

Homework #1

Tanner
Kogel

$$1) \ddot{y} - 4\dot{y} + 4y = 0$$

a) Show by direct calculation that both $y(t) = e^{2t}$ and $y(t) = te^{2t}$ are solutions

$$\begin{aligned} y(t) &= e^{2t} \\ \dot{y}(t) &= 2e^{2t} \\ \ddot{y}(t) &= 4e^{2t} \end{aligned}$$

Plug in to system:

$$\begin{aligned} (4e^{2t}) - 4(2e^{2t}) + 4(e^{2t}) &= 0 \\ e^{2t}(4 - 8 + 4) &= 0 \\ 4 - 8 + 4 &= 0 \checkmark \end{aligned}$$

$$\begin{aligned} \dot{y}(t) &= te^{2t} \\ \ddot{y}(t) &= 2te^{2t} + e^{2t} \\ \ddot{y}(t) &= 4te^{2t} + 2e^{2t} + 2e^{2t} = \underline{4te^{2t} + 4e^{2t}} \end{aligned}$$

Plug in to system:

$$\begin{aligned} (4te^{2t} + 4e^{2t}) - 4(2te^{2t} + e^{2t}) + 4(te^{2t}) &= 0 \\ e^{2t}(4t + 4 - 8t - 4 + 4t) &= 0 \\ t(4 - 8 + 4) + (4 - 4) &= 0 \checkmark \end{aligned}$$

both $y(t)$ definitions solve the second-order system

b) Write system in state space & find T such that $\tilde{A} = T^{-1}AT$
is in Jordan form

$$\begin{aligned} x_1 &= y & \dot{x}_1 &= \dot{y} = x_2 \\ x_2 &= \dot{x}_1 = \ddot{y} & \dot{x}_2 &= \ddot{y} = 4\dot{y} - 4y = 4x_2 - 4x_1 \end{aligned}$$

$$\boxed{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}$$

$$A = \begin{bmatrix} 0 & 1 \\ -4 & 4 \end{bmatrix}, \quad a=0, b=1 \quad p=a+d=0+4=4 \\ c=-4, d=4 \quad q=ad-bc=(0)(4)-(1)(-4)=4$$

$$\lambda = \frac{1}{2}p \pm \sqrt{p^2 - 4q} = \frac{1}{2}(4) \pm \sqrt{(4)^2 - 4(4)} = \underline{2}$$

$$(A - \lambda I)x = 0$$

$$\left(\begin{bmatrix} 0 & 1 \\ -4 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\lambda & 1 \\ -4 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$16 \text{ cont}) \quad \begin{cases} -2x_1 + x_2 = 0 \\ -4x_1 + (4-\lambda)x_2 = 0 \end{cases} \quad (\lambda = 2) \quad \begin{cases} -2x_1 + x_2 = 0 \\ -4x_1 + 2x_2 = 0 \end{cases}$$

$2x_1 = x_2$

$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$(A - \lambda I) y = x$$

$$\left(\begin{bmatrix} 0 & 1 \\ -4 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right) \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -\lambda & 1 \\ -4 & 4-\lambda \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{cases} -2y_1 + y_2 = 1 \\ -4y_1 + 2y_2 = 2 \end{cases} \Rightarrow \underline{y_2 = 1 + 2y_1}$$

$$T = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$$

$$T^{-1} = \frac{1}{(1)(3) - (1)(2)} \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix} = \frac{1}{3-2} \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$$

$$\bar{A} = T^{-1}AT = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -4 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$$x = Tz$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$x_1 = z_1 + z_2 = y$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$2) \dot{x} = Ax$$

Prove that if A is nonsingular, $\bar{x} = 0$ is the only equilibrium point

if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is nonsingular, $ad - bc \neq 0$, $\Rightarrow ad \neq bc$

equilibrium is found at $\bar{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}$ so that $\dot{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$,

$$\dot{x} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} ax_1 + bx_2 \\ cx_1 + dx_2 \end{bmatrix}$$

$$\begin{cases} ax_1 + bx_2 = 0 \Rightarrow x_1 = -\frac{b}{a}x_2 \\ cx_1 + dx_2 = 0 \end{cases}$$

$$\downarrow$$

$$-\frac{bc}{a}x_1 = -dx_2$$

$$bcx_1 = adx_2 \quad ①$$

in order for ① to be true, while A is nonsingular ($ad \neq bc$), the only value of x is $[0]$

$$3) \dot{x} = Ax$$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

Find the set of all equilibrium points of the system & if they are isolated or non-isolated

$$\det(A) = 4 - 4 = 0 \therefore A \text{ is singular}$$

$$\dot{x}_1 = x_1 + 2x_2 = 0 \quad x_1 = -2x_2$$

$$\dot{x}_2 = 2x_1 + 4x_2 = 0$$

points on the form $x_1 = -2x_2$ are equilibrium points (non-isolated).

$$4) \text{ a) } \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_1^4 + x_1^3 + 2x_1^2 + 2x_1 - x_2 \end{cases}$$

$$\begin{cases} 0 = x_2 \\ 0 = x_1^4 + x_1^3 + 2x_1^2 + 2x_1 - x_2 \end{cases} \quad \begin{matrix} (x_2=0) \\ \underline{\text{all}} \end{matrix}$$

$$\underline{x_1^3 + x_1^2 + 2x_1 + 2 = 0} \quad (x_2=0)$$

$$f(-1) = (-1)^3 + (-1)^2 + 2(-1) + 2 = -1 + 1 - 2 + 2 = 0$$

$$f(-2) = (-2)^3 + (-2)^2 + 2(-2) + 2 = -8 + 4 - 4 + 2 = -6$$

$$(x_1+1) \Rightarrow (x_1 = -1)$$

$$\begin{array}{r} x_1^2 + 2 \\ x_1 + 1 \quad \overline{)x_1^3 + x_1^2 + 2x_1 + 2} \\ -(x_1^3 + x_1^2) \\ \hline 2x_1 + 2 \\ - (2x_1 + 2) \\ \hline 0 \end{array} \quad (x_1+1)(x_1^2+2)=0$$

$$(x_1^2 + 2) = 0$$

$$x_1 = \pm \sqrt{-2} \quad (x_1 = \pm i\sqrt{2})$$

equilibria: all isolated

$$\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} i\sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} -i\sqrt{2} \\ 0 \end{bmatrix}$$

