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Homework 3 Fisherface

Hello Soft Clustering (GMM)

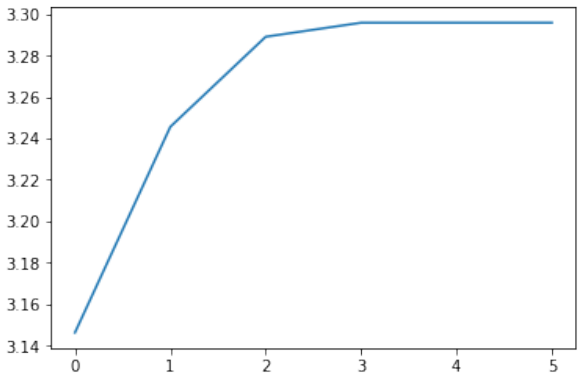
T1.

```
Iteration  1
Probabilities
[[1.19202922e-01 8.80797076e-01 1.81545808e-09]
 [7.31058579e-01 2.68941421e-01 1.69570706e-16]
 [2.68941421e-01 7.31058579e-01 1.01529005e-11]
 [9.99983299e-01 1.67014218e-05 2.03105874e-42]
 [9.99088949e-01 9.11051194e-04 5.37528453e-32]
 [9.99876605e-01 1.23394576e-04 3.30529272e-37]
 [2.31952283e-16 1.38879439e-11 1.00000000e+00]
 [2.31952283e-16 1.38879439e-11 1.00000000e+00]
 [3.30570063e-37 5.90009054e-29 1.00000000e+00]]
Weights
[0.45757242 0.20909425 0.33333333]
Means
[[ 5.78992692  5.81887265]
 [ 1.67718211  2.14523106]
 [-4.         -4.66666666]]
Covariances
[[4.53619412 0.        ]
 [0.         4.28700611]]
[[0.51645579 0.        ]
 [0.         0.13152618]]
[[4.66666668 0.        ]
 [0.         2.88888891]]
```

```
Iteration  2
Probabilities
[[3.16932821e-003 9.96824702e-001 5.96935641e-006]
 [6.55101207e-001 3.44898109e-001 6.84250685e-007]
 [5.77503537e-003 9.94223665e-001 1.30002282e-006]
 [1.00000000e+000 9.14501760e-073 4.53098372e-019]
 [1.00000000e+000 3.18241075e-032 5.49960655e-014]
 [1.00000000e+000 1.60282391e-050 1.66508966e-016]
 [4.73612484e-008 1.97981704e-052 9.99999953e-001]
 [3.08502694e-008 1.35874716e-067 9.99999969e-001]
 [5.39509443e-016 1.08758623e-168 1.00000000e+000]]
Weights
[0.40711618 0.25954961 0.33333421]
Means
[[ 6.27176215  6.27262711]
 [ 1.72091544  2.14764812]
 [-3.99998589 -4.6666488 ]]
Covariances
[[2.94672736 0.        ]
 [0.         2.93847196]]
[[0.49649261 0.        ]
 [0.         0.12584815]]
[[4.66673088 0.        ]
 [0.         2.88900236]]
```

```
Iteration  3
Probabilities
[[9.82897443e-005 9.99896667e-001 5.04302423e-006]
 [2.45965474e-001 7.54033295e-001 1.23094211e-006]
 [3.18013803e-004 9.99680992e-001 9.94302212e-007]
 [1.00000000e+000 9.43430508e-076 3.14527994e-019]
 [1.00000000e+000 1.86592722e-033 4.19732750e-014]
 [1.00000000e+000 1.37418755e-052 1.08248377e-016]
 [5.61748867e-013 6.98460925e-055 1.00000000e+000]
 [3.64921259e-013 1.02517456e-070 1.00000000e+000]
 [1.03044593e-025 1.72424695e-176 1.00000000e+000]]
Weights
[0.36070909 0.30595677 0.33333414]
Means
[[ 6.6962644   6.69629468]
 [ 1.91071238  2.27383436]
 [-3.99998673 -4.6666501 ]]
Covariances
[[1.73961067 0.        ]
 [0.         1.73929602]]
[[0.62898406 0.        ]
 [0.         0.1988491 ]]
[[4.66672942 0.        ]
 [0.         2.88899545]]
```

T2. Yes, the log likelihood goes up every iteration until converge at about iteration 4.



