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Вариант 18

Найти производную

$$y' = \left(\sin(\sqrt{x+4}) + \arccos^4 \ln \frac{\sqrt{4^x+x}}{x-\ln 2}\right)' =$$

$$= \cos(\sqrt{x+4}) \cdot \frac{1}{2\sqrt{x+4}} \cdot 1 + 4\arccos^3\left(\ln \frac{\sqrt{4^x+x}}{x-\ln 2}\right) \cdot \left(-\frac{1}{\sqrt{1-\ln^2 \frac{\sqrt{4^x-x}}{x-\ln 2}}}\right) \cdot$$

$$\cdot \frac{x-\ln 2}{\sqrt{4^x+x}} \cdot \frac{\frac{1}{2\sqrt{4^x-x}} \cdot (4^x \cdot \ln 4 + 1)(x-\ln 2) - \sqrt{4^x+x} \cdot (1-0)}{(x-\ln 2)^2} =$$

$$= \frac{\cos(\sqrt{x+4})}{2\sqrt{x+4}} - 4 \cdot \arccos^3\left(\ln \frac{\sqrt{4^x+x}}{x-\ln 2}\right) \cdot \frac{1}{\sqrt{1-\ln^2 \frac{\sqrt{4^x-x}}{x-\ln 2}}} \cdot \frac{x-\ln 2}{\sqrt{4^x+x}} \cdot$$

$$\cdot \frac{(4^x \cdot \ln 4 + 1)(x-\ln 2) - 2(4^x+x)}{2\sqrt{4^x+x}\,(x-\ln 2)^2} = \frac{\cos(\sqrt{x+4})}{2\sqrt{x+4}} - 2\arccos^3 \ln \frac{\sqrt{4^x+x}}{x-\ln 2} \cdot$$

$$\cdot \frac{x-\ln 2}{4^x+x} \cdot \frac{(4^x \cdot \ln 4 + 1)(x-\ln 2) - 2(4^x+x)}{(x-\ln 2)^2}$$

$$y = (x^3+\sqrt{x+9})^{\operatorname{arcctg}(x^2+\sqrt{x}-11)}$$

$$\ln y = \ln(x^3+\sqrt{x+9})^{\operatorname{arcctg}(x^2+\sqrt{x}-11)}$$

$$\ln y = \operatorname{arcctg}(x^2+\sqrt{x}-11) \cdot \ln(x^3+\sqrt{x+9}) \quad |\,()'$$

$$(\ln y)' = (\operatorname{arcctg}(x^2+\sqrt{x}-11) \cdot \ln(x^3+\sqrt{x+9}))'$$

$$\frac{y'}{y} = -\frac{1 \cdot (2x+\frac{1}{2\sqrt{x}})}{1+(x^2+\sqrt{x}-11)^2} \cdot \ln(x^3+\sqrt{x+9}) + \operatorname{arcctg}(x^2+\sqrt{x}-11) \cdot \frac{1 \cdot (3x^2+\frac{1 \cdot 1}{2\sqrt{x+9}})}{x^3+\sqrt{x+9}}$$

$$y' = y \cdot \left(-\frac{(2x+\frac{1}{2\sqrt{x}}) \cdot \ln(x^3+\sqrt{x+9})}{1+(x^2+\sqrt{x}-11)^2} + \frac{(3x^2+\frac{1}{2\sqrt{x+9}}) \cdot \operatorname{arcctg}(x^2+\sqrt{x}-11)}{x^3+\sqrt{x+9}}\right)$$

$$y' = -(x^3+\sqrt{x+9})^{\operatorname{arcctg}(x^2+\sqrt{x}-11)} \cdot \left(\frac{(2x+\frac{1}{2\sqrt{x}}) \cdot \ln(x^3+\sqrt{x+9})}{1+(x^2+\sqrt{x}-11)^2} +\right.$$

$$\left.+ \frac{(3x^2+\frac{1}{2\sqrt{x+9}}) \cdot \operatorname{arcctg}(x^2+\sqrt{x}-11)}{1+(x^2+\sqrt{x}-11)^2}\right)$$

$$\operatorname{ctg}(7xy+5x+3y)+\frac{xy+4x^3-x}{5y}=8x^2+2xy+y-3$$

$$\operatorname{ctg}(7xy+5x+3y)+\frac{xy+4x^3-x}{5y}-8x^2-2xy-y+3=0$$

$$F'_x(x;y)=-\frac{1\cdot(7y+5)}{\sin^2(7xy+5x+3y)}+\frac{(y+4\cdot 3x^2-1)\cdot 5y-(xy+4x^3-x)\cdot 0}{(5y)^2}-$$

$$-8\cdot 2\cdot x-2y=\frac{7y+5}{\sin^2(7xy+5x+3y)}+\frac{y+12x^2-1}{5y}-16x-2y$$

$$F'_y(x;y)=\frac{1\cdot(7x+3)}{\sin^2(7xy+5x+3y)}+\frac{x\cdot 5y-(xy+4x^3-x)\cdot 5}{(5y)^2}-2x-1=$$

$$=\frac{7x+3}{\sin^2(7xy+5x+3y)}+\frac{5x-20x^3}{(5y)^2}-2x-1$$

$$y'=-\frac{\frac{7y+5}{\sin^2(7xy+5x+3y)}+\frac{y+12x^2-1}{5y}-16x-2y}{\frac{7x+3}{\sin^2(7xy+5x+3y)}+\frac{5x-20x^3}{25y^2}-2x-1}$$