Haccont) linearization

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

$$\frac{\partial f_1}{\partial x_1} = 0$$

$$\frac{\partial f_1}{\partial x_2} = 1$$

$$\frac{\partial f_2}{\partial x_1} = 4\bar{x}_1^3 + 3\bar{x}_1^2 + 4\bar{x}_1 + 2$$

$$\frac{\partial f_2}{\partial x_2} = -1$$

about [0]

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} x$$

about [-7]

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} x$$

about $\begin{bmatrix} i\sqrt{2} \\ 0 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 1 \\ -4-4i\sqrt{2} & -1 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -4-4i\sqrt{2} & -1 \end{bmatrix} x$$

about $\begin{bmatrix} -i\sqrt{2} \\ 0 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 1 \\ -4+4i\sqrt{2} & -1 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -4+4i\sqrt{2} & -1 \end{bmatrix} x$$

$$46) \begin{cases} \dot{x}_1 = 2x_1 - x_1 x_2 \\ \dot{x}_2 = -\frac{1}{2}x_2 + x_1 x_2 \end{cases}$$

$$\begin{cases} 0 = 2x_1 - x_1 x_2 \\ 0 = x_2 - 2x_1 x_2 \end{cases} \Rightarrow \begin{cases} x_1 x_2 = 2x_1 \\ x_1 x_2 = \frac{1}{2}x_2 \end{cases}$$

$(0,0)$
 $(x_2=a)$

$$\begin{cases} x_1 a = 2x_1 \Rightarrow a=2 \\ x_1 a = \frac{1}{2}a \Rightarrow x_1 = \frac{1}{2} \end{cases}$$

$$\begin{cases} ax_2 = 2a \Rightarrow x_2 = 2 \\ ax_2 = \frac{1}{2}x_2 \Rightarrow a = \frac{1}{2} \end{cases} \quad (\frac{1}{2}, 2)$$

equilibria: all isolated

$$\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{2} \\ 2 \end{bmatrix}$$

linearization:

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

$$\frac{\partial f_1}{\partial x_2} = 2 - \bar{x}_2$$

$$\frac{\partial f_2}{\partial x_1} = -\bar{x}_1$$

$$\frac{\partial f_2}{\partial x_2} = \bar{x}_2$$

$$\frac{\partial f_1}{\partial x_1} = -\frac{1}{2} + \bar{x}_1$$

about $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} x$$

about $\begin{bmatrix} \frac{1}{2} \\ 2 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & -\frac{1}{2} \\ 2 & 0 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} 0 & -\frac{1}{2} \\ 2 & 0 \end{bmatrix} x$$

$$4c) \begin{cases} \dot{x}_1 = 2x_1(1-x_1) - x_1x_2 \\ \dot{x}_2 = -\frac{1}{2}x_2 + x_1x_2 \\ \begin{cases} 0 = -2x_1^2 + 2x_1 - x_1x_2 \\ 0 = x_1 - 2x_1x_2 \end{cases} \Rightarrow \begin{cases} 2x_1(1-x_1) = x_1x_2 \\ x_1 = 2x_1x_2 \end{cases} \end{cases}$$

(0, 0)

$(x_1 = a)$

$$\begin{cases} -2a^2 + 2a = ax_2 \Rightarrow x_2 = 2 - 2a \Rightarrow \frac{1}{2} = 2 - 2a \\ a = 2ax_2 \Rightarrow x_2 = \frac{1}{2} \end{cases}$$

$$4a = 4 - 1 = 3$$

$$a = \frac{3}{4}$$

$(\frac{3}{4}, \frac{1}{2})$

equilibria:

$$\left[\begin{array}{c} \bar{x}_1 \\ \bar{x}_2 \end{array} \right] = \left[\begin{array}{c} 0 \\ \bar{x}_2 \end{array} \right], \quad \left[\begin{array}{c} \frac{3}{4} \\ \frac{1}{2} \end{array} \right]$$

non-isolated isolated

linearization:

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

$$\frac{\partial f_1}{\partial x_1} = -4\bar{x}_1 + 2 - \bar{x}_2$$

$$\frac{\partial f_1}{\partial x_2} = -\bar{x}_1$$

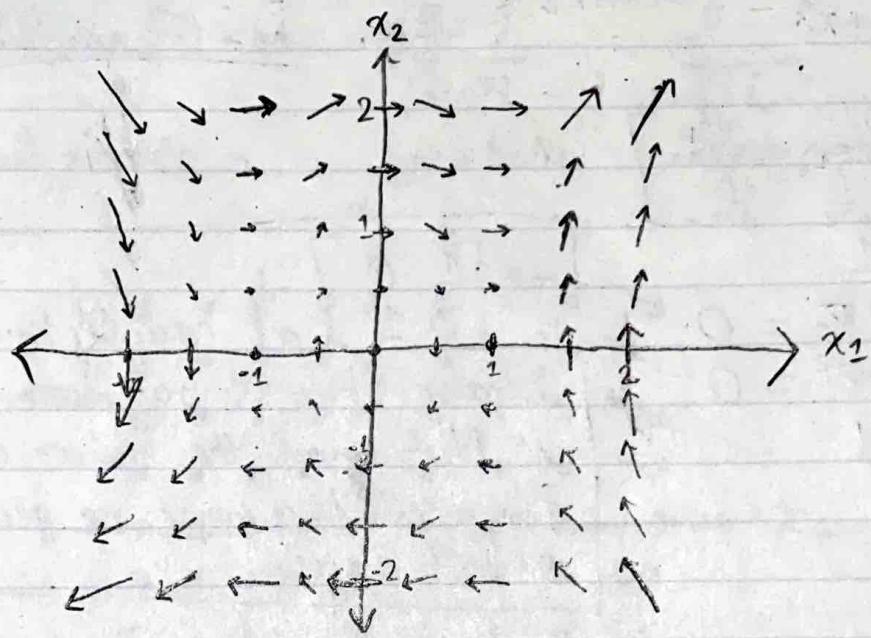
$$\frac{\partial f_2}{\partial x_1} = -\frac{1}{2} + \bar{x}_2$$

$$\frac{\partial f_2}{\partial x_2} = \bar{x}_1$$

about $\left[\begin{array}{c} 0 \\ \bar{x}_2 \end{array} \right]$ $A = \begin{bmatrix} 2 - \bar{x}_2 & 0 \\ \bar{x}_2 - \frac{1}{2} & 0 \end{bmatrix}$ $\dot{x} = \begin{bmatrix} 2 - \bar{x}_2 & 0 \\ \bar{x}_2 - \frac{1}{2} & 0 \end{bmatrix} x$	about $\left[\begin{array}{c} \frac{3}{4} \\ \frac{1}{2} \end{array} \right]$ $A = \begin{bmatrix} -\frac{3}{2} & -\frac{3}{4} \\ 0 & \frac{3}{4} \end{bmatrix}$ $\dot{x} = \begin{bmatrix} -\frac{3}{2} & -\frac{3}{4} \\ 0 & \frac{3}{4} \end{bmatrix} x$
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$$5) \ddot{y} + y - y^3 = 0$$

