

Comparison of Character Rankings Between Graphs & Centrality Methods

Table 1: Rankings of characters in terms of degree centrality

| Rank | <i>Lord of The Rings</i> - Unweighted | | <i>Lord of The Rings</i> - Weighted | | <i>Lord of The Rings & The Hobbit</i> - Weighted | |
|------|---------------------------------------|-------------------|-------------------------------------|-------------------|--|-------------------|
| | Character | Degree Centrality | Character | Degree Centrality | Character | Degree Centrality |
| 1 | Gandalf | 153 | Gandalf | 762 | Gandalf | 901 |
| 2 | Frodo | 144 | Frodo | 661 | Frodo | 661 |
| 3 | Aragorn | 140 | Aragorn | 632 | Aragorn | 632 |
| 4 | Pippin | 137 | Pippin | 606 | Pippin | 606 |
| 5 | Elrond | 125 | Elrond | 484 | Bilbo | 602 |

Interesting things which can be found with degree centrality, as seen in **Table 1**, is that in the graphs only considering *The Lord of the Rings*, the rankings do not change from the unweighted to weighted graphs. This effectively tells us that for at least the ranking of the top 5 most central characters, the proportion of which they share chapters with *different* characters is similar to the proportion of which they share chapters with other characters in general. Another important detail to be seen is that the degrees of Frodo, Aragorn, and Pippin remain the same whether or not *The Hobbit* is considered, meaning that these characters do not appear in *The Hobbit*, or they are the sole character of any chapters that they appear in in *The Hobbit*.

Table 2: Ranking of characters in terms of eigenvector centrality.

| Rank | <i>Lord of The Rings</i> - Unweighted | | <i>Lord of The Rings</i> - Weighted | | <i>Lord of The Rings & The Hobbit</i> - Weighted | |
|------|---------------------------------------|------------------------|-------------------------------------|------------------------|--|------------------------|
| | Character | Eigenvector Centrality | Character | Eigenvector Centrality | Character | Eigenvector Centrality |
| 1 | Gandalf | 0.1682 | Gandalf | 0.3412 | Gandalf | 0.3570 |

| Rank | <i>Lord of The Rings</i> - Unweighted | | <i>Lord of The Rings</i> - Weighted | | <i>Lord of The Rings & The Hobbit</i> - Weighted | |
|------|---------------------------------------|------------------------|-------------------------------------|------------------------|--|------------------------|
| | Character | Eigenvector Centrality | Character | Eigenvector Centrality | Character | Eigenvector Centrality |
| 2 | Aragorn | 0.1641 | Frodo | 0.2920 | Frodo | 0.2827 |
| 3 | Frodo | 0.1618 | Aragorn | 0.2908 | Aragorn | 0.2783 |
| 4 | Elrond | 0.1541 | Pippin | 0.2839 | Pippin | 0.2723 |
| 5 | Pippin | 0.1533 | Sauron | 0.2300 | Bilbo | 0.2287 |

The ranking of characters in terms of their eigenvector centrality is shown in **Table 2**. Unlike degree centrality, there is a large change in the character rankings when weights are considered. This is reflective of the fact that very important characters, such as Gandalf, who interact with a very large number of different characters, also spend more time interacting with other important characters than they do with less important characters. The rankings do not change much when considering *The Hobbit* in the graph. This can be due to the same reason three of the top five characters remained in the top 5 when considering *The Hobbit* despite not appearing in it, which is that *The Lord of The Rings* trilogy has over three times as many chapters as *The Hobbit*, and the importance of the characters from *The Lord of The Rings* is a larger factor when looking at the overall picture.

