What has been done?
Implementation of TransE ,TransE_fifth_triple, dismult, dismult_fifth_triple, TransH

TransE: for a triple (h,r,t), we have the vector representation (**h,r,t**) where bold symbols denote vector representation of head, relation and tail.

For example, the vector **h** is a d-dimensional vector i.e. **h** = $[h^1, h^2, \ldots, h^d]$, where h^i is the ith element of embedding vector of h (ith entry).

TransE Score: $||\mathbf{h} + \mathbf{r} - \mathbf{t}||$

TransH: this model uses embedding vectors of a triple i.e., (**h,r,t**) plus additional vector w_r for each relation r which is actually a normal vector of relation-specific hyperplane.
TransH projects each entity into the hyperplane by the following formula

$\mathbf{h_p = h - w_r^T h\, w_r}$
$\mathbf{t_p = t - w_r^T t\, w_r}$

Where $\mathbf{h_p}$, $\mathbf{t_p}$ are the projected vector of head and tail on the hyperplane denoted by normal vector of $\mathbf{w_r}$

TransH Score: $||\mathbf{h_p} + \mathbf{r} - \mathbf{t_p}||$

To be implemented

Model 1: Element-wise TransH (EW-TransH):

In TransH, we had a plain vector for head i.e., **h** = $[h^1, h^2, \ldots, h^d]$. In **element-wise TransH**, we have vector of vectors for embeddings of head, relation and tail i.e.,

***** how we will generate vectors of vector this part is not clear to me******

h_3*d

h = $[[h^{11}, h^{12}, h^{13}];[h^{21}, h^{22}, h^{23}];\ldots;[h^{d1}, h^{d2}, h^{d3}]]$
r = $[[r^{11}, r^{12}, r^{13}];[r^{21}, r^{22}, r^{23}];\ldots;[r^{d1}, r^{d2}, r^{d3}]]$
t = $[[t^{11}, t^{12}, t^{13}];[t^{21}, t^{22}, t^{23}];\ldots;[t^{d1}, t^{d2}, t^{d3}]]$

For simplicity of notation, I show **h** in the following form

$\mathbf{h} = [H^1;H^2;\ldots; H^d]$, Where, $H^i = [h^{i1} , h^{i2},h^{i3}]$, i=1,...,d.

$\mathbf{r} = [R^1;R^2;\ldots; R^d]$,

$R^i = [r^{i1} , r^{i2},r^{i3}]$, i=1,...,d.

$\mathbf{t} = [T^1;T^2;\ldots; T^d]$,

$T^i = [t^{i1} , t^{i2},t^{i3}]$, i=1,...,d.

Similar to TransH, we have relation-specific hyperplanes as follows:
$\mathbf{w_r} = [W^1;W^2;\ldots; W^d]$,

$W^i = [W^{i1} , W^{i2},W^{i3}]$, i=1,...,d.

Now, in each dimension, we have one TransH:

$\mathbf{H^i_p} = H^i - W^{i\wedge T} H^i W^i$, i=1,...,d
$\mathbf{T^i_p} = T^i - W^{i\wedge T} T^i W^i$, i=1,...,d

$Score_i = || \mathbf{H^i_p} + R^i - \mathbf{T^i_p}||$, i=1,...,d

$Score_{(h,r,t)} = Score_1 + \ldots + Score_d$

Model 2: Spatio-Temporal element-wise TransH (STEW-TransH):

Similar to previous the previous part (**element-wise TransH), we have**

$\mathbf{h} = [[h^{11} , h^{12},h^{13}];[h^{21} , h^{22},h^{23}];\ldots; [h^{d1} , h^{d2},h^{d3}]]$
$\mathbf{r} = [[r^{11} , r^{12},r^{13}];[r^{21} , r^{22},r^{23}];\ldots; [r^{d1} , r^{d2},r^{d3}]]$
$\mathbf{t} = [[t^{11} , t^{12},t^{13}];[t^{21} , t^{22},t^{23}];\ldots; [t^{d1} , t^{d2},t^{d3}]]$

Because we work on fiftiple (h,r,t, loc, tim), we have also location and time vectors as follows

$\mathbf{loc} = [[l^{11} , l^{12},l^{13}];[l^{21} , l^{22},l^{23}];\ldots; [l^{d1} , l^{d2},l^{d3}]]$
$\mathbf{tim} = [[m^{11} , m^{12},m^{13}];[m^{21} , m^{22},m^{23}];\ldots; [m^{d1} , m^{d2},m^{d3}]]$

For simplicity of illustration, we show the above vectors as follows

$\mathbf{h} = [H^{11};H^{21};\ldots; H^{d1}]$,

$H^{i} = [h^{i1}, h^{i2}, h^{i3}]$, i=1,..., d.

$\mathbf{r} = [R^{1};R^{2};\ldots; R^{d}]$,

$R^{i} = [r^{i1}, r^{i2}, r^{i3}]$, i=1,...,d.

$\mathbf{t} = [T^{1};T^{2};\ldots; T^{d}]$,

$T^{i} = [t^{i1}, t^{i2}, t^{i3}]$, i=1,..., d.