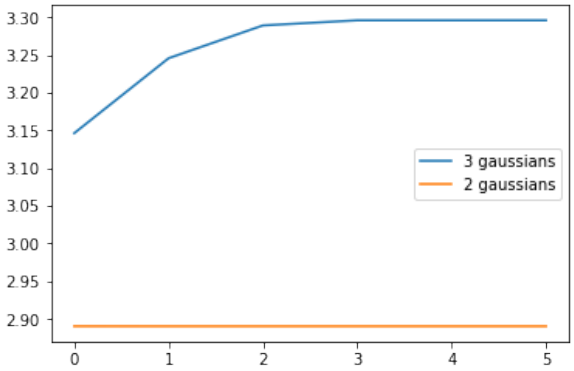
T3.

```
Iteration  1
Probabilities
[[9.99999985e-01 1.52299795e-08]
 [1.00000000e+00 2.31952283e-16]
 [1.00000000e+00 3.77513454e-11]
 [1.00000000e+00 2.03109266e-42]
 [1.00000000e+00 5.38018616e-32]
 [1.00000000e+00 3.30570063e-37]
 [2.31952283e-16 1.00000000e+00]
 [2.31952283e-16 1.00000000e+00]
 [3.30570063e-37 1.00000000e+00]]
Weights
[0.66666666 0.33333334]
Means
[[ 4.50000001  4.66666667]
 [-3.99999997 -4.66666663]]
Covariances
[[6.91666665 0.        ]
 [0.         5.88888889]]
[[4.66666677 0.        ]
 [0.         2.8888891 ]]
```

```
Iteration  2
Probabilities
[[9.99879274e-01 1.20725832e-04]
 [9.99999741e-01 2.59403362e-07]
 [9.99975922e-01 2.40783341e-05]
 [1.00000000e+00 9.39286607e-19]
 [1.00000000e+00 7.41043154e-14]
 [1.00000000e+00 2.98366370e-16]
 [2.41448223e-04 9.99758552e-01]
 [1.52869075e-04 9.99847131e-01]
 [5.22429300e-09 9.99999995e-01]]
Weights
[0.66669436 0.33330564]
Means
[[ 4.49961311  4.66620178]
 [-3.99993241 -4.66651231]]
Covariances
[[6.91944755 0.        ]
 [0.         5.89275124]]
[[4.66806942 0.        ]
 [0.         2.89103318]]
```

```
Iteration  3
Probabilities
[[9.99878589e-01 1.21411383e-04]
 [9.99999738e-01 2.61769575e-07]
 [9.99975770e-01 2.42295830e-05]
 [1.00000000e+00 9.63496021e-19]
 [1.00000000e+00 7.54839047e-14]
 [1.00000000e+00 3.04963776e-16]
 [2.42792697e-04 9.99757207e-01]
 [1.53838319e-04 9.99846162e-01]
 [5.28882554e-09 9.99999995e-01]]
Weights
[0.66669453 0.33330547]
Means
[[ 4.49961084  4.66619903]
 [-3.99993206 -4.66651141]]
Covariances
[[6.91946372 0.        ]
 [0.         5.8927741 ]]
[[4.66807754 0.        ]
 [0.         2.89104566]]
```

T4. The log likelihood of the model using 3 mixtures is better than the 2 mixtures' one.



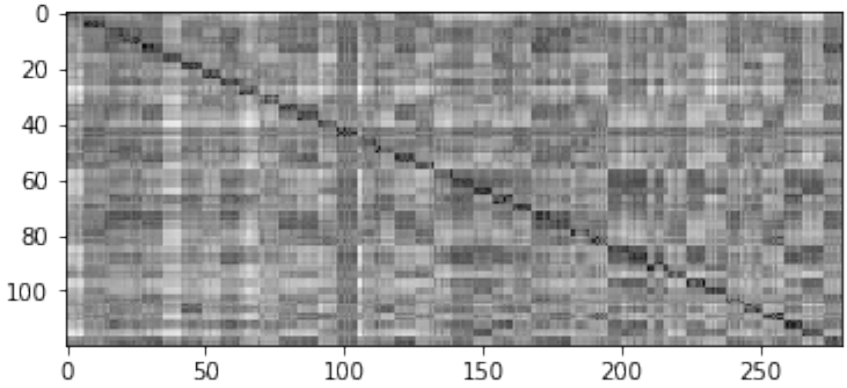
The face database

T5. The Euclidean distance between xf[0,0] and xf[0,1] is 10.04.

