical flux and observed time. In Section 4, we use our model to discuss the afterglows of GRB970228 and GRB970616. In the final section, we summarize our work.

## 2 THREE MEASURES OF TIME AND FIRST-ORDER RADIATIVE CORRECTION

Even though the source of GRBs is unknown, one assumes the following scenario for GRBs: a compact source ( $\sim 10^7$  cm) releases an energy  $E$  comparable to that observed in gamma rays,  $E \sim 10^{51}$  erg, over a time less than 100 s. The huge optical depth in the source results in an initial fireball that expands and accelerates to relativistic velocity. During the acceleration, the fireball energy is converted to bulk kinetic energy with a Lorentz factor  $\gamma \sim 300$ . Of course, internal shocks may be formed in this period. Subsequently, the expansion of the fireball starts to be significantly influenced by the swept-up medium and two external shocks are formed: a forward blast wave and a reverse shock (Rees & Mészáros 1992). A GRB may be produced by non-thermal processes in shocks. After the GRB, the relativistic blast wave continues to sweep up the medium. The fireball may go into an adiabatic expansion phase. For simplicity, we assume that the medium density varies with shock radius based on the expression

$$n_{\text{ext}} = n_0 \left( \frac{R}{R_0} \right)^{-k} \quad \text{for } R \geq R_0, \quad (1)$$

where  $n_0$  is the medium density at the radius  $R_0$  at which the fireball starts to be significantly influenced by the medium. As usual,  $R_0$  is defined as

$$R_0 = \left( \frac{3E}{4\pi n_0 m_p c^2 \eta^2} \right)^{1/3} = 10^{16} E_{51}^{1/3} n_0^{-1/3} \eta_{300}^{-2/3} \text{ cm}, \quad (2)$$

where  $E_{51} = E/10^{51}$  erg,  $\eta_{300} = \eta/300$ , and  $n_0$  is in units of  $1 \text{ cm}^{-3}$ .

The use of a power-law dependence of the density surrounding a GRB source is motivated by the following points. First, some

theoretical studies of supernova remnants (for a review see Ostriker & McKee 1988) suggest a power-law dependence of the remnant density at a radius larger than some fixed value. Secondly, the actual dependence might be more complicated than a power-law form. However, only in the case with a power-law form can we analytically study the evolution of a relativistic fireball. A study of the relativistic-fireball expansion in the non-uniform medium without a power-law dependence of the matter density on radius, needs numerical simulations and thus is beyond the scope of this paper.

### 2.1 Three measures of time

We assume that the expansion is ultra-relativistic, and that radiation losses are small. This implies that when most of the energy has been given to the medium, the energy in the shocked medium is constant and approximately equal to  $E$ . The rest mass of the shocked medium  $M \propto R^{3-k}$ . Since the medium was thermalized by the relativistic blast wave, its energy in the observer's frame is  $\sim M\gamma^2 \sim E$ , where  $\gamma$  is the Lorentz factor of the shocked medium just behind the shock. Thus, we have the scaling law

$$\gamma \propto R^{-(3-k)/2}. \quad (3)$$

It should be emphasized that, for a relativistic strong blast wave, the Lorentz factor of the shock  $\Gamma = \sqrt{2}\gamma$  (Blandford & McKee 1976).

According to the scaling law (3), we next investigate relations among three different measures of time denoted as  $t$ ,  $t_{\text{co}}$  and  $t_{\oplus}$ , which are measured in the rest frame of the burster, in the frame comoving with the fireball, and in the observer's frame, respectively. When the shock propagates a small distance  $\delta R \approx c\delta t$ , photons that are emitted from the shock will be observed on the observed time-scale of  $\delta t_{\oplus} = \delta R/(2\Gamma^2 c) = \delta R/(4\gamma^2 c)$ , while the change in time in the frame comoving with the fireball is  $\delta t_{\text{co}} = \delta R/(\gamma c)$ . Integrating these equations over time and using the scaling law (3), we obtain

$$t_{\oplus} = \frac{R}{4(4-k)\gamma^2 c} = \frac{t}{4(4-k)\gamma^2}, \quad (4)$$

and

$$t_{\text{co}} = \frac{2R}{(5-k)\gamma c} = \frac{2t}{(5-k)\gamma}. \quad (5)$$

In the case of a uniform medium, e.g.  $k = 0$ , equation (4) turns out to be consistent with the expression derived by Sari (1997). In this case, comparing equation (5) with the commonly used expression  $R/(\gamma c)$  (Waxman 1997a,b; Vietri 1997b), we see the present expression is a factor of 5/2 smaller.

