EECS 289A Introduction to Machine Learning

Final Project T:

Image Processing and Least Square Problems

- An application of image super-resolution

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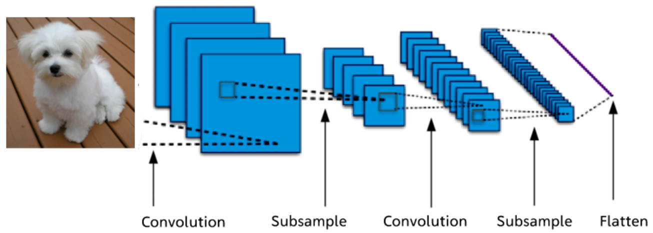
Introduction

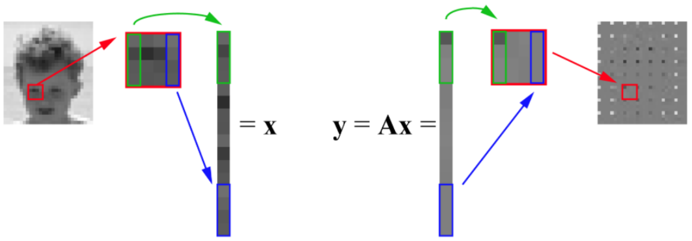
- Image processing:
  - Image denoting
  - Image blurring
  - Image restoration
- Objectives:

1) Implement image reconstruction to build a bridge between image processing and various types of least squares problems.
2) Train a learning-based model through solving some linear programming problems.

- Neural network



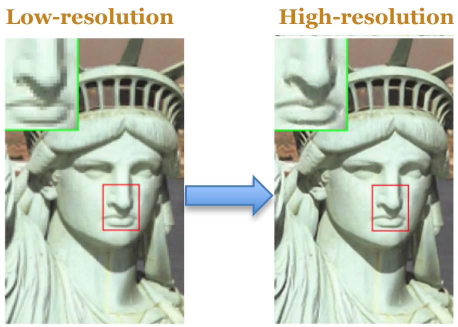
- conventional methods revoking linear algebra



Background

- Single-image super-resolution, based on sparse signal representation.

1) image patches can be well represented as a sparse linear combination of elements from an appropriately chosen over-complete dictionary
2) Procedure:



①Recover from the downsampled signals

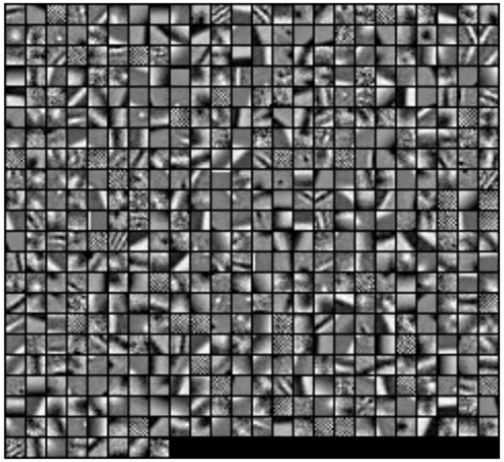
②Sparse representation of the low-resolution input

③jointly training two dictionaries for the low and high-resolution image patches

④use the coefficients of this representation of low-resolution image to generate the high-resolution output

Basic Ideas:

1) Under certain conditions, any sufficiently sparse linear representation of a high-resolution image patch can be recovered almost perfectly form the low-resolution image patch
2) $D \in R^{n \times K}$: overcomplete dictionary of K features
3) $x = D\alpha$: image patch, where $\alpha \in R^{K}$ is a sparse vector
4) $y = LD\alpha$:
   - L: projection matrix
   - x is a high-resolution image patch, y is its low-resolution counterpart
   - two coupled dictionaries: D_h and D_l
   - concatenate D_h and D_l with normalization for consistent sparse representation



The high-resolution image patch dictionary trained using 100,000 high resolution and low-resolution image patch pairs sampled from the generic training images. (Yang, J., Wright, J., Huang, T. S., & Ma, Y. (2010))