The Euclidean distance between xf[0,0] and xf[1,0] is 8.17.

No, I don't think that these number is useful for face verification since xf[0,0] and xf[0,1] are the same person, so it should get the smaller value than xf[0,0] and xf[1,0] which are different person.

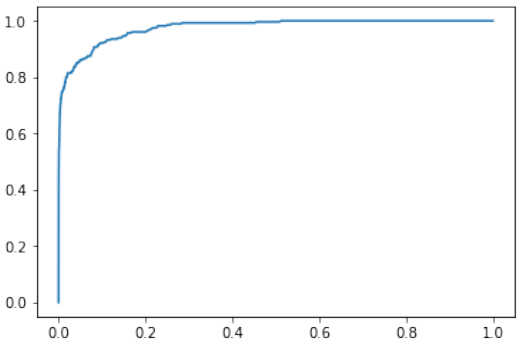
T6.



T7. It suggests that the picture 1-5 of the person number 2 are similar together because the Euclidean distance is near 0. While the picture from person number 1 is not close together.

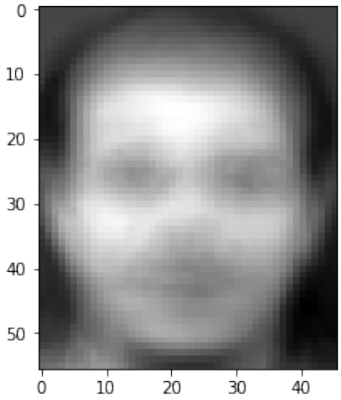
T8. True positive rate is 0.996 and false alarm rate is 0.456.

T9. The minimum threshold should be the minimum value of similarity matrix, 1.74.
The maximum threshold should be the maximum value of similarity matrix, 17.54.



T10. The EER (Equal Error Rate) is 0.0896.
The recall rate at 0.1% false alarm rate is 0.546.

T11.



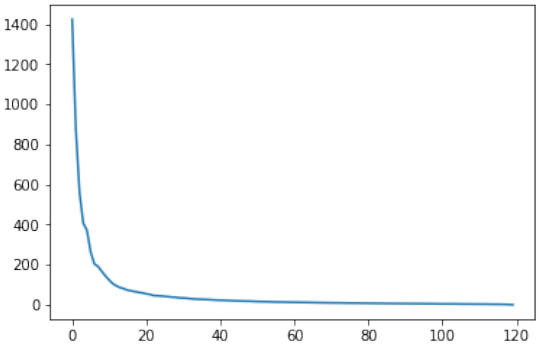
T12. The size of an image is 56*46 = 2576, so covariance matrix size is 2576*2576.
The number of images used in training data is 120, so the rank of covariance matrix is at most 119.

T13. Since the number of images used in training data is 120. Therefore, the size of the Gram matrix is 120*120. The number of non-zero eigenvalues is 119, same as the most rank of covariance matrix.

