ADDIS ABABA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Applied Mathematics IB (Math1041)
Exercise 1

1. Let A, B and C be non zero vectors in IR^3
   a) Is it possible $|A \cdot B| = \|A\|\|B\|$? Is it possible $\|A+B\| = \|A\| + \|B\|$? Explain!
   b) If $A \cdot B = A \cdot C$, then what can we say about B and C?
   c) If $A \times B = A \times C$, then what can we say about B and C?
   d) If $A \cdot B = A \cdot C$ and $A \times B = A \times C$, then what can we say about B and C?
   e) If $A + B + C = 0$, then show that $A \times B = B \times C = C \times A$.
2. Determine where the given vectors are coplanar or not.
   a) $2i - 3j + k, i - j, j - 3k$ b) $3i - 2j + k, 5i - 2j + k, j - k$ c) $i - 2j + k, -2i + 3k, -4j + 5k$
3. Show that the vectors $2i + j - k$, $3i + 7j + 13k$ and $20i - 29j + 11k$ are mutually orthogonal.
4. Let A and B be parallel vectors in space such that A=2i-j-2k and $\|B\| = 12$. Then find B.
5. If u and v are perpendicular unit vectors, show that $\|u - v\| = \sqrt{2}$.
6. For any vectors A and B, show that $\|A+B\|^2 \|A-B\|^2 = (\|A\|^2 + \|B\|^2)^2 - 4(A \cdot B)^2$
7. Let A and B be vectors in IR^3. Then show that $\|A \times B\|^2 = \|A\|^2\|B\|^2 - (A \cdot B)^2$.
8. Let A be a vector in the first octant such that $\|A\| = 3$ and the direction cosines with respect to $\vec{i}$ and $\vec{j}$ are 1/3 and 2/3 respectively. Find A.
9. Find two unit vectors that are parallel to the yz-plane and are perpendicular to the vector 3i-j+2k
10. Find the area and the cosines of the angles of the triangle with vertices at the points (0,0,0), (4,-1,3) and (1,2,3).
11. Show that the quadrilateral PQSR such that P=(1,-2,3), Q=(4,3,-1), R=(2,2,1) and S=(5,7,-3) is a parallelogram, and find its area.
12. Find the volume and the surface area of the parallelepiped three of whose edges are the vectors $A = \vec{i} - \vec{j} - 2\vec{k}$, $B = 3\vec{i} - 4\vec{j} + \vec{k}$ and $C = 3\vec{i} + \vec{j} + 2\vec{k}$.
13. Show that $(\frac{1}{2}, \frac{1}{3}, 0), (1,1,-1)$ and $(-2,-3,5)$ are collinear points, and find symmetric equation of the line containing them.
14. Show that the points $(-1,1,1), (0,2,1), (0,0,\frac{3}{2})$ and $(13,-1,5)$ lie on the same plane and find the equation of the plane containing them.

15. Find the equation of the line passing through the point (-2,5,-3) and perpendicular to the plane $\pi: 2x - 3y + 4z + 7 = 0$

16. Find the equation of the plane:

a) that is parallel to the z-axis and contains the points (3,-1,5) and (7,9,4).

b) that contains the point (-2,1,4) a line L: $x = 2 - 3t$, $y = 4 + 2t$, $z = 3 - 5t$.

c) that contains the point (4,0,-2) and perpendicular to each of the planes $x - y + z = 0$ and $2x + y - 4z = 5$.

17. Let π_1: $2x - 4y + 4z = 7$ and π_2: $6x + 2y - 3z = 2$ be two planes

a) Find parametric equations of the line of intersection of the planes π_1 and π_2.

b) Find the cosine of the acute angle of the intersecting planes π_1 and π_2.

18. Find the point of intersection of the line L: $x = 2 - 3t$, $y = 4 + 2t$, $z = 3 - 5t$, $t \in \mathbf{R}$ and the plane $2x + 3y + 4z = -8$.

