In [ ]:

```
import scipy
import scipy.integrate
import pylab as plt
```

In [ ]:

```
N = 1000
beta = 1.0  # infected person infects 1 other person per day
D = 4.0 # infections lasts four days
gamma = 1.0/D

S0, I0, R0 = 999, 1, 0  # initial conditions: one infected, rest susceptible
```

In [ ]:

```
def deriv(y, t, N, beta, gamma):
    S, I, R = y
    dSdt = -beta * S * I / N
    dIdt = beta * S * I / N - gamma * I
    dRdt = gamma * I
    return dSdt, dIdt, dRdt
```

In [ ]:

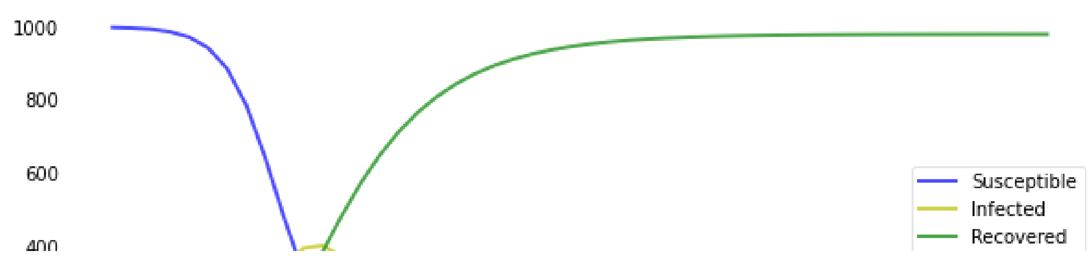
```
import numpy as np
import scipy
from scipy.integrate import odeint

t = np.linspace(0, 50, 50) # Grid of time points (in days)
y0 = S0, I0, R0 # Initial conditions vector

# Integrate the SIR equations over the time grid, t.
ret = odeint(deriv, y0, t, args=(N, beta, gamma))
S, I, R = ret.T
```

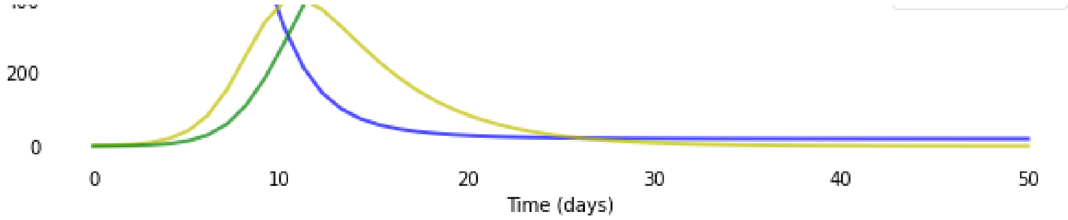
In [ ]:

```
def plotsir(t, S, I, R):
  f, ax = plt.subplots(1,1,figsize=(10,4))
  ax.plot(t, S, 'b', alpha=0.7, linewidth=2, label='Susceptible')
  ax.plot(t, I, 'y', alpha=0.7, linewidth=2, label='Infected')
  ax.plot(t, R, 'g', alpha=0.7, linewidth=2, label='Recovered')

  ax.set_xlabel('Time (days)')

  ax.yaxis.set_tick_params(length=0)
  ax.xaxis.set_tick_params(length=0)
  ax.grid(b=True, which='major', c='w', lw=2, ls='-')
  legend = ax.legend()
  legend.get_frame().set_alpha(0.5)
  for spine in ('top', 'right', 'bottom', 'left'):
      ax.spines[spine].set_visible(False)
  plt.show();
```

