Review Probability

Before Sebastian teaches you about localizing a robot, in the next couple concepts, we'll review how to represent uncertainty in robot motion and sensors. The next couple sections will include multiple videos and quizzes to check your understanding of probability and probability distributions. These are meant to prepare you for this lesson, so make sure to read this material carefully to become familiar with terminology and mathematical representations of uncertainty!

Probability Review

Before learning more about uncertainty in robot localization, let's review how to mathematically represent uncertainty using probability!

A classic example of an event with some certainty associated with it is a coin flip. A typical coin has two sides: heads and tails. Without me telling you anything else, what chance do you think a coin like this has of flipping and landing heads up?

[Heads and tails of a coin.]()

Formal Definition of Probability

The probability of an event, X, occurring is P(X). The value of P(X) must fall in a range of 0 to 1.

- 0 <= P(X) <= 1

An event, X, can have multiple outcomes which we might call X1, X2, .. and so on; the probabilities for all outcomes of X must add up to one. For example, say there are two possible outcomes, X1 and X2:

- If P(X1) = 0.2 then P(X2) = 0.8 because all possible outcomes must sum to 1.

Terminology

Independent Events

Events like coin flips are known as independent events; this means that the probability of a single flip does not affect the probability of another flip; P(H) will be 0.5 for each fair coin flip. When flipping a coin multiple times, each flip is an independent event because one flip does not affect the probability that another flip will land heads up or tails up.

Dependent Events

When two events are said to be dependent, the probability of one event occurring influences the likelihood of the other event. For example, say you are more likely to go outside if it's sunny weather. If the probability of sunny weather is low on a given day, the probability of you going outside will decrease as well, so the probability of going outside is dependent on the probability of sunny weather.

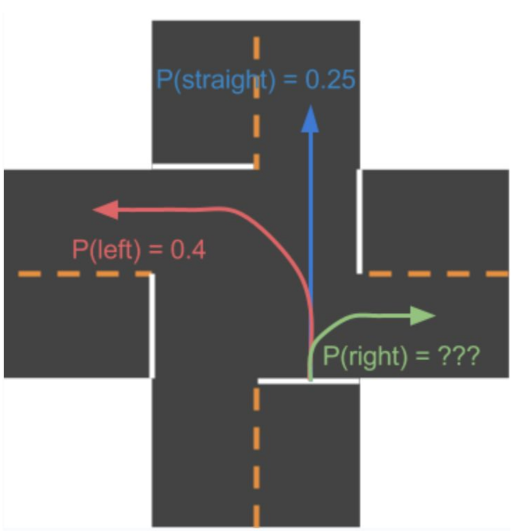
Joint Probability

The probability that two or more independent events will occur together (in the same time frame) is called a joint probability, and it is calculated by multiplying the probabilities of each independent event together. For example, the probability that you will flip a coin and it will lands heads up two times in a row can be calculated as follows:

- The probability of a coin flipping heads up, P(H) = 0.5
- The joint probability of two events (a coin landing heads up) happening in a row, is the probability of the first event times the probability of the second event: P(H)*P(H) = (0.5)*(0.5) = 0.25

Quantifying Certainty (and Uncertainty)

When we talk about being certain that a robot is at a certain location (x, y), moving a certain direction, or sensing a certain environment, we can quantify that certainty using probabilistic quantities. Sensor measurements and movement all have some uncertainty associated with them (ex. a speedometer that reads 50mph may be off by a few mph, depending on whether a car is moving up or down hill).