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Image Super-Resolution From sparsity

Original problem: $\min \left|\left|\alpha\right|\right|_0 \ s.t. \left|\left|FD_l\alpha - Fy\right|\right|_2^2 \leq \epsilon$

Donoho (2008): $\min \left|\left|\alpha\right|\right|_1 \ s.t. \left|\left|FD_l\alpha - Fy\right|\right|_2^2 \leq \epsilon$

Lagrange: $\min_\alpha \left|\left|FD_l\alpha - Fy\right|\right|_2^2 + \lambda\left|\left|\alpha\right|\right|_1$

Similar to LASSO regression

Compatibility of D_l and D_h: $\min_\alpha \left|\left|\widetilde{D}\alpha - \tilde{y}\right|\right|_2^2 + \lambda\left|\left|\alpha\right|\right|_1$ Where $\widetilde{D} = \begin{bmatrix} FD_l \\ PD_h \end{bmatrix}$, $\tilde{y} = \begin{bmatrix} Fy \\ \omega \end{bmatrix}$

Dictionary Training

Object:

$$D = \underset{D,Z}{\operatorname{argmin}} \left|\left|X - DZ\right|\right|_2^2 + \lambda \left|\left|Z\right|\right|_1 \; s.t. \left|\left|D_i\right|\right|_2^2 \leq 1 \,, i = 1, 2, \ldots, K$$

Algorithm:

1) Initialize D with a normalized Gaussian random matrix
2) Fix D and update Z by: $\mathrm{Z} = \underset{Z}{\operatorname{argmin}} \left|\left|X - DZ\right|\right|_2^2 + \lambda \left|\left|Z\right|\right|_1$
3) Fix Z and update D by: $\mathrm{D} = \underset{D,Z}{\operatorname{argmin}} \left|\left|X - DZ\right|\right|_2^2$ s.t. $\left|\left|D_i\right|\right|_2^2 \leq 1 \,, i = 1, 2, \ldots, K$
4) Repeat (2) and (3) until converge

An Efficient Sparse Coding Alogrithm for LASSO

1) L1-Regularized Least Square $\min_{x} ||\boldsymbol{y} - A\boldsymbol{x}||^2 + \gamma ||\boldsymbol{x}||_1$
2) If we know the signs (positive, zero or negative) of the x_i, we can replace each of the term $||\boldsymbol{x}||_1$ with: x_i if $(x_i > 0)$, $-x_i$ (if $x_i < 0$), or 0 (if $x_i = 0$);
3) Lasso regression reduces to a standard, unconstrained quadratic optimization problem, which can be solved analytically and efficiently.

Lee, H., Battle, A., Raina, R., & Ng, A. Y. (2007). Efficient sparse coding algorithms. In Advances in neural information processing systems (pp 801-808).

Constrained Least Squares with Lagrange Dual

1) Constrained Least Square.

$$\min_D \left|\left|X - DZ\right|\right|_F^2 \quad s.t. \sum_{i=1}^{k} D_{i,j}^2 \leq c, \quad \forall j = 1,2,\cdots,n$$

2) Or equivalently consider the Lagrangian:

$$L(D,\lambda) = trace\big((X - DZ)^T(X - DZ)\big) + \sum_{j=1}^{n} \lambda_i(\sum_{j=1}^{k} D_{i,j}^2 - c)$$

3) Where each $\lambda_i \geq 0$ is a dual variable, minimize over D analytically:

$$D(\lambda) = trace(X^T - XZ^T(ZZ^T + \Lambda)^{-1}(XZ^T)^T - c\Lambda), \qquad \Lambda = \mathrm{diag}(\lambda)$$

4) Computing the gradient and Hessian of $D(\lambda)$, Λ could be optimized using Newton's method or conjugate gradient.

