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Radio scintillation of gamma-ray-burst afterglows

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Abstract

Stars twinkle to the eye through atmospheric turbulence, but planets, because of their larger angular size, do not. Similarly, scintillation due to the local interstellar medium will modulate the radio flux of gamma-ray-burst afterglows and may permit indirect measurements of their angular sizes. The amplitude of refractive scintillation is of order ten percent at ten gigahertz unless the source size is much larger than the expected size, of order microarcseconds. Diffractive scintillation is marginally possible, depending sensitively on the source size, observing frequency, and scattering measure of the interstellar medium. © 1997 Elsevier Science B.V.

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1. Introduction

Recent identification of a few gamma-ray bursts (GRBs) with transient sources at longer electromagnetic wavelengths ("afterglows") has excited the field and raised hopes of insight into the nature of GRBs. The critical element in these developments has been the Italian-Dutch BeppoSAX satellite and its capability for wide-field, simultaneous gamma-ray and X-ray observations.

Measurements of the X-ray afterglow of GRB960720 provided a precise position for this burst (Piro et al., 1997). The duration of the event as observed at different energies was correlated with wavelength roughly as $\Delta t \propto \lambda^{0.46}$, a pattern that has been repeated, at least qualitatively, in several subsequent bursts. Six months later, BeppoSAX detected a second X-ray transient associated with GRB970111 (Costa et al., 1997a). A few weeks after that, GRB970228 became the first burst discovered to have an optical afterglow (Costa et al., 1997b,c; Van Paradijs et al., 1997). Both the optical and the X-ray event continued to be detectable at least hours, and possibly weeks, after the GRB (Sahu et al., 1997; Costa et al., 1997b,c; Yoshida et al., 1997).

The next major milestone was GRB970508. Originally localized by BeppoSAX (Costa et al., 1997d) and optically identified by Bond (1997), the afterglow of this GRB revealed optical absorption lines of iron and magnesium at $z = 0.835$, confirming the cosmological distance of the source (Metzger et al., 1997). GRB970508 was also the first burst convinc-

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ingly associated with a radio transient (Frail & Kulkarni, 1997; Pooley & Green, 1997).

These X-ray-to-radio afterglows are qualitatively consistent with models based on relativistic blastwaves at cosmological distances (Rees & Mészáros, 1992; Paczyński & Rhoads, 1993; Katz, 1994; Vietri, 1997; Wijers et al., 1997; Waxman, 1997a,b; Mészáros & Rees, 1997). In these models, the shock Lorentz factor is $\gtrsim 300$ at the time of the gamma-ray event and falls thereafter as $\Gamma_s \propto t^{-3/8}$, t being the time measured at Earth – or somewhat faster if radiative energy loss by the shock is taken into account (Sari, 1997). The bulk Lorentz factor of the postshock material is $\Gamma_{ps} = 2^{-1/2}\Gamma_s$ when $\Gamma_s \gg 1$. Most of the models assume that principle emission mechanism is synchrotron radiation, and that the instantaneous flux density F_ν peaks at a frequency $\nu_{max} \propto t^{-q}$ for various exponents $q > 0$.

A model-independent lower limit to the angular size $\theta_s = r_s/D$ of the radio source can be derived if the brightness temperature is less the Compton limit for incoherent synchroton radiation, $T_b \lesssim \Gamma_{ps}(1+\beta_{ps})T_{max}$ with $T_{max} \approx 10^{12}$ K (Kellermann & Pauliny-Toth, 1969):

$$\theta_s \geq 1.5 \left(\frac{1+z}{(1+\beta_{ps})\Gamma_{ps}}\right)^{1/2} \left(\frac{F_\nu}{\mathrm{mJy}}\right)^{1/2} \nu_{10}^{-1/2} \times \left(\frac{T_{max}}{10^{12}\ \mathrm{K}}\right)^{-1/2} \mu\,\mathrm{a.s.}, \tag{1}$$

where $\nu_{10} \equiv \nu/(10\ \mathrm{GHz})$.

In blastwave models, most of the emission is seen from a disk (spherical cap) of radius $\sim 2\Gamma_{ps}ct$ [the precise numerical factor depends on details (Sari, 1997; Waxman, 1997c)], so

$$\theta_s = \frac{ct}{(1+z)d} \times \begin{cases} 2\Gamma_{ps} & \Gamma_{ps} \gg 1; \\ (25/6)\beta_{ps} & \beta_{ps} \equiv (1-\Gamma_{ps}^{-2})^{1/2} \ll 1, \end{cases} \tag{2}$$

where d is the angular-diameter distance of the source, and the nonrelativistic alternative assumes the Sedov solution with adiabatic index 5/3. If we require that the angular size predicted by Eq. (1) be compatible with the lower limit Eq. (2) derived from the observed flux, we can derive lower bounds on Γ_s and θ_s. Thus for example, the radio counterpart of GRB970508 had a flux density of 0.61 ± 0.04 mJy at 8.46 GHz 6.2 days after the burst (Frail & Kulkarni, 1997). Smoothly interpolating between the ultrarelativistic and nonrelativistic limits, we find $\beta_{ps} \gtrsim 0.6$ and $\theta_s \gtrsim 1.5\ \mu$ a.s. at the time of this observation, assuming $z = 1$ and $(1+z)d = 10^{28}$ cm (3 Gpc).

On the other hand, it could be that the events are more jet-like than spherical and are seen as GRBs if the observer lies near the axis of the jet. Jet models predict angular sizes a few times smaller than corresponding spherical ones, and have less radio flux (Rhoads, 1997; Katz & Piran, 1997). Jet models generally predict that for every GRB with an afterglow, there should be many afterglows without an associated GRB, since the long-wavelength emission is less tightly beamed.