$$\begin{aligned} x_1 &= y \\ x_2 &= \dot{x}_1 = \dot{y} \end{aligned} \Rightarrow \boxed{\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_2^3 - x_1 \end{cases}} \text{ a. } \begin{bmatrix} 0 \\ 6 \end{bmatrix}, \begin{bmatrix} +1 \\ 0 \end{bmatrix}$$



$$6) \begin{cases} \dot{x}_1 = x_2^2 + x_1 \\ \dot{x}_2 = x_2 + x_1^3 \end{cases}$$

equilibriums: (0,0)

$$\begin{cases} 0 = x_2^2 + x_1 \\ 0 = x_2 + x_1^3 \end{cases} \Rightarrow (x_2=0) \quad \begin{cases} x_1 = -a^2 \\ x_1^3 = a \end{cases} \quad \underline{\text{no other equilibrium}}$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

$$\frac{\partial f_1}{\partial x_1} = 1$$

$$\frac{\partial f_1}{\partial x_2} = 2\bar{x}_2 = 0$$

$$\frac{\partial f_2}{\partial x_1} = 3\bar{x}_1^2 = 0$$

$$\frac{\partial f_2}{\partial x_2} = 1$$

$$J = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

at equilibrium
because there is no parameter
μ that causes the system to change
from a sink to a source, no periodic
orbit exists

$$7) \begin{cases} \dot{x}_1 = \beta + x_1^2 \\ \dot{x}_2 = -x_2 \end{cases}$$

equilibrium points:

$$\beta = -1$$

$$(\pm 1, 0)$$

nonhyperbolic

$$\beta = 0$$

$$(0, 0)$$

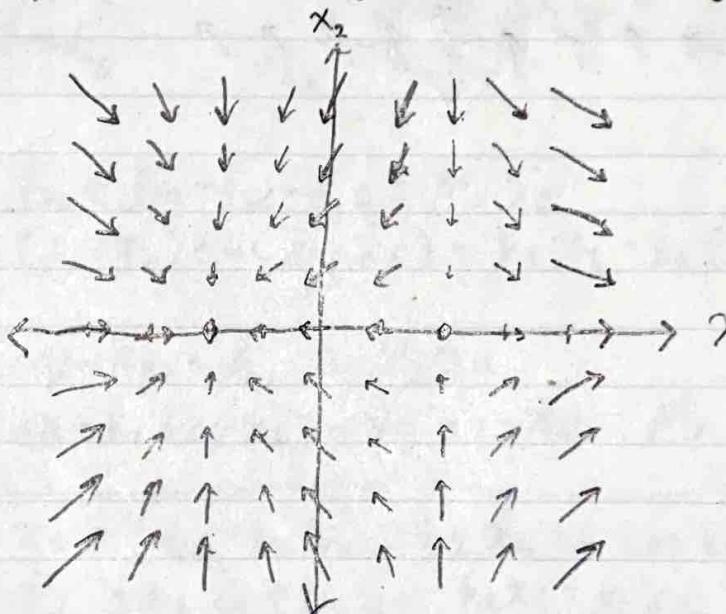
hyperbolic

$$\beta = 1$$

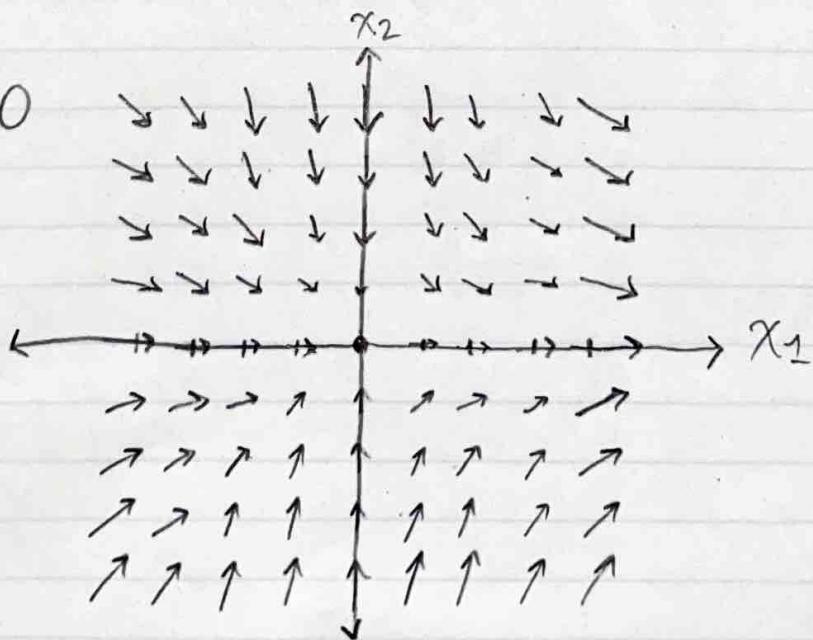
$$(\pm i, 0)$$

nonhyperbolic

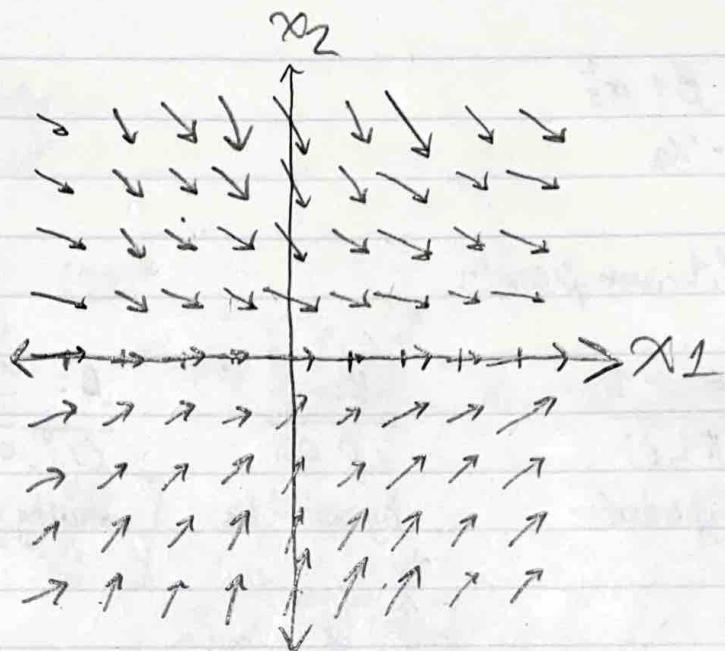
$$\beta = -1$$



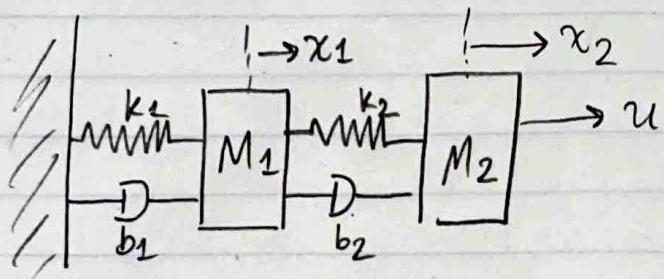
$$\beta = 0$$



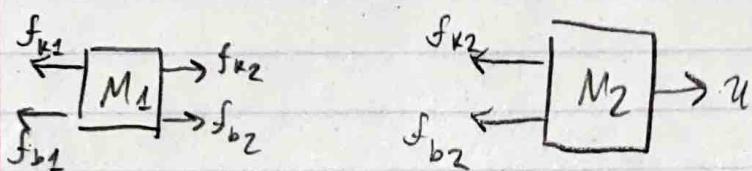
7 (cont)



8)



a) Derive EOMs



$$\sum F_{M1} = f_{k2} + f_{b2} - f_{k1} - f_{b1} = M_1 \ddot{x}_1$$

$$k_2(x_2 - x_1) + b_2(\dot{x}_2 - \dot{x}_1) - k_1 x_1 - b_1 \dot{x}_1 = M_1 \ddot{x}_1$$

$$\sum F_{M2} = u - f_{k2} - f_{b2} = M_2 \ddot{x}_2$$

$$u - k_2(x_2 - x_1) - b_2(\dot{x}_2 - \dot{x}_1) = M_2 \ddot{x}_2$$

$M_1 \ddot{x}_1 + (b_1 + b_2)\dot{x}_1 - b_2 \dot{x}_2 + (k_1 + k_2)x_1 - k_2 x_2 = 0$
$M_2 \ddot{x}_2 - b_2 \dot{x}_1 + b_2 \dot{x}_2 - k_2 x_1 + k_2 x_2 = u$

```
%% Homework 1
% MECH 6314.001
% Tanner Kogel
clear all; clc

%% Plot settings
set(0,'defaultLineLineWidth', 2)
set(0,'defaultAxesFontName' , 'Times')
set(0,'defaultTextFontName' , 'Times')
set(0,'defaultAxesFontSize' , 18)
set(0,'defaultTextFontSize' , 18)
set(0,'defaultTextInterpreter','latex')
set(0,'defaultLegendInterpreter','latex')
set(0,'defaultAxesGridLineStyle','-.')

%% problem 4a

figure(1)
clf
hold on
grid on
[x1,x2]=meshgrid(-3:0.125:3,-3:0.125:3);
x1_dot = x2;
x2_dot = x1.^4 + x1.^3 + 2*x1.^2 + 2*x1 - x2;
L = sqrt(x1_dot.^2 + x2_dot.^2);
q=quiver(x1,x2,x1_dot./L,x2_dot./L,0.4);
q.Color='black';
q.AutoScale='on';
q.MaxHeadSize=0.2;
axis([-2 2 -2 2]);
title('System 4a')
xlabel('$x_1$', 'interpreter', 'Latex'); ylabel('$x_2$', 'interpreter', 'Latex')