19. Let L_1: $x = 1 + 2t$, $y = 2 - t$, $z = 4 - 2t$ and L_2: $\dfrac{x-9}{1} = \dfrac{y-5}{3} = \dfrac{z+4}{-1}$ be two lines.

a) Show that L_1 and L_2 are intersecting lines.

b) Find the cosines of the acute angle between L_1 and L_2 at their intersection point.

c) Find symmetric equations for the line that is perpendicular to L_1 and L_2 and passes through their intersection point.

20. Let L be the line $\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z+5}{7}$ and π be the plane $2x + 2y - z + 4 = 0$. Find the two points on L at a distance 3 unit from π.

21. Let L_1: $\dfrac{x+2}{5} = \dfrac{y-1}{-2} = z + 4$ and L_2: $\dfrac{x-3}{-5} = \dfrac{y+4}{2} = \dfrac{z-3}{-1}$ be two lines. Then

a) Show that L_1 and L_2 are parallel and find the distance D between L_1 and L_2.

b) Find the equation of the plane that contains L_1 and L_2.

22. Find the distance between the parallel planes: $z = x + 2y + 1$ and $3x + 6y - 3z = 4$.

23. Let L_1: $x = 3 - t$, $y = 4 + 4t$, $z = 1 + 2t$ and L_2: $x = s$, $y = 3$, $z = 2s$ be two lines

a) Show that L_1 and L_2 are skew (neither intersecting nor parallel) lines.

b) Find two planes π_1 and π_2 such that π_1 is parallel to π_2, π_1 contains L_1 and π_2 contains L_2.

c) Find the distance D of the two parallel planes π_1 and π_2 (the planes you found in (b)). (Note that this distance D is the distance between the two skew lines L_1 and L_2)

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Exercise 2

1. Let V= $\{(a, b) | a,b \varepsilon IR\}$.Define addition in V and scalar multiplication by elements of IR as follows: (a, b) + (c, d) = (a + c, b + d) and k (a, b) = (ka, 0). Show that V is not a vector space over IR with respect to the operations defined. Which axiom(s) fail to hold?
2. Determine whether W is a subspace of V or not.
   a) $W = \{(x, y) \in IR^2 : x - 3y = 0\}$, $V = IR^2$ b) $W = \{(x, y) \in IR^2 : y = x^2\}$, $V = IR^2$
   c) W= $\{(a, b, c) \varepsilon IR^3 : a - 3b + 4c = 0\}$, $V = IR^3$ d) $W = \{(x, y, z) \in IR^3 : z = 2y\}$, $V = IR^3$
   e) W= $\{(a, b, c) \in IR^3$: b is an integer$\}$, $V = IR^3$ f) $W = \{(a, b, c) : a \le b \le c\}$, $V = IR^3$
3. Determine whether the following vectors are linearly independent or not.
   a) $(1,2), (3,2)$ in IR^2 b) $(-3,6), (1,-2)$ in IR^2 c) $(-1,2), (1,-2), (3,4)$ in IR^2
   d) $(1,1,3), (0,2,1)$ in IR^3 e) $(2,1,-2), (3,2,2), (5,3,0)$ in IR^3 f) $(1,0,2), (0,-1,1), (1,3,0)$ in IR^3
4. Let $P_n = P_n(IR)$ be the vector space of all polynomials of degree less than n, $\forall n \in IN$. Determine whether the following vectors are linearly independent or not.
   a) $1, x+2, 2x-1$ in P_2 b) $x-3, 1-2x$ in P_2 c) $x^2+1, x^2-x, x+1$ in P_3
   d) $3-2x, x^2+2x-1, x^2+2$ in P_3 e) $1, 2x-3, x^2+3x-5, 2x^2+2$ in P_3
5. Let V be the set of all continuous functions on IR\
   a) Show that V is a vector space under the usual addition and scalar multiplication of functions.
   b) Show that the set $W = \{f \in V : f \text{ is differentiable function}\}$ is a subspace of V.
   c) Show that $2^x, 2^{2x}, 2^{3x}$ are linearly independent vectors in V
   d) Show that $\sin 2x, \cos 2x, \cos x \sin x$ are linearly dependent vectors in V.
6. If possible, write **u** as a linear combination of **v** and **w**.
   a) $u = (1,-2)$, $v = (3,1)$, $w = (0.-1)$ b) $u = (-3,2)$, $v = (-1,3)$, $w = (2.-6)$
   c) $u = (0,-2,1)$, $v = (1,-1,2)$, $w = (0,2,4)$ d) $u = (8,-2,1)$, $v = (0,1,5)$, $w = (0,3,-1)$
   e) **u** = $3x^2 + 8x - 5$, **v** = $2x^2 + 3x - 4$, **w** = $x^2 - 2x - 3$
7. Determine whether or not β is a basis of V. If β is a basis, find the coordinate vector of any vector v in V with respect to β.
   (a) $\beta = \{(3,-1), (2,1)\}$, $V = IR^2$ (b) $\beta = \{(5,-15), (-1,3)\}$, $V = IR^2$
   (c) $\beta = \{(1,-2,1), (-1,1,0), (0,-1,1)\}$, $V = IR^3$ (d) $\beta = \{(1,-1,0), (0,2,1), (1,2,0)\}$, $V = IR^3$
8. Show that β is a basis of V and find the coordinate vector of the given vector v with respect to β.
   a) $\beta = \{2-t, 3t-1\}$ $V = P_2$, $v = at + b$ b) $\beta = \{1, 1+t, t^2+2t-1\}$, $V = P_3$, $v = 2t^2 + 3t - 4$
   c) $\beta = \{t, t^2+1, t^3+2t^2-1, t+2\}$, $V = P_4$, $v = at^3 + bt^2 + ct + d$

