



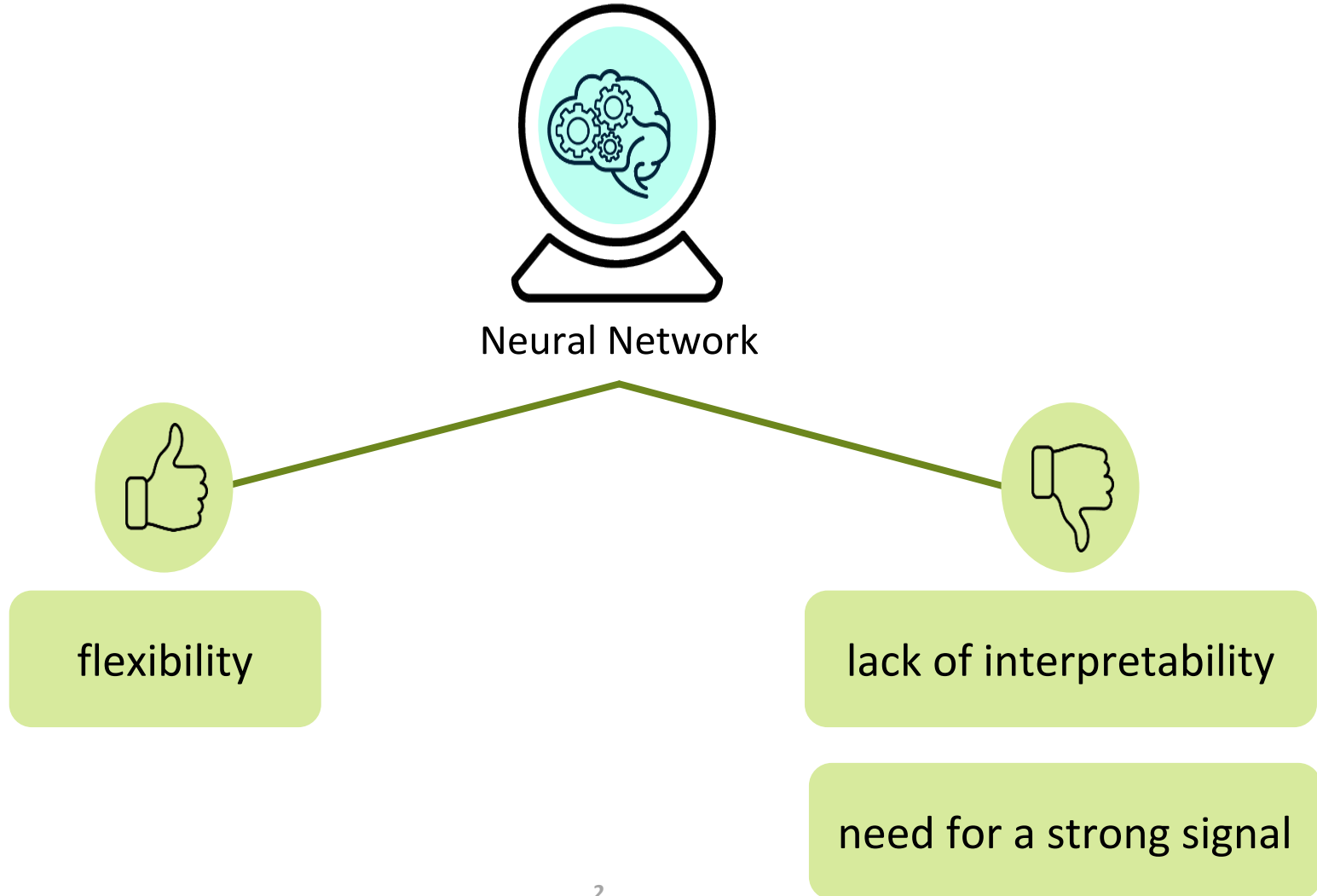
$$\overbrace{M^R \cdot M^b \cdot M^s}^M = \begin{pmatrix} YAY \\ \text{out} \end{pmatrix}$$

$$\begin{pmatrix} b & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} b & 1 \\ 1 & 0 \end{pmatrix} = M^R M^b M$$

