UNIVERZITET U TUZLI
FAKULTET ELEKTROTEHNIKE

ZADAĆA 1

Osnovi Telekomunikacija

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Kratak sadržaj

1 Zadatak 1

1.1 Zadatak 1 a)

Analitički odrediti energiju signala $x_1(t)$ i $x_2(t)$.

Korištenjem jednačine prave između dvije tačke, sa slike signala $|X_1(f)|$ i $|X_2(f)|$ možemo odrediti matematičke izraze datih signala:

$$X_1(f) = f \cdot \frac{2}{fm} \cdot [u(f) - u(f - \frac{fm}{2})] - (f - fm) \cdot \frac{2}{fm} \cdot [u(f - \frac{fm}{2}) - u(f - fm)] \tag{1}$$

$$X_2(f) = (f - \frac{fm}{2}) \cdot \frac{-2}{fm} \cdot [u(f) - u(f - \frac{fm}{2})] + (f - \frac{fm}{2}) \cdot \frac{2}{fm} \cdot [u(f - \frac{fm}{2}) - u(f - fm)] \tag{2}$$

Energiju računamo preko spektralne gustine energije signala:

$$\Psi(f) = |X(f)|^2 \tag{3}$$

$$E = \int_{-\infty}^{\infty} \Psi(f)\, df \tag{4}$$

Uzevši u obzir granice koje su date heavisideovom funkcijom, mijenjamo granice integracije i energiju možemo izračunati kao:

$$E_1 = \int_0^{\frac{fm}{2}} (f \cdot \frac{2}{fm})^2\, df + \int_{\frac{fm}{2}}^{fm} (2 - \frac{2f}{fm})^2\, df \tag{5}$$

$$E_1 = \frac{4}{fm^2} \cdot \int_0^{\frac{fm}{2}} f^2\, df + 4 \cdot \int_{\frac{fm}{2}}^{fm} df - \frac{8}{fm} \cdot \int_{\frac{fm}{2}}^{fm} f\, df + \frac{4}{fm^2} \cdot \int_{\frac{fm}{2}}^{fm} f^2\, df \tag{6}$$

$$E_1 = \frac{4}{fm^2} \cdot \frac{f^3}{3}\Big|_0^{\frac{fm}{2}} + 4 \cdot f\Big|_{\frac{fm}{2}}^{fm} - \frac{8}{fm} \cdot \frac{f^2}{2}\Big|_{\frac{fm}{2}}^{fm} + \frac{4}{fm^2} \cdot \frac{f^3}{3}\Big|_{\frac{fm}{2}}^{fm} \tag{7}$$

Uvrštavanjem granica i pojednostavljanjem jednačine dobijamo:

$$E_1 = \frac{4}{3} \cdot fm - fm = 1333.33 \tag{8}$$

Aanalogno istim postupkom dobijamo izraz za energiju signala $x_2(t)$:

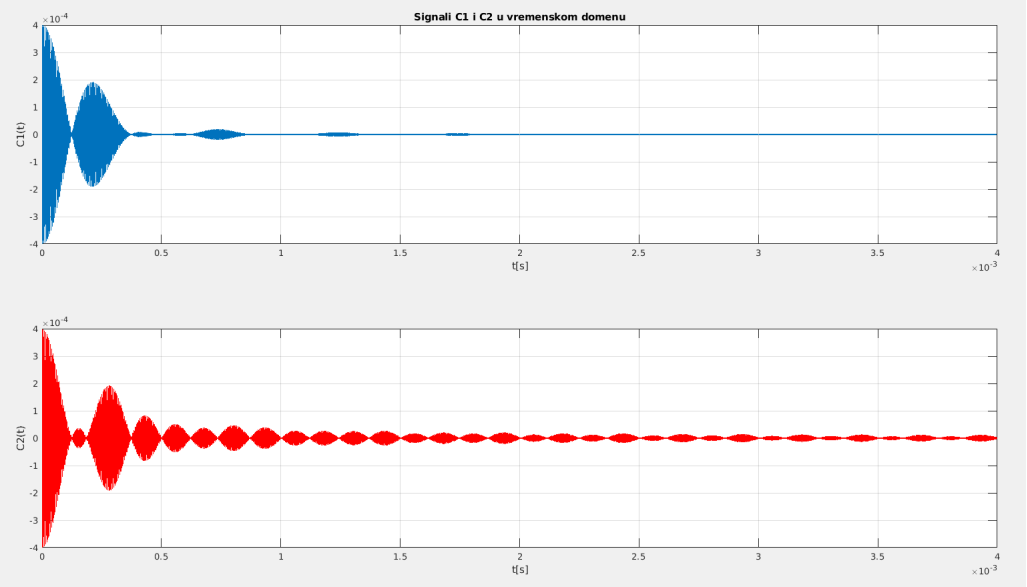
$$E_2 = \frac{fm}{2} - \frac{fm}{2} + \frac{fm}{6} + \frac{4fm}{3} - \frac{fm}{6} - 2fm + \frac{fm}{2} + \frac{fm}{2} \tag{9}$$

$$E_2 = \frac{4}{3} \cdot fm - fm = 1333.33 \tag{10}$$

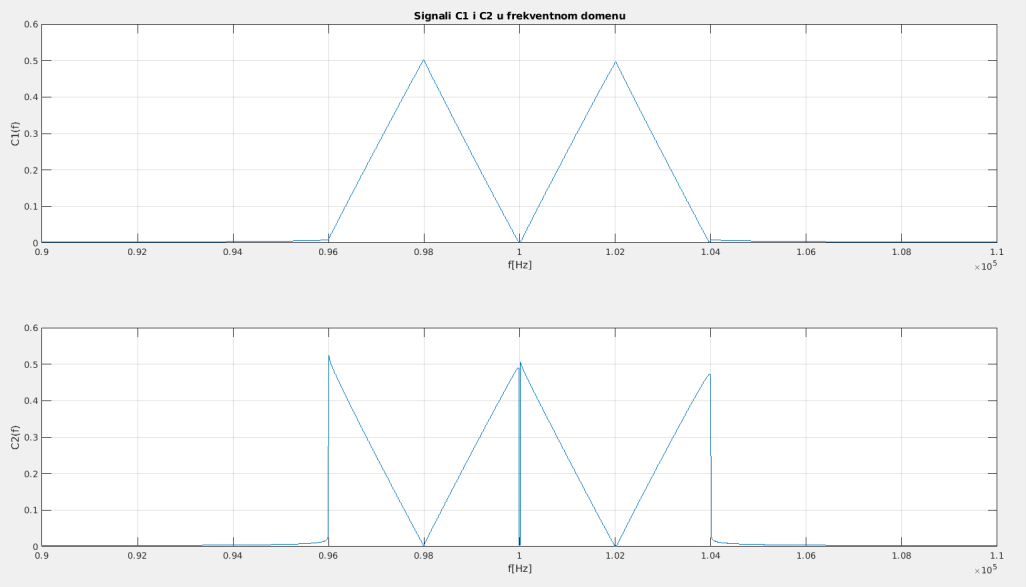
Zaključujemo da je energija signala $E_1 = E_2 = E$

