



$$M_b^q M_b^s M_b^r = \begin{pmatrix} YAR \\ t u 0 \end{pmatrix}$$

$$\begin{pmatrix} b \\ \frac{b}{17} - 1 \\ \frac{1}{17} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} b & 1 \\ 1 & 0 \end{pmatrix} = M_b^q M_b^r$$

## Lesson X : Regression Algorithms

# Model Essentials: Regressions

- ▶ **Predict new cases.**
- ▶ **Select useful inputs.**
- ▶ **Optimize complexity.**

**Prediction  
formula**

**Sequential  
selection**

**Best model  
from  
sequence**

# Linear Regression Prediction Formula

$$\hat{y} = \hat{w}_0 + \hat{w}_1^t x_1 + \hat{w}_2 x_2$$

*input measurement*  
*prediction estimate*  
*intercept estimate*   *parameter estimate*

Choose intercept and parameter estimates to *minimize*:

$$\sum_{\text{training data}} (y_i - \hat{y}_i)^2$$

*squared error function*

# Linear Regression Prediction Formula

$$\hat{y} = \hat{w}_0 + \hat{w}_1^t x_1 + \hat{w}_2 x_2$$

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*prediction estimate*  
*intercept estimate*   *parameter estimate*

Choose intercept and parameter estimates to *minimize*.

$$\sum_{\text{training data}} (y_i - \hat{y}_i)^2$$

*squared error function*

# Logistic Regression Prediction Formula

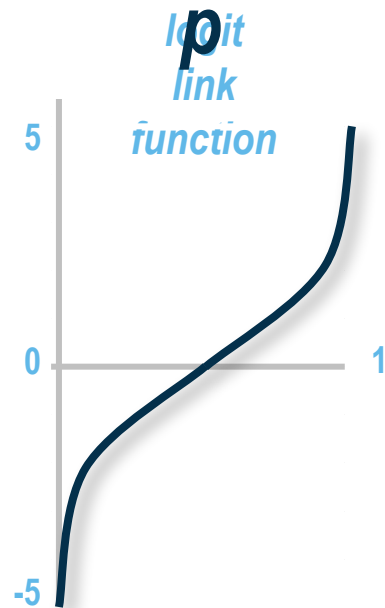
$$\log\left(\frac{p}{1-p}\right) = \hat{w}_0 + \hat{w}_1 x_1 + \hat{w}_2 x_2 \quad \text{logit scores}$$

# Logit Link Function

$$\log\left(\frac{p}{1-p}\right)$$

$$= \hat{w}_0 + \hat{w}_1 x_1 + \hat{w}_2 x_2$$

logit scores



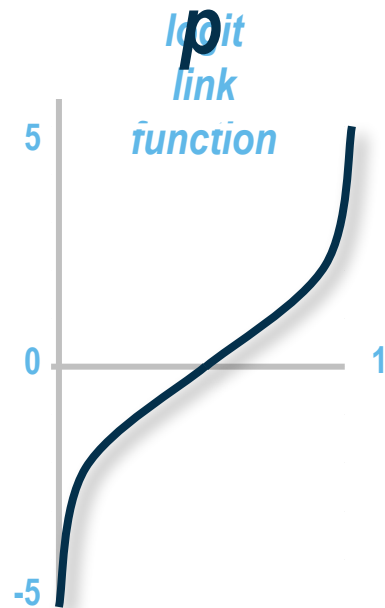
The logit link function transforms probabilities (between 0 and 1) to logit scores (between  $-\infty$  and  $+\infty$ ).

# Logit Link Function

$$\log\left(\frac{p}{1-p}\right)$$

$$= \hat{w}_0 + \hat{w}_1 x_1 + \hat{w}_2 x_2$$

logit scores



The logit link function transforms probabilities (between 0 and 1) to logit scores (between  $-\infty$  and  $+\infty$ ).

# Logit Link Function

$$\log\left(\frac{p}{1-p}\right) = \hat{w}_0 + \hat{w}_1 x_1 + \hat{w}_2 x_2 = \text{logit}(\hat{p})$$

$$\hat{p} = \frac{1}{1 + e^{-\text{logit}(\hat{p})}}$$

To obtain prediction estimates, the logit equation is solved<sup>^</sup> for  $p$ .