xic = [-1 0 0.5 -1 0.5 1 -1.5 1.5 0.25 0];
yic = [-2 -2 -2 -3 -1 -2 2.5 -2 -1 2];
for ic = [xic;yic]
[~,x] = ode45(@system4a,[0,12],ic);
plot(x(:,1),x(:,2),'k');
end

%% problem 4b

figure(2)
clf
hold on
grid on
[x1,x2]=meshgrid(-3:0.125:3,-3:0.1125:3);
x1_dot = 2*x1 - x1.*x2;
x2_dot = -x2/2 + x1.*x2;
L = sqrt(x1_dot.^2 + x2_dot.^2);      %normalize the length of the arrows
q=quiver(x1,x2,x1_dot./L,x2_dot./L,0.4);
```

```

q.Color='black';
q.AutoScale='on';
q.MaxHeadSize=0.2;
axis([-2 2 -1 3]);
title('System 4b')
xlabel('$x_1$', 'interpreter', 'Latex'); ylabel('$x_2$', 'interpreter', 'Latex')
xic = [0.5 1 0 0 0.5 -0.25 0.25];
yic = [1.5 1.5 3 -3 0.5 3 -0.5];
for ic = [xic;yic]
[~,x] = ode45(@system4b,[0,9],ic);
plot(x(:,1),x(:,2),'k');
end

%% problem 4c

figure(3)
clf
hold on
grid on
[x1,x2]=meshgrid(-3:0.125:3,-3:0.125:3);
x1_dot = 2*x1 - 2*x1.^2 - x1.*x2;
x2_dot = -x1/2 + x1.*x2;
L = sqrt(x1_dot.^2 + x2_dot.^2); %normalize the length of the arrows
q=quiver(x1,x2,x1_dot./L,x2_dot./L,0.4);
q.Color='black';
q.AutoScale='on';
q.MaxHeadSize=0.2;
axis([-2 2 -2 2]);
title('System 4c')
xlabel('$x_1$', 'interpreter', 'Latex'); ylabel('$x_2$', 'interpreter', 'Latex')
xic = [2 2 -0.125 0.125 2 2 0.125 0.125 0.125 2 -0.125 -0.125 -0.125 -0.125 -0.125];
yic = [1 0.5 0.5 0.375 0.375 0.625 0.625 1 0 0 2 -2 -1 1];
for ic = [xic;yic]
[~,x] = ode45(@system4c,[0,9],ic);
plot(x(:,1),x(:,2),'k');
end

%% problem 8b

z0 = [1; 0; -1; 0]; % define initial condition
u = @(t)0*t; % define input functino as 0

[t,z] = ode45(@(t,z)system8(u,t,z),[0,20],z0); % simulate response of the system

% plot position figures
figure(4)
clf