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Exercise 3

1. Let A, B and C be three square matrices of the same size. Explain why each of the following statements is false.
   a) $(A+B)^2 = A^2 + 2AB + B^2$
   b) If $AC = BC$, then $A = B$
   c) det(A+B)=det(A)+det(B)
   d) $(A' + B'C)' = A + BC'$

2. a) Let $A = \begin{pmatrix} 2 & -1 & 3 \\ 1 & 5 & 0 \\ 0 & 2 & 4 \end{pmatrix}$. Find a matrix B such that $(3B' - A)' = 2I_3$.

   b) Find a matrix D such that DA = B where $A = \begin{pmatrix} 1 & -1 \\ -2 & 3 \\ 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$.

   c) Find a matrix A such that $\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} A - A \begin{pmatrix} 1 & 4 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 5 & 4 \end{pmatrix}$

3. Let $A = \begin{pmatrix} 2 & 1 \\ 1 & -5 \end{pmatrix}$ and $B = \begin{pmatrix} b & c \\ c & d \end{pmatrix}$. Find a condition on b, c and d for which $AB = BA$

4. Find the n[th] power of the following matrices
   a) $\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$
   b) $\begin{pmatrix} 4 & 5 \\ -3 & -4 \end{pmatrix}$
   c) $\begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$

5. Let $A_n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$ for $n \in \mathbb{N}$. Then show that i) $A_nA_m = A_{n+m}$ ii) $A_n^{-1} = A_{-n}$

6. Write the following matrices as a sum of symmetric and skew-symmetric matrices.
   a) $\begin{pmatrix} 2 & -3 \\ 5 & 0 \end{pmatrix}$, b) $\begin{pmatrix} 2 & -1 & 3 \\ 1 & 5 & 1 \\ 0 & -2 & 4 \end{pmatrix}$ c) $\begin{pmatrix} 0 & 2 & 1 & -3 \\ 5 & 3 & -2 & 0 \\ 0 & 2 & 1 & 4 \\ 1 & 0 & -1 & 0 \end{pmatrix}$

7. Let $A = \begin{pmatrix} 2 & a-2b+2c & 2a+b+c \\ 3 & 5 & a+c \\ 0 & -2 & 7 \end{pmatrix}$. Find the values of a, b and c for which A is symmetric matrix.