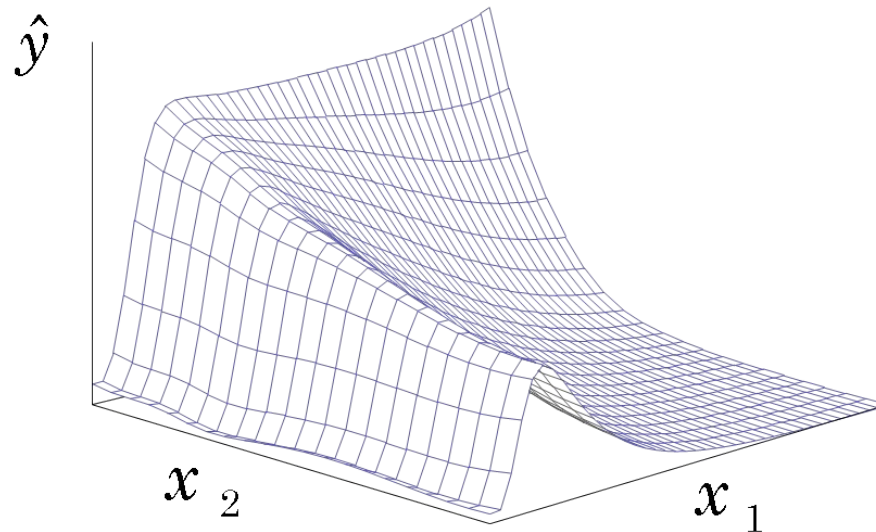
Lesson XII : Neural Network

Advantages and Disadvantages of Neural Networks



Universal Approximator

Given enough neurons and time, a neural network can model any input/output relationship, to any degree of precision.



Neural Network Prediction Formula

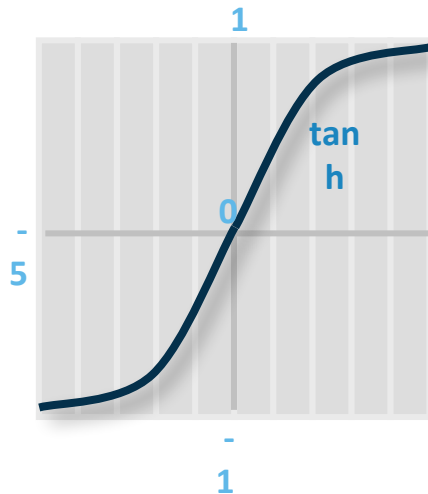
prediction estimate

hidden unit

bias estimate

weight estimate

$$\hat{y} = \hat{w}_{00} + \hat{w}_{01} H_1 + \hat{w}_{02} H_2 + \hat{w}_{03} H_3$$



$$H_1 = \tanh(\hat{w}_{10} + \hat{w}_{11} x_1 + \hat{w}_{12} x_2)$$

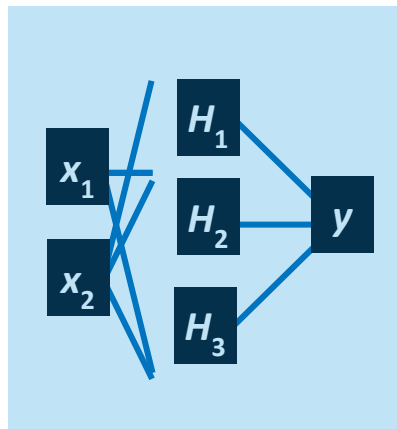
$$H_2 = \tanh(\hat{w}_{20} + \hat{w}_{21} x_1 + \hat{w}_{22} x_2)$$

$$H_3 = \tanh(\hat{w}_{30} + \hat{w}_{31} x_1 + \hat{w}_{32} x_2)$$

activation function

Neural Network Diagram

$$\log\left(\frac{\hat{p}}{1-\hat{p}}\right) = \hat{w}_{00} + \hat{w}_{01} H_1 + \hat{w}_{02} H_2 + \hat{w}_{03} H_3$$