In [ ]:

```
plotsir(t, S, I, R)
```





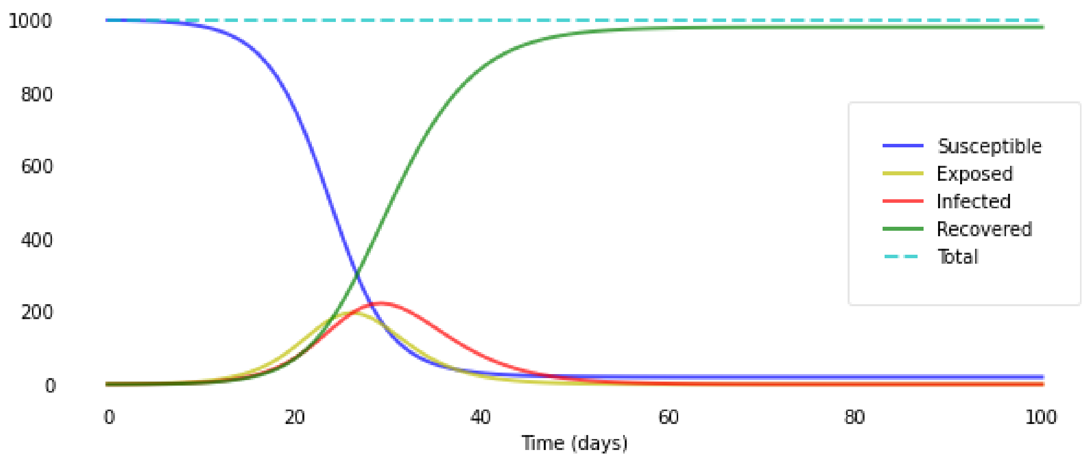
In [ ]:

```
def plotseird(t, S, E, I, R, D=None, L=None, R0=None, Alpha=None, CFR=None):
  f, ax = plt.subplots(1,1,figsize=(10,4))
  ax.plot(t, S, 'b', alpha=0.7, linewidth=2, label='Susceptible')
  ax.plot(t, E, 'y', alpha=0.7, linewidth=2, label='Exposed')
  ax.plot(t, I, 'r', alpha=0.7, linewidth=2, label='Infected')
  ax.plot(t, R, 'g', alpha=0.7, linewidth=2, label='Recovered')
  if D is not None:
    ax.plot(t, D, 'k', alpha=0.7, linewidth=2, label='Dead')
    ax.plot(t, S+E+I+R+D, 'c--', alpha=0.7, linewidth=2, label='Total')
  else:
    ax.plot(t, S+E+I+R, 'c--', alpha=0.7, linewidth=2, label='Total')

  ax.set_xlabel('Time (days)')

  ax.yaxis.set_tick_params(length=0)
  ax.xaxis.set_tick_params(length=0)
  ax.grid(b=True, which='major', c='w', lw=2, ls='-')
  legend = ax.legend(borderpad=2.0)
  legend.get_frame().set_alpha(0.5)
  for spine in ('top', 'right', 'bottom', 'left'):
      ax.spines[spine].set_visible(False)
  if L is not None:
      plt.title("Lockdown after {} days".format(L))
  plt.show();

  if R0 is not None or CFR is not None:
    f = plt.figure(figsize=(12,4))

  if R0 is not None:
    # sp1
    ax1 = f.add_subplot(121)
    ax1.plot(t, R0, 'b--', alpha=0.7, linewidth=2, label='R_0')

    ax1.set_xlabel('Time (days)')
    ax1.title.set_text('R_0 over time')
    # ax.set_ylabel('Number (1000s)')
    # ax.set_ylim(0,1.2)
    ax1.yaxis.set_tick_params(length=0)
    ax1.xaxis.set_tick_params(length=0)
    ax1.grid(b=True, which='major', c='w', lw=2, ls='-')
    legend = ax1.legend()
    legend.get_frame().set_alpha(0.5)
    for spine in ('top', 'right', 'bottom', 'left'):
      ax.spines[spine].set_visible(False)

  if Alpha is not None:
    # sp2
    ax2 = f.add_subplot(122)
    ax2.plot(t, Alpha, 'r--', alpha=0.7, linewidth=2, label='alpha')

    ax2.set_xlabel('Time (days)')
    ax2.title.set_text('fatality rate over time')
    # ax.set_ylabel('Number (1000s)')
    # ax.set_ylim(0,1.2)
    ax2.yaxis.set_tick_params(length=0)
    ax2.xaxis.set_tick_params(length=0)
    ax2.grid(b=True, which='major', c='w', lw=2, ls='-')
    legend = ax2.legend()
    legend.get_frame().set_alpha(0.5)
    for spine in ('top', 'right', 'bottom', 'left'):
      ax.spines[spine].set_visible(False)
```

```
    plt.show();
```

