Written Assignment #1

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1.1:   Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

True or false? In every instance of the Stable Matching Problem, there is a stable matching containing a pair (m, w) such that m is ranked first on the preference list of w and w is ranked first on the preference list of m.

This statement is false. There is at least one situation where there is no pair (m, w) possible where m is the first choice of w and w is the first choice of m. I think of it as the condition of wrap-around. When each m ∈ M has ranked first a unique w ∈ W, and vice versa, the selection can wrap around. The following table shows a particular situation, where each arrow points from a member to the first-ranked member of the opposite group. As can be seen, nowhere do these arrows agree. We needn't even provide a stable matching, since we have shown that no matching, stable or not, can fulfill the requirement.

M	Rankings		W	Rankings
m1	w1, w2, w3		w1	m2, m3, m1
m2	w2, w3, w1		w2	m3, m1, m2
m3	w3, w1, w2		w3	m1, m2, m3

1.2:   Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

> *True or false? Consider an instance of the Stable Matching Problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m. Then in every stable matching S for this instance, the pair (m, w) belongs to S.*

This statement is true. Here we can use a proof by contradiction. Were this not the case, then there must be some stable matching X that the pair (m,w) does not exist in. But this would mean that the pairs (m, w') and (m', w) must exist, where w' and m' are any other match in their preference list. But we know from the premise that m prefers w to w' (any other), and w prefers m to m' (any other). Therefore they would "elope". This fulfills one of the definitions of instability we established, and therefore X is not a stable matching, and the possibility is disproved.