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HYDRODYNAMICS OF GAMMA-RAY BURST AFTERGLOW

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ABSTRACT

The detection of delayed emission at X-ray optical and radio wavelengths ("afterglow") following gamma-ray bursts (GRBs) suggests that the relativistic shell that emitted the initial GRB as the result of internal shocks decelerates on encountering an external medium, giving rise to the afterglow. We explore the interaction of a relativistic shell with a uniform interstellar medium (ISM) up to the nonrelativistic stage. We demonstrate the importance of several effects that were previously ignored and must be included in a detailed radiation analysis. At a very early stage (few seconds), the observed bolometric luminosity increases as t^2. On longer timescales (more than ∼10 s), the luminosity drops as t^{-1}. If the main burst is long enough, an intermediate stage of constant luminosity will form. In this case, the afterglow overlaps the main burst; otherwise there is a time separation between the two. On the long timescale, the flow decelerates in a self-similar way, reaching nonrelativistic velocities after ∼30 days. Explicit expressions for the radial profiles of this self-similar deceleration are given. As a result of the deceleration and the accumulation of ISM material, the relation between the observed time, the shock radius, and its Lorentz factor is given by $t = R/16\gamma^2 c$, which is a factor of 8 different from the usual expression. We show that even though only a small fraction of the internal energy is given to the electrons, most of the energy can be radiated over time. If the fraction of energy in electrons is greater than ∼10%, radiation losses will significantly influence the hydrodynamical evolution at early times (less than ∼1 day).

Subject headings: gamma rays: bursts — hydrodynamics — relativity — shock waves

1. INTRODUCTION

The isotropy of the angular distribution of gamma-ray bursts (GRBs), combined with the nonhomogeneous distribution, suggests that GRBs originate from cosmological distances and therefore radiate energies of order of 10^{51} ergs. Considerations of optical depth then show that the bursts are produced by the dissipation of kinetic energy of highly relativistic shells with Lorentz factor $\eta > 100$ (see Piran 1996 for review). This dissipation can be caused by internal shocks or the surrounding ISM. Sari & Piran (1997) have shown that deceleration on the ISM could not give rise to the variability observed in the bursts (unless the process is very inefficient and involves much more than 10^{51} ergs), while internal shocks could produce the observed fluctuations efficiently (Kobayashi, Piran, & Sari 1997) if the "inner engine" has considerable fluctuations. It is therefore likely that the main GRB is due to internal shocks.

These cosmological models predict that after the main GRB event the ejecta decelerate, owing to interaction with the ISM, emitting radiation at longer and longer wavelengths (Paczyński & Rhoads 1993; Katz 1994; Mészáros & Rees 1997). This emission has been detected recently for several GRBs from an accurate determination of their position. The quantitative agreement between the deceleration models and the measurements is good (Waxman 1997a, 1997b; Wijers, Rees, & Mészáros 1997). Since the quality of data for the afterglows is higher than for the burst itself, more quantitative results are needed.

In this Letter we explore the hydrodynamics of the deceleration of a relativistic fireball on a uniform ISM. We discuss the relation between the rise time of the afterglow and the time of the main burst. We use the analytic solution found by Blandford & McKee (1976) to describe the swept-up ISM. We show that although only a small fraction of the internal energy is given to the radiating electrons, a considerable amount of the energy can be radiated over the deceleration period.

2. THE RISE OF THE AFTERGLOW

The problem of deceleration of a relativistic shell onto the ISM is determined by four parameters: the initial shell's Lorentz factor η, the energy of the shell $E = E_{52}10^{52}$ ergs, the width of the shell Δ, and the ISM density $n = n_1$ cm^{-3}. Here and throughout this Letter, all hydrodynamical quantities such as particle density, energy density, and pressure are measured in the fluid rest frame, while length is measured in the observer's frame. The basic details of the interaction between the shell and ISM were given in Sari & Piran (1995), and we briefly review the main ideas. The treatment in this section is approximate, and correction factors of order unity may need to be included in a more precise treatment.

