UNIVERSITY OF CAMBRIDGE
INTERNATIONAL EXAMINATIONS

SHANGHAI EXPERIMENTAL
FOREIGN LANGUAGE SCHOOL
上实剑桥外国语中学

Class	

Chinese Name		Pinyin		English Name	

Total Marks	

PureMath3 for 2020 Spring

Mid-term exam

Total Page： 3 Duration： 1 hour 45 minutes

Additional Materials： calculator, MF19,

Total mark: 75

For examiner's use only	
Question number	Mark
Total	

READ THESE INSTRUCTIONS FIRST:

1. Write your Chinese name, English name, and class on all the work you hand in.
2. Write in dark blue or black pen.
3. Do not use staples, paper clips, highlighters, glue or correction fluid.
4. Dictionaries are NOT allowed.
5. Write all your answers on the Question Paper

1 Solve the equation $\ln(x^2 + 4) = 2\ln x + \ln 4$, giving your answer in an exact form. [3]

2 Express the equation $\tan(\theta + 45°) - 2\tan(\theta - 45°) = 4$ as a quadratic equation in $\tan\theta$. Hence solve this equation for $0° \leqslant \theta \leqslant 180°$. [6]

3 The equation $x^5 - 3x^3 + x^2 - 4 = 0$ has one positive root.

(i) Verify by calculation that this root lies between 1 and 2. [2]

(ii) Show that the equation can be rearranged in the form

$$x = \sqrt[3]{\left(3x + \frac{4}{x^2} - 1\right)}.$$ [1]

(iii) Use an iterative formula based on this rearrangement to determine the positive root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

4 The polynomial $4x^3 + ax + 2$, where a is a constant, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$.

(i) Find the value of a. [2]

(ii) When a has this value,

(a) factorise $p(x)$, [2]

(b) solve the inequality $p(x) > 0$, justifying your answer. [3]

5 Let $I = \displaystyle\int_0^1 \frac{9}{(3 + x^2)^2}\,dx$.

(i) Using the substitution $x = (\sqrt{3})\tan\theta$, show that $I = \sqrt{3}\displaystyle\int_0^{\frac{1}{6}\pi} \cos^2\theta\,d\theta$. [3]

(ii) Hence find the exact value of I. [4]

6 A curve has equation

$$\sin y \ln x = x - 2\sin y,$$

for $-\frac{1}{2}\pi \leqslant y \leqslant \frac{1}{2}\pi$.

(i) Find $\dfrac{dy}{dx}$ in terms of x and y. [5]

(ii) Hence find the exact x-coordinate of the point on the curve at which the tangent is parallel to the x-axis. [3]

7 The variables x and y satisfy the differential equation

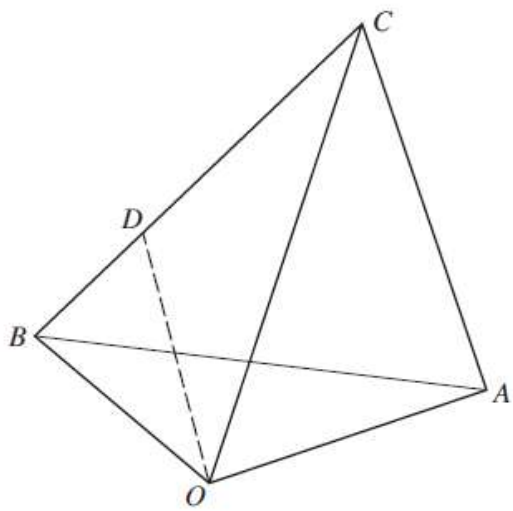
$$\frac{dy}{dx} = xe^{x+y},$$

and it is given that $y = 0$ when $x = 0$.

(i) Solve the differential equation and obtain an expression for y in terms of x. [7]

(ii) Explain briefly why x can only take values less than 1. [1]

8.



The diagram shows three points A, B and C whose position vectors with respect to the origin O are given by $\overrightarrow{OA} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$. The point D lies on BC, between B and C, and is such that $CD = 2DB$.

(i) Find the position vector of D. [1]

(ii) Show that the length of the perpendicular from A to OD is $\frac{1}{3}\sqrt{(65)}$. [4]

(iii) **Find the projection of BA onto BC** **[4]**

9 The complex number $1 + 2i$ is denoted by u.

(i) It is given that u is a root of the equation $2x^3 - x^2 + 4x + k = 0$, where k is a constant.

(a) Showing all working and without using a calculator, find the value of k. [3]

(b) Showing all working and without using a calculator, find the other two roots of this equation. [4]

(ii) On an Argand diagram sketch the locus of points representing complex numbers z satisfying the equation $|z - u| = 1$. Determine the least value of $\arg z$ for points on this locus. Give your answer in radians correct to 2 decimal places. [4]

10.

Let $\mathrm{f}(x) = \dfrac{x(6-x)}{(2+x)(4+x^2)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]