Table 3: Ranking of characters in terms of Katz centrality.

| Rank | <i>Lord of The Rings</i> - Unweighted | | <i>Lord of The Rings</i> - Weighted | | <i>Lord of The Rings & The Hobbit</i> - Weighted | |
|------|---------------------------------------|---------------------------------------|-------------------------------------|---------------------------------------|--|---------------------------------------|
| | Character | Katz Centrality ($\alpha = 0.0157$) | Character | Katz Centrality ($\alpha = 0.0032$) | Character | Katz Centrality ($\alpha = 0.0031$) |
| 1 | Gandalf | 292.9 | Gandalf | 81.5 | Gandalf | 86.0 |
| 2 | Aragorn | 285.6 | Frodo | 69.9 | Frodo | 68.1 |
| 3 | Frodo | 281.7 | Aragorn | 69.5 | Aragorn | 67.0 |
| 4 | Elrond | 268.2 | Pippin | 67.9 | Pippin | 65.5 |

| Rank | <i>Lord of The Rings</i> - Unweighted | | <i>Lord of The Rings</i> - Weighted | | <i>Lord of The Rings & The Hobbit</i> - Weighted | |
|------|---------------------------------------|--|-------------------------------------|--|--|--|
| | Character | Katz Centrality ($\alpha = 0.0157$) | Character | Katz Centrality ($\alpha = 0.0032$) | Character | Katz Centrality ($\alpha = 0.0031$) |
| 5 | Pippin | 266.9 | Sauron | 55.1 | Bilbo | 55.6 |

The ranking of characters in terms of their Katz centrality is shown in **Table 3**, where α is set to a value very close to λ_1^{-1} , where λ_1 is the largest eigenvalue of A . The results between Katz centrality and eigenvalue centrality are very similar, which is to be expected as all of the graphs are undirected. Thus, all of the trends and observations made for eigenvector centrality apply to Katz centrality as the results yielded are very similar. The value of α changes from graph to graph to satisfy the expression and give the most accurate result. During the tests it is seen with varying values of α , the rankings of the characters do not change from what has been seen, but the centrality values approach 1 as α approaches 0, and the difference between the centrality values for each character decreases.

Table 4: Ranking of characters in terms of PageRank when $\alpha = 0.95$.

| Rank | <i>Lord of The Rings</i> - Unweighted | | <i>Lord of The Rings</i> - Weighted | | <i>Lord of The Rings & The Hobbit</i> - Weighted | |
|------|---------------------------------------|----------|-------------------------------------|----------|--|----------|
| | Character | PageRank | Character | PageRank | Character | PageRank |
| 1 | Gandalf | 68.09 | Gandalf | 145.77 | Gandalf | 160.20 |
| 2 | Frodo | 64.18 | Frodo | 127.53 | Frodo | 118.77 |
| 3 | Aragorn | 61.39 | Aragorn | 120.04 | Aragorn | 111.77 |
| 4 | Pippin | 60.88 | Pippin | 116.34 | Pippin | 108.42 |
| 5 | Bilbo | 54.95 | Elrond | 92.52 | Bilbo | 107.69 |

The ranking of characters in terms of their PageRank when $\alpha = 0.95$ is shown in **Table 4**. The fact that Elrond replaces Bilbo as the last character on the list when weights are considered, but regains that position once *The Hobbit* is considered can be attributed to a few things with the measurement of PageRank. PageRank is used to reduce the effect of high-degree nodes on their low-degree neighbors

when calculating centrality. Elrond being able to take the last spot on the list from bilbo when weights are considered can mean that Bilbo does not have as many interactions as Elrond, which is confirmed by the degree centrality, but those interactions that Bilbo does have are with other characters that are highly central. Bilbo commonly increases on the ranks when *The Hobbit* is introduced, which is likely a product of the fact that Bilbo is a more central character in *The Hobbit* than in *The Lord of The Rings* trilogy.

Table 5: Ranking of characters in terms of Betweenness Centrality.

| Rank | <i>Lord of The Rings</i> - Unweighted | | <i>Lord of The Rings</i> - Weighted | | <i>Lord of The Rings & The Hobbit</i> – Weighted | |
|------|---------------------------------------|------------|-------------------------------------|------------|--|------------|
| | Character | Centrality | Character | Centrality | Character | Centrality |
| 1 | Gandalf | 0.0797 | Gandalf | 0.0437 | Gandalf | 0.0498 |
| 2 | Frodo | 0.0675 | Pippin | 0.0366 | Pippin | 0.0370 |
| 3 | Pippin | 0.0613 | Aragorn | 0.0345 | Aragorn | 0.0339 |
| 4 | Aragorn | 0.0486 | Frodo | 0.0289 | Frodo | 0.0296 |
| 5 | Bilbo | 0.0393 | Bilbo | 0.0270 | Bilbo | 0.0292 |