At a particular intersection, cars can either:

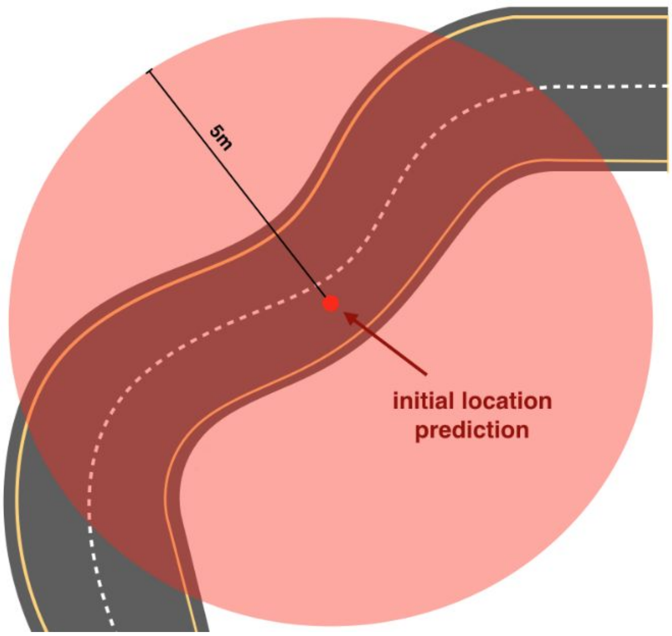
1. Turn left
2. Go straight
3. Turn right

Cars tend to turn left with a probability of 0.4 and go straight with a probability of 0.25. What is the probability that a car turns right?

Ans: 0.35

Bayes' Rule

Bayes' Rule is extremely important in robotics and it can be summarized in one sentence: given an initial prediction, if we gather additional data (data that our initial prediction depends on), we can improve that prediction!



Initial Scenario

[Map of the road and the initial location prediction.](#)
We know a little bit about the map of the road that a car is on (pictured above). We also have an initial GPS measurement; the GPS signal says the car is at the red dot. However, this GPS measurement is inaccurate up to about 5 meters. So, the vehicle could be located anywhere within a 5m radius circle around the dot.

Sensors

Then we gather data from the car's sensors. Self-driving cars mainly use three types of sensors to observe the world:

- Camera, which records video,
- Lidar, which is a light-based sensor, and
- Radar, which uses radio waves.

All of these sensors detect surrounding objects and scenery.
Autonomous cars also have lots of internal sensors that measure things like the speed and direction of the car's movement, the orientation of its wheels, and even the internal temperature of the car!

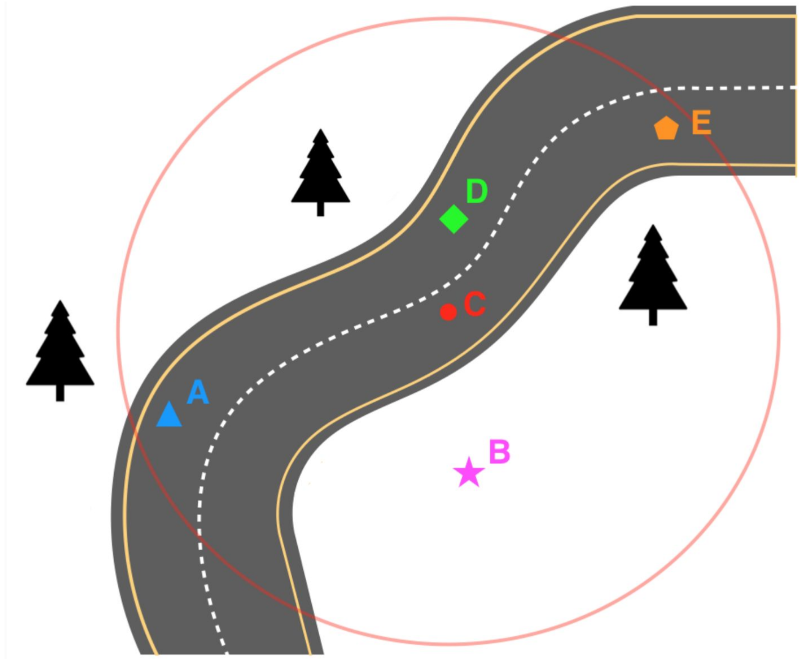
Sensor Measurements

Suppose that our sensors detect some details about the terrain and the way our car is moving, specifically:

- The car could be anywhere within the GPS 5m radius circle,
- The car is moving upwards on this road,

- There is a tree to the left of our car, and
- The car's wheels are pointing to the right.