8. Let A and B be square matrices of order n. Express the following in terms of det A and det B
   a) $\det(4A'B^5)$
   b) $\det(adjA)$
   c) $\det(A^2B^{-1})$ (assuming B^{-1} exists)

9. Given that $\begin{pmatrix} 1 & -2 & -3 \\ 2 & 1 & -2 \\ 0 & 2 & 0 \end{pmatrix} A \begin{pmatrix} 1 & 0 & 3 \\ -2 & 1 & -8 \\ 3 & 0 & 7 \end{pmatrix} = 2I_3$, find det(A).

10. If $A = \begin{pmatrix} a & 1 & 1 \\ -2 & 3 & b \\ -9 & c & 0 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 7 & d & 4 \\ e & 9 & -37 \\ 29 & -4 & f \end{pmatrix}$, find the values of a, b, c, d, e, and f.

11. Find the determinant of the following matrices

a) $\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} 5 & -2 \\ 4 & -1 \end{pmatrix}$ b) $\begin{pmatrix} abc & a^2 & ac \\ bc & ac & c^2 \\ b^2 & ab & bc \end{pmatrix}$ c) $\begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix}$ d) $\begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix}$

12. Let $A = \begin{pmatrix} x+y-1 & 0 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 3 \\ 0 & 2x-3y-7 \end{pmatrix}$. Find all values of x and y for which A and B are not invertible.

13. Find the rank of the following matrices

(a) $\begin{pmatrix} -1 & 0 \\ 2 & 5 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & -3 & 1 \\ -4 & 6 & -2 \end{pmatrix}$ (c) $\begin{pmatrix} -1 & 0 & -1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \\ 2 & 3 \\ 2 & 3 \end{pmatrix}$

14. If it exists, find the inverse of the following matrices.

(a) $\begin{pmatrix} 2 & 4 \\ 6 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & 3 \end{pmatrix}$ (c) $\begin{pmatrix} 4 & -2 & 5 \\ 1 & 1 & -2 \\ 6 & 1 & -1 \end{pmatrix}$

15. Let $A = \begin{pmatrix} 2 & 2 & t \\ 0 & -t & -3 \\ t & 0 & -1 \end{pmatrix}$. Find the set of values of t for which the homogeneous system of linear equations $AX = 0$ has non trivial solution.

16. Let $M = \begin{pmatrix} a & 0 & b & 2 \\ a & a & 4 & 4 \\ 0 & a & 2 & b \end{pmatrix}$ be the augmented matrix for a linear system. For what values of a and b the system will have: a) no solution? b) unique solution? c) infinitely many solutions?

17. Solve the following system of equations using either Gauss's Method or Cramer's Rule, whenever possible.

(a) $\begin{cases} 3x-7y=2 \\ 2x-5y=-1 \end{cases}$ (b) $\begin{cases} 4x+6y+z=2 \\ 2x+y-4z=3 \\ 3x-2y+5z=8 \end{cases}$ (c) $\begin{cases} 3x+3y+4z=3 \\ 4x+6z=2 \\ 8x+z=2 \end{cases}$

(d) $\begin{cases} 3x-9y=1 \\ -2x+6y=-3 \end{cases}$ (e) $\begin{cases} 2x+3y+z=1 \\ x+4y-z=0 \end{cases}$ (f) $\begin{cases} x+y+2z-w=-2 \\ 2x+y-2z-2w=-2 \\ 3x-3w=-3 \end{cases}$

17. Find the characteristic polynomial, eigenvalues and eigenvectors of

a) $A = \begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix}$ b) $A = \begin{pmatrix} 3 & 4 \\ 1 & 3 \end{pmatrix}$ c) $B = \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix}$ d) $B = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ -2 & 0 & -1 \end{pmatrix}$

