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ECE 3101 Signals and Systems
Lab 2: Systems

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1. Introduction

The goal of this laboratory assignment is to provide you an opportunity to learn to use MATLAB to study various issues related to continuous-time linear time-invariant systems. Again, we will be using MATLAB in the lab and I will supply you some examples to get you started. Also, you will need to bring your Lab 1 knowledge to solve some of the problems here.

2. Solving Differential Equations

Solving differential equation is a big deal in this chapter, namely, zero-input response and unit impulse response. Therefore, we will start our discussion on this topic. The method to use is the Symbolic tool in MATLAB.

2.2-4 An LTIC system is specified by the equation

$$(D^2 + 5D + 6)y(t) = (D + 1)x(t)$$

(a) Find the characteristic polynomial, characteristic equation, characte roots, and characteristic modes of this system.

(b) Find $y_0(t)$, the zero-input component of the response $y(t)$ for $t \geq 0$, the initial conditions are $y_0(0^-) = 2$ and $\dot{y}_0(0^-) = -1$.

Part (a): Characteristic Polynomial, Roots, and Modes

```
syms D
% Define the characteristic polynomial
charPoly = D^2 + 5*D + 6;

% Find roots of the characteristic polynomial
charRoots = roots(sym2poly(charPoly));

% Display the characteristic polynomial and roots
fprintf('Characteristic Polynomial: %s\n', charPoly);
```

```
Characteristic Polynomial: 5*D + D^2 + 6
```

```
disp('Roots of the characteristic polynomial:');
```

```
Roots of the characteristic polynomial:
```

```
disp(charRoots);
```

```
   -3.0000
   -2.0000
```

```

% Calculate and display the characteristic modes
syms t
modes = exp(charRoots*t);
disp('Characteristic modes:');
```

```
Characteristic modes:
```

```
disp(modes);
```

$$\begin{pmatrix} e^{-3t} \\ e^{-2t} \end{pmatrix}$$

Part (b): Zero-Input Component of the Response

```
syms y(t)
Dy = diff(y);
D2y = diff(y,2);
% Homogeneous differential equation corresponding to zero-input response
ode = D2y + 5*Dy + 6*y == 0;
% Initial conditions
cond1 = y(0) == 2;
cond2 = Dy(0) == -1;
% Solve the differential equation
ySol(t) = dsolve(ode, [cond1, cond2]);
% Display the solution
disp('Zero-input component of the response y0(t):');
```

```
Zero-input component of the response y0(t):
```

```
disp(ySol(t));
```

$5\,e^{-2t} - 3\,e^{-3t}$

> 2.2-5 Repeat Prob. 2.2-4 for
>
> $$(D^2 + 4D + 4)y(t) = Dx(t)$$
>
> and $y_0(0^-) = 3$, $\dot{y}_0(0^-) = -4$.

Part (a): Characteristic Polynomial, Roots, and Modes

```
% Part (a): Characteristic Polynomial, Roots, and Modes for 2.2-5
syms D y(t) t

% Define the differential equation (homogeneous part)
homogeneous_eqn_225 = (D^2 + 4*D + 4)*y(t) == 0;

% Extract the characteristic polynomial
char_poly_225 = simplify(lhs(homogeneous_eqn_225));

% Solve for the roots of the characteristic equation
char_roots_225 = solve(char_poly_225 == 0, D);

% Determine the characteristic modes based on the roots
char_modes_225 = exp(char_roots_225 * t);
```

```
% Display results
disp('Characteristic Polynomial for 2.2-5:');
```

```
Characteristic Polynomial for 2.2-5:
```

```
disp(char_poly_225);
```

$y(t)\ (\mathrm{D}+2)^2$

```
disp('Characteristic Equation for 2.2-5:');
```

```
Characteristic Equation for 2.2-5:
```

```
disp(char_poly_225 == 0);
```

