

Fundamentals of GPS - Homework 2

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Problem 1

Consider a simple estimation of a single parameter: $\hat{\theta} = \bar{y}$, where the measurement noise has unit variance.

Find:

- **Part A:** Determine the accuracy of the parameter estimate as a function of the number of measurements
- **Part B:** Verify the results through a Monte Carlo simulation

Solution:

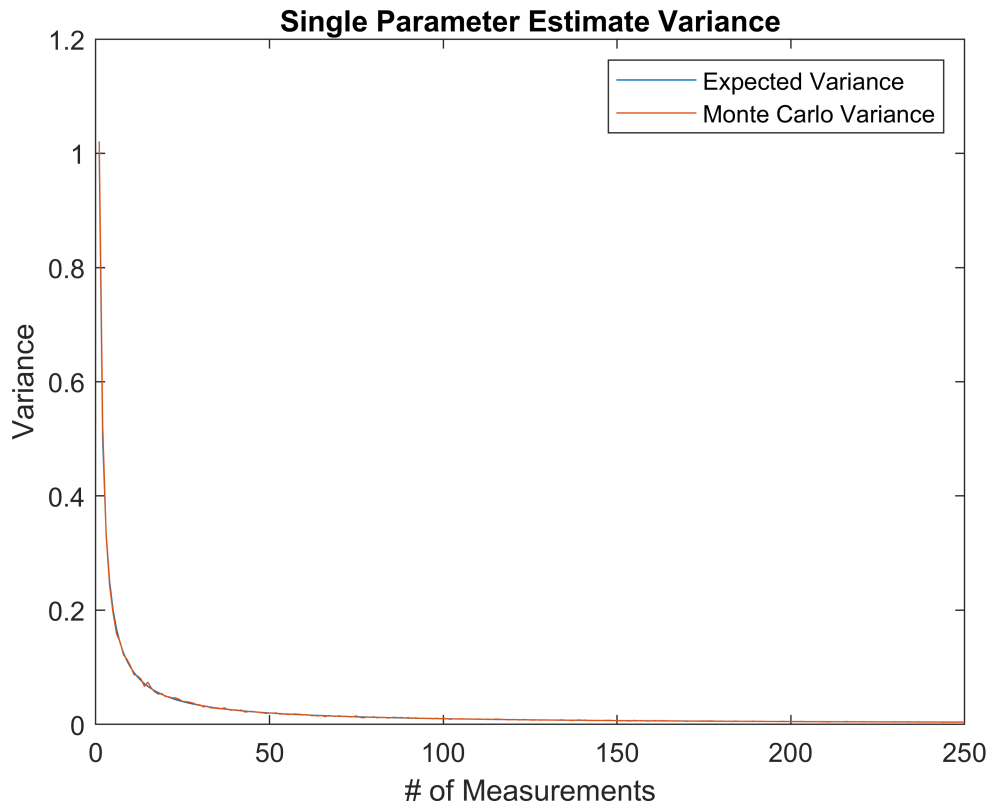
- **Part A:** An actual value of 5 was chosen for this single parameter estimation. Random Gaussian noise with unit variance was added to each of 1 to 250 measurements. The expected variance (accuracy), P , was calculated using

$$P = \frac{\sigma^2}{N}$$

where σ^2 is variance and N is the number of measurements.

- **Part B:** The 1 to 250 measurements were simulated 1000 times. After each simulation, the mean (Least Squares estimate) and the estimate's error from the actual value were calculated and logged. Then, the variance of the estimate errors for each simulation was calculated and repeated for each measurement 1 to 250. These Monte Carlo variances were plotted against P from Part A. The plot shows the expected variance relationship is the same as the simulated relationship with some small noise.

Part A & B)



Problem 2

Given the set of data in the table, perform a Least Squares fit to solve for the model coefficients. Provide the estimate of the coefficients and the estimated/predicted estimation error (1-sigma) for the coefficient a . Assume the 1-sigma measurement noise on y is 0.4.

Find:

- **Part A:** $y = a + bx$
- **Part B:** $y = a + bx + cx^2$
- **Part C:** $y = a + bx + cx^2 + dx^3$
- **Part D:** Is the estimate for a consistent? Why or why not? Which is probably the correct prediction of the estimation error on coefficient a ?

Solution:

- **Part A, B, and C:** An observation matrix based on the provided model was determined and then used in a Least Squares estimate to determine the a , b , and/or c and d values.
- **Part D:** The estimate for a is not consistent because the addition of higher order terms increases the variance of a as they begin to dominate the fit. According to the predicted estimate error (σ_a), the first a value calculated is most accurate and this is likely because the linear model doesn't fit to as much noise in the measurements as the higher order models.

