MECH 7970-004 Project 2

Instructions:

- Part I: Perform individually and do not consult others for solutions. This part should be submitted on the last day of class separate from Part II.
- Part II: Form a group of 2 (groups can be different than for Project 1). Submit 1 copy per group.

Due Date: December 1st, 2022

Part I: Individual Portion

Problem 1:

- Consider the state equation $\dot{x} = Ax + Bu$, where

$$A = \begin{bmatrix} -1 & 1 & 1 \\ 2 & -1 & 2 \\ -2 & -1 & -4 \end{bmatrix}, B = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}.$$

1. Determine if Lyapunov's test for controllability can be applied. If so, then apply the test to determine the controllability of the system. Do NOT use lyap to solve. Show your work.
2. Apply the eigenvector test for controllability.

Problem 2:

- Consider the state equation $\dot{x} = Ax + Bu$, $y = Cx$, where

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}.$$

Find the basis vectors for each of the following:

1. Stable and Unstable Subspaces
2. Controllable and Uncontrollable Subspaces
3. Observable and Unobservable Subspaces
4. Is the system stabilizable?
5. Is the system detectable? (observability dual to Stabilizability: unstable subspace is in the observable subspace, and the unobservable subspace is in the stable subspace)

Part II: Group Portion

Given: (Same model as the previous project)

- The cart-pendulum system that is depicted in Figure 1.

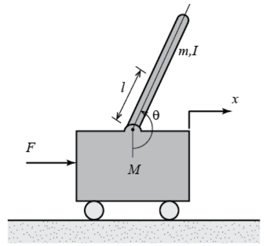

Figure 1: Image of the cart-pendulum system.

- Assume the following:

Table 1: Cart-Pendulum Relevant Terms

Term	Description	Value
F	force applied to the cart	-
x	cart position coordinate	-
x^{eq}	cart equilibrium position	$0\ m$
θ	pendulum angle from downward vertical	-
θ^{eq}	pendulum equilibrium angle	$\pi\ rad$

- In this project, reuse the nonlinear and linear models you constructed during Project 1.
- Recall that the linearized model takes the following form: $\dot{z} = Az + Bu$, $y = Cz$,where

$$z = \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix}, \quad \text{and } \theta = \pi + \phi.$$

Design Requirements:

1. 2% settling time for x and θ of less than 5 seconds.
   (a) The cart gets to within 2% of its desired position within 5 seconds.
   (b) The pendulum returns to within 2% of its upright position within 5 seconds.
2. Pendulum angle θ is never more than 35 degrees (0.61 rad) from the vertical.
3. Steady-state error of less than 2% for x and θ.

Initial Conditions:

When designing the following controllers to satisfy the design requirements, use the following initial conditions:

$$z(0) = \begin{bmatrix} -0.5\,m \\ 0 \\ 30\,\text{deg} \\ 0 \end{bmatrix}.$$

To get a better feel for the controller's performance, you can modify the initial conditions to see how the performance changes.

Problem 3: LQR State Feedback Regulation

- Design a state feedback controller

$$u = -Kz \tag{1}$$

to drive the states to 0 while satisfying the aforementioned design requirements.

- Specifically state the values you used for Q, R, and K and your motivation for your specific selections of Q and R.

1. Design the controller for the linear model.
   (a) Provide the controller gain, plots of the outputs, and plots of the input.
   (b) What are the eigenvalues of the closed-loop system? (i.e., the eigenvalues of $A - BK$)
2. Design the controller for nonlinear model.
   (a) Provide the controller gain, plots of the outputs, and plots of the input.
3. Discuss differences between tuning the model for the linear and nonlinear model.

Problem 4: LQR State Feedback Regulation

- Choose 5 separate combinations of Q and R.
  - For at least one of your combinations, design $Q = C^T Q_1 C$ to achieve output regulation.
- Provide your selections of Q, R, and K.
  - Implement each controller on the linear model.
    * Provide the controller gain, plots of the outputs, and plots of the input.
    * What are the eigenvalues of the closed-loop system? (i.e., the eigenvalues of $A - BK$)
- Provide comments on the effect of your selection of Q and R on the system errors and control input.

Problem 5: LQR State Feedback Setpoint Tracking

- Design a state feedback controller to drive $\phi \to 0$ and $z \to 0.5\,m$ and to satisfy the aforementioned design requirements.

1. Design the controller for the linear model.
   (a) Provide the controller gain, plots of the outputs, and plots of the input.
   (b) What are the eigenvalues of the closed-loop system? (i.e., the eigenvalues of $A - BK$)
2. Design the controller for nonlinear model.
   (a) Provide the controller gain, plots of the outputs, and plots of the input.

Problem 6: LQR Output Feedback Regulation (Full-Order)

- Design a full-order observer to estimate the deviation state z and design an LQR output feedback controller

$$u = -K\hat{z}. \tag{2}$$

- Use the following initial state estimate:

$$\hat{z}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{3}$$

- Specifically state the values you used for Q, R, and K and your motivation for your specific selections of Q and R.

1. Design an observer gain L and a controller gain K for the linear model to satisfy the design requirements.
   (a) What are the eigenvalues of $A - LC$?
   (b) Do you satisfy the design requirements?
   (c) Provide the observer gain, plots of the outputs, plots of the state estimation errors, and plots of the input.
2. Design the observer gain L and the controller gain K for the nonlinear model to satisfy the design requirements.
   (a) Provide the observer and controller gains, plots of the outputs, plots of the state estimation errors, and plots of the input.
   (b) What are the eigenvalues of $A - LC$ and $A - BK$?
3. Compare the responses of the system under state feedback to the responses of the system under output feedback.
   (a) Discuss for both the linear and nonlinear models.