```

```
hold on
grid on
plot(t,z(:,1));
plot(t,z(:,3));
title('Position of System 8b')
xlabel('Time'); ylabel('Position')
legend('$x_1$', '$x_2$', 'interpreter', 'Latex');

% plot velocity figures
figure(5)
clf
hold on
grid on
plot(t,z(:,2), 'LineWidth', 2);
plot(t,z(:,4), 'Linewidth', 2);
title('Velocity of System 8b')
xlabel('Time'); ylabel('Velocity')
legend('$v_1$', '$v_2$', 'interpreter', 'Latex');

%% problem 8c

% define constants and initial conditions
A = 5;
omega = pi;
z0 = [0;0;0;0];

u = @(t)A*sin(omega*t); % define input function

[t,z] = ode45(@(t,z)system8(u,t,z),[0,20],z0); % simulate response of the system

% plot position figures
figure(6)
clf
hold on
grid on
plot(t,z(:,1));
plot(t,z(:,3));
title('Position of System 8c')
xlabel('Time'); ylabel('Position')
legend('$x_1$', '$x_2$', 'interpreter', 'Latex');

% plot velocity figures
figure(7)
clf
hold on
grid on
plot(t,z(:,2), 'LineWidth', 2);
plot(t,z(:,4), 'Linewidth', 2);
title('Velocity of System 8c')
xlabel('Time'); ylabel('Velocity')
```

```
legend('$v_1$', '$v_2$', 'interpreter', 'Latex');

%% Functions

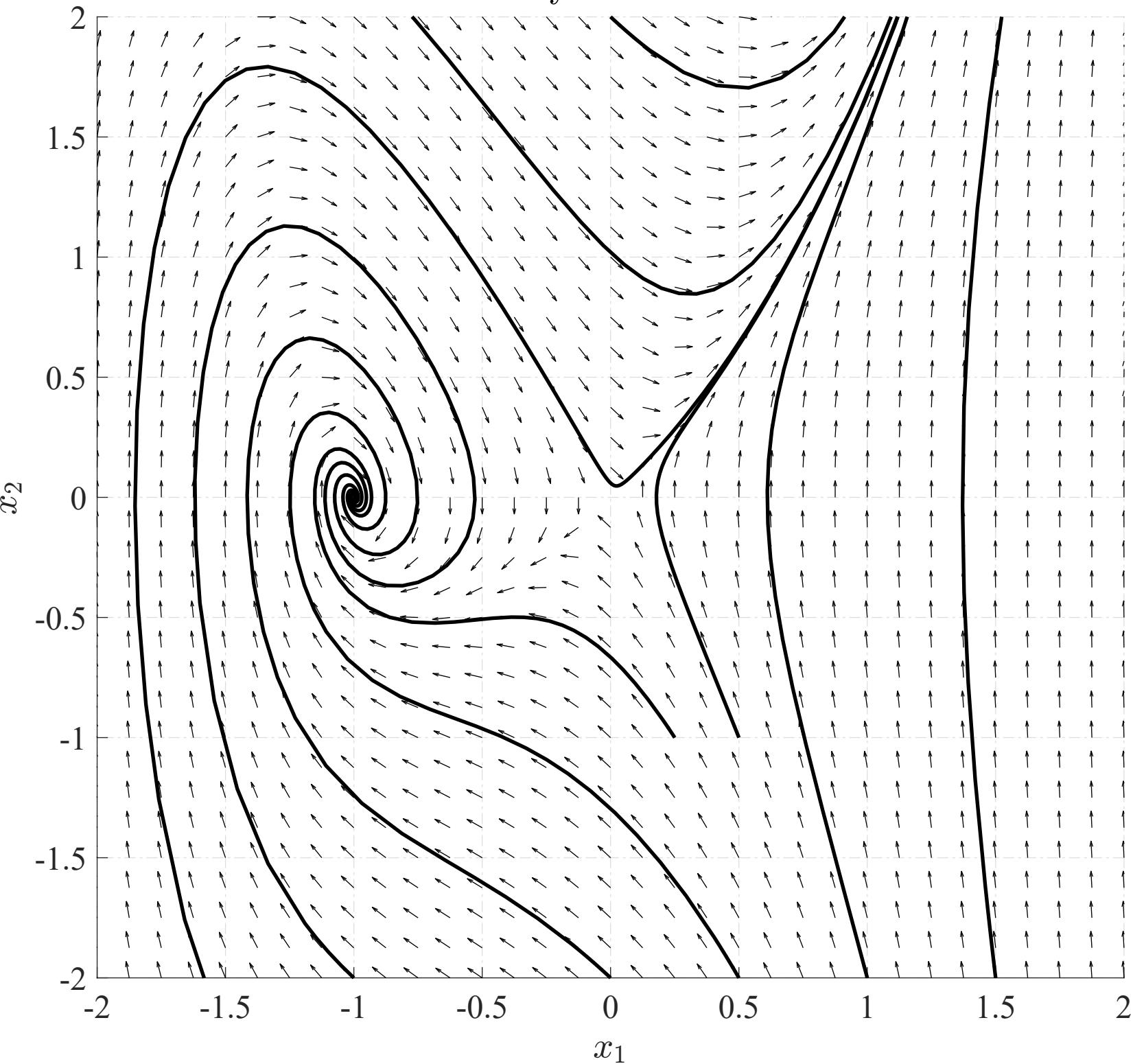
%ode45 function for system 4a
function dxdt = system4a(~,x)
x1d=x(1);
x2d=x(2);
dxdt = [x2d; (x1d^4 + x1d^3 + 2*x1d^2 + 2*x1d - x2d) ];
end

%ode45 function for system 4b
function dxdt = system4b(~,x)
x1d=x(1);
x2d=x(2);
dxdt = [ (2*x1d - x1d*x2d) ; (x1d*x2d - x2d/2) ];
end

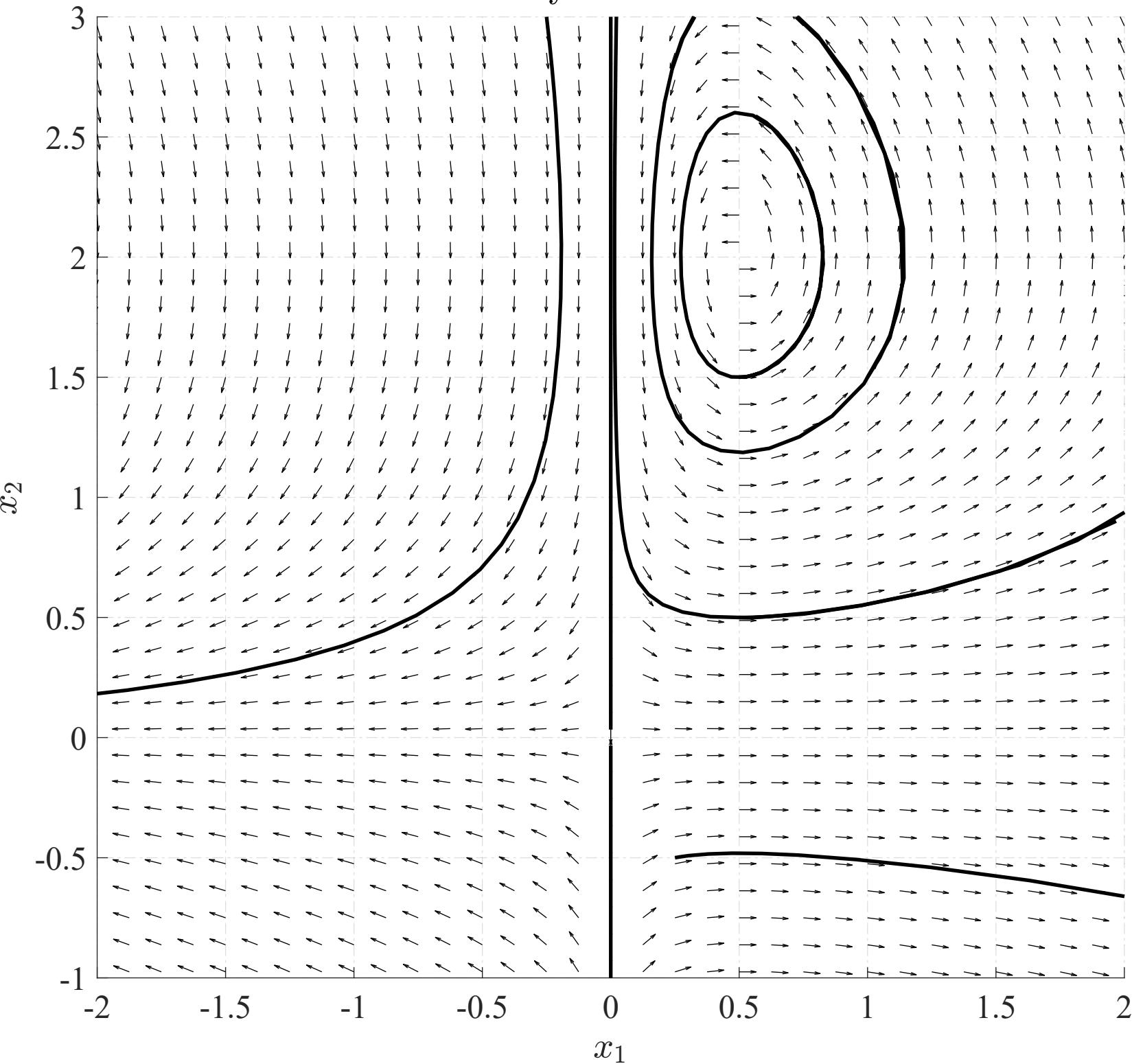
%ode45 function for system 4b
function dxdt = system4c(~,x)
x1d=x(1);
x2d=x(2);
dxdt = [ (2*x1d - 2*x1d^2 - x1d*x2d) ; (x1d*x2d - x1d/2) ];
end

%ode45 function for system 8
function dxdt = system8(u,t,z)
z1=z(1);
z2=z(2);
z3=z(3);
z4=z(4);
% define constants
b1 = 1;
b2 = 1;
k1 = 1;
k2 = 1;
M1 = 1;
M2 = 1;
dxdt = [ z2 ; ( -z1*(k1+k2)/M1 - z2*(b1+b2)/M1 + z3*k2/M1 + z4*b2/M1 ) ; z4 ; ( z1*k2/M2 ↴
+ z2*b2/M2 - z3*k2/M2 - z4*b2/M2 + u(t)/M2 ) ];
end
```