input hidden target
layer layer layer

$$H_1 = \tanh(\hat{w}_{10} + \hat{w}_{11} x_1 + \hat{w}_{12} x_2)$$

$$H_2 = \tanh(\hat{w}_{20} + \hat{w}_{21} x_1 + \hat{w}_{22} x_2)$$

$$H_3 = \tanh(\hat{w}_{30} + \hat{w}_{31} x_1 + \hat{w}_{32} x_2)$$

Prediction Illustration: Neural Networks

1. Standardize (scale) the input variables.

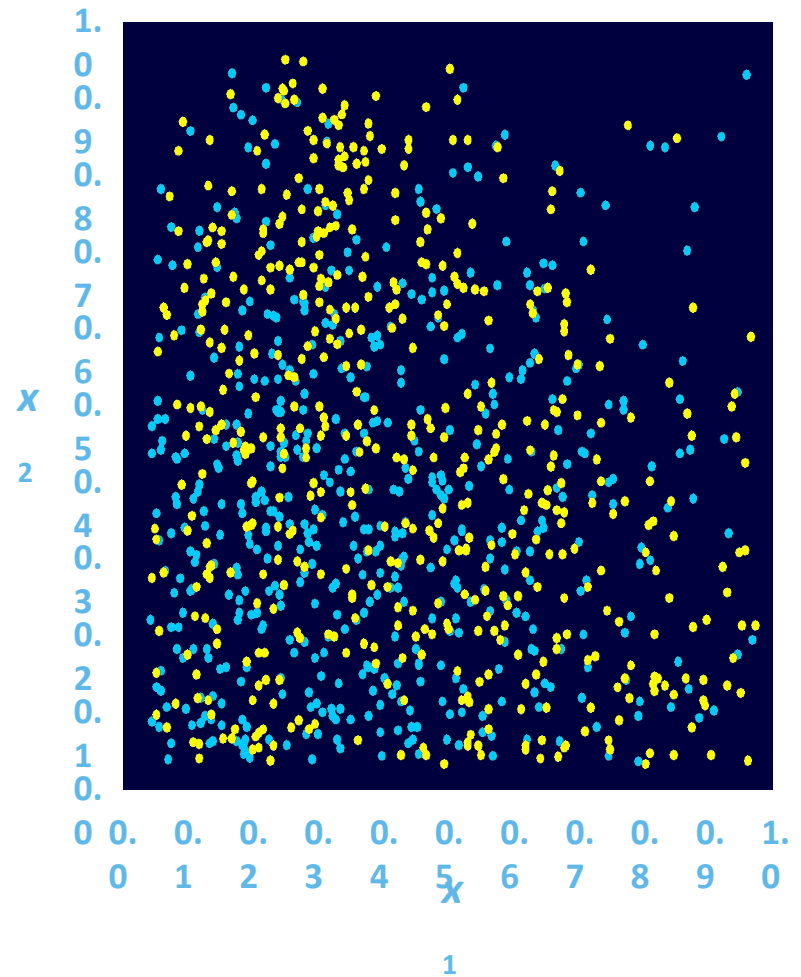
logit

$$\text{logit}(\hat{p}) = \hat{w}_{00} + \hat{w}_{01} H_1 + \hat{w}_{02} H_2 + \hat{w}_{03} H_3$$

$$H_1 = \tanh(\hat{w}_{10} + \hat{w}_{11} x_1 + \hat{w}_{12} x_2)$$

$$H_2 = \tanh(\hat{w}_{20} + \hat{w}_{21} x_1 + \hat{w}_{22} x_2)$$

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Standardization Methods

Midrange

- $midrange = \frac{(Max+Min)}{2}$
- $x_{midrange} = \frac{(x-midrange)}{range/2}$
- Midrange is 0. Half range is 1.

Z-Score

- $\mu = 0$ and $\sigma = 1$
- $x_{std} = Z = \frac{x-\mu}{\sigma}$

Standardization can be defined for hidden and target layers.

Prediction Illustration: Neural Networks

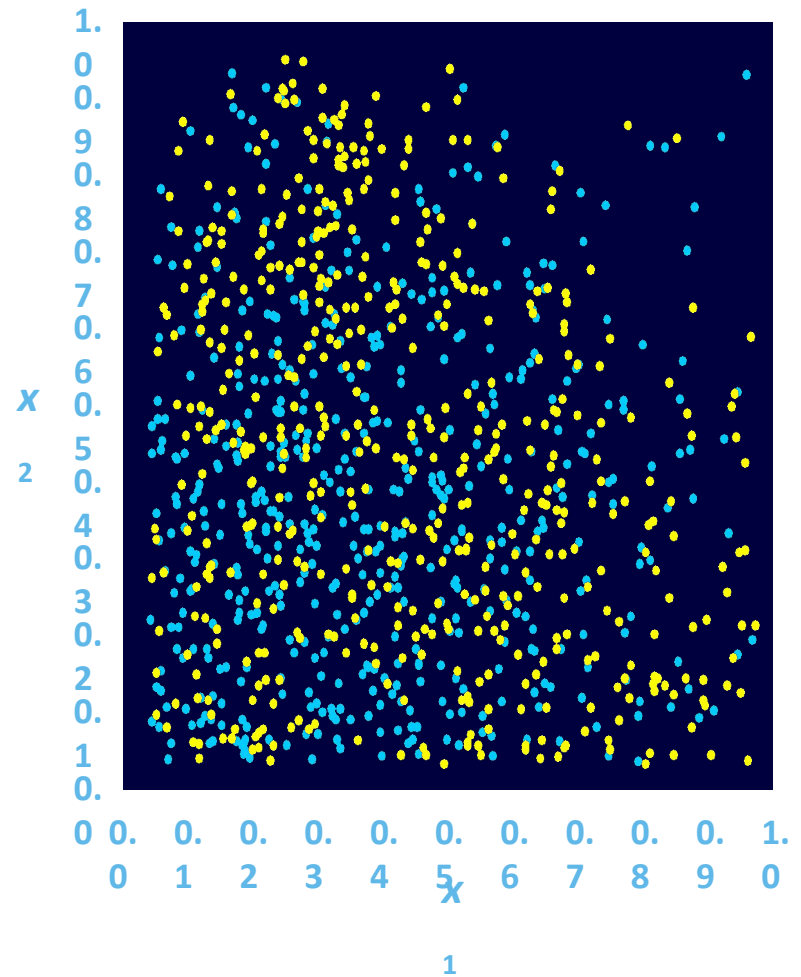
- Find the weight estimates.