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Exercise 4

1. How large must a δ be chosen so that

   a) $0<|x-2|<\delta \Rightarrow \left|\sqrt{x+2}-4\right|<0.001$ b) $0<|x-2|<\delta \Rightarrow \left|\dfrac{1}{x-1}-1\right|<0.001$

2. Using $\varepsilon-\delta$ definition of a limit, show the following

   a) $\lim\limits_{x\to -2}(7-2x)=11$ b) $\lim\limits_{x\to 5}(x^2-3x)=10$ c) $\lim\limits_{x\to 1}\left(\dfrac{x+1}{x+2}\right)=\dfrac{2}{3}$ d) $\lim\limits_{x\to 0}\sin 3x=0$

3. If it exists, evaluate following limits

   a) $\lim\limits_{x\to 0}h^2\left(1+\dfrac{1}{h^2}\right)$ $= 1$ b) $\lim\limits_{y\to 1}\left(\dfrac{y^3-1}{\sqrt{y}-1}\right)$ c) $\lim\limits_{x\to 8}\dfrac{\sqrt{7+\sqrt[3]{x}}-3}{x-8}$

   d) $\lim\limits_{y\to 8}\left(\dfrac{y^{\frac{1}{3}}-2}{y-8}\right)$ e) $\lim\limits_{x\to -2}\left(\dfrac{x^2-4}{x^3-8}\right)$ $=0$ f) $\lim\limits_{x\to 0}\dfrac{x^2}{\sec x-1}$

   g) $\lim\limits_{x\to 0}\dfrac{2x^2+x}{\sin x}$ h) $\lim\limits_{x\to 0}x^2(1+\cot^2 3x)$ i) $\lim\limits_{x\to \pi}\left(\dfrac{\sin^3 x+\sin x\cos x^2}{x-\pi}\right)$

   j) $\lim\limits_{x\to \infty}\left(\dfrac{2+\sin^2 x}{x^2}\right)$ k) $\lim\limits_{x\to \infty}\left(1-\dfrac{1}{3x}\right)^{2x+1}$ l) $\lim\limits_{x\to \infty}\dfrac{e^x-e^{-x}}{e^{2x}-e^{-2x}}$

   m) $\lim\limits_{x\to -\infty}\left(\dfrac{3x+4}{\sqrt{2x^2-5}}\right)$ n) $\lim\limits_{t\to \infty}\left(\sqrt{t^2+t}-\sqrt{t^2+4}\right)$ o) $\lim\limits_{x\to \infty}\dfrac{(x^3+1)^{\frac{1}{3}}}{\sqrt{x^2-3}}$

   p) $\lim\limits_{x\to \infty}\dfrac{3^x-5^x}{3^x+5^x}$ q) $\lim\limits_{x\to -1}\left(\dfrac{|x+2|-1}{1-|x|}\right)$ r) $\lim\limits_{x\to 0}\dfrac{|x|^3-x^2}{x^3+x^2}$

4. Investigate the one sided limits at the given point a and decide whether or nor $\lim\limits_{x\to a}f(x)$ exists?

   a) $f(x)=[x]+[-x]$: a= 2 b) $f(x)=\dfrac{|2+x|-|x|-2}{x}$: a = -2 , 0

   c) $f(x)=\begin{cases}\dfrac{|x+4|}{x-4} & for\ x\neq 4\\ 1 & for\ x=4\end{cases}$ a=4 d) $f(x)=\dfrac{4x+|x|}{5x-3|x|}$: a=0

   e) $f(x)=(x-1)[x]$: a=1 f) $f(x)=[1-x^2]$: a=2

5. Using squeezing theorem evaluate

   a) $\lim\limits_{x\to \pi}(x-\pi)\cos^2\left(\dfrac{1}{x-\pi}\right)$ b) $\lim\limits_{x\to 0}\sin x\cos\dfrac{1}{x}$ c) $\lim\limits_{x\to 0}x^4\sin\dfrac{1}{\sqrt[5]{x^3}}$

   d) $\lim\limits_{x\to -2}g(x)$, if $|g(x)-3|<5(x+2)^2\ \forall x$ e) $\lim\limits_{x\to \frac{-\pi}{2}}f(x)$, if $4-2\sin x\leq 2f(x)+4\leq 8+2\sin x\ \forall x\in(-\pi,0)$