To summarize the above discussion, we may expect radio afterglows of typical (i.e. distant) GRBs to have angular sizes on the order of one to a few more micro-arcseconds (μ a.s.). If this is correct, then most radio afterglows will be unresolved by VLBI in the foreseeable future. An upper limit of 300 μ a.s. has been set by the VLBA eight days after GRB970508 (Taylor et al., 1997).

Afterglows may, however, be resolved indirectly by interstellar scattering. Interstellar propagation effects will cause the flux of a compact source to vary (scintillate), whereas a more extended source will be steady. Hence by monitoring the flux, one can obtain indirect information on the angular size, in much the same way that the eye can distinguish stars from planets by whether or not they twinkle.

For the sake of completeness, Section 2 offers a brief pedagogical overview of the theory of interstellar scintillation. Section 3 applies scintillation theory to GRB afterglows. Some modest extensions (outlined in the Appendix) of the standard results are required because the radio afterglows are predicted to lie near the boundary between two or three asymptotic regimes of scintillation. Section 4 summarizes the main results and uncertainties.

2. Interstellar scintillation

This section is intended as a brief review of the main features of the subject. A more comprehensive review can be found in Rickett (1990) and references therein. Table 1 summarizes the most important symbols we will use.

Because of free electrons in the interstellar medium (ISM), the index of refraction for radio waves is less than unity:

$$n \equiv \frac{c}{\lambda\nu} = \left(1 - \frac{\nu_p^2}{\nu^2}\right)^{1/2} \approx 1 - \frac{\nu_p^2}{2\nu^2} \qquad (3)$$

where ν is the frequency, λ the wavelength, and

Table 1
Key to important symbols

Symbol	Meaning	Where defined
b	galactic latitude of source	–
C_N^2	turbulence strength	Eq. (5)
$D_N(\vec{s})$	electron-density structure function	Eq. (6)
$d_{\rm scr}$	distance of scattering screen	Section 2
$\Delta\nu_{\rm dc}$	decorrelation bandwidth	Eq. (17)
$\delta N_e(s)$	electron-density fluctuation on scale s	Section 2
$f(s)$	focal length of $\delta N_e(s)$	Section 2
Γ_s	Lorentz factor of shock	Section 1
$\Gamma_{\rm ps}$	Lorentz factor of postshock gas	Section 1
H	ISM scale height	Section 2
λ	radio wavelength	Section 2
m_R	refractive modulation index	Eqs. (20) and (A.5)
N_e	electron density	Section 2
n	refractive index	Eq. (3)
ν	radio frequency	Section 1
ν_{10}	$\nu/(10^{10}\text{Hz})$	Section 2
ν_p	plasma frequency	Eq. (4)
$\nu_{\rm peak}$	ν for maximum m_R	Eq. (23)
$\nu_{\rm quench}$	minimum frequency for DISS	Section 3.2
$\nu_{\rm ss}$	maximum ν for strong scattering	Eq. (13)
$\vec{q}$	spatial wavenumber	Section 2
$\Phi_{N_e}(\vec{q})$	electron-density power spectrum	Eq. (5)
r_e	classical radius of the electron: e^2/m_ec^2	Section 2
SM	scattering measure	Eq. (11)
$SM_{-3.5}$	$SM/(10^{-3.5}m^{-20/3}\text{ kpc})$	Section 2
$\vec{s}$	spatial separation	Section 2
s	spatial scale in turbulence	Section 2
$s_{\rm d}$	diffractive length	Eq. (9)
$s_{\rm r}$	refractive length	Section 2
$t_{\rm diff}$	diffractive timescale	Eq. (16)
$t_{\rm ref}$	refractive timescale	Eq. (19)
$\theta_{\rm eff}$	effective angular size	Eq. (18)
θ_F	Fresnel angle	Eq. (12)
$\theta_{\rm s}$	source angular size	Section 2
$\theta_{\rm d}$	scattering angle	Eq. (10)
$v_\perp$	transverse velocity of line of sight	Eq. (16)

$$\nu_p^2 = \frac{N_e e^2}{\pi m_e} \approx (9.0\ \mathrm{kHz})^2 \left(\frac{N_e}{\mathrm{cm}^{-3}}\right) \tag{4}$$

is the square of the plasma frequency, which is proportional to the density of free electrons (N_e). A local concentration of electrons acts as a diverging lens, since $\delta n = -(r_e\lambda^2/2\pi)\delta N_e$, where $r_e \equiv e^2/m_e c^2$ is the classical radius of the electron.

Although it has not been satisfactorily explained, there is extensive evidence observational evidence showing that electron-density fluctuations are well described by a power-law power spectrum across a broad range of spatial scales (at least 10^9 to 10^{14} cm):

$$\Phi_{N_e}(\vec{q}) \equiv (2\pi)^{-3}\int d^3\vec{s}\langle\delta N_e(\vec{x})\delta N_e(\vec{x}+\vec{s})\rangle e^{i\vec{q}\cdot\vec{s}} = C_N^2 q^{-11/3}, \tag{5}$$

where $\vec{q}$ is the spatial wavenumber (Armstrong et al., 1995). The strength of the inhomogeneities is determined by C_N^2. Although this parameter is locally constant, it varies considerably on scales of parsecs or more. A typical value near the Sun is $C_N^2 \sim 10^{-3.5}\ \mathrm{m}^{-20/3}$.