The ranking of characters in terms of their betweenness centrality is shown in **Table 5**. We see a drastic change in the order when weights are considered, which is to be expected. The edges between highly central characters are likely to have a very large weight as main characters likely interact very frequently throughout the series. Betweenness centrality is calculated using shortest paths, and as weights, which are likely very large, are considered, it is very likely that these shortest paths will have significant changes, thus changing the nodes which are passed through frequently by these shortest paths. There is virtually no change in the betweenness centrality when *The Hobbit* is considered, which is likely due to the fact that *The Hobbit* has many less chapters than *The Lord of The Rings* trilogy, and thus contributes much less to the centrality of the network.

Lord of The Rings Unweighted Network Print

The following is the output of the lab3_tjk190000.ipynb file when analyzing the unweighted *Lord of The Rings* graph.

top 5 degree centrality characters

1. gandalf (153.0000)
2. frodo (144.0000)
3. aragorn (140.0000)
4. pippin (137.0000)
5. elrond (125.0000)

Eigenvector Centrality (by NetworkX):

1. gandalf (0.1682)
2. aragorn (0.1641)
3. frodo (0.1618)
4. elrond (0.1541)
5. pippin (0.1533)

Eigenvector Centrality (by linear algebra):

1. gandalf (0.1682)
2. aragorn (0.1641)
3. frodo (0.1618)
4. elrond (0.1541)
5. pippin (0.1533)

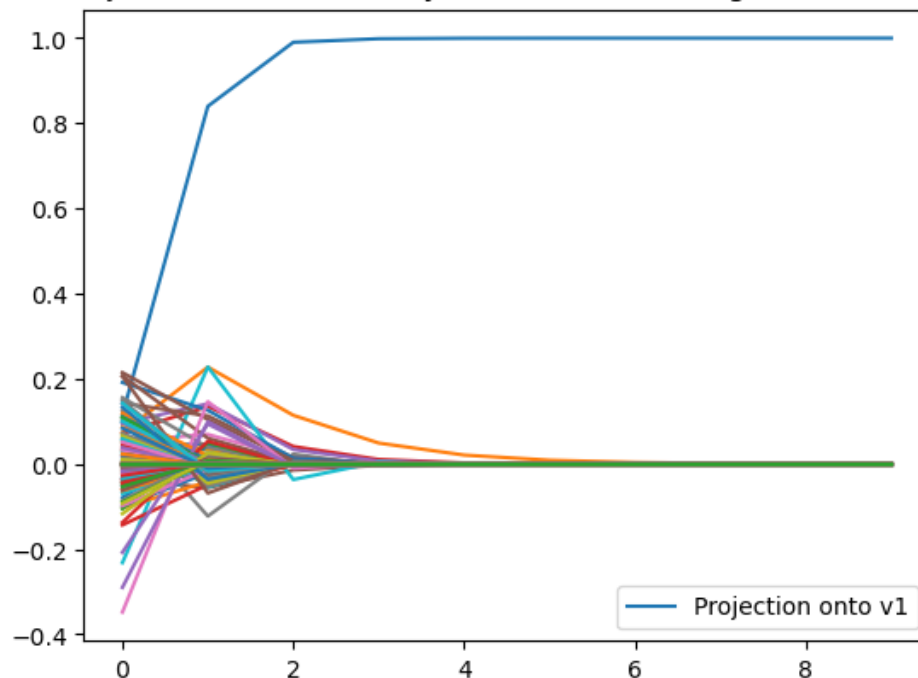
Confirming that eigenvector centrality is a steady-state of sorts for node aragorn:

Eigenvector centrality for node aragorn: 0.1641

Sum of the centralities of neighbors of aragorn normalized by the largest eigenvalue: 0.1641

Showing the converge of eigenvector centrality...