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6. Find all vertical asymptotes of graph of f.

   a) $f(x)=\dfrac{x^2-5x+6}{x^3-8}$ b) $f(x)=\dfrac{8x-2x^2}{x^2-16}$ c) $f(x)=\dfrac{\sin(x^2-1)}{x^3-x}$

7. Find all horizontal asymptotes of graph of f.

   a) $f(x)=\dfrac{x^2-5x+6}{x^3-8}$ b) $f(x)=\dfrac{(1-x^2)(x+1)}{x^2(1-2x)}$ c) $f(x)=\dfrac{x}{\sqrt{x^2+1}}$

8. Show each of the following.

   a) $\lim\limits_{x\to a} f(x)=0$ if and only if $\lim\limits_{x\to a}|f(x)|=0$. B) If $\lim\limits_{x\to a} f(x)=L$, then $\lim\limits_{x\to a}|f(x)|=|L|$.

9. a) Suppose $\lim\limits_{x\to a} f(x)=L\in IR$ and $\lim\limits_{x\to a}(f(x)g(x))=1$. What can we say about $\lim\limits_{x\to a} g(x)$?

   b) If $\lim\limits_{x\to a}(f(x)+g(x))$ and $\lim\limits_{x\to a} f(x)$ exist, what can we say about $\lim\limits_{x\to a} g(x)$?

10. Prove that if there is a number M such that $\left|\dfrac{f(x)-L}{x-a}\right|\le M, \forall x\ne c$, then $\lim\limits_{x\to a} f(x)=L$.

11. If $\lim\limits_{x\to a} f(x)=0$ and $|g(x)|\le M\ \forall x\ne a$, then show that $\lim\limits_{x\to a}(fg)(x)=0$.

12. Let $f(x)=\begin{cases} ax-b & x\le 1 \\ 3x & 1<x<2 \\ bx^2-a & 2\le x \end{cases}$. Find numbers a and b such that f is continuous on IR.

13. Find the value of k so that $f(x)=\begin{cases} \dfrac{\cos x-1}{\sin 2x}, & x\neq 0 \\ k, & x=0 \end{cases}$ is continuous at 0.

14. Determine for which of the following functions we can define f(a) so as to make f continuous at a.

    a) $f(x)=\dfrac{\tan 3x}{2x^2-5x}; a=0$ b) $f(x)=\dfrac{x^2-1}{|x-1|}$; a=1 c) $f(x)=\begin{cases} x^2+1 & for\ x<-1 \\ x & for\ x>-1 \end{cases}$; a= -1

15. Using the intermediate value theorem show that the equation has a root in the given interval.

    a) $2x^3-4x^2+5x=4$, $[1,2]$ b) $2\cos x=x^2$, $[0,\frac{\pi}{2}]$ c) $\tan x=1-x$, $[0,\frac{\pi}{4}]$

16. Show that if f is continuous and $0\le f(x)\le 1\ \forall x\in[0,1]$, then there exists at least one number c in $[0,1]$ for which $f(c)=c$. (**Hint** apply IVT on $g(x)=x-f(x)$)

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Exercise 5

1. Find the set of all points where the derivative exists for

   a. $f(x)=|\sin x|$ b. $f(x)=[x]$ c. $f(x)=\begin{cases}\sqrt{x} & for\, 0\le x\le 1\\ x^2 & else\, where\end{cases}$ d. $f(x)=\begin{cases}x\cos(\frac{1}{x}), & x\neq 0\\ 0 & ,x=0\end{cases}$

2. If f and g are two functions with $f(a)=1, f'(a)=2, g(a)=-1$ and $g'(a)=3$, then find $(fg+f)'(a)$ and $\left(\frac{f}{f-g}\right)'(a)$

3. Let f be a differentiable function such that $f'(x)=e^x$, then find $\frac{d}{dx}(f(\ln(\ln 2x)))$

4. Find the equation of the tangent line and the normal line to the graph of $f(x)=1-(x+1)^{\frac{1}{3}}$ at $(-1,f(-1))$.