A spectrum of the form Eq. (5) governs a passive scalar contaminant (e.g. dye) in the inertial range of incompressible fluid turbulence (Obukhoff, 1949). Atmospheric density fluctuations that are responsible for optical seeing have such a spectrum (Coulman, 1985). Since ISM turbulence is magnetized and often supersonic or super-alfvenic, the apparent universality of the spectrum Eq. (5) remains mysterious (Goldreich & Sridhar, 1995).

Unless the power-law form of $\Phi_{N_e}(q)$ is restricted by a cutoff at small q, the inverse Fourier transform for the correlation function $C(\vec{s}) \equiv \langle\delta N_e(\vec{x})\delta N_e(\vec{x}+\vec{s})\rangle$ is ill-defined. But the physically important quantity for scintillation is the *structure function*

$$D_N(\vec{s}) \equiv \langle[N_e(\vec{x}) - N_e(\vec{x}+\vec{s})]^2\rangle = 2\int d^3\vec{q}\ \Phi_{N_e}(\vec{q})\left(1 - e^{-i\vec{q}\cdot\vec{s}}\right), \tag{6}$$

which is well-defined without the cutoff: in fact $D_N(\vec{s}) \propto s^{2/3}$ for $s \ll q_{\min}^{-1}$. The structure function is related to the correlation function by $D(\vec{s}) = 2C(0) - 2C(\vec{s})$. Expressions such as "the characteristic electron-density fluctuation on scale s" and "$\delta N_e(s)$" are to be interpreted as $[D_N(\vec{s})]^{1/2}$. It is often assumed that the spectrum and structure function are isotropic, as implied by Eq. (5), so that they can be regarded as functions of q and s rather than $\vec{q}$ and $\vec{s}$. Corresponding to Φ_{N_e} and D_N are the spectrum Φ_n and structure function D_n of the refractive index, which are obtained from the former upon multiplying by $\lambda^4(r_e/2\pi)^2$. Hence scattering is much stronger at low frequencies than high.

Satisfactory treatments of scintillation require detailed calculations with physical optics, but the main effects can be understood heuristically. As applied to interstellar scintillation of pulsars, these ideas were first developed by Scheuer (1968) and extended to power-law spectra by Lovelace (1970). An individual spherical inhomogeneity of size s and amplitude $\delta N_e(s)$ deflects a ray through a very small angle of order $\delta n(s) \propto \lambda^2 s^{1/3}$. There are $\approx L/s$ such inhomogeneities along a line of sight of length L, and their deflections add incoherently. Hence the net deflection due to scale s is

$$\theta(s) = (r_e/2\pi)\lambda^2 \Delta N_e(s)\sqrt{\frac{L}{s}} \propto \lambda^2 (C_N^2 L)^{1/2} s^{-1/6}. \tag{7}$$

Smaller scales therefore scatter through larger angles, but very small scales are ineffective, for this reason: The accumulated phase shift due to inhomogeneities of scale s is

$$\phi(s) \approx \delta n(s)\frac{2\pi s}{\lambda}\sqrt{\frac{L}{s}} \propto \lambda (C_N^2 L)^{1/2} s^{+5/6}. \tag{8}$$

If $\phi(s) < 1$, then geometric optics does not apply on the scale s; in this case, physical optics shows that only a fraction $\sim\phi^2$ is scattered. Therefore, the characteristic scattering angle is determined by the scale s_d determined by

$$\phi(s_\mathrm{d}) = 1. \tag{9}$$

For reasons to be explained later, s_d is often called the *diffractive scale*.

The characteristic scattering angle is obtained by substituting s_d for s in Eq. (7):

$$\theta_d = \frac{\lambda}{2\pi s_d} = 2.93\left(\frac{\nu}{10\ \mathrm{GHz}}\right)^{-11/5} \times\left(\frac{SM}{10^{-3.5}\ \mathrm{m}^{-20/3}\ \mathrm{kpc}}\right)^{3/5}\ \mu\ \mathrm{a.s.} \quad (10)$$

The numerical constants follow from a more precise treatment. The *scattering measure SM* is the integral of C_N^2 along the line of sight:

$$SM \equiv \int_0^d dx C_N^2(x), \quad (11)$$

where x is a coordinate along the line of sight and d is the distance to the source. For an extragalactic source, the effective path length through the scattering medium, L, is $\ll d$. We have scaled the result Eq. (10) to a frequency of interest for GRB afterglows. Pulsars, which have steep spectra, are usually observed at 300 MHz or 1.4 GHz, and at these lower frequencies θ_d is typically of order a milliarcsecond.

For a pointlike source, θ_d can represent either a displacement in the apparent position of the source, or a broadening of its angular size. This depends on the focal length of the dominant scatterers, $f_d \sim s_d/\theta_d$, relative to their typical distance, d_{scr}. (When the source and observer are embedded in a statistically homogeneous scattering medium, then $d_{scr} \approx d/2$; when the source is a high-latitude extragalactic source, $d_{scr} \sim L$ is of order the local ISM scale height.) If $d_{scr} \ll f_d$, then the scatterers can displace the image but not divide it. This is the *weak scattering* regime. If $d_{scr} \gg f_d$, then multiple images of the source are formed, the number scaling as $(d_{scr}\theta_d/s_d)^2$. This is the *strong scattering* regime, and in this case θ_d can be interpreted as the effective size of the scatter-broadened source image (which actually consists of many sub-images). Since $d_{scr}/f_d = 2\pi d_{scr}\theta_d^2/\lambda$, strong scattering occurs when θ_d is larger than the *Fresnel angle*

$$\theta_F \equiv \left(\frac{\lambda}{2\pi d_{scr}}\right)^{1/2} \approx 2.57\nu_{10}^{-1/2} d_{scr,\ kpc}^{-1/2}\ \mu\ \mathrm{a.s.}, \quad (12)$$

where $\nu_{10} \equiv \nu/(10\ \mathrm{GHz})$ and $d_{scr,\ kpc} \equiv d_{scr}/\ \mathrm{kpc}$.