$$\text{logit}(\hat{p}) = \hat{w}_{00} + \hat{w}_{01} H_1 + \hat{w}_{02} H_2 + \hat{w}_{03} H_3$$

$$H_1 = \tanh(\hat{w}_{10} + \hat{w}_{11} x_1 + \hat{w}_{12} x_2)$$

$$H_2 = \tanh(\hat{w}_{20} + \hat{w}_{21} x_1 + \hat{w}_{22} x_2)$$

$$H_3 = \tanh(\hat{w}_{30} + \hat{w}_{31} x_1 + \hat{w}_{32} x_2)$$



Prediction Illustration: Neural Networks

- Find the weight estimates.

$$\text{logit}(\hat{p}) = \hat{w}_{00} + \hat{w}_{01} H_1 + \hat{w}_{02} H_2 + \hat{w}_{03} H_3$$

$$H_1 = \tanh(\hat{w}_{10} + \hat{w}_{11} x_1 + \hat{w}_{12} x_2)$$

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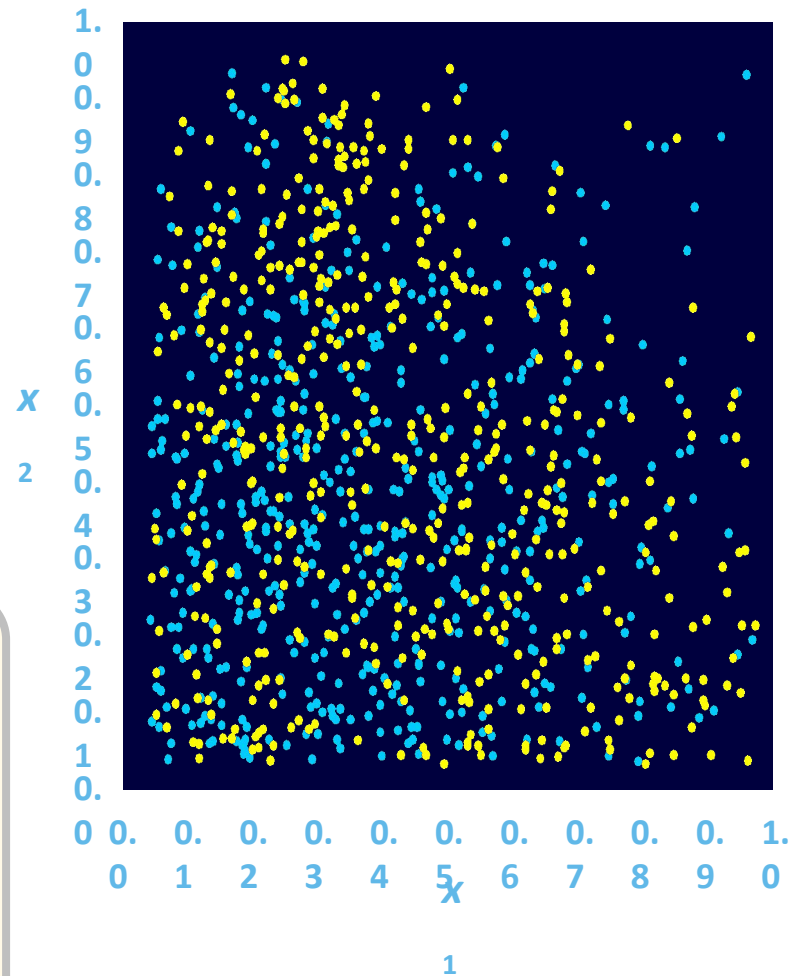
Binary Target

