By definition of a connected, undirected graph, the adjacency matrix is symmetric, and no nodes have an out degree of 0, and out-degree is equivalent to degree. There fore the following definitions can be defined:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{12} & a_{22} & & \vdots \\ \vdots & & \ddots & \\ a_{1n} & & \cdots & a_{nn} \end{bmatrix} \qquad D = \begin{bmatrix} k_1 & & & 0 \\ & k_2 & & \\ & & \ddots & \\ 0 & & & k_n \end{bmatrix}$$

$$v_1 = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix} \qquad k_i = \sum_{j=1}^{n} a_{ij}$$

Because D is a diagonal matrix, its inverse can easily be found.

$$D^{-1} = \begin{bmatrix} k_1^{-1} & & & 0 \\ & k_2^{-1} & & \\ & & \ddots & \\ 0 & & & k_n \end{bmatrix}$$

The definition of an eigenvector v, of AD^{-1}, is that $AD^{-1}v = \lambda v$ for some eigenvalue λ that is a scalar. To prove that v_1 is an eigenvector of AD^{-1}, $AD^{-1}v_1$ will be found.

$$AD^{-1}v_1 = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{12} & a_{22} & & \vdots \\ \vdots & & \ddots & \\ a_{1n} & & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} k_1^{-1} & & & 0 \\ & k_2^{-1} & & \\ & & \ddots & \\ 0 & & & k_n \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix}$$

$$AD^{-1}v_1 = \begin{bmatrix} a_{11}k_1^{-1} & a_{12}k_2^{-1} & \cdots & a_{1n}k_n^{-1} \\ a_{12}k_1^{-1} & a_{22}k_2^{-1} & & \vdots \\ \vdots & & \ddots & \\ a_{1n}k_1^{-1} & \cdots & & a_{nn}k_n^{-1} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{n} a_{1j} \\ \sum_{j=1}^{n} a_{2j} \\ \vdots \\ \sum_{j=1}^{n} a_{nj} \end{bmatrix}$$

$$AD^{-1}v_1 = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix} \qquad \therefore AD^{-1}v_1 = (1)\,v_1$$

By definition, v_1 is an eigenvalue of AD^{-1} with an eigen value of 1.

α must be chosen as, $\alpha < \lambda_1^{-1}$, where λ_1 is the largest eigenvalue of AD^{-1}, and in this case, that value is 1.

The definition of an eigen vector, and its corresponding eigen value of A is:

$$Av = \lambda v$$

where A is the matrix of interest, v is an eigen vector, & λ is the corresponding eigen value.
By definition of authority & hub centrality:

$$AA^T x = \lambda x \qquad A^T A y = \lambda y$$

where A is the adjacency matrix, x is the authority eigenvector(s) and y is the hub eigen vector(s).

$$AA^T x = \lambda x$$
$$A^T (AA^T x) = A^T (\lambda x)$$

Because λ is a scalar, A^T can commute to create:

$$A^T A (A^T x) = \lambda (A^T x)$$

where it can be seen that $A^T x$ is an eigen value of $A^T A$. From the definition we know that the eigen vectors of $A^T A$ are the hub eigen vectors, Thus:

$$\boxed{y = A^T x}$$