

EECS 289A Introduction to Machine Learning

Final Project T:

Image Processing and Least Square Problems

- An application of image super-resolution



Presented By:

Peng TAN

Han LIU

Jinyan ZHAO

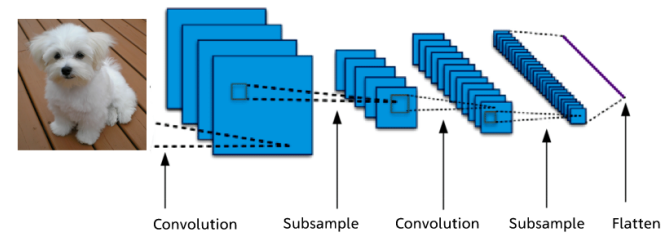
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Date: Dec. 2020

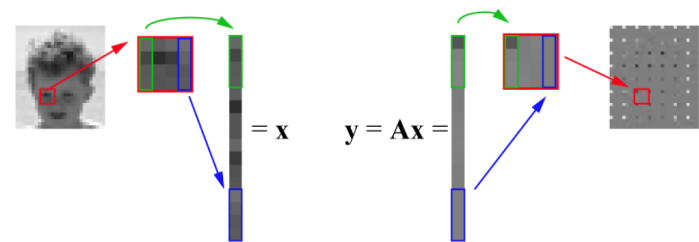
Introduction

- Image processing:
 - Image denoting
 - Image blurring
 - Image restoration
- Objectives:
 - 1) Implement image reconstruction to build a bridge between image processing and various types of least squares problems.
 - 2) Train a learning-based model through solving some linear programming problems.

- Neural network

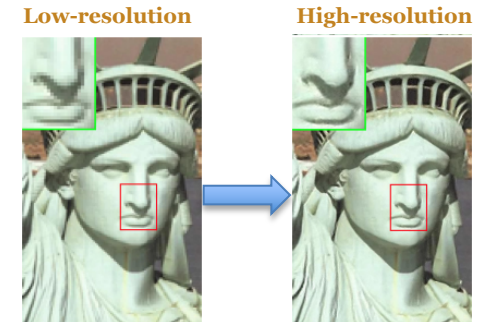
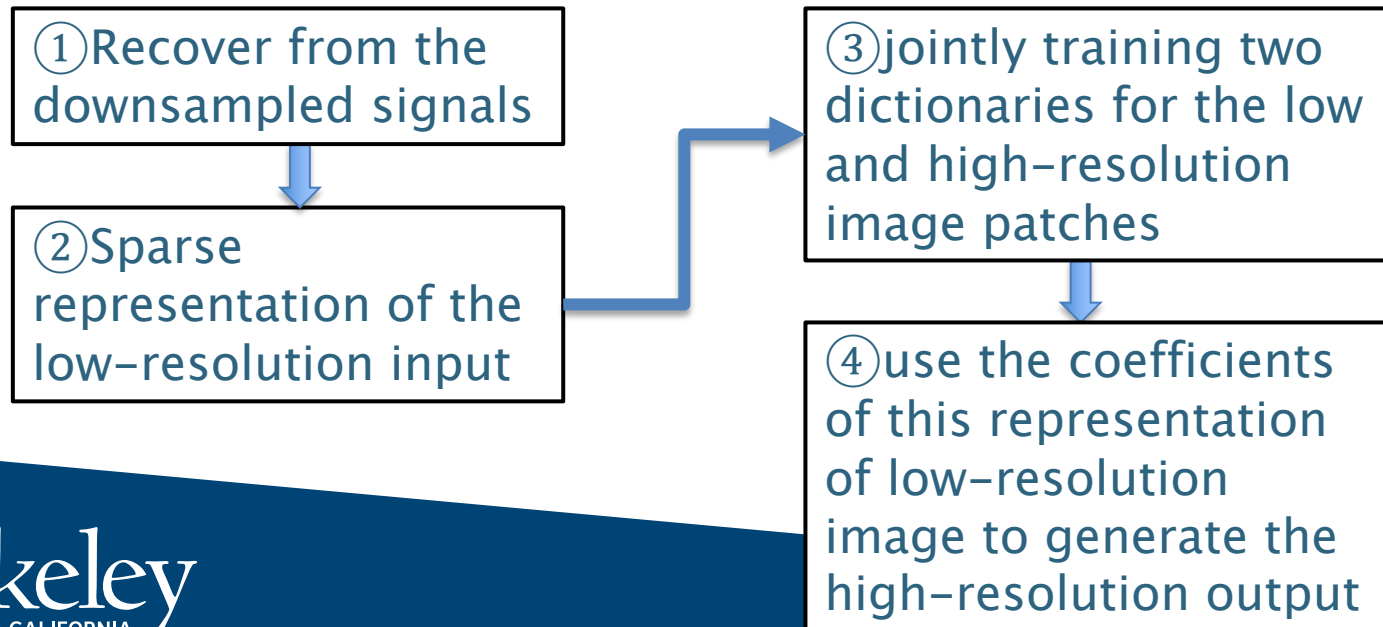