$\mathbf{loc} = [L^{1};L^{2};\ldots; L^{d}]$,

$L^{i} = [l^{i1}, l^{i2}, l^{i3}]$, i=1,... , d.

$\mathbf{tim} = [M^{1};M^{2};\ldots; M^{d}]$,

$M^{i} = [m^{i1}, m^{i2}, m^{i3}]$, i=1,..., d.

Step 1: Cross product between time and location:

In ith element, the cross product (shown by *) of L^{i} and M^{i} is computed as follows:

$W^{i} = L^{i} * M^{i} = [(l^{i2} m^{i3} - l^{i3} m^{i2}), (l^{i3} m^{i1} - l^{i1} m^{i3}), (l^{i1} m^{i2} - l^{i2} m^{i1})]$

For each dimension i=1,...,d compute the above equation

Step 2: Compute $\mathbf{w}_{loctime} = [W^{1};W^{2};\ldots; W^{d}]$,

Step 3: Project head and tail into location-time plane as follows

$\mathbf{H^{i}_{p}} = H^{i} - W^{i\wedge T} H^{i} W^{i}$, i=1,..., d
$\mathbf{T^{i}_{p}} = T^{i} - W^{i\wedge T} T^{i} W^{i}$, i=1,..., d

Step 4: we now have projected head and tail in the location-time plane. Therefore, we can compute per dimension score as follows

Score $_i$ = || $\mathbf{H}^i_p$ **+** R ^{i} - $\mathbf{T}^i_p$||, i=1,... ,d

Step 5: we can compute not the total score as follows

Score = $Score_1$ + …+ $Score_d$

Score shows the score of fiftiple (h,r,t, loc, time)

Model 3: Temporal Matrix Mapping TransR (TMM-TransR):

For each fiftiple (h,r,t, loc, time), we have the following vectors:

$\mathbf{h} = [h^1 , h^2, \dots, h^d]$
$\mathbf{r} = [r^1 , r^2, \dots, r^d]$
$\mathbf{t} = [t^1 , t^2, \dots, t^d]$

$\mathbf{loc} = [l^1 , l^2, \dots, l^d]$
$\mathbf{tim} = [m^1 , m^2, \dots, m^d]$.

Step 1:
We compute location-time matrix as follows

$Q_{loctime}$ = **eye** + (**loc**^{T} X **Tim). eye** is the identity matrix (a d X d matrix whose diagonal is 1, and non-diagonal entries are zeros), X is multiplication between two vectors which results in a (d X d) matrix. Therefore, $Q_{loctime}$ is a (d X d) matrix.

Step 2: Compute matrix projected head and tail based on location-time matrix as follows:

$\mathbf{h}_p = Q_{loctime}\, \mathbf{h}^T$
$\mathbf{t}_p = Q_{loctime}\, \mathbf{t}^T$

Step 3: compute score as follows:

Score: $||\mathbf{h}_p + \mathbf{r} - \mathbf{t}_p||$

Background about Complex for model 4:
The above mentioned models are translation based models which have been derived from the baseline TransE.
Now we want to extend the already proposed model ComplEx to fiftiple. Before introducing the fiftiple version of ComplEx, I recall the base model ComplEx working on triples.

Complex numbers: $x = x_R + i\,x_I$
x_R is real part of x, and x_I is imaginary part of x.

If we have two complex number x, y, the complex product (denoted by * here) is computed as follows:
$x*y = (x_R + i\,x_I) * (y_R + i\,y_I) = (x_R\,y_R - x_I\,y_I) + i\,(x_R\,y_I + x_I\,y_R)$.

Therefore, a complex vector is shown as

$\mathbf{x} = [x_R^1 + ix_I^1, x_R^2 + ix_I^2, \ldots, x_R^d + ix_I^d]$, or equivalently, we can show that by

$\mathbf{X} = \mathbf{x_R} + i\mathbf{x_I}$
$\mathbf{x_R} = [x_R^1, x_R^2, \ldots, x_R^d]$
$\mathbf{x_I} = [x_I^1, x_I^2, \ldots, x_I^d]$.

ComplEx:

The ComplEx model considers entities and relations as complex vectors as follows:

$\mathbf{h} = [h_R^1 + ih_I^1, h_R^2 + ih_I^2, \ldots, h_R^d + ih_I^d]$,
$\mathbf{t} = [t_R^1 + it_I^1, t_R^2 + it_I^2, \ldots, t_R^d + it_I^d]$,
$\mathbf{r} = [r_R^1 + ir_I^1, r_R^2 + ir_I^2, \ldots, r_R^d + ir_I^d]$

ComplEx Score: Re(**h*r*Conj(t)**),

Conj(t) = $[t_R^1 - it_I^1, t_R^2 - it_I^2, \ldots, t_R^d - it_I^d]$,

Mirza already implemented ComplEx, please check his code!