Найти интегралы

$$\int \frac{x\,dx}{(5-3x^2)^7} = \left[\begin{array}{l} t = 5-3x^2 \\ dt = (5-3x^2)'dx = -3\cdot 2\cdot x\,dx = -6x\,dx \Rightarrow x\,dx = \frac{dt}{-6} \end{array}\right] =$$

$$= \int \frac{dt}{(-6)\cdot t^7} = -\frac{1}{6}\int t^{-7}dt = -\frac{1}{6}\cdot\frac{t^{-6}}{(-6)} + C = \frac{1}{36t^6} + C =$$

$$= \frac{1}{36(5-3x^2)^6} + C$$

$$\int \frac{dx}{2\cos^2 x + 3\sin^2 x} = \left[\begin{array}{l} R(\sin x;\cos x) = R(-\sin x;-\cos x) \quad x = \operatorname{arctg} t \\ t = \operatorname{tg} x \Rightarrow dx = \frac{dt}{1+t^2}, \ \sin x = \frac{t}{\sqrt{1+t^2}}, \ \cos x = \frac{1}{\sqrt{1+t^2}} \end{array}\right] =$$

$$= \int \frac{\frac{dt}{1+t^2}}{2\cdot\left(\frac{1}{\sqrt{1+t^2}}\right)^2 + 3\left(\frac{t}{\sqrt{1+t^2}}\right)^2} = \int \frac{\frac{dt}{1+t^2}}{\frac{2\cdot 1 + 3\cdot t^2}{1+t^2}} = \int \frac{dt}{3t^2+2} = \frac{1}{3}\int \frac{dt}{t^2+\frac{2}{3}} =$$

$$= \frac{1}{3}\cdot\sqrt{\frac{3}{2}}\cdot\operatorname{arctg}\left(\sqrt{\frac{3}{2}}\cdot t\right) + C = \frac{1}{\sqrt{6}}\operatorname{arctg}\left(\sqrt{\frac{3}{2}}\operatorname{tg} x\right) + C$$

$$\int (x^3+4x)\ln x\,dx = \left[\begin{array}{l} u = \ln x \Rightarrow du = \frac{1}{x} \\ v' = x^3+4x \Rightarrow v = \int (x^3+4x)dx = \frac{x^4}{4} + \frac{4x^2}{2} \end{array}\right] =$$

$$= \ln x\cdot\left(\frac{x^4}{4} + \frac{4x^2}{2}\right) - \int \frac{1}{x}\cdot\left(\frac{x^4}{4} + 2x^2\right)dx = \ln x\cdot\left(\frac{x^4}{4} + 2x^2\right) -$$

$$- \int \frac{x^3}{4}dx - \int 2x\,dx = \left(\frac{x^4}{4} + 2x^2\right)\ln x - \frac{x^4}{4\cdot 4} - \frac{2x^2}{2} =$$

$$= \left(\frac{x^4}{4} + 2x^2\right)\ln x - \frac{x^4}{16} - x^2$$

$$\int x\arcsin(2x)\,dx = \left[\begin{array}{l} u = \arcsin(2x) \Rightarrow u' = 2\cdot\frac{1}{\sqrt{1-(2x)^2}} \\ v' = x \Rightarrow dv = \int x\,dx = \frac{x^2}{2} \end{array}\right] =$$

$$= \arcsin(2x)\cdot\frac{x^2}{2} - \int \frac{2}{\sqrt{1-4x^2}}\cdot\frac{x^2}{2}dx = \frac{x^2}{2}\arcsin 2x - \int \frac{x^2}{\sqrt{1-4x^2}}dx =$$

$$= \left[\begin{array}{l} x = \frac{\sin t}{2}, \ dx = \frac{\cos t}{2}dt \\ t = \arcsin\frac{x}{2} \end{array}\right] = \frac{x^2}{2}\arcsin 2x - \int \frac{\frac{\sin^2 t}{4}\cdot\frac{\cos t}{2}dt}{\sqrt{1-4\cdot\frac{\sin^2 t}{4}}} =$$