# Logit Link Function

$$\log\left(\frac{p}{1-p}\right) = \hat{w}_0 + \hat{w}_1 x_1 + \hat{w}_2 x_2 = \text{logit}(\hat{p})$$

$$\hat{p} = \frac{1}{1 + e^{-\text{logit}(\hat{p})}}$$

To obtain prediction estimates, the logit equation is solved<sup>^</sup> for  $p$ .

# Logit Link Function

$$\text{logit}(\hat{p}) = \hat{w}_0 + \hat{w}_1 \cdot x_1 + \hat{w}_2 \cdot x_2$$

$$\hat{p} = \frac{1}{1 + e^{-\text{logit}(\hat{p})}}$$

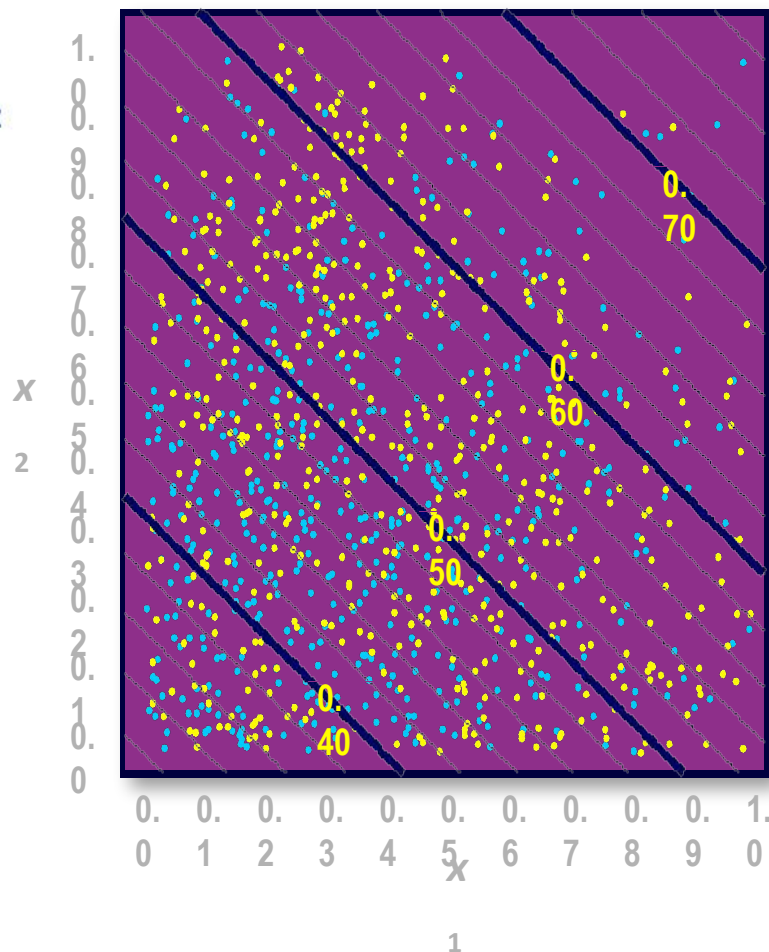
# Simple Prediction Illustration: Regressions

$$\text{logit}(\hat{p}) = \hat{w}_0 + \hat{w}_1 x_1 + \hat{w}_2 x_2$$

$$\hat{p} = \frac{1}{1 + e^{-\text{logit}(\hat{p})}}$$

You need intercept and parameter estimates.