$y(t)\ (\mathrm{D}+2)^2 = 0$

```
disp('Roots for 2.2-5:');
```

```
Roots for 2.2-5:
```

```
disp(char_roots_225);
```

$$\begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

```
disp('Characteristic Modes for 2.2-5:');
```

```
Characteristic Modes for 2.2-5:
```

```
disp(char_modes_225);
```

$$\begin{pmatrix} e^{-2t} \\ e^{-2t} \end{pmatrix}$$

Part (b): Solve the Full Differential Equation (Inhomogeneous)

```
syms y(t) Dy
D2y = diff(y, t, 2);  % Define the second derivative
Dy = diff(y, t);       % First derivative

% Define the homogeneous differential equation corresponding to zero-input response
hom_eqn_225 = D2y + 4*Dy + 4*y == 0;

% Initial conditions provided in the problem statement
cond1_225 = y(0) == 3;
cond2_225 = Dy(0) == -4;  % Correct derivative notation used here

% Solve the differential equation using defined derivatives
ySol_225(t) = dsolve(hom_eqn_225, cond1_225, cond2_225);

% Display the zero-input response
disp('Zero-input component of the response for 2.2-5:');
```

```
Zero-input component of the response for 2.2-5:
```

```
disp(vpa(ySol_225(t), 4));  % Using vpa for better numerical precision in display
```

$3.0\,e^{-2.0\,t} + 2.0\,t\,e^{-2.0\,t}$

2.2-7 Repeat Prob. 2.2-4 for

$$(D^2 + 9)y(t) = (3D + 2)x(t)$$

and $y_0(0^-) = 0$, $\dot{y}_0(0^-) = 6$.

Part (a): Characteristic Polynomial, Roots, and Modes

```
% Part (a): Characteristic Polynomial, Roots, and Modes for 2.2-7
syms D y(t) t

% Define the differential equation (homogeneous part)
homogeneous_eqn_227 = (D^2 + 9)*y(t) == 0;

% Extract the characteristic polynomial
char_poly_227 = simplify(lhs(homogeneous_eqn_227));

% Solve for the roots of the characteristic equation
char_roots_227 = solve(char_poly_227 == 0, D);

% Determine the characteristic modes based on the roots
char_modes_227 = exp(char_roots_227 * t);

% Display results
disp('Characteristic Polynomial for 2.2-7:');
```

```
Characteristic Polynomial for 2.2-7:
```

```
disp(char_poly_227);
```

$y(t)\ (D^2 + 9)$

```
disp('Characteristic Equation for 2.2-7:');
```

```
Characteristic Equation for 2.2-7:
```

```
disp(char_poly_227 == 0);
```

$y(t)\ (D^2 + 9) = 0$

```
disp('Roots for 2.2-7:');
```

Roots for 2.2-7:

```
disp(char_roots_227);
```

$$\begin{pmatrix} -3\,\mathrm{i} \\ 3\,\mathrm{i} \end{pmatrix}$$

```
disp('Characteristic Modes for 2.2-7:');
```

Characteristic Modes for 2.2-7:

```
disp(char_modes_227);
```

$$\begin{pmatrix} \mathrm{e}^{-3\,t\,\mathrm{i}} \\ \mathrm{e}^{3\,t\,\mathrm{i}} \end{pmatrix}$$

Part (b): Zero-Input Response

```
% Part (b): Zero-input response for 2.2-7
syms y(t) Dy
D2y = diff(y, t, 2);  % Define the second derivative
Dy = diff(y, t);       % First derivative

% Define the homogeneous differential equation corresponding to zero-input response
hom_eqn_227 = D2y + 9*y == 0;

% Initial conditions provided in the problem statement
cond1_227 = y(0) == 0;
cond2_227 = Dy(0) == 6;  % Correct derivative notation used here

% Solve the differential equation using defined derivatives
ySol_227(t) = dsolve(hom_eqn_227, [cond1_227, cond2_227]);

% Display the zero-input response
disp('Zero-input component of the response for 2.2-7:');
```