$$= \frac{x^2}{2}\arcsin 2x - \frac{1}{8}\int \frac{\sin^2 t\cdot\cos t\,dt}{\cos t} = \frac{x^2}{2}\arcsin 2x -$$

$$- \frac{1}{8}\int \sin^2 t\,dt = \frac{x^2}{2}\arcsin 2x - \frac{1}{8}\int \frac{1-\cos 2t}{2}dt = \frac{x^2}{2}\arcsin 2x -$$

$$- \frac{1}{8}\int \frac{1}{2}dx + \frac{1}{8\cdot 2}\int \cos 2t\cdot\frac{1}{2}d(2t) = \frac{x^2}{2}\arcsin 2x - \frac{1}{16}x + \frac{1}{32}\sin 2t + C =$$

$$= \frac{x^2}{2} \arcsin 2x - \frac{1}{16}x + \frac{1}{32} \cdot 2\sin t \cdot \cos t + C =$$

$$= \frac{x^2}{2} \arcsin 2x - \frac{x}{16} + \frac{1}{16} \sin(\arcsin \frac{x}{2}) \cdot \cos(\arcsin \frac{x}{2}) + C =$$

$$= \frac{x^2}{2} \arcsin 2x - \frac{x}{16} + \frac{1}{16} \cdot \frac{x}{2} \cdot \sqrt{1-(\frac{x}{2})^2} + C =$$

$$= \frac{x^2}{2} \arcsin 2x - \frac{x}{16} + \frac{x}{32} \cdot \sqrt{1-\frac{x^2}{4}} + C$$

$$\int \frac{2x+5}{x^3-x^2+2x-2}\,dx = \int \frac{2x+5}{x^2(x-1)+2(x-1)}\,dx = \int \frac{2x+5}{(x-1)(x^2+2)} =$$

$$= \left[\begin{array}{l} (x-1)(x^2+2)=0 \\ \downarrow \\ \text{есть решение} \end{array} \right. \Rightarrow \frac{2x+5}{(x-1)(x^2+2)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2}$$

$$2x+5 = A(x^2+2)+(Bx+C)(x-1)$$

I сп.

$$2x+5 = Ax^2+2A+Bx^2-Bx+Cx-C$$

$$2x+5 = (A+B)x^2+(C-B)x+(2A-C)$$

$$\begin{cases} A+B=0, \\ C-B=2, \\ 2A-C=5; \end{cases} \begin{cases} A=-B, \\ C-B=2, \\ -C-2B=5; \end{cases} \begin{cases} A=-B, \\ C=B+2, \\ -3B=7; \end{cases} \begin{cases} B=-\frac{7}{3}, \\ A=\frac{7}{3}, \\ C=-\frac{7}{3}+2; \end{cases} \begin{cases} A=\frac{7}{3}, \\ B=-\frac{7}{3}, \\ C=-\frac{1}{3}. \end{cases}$$

II сп.

$$x=1:\ 2\cdot 1+5 = A(1^2+2)+(B\cdot 1+C)\cdot(1-1)$$

$$7=3A$$

$$A=7/3$$

$$x=0:\ 2\cdot 0+5 = A(0^2+2)+(B\cdot 0+C)(0-1)$$

$$5=2A-C$$

т.к. $A=7/3$, то

$$5=2\cdot\frac{7}{3}-C$$

$$C=\frac{14}{3}-5=-\frac{1}{3}$$

$$x=-1:\ 2\cdot(-1)+5 = A((-1)^2+2)+(B\cdot(-1)+C)(-1-1)$$

$$3=3A+2B-2C$$

т.к. $A=\frac{7}{3}$, $C=-\frac{1}{3}$:

$$3=3\cdot\frac{7}{3}+2\cdot B-2\cdot\left(-\frac{1}{3}\right)$$

$$3=7+2B+\frac{2}{3}$$

$$2B=-4-\frac{2}{3}$$

$$2B=-\frac{14}{3}$$

$$B=-\frac{7}{3} \quad \Big] =$$

$$= \int \left(\frac{\frac{7}{3}}{x-1} + \frac{-\frac{7}{3}x - \frac{1}{3}}{x^2+2} \right) dx = \frac{7}{3} \int \frac{dx}{x-1} - \frac{1}{3} \int \frac{7x+1}{x^2+2} dx =$$