Part A)

```
estcoeff_a = 2x1
    1.1866
    0.6933

sigmaa_a = 0.3098
```

Part B)

```
estcoeff_b = 3x1
    0.4039
    2.2587
   -0.3914

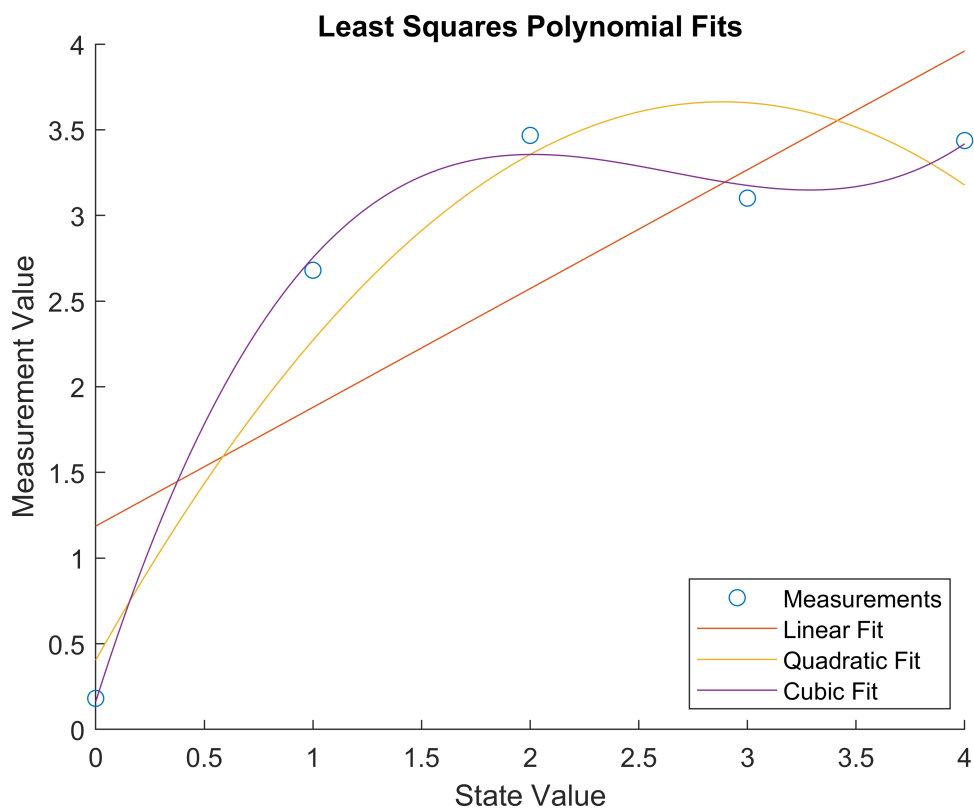
sigmaa_b = 0.3764
```

Part C)

```
estcoeff_c = 4x1
    0.1625
    3.9888
   -1.5984
    0.2012

sigmaa_c = 0.3971
```

Part D)



Problem 3

Given the following range equation $r^2 = (x - a)^2 + (y - b)^2$ with a range error of 0.5 meters (1- sigma).

Find:

- **Part A:** Find Jacobian matrix for this System (i.e. the geometry matrix)
- **Part B:** What is the expected solution uncertainty (1-sigma)?
- **Part C:** What is the solution for x and y ?
- **Part D:** Perform a monte-carlo simulation and verify part b

Solution:

- **Part A:** The range equation was linearized and the estimated squared range was subtracted from the range squared measurements to create the measurement vector y . The G matrix was constructed from the rest of the linearization (the partial derivatives) as so:

$$G = \begin{bmatrix} 2x_0 & 2y_0 \\ 2(x_0 - 10) & 2y_0 \\ 2x_0 & 2(y_0 - 10) \\ 2(x_0 - 10) & 2(y_0 - 10) \end{bmatrix}$$

The measurement vector is:

$$y = \begin{bmatrix} 25 - x_0^2 - y_0^2 \\ 65 - (x_0 - 10)^2 - y_0^2 \\ 45 - x_0^2 - (y_0 - 10)^2 \\ 85 - (x_0 - 10)^2 - (y_0 - 10)^2 \end{bmatrix}$$

The state vector is:

$$x = \begin{bmatrix} x \\ y \end{bmatrix}$$

- **Part B:** The expected solution uncertainty was calculated using the solution ($x = 3, y = 4$) found in Part C as:

Part B)

```
sigma_exp = 2x1
0.0116
0.0123
```

- **Part C:** The solution was found using Least Squares (demonstrated in Part D) and was determined to be $x = 3m, y = 4m$.
- **Part D:** A Monte Carlo simulation with noise added to the measurement was conducted and the corresponding position variances and sigmas were calculated to be compared to the expected solution uncertainty.