1.2 Zadatak 1 b)

Tačke C_1 i C_2

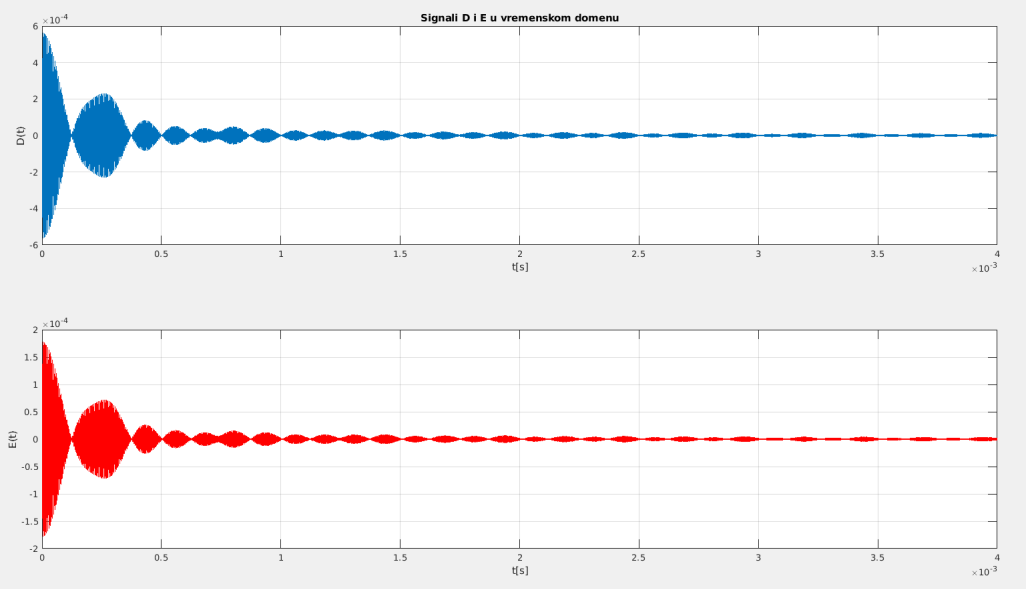


Slika 1: Signali u tačkama C_1 i C_2 u vremenskom domenu

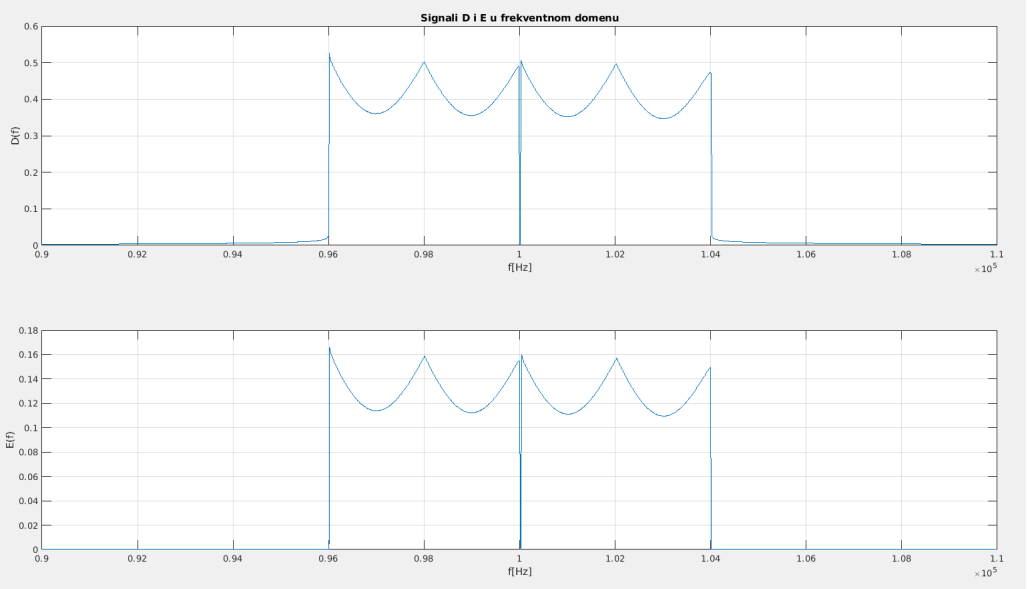


Slika 2: Signali u tačkama C_1 i C_2 u frekventnom domenu

Tačke D i E

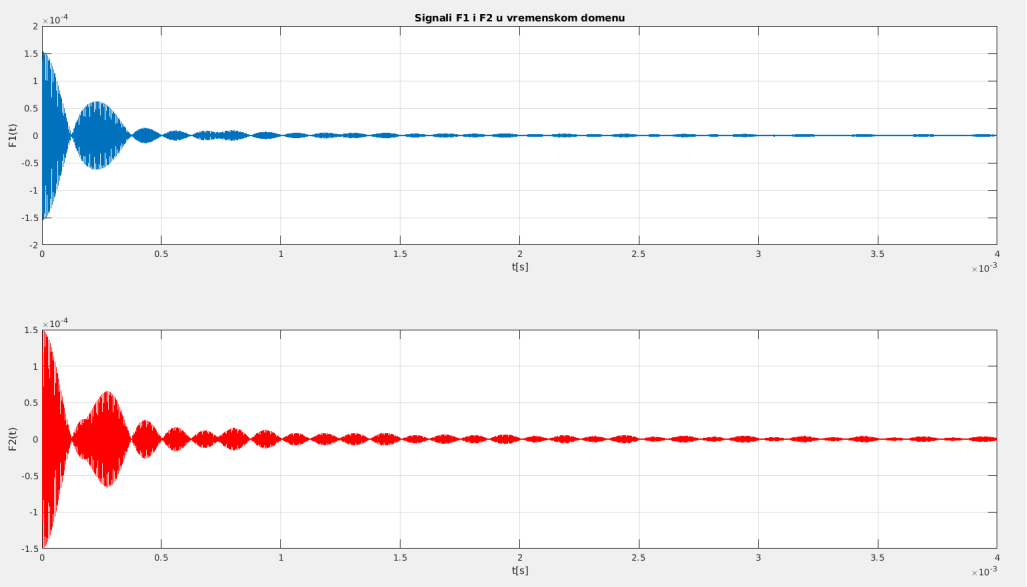


Slika 3: Signali u tačkama D i E u vremenskom domenu

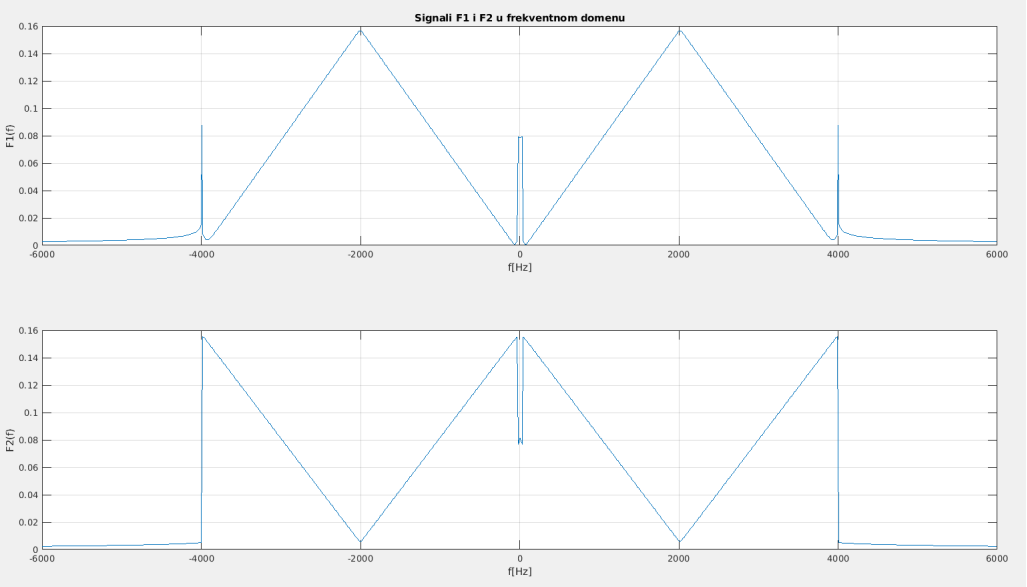


Slika 4: Signali u tačkama D i E u frekventnom domenu

Tačke F_1 i F_2

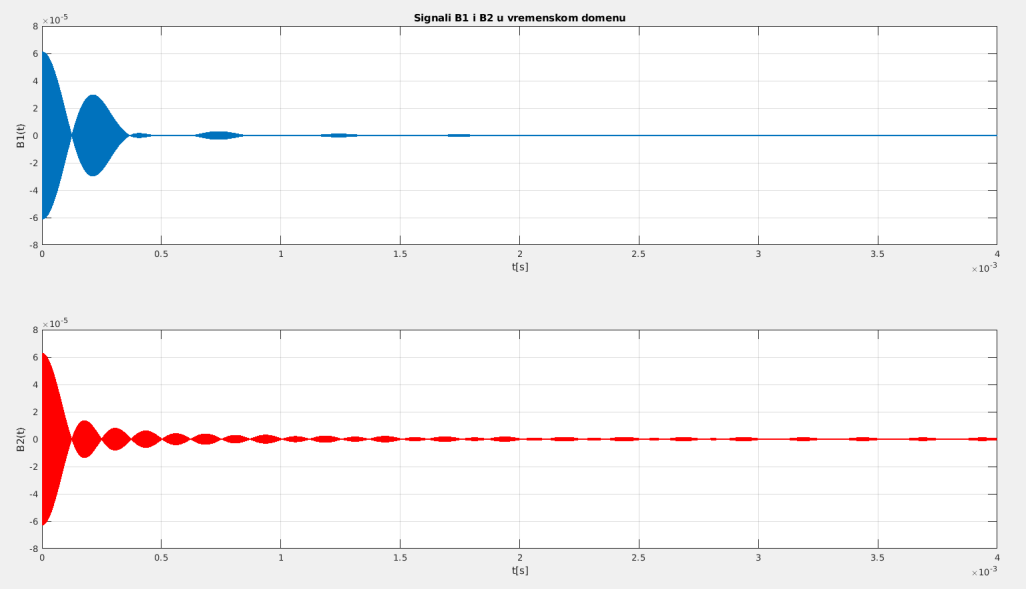


Slika 5: Signali u tačkama F_1 i F_2 u vremenskom domenu

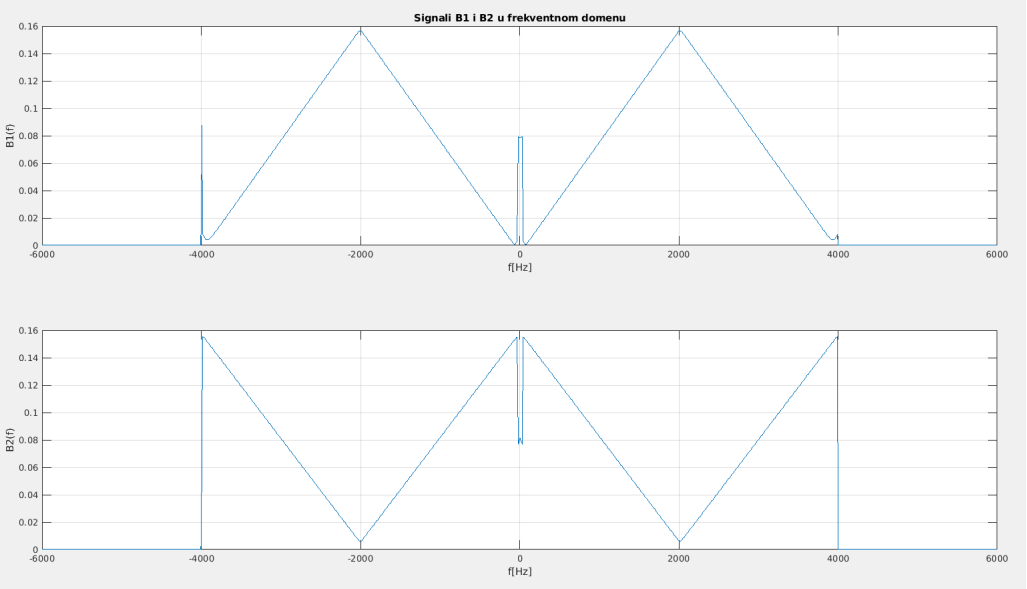