Knowing only these sensor measurements, examine the map below and answer the following quiz question.



QUIZ QUESTION

After considering the sensor measurements and the initial location prediction, which point on the above map is the best estimate for our car's location?

Ans: A

What is a Probability Distribution?

Probability distributions allow you to represent the probability of an event using a mathematical equation. Like any mathematical equation:

- probability distributions can be visualized using a graph especially in 2-dimensional cases.
- probability distributions can be worked with using algebra, linear algebra and calculus.

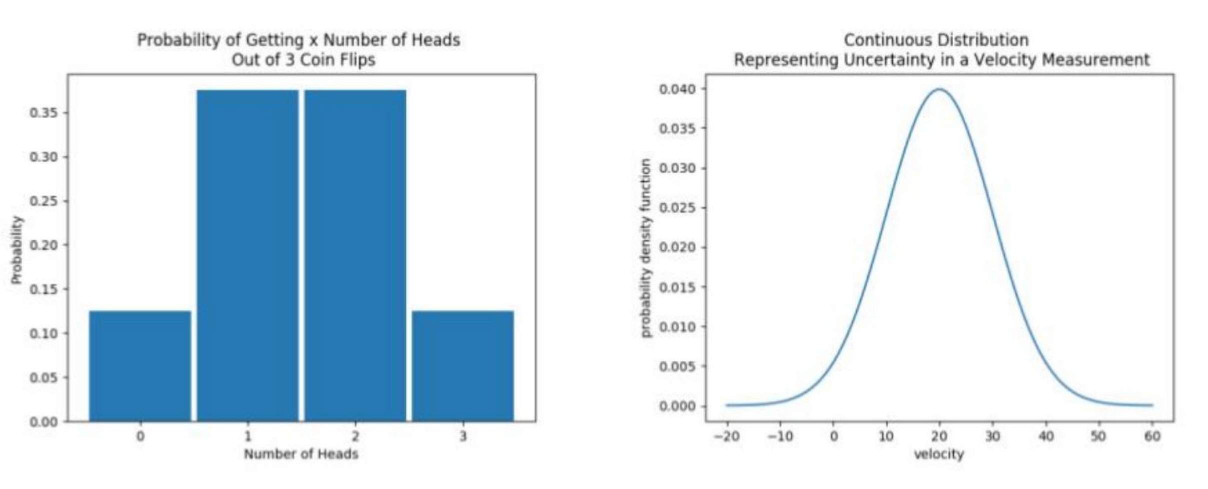
These distributions make it much easier to understand and summarize the probability of a system whether that system be a coin flip experiment or the location of an autonomous vehicle.

Types of Probability Distributions

Probability distributions are really helpful for understanding the probability of a system. Looking at the big pictures, there are two types of probability distributions:

- discrete probability distributions
- continuous probability distributions

Before we get into the details about what discrete and continuous mean, take a look at these two visualizations below. The first image shows a discrete probability distribution and the second a continuous probability distribution. What is similar and what is different about each visualization?



[Discrete Distribution (left) and Continuous Distribution (right).]

More terminology

- Prior - a prior probability distribution of an uncertain quantity, such as the location of a self-driving car on a road. This is the probability distribution that would express one's beliefs about the car's location before some sensor measurements or other evidence is taken into account.
- Posterior - the probability distribution of an uncertain quantity, after some evidence (like sensor measurements) have been taken into account.

