1) YIQ Color Model

a)

$$A: \begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & -0.321 \\ 0.211 & -0.523 & 0.312 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Step1: find the determinant

$$\begin{array}{ccccc} & & -0.007 & 0.050 & 0.109 \\ 0.299 & 0.587 & 0.114 & 0.299 & 0.587 \\ 0.596 & -0.274 & -0.321 & 0.596 & -0.274 \\ 0.211 & -0.523 & 0.312 & 0.211 & -0.523 \\ & & -0.026 & -0.039 & -0.036 \end{array}$$

$$\text{Determinant} = (-0.026 \quad -0.039 \quad -0.036) - (-0.007 + 0.050 + 0.109) =$$

$$\text{Determinant} = (-0.101) - (0.152) = -0.253$$

Step2: make 2x2 matrices

$$N: \begin{bmatrix} \begin{bmatrix} -0.274 & -0.321 \\ -0.523 & 0.312 \end{bmatrix} & \begin{bmatrix} 0.596 & -0.321 \\ 0.211 & 0.312 \end{bmatrix} & \begin{bmatrix} 0.596 & -0.274 \\ 0.211 & -0.523 \end{bmatrix} \\ \begin{bmatrix} 0.587 & 0.114 \\ -0.523 & 0.312 \end{bmatrix} & \begin{bmatrix} 0.299 & 0.114 \\ 0.211 & 0.312 \end{bmatrix} & \begin{bmatrix} 0.299 & 0.587 \\ 0.211 & -0.523 \end{bmatrix} \\ \begin{bmatrix} 0.587 & 0.114 \\ -0.274 & -0.321 \end{bmatrix} & \begin{bmatrix} 0.299 & 0.114 \\ 0.596 & -0.321 \end{bmatrix} & \begin{bmatrix} 0.299 & 0.587 \\ 0.596 & -0.274 \end{bmatrix} \end{bmatrix}$$

Step 3: alternate +/-

$$N: \begin{bmatrix} {}^{+}-0.253 & {}^{-}0.254 & {}^{+}-0.254 \\ {}^{-}0.243 & {}^{+}0.069 & {}^{-}-0.280 \\ {}^{+}-0.157 & {}^{-}-0.164 & {}^{+}-0.432 \end{bmatrix}$$

result

Step 4: Reflect accross diagonal

$$N: \begin{bmatrix} -0.253 & -0.254 & -0.254 \\ -0.243 & 0.069 & 0.280 \\ -0.157 & 0.164 & -0.432 \end{bmatrix}$$

reflect

$$N: \begin{bmatrix} -0.253 & -0.243 & -0.157 \\ -0.254 & 0.069 & 0.164 \\ -0.254 & 0.280 & -0.432 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\text{Detem}} [N] = \frac{1}{-0.253} \begin{bmatrix} -0.253 & -0.243 & -0.157 \\ -0.254 & 0.069 & 0.164 \\ -0.254 & 0.280 & -0.432 \end{bmatrix}$$

Step 5: Plug in determinant and N to solve

$$A^{-1} = \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 & 0.960 & 0.621 \\ 1.004 & -0.273 & -0.648 \\ 1.004 & -1.107 & 1.708 \end{bmatrix} \begin{bmatrix} Y \\ I \\ Q \end{bmatrix}$$

b) Convert the normalized RGB color <.25, 1, .75> to the YIQ color space.

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & -0.321 \\ 0.211 & -0.523 & 0.312 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$RGB = \langle 0.25,\ 1,\ 0.75 \rangle$$

$$YIQ = \langle 0.74725, -0.36575, -0.23625 \rangle$$

$$Y = (0.299(0.25)) + (0.587(1)) + (0.114(0.75)) = 0.74725$$

$$I = (0.596(0.25)) + (-0.274(1)) + (-0.321(0.75)) = -0.36575$$

$$Q = (0.211(0.25)) + (-0.523(1)) + (0.312(0.75)) = -0.23625$$

c) Convert the normalized YIQ color <.25, 1, .75> to the 8-bit (non normalized) RGB color space.