In terms of the frequency, the condition for strong scattering is

$$\nu < 10.4(SM_{-3.5})^{6/17} d_{scr,\ kpc}^{5/17}\ \mathrm{GHz} \equiv \nu_{ss}, \quad (13)$$

where $SM_{-3.5} \equiv (SM/10^{-3.5}\ \mathrm{m}^{-20/3}\ \mathrm{kpc})$. Here the numerical coefficient, but not the exponents, depends upon geometrical details. We have adopted the *thin-screen approximation*, in which one imagines that all of the scattering occurs in a narrow layer at distance d_{scr}, and we have taken $d \gg d_{scr}$. Since pulsars are observed at $\nu \ll 10$ GHz, their scintillation is usually well into the strong regime.

Since the boundary between weak and strong scattering can be derived from the focal length of the dominant scatterers, it is not surprising that strong scattering can involve large modulations of the flux. There are actually two forms of flux scintillation, *refractive* and *diffractive*.

Refractive scintillation is a geometric-optics effect – simply the random magnification of individual subimages. In strong scattering, refractive scintillation of a pointlike source actually declines (the flux varies less) as the scattering strength increases. This occurs because as the subimages proliferate, their individual magnifications tend to average out. For a smooth extended source of intrinsic angular size θ_s, refractive scintillation is negligible until the scattering is so strong that $\theta_d > \theta_s$, because refractive magnification conserves surface brightness.

Diffractive scintillation is a physical-optics interference effect. Each subimage corresponds to a ray path from source to observer. The k^{th} path is associated with a phase delay ϕ_k that depends on the length and refractive index along the path. If the receiving telescope is too small to separate these subimages (i.e., its angular resolution is larger than θ_d), then the received flux involves the coherent sum of the contributions from all paths:

$$F = \left|\sum_{k=1}^{N} M_k^{1/2} e^{i\phi_k}\right|^2,$$

where the $\{M_k\}$ are the path magnifications. In geometric optics, the individual magnifications could become infinite, but because of the finite radio wavelength and the small size of the scatterers, the magnifications are bounded and their mean values are $\sim \bar{F}/N$ if $\bar{F}$ is the mean flux of the source. In strong scattering, $N \gg 1$ and the phases are independent and large. It can then be shown that F has the probability density $p(F) = \bar{F}^{-1}\exp(-F/\bar{F})$, whence $\langle F^2 \rangle = 2\bar{F}^2$ (*Rayleigh statistics*). On an imaginary plane perpendicular to the line of sight and passing through the observer, the flux forms a speckle pattern – a chaotic distribution of bright patches separated by dark lanes, with a root-mean-square fluctuation in the flux equal to its mean. Speckle patterns commonly result from scattering of pointlike sources by random media. They can be produced, for example, by illuminating the ground or other surface with a laser beam that passes through a long path of turbulent air.

Speckle can be produced by incoherent sources if they are sufficiently compact (Born & Wolf, 1980; Marathay, 1982). There is no interference between the radiation from distinct points on the source, since the points are mutually incoherent. Hence each point produces its own speckle pattern. For reasonably compact sources, these speckle patterns are almost identical in form since they pass through the same medium, but the patterns are shifted with respect to one another in the observer's plane by distances of order $d_{\rm scr}\theta_s$, where θ_s is the angular size of the source. If these shifts are smaller than the characteristic size of a speckle, then speckles are visible in the total flux.

Two slits of separation a illuminated by a plane wave produce fringe spacing $\lambda d/a = \lambda/\Delta\theta$ on a screen at distance d, where $\Delta\theta \equiv a/d$ is the angular separation of the slits. In the problem at hand, each subimage is analogous to a slit, and the speckle pattern is a two-dimensional, chaotic distribution of fringes. The characteristic size of a speckle is

$$\frac{\lambda}{2\pi\theta_{\rm d}} = s_{\rm d}. \tag{14}$$

This explains why $s_{\rm d}$ is called the diffractive scale. Two points on the source separated by angle $\Delta\theta$ produce fringe patterns on the observer's plane that are almost identical but displaced by $d_{\rm scr}\Delta\theta$. In order that the speckle pattern not be smoothed out by the angular size of the source, it is necessary that $d_{\rm scr}\theta_s < s_{\rm d}$). With Eq. (10) and Eq. (14), the source size limit for diffractive scintillation becomes

$$\theta_s < 2.25\left(\frac{\nu}{10\ \rm GHz}\right)^{6/5}\left(\frac{d_{\rm scr}}{\rm kpc}\right)^{-1} \times (SM_{-3.5})^{-3/5}\ \mu\ \text{a.s.}, \tag{15}$$

This limit becomes more severe (smaller) as $\theta_{\rm d}$ becomes larger (stronger scattering). Pulsars are almost always sufficiently compact to scintillate diffractively, whereas bright ($F_\nu \sim$ Jy) extragalactic sources are not.