Projection of the centrality vector x onto the eigenvectors of A



Katz Centrality:

Top 5 central characters using Katz centrality ($\alpha = 0.0158$):

1. gandalf (1549115332298075.5000)
2. aragorn (1510834676614963.0000)
3. frodo (1489883525045676.5000)
4. elrond (1419026387096076.5000)
5. pippin (1411232561834083.5000)

Top 5 central characters using Katz centrality ($\alpha = 0.0157$):

1. gandalf (292.9143)
2. aragorn (285.5536)
3. frodo (281.7088)
4. elrond (268.1689)
5. pippin (266.9178)

Top 5 central characters using Katz centrality ($\alpha = 0.0079$):

1. gandalf (3.1165)
2. frodo (3.0126)
3. aragorn (2.9915)
4. pippin (2.9150)
5. elrond (2.8220)

Top 5 central characters using Katz centrality ($\alpha = 0.0053$):

1. gandalf (2.1064)
2. frodo (2.0484)
3. aragorn (2.0306)
4. pippin (1.9979)
5. elrond (1.9351)

Top 5 central characters using Katz centrality ($\alpha = 0.0040$):

1. gandalf (1.7543)
2. frodo (1.7135)
3. aragorn (1.6993)
4. pippin (1.6791)
5. elrond (1.6319)

| PageRank | Betweenness Centrality |
|----------------------|------------------------|
| 1. gandalf (68.0983) | 1. gandalf (0.0797) |
| 2. frodo (64.1828) | 2. frodo (0.0675) |
| 3. aragorn (61.3929) | 3. pippin (0.0613) |
| 4. pippin (60.8781) | 4. aragorn (0.0486) |
| 5. bilbo (54.9509) | 5. bilbo (0.0393) |

Lord of The Rings Weighted Network Print

The following is the output of the lab3_tjk190000.ipynb file when analyzing the weighted *Lord of The Rings* graph.

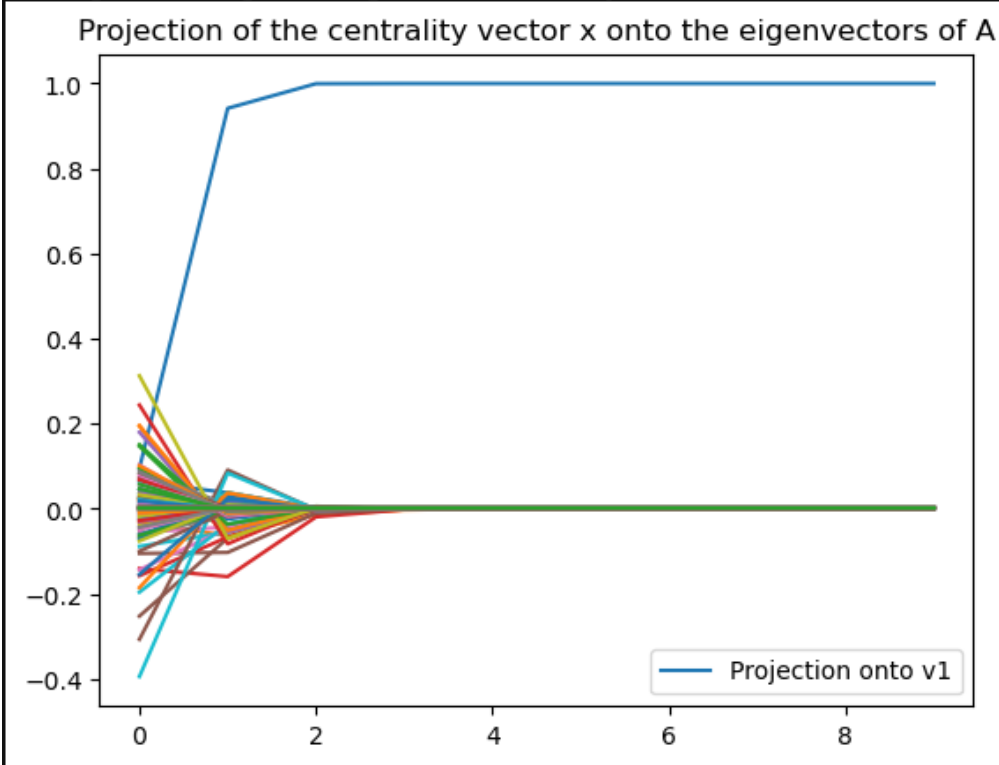
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top 5 degree centrality characters   Eigenvector Centrality (by NetworkX):   Eigenvector Centrality (by linear algebra):
1. gandalf (762.0000)               1. gandalf (0.3412)                     1. gandalf (0.3412)
2. frodo (661.0000)                 2. frodo (0.2920)                       2. frodo (0.2920)
3. aragorn (632.0000)                3. aragorn (0.2908)                     3. aragorn (0.2908)
4. pippin (606.0000)                 4. pippin (0.2839)                     4. pippin (0.2839)
5. elrond (484.0000)                 5. sauron (0.2300)                      5. sauron (0.2300)
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Confirming that eigenvector centrality is a steady-state of sorts for node aragorn:

Eigenvector centrality for node aragorn: 0.2908

Sum of the centralities of neighbors of aragorn normalized by the largest eigenvalue: 0.2908

Showing the converge of eigenvector centrality...



Katz Centrality:

Top 5 central characters using Katz centrality ($\alpha = 0.0033$):

1. ghan (-27288611503717.1094)
2. orome (-27288611503717.1094)
3. twofoot (-34698631076483.9375)
4. proudfoot (-34698631076483.9414)
5. adelard (-34698631076483.9414)

Top 5 central characters using Katz centrality ($\alpha = 0.0032$):

1. gandalf (81.4709)
2. frodo (69.9094)
3. aragorn (69.5233)
4. pippin (67.8687)
5. sauron (55.1369)

Top 5 central characters using Katz centrality ($\alpha = 0.0017$):

1. gandalf (3.5108)
2. frodo (3.1648)
3. aragorn (3.1121)
4. pippin (3.0444)
5. sauron (2.6388)

Top 5 central characters using Katz centrality ($\alpha = 0.0011$):

1. gandalf (2.2607)
2. frodo (2.0893)
3. aragorn (2.0557)
4. pippin (2.0187)
5. sauron (1.8135)

Top 5 central characters using Katz centrality ($\alpha = 0.0008$):

1. gandalf (1.8423)
2. frodo (1.7285)
3. aragorn (1.7037)
4. pippin (1.6780)
5. sauron (1.5403)

| PageRank | Betweenness Centrality |
|-----------------------|------------------------|
| 1. gandalf (145.7734) | 1. gandalf (0.0437) |
| 2. frodo (127.5262) | 2. pippin (0.0366) |
| 3. aragorn (120.0421) | 3. aragorn (0.0345) |
| 4. pippin (116.3430) | 4. frodo (0.0289) |
| 5. elrond (92.5220) | 5. bilbo (0.0270) |

Lord of The Rings & The Hobbit Weighted Network Print-out

The following is the output of the lab3_tjk190000.ipynb file when analyzing the weighted *Lord of The Rings & The Hobbit* graph.