Zero-input component of the response for 2.2-7:

```
disp(vpa(ySol_227(t), 4));  % Using vpa for better numerical precision in display
```

$$2.0\sin(3.0\,t)$$

2.4-16 The unit impulse response of an LTIC system is

$$h(t) = e^{-t}u(t)$$

Find this system's (zero-state) response $y(t)$ if the input $x(t)$ is:

(a) $u(t)$

(b) $e^{-t}u(t)$

(c) $e^{-2t}u(t)$

(d) $\sin 3t u(t)$

Use the convolution table (Table 2.1) to find your answers.

```
% Unit impulse response: h(t) = e^(-t)u(t)
syms t tau u(t) h(t) x(t) y(t)
h(t) = exp(-t) * heaviside(t);  % Impulse response

% Inputs for different parts:
% (a) u(t)
x(t) = heaviside(t);
y(t) = int(h(tau) * subs(x(t), t, t-tau), tau, -inf, inf);
disp(simplify(y(t)));
```

$$\frac{e^{-t}\,(e^{t}-1)\,(\mathrm{sign}(t)+1)}{2}$$

```
% (b) e^(-t)u(t)
x(t) = exp(-t) * heaviside(t);
y(t) = int(h(tau) * subs(x(t), t, t-tau), tau, -inf, inf);
disp(simplify(y(t)));
```

$$\frac{t\,e^{-t}\,(\mathrm{sign}(t)+1)}{2}$$

```
% (c) e^(-2t)u(t)
x(t) = exp(-2*t) * heaviside(t);
y(t) = int(h(tau) * subs(x(t), t, t-tau), tau, -inf, inf);
disp(simplify(y(t)));
```

$$\frac{e^{-2t}\,(e^{t}-1)\,(\mathrm{sign}(t)+1)}{2}$$

```
% (d) sin(3t)u(t)
```

```
x(t) = sin(3*t) * heaviside(t);
y(t) = int(h(tau) * subs(x(t), t, t-tau), tau, -inf, inf);
disp(simplify(y(t)));
```

$$-\left(\frac{\mathrm{sign}(t)}{2}+\frac{1}{2}\right)\left(-\frac{3\,\mathrm{e}^{-t}}{10}+\mathrm{e}^{-3\,t\,\mathrm{i}}\left(\frac{3}{20}-\frac{1}{20}\mathrm{i}\right)+\mathrm{e}^{3\,t\,\mathrm{i}}\left(\frac{3}{20}+\frac{1}{20}\mathrm{i}\right)\right)$$

