Cern Root Practice Report Muon energy loss in matter

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1 Introduction

The muon is an elementary particle whose charge (-1 e) and spin (1/2) are equal to that of the electron. It is sometimes regarded as a "heavy" electron, because its mass is 207 times the electron mass and its interactions with matter are very similar to those of electrons. Muon interactions with matter differ significantly from electron interactions purely as a result of its much greater mass. For example, the bremsstrahlung process dominates the stopping power for electrons, particularly in the high-energy regime,, which is not the case for muons unless the energies are in the multi-GeV range. On the other hand, in this multi-GeV regime radioactive processes are more pronounced than for other heavy charged particles and ions.

For muons, the stopping power can be calculated by

$$S_e = K\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_e c^2\beta^2\gamma^2 Q_{\max}}{I^2} - \beta^2 - \frac{\delta}{2} + \frac{1}{8}\frac{Q_{\max}^2}{(\gamma Mc^2)}\right] + \Delta\left|\frac{dE}{dx}\right|$$

where

K/A=0.307075 MeV/g/cm^2 for A=1g/mol,

M is the muon mass,

δ is the density factor,

$$Q_{\max} = \frac{2m_e c^2\beta^2\gamma^2}{1+2\gamma m_e/M + (m_e/M)^2},$$

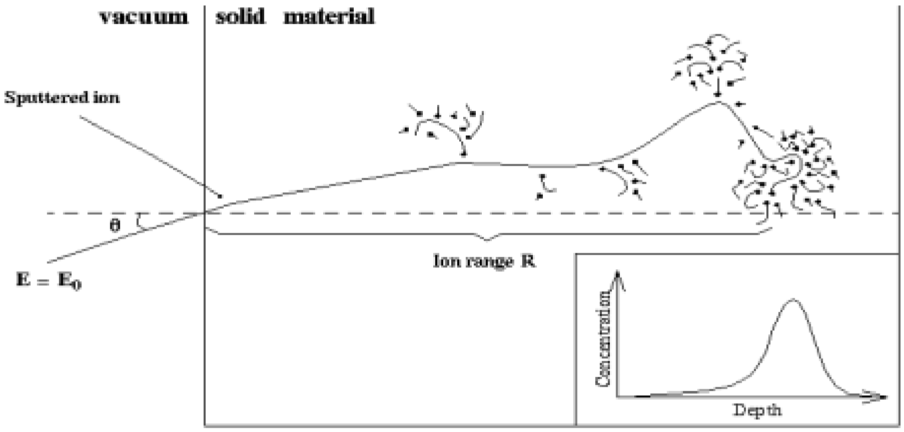
$\Delta\left|\frac{dE}{dx}\right|$ is the bremstrahlung from atomic collisions and is given by

$$\Delta\left|\frac{dE}{dx}\right| = \frac{K}{4\pi}\frac{Z}{A}\alpha\left[\ln\frac{2E}{Mc^2} - \frac{1}{3}\ln\frac{2Q_{\max}}{m_e c^2}\right]\left(\ln\frac{2Q_{\max}}{m_e c^2}\right)^2,$$

and α=1/137.035999 is the fine structure constant.

Radiative stopping power results from the interaction of muons with the coulomb field of the nucleus. This is important only at extremely high energies, i.e. more than 100 GeV for Uranium and more than 2.5 TeV for hydrogen.

Stopping power in nuclear physics is defined as the retarding force acting on charged particles due to interaction with matter, resulting in loss of particle energy. Beyond the maximum, stopping power decreases approximately like $1/v^2$ with increasing particle velocity v, but after a minimum, it increases again.A minimum ionizing particle (MIP) is a particle whose mean energy loss rate through matter is close to the minimum. In many practical cases, relativistic particles (e.g., cosmic ray muons) are minimum ionizing particles. The figure is taken from Wikipedia for a simple illustration of stopping of a heavy ion any material.