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 & 0.960 & 0.621 \\ 1.004 & -0.273 & -0.648 \\ 1.004 & -1.107 & 1.708 \end{bmatrix} \begin{bmatrix} Y \\ I \\ Q \end{bmatrix}$$

$$YIQ = \langle 0.25, 1, 0.75 \rangle$$

$$RGB = \langle 1.67575, -0.508, 0.425 \rangle$$

$$R = (1(0.25)) + (0.960(1)) + (0.621(0.75)) = 1.67575$$

$$G = (1.004(0.25)) + (-0.273(1)) + (-0.648(0.75)) = -0.508$$

$$B = (1.004(0.25)) + (-1.107(1)) + (1.708(0.75)) = 0.425$$

2) Color Metrics

a) Give a formal proof that the maximum L1 distance in HSB space is (1 + 2√2).

brightness (0-1)
Saturation (0-1)
hue (0- 1)
normalized

$C = (h, s, b)$
$C_1 = (1/8, 1, 1)$
$C_2 = (5/8, 1, 1)$

$$
\begin{aligned}
L_1(h,s,b) &= |s_1 \cos(2\pi h_1) - s_2 \cos(2\pi h_2)| + |s_1 \sin(2\pi h_1) - s_2 \sin(2\pi h_2)| + |b_1 - b_2| \\
&= |1 \cdot \cos(2\pi \cdot 1/8) - 1 \cos(2\pi \cdot 5/8)| + |1 \cdot \sin(2\pi \cdot 1/8) - 1 \cdot \sin(2\pi \cdot 5/8)| + |0 - 1| \\
&= |\cos(\pi/4) - \cos(5\pi/8)| + |\sin(\pi/4) - \sin(5\pi/4)| + |-1| \\
&= |\sqrt{2}/2 - (-\sqrt{2}/2)| + |\sqrt{2}/2 - (-\sqrt{2}/2)| + 1 \\
&= \frac{2\sqrt{2}}{2} + \frac{2\sqrt{2}}{2} + 1 \\
&= \boxed{1 + 2\sqrt{2}}
\end{aligned}
$$

b) Give a formal proof that the maximum L2 distance in HSB space is √5.

brightness (0-1)
Saturation (0-1)
hue (0-360)

$C = (h, s, b)$
$C_1 = (1/8, 1, 1)$
$C_2 = (5/8, 1, 1)$

$$
\begin{aligned}
L_2(h,s,b) &= \sqrt{(s_1 \cos(2\pi h_1) - s_2 \cos(2\pi h_2))^2 + (s_1 \cdot \sin(2\pi h_1) - s_2 \sin(2\pi h_2))^2 + (b_1 - b_2)^2} \\
&= \sqrt{(1 \cdot \cos(2\pi \cdot 1/8) - 1 \cos(2\pi \cdot 5/8))^2 + (1 \cdot \sin(2\pi \cdot 1/8) - 1 \cdot \sin(2\pi \cdot 5/8))^2 + (0-1)^2} \\
&= \sqrt{(\cos(\pi/4) - \cos(5\pi/4))^2 + (\sin(\pi/4) - \sin(\tfrac{5\pi}{4}))^2 + (-1)^2} \\
&= \sqrt{\left(\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right)^2 + \left(\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right)^2 + 1} \\
&= \sqrt{2 + 2 + 1} \\
&= \boxed{\sqrt{5}}
\end{aligned}
$$

c) Give two colors C1 and C2 such that the L1 distance between them in HSB space is maximal.

$C = (h, s, b)$
$C_1 = (1/8, 1, 1)$
$C_2 = (5/8, 1, 1)$

d) Give two colors C1 and C2 such that the L2 distance between them in HSB space is maximal.

$C = (h, s, b)$
$C_1 = (1/8, 1, 1)$
$C_2 = (5/8, 1, 1)$