minimize

$$-2 \cdot \left[\sum \log(\hat{p}_i) + \sum \log(1 - \hat{p}_i) \right]$$

primary
outcome
training cases

secondary
outcome training
cases



Prediction Illustration: Neural Networks

3. Obtain a prediction.

$$\text{logit}(\hat{p}) = -0.5 + -2.6 H_1 + -1.9 H_2 + -0.63 H_3$$

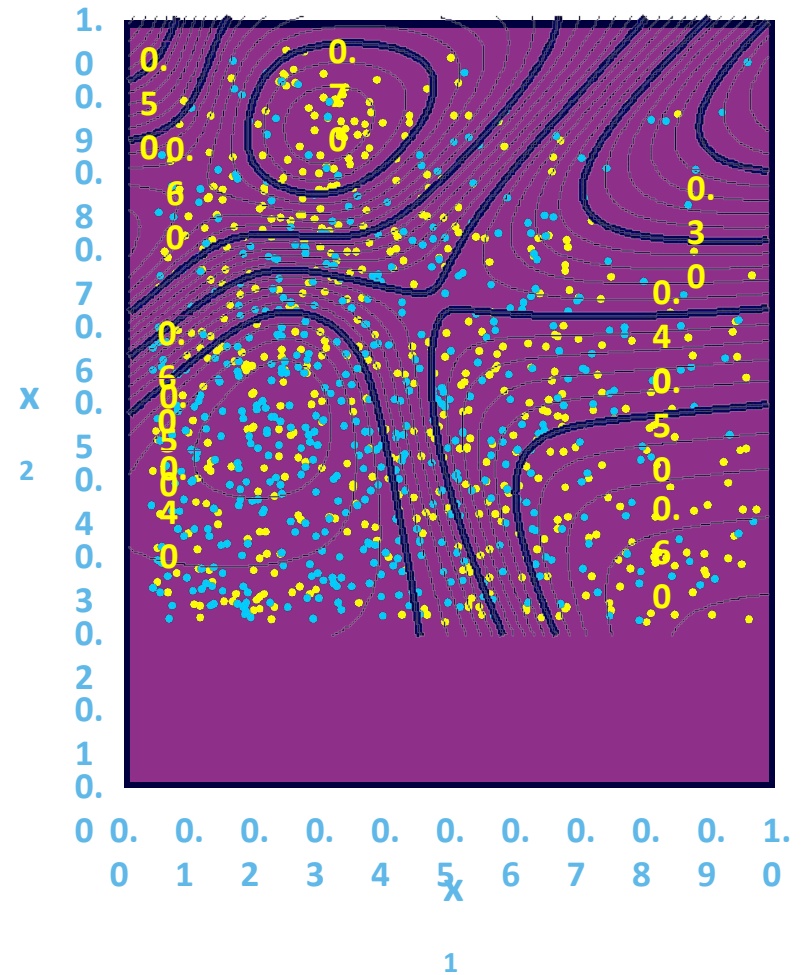
$$H_1 = \tanh(-1.8 + 0.25 x_1 + -1.8 x_2)$$

$$H_2 = \tanh(2.7 + 2.7 x_1 + -5.3 x_2)$$

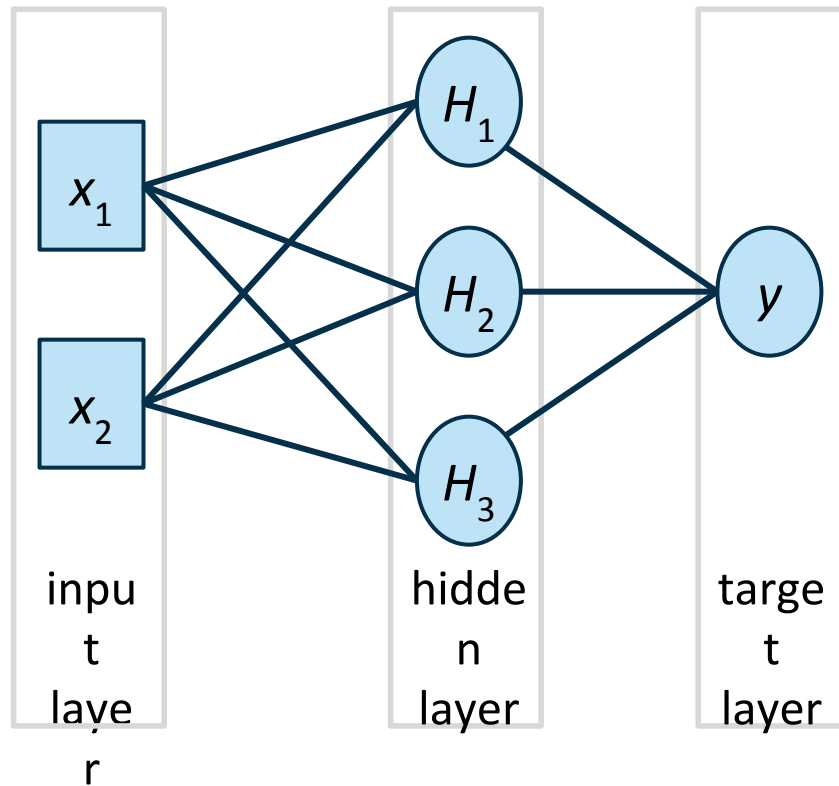
$$H_3 = \tanh(-5.0 + 8.1 x_1 + 4.3 x_2)$$