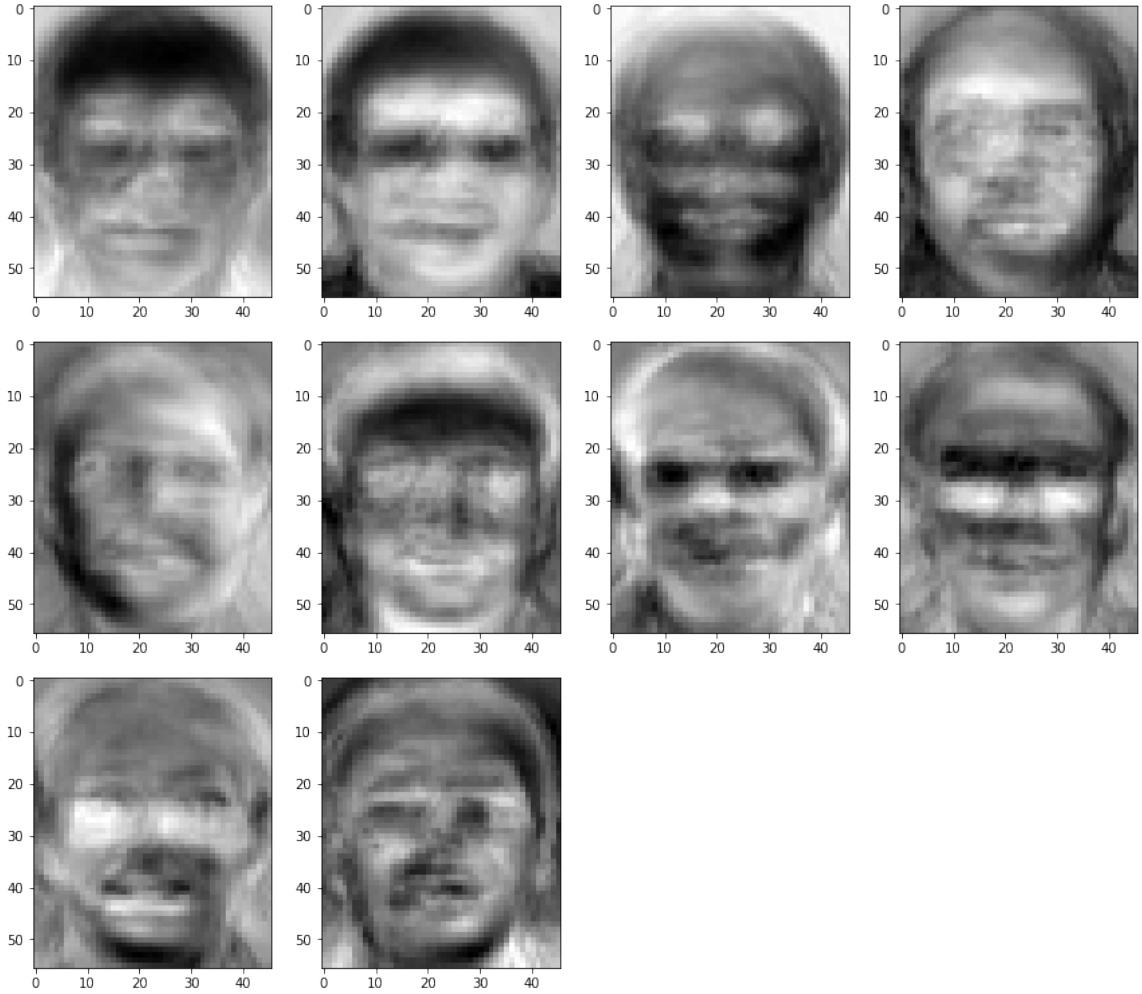
T14. Gram matrix is symmetric because $(X^TX)^T = X^TX$

T15. The number of non-zero eigenvectors is 119.

T16. If we want to keep 95% of the variance in the training data, we should keep 64 eigenvators.



T17.



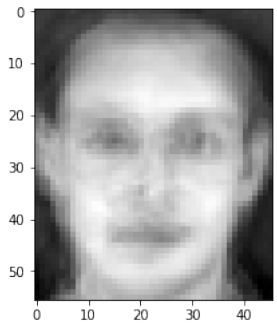
T18. The first eigenvector captures face without hair, eyes and mouth by using white label, others are black. The second eigenvector is similar to the first one but has larger white area in the face. Look at the original images, these 2 eigenvectors are not capture some image with little hair, so the forehead of these eigenfaces should have larger white area.

T19. The EER (Equal Error Rate) is 0.0786.