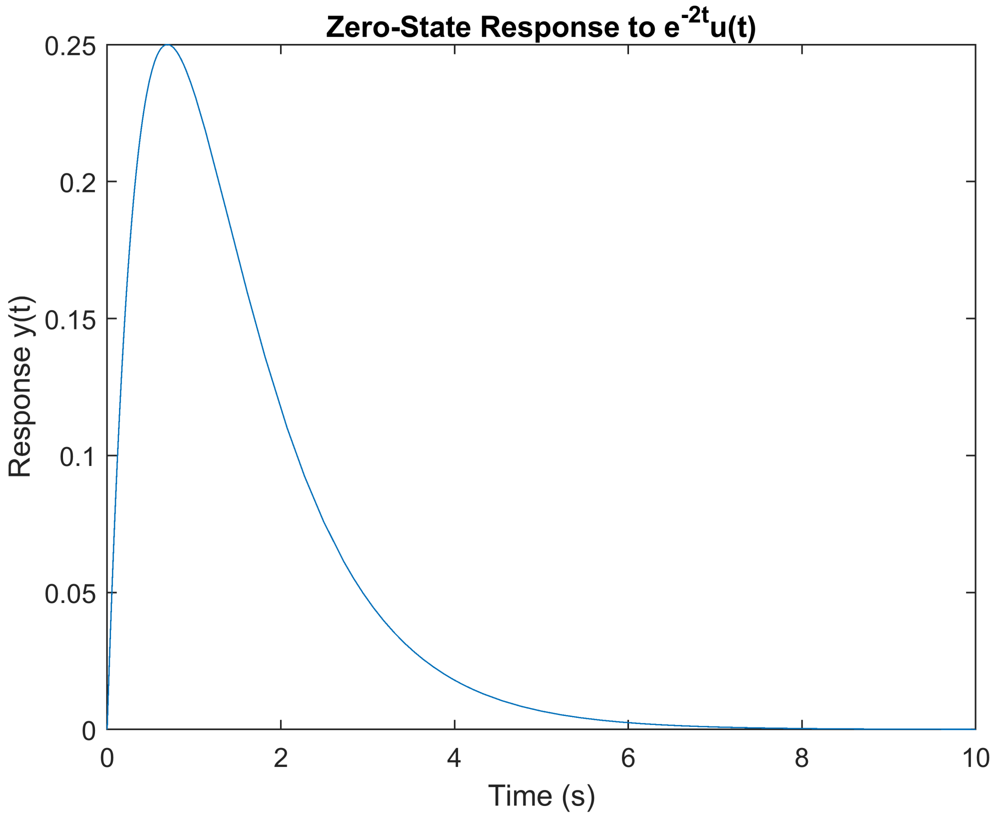
4-11 Repeat Prob. 2.4-7 if

$$h(t) = e^{-t}u(t)$$

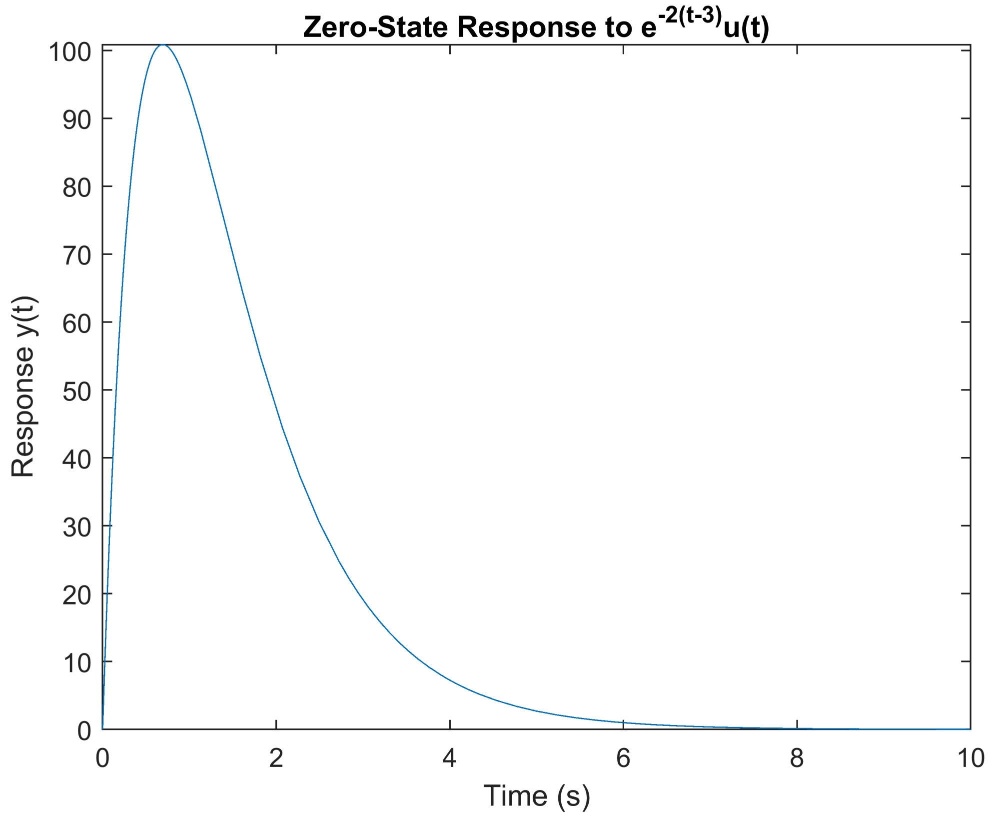
and if the input $f(t)$ is: **(a)** $e^{-2t}u(t)$ **(b)** $e^{-2(t-3)}u(t)$ **(c)** $e^{-2t}u(t-3)$ **(d)** the gate pulse depicted in Fig. P2.4-11. For **(d)**, sketch $y(t)$.