Slika 6: Signali u tačkama F_1 i F_2 u frekventnom domenu

Tačke B_1 i B_2



Slika 7: Signali u tačkama B_1 i B_2 u vremenskom domenu



Slika 8: Signali u tačkama B_1 i B_2 u frekventnom domenu

1.3 Zadatak 1 c)

$$u_{n1}(t) = sin(\omega_0 t) \tag{11}$$

$$u_{n2}(t) = sin(\omega_0 t + \theta) \tag{12}$$

$$u_{n12}(t) = sin(\omega_0 t + \Phi)$$
$$\Phi = -\frac{\pi}{2}$$
$$u_{n12}(t) = sin(\omega_0 t - \frac{\pi}{2}) = -cos(\omega_0 t)$$

Tačka C_1

$$C_1(t) = x_1(t)sin(\omega_0 t) \tag{13}$$

Tačka C_2

$$C_2(t) = x_2(t)sin(\omega_0 t - \frac{\pi}{2}) = -x_2(t)cos(\omega_0 t) \tag{14}$$

Tačka D

$$D(t) = C_1(t) + C_2(t) \tag{15}$$

$$D(t) = x_1(t)sin(\omega_0 t) - x_2(t)cos(\omega_0 t) \tag{16}$$

Kako je $H(j\omega) = -10dB$, a znamo da je $H(j\omega) = 20\log(\frac{A_E}{A_D})$ gdje su A_E i A_D amplitude signala E i D imamo da je:

$$-10 = 20\log(\frac{A_E}{A_D})$$
$$-0.5 = \log(\frac{A_E}{A_D})$$
$$10^{-0.5} = \frac{A_E}{A_D}$$
$$A_E = A_D * 10^{-0.5} = \frac{A_D}{\sqrt{10}}$$

Tačka E

$$E(t) = \frac{x_1(t)}{\sqrt{10}}sin(\omega_0 t) - \frac{x_2(t)}{\sqrt{10}}cos(\omega_0 t) \tag{17}$$

Tačka F_1

$$F_1(t) = E(t)sin(\omega_0 t + \theta) \tag{18}$$

$$\begin{aligned}
F_1(t) &= \frac{x_1(t)}{\sqrt{10}}sin(\omega_0 t)sin(\omega_0 t + \theta) - \frac{x_2(t)}{\sqrt{10}}cos(\omega_0 t)sin(\omega_0 t + \theta) \\
&= \frac{x_1(t)}{2\sqrt{10}}[cos(\omega_0 t - \omega_0 t - \theta) - cos(\omega_0 t + \omega_0 t + \theta)] - \frac{x_2(t)}{2\sqrt{10}}[sin(\omega_0 t + \theta + \omega_0 t) + sin(\omega_0 t + \theta - \omega_0 t)] \\
&= \frac{x_1(t)}{2\sqrt{10}}[cos(\theta) - cos(2\omega_0 t + \theta)] - \frac{x_2(t)}{2\sqrt{10}}[sin(2\omega_0 t + \theta) + sin(\theta)] \\
&= \frac{x_1(t)}{2\sqrt{10}}cos(\theta) - \frac{x_2(t)}{2\sqrt{10}}sin(\theta) - \frac{x_1(t)}{2\sqrt{10}}cos(2\omega_0 t + \theta) - \frac{x_2(t)}{2\sqrt{10}}sin(2\omega_0 t + \theta)
\end{aligned}$$