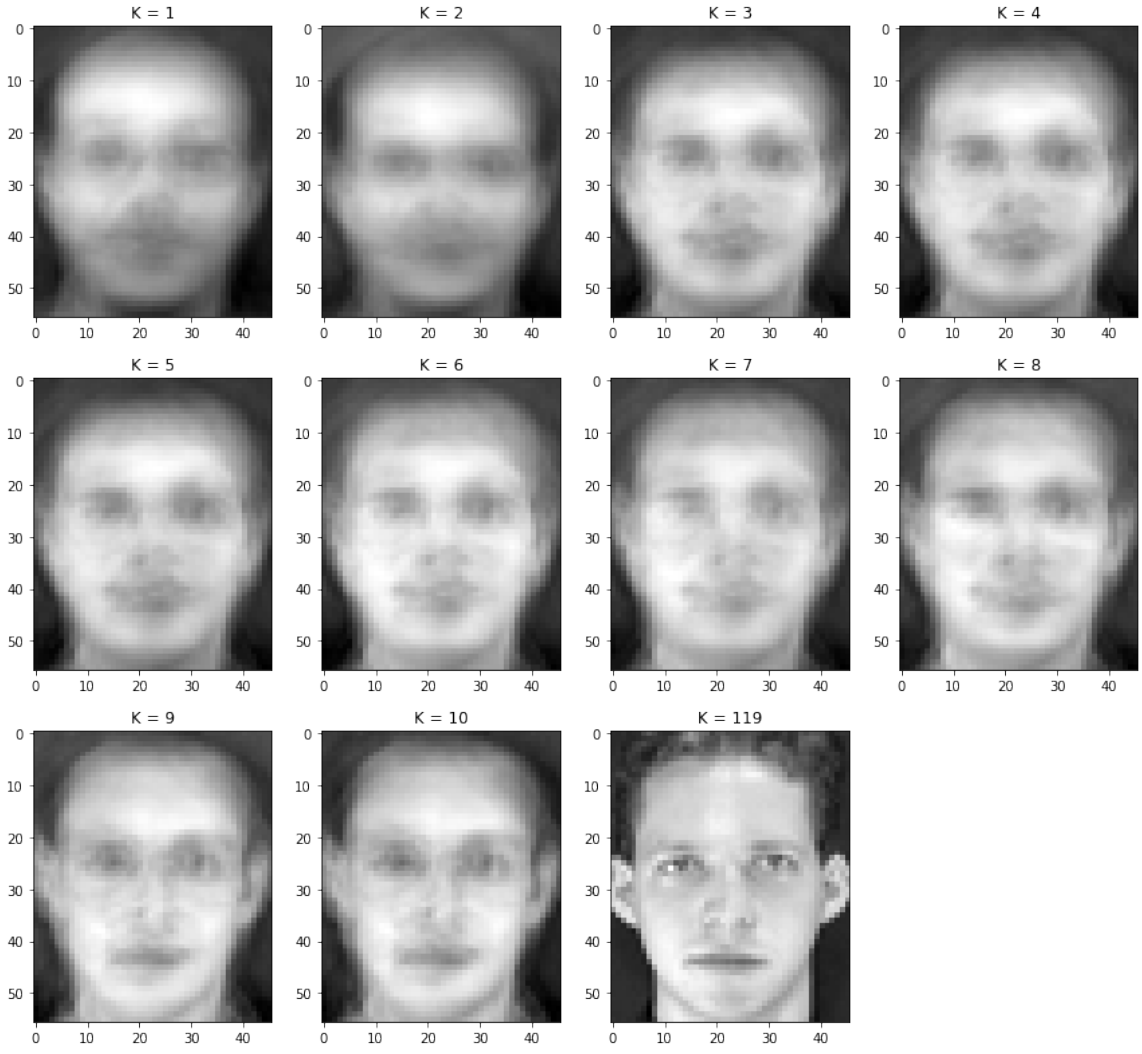
The recall rate at 0.1% false alarm rate is 0.518.

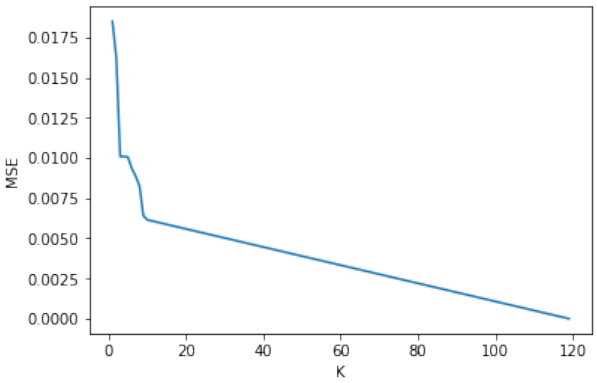
T20. The k that give the best EER is 11, which give EER = 0.0784.

OT1. The MSE is 0.00615.



OT2. The reconstructed images from k = 1, 2, 3, ..., 10, 119 respectively.



