Q1) [20 points]
Find the singularities of the SCARA manipulator in Fig.1

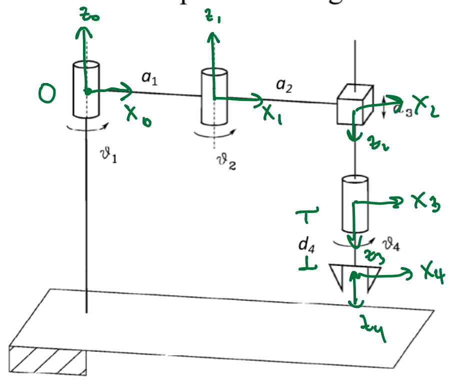


Fig.1

		d_i	θ_i	a_i	α_i
$^{0}_{1}$	1	0	θ_1	a_1	0
$^{1}_{2}$	2	0	θ_2	a_2	π
$^{2}_{3}$	3	d_3	0	0	0
$^{3}_{4}$	4	d_4	θ_4	0	0

d_i - translation along z_{i-1}
θ_i - rotation about z_{i-1}
a_i - translation along x_i
α_i - rotation about x_i

Jacobian

rev : $\begin{matrix} z_{i-1} \times (P_e - P_{i-1}) \\ z_{i-1} \end{matrix}$ prismatic : $\begin{matrix} z_{i-1} \\ 0 \end{matrix}$

$$J(q) = \begin{bmatrix} z_0 \times (P_e - P_0) & z_1 \times (P_e - P_1) & z_2 & z_3 \times (P_e - P_3) \\ z_0 & z_1 & 0 & z_3 \end{bmatrix}$$

Need many of these vectors, so now homogeneous transforms will be made.

$$A_i^{i-1} = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 1 :

	d_i	θ_i	a_i	α_i
1	0	θ_1	a_1	0

$$A_1^0 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & a_1 C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & a_1 S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

<u>Link 2</u>:

	d_i	θ_i	a_i	α_i
2	0	θ_2	a_2	π

$$A_2^1 = \begin{bmatrix} C\theta_2 & S\theta_2 & 0 & a_2 C\theta_2 \\ S\theta_2 & -C\theta_2 & 0 & a_2 S\theta_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 3:

	d_i	θ_i	a_i	α_i
3	d_3	0	0	0

$$A_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 4

	d_i	θ_i	a_i	α_i
4	d_4	θ_4	0	0

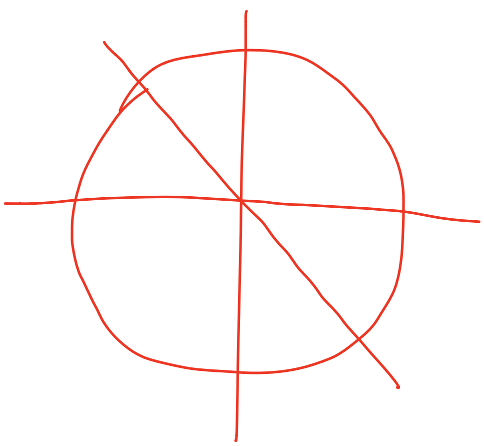
$$A_4^3 = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & 0 \\ s\theta_4 & c\theta_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

At this point I used MATLAB to calculate T_1^0, T_2^0, T_3^0, T_e. Then I extracted z_0, z_1, z_2, z_3, p_1, p_2, p_3, & p_3 which is used in the Jacobian calculation.

$$J = \begin{bmatrix} -a_2 \sin(\theta_1+\theta_2) - a_1 \sin(\theta_1) & a_2 \sin(\theta_1+\theta_2) & 0 & 0 \\ a_2 \cos(\theta_1+\theta_2) + a_1 \cos\theta_1 & -a_2 \cos(\theta_1+\theta_2) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix}$$

Singularities occur when J is rank deficient. This will occur if rows 1 & 2 are linearly dependent.

This would occur when
$$\begin{cases} \sin(\theta_1) = -\cos(\theta_1) \\ \sin(\theta_1+\theta_2) = -\cos(\theta_1+\theta_2) \end{cases}$$



$$\boxed{\begin{aligned} \theta_1 &= \frac{3\pi}{4}, \frac{7\pi}{4} \\ \theta_2 &= 0, \pi \end{aligned}}$$

Q2) [20 points]

Find the transformation matrix $T(\phi_e)$ (where $\omega_e = T(\phi_e)\,\dot{\phi}_e$) in the case of Z(φ)Y(θ)X(ψ) rotations in the current frame.

$$R_x(\psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\psi & -s\psi \\ 0 & s\psi & c\psi \end{bmatrix} \qquad R_y(\theta) = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}$$

$$R_z(\phi) = \begin{bmatrix} c\phi & -s\phi & 0 \\ s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_e = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\dot{\phi} + R_z(\theta)\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\dot{\theta} + R_z(\phi)\,R_y(\theta)\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\dot{\psi}$$

$$T_e = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\dot{\phi} + \begin{bmatrix} -s\phi \\ c\phi \\ 0 \end{bmatrix}\dot{\theta} + \begin{bmatrix} c\phi & -s\phi & 0 \\ s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\dot{\psi}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\dot{\phi} + \begin{bmatrix} -s\phi \\ c\phi \\ 0 \end{bmatrix}\dot{\theta} + \begin{bmatrix} c\phi\, c\theta & -s\phi & c\phi\, s\theta \\ s\phi\, c\theta & c\phi & s\phi\, s\theta \\ -s\theta & 0 & c\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\dot{\varphi}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\dot{\phi} + \begin{bmatrix} -s\phi \\ c\phi \\ 0 \end{bmatrix}\dot{\theta} + \begin{bmatrix} c\phi\, c\theta \\ s\phi\, c\theta \\ -s\theta \end{bmatrix}\dot{\varphi}$$

$$\boxed{T_e = \begin{bmatrix} 0 & -s\phi & c\phi\, c\theta \\ 0 & c\phi & s\phi\, c\theta \\ 1 & 0 & -s\theta \end{bmatrix}}$$

Q3) [60 points]

Use MATLAB to solve inverse kinematics problem for the SCARA robot presented in Fig.1 using closed loop inverse Jacobian. Also, check your work using robotics toolbox (use version 9 for consistency).

Hint: In robotics toolbox, use ‘workspace’ option for plot and mask matrix for ikine (check syntax on the user manual).

$a_1 = a_2 = d_4 = 0.3$ m

Initial posture: $q = [\pi/4 \text{ rad}, \pi/2 \text{ rad}, 0.2 \text{ m}, 0 \text{ rad}]^T$

$P_d(t) = [0.42*\cos(t*\pi/10), 0.42*\sin(t*\pi/10), 0.1*(1+\sin(t))]^T$ $\quad 0 \le t < 2.5$ sec

$\phi(t) = t*\pi/10 + 7\pi/12$ $\quad 0 \le t < 2.5$ sec