When the shell encounters the ISM, two shocks are formed: a forward shock accelerating the ISM and a reverse shock decelerating the shell. The forward shock is always highly relativistic since the initial Lorentz factor $\eta \gg 1$. Let f be the density ratio between the preshock fluid in the shell and in the ISM, given by

$$f = E/(16\pi\eta^2 \Delta n m_p c^4 \gamma^4 t^2), \qquad (1)$$

where we used $R = 2c\gamma^2 t$ for the radius of the shell and t is the observed time. The reverse shock is relativistic if $f < \eta^2$, reducing the Lorentz factor of the shocked fluid from η to $\gamma = \eta^{1/2} f^{1/4}/\sqrt{2}$. The reverse shock is Newtonian if $f > \eta^2$, making only a negligible change to the shell's Lorentz factor, i.e., $\gamma = \eta$. At early stages, $f \gg \eta^2$, so the reverse shock is Newtonian and the shell's Lorentz factor equals its initial value $\gamma = \eta$. However, due to the increase in the area of the shell, it produces internal energy in an increasing rate of

$$L = 32\pi c^5 n m_p \gamma^8 t^2 = 2.5 \times 10^{50} \gamma_{300}^8 n_1 t_s^2 \text{ ergs s}^{-1}, \qquad (2)$$

where we use t_s for the time in seconds. Assuming that the cooling is fast (Sari, Narayan, & Piran 1996), the observed

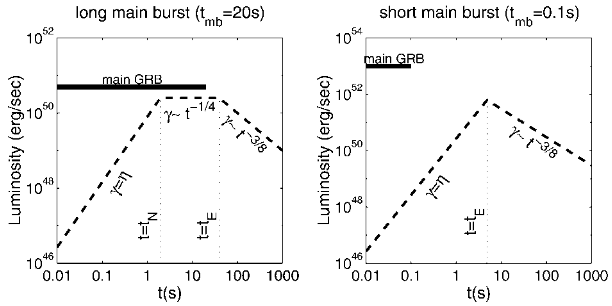

FIG. 1.—Luminosity from the ISM as function of time in the relativistic (*left*) and Newtonian (*right*) cases is drawn as a dashed line. At the early stage, the Lorentz factor is constant and the luminosity increases due to the increase in shell area. When the ISM has energy comparable with the total energy ($t = t_E$), a self-similar solution is established and the luminosity drops as t^{-1}. If the shell is thick (typical for long main bursts), the reverse shock becomes relativistic at t_N, before the self-similar solution is established and some deceleration begins, leading to constant luminosity. The solid line gives the luminosity of the main GRB. Both frames use $E = 10^{52}$ ergs and $\eta = 300$. The behavior $L \sim t^{-1}$ and $\gamma \sim t^{-3/8}$ continues up to the nonrelativistic stage, which is about 30 days.

bolometric luminosity is proportional to the internal energy increase rate and is therefore also given by equation (2). This behavior will continue until either the shell has given the ISM an energy comparable to its initial energy at

$$t_E = \left(\frac{3E}{32\pi c^5 n m_p \eta^8}\right)^{1/3} = 5\ E_{52}^{1/3}\eta_{300}^{-8/3}n_1^{-1/3}\ \mathrm{s} \qquad (3)$$

or until the reverse shock is no longer Newtonian, i.e., $f = \eta^2$ at

$$t_N = \left(\frac{E}{16\pi\Delta n m_p c^4 \eta^8}\right)^{1/2}$$
$$= 2\ E_{52}^{1/2}\left(\frac{\Delta}{6\times10^{11}\ \mathrm{cm}}\right)^{-1/2}\eta_{300}^{-4}n_1^{-1/2}\ \mathrm{s}, \qquad (4)$$

whichever comes first. As in Sari & Piran (1995), we define the ratio between the two expressions as

$$\xi \equiv \frac{t_N}{t_E} = \left(\frac{E}{36\pi n m_p c^2 \Delta^3 \eta^8}\right)^{1/6}$$
$$= 0.4\ E_{52}^{1/6}\left(\frac{\Delta}{6\times10^{11}\ \mathrm{cm}}\right)^{-1/2}\eta_{300}^{-4/3}n_1^{-1/6}. \qquad (5)$$