In summary, the classical way to describe the average muon energy loss is sum of energy losses due to ionization loss + bremsstrahlung + production of electron positron pairs + photo-productions. Later four terms are called radioactive loss in together. Muon stopping power can be summarized in the following figure:

The minimum energy E_m of a muon to penetrate at depth X before it stops can be calculated in the following way:

$$\left\langle \frac{dE_\mu}{dx} \right\rangle = -(a + bE_\mu) \rightarrow X = \int_{E_{\mu,min}}^{0} \frac{dE}{dE/dx} = \int_{0}^{E_{\mu,min}} \frac{1}{a+bE} = \frac{1}{b}\ln\left(1 + \frac{b}{a}E_{\mu,min}\right)$$

$$\varepsilon = \frac{a}{b} \rightarrow bX = \log\left(\frac{a + bE_{\mu,min}}{a}\right) = \log\left(\frac{\varepsilon + E_{\mu,min}}{\varepsilon}\right) \rightarrow e^{bX} = \frac{\varepsilon + E_{\mu,min}}{\varepsilon} \rightarrow E_{\mu,min} = \varepsilon\left(e^{bX} - 1\right)$$

'a' is MIP/(gm/cm^2) and 'b' is approximately 4x10^{-6}/(gm/cm^2);

The time, the particle is close enough to a "target" electron or nucleus to affect it is sufficiently short that it is usually reasonable to use the impulse approximation. This assumes that the target does not move significantly during the collision, so that the integrated effect parallel to the direction of motion of the "projectile" particle is zero, and

Figure 1:

1.1 Root Programme

```

#include<iostream>
#include<iomanip>
#include<cmath>
using namespace std;


double df(double x, double y)
{ double a=13.8;
  double b=4*TMath::Power(10,-6);
    double p = -(a) - b*(y);
  //double p=y;
    return p;
}
void diff()
{
    int n,i;
    double x0,y0,x,y,h,xn;
  double X[30000],Y[30000],Z[30000];
  X[0]=0;
```

```
  //Y[0]=1.6*TMath::Power(10,-11);
  Y[0]=10000;
    cout<<"\nFor what value of x do you want to find the value of y\n";
    cin>>xn;
    cout<<"\nEnter the width of the sub-interval:\n";
    cin>>h;
    cout<<"x"<<setw(19)<<"y\n";
  n = (xn-X[0])/h;
    for(i=0;i<=n;i++)
    {
        Y[i+1]=Y[i]+(h*df(X[i],Y[i]));
    X[i+1]=X[0]+(i+1)*h;
        cout<<X[i+1]<<setw(16)<<Y[i+1]<<endl;
    }
  TGraph *gr1 = new TGraph (n,X,Y );
  gr1->SetTitle("our graph; Distance; energy");
  gr1->SetMarkerColor(kRed);
  gr1->Draw("AC");
}
```

Programme to produce landau distribution

```

#include<iostream>
#include<iomanip>
#include<cmath>
using namespace std;
void diff()
{
    int n,i;
    double dx,a,b;
  double x[300],Y[30000],Z[30000],T[30000],dE[30000],dE_mean[300];
  a = 1.403;
  b = 0.000004;
  dx = 1;
  dE[0]=0;
  T[0]=1000;
  Y[0]=T[0];

  TH1D *h1 = new TH1D("h1","landau",1000,0,50);
  TF1 *f1 = new TF1("f1","TMath::Landau(x,[0],[1],0)",0,50);
  for (i=0;i<300/dx;i++)
{
  x[i] =i;
}

for (i=1;i<300;i++)
{
  T[i] = Y[i-1]-(a+(b*Y[i-1]))*dx;

  dE[i] = Y[i-1] - T[i-1];
  f1->SetParameters(dE[i],0.1*dE[i]);
for (int j=0;j<100;j++)
{
  h1->Fill(f1->GetRandom());
}
  Y[i] = Y[i-1]-(f1->GetRandom());
cout<<Y[i]<<endl;
  dE_mean[i-1] = h1->GetMean();
  if (Y[i]<0){break;}
cout<<dE_mean[i-1]<<endl;
}
  TGraph*gr = new TGraph(299,x,dE_mean);
  gr->Draw();



}
```