Predict dot color for each  $x_1$  and

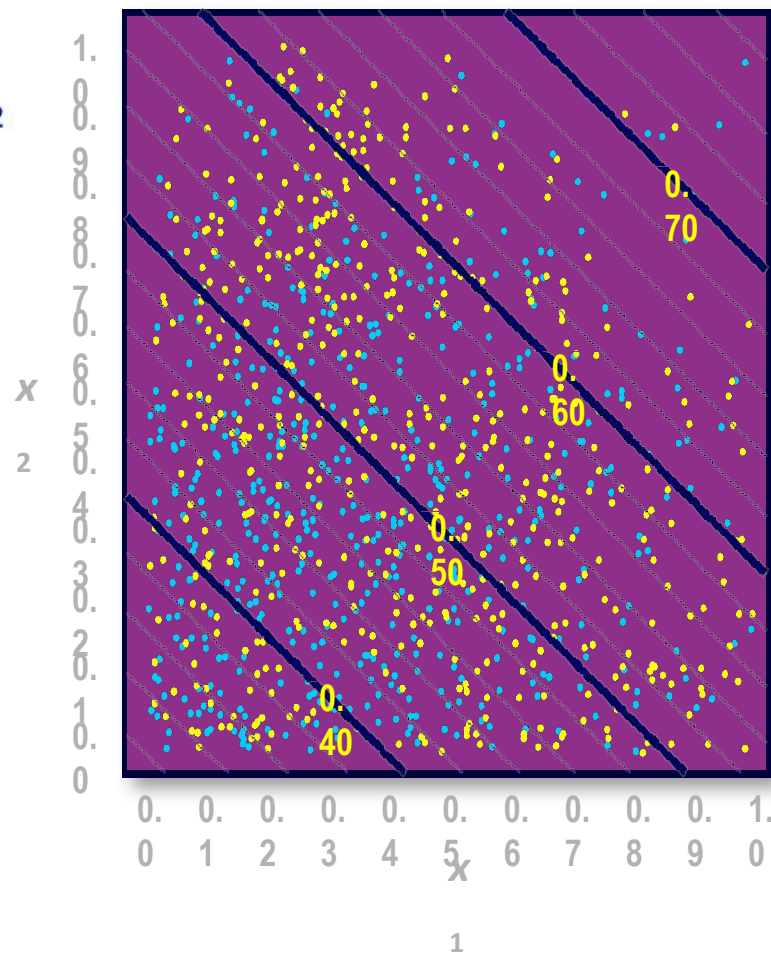


# Simple Prediction Illustration: Regressions

$$\text{logit}(\hat{p}) = \hat{w}_0 + \hat{w}_1 x_1 + \hat{w}_2 x_2$$

$$\hat{p} = \frac{1}{1 + e^{-\text{logit}(\hat{p})}}$$

**You need intercept and parameter estimates.**



# Simple Prediction Illustration: Regressions

$$\text{logit}(\hat{p}) = \hat{w}_0 + \hat{w}_1 x_1 + \hat{w}_2 x_2$$

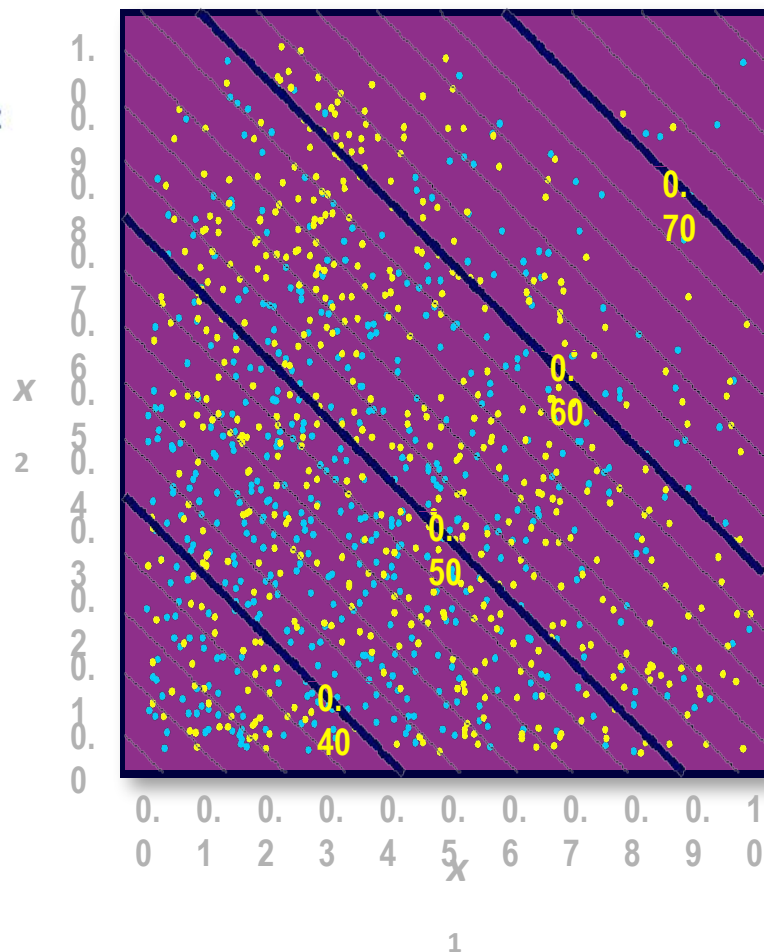
$$\hat{p} = \frac{1}{1 + e^{-\text{logit}(\hat{p})}}$$