5. Find the points where the tangent line is parallel to the x-axis for the curve, $25y^2+12xy+4x^2=1$

6. Find the point on the curve $y=x^3+x^2+x$ where the tangent line is parallel to $y=2x+3$.

7. Find the equations for the tangent line to the ellipse $4x^2+y^2=72$ that are perpendicular to the line $x+2y+3=0$

8. Find the equation for the normal line to the hyperbola $4x^2-y^2=36$ that are parallel to the line $2x+5y-4=0$

9. Let $f(x)=x^2|x|$. Show that $f''(x)$ is continuous at 0 but not differentiable at 0.

10. Discuss the continuity and differentiability of each function at the given point a

    a. $f(x)=\sqrt[3]{x}-x$ at a = 0 b. $f(x)=\begin{cases}\frac{\sqrt{x-1}}{x-1} & for\, x\neq 1\\ 1/2 & for\, x=1\end{cases}$ at a = 1 c. $f(x)=\begin{cases}x^2\sin(\frac{1}{x}) & if\, x\neq 0\\ 0 & if\, x=0\end{cases}$ at a=0

11. Show that the derivative of even function is odd and vice versa.

12. Assuming $f'(a)$ exists express the following in terms of $f'(a)$

    a. $\lim\limits_{h\to 0}\frac{f(a-h)-f(a)}{h}$ b. $\lim\limits_{h\to 0}\frac{f(a+h)-f(a-h)}{h}$ c. $\lim\limits_{x\to a}\frac{f(x)-f(a)}{\sqrt{x}-\sqrt{a}}, a>0$

13. Using implicit differentiation find the first and second derivative at the indicated point.

    a. $\sqrt{x}+\sqrt{y}=3$ at $(1,4)$ b. $\frac{y^2}{x+y}=1-x^2$ $(0,1)$ c. $y^2\sin 2x=-2y$ $(\frac{\pi}{4},-2)$

14. Find $f'(x)$ at each point where the derivative exists.

a. $f(x)=\dfrac{-x}{(x.-1)^2}$ b. $f(x)=xe^{-\ln x}$ c. $f(x)=x+|x|$

d. $f(x)=(x-7)|x-7|$ e. $f(x)=\dfrac{e^x}{\sqrt{3^{2x}-1}}$ f. $f(x)=\log_{x^2}(x^2e^x)$

g. $f(x)=(\ln x)^x$ h. $f(x)=3^{2x}\log_3(x^2+3)$ i. $f(x)=x^{x^2}$

15. Given $f(x)=\begin{cases}\cos x & x\geq 0\\ ax+b & x<0\end{cases}$ find the values of a and b such that $f'(0)$ exists

16. Given $f(x)=\begin{cases}x^3 & if\ x<1\\ ax^2+bx+c & if\ x\geq 1\end{cases}$ find the values of a and b such that $f''(1)$ exists.