- conventional methods revoking linear algebra



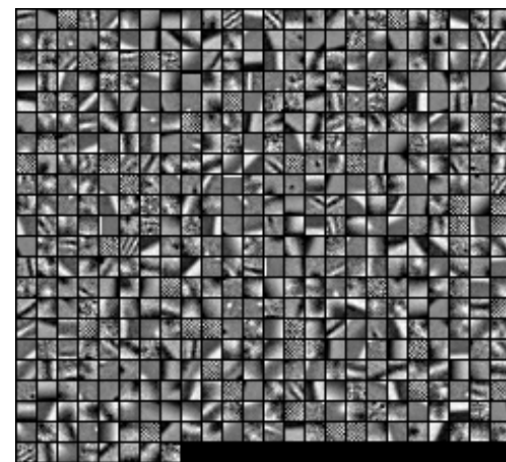
Background

- Single-image super-resolution, based on sparse signal representation.
- 1) image patches can be well represented as a sparse linear combination of elements from an appropriately chosen over-complete dictionary
 - 2) Procedure:



Basic Ideas:

- 1) Under certain conditions, any sufficiently sparse linear representation of a high-resolution image patch can be recovered almost perfectly from the low-resolution image patch
- 2) $D \in \mathbb{R}^{n \times K}$: overcomplete dictionary of K features
- 3) $x = D\alpha$: image patch, where $\alpha \in \mathbb{R}^K$ is a sparse vector
- 4) $y = LD\alpha$:
 - L : projection matrix
 - x is a high-resolution image patch, y is its low-resolution counterpart
 - two coupled dictionaries: D_h and D_l
 - concatenate D_h and D_l with normalization for consistent sparse representation



The high-resolution image patch dictionary trained using 100,000 high resolution and low-resolution image patch pairs sampled from the generic training images. (Yang, J., Wright, J., Huang, T. S., & Ma, Y. (2010))

Image Super-Resolution From sparsity

Original problem: $\min \|\alpha\|_0 \text{ s.t. } \|FD_l\alpha - Fy\|_2^2 \leq \epsilon$



Donoho (2008): $\min \|\alpha\|_1 \text{ s.t. } \|FD_l\alpha - Fy\|_2^2 \leq \epsilon$



Lagrange: $\min_{\alpha} \|FD_l\alpha - Fy\|_2^2 + \lambda \|\alpha\|_1$



Compatibility of D_l and D_h : $\min_{\alpha} \|\tilde{D}\alpha - \tilde{y}\|_2^2 + \lambda \|\alpha\|_1$ Where $\tilde{D} = \begin{bmatrix} FD_l \\ PD_h \end{bmatrix}$, $\tilde{y} = \begin{bmatrix} Fy \\ \omega \end{bmatrix}$

Similar to LASSO regression



Dictionary Training

Object:

$$D = \underset{D, Z}{\operatorname{argmin}} \|X - DZ\|_2^2 + \lambda \|Z\|_1 \text{ s.t. } \|D_i\|_2^2 \leq 1, i = 1, 2, \dots, K$$

Algorithm:

- 1) Initialize D with a normalized Gaussian random matrix
- 2) Fix D and update Z by: $Z = \underset{Z}{\operatorname{argmin}} \|X - DZ\|_2^2 + \lambda \|Z\|_1$
- 3) Fix Z and update D by: $D = \underset{D, Z}{\operatorname{argmin}} \|X - DZ\|_2^2 \text{ s.t. } \|D_i\|_2^2 \leq 1, i = 1, 2, \dots, K$
- 4) Repeat (2) and (3) until converge

An Efficient Sparse Coding Algorithm for LASSO

- 1) L1-Regularized Least Square $\min_x ||\mathbf{y} - A\mathbf{x}||^2 + \gamma ||\mathbf{x}||_1$
- 2) If we know the signs (positive, zero or negative) of the x_i , we can replace each of the term $||\mathbf{x}||_1$ with: x_i if $(x_i > 0)$, $-x_i$ (if $x_i < 0$), or 0 (if $x_i = 0$);
- 3) Lasso regression reduces to a standard, unconstrained quadratic optimization problem, which can be solved analytically and efficiently.

Lee, H., Battle, A., Raina, R., & Ng, A. Y. (2007). Efficient sparse coding algorithms. In Advances in neural information processing systems (pp 801-808).

Constrained Least Squares with Lagrange Dual

1) Constrained Least Square.

$$\min_D \|X - DZ\|_F^2 \quad s.t. \sum_{i=1}^k D_{i,j}^2 \leq c, \quad \forall j = 1, 2, \dots, n$$

2) Or equivalently consider the Lagrangian:

$$L(D, \lambda) = \text{trace}((X - DZ)^T(X - DZ)) + \sum_{j=1}^n \lambda_j \left(\sum_{i=1}^k D_{i,j}^2 - c \right)$$

3) Where each $\lambda_i \geq 0$ is a dual variable, minimize over D analytically:

$$D(\lambda) = \text{trace}(X^T - XZ^T(ZZ^T + \Lambda)^{-1}(XZ^T)^T - c\Lambda), \quad \Lambda = \text{diag}(\lambda)$$

4) Computing the gradient and Hessian of $D(\lambda)$, Λ could be optimized using Newton's method or conjugate gradient.

