Optimal Estimation - Homework 4

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# Problem 1

Develop a model for a pendulum with inertia 𝐽𝑝 =2.5 𝑁𝑚 𝑟𝑎𝑑/𝑠^2 (at pin), mass 𝑚=1.6 𝑘𝑔, and length 𝐿=1 𝑚. The pin introduces damping in the system that should be modeled as 𝜏𝑏 =𝑏𝜃̇3 where 𝑏 =1.25 𝑁𝑚 r𝑎𝑑/𝑠. The input to the system is a torque at the pin given by 𝜏 =12𝑁𝑚. Assume system is acted on by a horizontal disturbance force at the end of the pendulum (𝑓(𝑡)=5+𝜂 where 𝜂~𝑁(0,2)). The measurement of the angle of the pendulum is corrupted by zero mean Gaussian white noise with variance of 1 degree.

## Find:

* **Part A:** Develop a simulation of the system.
* **Part B:** Develop an extended Kalman filter to estimate the position and velocity (and any additional needed parameters) of the pendulum given measurement as described.
* **Part C:** Develop an unscented Kalman filter to estimate the position and velocity (and any additional needed parameters) of the pendulum given measurement as described.
* **Part D:** Use Monte Carlo simulations to compare the performance of the EKF and UKF. Be sure to compare expected covariance to sampled covariance from Monte Carlo simulations.

## Solution:

* **Part A:** The following code simulates the pendulum system and the plot shows the simulated system and measurement.

J = 2.5;

m = 1.6;

l = 1;

b = 1.25;

tau = 12;

g = 9.81;

% Jddot + bddot^3 + mglsin(theta) = T + dist

dt = 0.001;

tEnd = 20;

t = 0:dt:tEnd - dt;

dot = 0;

ddot = 0;

theta = 0;

for i = 1:length(t)

fD = 5 + sqrt(2)\*randn;

ddot = (tau + fD\*l\*cos(theta) - m\*g\*l\*sin(theta) - b\*dot^3)/J;

dot = dot + ddot\*dt;

theta = theta + dot\*dt;

thetaL(i) = theta;

dotL(i) = dot;

y(i) = theta + sqrt(deg2rad(1))\*randn;

end

figure

plot(t,y)

hold on

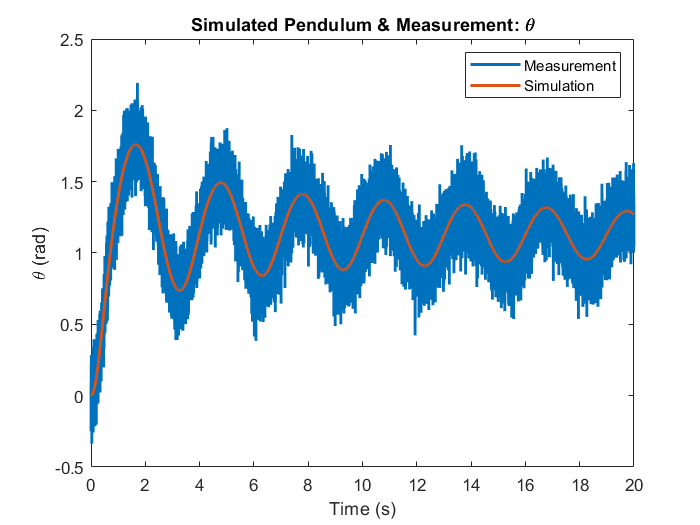
plot(t,thetaL)

title('Simulated Pendulum & Measurement: \theta')

legend('Measurement','Simulation')

xlabel('Time (s)')

ylabel('\theta (rad)')



* **Part B:** The following code implements an EKF to estimate the position and velocity of the pendulum system. The EKF could use a lot of work as it was a quick implementation. The bias from the disturbance is poorly estimated causing the position and velocity estimate to be biased. With more time, I feel confident I could tune the EKF and adjust the time update to estimate the sates correctly.