Exercise Repository

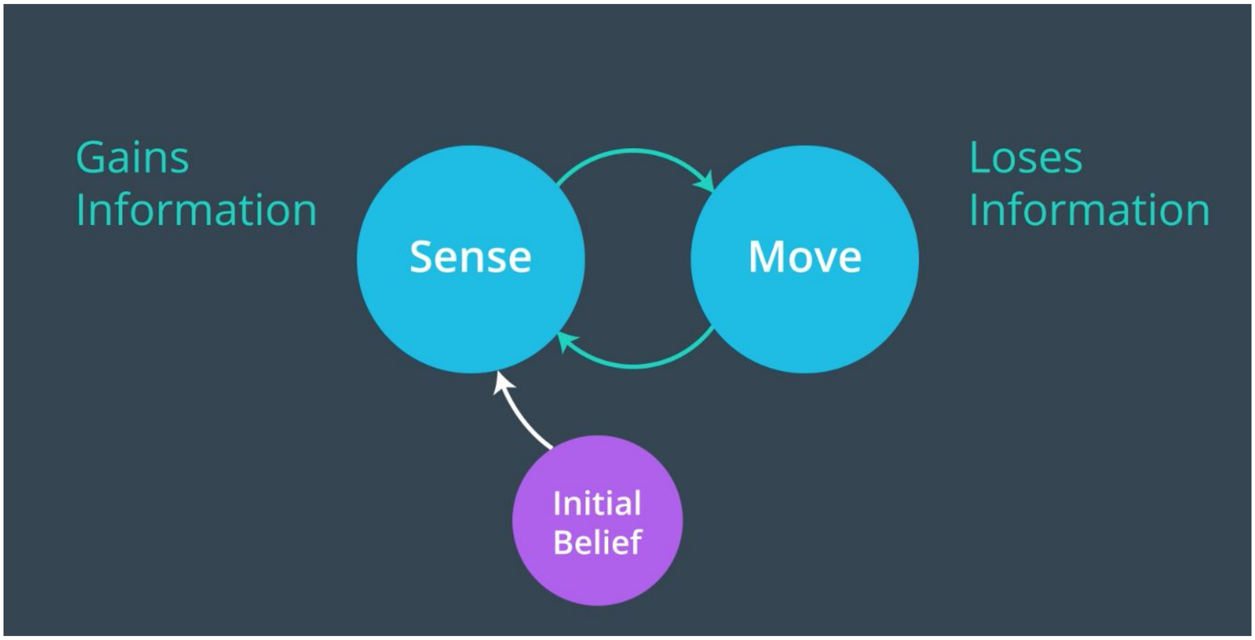
Most of the code notebooks in this lesson can be run, locally, by downloading the files in this Github exercise repository.

QUIZ QUESTION

For a distribution that starts as [1,0,0,0,0], what will this distribution look like after a robot moves infinitely many times (without sensing)? It may help to think about what happens as uncertainty accumulates over 1 motion to the right, then, 2, then 1000, and so on!
Ans: [0.2 , 0.2 , 0.2 , 0.2 , 0.2]
After infinite motions (without any sensor measurements), the accumulated uncertainty will bring us back to knowing nothing: our uniform distribution, which indicates maximum confusion!

Localization

Now, you've learned the foundation of all localization techniques! You know that: first, a robot starts out with some certainty/uncertainty about its position in a world, which is represented by an initial probability distribution, often called the initial belief or the prior. Then it cycles through sensor measurements and movements.