Logistic Function

$$\hat{p} = \frac{1}{1 + e^{-\text{logit}(\hat{p})}}$$



Network Architecture



different connection types

number of layers

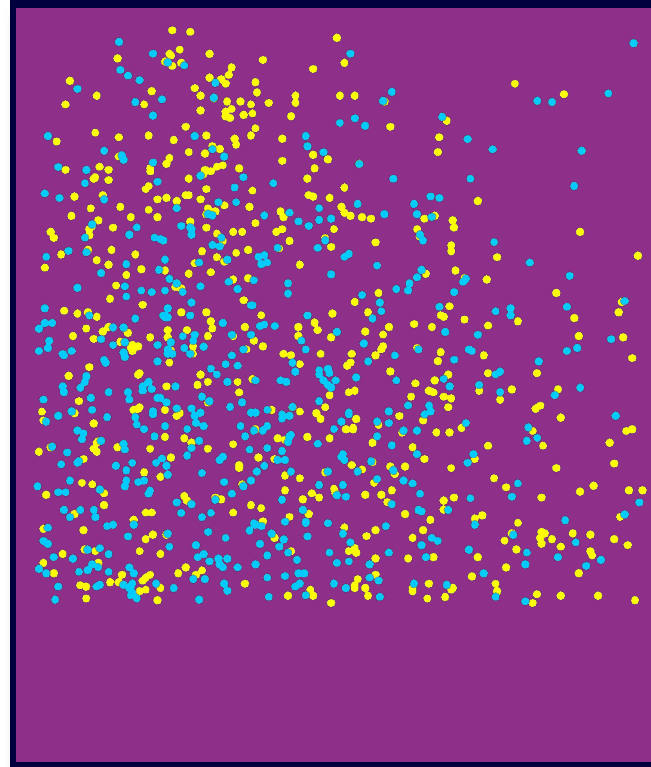
activation functions

number of neurons in each layer

Early Stopping

Initial hidden unit weights

$$\begin{aligned} \text{logit}(\hat{p}) &= 0 + 0 \cdot H_1 + 0 \cdot H_2 + 0 \cdot H_3 \\ &= 0 \end{aligned}$$