Find parameter estimates  
by *maximizing*

$$\sum \log(\hat{p}_i) + \sum \log(1 - \hat{p}_i)$$

primary  
outcome  
training cases
secondary  
outcome  
training cases

*log-likelihood  
function*



# Simple Prediction Illustration: Regressions

$$\text{logit}(\hat{p}) = \hat{w}_0 + \hat{w}_1 x_1 + \hat{w}_2 x_2$$

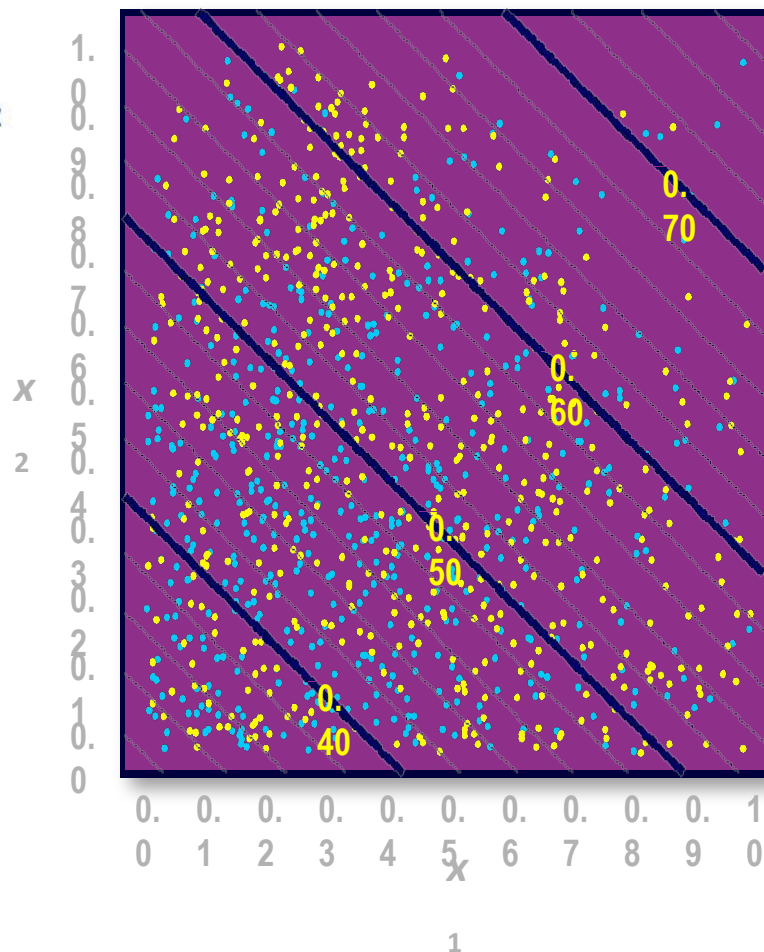
$$\hat{p} = \frac{1}{1 + e^{-\text{logit}(\hat{p})}}$$