The timescale for diffractive scintillation is usually assumed to be dominated by the transverse motion of the line of sight through the speckle pattern:

$$t_{\rm diff} = \frac{s_{\rm d}}{v_\perp} = \frac{\lambda}{2\pi\theta_{\rm d}v_\perp} = 3.1\nu_{10}^{6/5}(SM_{-3.5})^{-3/5}\left(\frac{v_\perp}{30\ \rm km\ s^{-1}}\right)^{-1}\ \rm hr. \tag{16}$$

Another important measure of diffractive scintillation is the *decorrelation bandwidth*. In the thin-screen approximation, the difference in path length between a ray that passes through the center of the image and one that passes at distance $\theta_{\rm d}$ from the center is $\Delta l = d_{\rm scr}\theta_{\rm d}^2/2$. In addition to the geometric delay, there is a comparable delay due to the refractive index. Hence, the characteristic spread in ray propagation times is $\tau_p = d_{\rm scr}\theta_{\rm d}^2/c$. The relative phase between two typical rays changes with frequency roughly as $2\pi\tau_p\delta\nu$, which alters the speckle pattern even if the positions of the subimages remain fixed. Hence to show diffractive scintillation, the source must be observed with a bandwidth less than

$$\Delta\nu_{\rm dc} = \frac{1}{2\pi\tau_p} = \frac{c}{2\pi\theta_{\rm d}^2 d_{\rm scr}} \approx 7.6\nu_{10}^{22/5}(SM_{-3.5})^{-6/5}d_{\rm scr,\ kpc}^{-1}\ \rm GHz. \tag{17}$$

The relative decorrelation bandwidth $\Delta\nu_{\rm dc}/\nu$ decreases as scattering increases; at the boundary between weak and strong scintillation, $\Delta\nu_{dc} = \nu$ [cf. Eq. (13)].

We now return to refractive scintillation.

When analyzing refractive scintillation, it is often useful to treat the scatter-broadened image as though it were an extended incoherent source. This is most easily justified when diffractive scintillation has been quenched – by a source size larger than the limit Eq. (15), or an observing bandwidth is larger than Eq. (17) – because then the subimages are mutually incoherent. But for the purposes of refractive scintillation, it turns out that the effective image can be treated as incoherent even when it really is not, because refractive and diffractive effects are almost decoupled in strong scattering (Blandford & Narayan, 1985; Goodman & Narayan, 1985).

The effective image of the source is the convolution of its intrinsic surface brightness distribution with the scatter-broadened image of a point source. The former has angular size θ_s, and the latter, θ_d. A useful measure of the effective size of the source is therefore

$$\theta_{\rm eff} \equiv [\theta_s^2 + (0.71\theta_d)^2 + (0.85\theta_F)^2]^{1/2}, \tag{18}$$

(cf. the Appendix). The presence here of the Fresnel angle [defined in Eq. (12)] allows subsequent formulae to be used for weak as well as strong scattering.

As an incoherent source, the effective image can be lensed only by inhomogeneities at least as large as the projected linear size of the image, $s \geq d_{\rm scr}\theta_{\rm eff} \equiv s_r$. From the discussion above, it follows that the focal length scales as $s^{7/6}$, so very-large-scale inhomogeneities are ineffective. Therefore, inhomogeneities near the refractive scale s_r dominate refractive scintillation. The characteristic timescale of the modulation is

$$t_{\rm ref} = \frac{s_r}{v_\perp} = 14\left(\frac{\theta_{\rm eff}}{10\ \mu\ \text{a.s.}}\right)\left(\frac{d_{\rm scr}}{\text{kpc}}\right)\left(\frac{v_\perp}{30\ \text{km s}^{-1}}\right)^{-1}\ \text{hr}, \tag{19}$$

For strong scattering of a pointlike source, $\theta_{\rm eff} \approx \theta_d \gg \theta_F$, and it follows from Eq. (16) and Eq. (19) that $t_{\rm ref}/t_{\rm diff} = (\theta_d/\theta_F)^2 \gg 1$. Thus when scintillation is strong, the diffractive and refractive timescales are well separated. When scintillation is weak, there is only one timescale, which may be identified with $t_{\rm ref}$ if $t_{\rm ref}$ is defined in terms of the $\theta_{\rm eff}$ as above.

The strength of refractive scintillation is measured by the *modulation index* m_R, the root-mean-square refractive variation in the flux normalized by the mean flux. Using results from the Appendix, we have

$$m_R = 0.114\nu_{10}^{-2}(SM_{-3.5})^{1/2}\left(\frac{\theta_{\rm eff}}{10\ \mu\ \text{a.s.}}\right)^{-7/6} \times\left(\frac{d_{\rm scr}}{1\ \text{kpc}}\right)^{-1/6}. \tag{20}$$

The $\theta_{\rm eff}^{-7/6}$ scaling above reflects the behavior of the focal length of the inhomogeneities on the refractive scale: $f(s_r) \propto s_r^{7/6}$. Eq. (20) implies that in the strong scattering regime, the modulation index of an extended source increases with scattering measure and decreases with frequency as long as $\theta_{\rm eff} \approx \theta_s \gg \theta_d$. For a pointlike source, however, the trends are reversed: $\theta_{\rm eff} \approx \theta_d \propto SM^{3/5}\nu^{-11/5}$ [Eq. (10)], so $m_R \propto SM^{-1/5}\nu^{17/30}$.

3. Scintillation of GRB radio afterglows

This section applies the general results of the last section to the particular case of GRB afterglows. The goal is to expose what might be learned about the angular sizes of these afterglows by observing scintillation.

3.1. Strong or weak scattering?

As remarked above, these are two qualitatively different regimes for scintillation, and it is therefore important to decide which is relevant for GRBs.