Part D)

```
sigma = 2x1
0.0117
0.0123
```

Problem 4

On the class website, download the data file for HW #2. This is a file of actual GPS satellites with simulated psuedoranges to a known location. The file also has simulated SOOP (signal of opportunity) stations with their simulated psuedoranges. The clocks on the SOOP are assumed to be synchronized with the GPS satellites (using GPS time). The receiver has a clock bias and measures the ranges with an 1 sigma accuracy of 0.5 meters.

Find:

- **Part A:** Calculate the position and expected horizontal and vertical error using the first 4 GPS satellites. Initialize your position to the center of the earth. How many iterations does it take to solve for your position?
- **Part B:** Repeat part a with all 9 GPS satellites
- **Part C:** Repeat part a assuming a perfect clock (note you should correct the pseudo-ranges with the clock bias from part b)
- **Part D:** Calculate the position using 2 GPS satellites and 2 SOOPs. What happens and why?
- **Part E:** Repeat part c, but start with an initial guess of [423000, -5362000, 3417000]

Solution:

- **Part A:** The position was calculated using 3 functions. One that calculates the pseudorange error vector, one that calculates the geometry matrix, and one that performs the Least Squares and Newton-Raphson estimation. This part uses the first four GPS satellites in the data file. The position, horizontal error, and vertical error are found below. The Least Squares estimate took 5 iterations to complete using a convergence criteria that exited the estimate when the norm of the position/clock error vector was below the variance of the measurements.

Part A)

```
pos = 3x1
106 x
0.4233
-5.3617
3.4173

clock_bias = 100.0000

itr = 5

hor_err = 5.0481

vert_err = 1.5607

lla = 1x3
32.6055 -85.4856 200.8072
```

- **Part B:** The same functions from Part A were used in Part B and the position calculation was repeated with all 9 satellites. The position, horizontal error, and vertical error are found below. The position solution is the exact same, but the predicted error estimates were drastically decreased as expected with more measurements.

Part B)

```
pos = 3×1
106 ×
    0.4233
   -5.3617
    3.4173

clock_bias = 100.0000

itr = 5

hor_err = 0.6538

vert_err = 0.4279

lla = 1×3
    32.6055  -85.4856  200.8072
```

- **Part C:** The functions from Parts A & B were modified to construct an $mx3$ G that assumes a perfect clock. This means no clock bias is estimated. The position calculation was repeated after subtracting the previous clock bias from the pseudoranges. The position, horizontal error, and vertical error are found below. The position solution is the exact same, but the associated errors are less than they were when the clock was assumed to have bias and the pseudoranges weren't corrected. Ideally, if the clock bias that was subtracted from the pseudorange was deterministic, only 3 satellite measurements would be necessary to determine position.

Part C)

```
pos = 3×1
106 ×
    0.4233
   -5.3617
    3.4173

itr = 5

hor_err = 0.5637

vert_err = 1.3407

lla = 1×3
    32.6055  -85.4856  200.8072
```

- **Part D:** The same functions from Parts A & B were used in Part D and the position calculation was attempted. The algorithm never produced a position solution because the geometry of the 2 GPS satellites and SOOPs is poor enough to essentially decrease the number of usable measurements. The observability of the user's position is poor, therefore, no solution is produced.

- **Part E:** The same functions from Part C were used but the estimated position parameter was modified to take an initial position other than the center of the earth. The position calculation was repeated after subtracting the clock bias from the pseudoranges and was determined to be the exact same, but the Least Squares estimate converged in 2 iterations instead of 5. The position, horizontal error, and vertical error are found below.

Part E)

```
pos = 3x1
106 x
    0.4233
   -5.3617
    3.4173

itr = 2

hor_err = 0.5637

vert_err = 1.3407

lla = 1x3
    32.6055  -85.4856  200.8072
```

Problem 5

One of the sources of error in the GPS pseudoranges comes from the ionosphere (from 50km to 1000km height above the Earth). In effect, the amount of the ionospheric delay is a function of how much atmosphere the signal passes through, which is a function of the satellite elevation angle, θ . A simple model of the an ionosphere error (described in class) is:

$$I(\theta) = A_1 * (1 + 16 * (0.53 - \frac{\theta}{180})^3)$$

where $A_1 = 5 \times 10^{-9}$ seconds and θ is in degrees.

Find:

- Using the 9 GPS satellite positions in the data file and the Ionospheric delay model, generate a plot of position accuracy vs mask angle (note that the mask angle can only be increased until less than four satellites are in view). Comment on this plot.

Solution:

- A plot of position variance versus mask angle was produced along with the Ionosphere error in meters. It can be seen that the variance increases significantly while the Ionosphere error gradually decreases with an increase in mask angle. This inverse relationship implies the geometry of the satellites and number of available measurements plays a much larger role in the predicted variance of the position solution than the Ionosphere error. This implication means it may be slightly beneficial to institute a low mask angle to rid the solution of a small quantity of variance, but over masking may introduce more variance by eliminating overhead measurements.