Find parameter estimates  
by *maximizing*

$$\sum \log(\hat{p}_i) + \sum \log(1 - \hat{p}_i)$$

primary  
outcome  
training cases
secondary  
outcome  
training cases

*log-likelihood  
function*

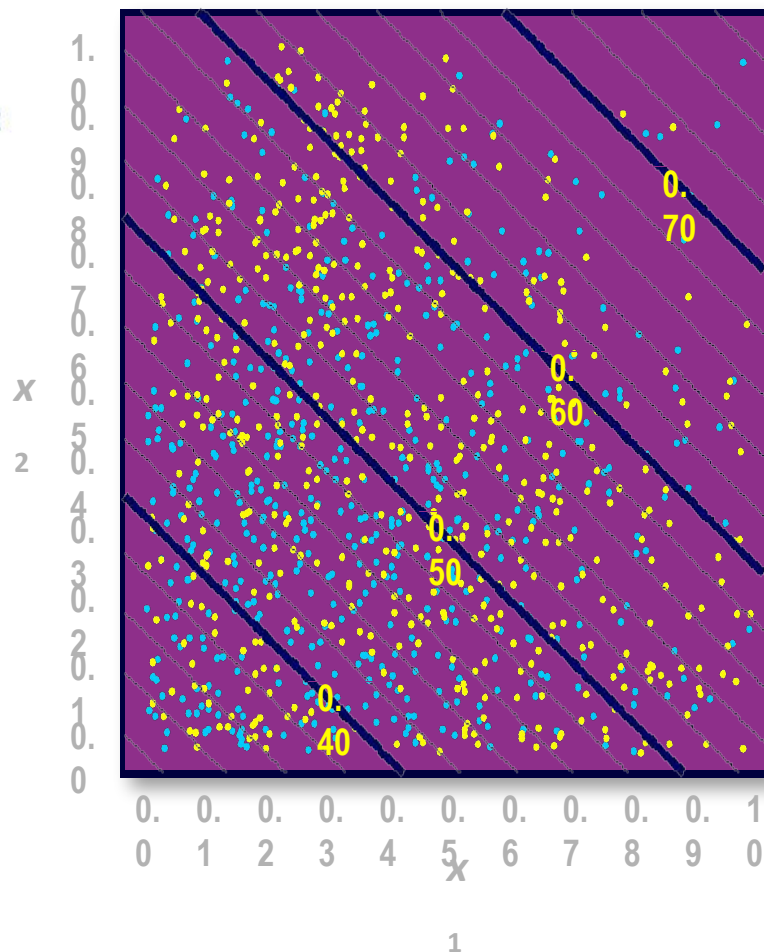


# Simple Prediction Illustration: Regressions

$$\text{logit}(\hat{p}) = -0.81 + 0.92x_1 + 1.11x_2$$

$$\hat{p} = \frac{1}{1 + e^{-\text{logit}(\hat{p})}}$$

Using the maximum likelihood estimates, the prediction formula assigns a logit score to each  $x_1$  and  $x_2$ .