The borderline between strong and weak scattering depends not only on the medium but also on the frequency of observation. To date, only GRB970508 has been positively identified with a radio source. In the first week after the gamma-ray event, the radio spectrum was inverted (flux rising with frequency), and detections were made at $\nu \geq 8$ GHz (Section 1). At this frequency, according to formula Eq. (13), the scattering should be near the the borderline between weak and strong for a typical line of sight. This is the most difficult regime in which to give a tidy theoretical account of the phenomena to be expected. Rather than focus too much on this one afterglow, we offer a general discussion in which we presume that the various asymptotic regimes – weak versus strong, diffractive versus refractive – can be separated. This will be possible with future afterglows if observations are made at multiple frequencies.

The distinction between diffractive and refractive scintillation makes sense only if the scattering is strong. In principle, therefore, there are three regimes to discuss:

1. diffractive scintillation
2. refractive scintillation
3. weak scattering

We will treat the last two together.

3.2. Diffractive scintillation

Diffractive interstellar scintillation (henceforth DISS) occurs only if the angular size of the source, the scattering strength, and the observing frequency and bandwidth satisfy the conditions explained in Section 2. But if DISS does occur, it will be very informative.

The most critical source parameter is the intrinsic angular size of the source, θ_s, which must satisfy the condition Eq. (15). Since the limit on the angular size is $\propto \nu^{6/5}$, if DISS occurs at some frequency, it will be quenched at a lower frequency, ν_{quench}. By measuring ν_{quench}, one can use the relation Eq. (15) to constrain θ_s.

Eq. (15) also involves the uncertain ISM parameters SM and d_{scr}. In particular, the scattering measure SM can vary by an order of magnitude among high-latitude lines of sight (Spangler et al., 1993). Fortunately, one can estimate SM independently from the diffractive time Eq. (16). In fact, Eq. (16) and Eq. (15) can be combined into

$$\theta_{\text{s,max}} = \frac{v_\perp t_{\text{diff}}}{d_{\text{scr}}}, \tag{21}$$

where d_{scr} is the effective distance of the scattering medium, and $v_\perp$ is the transverse velocity of the line of sight (Section 2). Even if the source is mildly superluminal, $v_\perp$ is dominated by the Earth's orbital motion and the peculiar velocity of the Sun with respect to the ISM; internal motions of the ISM and galactic differential rotation are probably small in comparison to these, except at very low galactic latitudes. Hence $v_\perp$ can be regarded as a known quantity.

In principle, the decorrelation bandwidth Eq. (17) measures another combination of SM and d_{scr}, but $\Delta\nu_{\text{dc}}$ is really only well defined in very strong scattering, where $\Delta\nu_{\text{dc}} \ll \nu$; for the conditions of interest here, the scattering is at best only marginally strong, so that $\Delta\nu_{\text{dc}} \sim \nu$.

As explained in Section 2, DISS does not properly exist unless the scattering is strong, and then only for sufficiently compact sources. Eq. (13) is one among several equivalent conditions for strong scattering (another is $\theta_{\text{d}} > \theta_F$). The limiting angular size Eq. (15) increases with frequency, but only as long as Eq. (13) is satisfied. Hence there is a critical intrinsic size θ_{crit} above which DISS cannot be seen at any frequency, which can be obtained by substituting ν_{ss} for ν in Eq. (15):

$$\theta_{\text{crit}} = 2.35(SM_{-3.5})^{-3/17} d_{\text{scr, kpc}}^{-11/17}\ \mu\text{ a.s.} \tag{22}$$

It is interesting that for plausible ISM parameters (as shown), θ_{crit} is comparable to the values expected from cosmological blastwave solutions (Section 1).

We have assumed that scattering is dominated by the local ISM. We have neglected contributions from the burst's host galaxy (if any), from the intergalactic medium (IGM), or from intervening objects such as the one responsible for the optical absorption lines in the afterglow of GRB970508 (Metzger et al., 1997). One might think that DISS in particular is very sensitive to distant scatters, because the angular size limit Eq. (15) for DISS varies inversely with the distance to the scattering screen. That formula, however, was derived in the approximation that $d_{source} \gg d_{scr}$ (scattering of a plane wave). More generally, the apparent size of a point source due to scattering is related to the scattering angle θ_d by

$$\theta_{app} = \frac{d_{source} - d_{scr}}{d_{source}} \theta_d .$$

For scattering in the host, the numerator above is ~1 kpc and the denominator is ~1 Gpc, so the contribution to θ_{app} is suppressed by 10^{-6}.

Scattering by the IGM is negligible presuming (in the absence of better information) that the scattering measure scales as the path length times the square of the mean electron density:

$$\frac{SM_{IGM}}{SM_{ISM}} \sim \frac{1\ \text{Gpc}}{1\ \text{kpc}} \left(\frac{2 \times 10^{-7}\ \text{cm}^{-3}}{0.02\ \text{cm}^{-3}} \right)^2 \sim 10^{-4}.$$

We have assumed a fully-ionized IGM containing most of the baryons allowed by primordial nucleosynthesis, $\Omega_b h_{50}^2 = 0.05 \pm 0.01$ (Izotov, Thuan, & Lipovetsky, 1997) and taken the local electron density from Taylor & Cordes (1993). In the case of GRB970508, scattering by an intervening galaxy associated with the $z = 0.835$ absorption system might be comparable to that due to the local ISM, but there appears to be no intrinsically bright galaxy on the line of sight (Fruchter et al., 1997), so it is likely that the absorption comes from the outer halo of a galaxy, where the electron density and scattering measure are likely to be small.

So distant scatterers are not expected to increase θ_{app} much above the intrisic source size θ_s. If this is the case, then the criterion for diffractive scintillation by the local ISM is unaffected, since the criterion already assumes an incoherent source.