### 2.2 First-order radiative correction

After establishing equations (4) and (5), we can study the adiabatic evolution of the fireball in the non-uniform medium, and investigate the effect of the first-order radiative correction on the evolution.

The starting point of our study is equation (69) of Blandford & McKee (1976), concerning the total energy of the fireball,

$$E = \frac{16\pi n_{\text{ext}} m_p c^2 \gamma^2 R^3}{17 - 4k}, \quad (6)$$

which is constant (to lowest order). Here  $n_{\text{ext}}$  is to be interpreted as the external density at the position of the shock. Combining equation (6) with equations (1) and (4), we get

$$\gamma = \gamma_0 \left( \frac{t_{\oplus}}{t_0} \right)^{-(3-k)/(8-2k)}, \quad (7)$$

**Table 1.** The typical values of some parameters for different  $k$  in the case of adiabatic evolution.

	$\gamma_0$	$t_0$ (s)	$t_{1,a}$ (s)	$t_{1,b}$ (s)	$f_a$	$f_b$	$t_{2,a}$ (s)	$t_{2,b}$ (s)
$k = 0$ .....	4.67	$3.31 \times 10^4$	200	1800	0.71	0.20	12.5	1013
$k = 1$ .....	7.46	$4.41 \times 10^4$	30	140	0.81	0.42	3.75	90.9
$k = 2$ .....	18.56	$6.61 \times 10^4$	5.0	15.0	0.94	0.73	1.25	11.3

Notes —  $\gamma_0$ ,  $t_0$ ,  $t_1$ ,  $f$  and  $t_2$  are defined by equations (8), (9), (12), (16) and (23) in the text, respectively. The values are computed for  $E_{51} = 4$ ,  $\eta_{300} = 1$ ,  $n_0 = 1 \text{ cm}^{-3}$  and  $\xi_B = 0.1$ . The subscripts ‘a’ and ‘b’ represent  $\xi_e = 0.1$  and  $0.3$ , respectively.

where  $\gamma_0$  and  $t_0$  are defined as

$$\gamma_0 = \left[ \frac{(17 - 4k)E}{16\pi n_0 m_p c^2 R_0^3} \right]^{1/(8-2k)} \quad (8)$$

and

$$t_0 = \frac{R_0}{4(4-k)c}. \quad (9)$$

Accordingly, the shock radius can be written as

$$R = R_0 \gamma_0^2 \left( \frac{t_\oplus}{t_0} \right)^{1/(4-k)}. \quad (10)$$

It is easily seen that in the case of  $k = 0$ , equations (7) and (10) are in agreement with those derived by Sari (1997).

In the following we consider only synchrotron emission from the accelerated electrons. We neglect the contribution of inverse-Compton (IC) emission from these electrons. This is because IC emission is not of importance, particularly at late times of the fireball expansion (for a discussion, see the final paragraph of this subsection). In order to calculate the effect of synchrotron emission, we need to determine the magnetic field strength and electron energy. As usual, we assume that the magnetic energy density in the comoving frame is a fraction  $\xi_B$  of the total thermal energy density  $e' = 4\gamma^2 n_{\text{ext}} m_p c^2$ , i.e.,  $B' = (8\pi \xi_B e')^{1/2}$ , and that the electrons carry a fraction  $\xi_e$  of the energy. This implies that the Lorentz factor of the random motion of a typical electron in the comoving frame is  $\gamma_{\text{em}} = \xi_e \gamma m_p / m_e$ . The ratio of the comoving-frame expansion time,  $t_{\text{co}} = 2R/(5-k)\gamma c$ , to the synchrotron cooling time,  $t_{\text{syn}} = 6\pi m_e c / \sigma_T \gamma_{\text{em}} B'^2$ , is

$$\frac{t_{\text{co}}}{t_{\text{syn}}} = \left( \frac{t_1}{t_\oplus} \right)^{2/(4-k)}, \quad (11)$$

where  $t_1$  has been defined based on

$$t_1^{2/(4-k)} = \frac{128(4-k)}{3(5-k)} \left( \frac{m_p}{m_e} \right)^2 \xi_e \xi_B (\sigma_T n_0 c t_0) \gamma_0^{3-2k} t_0^{2/(4-k)}. \quad (12)$$