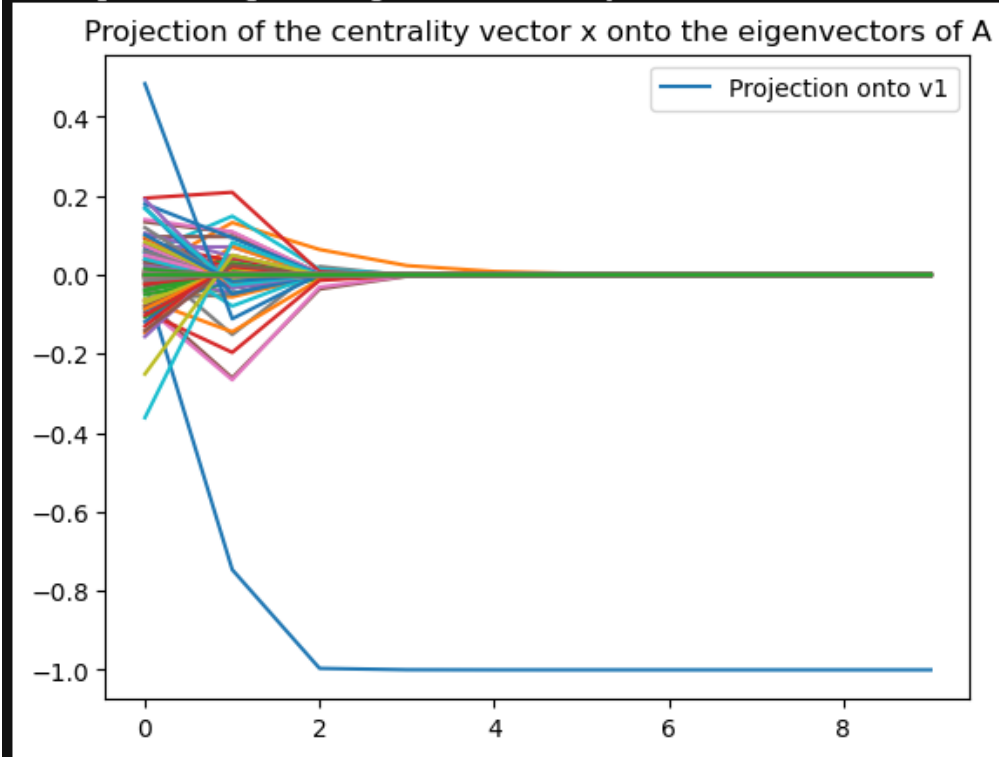
| top 5 degree centrality characters | Eigenvector Centrality (by NetworkX): | Eigenvector Centrality (by linear algebra): |
|------------------------------------|---------------------------------------|---|
| 1. gandalf (901.0000) | 1. gandalf (0.3570) | 1. gandalf (0.3570) |
| 2. frodo (661.0000) | 2. frodo (0.2827) | 2. frodo (0.2827) |
| 3. aragorn (632.0000) | 3. aragorn (0.2783) | 3. aragorn (0.2783) |
| 4. pippin (606.0000) | 4. pippin (0.2723) | 4. pippin (0.2723) |
| 5. bilbo (602.0000) | 5. bilbo (0.2287) | 5. bilbo (0.2287) |

Confirming that eigenvector centrality is a steady-state of sorts for node aragorn:

Eigenvector centrality for node aragorn: 0.2783

Sum of the centralities of neighbors of aragorn normalized by the largest eigenvalue: 0.2783

Showing the converge of eigenvector centrality...



Katz Centrality:

Top 5 central characters using Katz centrality ($\alpha = 0.0032$):

1. gandalf (3506233965768915.0000)
2. frodo (2776134896343451.5000)
3. aragorn (2733443552956267.5000)
4. pippin (2674606740861643.0000)
5. bilbo (2246654230718593.0000)

Top 5 central characters using Katz centrality ($\alpha = 0.0031$):

1. gandalf (85.9849)
2. frodo (68.0973)
3. aragorn (66.9911)
4. pippin (65.5381)
5. bilbo (55.6171)

Top 5 central characters using Katz centrality ($\alpha = 0.0016$):

1. gandalf (3.8101)
2. frodo (3.1354)
3. aragorn (3.0695)
4. pippin (3.0064)
5. bilbo (2.8592)

Top 5 central characters using Katz centrality ($\alpha = 0.0011$):

1. gandalf (2.4204)
2. frodo (2.0663)
3. aragorn (2.0286)
4. pippin (1.9938)
5. bilbo (1.9446)

Top 5 central characters using Katz centrality ($\alpha = 0.0008$):

1. gandalf (1.9521)
2. frodo (1.7105)
3. aragorn (1.6839)
4. pippin (1.6595)
5. bilbo (1.6343)

| PageRank | Betweenness Centrality |
|-----------------------|------------------------|
| 1. gandalf (160.2037) | 1. gandalf (0.0498) |
| 2. frodo (118.7663) | 2. pippin (0.0370) |
| 3. aragorn (111.7720) | 3. aragorn (0.0339) |
| 4. pippin (108.4245) | 4. frodo (0.0296) |
| 5. bilbo (107.6881) | 5. bilbo (0.0292) |