5) The optimum D is obtained:

$$D^T = (ZZ^T + \Lambda)^{-1}(XZ^T)^T$$

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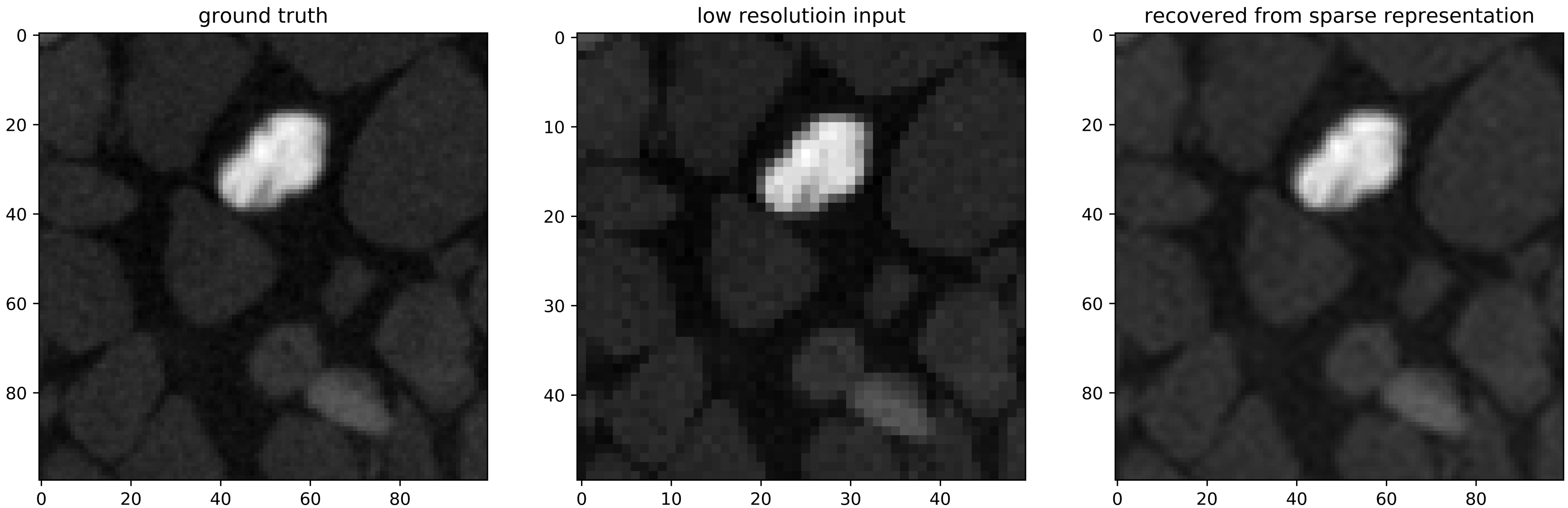
Experimental Results

Super-resolution results obtained by applying the previously mentioned methods on X-ray tomography images on granular material.

- Visually satisfactory.
- Outperforms current conventional interpolation-based method. (Mean Squared Error **2.98** vs. **3.16**).
- More details in note and Jupyter-notebooks.



Conclusion

- A novel learning-based approach is presented toward single image super-resolution based using sparse representations.
- Two convex optimization least square problems are associated.
- An efficient algorithm is adopted and implemented to solve LASSO regression.
- Constrained least square problem is solved with Lagrange dual.
- Supplementary Jupyter-notebook covers feature engineering, algorithm implementation, tuning hyper-parameters and numerical experiment details.

Reference:

Donoho, D. L. (2006). For most large underdetermined systems of linear equations the minimal $\ell 1$-norm solution is also the sparsest solution. *Communications on Pure and Applied Mathematics: A Journal Issued by the Courant Institute of Mathematical Sciences*, *59* (6), 797–829.

Lee, H., Battle, A., Raina, R., & Ng, A. (2006). Efficient sparse coding algorithms. *Advances in neural information processing systems*, *19*, 801–808.

Yang, J., Wright, J., Huang, T. S., & Ma, Y. (2010). Image super-resolution via sparse representation. *IEEE transactions on image processing*, *19*(11), 2861–2873.

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