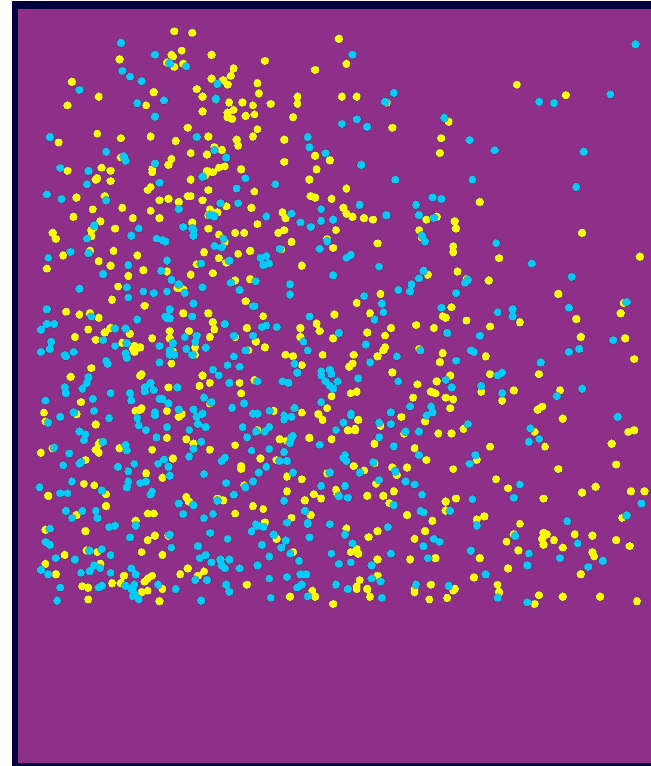


Early Stopping

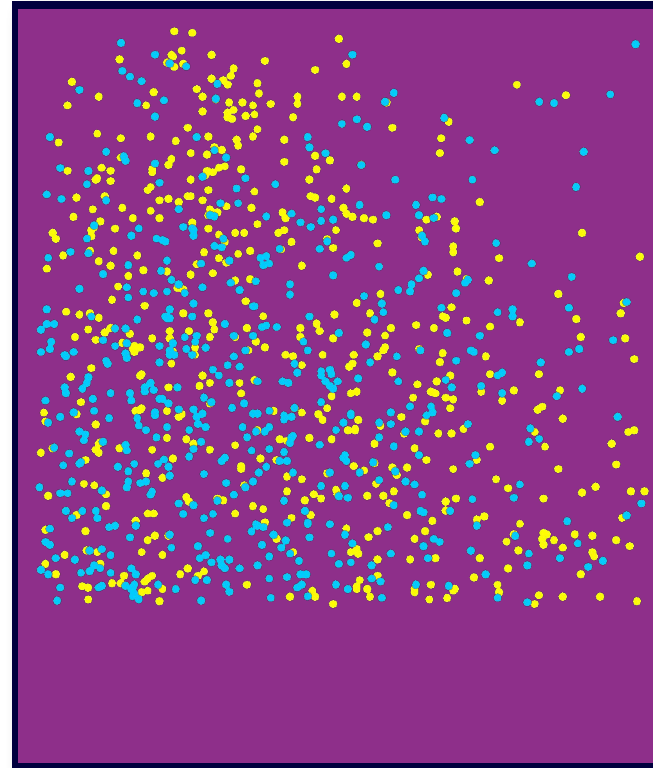
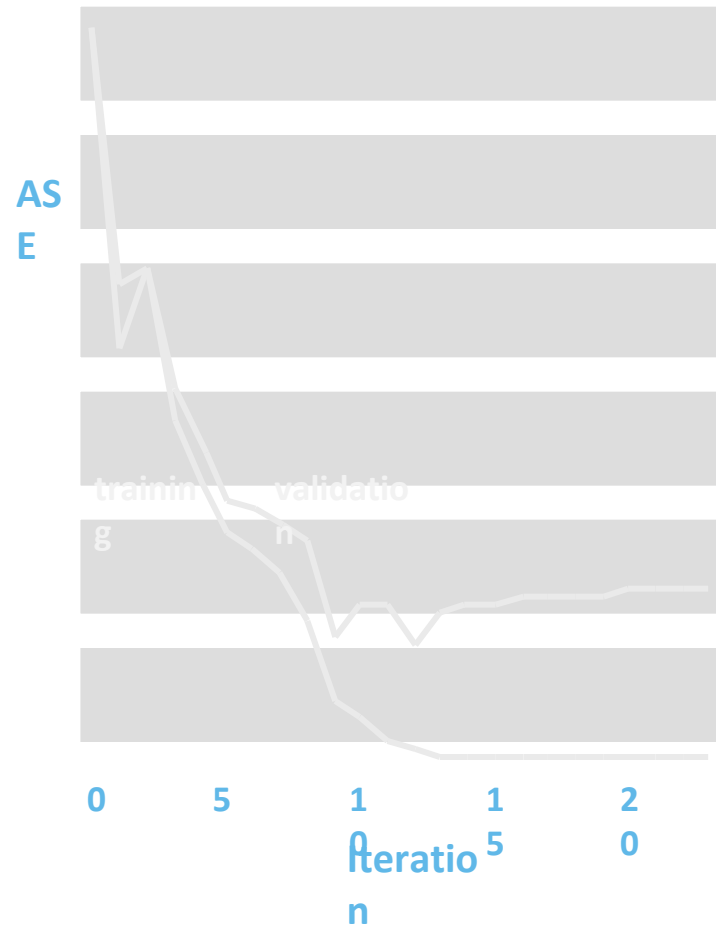
Initial hidden unit weights

$$\begin{aligned} \text{logit}(\hat{p}) &= 0 + 0 \cdot H_1 + 0 \cdot H_2 + 0 \cdot H_3 \\ H_1 &= \tanh(-1.5 - .03x_1 - .07x_2) \\ H_2 &= \tanh(.79 - .17x_1 - .16x_2) \\ H_3 &= \tanh(.57 + .05x_1 + .35x_2) \end{aligned}$$

