YSC2229: Assignment 2

Tentative Deadline: March 8, 2021 at 5pm

(Strict Deadline: No Late Submission Accepted)

Submit your work in the jupyter-notebook format via Canvas by the deadline. Include your non-code answers (in pdf) on your submitted jupyter notebook. This is an individual assignment. No collaboration with anyone is allowed. **Violation on this academic integrity is a serious offence and will be reported to the College's academic integrity committee**.

Part 1:

1. Considering the maximum subarray problem, which is discussed in the textbook Sec.4.1 (page 68), answer the following questions:
   (a) Write pseudocode for the brute-force method that runs in $O(n^2)$ time.
   (b) Write code in python based on your pseudocode.
   (c) Prove that your implemented brute force algorithm is $O(n^2)$
   (d) Write code in python to find the maximum subarray using a divide-and-conquer approach as described in the textbook.
   (e) Calculate the big-O runtime of your maximum-subarray implementation.

2. Suppose that you are given n points on a planar surface and their location, where you can assume the given locations are in form of (x, y). Regarding this:
   (a) Write pseudocode the find the closest pair using a divide-and-conquer approach.
   (b) Write code in python based on your pseudocode.
   (c) Calculate the big-O runtime of your implementation.

   Note: we learned in Exercise 2 that the runtime of a brute force approach for this problem is $O(n^2)$; hence your divide-and-conquer algorithm should be more efficient than this.

Part 2:

1. Prove or disprove the following statements using mathematical induction:
   (a) $T(n) = 2T(n/2 + 17) + n = O(n \log n)$
   (b) $T(n) = 4T(n/3) + n = \Omega(n^{\log_3 4})$
   (c) $T(n) = 4T(n/2) + n^2 = O(n^2)$

   Note: You must show the conditions of the equations (n_0 and c, or a in our discussion), particularly when you prove them true.

2. Given $T(n) = 5T(n/2) + 3$, and $T(1) = 7$, find the most possible tight big-O, and prove. Note, you must argue (or provide justification) why you think it is the tightest big-O.