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Homework 5

Problem 1

1. From eq 5.109, we can write:

$$\frac{d^2x}{dt^2} = \frac{x(t+h) - 2x(t) + x(t-h)}{h^2} \tag{1}$$

So:

$$\frac{x(t+h) - 2x(t) + x(t-h)}{h^2} = -g \tag{2}$$

$$x(t) = \frac{1}{2}(x(t+h) + x(t-h) + gh^2) \tag{3}$$

So the relaxation equation is:

$$x'(t) = \frac{1}{2}(x(t+h) + x(t-h) + gh^2) \tag{4}$$

2. The boundary conditions are set as x=0 at t=0 and t=10. Let $g = 9.8m/s$, and the maximum height is easily calculated, $h_{max} = 122.5m$. Now, using the relaxation method and solve for the trajectory:

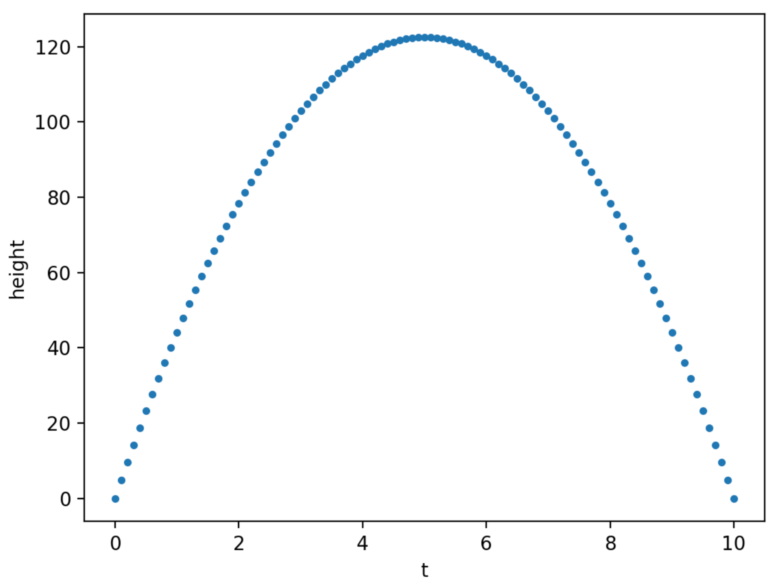

Figure 1: Trajectory plot for t=0 to t=10

And from the relaxation result, the maximum height is 122.49797423 m, so the simulated result is quite accurate.

Problem 2

1. I have downloaded the file, and applied CIC method. After plugging in the charge for electron and the permittivity of free space, here is the resulting grid:

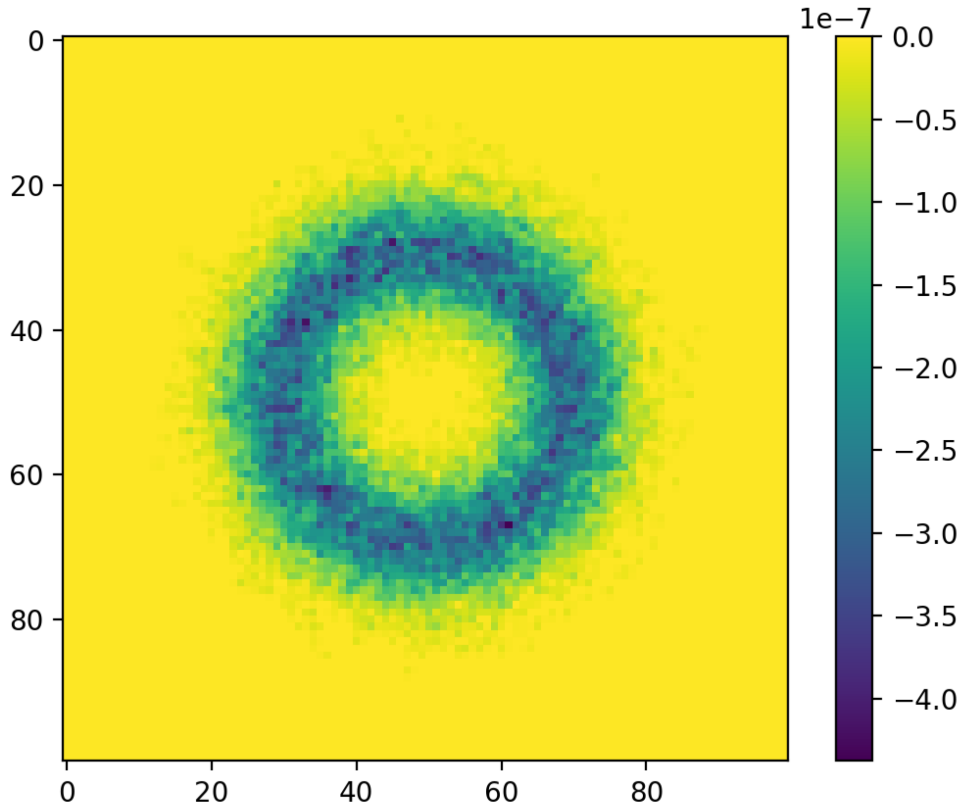

Figure 2: Resulting charge density after applying CIC

2. I have applied the normal relaxation method, and it took 11848 iterations to converge to 10^{-10}. So it was a very long time. The resulting image looks like this:

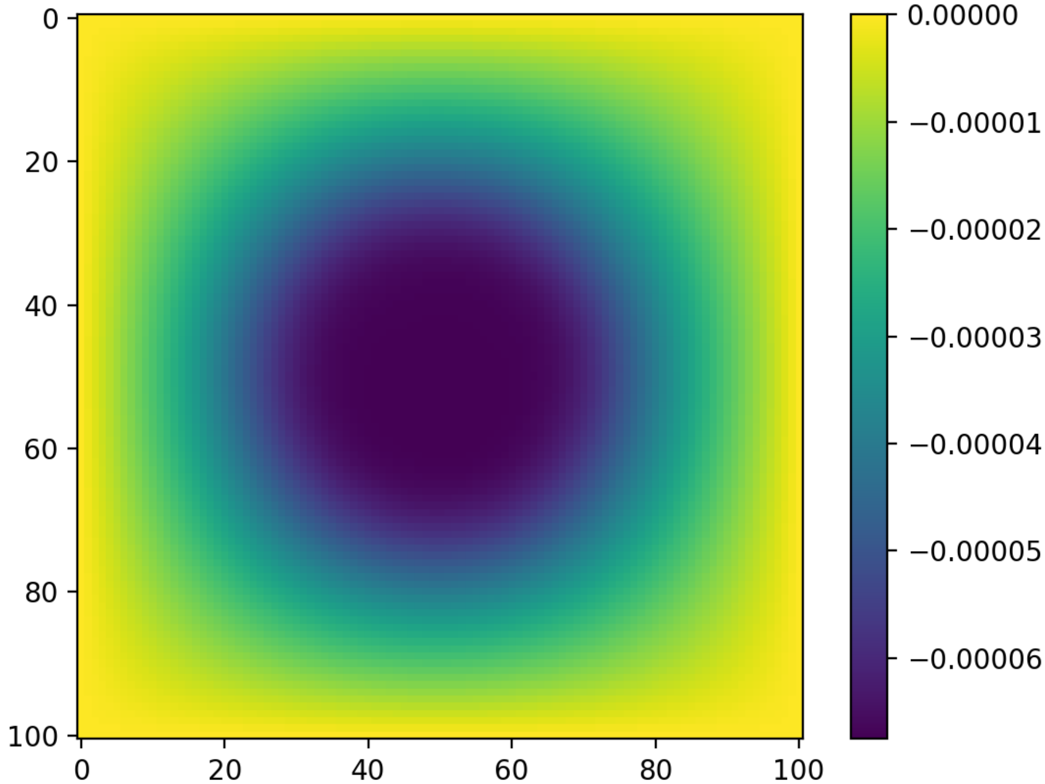

Figure 3: Resulting potential field, using relaxation method

3. Now, I have altered the code to apply Gauss-Seidel overrelaxation method. To start with, I have tried a range of ω from 0.93 to 0.95, and made a plot:

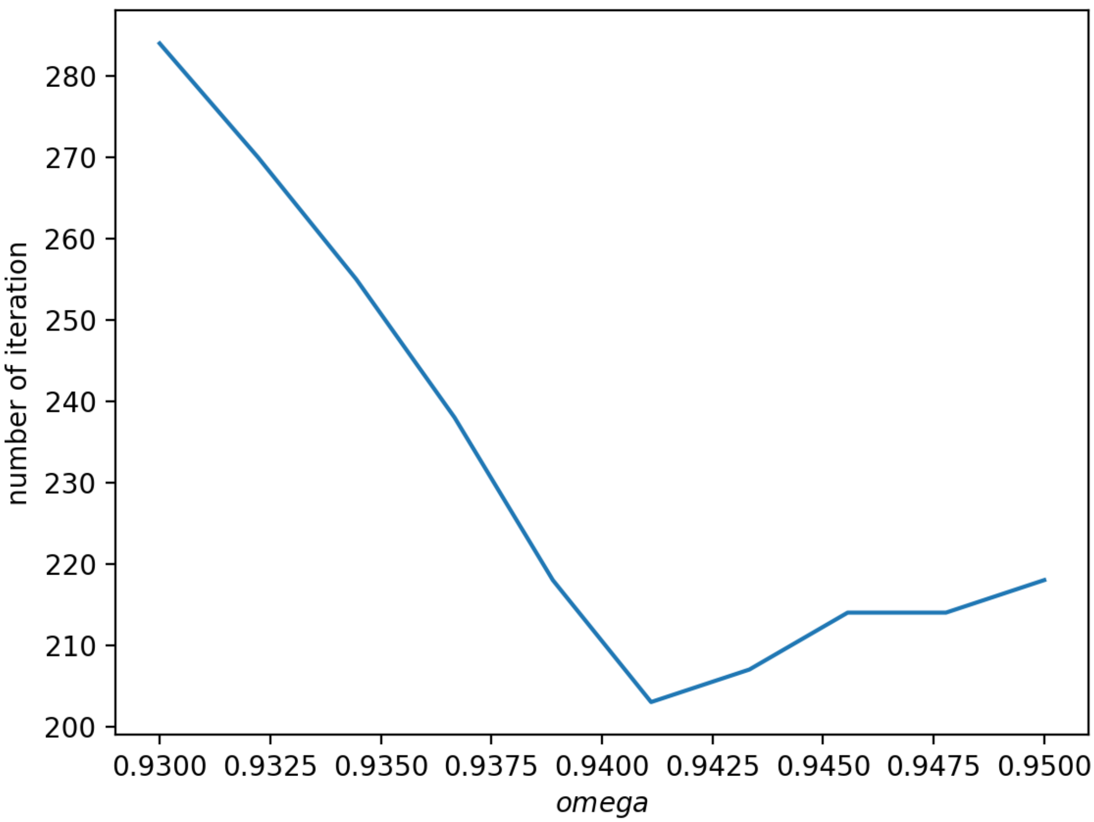

Figure 4: Plot of ω against number of iteration

Already, we can see a huge drop in number of iteration needed to reach the level of accuracy needed. There is clearly a local minimum near $\omega = 0.94$, so I would do a golden ratio search near there. So I have plotted the number of iteration in each golden ratio search iteration, and the following is the plot:

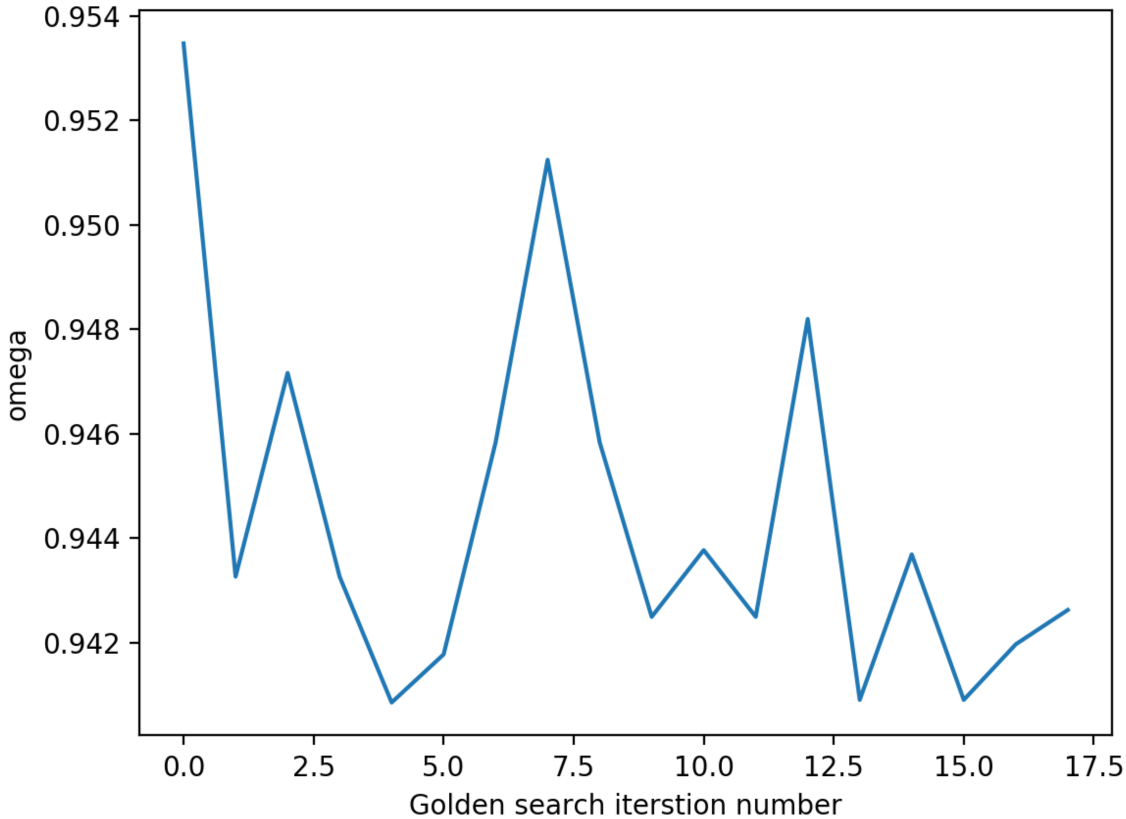

Figure 5: Plot that shows convergence in golden ratio search

The final value of ω it converged to is:

$$\omega = 0.9423 \tag{5}$$

It does look like it's having trouble converging though, maybe the algorithm isn't very stable.