Thus, we can define the radiative efficiency as

$$\xi = \frac{t_{\text{syn}}^{-1}}{t_{\text{co}}^{-1} + t_{\text{syn}}^{-1}} = \frac{(t_1/t_\oplus)^{2/(4-k)}}{(t_1/t_\oplus)^{2/(4-k)} + 1}. \quad (13)$$

This parameter accounts for a fraction of the energy of the accelerated electrons which is radiated away by the synchrotron emission. For  $t_\oplus \ll t_1$ ,  $\xi \approx 1$ ; but for  $t_\oplus \gg t_1$ ,  $\xi \approx (t_1/t_\oplus)^{2/(4-k)}$ . Following equation (84) of Blandford & McKee (1976), we derive the energy loss rate during the deceleration, which is given by  $4^4(4-k)^2 \pi \gamma^8 c^3 t_\oplus^2 n_{\text{ext}} m_p c^2$  multiplied by  $\xi_e \xi$ , where equation (4) has been used. Using equations (7)–(9), we get the total power radiated per unit time,

$$\frac{dE}{dt_\oplus} = -\xi_e \xi \frac{17-4k}{4(4-k)} \frac{E}{t_\oplus}. \quad (14)$$

Integrating this equation over time, we further obtain

$$E(t_\oplus) = E_0 \left[ \frac{(t_1/t_\oplus)^{2/(4-k)} + 1}{(t_1/t_{\text{in}})^{2/(4-k)} + 1} \right]^{\frac{17-4k}{8} \xi_e}, \quad (15)$$

where  $t_{\text{in}}$  is the observed initial time of the afterglow which is of the order of magnitude  $\sim 10$  s. We now define the parameter  $f$  as

$$f = \left[ \left( \frac{t_1}{t_{\text{in}}} \right)^{2/(4-k)} + 1 \right]^{-\frac{17-4k}{8} \xi_e}. \quad (16)$$

This parameter is in fact the ratio of the fireball energy at  $t_\oplus \gg t_1$  to the initial total energy, and thus accounts for the effect of the first-order radiative correction. To estimate  $f$ , we adopt the following values:  $E_{51} = 4$ ,  $\eta_{300} = 1$ ,  $n_0 = 1 \text{ cm}^{-3}$ ,  $\xi_B = 0.1$ , and  $\xi_e \sim 0.1$ – $0.3$ . Table 1 gives the values of  $\gamma_0$ ,  $t_0$ ,  $t_1$  and  $f$  for different  $k$ . It can be seen from this table that for  $\xi_e \sim 0.1$  the effect of the radiative corrections may be insignificant, and for  $\xi_e \sim 0.3$  this effect may not be of importance in the case of  $k > 0$ . This conclusion disagrees with that of Sari (1997), who did not consider the factor  $\xi$  in our equation (14). For this reason, we will not take into account this effect in the following.

In the remainder of this subsection, we want to discuss the validity of two of our assumptions.

First, the fireball expansion has been assumed to be adiabatic. As described in the introduction, this assumption in fact requires that the time-scale for the cooling of the accelerated electrons behind the shocks rapidly becomes longer than the fireball expansion time-scale. Now we assume that the initial expansion of the fireball is radiative. In this case (see, e.g., Vietri 1997b), the relations among the shock radius, comoving-frame time and observed time are  $R = 4(7-2k)\gamma^2 c t_\oplus$  and  $R = (4-k)\gamma c t_{\text{co}}$ , and the Lorentz factor of the fireball decreases as

$$\gamma = \eta \left( \frac{t_\oplus}{t_{0r}} \right)^{-\frac{3-k}{7-2k}}, \quad (17)$$

where

$$t_{0r} = \frac{R_0}{(7-2k)4\eta^2 c} = \frac{1}{7-2k} E_{51}^{1/3} n_0^{-1/3} \eta_{300}^{-8/3} \text{ s}. \quad (18)$$