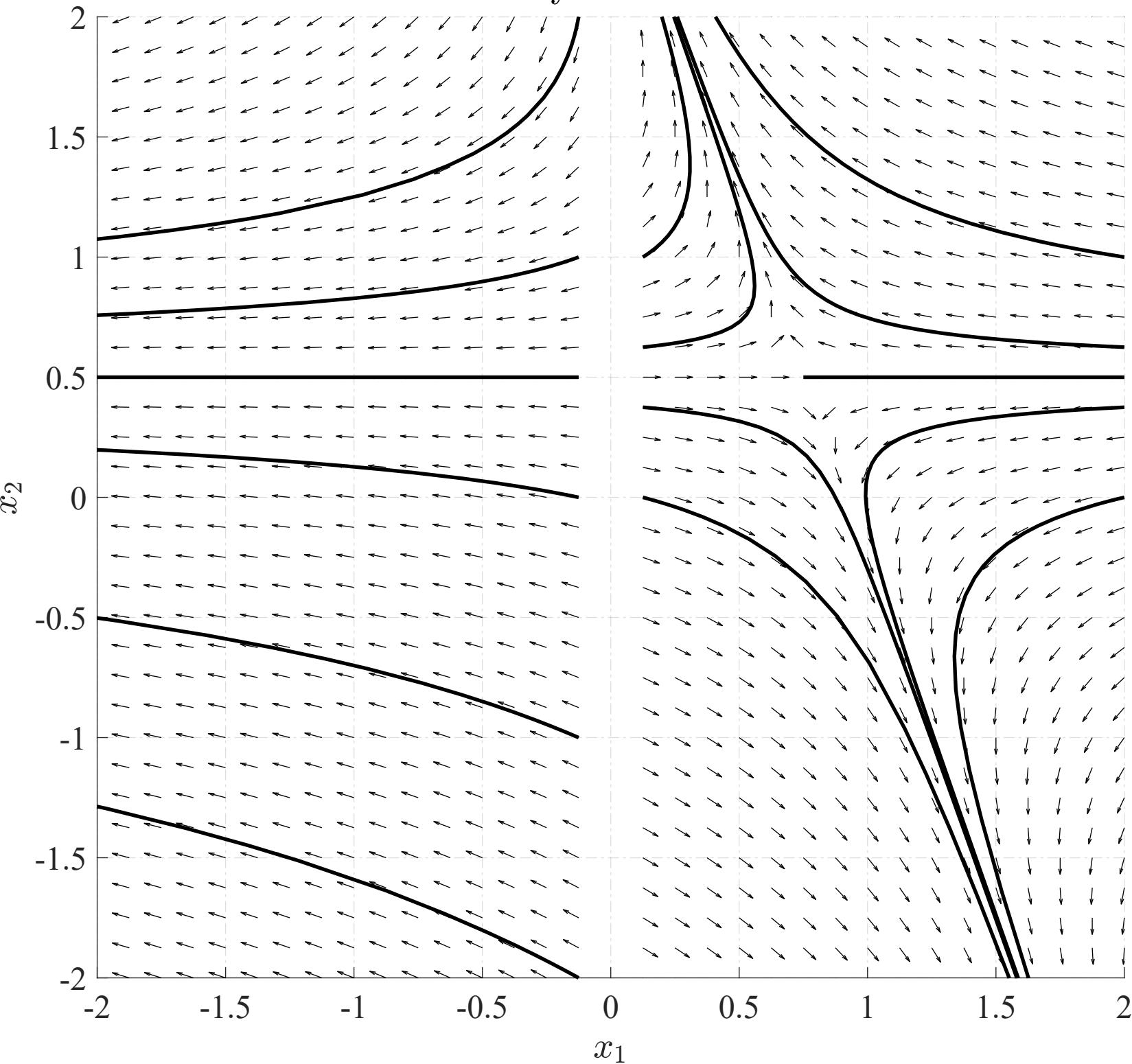
System 4a



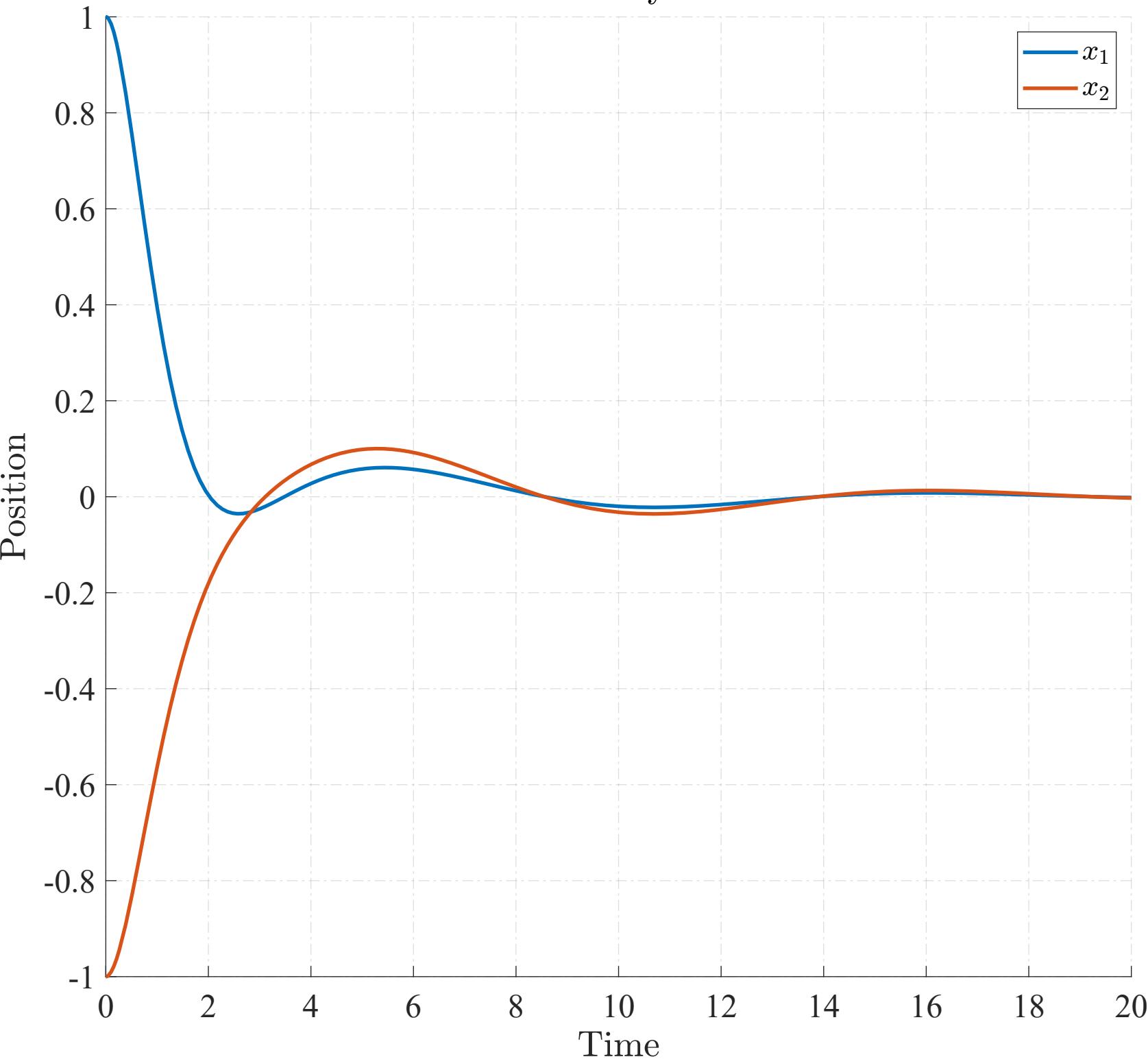
System 4b



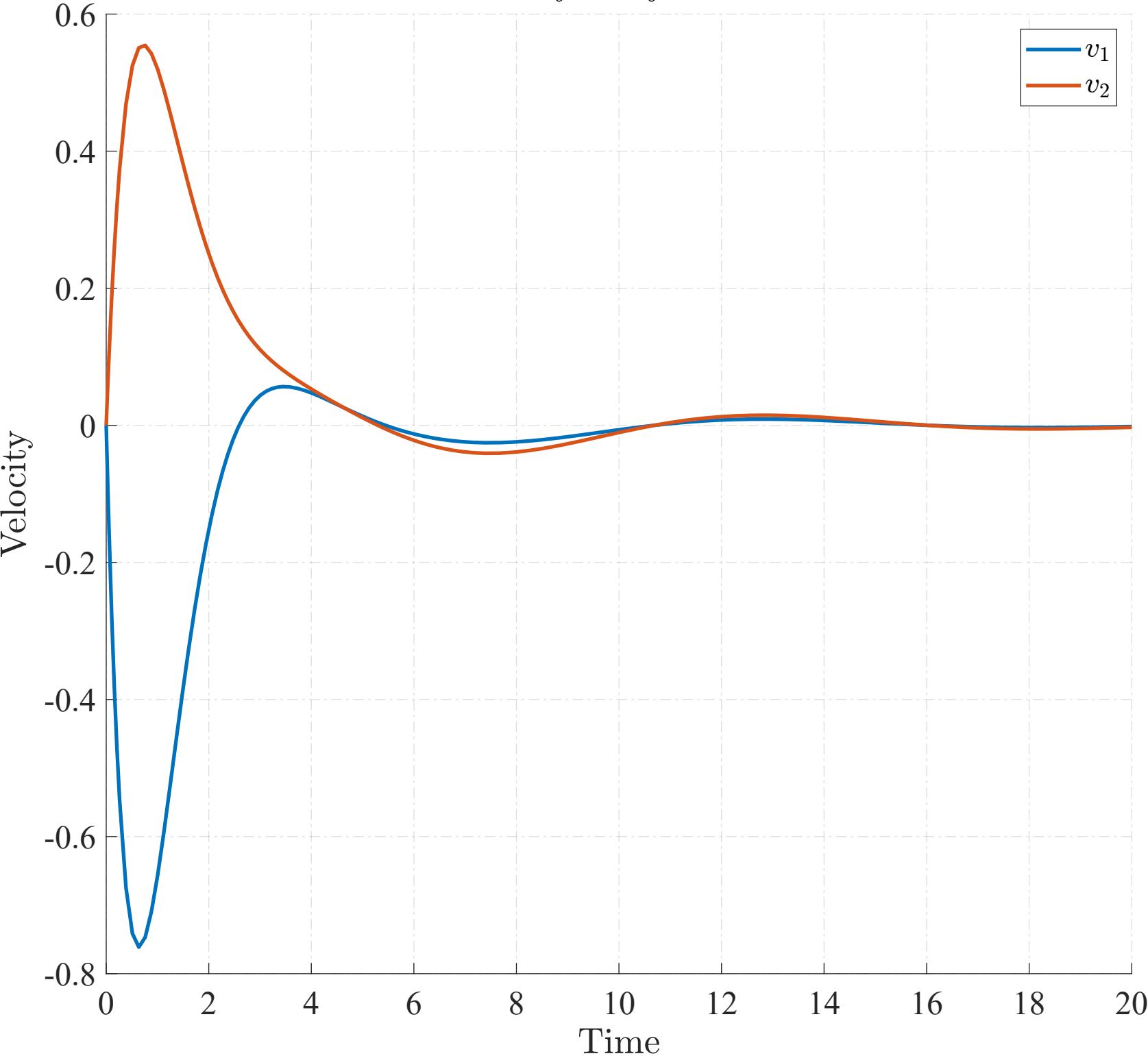