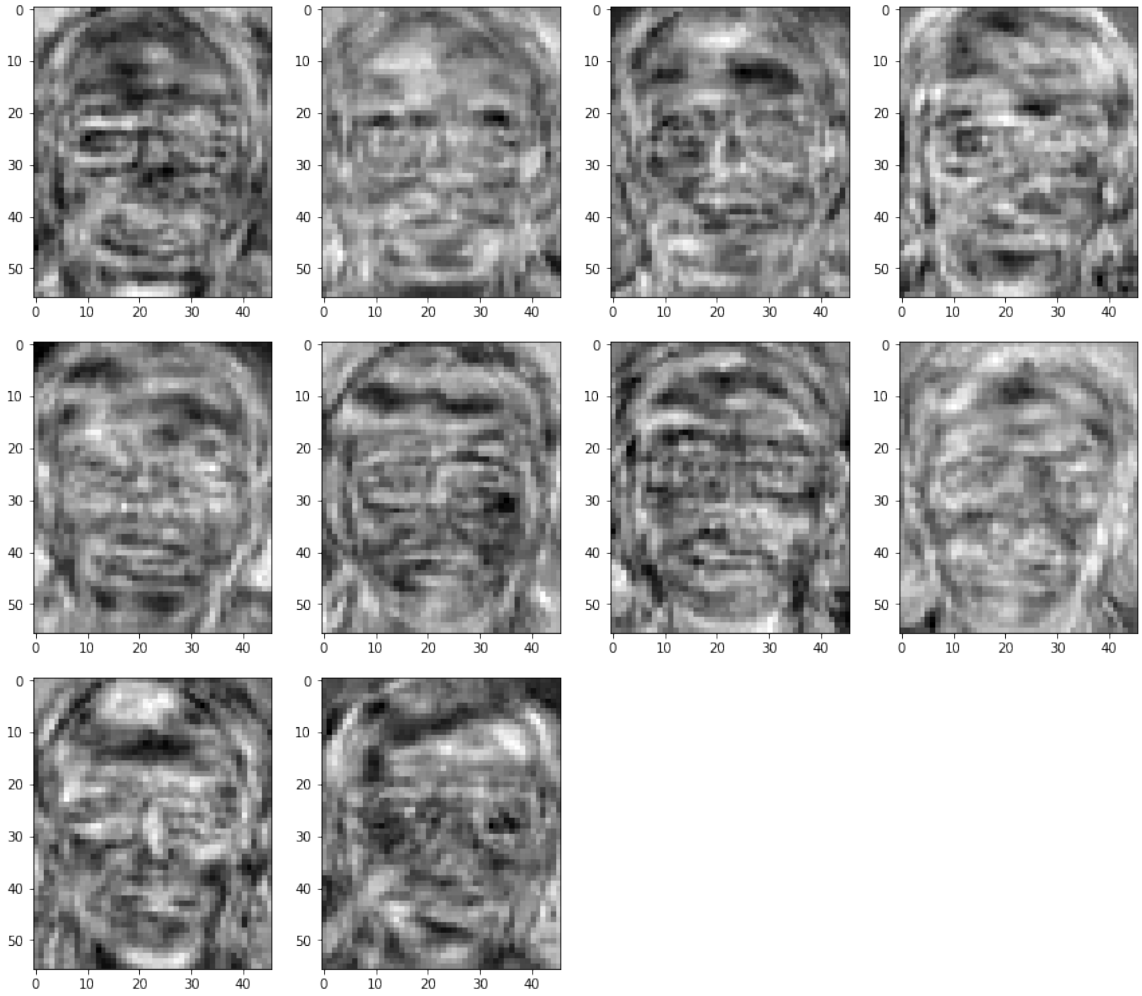


OT3. Size of meanface and each eigenface are 56 * 24 = 2576, total of (1+10) * 2576 = 28,336. In addition, each image uses 10 numbers to represent projection, total of 10 * 1,000,000 = 10,000,000. So it takes 32 * (10,000,000 + 28,336) = 320,906,752 bits or about 400 kB.

T21. Since we have 120 training data and can be classified into 40 classes(persons), so we need 80 dimensions to keep in order for S_w to be full rank.

T22. We cannot use "numpy.linalg.eigh" because $S_W^{-1} S_B$ is not symmetric. The number of non-zero eigenvalues is 39.

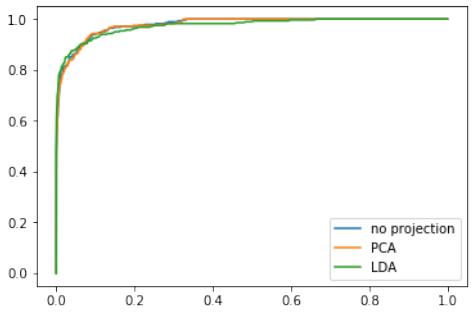
T23.



T24. The EER (Equal Error Rate) is 0.0857.

The recall rate at 0.1% false alarm rate is 0.618.

T25. The RoC of all three experiments are very close together but with PCA projection seems to be slightly the best among all experiments.



OT4. From the plots, we found that they both can classify the clustering of each person in a good way. Despite, they have some points which intersect with other clusters.