3.3. Refractive and weak scintillation

Even if diffractive scintillation is quenched, refractive interstellar scintillation (henceforth RISS) exists and may constrain the source size. The modulation index (m_R) of RISS depends on both frequency and source size through the effective size parameter Eq. (18). According to Eq. (10) and Eq. (18), at sufficiently low frequencies $\theta_{eff} \approx 0.71\theta_d \gg \theta_s$, and therefore according to Eq. (20), $m_R \propto \nu^{17/30}$: RISS increases with frequency at low frequencies. At sufficiently high frequencies, $\theta_{eff} \approx \theta_s =$ constant (if the source size is independent of frequency), so that $m_R \propto \nu^{-2}$. Therefore m_R achieves a maximum at some intermediate frequency, ν_{peak}.

It can be shown that if the source size is larger than the value θ_{crit} at which DISS is quenched [Eq. (22)], then the peak occurs in the strong scattering regime [Eq. (13)] and is determined by the condition $\theta_d(\nu_{peak}) = 2.7\theta_s$. From Eq. (10) and Eq. (20),

$$\nu_{peak} = 3.7 \left(\frac{\theta_s}{10\ \mu\ \text{a.s.}} \right)^{-5/11} (SM_{-3.5})^{3/11}\ \text{GHz},$$

$$m_{R,\ peak} = 0.35 \left(\frac{\theta_s}{10\ \mu\ \text{a.s.}} \right)^{-17/66} \times d_{scr,kpc}^{-1/6} (SM_{-3.5})^{-1/22} \qquad (23)$$

For a sufficiently compact source, ν_{peak} is determined by the condition $\theta_d \sim \theta_F$ rather than $\theta_d \sim \theta_s$:

$$\nu_{peak}^{pt.src.} = 6.8 (SM_{-3.5})^{6/17} d_{scr,kpc}^{5/17}\ \text{GHz},$$

$$m_{R,\ peak}^{pt.src.} = 0.47 \qquad (24)$$

[This frequency differs by only a constant factor from the frequency Eq. (13) dividing the strong and weak scattering regimes.] Thus if $\theta_s \ll \theta_{crit}$, the peak in RISS yields no information about the source size. In principle, at some higher frequency where $\theta_F \sim \theta_s$, the slope $d\ln m_R / d\ln \nu$ breaks from $-17/12$ to -2, but in practice, the break may be difficult to identify. On the other hand, the regime $\theta_s < \theta_{crit}$ is also that in which DISS can be observed and used to constrain θ_s.

The timescale t_{ref} for RISS yields additional information about θ_s, SM, and d_{scr} via Eq. (19). At high frequencies, $t_{ref} \approx \theta_s d_{scr} / v_\perp$. At low frequencies, $t_{ref} \approx \theta_d d_{scr} / v_\perp \propto d_{scr} SM^{3/5}$ [Eq. (10)].

4. Discussion

If radio afterglows of gamma-ray bursts are indeed produced by relativistic blastwaves at cosmological distances, then their small angular size and high surface brightness puts them in an extremely interesting part of parameter space with respect to interstellar scintillation. Both diffractive and refractive scintillation are possible for these sources.

Diffractive scintillation is sensitive to the radio frequency of observation, ν, and the angular size of the source, θ_s. It may have been possible to have seen diffractive scintillation in GRB970508 at $\nu \approx$ 10 GHz if $\theta_s \lesssim 2$ μ a.s. [Eq. (15)]. Future afterglows will present other opportunities for observing diffractive scintillation, which will be recognized because

- the flux variations are of unit strength, $\langle F^2 \rangle = 2\langle F \rangle^2$ [but contrary to the situation with pulsars, scintillation will be correlated across a broad frequency band, cf. Eq. (17)];
- the timescale decreases with decreasing radio frequency, and is on the order of a few hours [Eq. (16)];
- the effect vanishes abruptly below a frequency determined by the scattering measure and the angular size of the source [Eq. (13)].

Diffractive scintillation will allow the interstellar medium to be used as a Michelson stellar interferometer to resolve angular sizes measured in micro-arcseconds, too small for conventional VLBI. The angular size obtained by these means will be somewhat uncertain because it depends upon the scattering measure, whose value fluctuates by an order of magnitude or so among high-latitude lines of sight. But the dependence is not very strong [cf. Eqs. (15) and (22)], and furthermore the scattering measure can be constrained or eliminated from the angular size limit if the diffractive timescale is well measured [Eqs. (16) and (21)].

Refractive scintillation will occur at all frequencies and for all angular sizes, though its amplitude depends algebraically on both parameters. The timescale will range upward from a few hours. The amplitude, though less than unity, will be large ($\gtrsim 10\%$) unless θ_s, and hence the Lorentz factor of the blastwave, are much larger than predicted by current models [Eq. (A.5)]. Refractive scintillation therefore can also be used to constrain θ_s and Γ_s, though it will not provide quite so sharp a test as diffractive scintillation.

Some implications for observing strategies are the following:

One wants to collect data at several frequencies, roughly in the range 2–20 GHz.

For diffractive scintillation, the flux should be sampled on timescales ~ 1 hr, and the first data points should be taken as early as the sensitivity of the equipment permits, since the source will be smaller and less likely to be overresolved by the scattering [see Eqs. (15) and (22)]. One should probably attempt to see diffractive scintillation in sources at high galactic latitude ($\gtrsim 30°$) only, once again because these are least likely to be overresolved.