17. Find $(f^{-1})'(c)$ where i) $f(x)=x+\sqrt{x}$ and $c=2$ ii) $f(x)=2x+\sin x$ and $c=0$.

18. Assuming y is a differentiable function of x find $\dfrac{dy}{dx}$.

a. $xy^3+\tan(x+y)=1$ b. $\sec(x+2y)+\cos(x-2y)=2$

c. $-7x^2+48xy+7y^2=25$ d. $\sin^{-3}(xy)+\cos(x+y)+x=5$

e. $y\sinh^{-1}x+e^y=y^5$

19. Evaluate : a) $\cot(\arccos(-0.5))$ b) $\sin(2\tan^{-1}3)$ c) $\cos(\arccos(0.02))$ d) $\arctan(\tan\frac{5\pi}{3})$

20. If $\sinh a=\frac{4}{3}$, find the value of the other hyperbolic functions at a.

21. Find the domain and the derivative of the following functions

a) $f(x)=x\arcsin x$ b) $f(x)=\cosh\sqrt{x+1}$ c) $f(x)=\sec^{-1}(x^2-1)$

22. Find the 3rd derivative of the following functions.

a. $f(x)=x^2e^{-3x}$ b. $f(x)=\ln(\ln(5x))$ c $f(x)=\cos^2x-e^{\tan x}$

23. Find a formula for the n^{th} derivative of the following functions.

a. $f(x)=x\sin x$ b. $f(x)=\ln(x+1)$ c. $f(x)=xe^x$

24. Find the value of the constant A so that $y=A\sin 3t$ satisfies the equation $\dfrac{d^2y}{dt^2}+2y=4\sin 3t$

Academic Year 2013/14, Semester I

ADDIS ABABA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Applied Mathematics IB (Math 2241) Extension Program
Exercise 6

1. Suppose g is the inverse of a differentiable function f and let G(x) = 1/g(x). If f(3) = 2 and f$'$(3) = 1/9 then find G$'$(2).
2. Let $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2 + \frac{3}{2}$. Find intervals on which f has inverse and find the corresponding derivatives.
3. Let $f(x) = \ln\left(\frac{\pi}{2} + \tan^{-1} x\right)$. Show that f^{-1} exists, and find a formula for f^{-1}. What is domain and range of f^{-1}?
4. Explain why the Mean Value Theorem (MVT) does not apply fo $f(x) = x^{2/3}$ on [-1,1]
5. Using the MVT show that
   a. if $|f'(x)| \le 1\ \forall x$ in some interval I, then $|f(x_1) - f(x_2)| \le |x_1 - x_2|\ \forall x_1, x_2 \in I$
   b. $|\sin x| \le |x|\ \forall x \in \mathbb{R}$.
6. Find the asymptote(s), intervals of monotonicity, critical points, the local extreme points, intervals of concavity and inflection point(s) of the following functions. Sketch the graph of each.
   a) $f(x) = -3x^4 + 4x^3$ b) $f(x) = |x^2 + x - 2|$ c) $f(x) = (x-2)^{2/3}$
   d) $f(x) = x^2 + \frac{2}{x}$ e) $f(x) = \frac{x^2}{x-2}$ f) $f(x) = \frac{x^2 - 6x}{(x+1)^2}$
   g) $f(x) = x^2e^{-x}$ h) $f(x) = |x-2| + |x-4|$ i) $f(x) = x\sqrt{1-x}$
7. Of all the triangles that pass through the point (1,1) and have sides lying on the coordinate axes, one has the smallest area. Determine the lengths of its sides.
8. A ladder is to reach over a fence 8 feet high to a wall 1 foot behind the fence. What is the length of the shortest ladder that can be used?
9. A rectangle of the greatest possible area is inscribed in a triangle whose base is a and altitude h. Determine the area of the rectangle.
10. Find the dimension of the right circular cylinder of the largest volume that can be inscribed in a sphere of radius 10 units.
11. Find the point on the graph of the line $y^2 = 4x$ which is nearest to the point (2,1).
12. Evaluate the limit, if it exists.
    a) $\lim_{x \to 0} \frac{x^2 - x}{\tan x}$ b) $\lim_{x \to 3} \frac{x^n - 3^n}{x - 3}$ c) $\lim_{x \to 0} \frac{\sin 2x}{e^x - 1}$
    d) $\lim_{x \to \infty} \frac{(\ln x)^3}{x - 1}$ e) $\lim_{x \to \pi/2} \frac{\tan x + 3}{\sec x - 1}$ f) $\lim_{x \to 0} (\frac{1}{x} - \csc x)$
    g) $\lim_{x \to 0} (x + e^{2x})^{1/x}$ h) $\lim_{x \to 0^+} x^{\sin x}$ i) $\lim_{x \to -\infty} (\frac{x}{x+1})^{-2x}$