Thus, the ratio of the comoving-frame expansion time-scale to the synchrotron cooling time-scale is given by

$$\frac{t_{\text{co}}}{t_{\text{syn}}} = \left( \frac{t_{1r}}{t_\oplus} \right)^{\frac{5-k}{7-2k}}, \quad (19)$$

where  $t_{1r}$  is defined through the expression

$$t_{1r}^{(5-k)/(7-2k)} = \frac{64(7-2k)}{3(4-k)} \left( \frac{m_p}{m_e} \right)^2 \xi_e \xi_B (\sigma_T n_0 c t_{0r}) \eta^4 t_{0r}^{(5-k)/(7-2k)}. \quad (20)$$

The values of  $t_{1r}$  for different  $k$  are calculated in Table 2. According to this table and to equation (19), we see that  $t_{\text{syn}} > t_{\text{co}}$  for  $t_\oplus > t_{1r}$ , and therefore conclude that even if the fireball starts off with

**Table 2.** The typical values of some parameters for different  $k$  in the case of radiative evolution.

	$t_{0r}(s)$	$t_{1r,a}(s)$	$t_{1r,b}(s)$
$k = 0$ .....	0.227	47.3	220
$k = 1$ .....	0.317	53.2	210
$k = 2$ .....	0.529	47.9	144

Notes —  $t_{0r}$  and  $t_{1r}$  are defined by equations (18) and (20) in the text, respectively. The values of the parameters of the fireball and medium are taken as in Table 1. The subscripts ‘a’ and ‘b’ represent  $\xi_e = 0.1$  and  $0.3$ , respectively.

radiative dynamics, the transition to adiabatic evolution comes soon after the GRB.

Secondly, we have not considered the contribution of IC emission to the cooling of the accelerated electrons. It is well known that whether IC emission is important or not depends on the ratio of the IC power to synchrotron power,

$$y = \frac{e'_S}{e'_B}, \quad (21)$$

where  $e'_S$  and  $e'_B$  are the synchrotron-photon and magnetic-field energy densities. For  $t_{co} < t_{syn}$  and when emission is dominated by the synchrotron process, as argued by Waxman (1997a), the energy density  $e'_S$  is a fraction  $t_{co}/4t_{syn}$  of the electron energy density, and thus the ratio  $y$  is given by

$$y = \frac{\xi_e}{\xi_B} \frac{t_{co}}{4t_{syn}} = \left( \frac{t_2}{t_\oplus} \right)^{2(4-k)}, \quad (22)$$

where  $t_2$  has been defined by

$$t_2 = \left( \frac{\xi_e}{4\xi_B} \right)^{(4-k)/2} t_1. \quad (23)$$

Therefore, we can conclude from Table 1 that IC emission is not an important process for the cooling of the accelerated electrons behind the shocks for  $t_\oplus > t_2$ .

### 3 THE X-RAY AND OPTICAL RADIATION

We first study the X-ray flux from a relativistic fireball expanding in the non-uniform medium. The total power radiated per unit time has been given by equation (14). In order to calculate the X-ray flux, we need to consider radiation mechanisms of the accelerated electrons behind the shock. As shown in the last section, the synchrotron emission is the main mechanism for the cooling of these electrons. We assume that the electron distribution behind the shock is a power law,

$$\frac{dN_e}{d\gamma_e} \propto \gamma_e^{-p}, \quad \text{for } \gamma_{em} \leq \gamma_e \leq \gamma_{e,max}. \quad (24)$$

The estimate of  $\gamma_{e,max}$  can be obtained by equating the electron acceleration time-scale,  $t_a = \gamma_e m_e c / eB'$ , to the synchrotron cooling time-scale so that

$$\gamma_{e,max} = \left( \frac{6\pi e}{\sigma_T B'} \right)^{1/2}. \quad (25)$$

It should be pointed out that even though the maximum Lorentz

factor estimated by this equation is rather high, the effect of IC emission on  $\gamma_{e,max}$  can be neglected, as seen in equation (22). For the electron distribution with equation (24), the fraction of synchrotron power radiated in the X-ray region is (Vietri 1997a,b)

$$f_x = \frac{\epsilon_u^{(3-p)/2} - \epsilon_l^{(3-p)/2}}{\epsilon_{max}^{(3-p)/2}}, \quad (26)$$

where  $\epsilon_u$  and  $\epsilon_l$  are the upper and lower limits of the *BeppoSAX* instruments, 2 and 10 keV respectively, and  $\epsilon_{max}$  is given by

$$\epsilon_{max} = \frac{\hbar e B'}{m_e c} \gamma_{e,max}^2 \gamma = 160 \gamma \text{ MeV}. \quad (27)$$