For $\xi > 1$, the energy is dissipated to internal energy before the Lorentz factor of the shell is reduced considerably. If $\xi < 1$, then the reverse shock turns relativistic before the kinetic energy of the shell was emitted. In this case the Lorentz factor decreases with time according to

$$\gamma(t) = \eta^{1/2}f(t)^{1/4}/\sqrt{2}$$
$$= 300\ E_{52}^{1/8}\left(\frac{\Delta}{6\times10^{11}\ \mathrm{cm}}\right)^{-1/8}n_1^{-1/8}t_s^{-1/4}. \qquad (6)$$

Note that at this stage the Lorentz factor γ is independent of its initial value η. Substituting this in equation (2), we see that the luminosity is constant in time and is given by

$$L = \frac{E}{2\Delta/c}. \qquad (7)$$

This stage will continue until the shell has given the shocked ISM energy comparable with its own at

$$t_E = 2\Delta/c. \qquad (8)$$

At this time the shell has decelerated to Lorentz factor of

$$\gamma = \left(\frac{E}{256\pi\Delta^3 n m_p c^2}\right)^{1/8}$$
$$= 120\ E_{52}^{1/8}\left(\frac{\Delta}{6\times10^{11}\ \mathrm{cm}}\right)^{-3/8}n_1^{-1/8}, \qquad (9)$$

independent of the initial Lorentz factor η. After the time t_E, given by equations (3) or (8), the flow will be described by a self-similar solution since the ISM energy is now constant and comparable to the initial energy of the shell E. As we shall show in § 4, from this time $\gamma \propto R^{-3/2} \propto t^{-3/8}$, and therefore the observed luminosity decreases as t^{-1}. This behavior is illustrated in Figure 1.

3. RELATION WITH THE MAIN BURST

If the main burst is produced by internal shocks, then the width of the shell, Δ, can be inferred directly from the observed main burst duration $\Delta = ct_{\mathrm{mb}}$. For long bursts, $t_{\mathrm{mb}} \sim 20$ s and $\Delta \sim 6 \times 10^{11}$ cm, while for short bursts $t_{\mathrm{mb}} \sim 0.1$ s and $\Delta \sim 3 \times 10^9$ cm. The initial Lorentz factor must satisfy $\eta > 100$ for the emission of the main burst not to be opaque. The reverse shock is therefore likely to be Newtonian for short bursts and might be relativistic for long bursts. Both cases are therefore of physical interest.

If the reverse shock is relativistic, then the observed peak of the afterglow emission is flat and overlaps the observed GRB emitted by internal shocks. Both end after an observed time of $\sim\Delta/c$. If the reverse shock is Newtonian, then the afterglow peaks on t_E given by equation (3), which is longer by a factor of

$$\frac{3}{2}\xi^2 > 1 \qquad (10)$$

than the main burst duration Δ/c. The duration and luminosity of the main burst and the afterglow rise are shown for the Newtonian and relativistic cases in Figure 1.

In both cases, the properties of the main burst and the afterglow are very different. The main burst is usually highly variable (depending on the internal structure of the shell), while the afterglow, which is due to external shocks, is expected to be smooth (Sari & Piran 1997). The afterglow's spectrum should peak, in the beginning, around 30 keV–10 MeV, depending on the fraction of internal energy in electrons and magnetic field (Sari, Narayan, & Piran 1996). If the peak energy is too high, it might not be observed in the first stage by the BATSE equipment. However, later as the ejecta decelerates, the emission peak decreases in time and should cross the soft γ-ray region.