System 4c



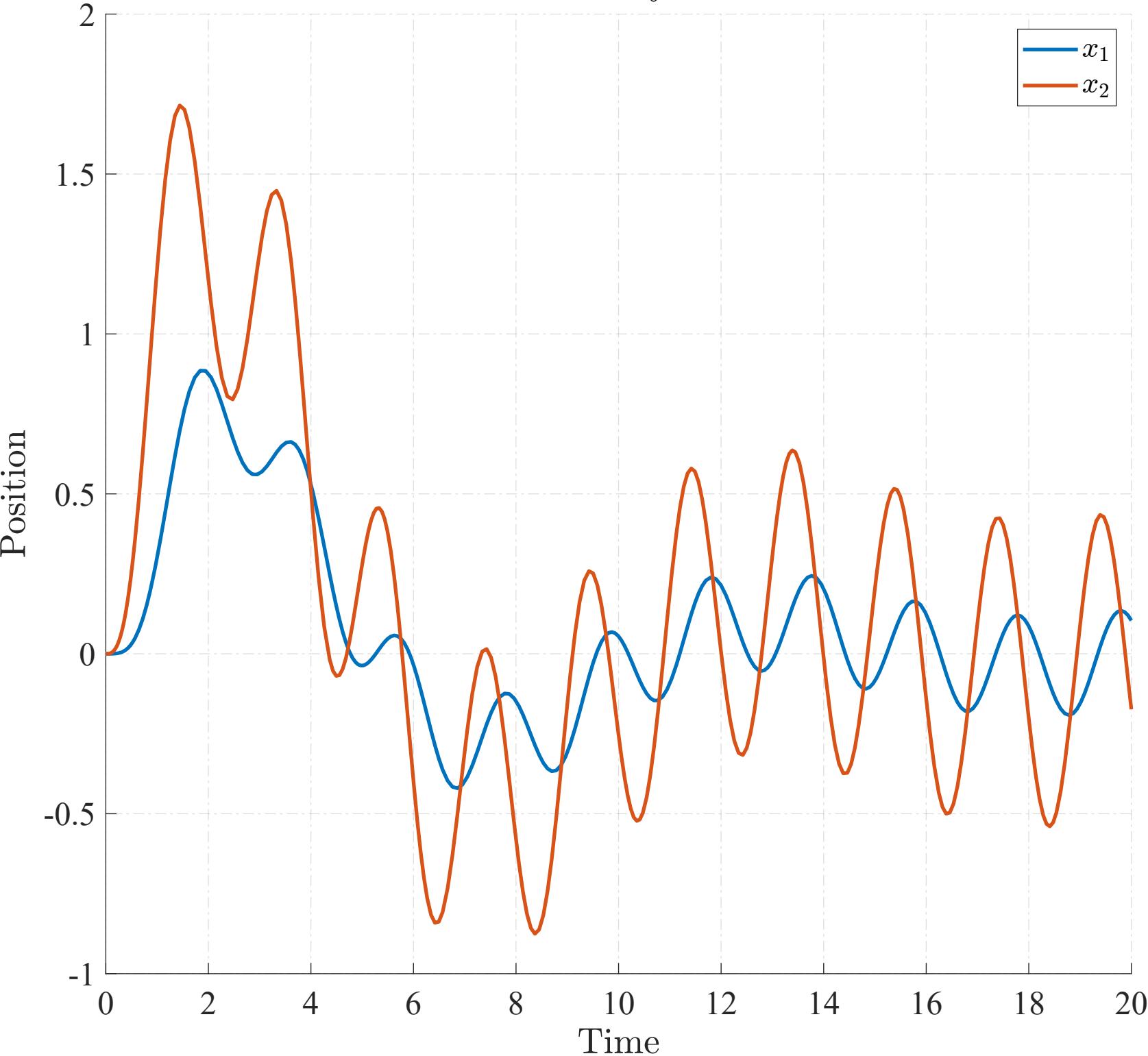
Position of System 8b



Velocity of System 8b



Position of System 8c



Velocity of System 8c