Inserting equation (27) into equation (26), we find

$$f_x \approx [6 \times 10^{-5} / \gamma]^{(3-p)/2}, \quad (28)$$

where the number in the brackets is two orders smaller than that of Vietri (1997b). Therefore, the expected X-ray flux is

$$F_x = \frac{dE}{dt_\oplus} \frac{f_x}{4\pi d^2} = 1.0 \times 10^{-6} \text{ erg s}^{-1} \text{ cm}^{-2} \left( \frac{d}{1 \text{ Gpc}} \right)^{-2} \left( \frac{t_\oplus}{1 \text{ s}} \right)^{-a}, \quad (29)$$

where  $d$  is the source distance, the constant has been computed for  $k = 0$ ,  $p = 2.5$ ,  $\xi_e = 0.1$ , and the values of the other parameters used in Table 1. For  $t_\oplus \ll t_1$  in the above equation,

$$a = 1 - \frac{3-k}{8-2k} \frac{3-p}{2}; \quad (30)$$

but for  $t_\oplus \gg t_1$ ,

$$a = \frac{6-k}{4-k} - \frac{3-k}{8-2k} \frac{3-p}{2}. \quad (31)$$

We next discuss the optical flux from the accelerated electrons behind the shock. For the electron distribution with equation (24), the observed frequency of synchrotron emission at peak flux is

$$\begin{aligned} \nu_m &= \frac{3}{4\pi} \gamma_{em}^2 \frac{eB'}{m_e c} = \frac{6}{\sqrt{2\pi}} \xi_e^2 \xi_B^{1/2} \gamma_0^{4-k} \left( \frac{m_p}{m_e} \right)^2 \frac{e(n_0 m_p c^2)^{1/2}}{m_e c} \left( \frac{t_\oplus}{t_0} \right)^{-3/2} \\ &= 8.0 \times 10^{21} \xi_e^2 \xi_B^{1/2} E_{51}^{1/2} \left( \frac{17-4k}{17} \right)^{1/2} \left( \frac{4}{4-k} \right)^{3/2} \left( \frac{t_\oplus}{1 \text{ s}} \right)^{-3/2} \text{ Hz}, \end{aligned} \quad (32)$$

where equations (8) and (9) have been used. Equation (32) shows that  $\nu_m$  is weakly dependent on  $k$ . The typical spectrum of the synchrotron emission has the form  $F_\nu \propto \nu^\alpha$ , where  $\alpha = 1/3$  for  $\nu < \nu_m$ , and  $\alpha = -(p-1)/2$  for  $\nu > \nu_m$ . Since the comoving electron density

$$n'_e \propto \gamma n_{ext} \propto t_\oplus^{-(3+k)/(8-2k)},$$

and the comoving width of the fireball

$$\Delta R' \propto R/\gamma \propto t_\oplus^{(5-k)/(8-2k)},$$

then the comoving intensity

$$I'_\nu \propto n'_e B' \Delta R' \propto t_\oplus^{-(1+2k)/(8-2k)}$$

(Mészáros & Rees 1997; Wijers et al. 1997). Thus, the observed peak flux as a function of time is

$$F_{\nu_m} \propto t_\oplus^2 \gamma^5 I'_{\nu_m} \propto t_\oplus^{-k/(8-2k)}.$$

Once  $\nu_m$  has entered the optical region, the observed flux at any frequency must vary according to

$$F_\nu = F_{\nu_m} (\nu/\nu_m)^\alpha \propto t_{\oplus}^{-b}, \quad (33)$$

where for  $\nu < \nu_m$ ,

$$b = -\frac{2-k}{4-k}; \quad (34)$$

for  $\nu > \nu_m$ ,

$$b = \frac{k}{8-2k} + \frac{3(p-1)}{4}. \quad (35)$$

It can be seen from equations (33)–(35) that only for  $k < 2$  the observed optical flux first increases and then decreases, but for  $k = 2$  the flux is first kept constant and subsequently declines.