1.2 Simulation result

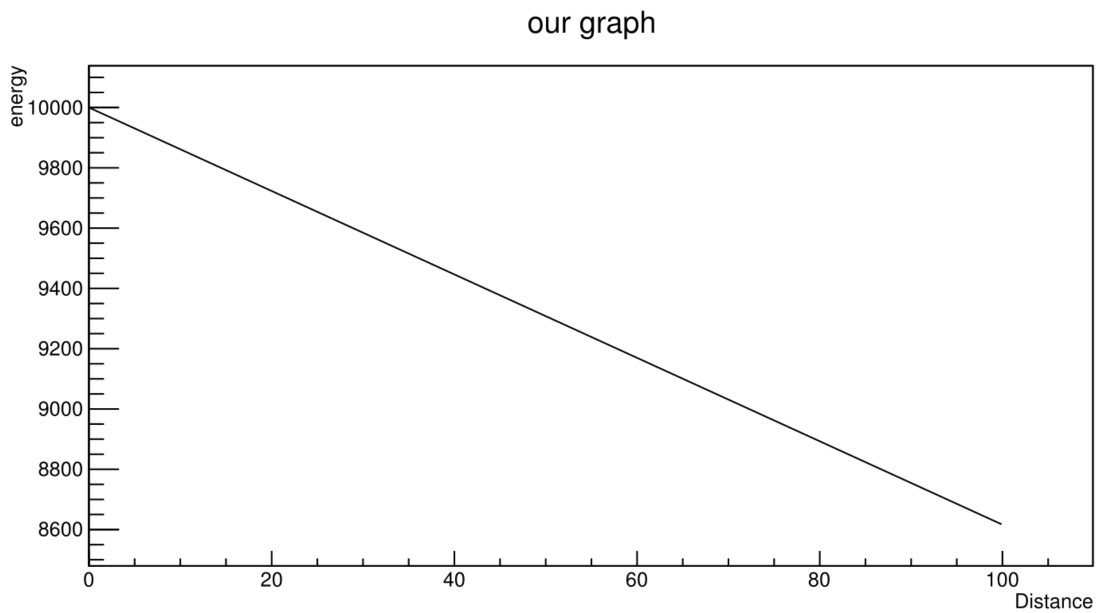


Figure 2: Fe

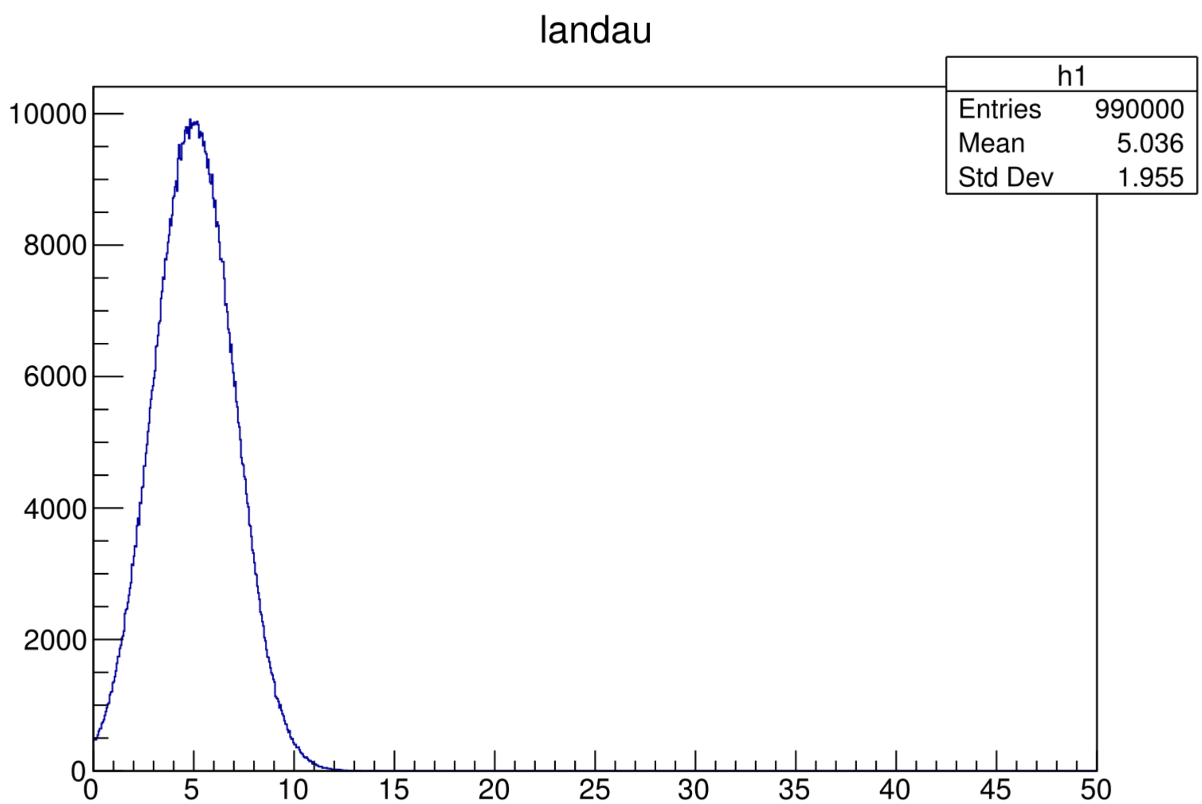


Figure 3: landau distribution of Fe