Model 4: Spatio-Temporal ComplEx (ST-ComplEx):

We have fiftiple (h,r,t, loc, time),
We have embedding vectors in complex space as follows

h = $[h_R^1 + ih_I^1 , h_R^2 + ih_I^2, \ldots, h_R^d + ih_I^d]$,
t = $[t_R^1 + it_I^1 , t_R^2 + it_I^2, \ldots, t_R^d + it_I^d]$,
r = $[r_R^1 + ir_I^1 , r_R^2 + ir_I^2, \ldots, r_R^d + ir_I^d]$

loc = $[l_R^1 + il_I^1 , l_R^2 + il_I^2, \ldots, l_R^d + il_I^d]$,
tim = $[m_R^1 + im_I^1 , m_R^2 + im_I^2, \ldots, m_R^d + im_I^d]$.

Step1: Complex product of head by time:

$\mathbf{h_{tim} = h * tim}$

Step2: Complex product of tail by location:
$\mathbf{t_{loc} = t * loc}$

Step 3: compute score of the fiftiple as follows

Spatio-Temporal ComplEx Score: Re($\mathbf{h_{tim}*r*Conj(t_{loc})}$),

******************* Results of the proposed score function *********************

************** transE orginal *********************

#

epoch = 200, learning rate = .1, margin = 100, last lose = 0.0165

Mean Rank: 327, 324
Mean RR: 0.3893, 0.6621
Hit@1: 0.2631, 0.6373
Hit@3: 0.4606, 0.6744
Hit@5: 0.5550, 0.6906
Hit@10: 0.6510, 0.7083

****

transE fifth triple

epoch = 200 , learning rate = .1 , margin = 100 , last lose = 0.0310

Mean Rank: 117, 115
Mean RR: 0.5202, 0.7016
Hit@1: 0.4177, 0.6458
Hit@3: 0.5504, 0.7274
Hit@5: 0.6535, 0.7675
Hit@10: 0.7670, 0.8194

*****************dismult orginal *********************************

epoch = 200 , learning rate = .1 , margin = 100 , last lose = 0.0683

Mean Rank: 1708, 1706
Mean RR: 0.0023, 0.0023
Hit@1: 0.0005, 0.0005
Hit@3: 0.0010, 0.0010
Hit@5: 0.0015, 0.0015
Hit@10: 0.0031, 0.0031

***************** dismult fifth triple *******************
# epoch = 200 , learning rate = .1 , margin = 100 , last lose = 0.0362

Mean Rank: 1788, 1787
Mean RR: 0.0023, 0.0023
Hit@1: 0.0008, 0.0008
Hit@3: 0.0018, 0.0018
Hit@5: 0.0018, 0.0018
Hit@10: 0.0023, 0.0023

******************** transH element for triple ******************************
#

epoch = 200 , learning rate = .1 , margin = 100 , last lose = 0.0157
Mean Rank: 341, 339
Mean RR: 0.3992, 0.6795
Hit@1: 0.2762, 0.6638
Hit@3: 0.4676, 0.6857
Hit@5: 0.5635, 0.6942
Hit@10: 0.6507, 0.7073

******************** transH element for fifth triple *******************************
#

epoch = 200 , learning rate = .1 , margin = 100 , last lose =   0.0304
Mean Rank: 323, 322
Mean RR: 0.4437, 0.6098
Hit@1: 0.3269, 0.5520
Hit@3: 0.5111, 0.6466
Hit@5: 0.6024, 0.6821
Hit@10: 0.6818, 0.7163

******************** compleX for triple *******************************
# epoch = 200 , learning rate = 0.1 , margin = 100 , last lose = 0.0357

Mean Rank: 707, 705
Mean RR: 0.3647, 0.6012
Hit@1: 0.2608, 0.5869
Hit@3: 0.4293, 0.6106
Hit@5: 0.5054, 0.6155
Hit@10: 0.5766, 0.6219

******************** compleX for fifth triple *******************************
# epoch = 200 , learning rate = 0.1 , margin = 100 , last lose = 0.0259
Mean Rank: 1629, 1629
Mean RR: 0.0043, 0.0043
Hit@1: 0.0021, 0.0021
Hit@3: 0.0031, 0.0031
Hit@5: 0.0041, 0.0041
Hit@10: 0.0057, 0.0057

******************** TransR for fifth triple *******************************
# epoch = 200 , learning rate = 0.1 , margin = 100 , last lose = 0.0037

Mean Rank: 1701, 1700
Mean RR: 0.0021, 0.0021
Hit@1: 0.0000, 0.0000
Hit@3: 0.0005, 0.0005
Hit@5: 0.0013, 0.0013
Hit@10: 0.0015, 0.0015

From the above result we can say TransR , compleX for fifth triple , dismult for fifth triple are not performing well

*********************** Working for the improvement of those model **************************

******************** Dismult Orginal *******************************

It perform well after adding the Regulariation

epoch = 200 , learning rate = 0.1 , margin = 100 , last lose = 0.1400

```
Mean Rank: 688, 685
Mean RR: 0.2877, 0.4261
Hit@1: 0.1839, 0.3570
Hit@3: 0.3387, 0.4699
Hit@5: 0.4223, 0.5069
Hit@10: 0.5157, 0.5584
```