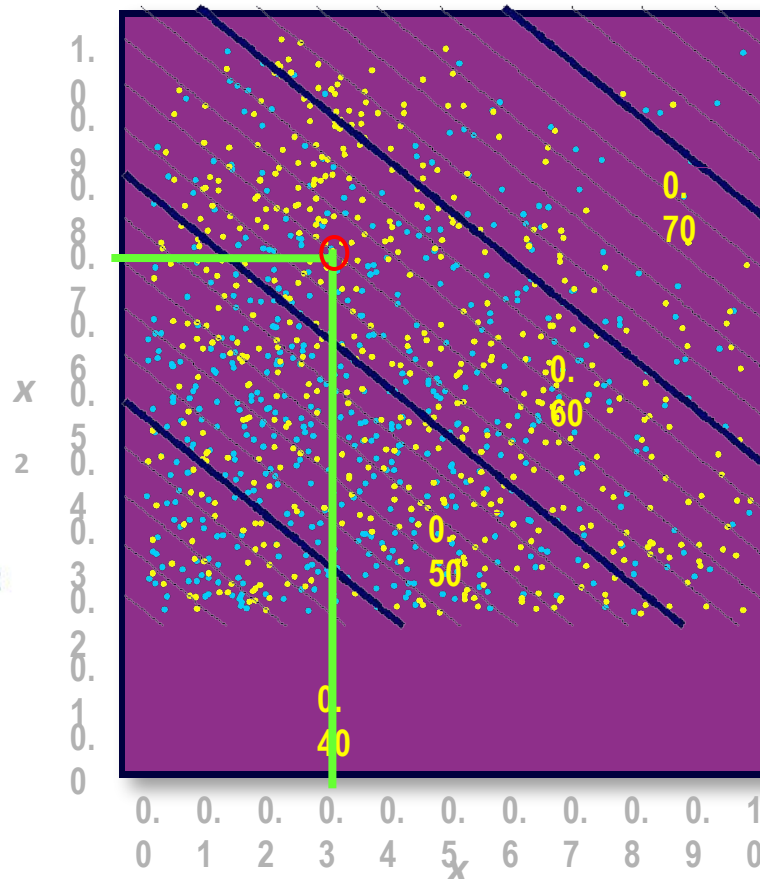
## 4.01 Multiple Choice Question

What is the logistic regression prediction for the indicated point?

- a. 0.243
- b. 0.56
- c. yellow
- d. It depends.

$$\text{logit}(\hat{p}) = -0.81 + 0.92x_1 + 1.11x_2$$

$$\hat{p} = \frac{1}{1 + e^{-\text{logit}(\hat{p})}}$$





## 4.01 Multiple Choice Question – Correct Answer

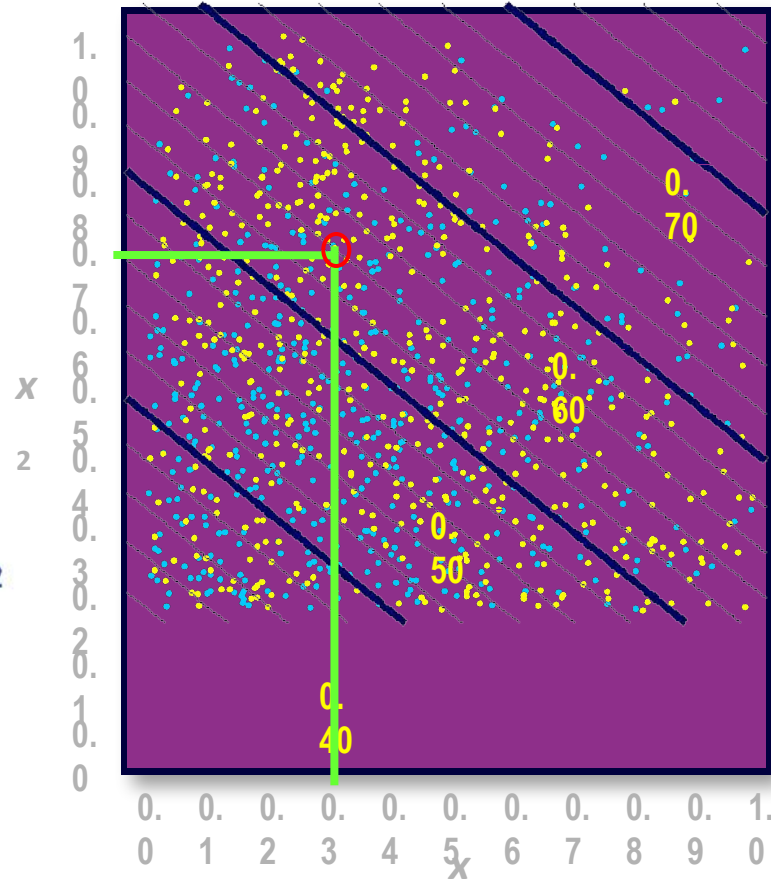
What is the logistic regression prediction for the indicated point?

- a. 0.243
- b. 0.56
- c. yellow
- d. It depends.



$$\text{logit}(\hat{p}) = -0.81 + 0.92x_1 + 1.11x_2$$

$$\hat{p} = \frac{1}{1 + e^{-\text{logit}(\hat{p})}}$$



# Regressions: Beyond the Prediction Formula

- ▶ **Manage missing values.**
- ▶ **Interpret the model.**
- ▶ **Handle extreme or unusual values.**
- ▶ **Use nonnumeric inputs.**
- ▶ **Account for nonlinearities.**



# Construct a Regression using SKlearn

ADSUP Lesson VIII Ra

1- Construct a Regression model for Fitness Dataset



## Practice

This practice reinforces the concepts discussed previously.

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