For refractive scintillations, many of the same considerations apply. In this case, one can afford to make observations later in the lifecycle of the afterglow, at longer time intervals, and at lower galactic latitude. It will be important to minimize or accurately subtract the contribution of receiver and atmospheric noise to the flux variations, since the amplitude of the modulation index is critical to the estimate of the angular size.

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Appendix A. Detailed results for refractive scintillation

Following Coles et al. (1987), the normalized spatial correlation of the flux due to refractive scintillation is

$$C(\vec{s}) \equiv \frac{\langle F_\nu(\vec{x})F_\nu(\vec{x}+\vec{s})\rangle}{\langle F_\nu\rangle^2} - 1$$

$$= 8\pi r_e^2 \lambda^2 \int_0^\infty dz \int d^2\vec{q}\, e^{i\vec{q}\cdot\vec{s}} \Phi_{N_e}(q_x,q_y,0;z)$$

$$\times \left| V\left(\frac{\vec{q}z}{k}\right)\right|^2 \exp\left[-\int_0^\infty d\bar{z} D'_\phi\left(\frac{\vec{q}z_<}{k},\bar{z}\right)\right]$$

$$\times \sin^2\left(\frac{q^2 z}{2k}\right). \qquad \text{(A.1)}$$

Here the line of sight runs from the observer at $z=0$ to the source at (effectively) $z=\infty$. The electron-density spectrum Φ_{N_e} [Eq. (5)] depends on z via the coefficient $C_N^2 \to C_N^2(z)$, which reflects changes in the strength of the turbulence along the line of sight. We take a gaussian profile for the scattering layer, $C_N^2(z) = C_N^2(0)\exp[-(z\sin b/H)^2]$, where $C_N^2(0)$ is the local value near the Sun, and $z\sin b$ is the height above the plane. The flux correlation depends upon the average (or sum) of the phase fluctuations along the line of sight, hence the z component of $\vec{q}$ is set to zero in Eq. (A.1).

The quantity $V(\vec{r})$ is the visibility of the source on baseline $\vec{r}$, which like $\vec{q}$ and $\vec{s}$ in this formula, is a vector transverse to the line of sight. We assume a gaussian image brightness distribution for the source, $I_\nu(\vec{\theta}) \propto \exp(-\vec{\theta}^2/2\theta_s^2)$, so that $V(\vec{q}z/k) = \exp(-q^2z^2 \times \theta_s^2/2)$.

Finally,

$$D'_\phi(\vec{r},z) =$$

$$4\pi r_e^2\lambda^2 \int d^2\vec{q}\,\Phi_{N_e}(q_x,q_y;q_z=0,z)\left[1-e^{i\vec{q}\cdot\vec{s}}\right]$$

$$= 2^{4/3}\frac{3\pi^2\Gamma(7/6)}{5\,\Gamma(11/6)} C_N^2(z) r_e^2 \lambda^2 s^{5/3} \qquad \text{(A.2)}$$

is the differential phase structure function. In other words, $D'_\phi(\vec{r},z)dz$ is the contribution of the slab $(z,z+dz)$ to the mean-square phase difference between lines of sight separated by $\vec{r}$. The quantity $z_< \equiv \min(z,\bar{z})$.

It is convenient to regard the three final factors in Eq. (A.1) as the squares of "visibilities," due respectively to the intrinsic source size, to the scattering, and to Fresnel optics. The characteristic angular scales associated with these visibilities at $d_{\rm scr} \approx H\csc b/2$ are θ_s, the scattering size θ_d [Eq. (10)], and the Fresnel angle Eq. (12). It is convenient to define a running Fresnel angle for each point along the line of sight:

$$\hat{\theta}_F(z) \equiv (kz\csc b)^{-1/2}, \qquad \text{(A.3)}$$

The Fresnel visibility at height z is

$$V_F(\vec{r}) \equiv \frac{\sin(k\hat{\theta}_F r/2)}{k\hat{\theta}_F r/2}. \qquad \text{(A.4)}$$

All three visibilities have the effect of restricting the range of wavenumbers $\vec{q}$ that contribute to $C(\vec{s})$. They occur multiplicatively in Eq. (A.1) because the effective source size seen by the observer is a convolution of the intrinsic surface brightness distribution $I_\nu(\vec{\theta})$ with the scatter-broadened image of a point source, and with a sort of Fresnel image.

The root-mean-square flux variation relative to the mean is $\sqrt{C(0)} \equiv m_R$, the *modulation index*. From Eq. (A.1), we have

$$m_R = \left[\tfrac{1}{4}\Gamma(7/6)\Gamma(1/3)\right]^{1/2} r_e \lambda^2 \theta_{\rm eff}^{-7/6} C_N^2(0)^{1/2}$$

$$\times (H\csc b)^{1/3}. \qquad \text{(A.5)}$$

Numerically evaluating this expression for $d_{\rm scr} \equiv (H\csc b)/2$ and $SM = (\sqrt{\pi}/2)C_N^2(0)H$, we obtain Eq. (20). Eq. (A.5) is not an exact consequence of Eq. (A.1) in general, but the numerical coefficients in the definition Eq. (18) of $\theta_{\rm eff}$ have been chosen so that Eq. (A.5) is correct when any one of the three characteristic angles $(\theta_s,\theta_d,\theta_F)$ is much larger than the other two. This extends the results of Coles et al. (1987), who gave explicit formulae for m_R in the limit of large intrinsic size, $\theta_s \gg \theta_d,\theta_F$. So our result, Eq. (A.5) and Eq. (18), is only an interpolation formula, but we believe that it is a serviceable one. If warranted by the data and one's confidence in the

ISM model, direct numerical evaluation of Eq. (A.1) is straightforward.

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