Tačka F_2

$$F_2(t) = E(t)sin(\omega_0 t + \theta - \frac{\pi}{2}) = -E(t)cos(\omega_0 t) \tag{19}$$

$$\begin{aligned} F_2(t) &= -\frac{x_1(t)}{\sqrt{10}}sin(\omega_0 t)cos(\omega_0 t + \theta) + \frac{x_2(t)}{\sqrt{10}}cos(\omega_0 t)cos(\omega_0 t + \theta) \\ &= -\frac{x_1(t)}{2\sqrt{10}}[sin(2\omega_0 t + \theta) - sin(\theta)] + \frac{x_2(t)}{2\sqrt{10}}[cos(\theta) + cos(2\omega_0 t + \theta)] \\ &= \frac{x_1(t)}{2\sqrt{10}}sin(\theta) + \frac{x_2(t)}{2\sqrt{10}}cos(\theta) - \frac{x_1(t)}{2\sqrt{10}}sin(2\omega_0 t + \theta) + \frac{x_2(t)}{2\sqrt{10}}cos(2\omega_0 t + \theta) \end{aligned}$$

Prolaskom kroz filter čija je granična frekvencija fm, dobijamo da izraze za B_1 i B_2

Tačka B_1

$$B_1(t) = \frac{x_1(t)}{2\sqrt{10}}cos(\theta) - \frac{x_2(t)}{2\sqrt{10}}sin(\theta) \tag{20}$$

Tačka B_2

$$B_2(t) = \frac{x_1(t)}{2\sqrt{10}}sin(\theta) + \frac{x_2(t)}{2\sqrt{10}}cos(\theta) \tag{21}$$

Kada je $\theta = 0$ imamo da je:

$$B_1(t) = \frac{x_1(t)}{2\sqrt{10}}$$
$$B_2(t) = \frac{x_2(t)}{2\sqrt{10}}$$

Ovim smo dokazali da su B_1 odnosno B_2 samo skalirani A_1 odnosno A_2.

1.4 Zadatak 1 d)

Iz dijela zadataka pod c) vidimo da u izrazu za B_1 imamo 2 člana, odnosno član koji potiče od signala A_1 i član koji potiče od signala A_2.

Prvi član:

$$x_{B11}(t) = \frac{x_1(t)}{2\sqrt{10}}cos(\theta) \tag{22}$$

Drugi član:

$$x_{B12}(t) = -\frac{x_2(t)}{2\sqrt{10}}sin(\theta) \tag{23}$$

Srednja energija prvog člana:

$$\begin{aligned} E_{B11} &= \int_{-\infty}^{\infty} |\frac{x_1(t)}{2\sqrt{10}}cos(\theta)|^2 \, dt \\ &= \int_{-\infty}^{\infty} \frac{x_1(t)^2}{40}cos^2(\theta) \, dt \\ &= \frac{1}{40}cos^2(\theta)\int_{-\infty}^{\infty} x_1(t)^2 \, dt \\ &= \frac{E_1}{40}cos^2(\theta) \end{aligned}$$

Srednja energija drugog člana:

$$\begin{aligned} E_{B12} &= \int_{-\infty}^{\infty} |\frac{x_2(t)}{2\sqrt{10}} sin(\theta)|^2 \, dt \\ &= \int_{-\infty}^{\infty} \frac{x_2(t)^2}{40} sin^2(\theta) \, dt \\ &= \frac{1}{40} sin^2(\theta) \int_{-\infty}^{\infty} x_2(t)^2 \, dt \\ &= \frac{E_2}{40} sin^2(\theta) \end{aligned}$$

Iz dijela zadatka pod a) vidjeli smo da su energija signala $x_1(t)$ i $x_2(t)$ jednake. Da bi bio ispunjen uslov koji se traži u ovom dijelu zadatka, imamo da je:

$$\begin{aligned} 10 \log \frac{E_{B11}}{E_{B12}} &\geq 26dB \\ 10 \log \frac{\frac{E}{40} cos^2(\theta)}{\frac{E}{40} sin^2(\theta)} &\geq 26dB \\ 20 \log \cot \theta &\geq 26dB \\ \log \cot \theta &\geq 1.3 \\ \cot \theta &\geq 20 \\ \theta &\leq 3^\circ \end{aligned}$$