In [ ]:

```
def deriv2(y, t, N, beta, gamma, delta):
    S, E, I, R = y
    dSdt = -beta * S * I / N
    dEdt = beta * S * I / N - delta * E
    dIdt = delta * E - gamma * I
    dRdt = gamma * I
    return dSdt, dEdt, dIdt, dRdt


N = 1000
beta = 1.0  # infected person infects 1 other person per day
D = 4.0 # infections lasts four days
gamma = 1.0 / D
delta = 1.0 / 3.0  # incubation period of three days

S0, E0, I0, R0 = 999, 1, 0, 0  # initial conditions: one exposed, rest susceptible

t = np.linspace(0, 100, 100) # Grid of time points (in days)
y0 = S0, E0, I0, R0 # Initial conditions vector

# Integrate the SIR equations over the time grid, t.
ret = odeint(deriv2, y0, t, args=(N, beta, gamma, delta))

S, E, I, R = ret.T

plotseird(t, S, E, I, R)
```



Programming the Dead-Compartment

In [ ]:

```
def deriv(y, t, N, beta, gamma, delta, alpha, rho):
    S, E, I, R, D = y
    dSdt = -beta * S * I / N
    dEdt = beta * S * I / N - delta * E
    dIdt = delta * E - (1 - alpha) * gamma * I - alpha * rho * I
    dRdt = (1 - alpha) * gamma * I
    dDdt = alpha * rho * I
    return dSdt, dEdt, dIdt, dRdt, dDdt
```

In [ ]:

```
N = 1_000_000
D = 4.0 # infections lasts four days
gamma = 1.0 / D
delta = 1.0 / 5.0  # incubation period of five days
R_0 = 5.0
beta = R_0 * gamma  # R_0 = beta / gamma, so beta = R_0 * gamma
```

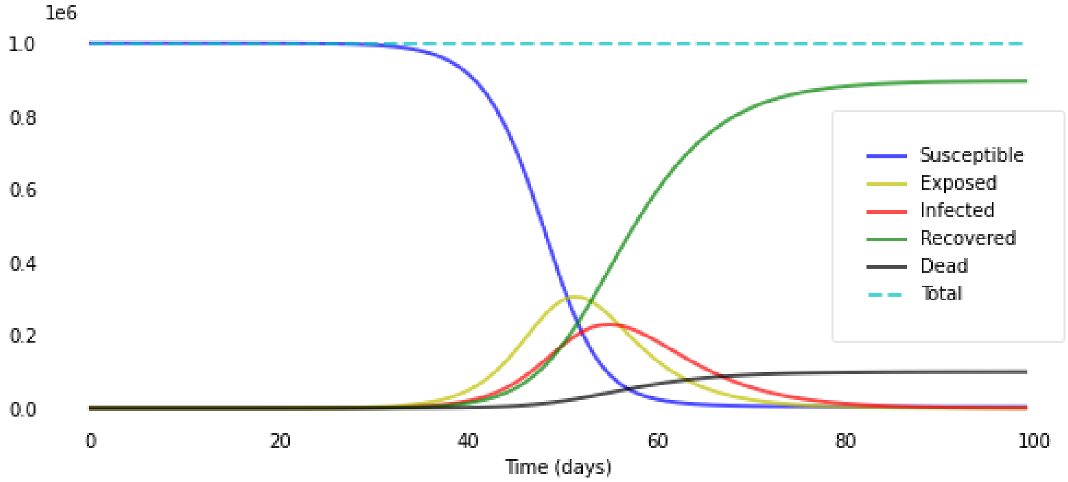
```
alpha = 0.2  # 20% death rate
rho = 1/9  # 9 days from infection until death
S0, E0, I0, R0, D0 = N-1, 1, 0, 0, 0  # initial conditions: one exposed
```