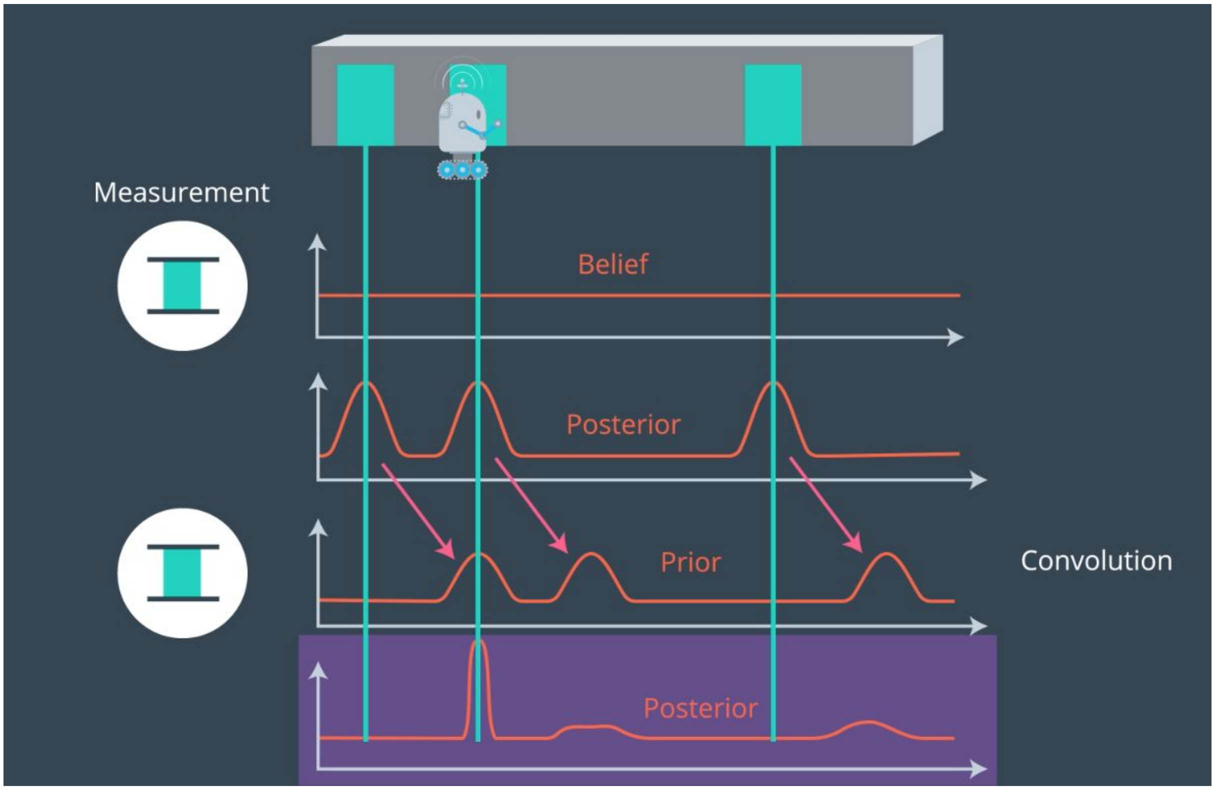


Sense/move cycle.

Sense/Move Cycle

1. When a robot senses, a measurement update happens; this is a simple multiplication that is based off of Bayes' rule, which says that we can update our belief based on measurements! This step was also followed by a normalization that made sure the resultant distribution was still valid (and added up to 1).
2. When it moves, a motion update or prediction step occurs; this step is a convolution that shifts the distribution in the direction of motion.

After this cycle, we are left with an altered posterior distribution!



[A move sense cycle in action, with an initial belief at the top.]

Elective: Learn C++

Now that you've seen how to program a histogram (aka Monte Carlo) localization filter, we have provided an elective section: C++Programming, which shows you how to translate this code into efficient C++ code.

Why Learn C++ ?

If you are looking for a job as a computer vision engineer (rather than just a software developer or researcher), you may find that many job posting require that you know image analysis, deep learning techniques, SLAM, and have proficiency in C++. This is because C++ is the language of hardware; most machines like robots, mobile phones, NVIDIA hardware, or autonomous vehicles, run on C++.
C++ is closer to machine code and can run much faster on hardware than Python can and this is critical for time-sensitive applications. So, should you want to learn more about C++ programming, check out the Elective section to learn the basics of C++ and implement a histogram filter!

This elective course is a very good starting point for learning C++, especially if you are more comfortable with Python and want to expand your knowledge of programming languages. So, if you have any extra time, I highly recommend it!