~~= [$x^2+2=0$ → III тип ⇒ $A=7$, $B=1$ $7x+1 = \frac{7}{2}(2x+1$~~
~~нет корней $p=0, q=2$]~~

$$= \frac{7}{3} \int \frac{dx}{x-1} - \frac{1}{3} \left(\int \frac{7x}{x^2+2} dx + \int \frac{dx}{x^2+2} \right) = \frac{7}{3} \int \frac{dx}{x-1} - \frac{7}{3} \int \frac{x\,dx}{x^2+2} -$$

$$- \frac{1}{3} \int \frac{dx}{x^2+2} = \left[2)\ t = x^2+2 \Rightarrow dt = 2x\,dx \Rightarrow x\,dx = \frac{dt}{2} \right] =$$

$$= \frac{7}{3} \int \frac{d(x-1)}{x-1} - \frac{7}{3 \cdot 2} \int \frac{dt}{t} - \frac{1}{3} \int \frac{dx}{x^2+(\sqrt{2})^2} =$$

$$= \frac{7}{3} \ln|x-1| - \frac{7}{6} \ln|t| - \frac{1}{3} \cdot \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x}{\sqrt{2}} + C =$$

$$= \frac{7}{3} \ln|x-1| - \frac{7}{6} \ln|x^2+2| - \frac{1}{3\sqrt{2}} \operatorname{arctg} \frac{x}{\sqrt{2}} + C =$$

$$= \frac{7}{3} \ln|x-1| - \frac{7}{6} \ln(x^2+2) - \frac{1}{3\sqrt{2}} \operatorname{arctg} \frac{x}{\sqrt{2}} + C$$

$$\int_{-3}^{-2} \frac{dx}{\sqrt{5-4x-x^2}} = \int_{-3}^{-2} \frac{dx}{\sqrt{9-4-4x-x^2}} = \int_{-3}^{-2} \frac{dx}{\sqrt{3^2-(x^2+4x+4)}} = \int_{-3}^{-2} \frac{dx}{\sqrt{3^2-(x+2)^2}} =$$

$$= \int_{-3}^{-2} \frac{d(x+2)}{\sqrt{3^2-(x+2)^2}} = \arcsin \frac{x+2}{3} \Big|_{-3}^{-2} = \arcsin \frac{-2+2}{3} -$$

$$- \arcsin \frac{-3+2}{3} = \arcsin 0 - \arcsin\left(-\frac{1}{3}\right) = \arcsin \frac{1}{3}$$

$$\int \frac{dx}{1+\sqrt{2x+1}} = \left[\begin{array}{l} t^2 = 2x+1 \Rightarrow x = \frac{t^2-1}{2} \\ dx = \frac{2t}{2} dt = t\,dt \end{array} \right] = \int \frac{t\,dt}{1+\sqrt{t^2}} = \int \frac{t\,dt}{1+t} =$$

$$= \int \frac{t+1-1}{t+1} dt = \int \frac{t+1}{t+1} dt - \int \frac{dt}{t+1} = \int dt - \int \frac{d(t+1)}{t+1} =$$

$$= t - \ln|t+1| + C = \sqrt{2x+1} - \ln|\sqrt{2x+1}+1| + C$$

ДУ

$$y' = -2y \quad , \quad y(0) = 19$$

$$\frac{dy}{dx} = -2y \quad , \quad \frac{dy}{y} = -2dx$$

$$\int \frac{dy}{y} = \int (-2dx) = -2 \int dx$$

$\ln|y| = -2x + C$

$y = e^{-2x+C}$

$y = e^{C} \cdot e^{-2x}$

$y = C \cdot e^{-2x}$

$19 = C \cdot e^{-2 \cdot 0}$

$19 = C \cdot e^{0}$

$C = 19$

Частное решение: $y = 19\, e^{-2x}$

$xy' = 2\sqrt{3x^2 + y^2} + y$

$y' = \dfrac{2\sqrt{3x^2 + y^2} + y}{x}$

$y = tx$, тогда

$y' = (tx)' = t'x + tx' = t'x + t$

$t'x + t = \dfrac{2\sqrt{3x^2 + tx} + tx}{x}$

$t' = \dfrac{dt}{dx}$, тогда

$\dfrac{dt}{dx} x + t = \dfrac{2\sqrt{3 + t} + t}{1}$

$\dfrac{dt}{dx} x = 2\sqrt{3+t} + t - t$

$\dfrac{dt}{dx} x = 2\sqrt{3+t}$

$\dfrac{x}{dx} = \dfrac{2\sqrt{3+t}}{dt} = \dfrac{dt}{2\sqrt{3+t}} = \dfrac{dx}{x}$

Интегрируем

$$\int \frac{dt}{2\sqrt{3+t}} = \int \frac{dx}{x}$$

$$\frac{1}{2}\int \frac{d(t+3)}{\sqrt{t+3}} = \int \frac{dx}{x}$$

$$\frac{1}{2} \cdot \frac{1}{2}\sqrt{t+3} = \ln|x| + C$$, т.к. $y = tx \Rightarrow t = \frac{y}{x}$

$$\frac{1}{4}\sqrt{\frac{y}{x}+3} = \ln|x| + C$$

$$\frac{1}{4}\sqrt{\frac{y}{x}+3} - \ln|x| = C$$ – общий вид