Hint: The input in **(d)** can be expressed as $u(t) - u(t-1)$. For parts **(c)** and **(d)**, use the shift property (2.34) of convolution. (Alternatively, you may want to invoke the system's time-invariance and superposition properties.)

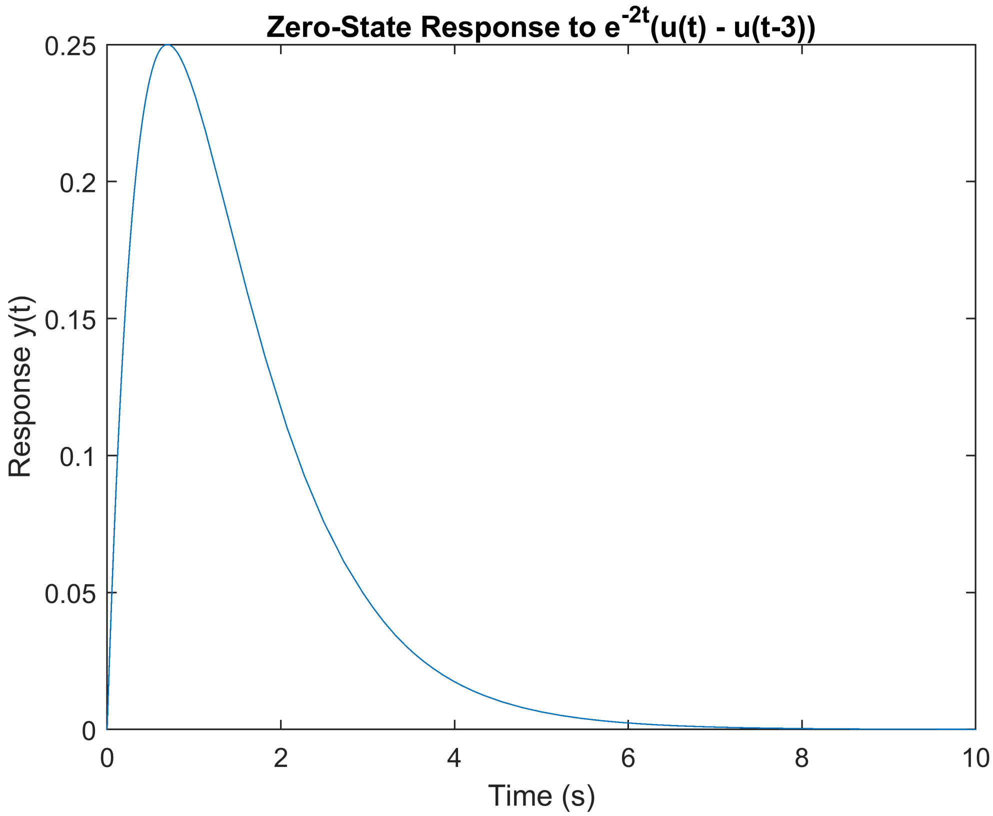
```
syms t tau u(t) h(t) f(t) y(t)
u(t) = heaviside(t);  % Heaviside step function
h(t) = exp(-t) * u(t);  % Unit impulse response

% (a) Input: f(t) = e^(-2t)u(t)
f(t) = exp(-2*t) * u(t);
y(t) = int(h(tau) * subs(f(t), t, t-tau), tau, -inf, inf);
figure;
fplot(y(t), [0, 10]);
title('Zero-State Response to e^{-2t}u(t)');
xlabel('Time (s)');
ylabel('Response y(t)');
```