4. THE SELF-SIMILAR SOLUTION

We begin with a simple consideration based upon conservation of energy. When most of the energy has been given to the ISM, and assuming that radiation losses are small, the energy in the shocked ISM is constant and approximately equal to the initial kinetic energy E. The shocked ISM rest mass is $M \propto R^3$. Since it was heated by a relativistic shock, its energy in the observer frame is $\sim M\gamma^2$. Comparing this with the constant total energy of the system E, we get that

$$\gamma \propto R^{-3/2}. \tag{11}$$

This is also the scaling law for the shock wave Lorentz factor Γ since for a relativistic shock $\Gamma = \sqrt{2}\gamma$.

The scaling law (eq. [11]), implies a quantitative but important change in the relation between t, R, and γ. Photons that were emitted from the shock while it has propagated a small distance δR will be observed on timescale of $\delta t \sim \delta R/2\Gamma^2 c$. Integrating this over time using the scaling law, we get $t = R/8\Gamma^2 c$, or

$$t = \frac{R}{16\gamma^2 c}. \tag{12}$$

Compared with the commonly used expression $R/2\gamma^2 c$ (Mészáros & Rees 1997; Waxman 1997a, 1997b; Wijers et al. 1997), this expression is factor of 8 smaller. This difference is important when trying to fit quantitatively the observed afterglow data. Note, however, that the differential relation is independent of the deceleration and is therefore given by $\delta t = \delta R/4\gamma^2 c$.

Blandford & McKee (1976) have described an analytical solution for the case in which the scaling law (eq. [11]) applies. Using their solution with several simplifications and some algebraic manipulations, we get

$$\begin{aligned} n(r, t) &= 4n\gamma\,[1 + 16\gamma^2(1 - r/R)]^{-5/4}, \\ \gamma(r, t) &= \gamma\,[1 + 16\gamma^2(1 - r/R)]^{-1/2}, \\ e(r, t) &= 4nm_p c^2\gamma^2\,[1 + 16\gamma^2(1 - r/R)]^{-17/12}, \end{aligned} \tag{13}$$

where $n(r, t)$, $\gamma(r, t)$, and $e(r, t)$ are, respectively, the density, Lorentz factor, and energy density of the material behind the shock [not to be confused with the ISM density n and the Lorentz factor of material just behind the shock, $\gamma(t) = \gamma(R, t)$]. This solution was derived by assuming extreme relativistic motion both of the shock and the fluid behind it. Equation (13) is therefore only valid for values of r and t for which $\gamma(r, t) \gg 1$.

The scaling laws of $R(t)$ and $\gamma(t)$ can be found using these profiles and requiring that the total energy in the flow would be equal to E:

$$\begin{aligned} R(t) &= \left(\frac{17Et}{\pi m_p nc}\right)^{1/4} = 3.2 \times 10^{16} E_{52}^{1/4} n_1^{-1/4} t_s^{1/4}\ \mathrm{cm}, \\ \gamma(t) &= \frac{1}{4}\left(\frac{17E}{\pi n m_p c^5 t^3}\right)^{1/8} = 260 E_{52}^{1/8} n_1^{-1/8} t_s^{-3/8}. \end{aligned} \tag{14}$$

This solution can serve as a starting point for detailed radiation emission calculations and comparison with observations. The scalings given by equation (14) are, of course, consistent with the scalings given by equations (11) and (12), which were derived using conservation of energy but supply the exact coefficient that cannot be produced otherwise. The time at which the flow behind the shock becomes subrelativistic follows from equation (14) as

$$t = 30 E_{52}^{1/3} n_1^{-1/3}\ \mathrm{days}. \tag{15}$$

5. RADIATIVE CORRECTIONS

In the previous sections we have assumed that the energy in the system is constant. This assumption cannot be strictly correct since the radiation takes some energy from the relativistic shell. We define ϵ_e to be the fraction of the internal energy that is radiated and lost from the system. Typically this should be the fraction of energy given by the shock to electrons and is estimated to be around 10% (Waxman 1997a, 1997b). This number seems to be negligible, and therefore the energy loss was neglected by previous analyses. However, the deceleration occurs over several orders of magnitude in time and Lorentz factor, and the fireball energy E is given again and again to newly heated material, leading to more and more energy losses.