Bc = [0; 1/J; 0];

R = deg2rad(1);

H = [1 0 0];

fD = 4;

theta = 0;

dot = 0;

X = [theta; dot; fD];

P = eye(3);

for i = 1:length(t)

% Time Update

ddot = (tau + X(3)\*l\*cos(X(1)) - m\*g\*l\*sin(X(1)) - b\*X(2)^3)/J;

dot = dot + ddot\*dt;

theta = theta + dot\*dt;

X = [theta; dot; fD];

F = [0 1 0;

-((m\*g\*l)/J)\*sin(X(1)) -(3\*b)/J\*X(2)^2 (l/J)\*cos(X(1));

0 0 0];

% A = expm(F\*dt);

[A,B,H,D] =c2dm(F,Bc,H,0,dt);

BW = [0; 0; l\*cos(X(1))/J];

QD = BW\*2\*BW'\*dt;

P = A\*P\*A' + QD;

% Measurement Update

K = P\*H'\*inv(H\*P\*H' + R);

X = X + K\*(y-H\*X);

P = (eye(3) - K\*H)\*P;

xL(i) = X(1);

xdotL(i) = X(2);

biasL(i) = X(3);

% PL(i) = P;

end

figure

plot(t,y)

hold on

plot(t,thetaL)

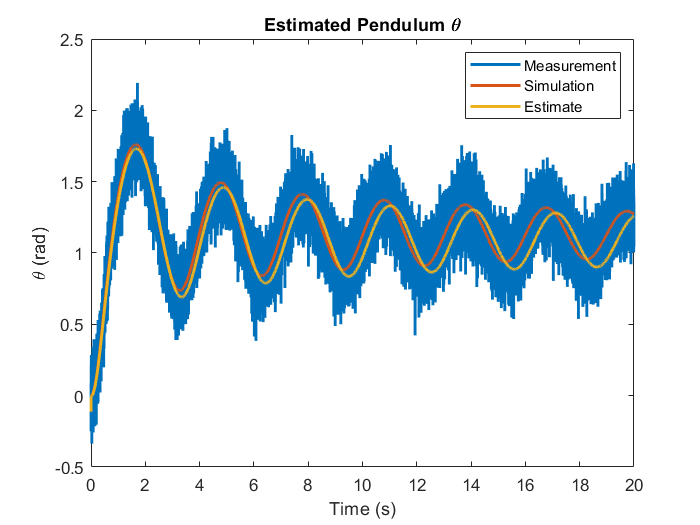
plot(t,xL)

title('Estimated Pendulum \theta')

legend('Measurement','Simulation','Estimate')

xlabel('Time (s)')

ylabel('\theta (rad)')



figure

plot(t,dotL)

hold on

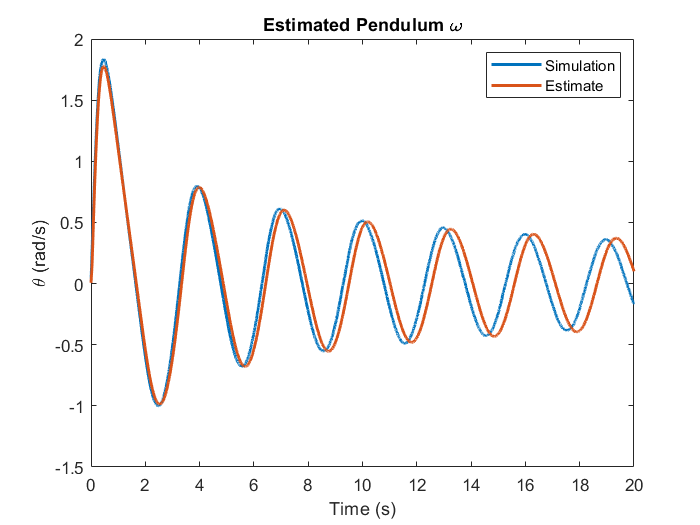
plot(t,xdotL)

title('Estimated Pendulum \omega')

legend('Simulation','Estimate')

xlabel('Time (s)')

ylabel('\theta (rad/s)')



* **Part C:**  I was unable to develop the Unscented Particle as I wasn't able to get the EKF done as quickly as I hoped.
* **Part D:** I was unable to compare the covariances because I couldn't complete the UKF.

# Problem 2

Refer to problem 3 from homework 3 (the “Navigation” filter). Design a particle filter to estimate the East and North position, radar and gyro bias using the data (hw3\_3 from canvas). Compare the performance of the particle filter to the performance of your estimator from homework 3 using at least 3 different numbers of particle (e.g. N=50, 100,1000). Provide plots of estimation error, analytical covariance (from EKF) and numerical covariance (from particle filter).

## Solution:

* Unable to complete in time.