In [ ]:

```
t = np.linspace(0, 99, 100) # Grid of time points (in days)
y0 = S0, E0, I0, R0, D0 # Initial conditions vector

# Integrate the SIR equations over the time grid, t.
ret = odeint(deriv, y0, t, args=(N, beta, gamma, delta, alpha, rho))
S, E, I, R, D = ret.T
```

In [ ]:

```
plotseird(t, S, E, I, R, D)
```



Time-Dependent R_0

Simple Approach: Single Lockdown

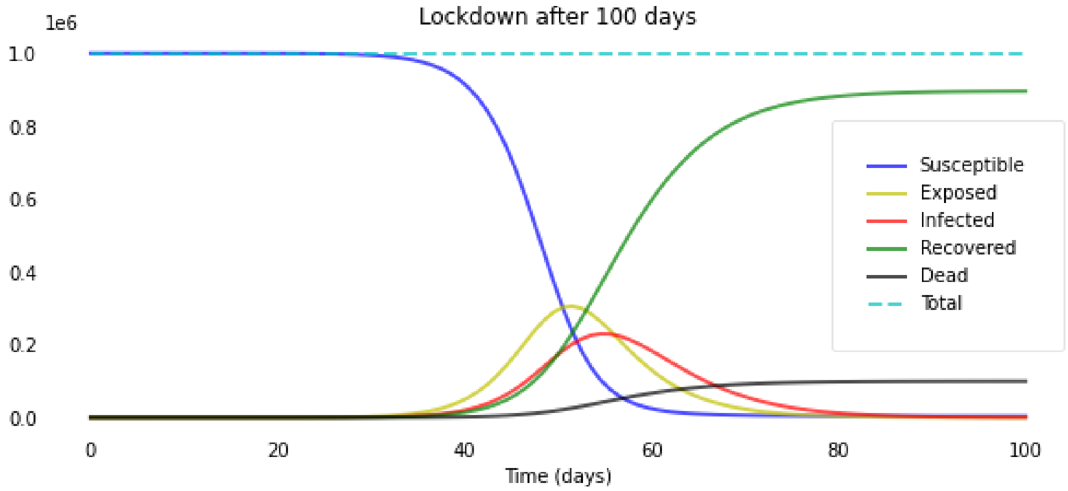
In [ ]:

```
def deriv(y, t, N, beta, gamma, delta, alpha, rho):
    S, E, I, R, D = y
    dSdt = -beta(t) * S * I / N
    dEdt = beta(t) * S * I / N - delta * E
    dIdt = delta * E - (1 - alpha) * gamma * I - alpha * rho * I
    dRdt = (1 - alpha) * gamma * I
    dDdt = alpha * rho * I
    return dSdt, dEdt, dIdt, dRdt, dDdt
```

In [ ]:

```
L = 100
N = 1000000
D = 4.0 # infections lasts four days
gamma = 1.0 / D
delta = 1.0 / 5.0  # incubation period of five days
def R_0(t):
    return 5.0 if t < L else 0.9
def beta(t):
    return R_0(t) * gamma

alpha = 0.2  # 20% death rate
rho = 1/9  # 9 days from infection until death
S0, E0, I0, R0, D0 = N-1, 1, 0, 0, 0  # initial conditions: one exposed
t = np.linspace(0, 100, 100) # Grid of time points (in days)
y0 = S0, E0, I0, R0, D0 # Initial conditions vector

# Integrate the SIR equations over the time grid, t.
ret = odeint(deriv, y0, t, args=(N, beta, gamma, delta, alpha, rho))
```

```
S, E, I, R, D = ret.T
plotseird(t, S, E, I, R, D, L)
```



In [ ]:

```

```