The energy loss rate during the deceleration is given by $4096\pi c^5 n m_p \gamma^8 t^2$ (the coefficient is different from eq. [2], owing to the relation $R = 16\gamma^2 ct$, which is relevant in the deceleration stage) multiplied by ϵ_e. Substituting the expression for $\gamma(t)$ from the self-similar solution (eq. [14]), we get

$$\frac{dE}{dt} = -\frac{17}{16}\epsilon_e \frac{E}{t}, \tag{16}$$

so that

$$E(t) = E_0 \left(\frac{t}{t_0}\right)^{-17\epsilon_e/16}. \tag{17}$$

Since the observed initial time of the afterglow is about 10 s, after about 1 week the energy is reduced by a factor of $\sim$3 for $\epsilon_e = 0.1$ or a factor of $\sim$30 if $\epsilon_e = 0.3$. These factors must be taken into account given the accuracy of current data.

The derivation of the above exponent, $-17\epsilon_e/16$, used the exact coefficients in equation (14), which were obtained from the self-similar solution. Without this solution the exponent could only be estimated approximately. Note that the use of equation (14) is valid as long as ϵ_e is small enough that the energy loss could be approximated as a small "radiative correction."

These radiation losses will also slightly affect the scaling of the shock radius and Lorentz factor as function of time. The approximate scaling including the radiation losses can be obtained by substituting equation (17) into equation (14).

6. DISCUSSION

We have explored the early evolution of the interaction of a relativistic shell with the ISM. If the main GRB is short enough, separation is expected between the main burst and the afterglow luminosity peak, while if it is long enough an overlap is expected. This property might be detectable in BATSE's data.

We have used the self-similar solution derived by Blandford

& McKee (1976) to obtain an explicit expression for the radial profile in the self-similar stage. This solution can be used in further analyses when considering a more detailed calculation of the radiation from the heated ISM.

The relation between the shock's radius, the material Lorentz factor, and the observed time was found to be $t = R/16\gamma^2 c$ instead of the commonly used expression $t = R/2\gamma^2 c$ because ISM is collected so the shock moves faster than the material behind it, and because of the deceleration of the shell, having higher Lorentz factor at earlier time. This relation was obtained assuming that the radiation is emitted from the shock front. On long timescales of ∼1 day, when the cooling of electrons is not fast enough, it might be that the width of the radiating zone will smear the observed radiation over longer timescales.

The role of energy loss due to the radiation was found to be nonnegligible even if the part of the internal energy that is radiated at each time is small. Thus, radiation can reduce the total energy in the system after 1 week by a factor of 3 if $\epsilon_e = 0.1$ or by a factor of 30 if $\epsilon_e = 0.3$.

The data of GRB 970228 and GRB 970508 fitted radiation models are within a factor of 2, without taking into account radiation losses (Waxman 1997a, 1997b). We can therefore roughly estimate $\epsilon_e \leq 0.1$. This estimate might change with a more self-consistent calculation that includes the effects described here. On timescale more than ∼1 day, the electron's cooling time is long, so only a small fraction of their energy is radiated. Since most of the observations were made after ∼1 day, the fraction of the energy that is given to the electrons can be high (more than 10%) without leading to considerable energy loss and with no effect on the observations made after ∼1 day. However, in such a case, the energy losses at earlier times will be considerable and will therefore require a much higher initial energy.

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