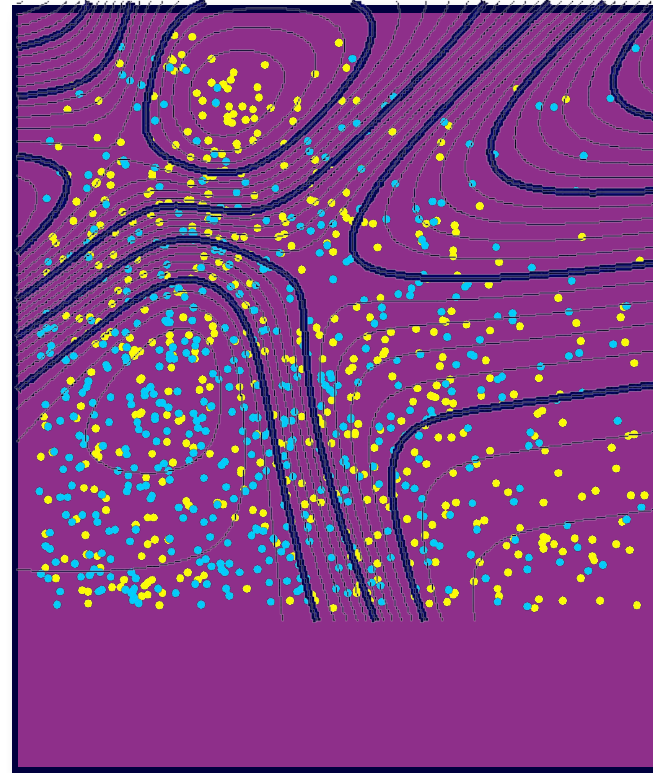
Random initial
input weights and
biases



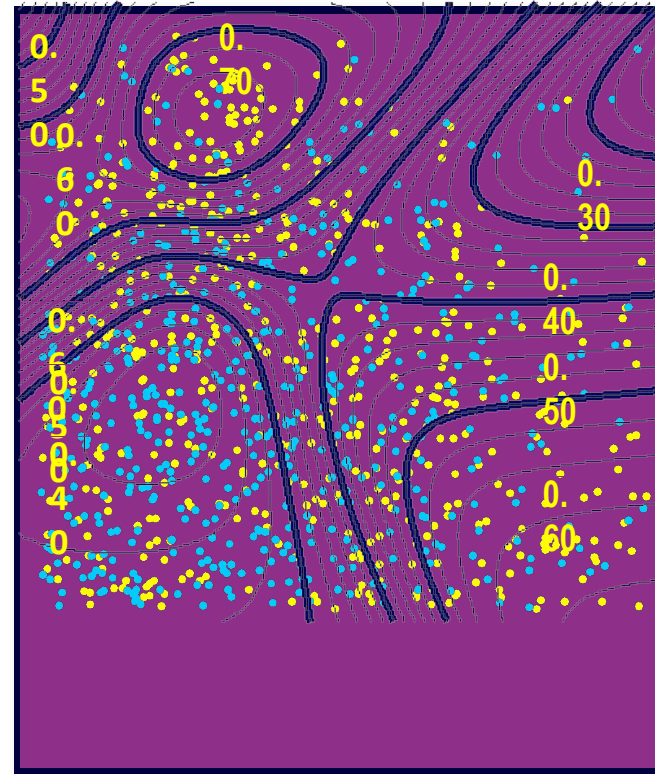
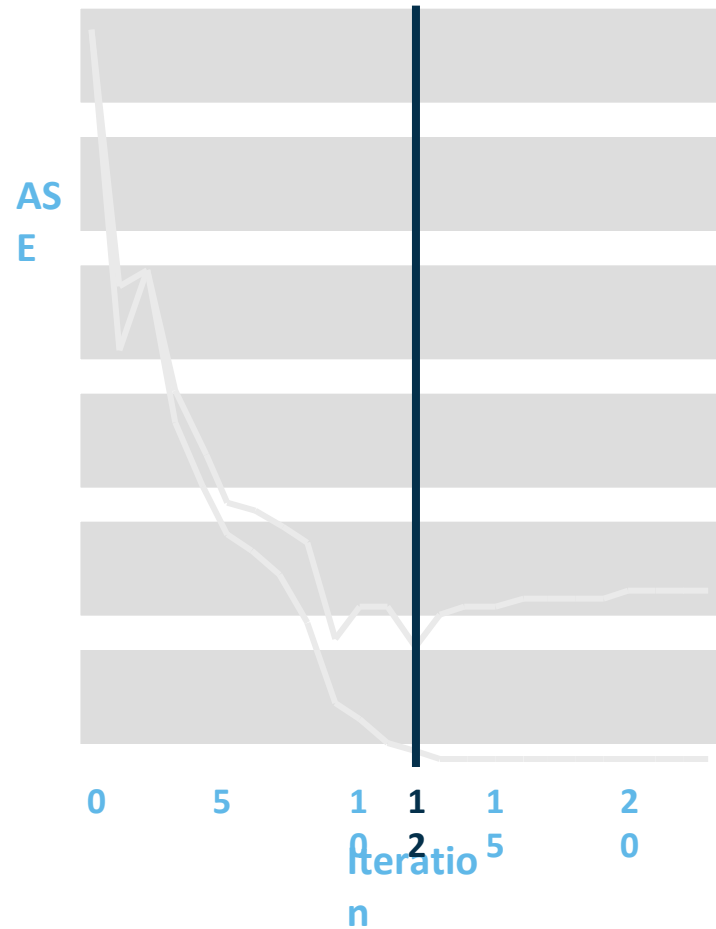
Early Stopping



Early Stopping



Early Stopping



Parameter Estimation: Example

$$\text{logit}(\hat{p}) = \hat{w}_{00} + \hat{w}_{01} H_1 + \hat{w}_{02} H_2 + \hat{w}_{03} H_3$$

$$H_1 = \tanh(\hat{w}_{10} + \hat{w}_{11} x_1 + \hat{w}_{12} x_2)$$

$$H_2 = \tanh(\hat{w}_{20} + \hat{w}_{21} x_1 + \hat{w}_{22} x_2)$$

$$H_3 = \tanh(\hat{w}_{30} + \hat{w}_{31} x_1 + \hat{w}_{32} x_2)$$

Binary Target

log-likelihood function

minimize

$$2 \cdot \left[\sum \log(\hat{p}_i) + \sum \log(1 - \hat{p}_i) \right]$$

primary
outcome
training cases

secondary
outcome training
cases

The error function is
always a deviance
function.



Bernoulli
function



Construct a NN using SKlearn

ADSUP Lesson 11 SVM

1- Construct a NN model for Organics Dataset



Practice

This practice reinforces the concepts in relation to neural network modeling. You use Model Studio to build a neural network model based on hyperparameter autotune.

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OFİS

Telefon +90 (216) 343 6623

Tophanelioğlu Cad. Korukent Köşk

No:6/5 Koşuyolu

İstanbul TÜRKİYE