Fuzzy set operation

Given x to be the universe of discourse and $\bar{A}$ and $\bar{B}$ to be fuzzy sets with $\mu_A(x)$ and $\mu_B(x)$ as their respective membership function, the basic fuzzy set operations are as follows:

(1) **Union** :-

$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$$

Let $A = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0)\}$ and
$B = \{(x_1, 0.8), (x_2, 0.2), (x_3, 1)\}$

$$\mu_{\bar{A} \cup \bar{B}}(x) = \max(\mu_{\bar{A}}(x), \mu_{\bar{B}}(x))$$

$$= \max\{(x_1, 0.8), (x_1, 0.5), (x_2, 0.7), (x_2, 0.2), (x_3, 0), (x_3, 1)\}$$

$$= \{(x_1, 0.8), (x_2, 0.7), (x_3, 1)\}$$

(2) **Intersection** :-

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

$$= \min\{(x_1, 0.5), (x_1, 0.7), (x_2, 0.7), (x_2, 0.2), (x_3, 0), (x_3, 1)\}$$

$$= \{(x_1, 0.5), (x_2, 0.2), (x_3, 0)\}$$

Union

$A = \{$ $\}$

$B = \{$ $\}$

$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$

$= \max((x_1, 0.5), (x_1, 0.8))$

$= (x_1, 0.8)$

intersection

$A \cap B$

$= \min(\mu_{\bar{A}}(x), \mu_{\bar{B}}(x))$

$= \min((x_1, 0.5)(x_1, 0.8))$

$= (x_1, 0.5)$

③ Complement :-

$$\mu_{\bar{A}^c}(x) = 1 - \mu_{\bar{A}}(x)$$

Let $\bar{A} = \{(x_1, 0.5)\ (x_2, 0.7)\ (x_3, 0)\}$

$\bar{A}^c = \{(x_1, 0.5)\ (x_2, 0.3)\ (x_3, 1)\}$

④ Product of two fuzzy sets :

$$\mu_{\bar{A}\cdot\bar{B}}(x) = \mu_{\bar{A}}(x) \cdot \mu_{\bar{B}}(x)$$

$\bar{A} = \{(x_1, 0.2)\ (x_2, 0.8)\ (x_3, 0.4)\}$

$\bar{B} = \{(x_1, 0.4)\ (x_2, 0)\ (x_3, 0.1)\}$

$\mu_{\bar{A}\cdot\bar{B}} = \{(x_1, 0.8)\ (x_2, 0)\ (x_3, 0.04)\}$

⑤ Equality :-

$$\mu_{\tilde{A}}(x_1) = \mu_{\tilde{B}}(x_1)$$

$\tilde{A} = \{(x_1, 0.2)\ (x_2, 0.8)$ $\tilde{B} = \{(x_1, 0.6)\ (x_2, 0.8)\}$

$\tilde{C} = \{(x_1, 0.2)\ (x_2, 0.8)\}$

$\mu_{\tilde{A}}(x_1) \neq \mu_{\tilde{B}}(x_1)$, $x_1 \neq (x_1, 0.2) \neq (x_1, 0.6)$

but $\mu_{\tilde{A}}(x_1) = \mu_{\tilde{C}}(x_1)$, $(x_1, 0.2) = (x_1, 0.2)$

$\mu_{\tilde{A}}(x_2) \neq \mu_{\tilde{B}}(x_2)$ but $\mu_{\tilde{B}}(x_2) = \mu_{\tilde{C}}(x_2)$

$\mu_{\tilde{A}}(x_3) = \mu_{\tilde{C}}(x_3)$

Complement

$\mu_{\bar{A}^c}(x) = 1 - \mu_{\bar{A}}(x)$

$\mu_{\bar{A}^c}(x)$ = Complement of fuzzy set

$\mu_{\bar{A}\cdot\bar{B}}(x) = \mu_{\bar{A}}(x) \cdot \mu_{\bar{B}}(x)$

Equality

$\neq$ $=$

$\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)$

$\mu_{\tilde{A}}(x) \neq \mu_{\tilde{B}}(x)$

(6) Product of a fuzzy set with a crisp set number.

$$\mu_{a.\bar{A}}(x) = a.\mu_{\bar{A}}(x)$$

$\bar{A} = \{(x_1, 0.4)(x_2, 0.6)(x_3, 0.8)\}$

$a = 0.3$

$\mu_{a.\bar{A}}(x) = 0.3.\{(x_1, 0.4)(x_2, 0.6)(x_3, 0.8)\}$

$= \{(x_1, 0.12)(x_2, 0.18)(x_3, 0.24)\}$

(7) Power of a fuzzy set :-

$$A^a$$

$$\mu_{A^a}(x) = (\mu_A(x))^a$$

$\bar{A} = \{(x_1, 0.4)(x_2, 0.2)(x_3, 0.7)\}$

$a = 2$

$\mu_{A^a}(x) = \{(x_1, 0.16)(x_2, 0.04)(x_3, 0.49)\}$

(8) Difference :-

$\bar{A} - \bar{B} = (\bar{A} \cap \bar{B}^c)$

$\tilde{A} = \{(x_1, 0.2)(x_2, 0.5)(x_3, 0.6)$, $\tilde{B} = \{(x_1, 0.1)(x_2, 0.4)(x_3, 0.8)\}$

$\tilde{B}^c = \{(x_1, 0.9)(x_2, 0.6)(x_3, 0.5)\}$

$\tilde{A} - \tilde{B} = \tilde{A} \cap \tilde{B}^c = \{(x_1, 0.2)(x_2, 0.5)(x_3, 0.5)\}$

(9) Disjunctive Sum :-

$\bar{A} \oplus \bar{B} = (\bar{A}^c \cap \bar{B}) \cup (\bar{A} \cap \bar{B}^c)$

With Product (with some value)

$\alpha = a = 1, 2 = 1$

$\mu_{\bar{A}}(x) \cdot a$

$\mu_{a.\bar{A}}(x) = 0.3 \times 0.4$

$A \oplus B$

$(A \cdot \bar{B} + \bar{B} \cdot \bar{A})$

Power

$A^{\alpha = a}$

$a = 2$

$\mu_A{}^{a(x)}$

$= (0.4)^2$

$= .16$

Difference

$A - B = \bar{A} \cap \bar{B}^c$

$B^c = \{\ .\ \}$

$\bar{A} \cap \bar{B} = \min(\ )$