#### 4 DISCUSSION

We first use our model to discuss the X-ray and optical afterglow of GRB 970228. According to the observational results summarized by Wijers et al. (1997), we find  $p \approx 2.4$  and  $k \approx 0$  by solving equations (31) and (35), which is consistent with the results of Waxman (1997a). The result of  $p \approx 2.4$  is consistent with the mean spectral index ( $p = 2.5$ ) of GRBs measured in Band et al. (1993). This shows that the spectral index of the electron distribution due to shock acceleration is likely to be similar for GRBs and their afterglows.

Our model can also be applied to discussing the X-ray afterglow of GRB 970616. This burst was detected by BATSE on 16.757 June UT. About 20 min after the initial trigger, a transient X-ray *RXTE* (*Rossi X-ray Timing Explorer*) source was found in the error box of this burst (Connaughton et al. 1997), and 4 h after the burst, scanning observations with the Proportional Counter Array on the *RXTE* revealed an X-ray afterglow in the band 2–10 keV with a flux  $\sim 1.1 \times 10^{-11} \text{ erg s}^{-1} \text{ cm}^{-2}$  (Marshall et al. 1997). On 20.35 June UT, *ASCA* detected an X-ray flux from the XTE/IPN error box of GRB970616, with  $\sim 3.7 \times 10^{-14} \text{ erg s}^{-1} \text{ cm}^{-2}$  in the band 0.7–7 keV (Murakami et al. 1997). Using these values of the X-ray flux in equation (29), we get  $a \approx 1.86$ . After knowing the value of  $a$  and assuming that  $p$  is equal to the mean spectral index of GRBs measured in Band et al. (1993), that is,  $p \approx 2.5$ , we solve equation (35) and find  $k = 2$ . This result implies non-uniformity of the surrounding medium.

We have found that the afterglows of GRB 970616 and GRB 970228 are well explained by assuming two cases with  $k = 2$  and 0 respectively. We easily understand these two cases. In the first case, a neutron star (as the GRB source) has lain in a supernova remnant and/or a stellar wind because of the low velocity of the star, so the postburst fireball has expanded in this non-uniform medium; but in the second case, we conjecture that since the velocity of a neutron star as the progenitor of the GRB was very high, the GRB source has left a supernova remnant and/or a stellar wind, and the fireball has met the uniform interstellar medium.

#### 5 SUMMARY

A GRB has been commonly believed to result from the dissipation of the kinetic energy of a relativistically expanding fireball, and its X-ray, optical and/or radio afterglow is a result of the continuous deceleration of the fireball. In this paper, we have assumed that the expansion of the fireball is adiabatic and ultra-relativistic. If compact objects (neutron stars or black holes) are the origin of

the GRB, the surrounding medium of the fireball may be non-uniform due to the existence of a stellar wind and/or a supernova remnant. For simplicity, we have assumed that the density of the medium is a power-law function of shock radius, i.e.  $n_{\text{ext}} \propto R^{-k}$ . In addition, radiation losses may significantly influence the hydrodynamical evolution of the fireball (Sari 1997). In view of these two important arguments, we have analytically studied the effects of the first-order radiative correction and the non-uniformity of the medium on the GRB afterglow in this paper. The results of our study are summarized as follows.

First, we have derived a new relation among the observed time, the shock radius and the Lorentz factor of the fireball. We have also obtained a new relation among the comoving time, the shock radius and the Lorentz factor of the fireball. We have shown that, in the case of a uniform medium, the former relation turns out to be consistent with the expression derived by Sari (1997), but the latter relation is smaller than the usually used expression by a factor of 5/2.

Secondly, we have used the analytic solution of Blandford & McKee (1976) to derive the Lorentz factor of the fireball and the shock radius as functions of observed time, which show that the non-uniformity of the medium must shorten ( $k < 0$ ) or prolong ( $k > 0$ ) the relativistic expansion of the fireball. Using these functions, we have further derived the radiation energy loss rate, and found that the first-order radiative correction may be insignificant. This conclusion disagrees with that of Sari (1997), who neglected the radiative efficiency defined in equation (13).

Thirdly, we have derived new scaling laws both between the X-ray flux and observed time and between the optical flux and observed time. We have found that only for  $k < 2$  does the observed optical flux first increase and then decrease, but for  $k = 2$  the flux is first kept constant and subsequently declines.

Finally, we have used our model to discuss the afterglows of GRB 970616 and GRB 970228. We have seen that the afterglow of GRB 970616 is well fitted by assuming  $k = 2$ . This value implies of the non-uniformity of the medium.

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