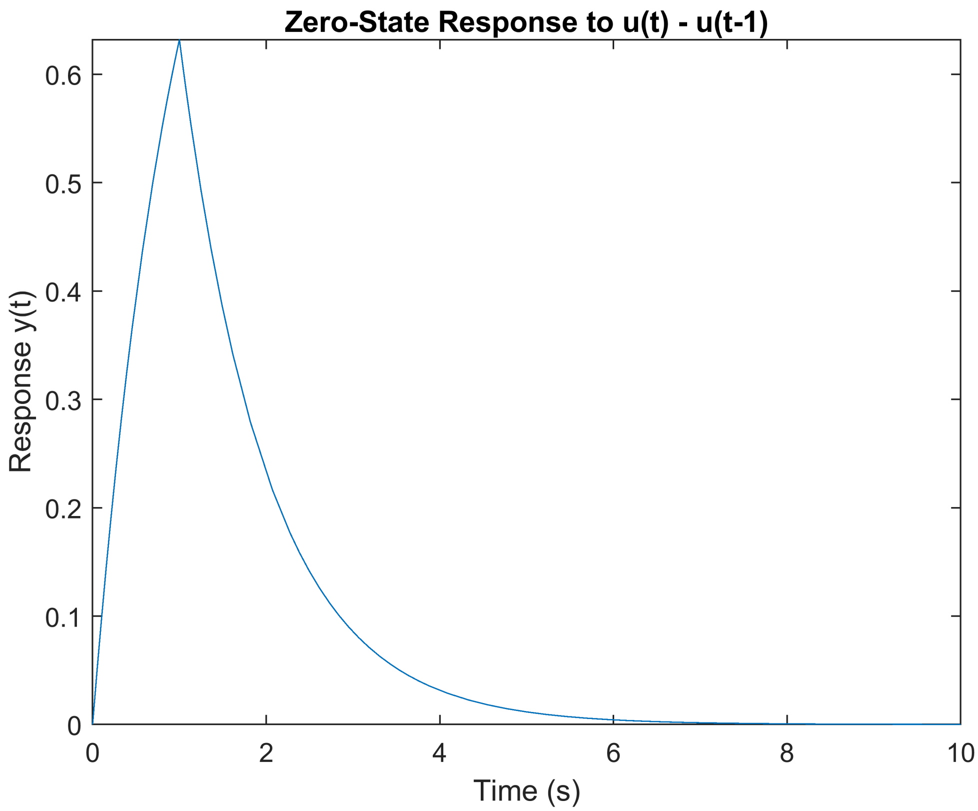

```
% (b) Input: f(t) = e^(-2*(t-3))u(t)
f(t) = exp(-2*(t-3)) * u(t);
y(t) = int(h(tau) * subs(f(t), t, t-tau), tau, -inf, inf);
figure;
fplot(y(t), [0, 10]);
title('Zero-State Response to e^{-2(t-3)}u(t)');
xlabel('Time (s)');
ylabel('Response y(t)');
```




```
% (c) Input: f(t) = e^(-2t)(u(t) - u(t-3))
f(t) = exp(-2*t) * (u(t) - u(t-3));
y(t) = int(h(tau) * subs(f(t), t, t-tau), tau, -inf, inf);
figure;
fplot(y(t), [0, 10]);
title('Zero-State Response to e^{-2t}(u(t) - u(t-3))');
xlabel('Time (s)');
ylabel('Response y(t)');
```




```
% (d) Input: f(t) = u(t) - u(t-1)
f(t) = u(t) - u(t-1);
y(t) = int(h(tau) * subs(f(t), t, t-tau), tau, -inf, inf);
figure;
fplot(y(t), [0, 10]);
title('Zero-State Response to u(t) - u(t-1)');
xlabel('Time (s)');
ylabel('Response y(t)');
```



Zero-State Response to u(t) - u(t-1)
Response y(t)
Time (s)
0
0.1
0.2
0.3
0.4
0.5
0.6
0
2
4
6
8
10