5) The optimum D is obtained:

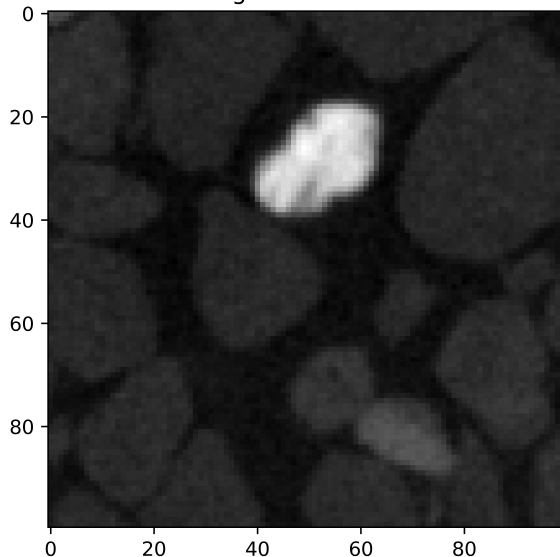
$$D^T = (ZZ^T + \Lambda)^{-1}(XZ^T)^T$$

Experimental Results

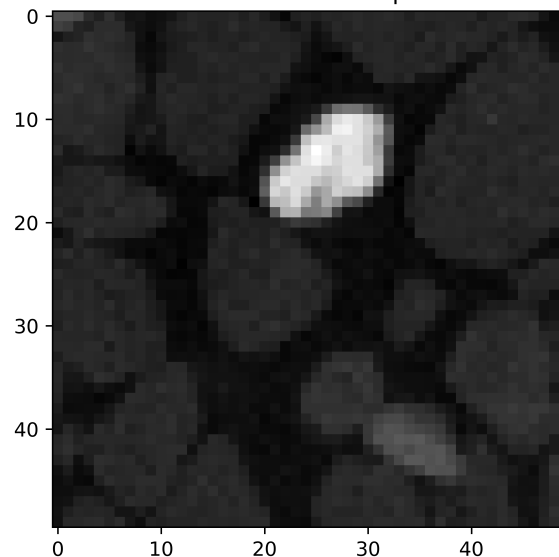
Super-resolution results obtained by applying the previously mentioned methods on X-ray tomography images on granular material.

- Visually satisfactory.
- Outperforms current conventional interpolation-based method. (Mean Squared Error **2.98** vs. **3.16**).
- More details in note and Jupyter-notebooks.

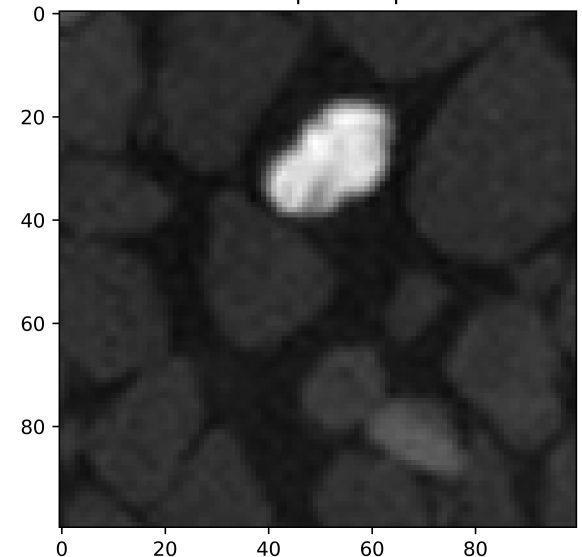
ground truth



low resolution input



recovered from sparse representation



Conclusion

- A novel learning-based approach is presented toward single image super-resolution based using sparse representations.
- Two convex optimization least square problems are associated.
- An efficient algorithm is adopted and implemented to solve LASSO regression.
- Constrained least square problem is solved with Lagrange dual.
- Supplementary Jupyter-notebook covers feature engineering, algorithm implementation, tuning hyper-parameters and numerical experiment details.
- <https://github.com/tanpeng1995/289FinalProject.git>

Reference:

Donoho, D. L. (2006). For most large underdetermined systems of linear equations the minimal ℓ_1 -norm solution is also the sparsest solution. *Communications on Pure and Applied Mathematics: A Journal Issued by the Courant Institute of Mathematical Sciences*, 59 (6), 797–829.

Lee, H., Battle, A., Raina, R., & Ng, A. (2006). Efficient sparse coding algorithms. *Advances in neural information processing systems*, 19, 801–808.

Yang, J., Wright, J., Huang, T. S., & Ma, Y. (2010). Image super-resolution via sparse representation. *IEEE transactions on image processing*, 19(11), 2861–2873.