SEEM Postgraduate Admission Quiz
(Operational Research and Financial Engineering)

July 2022

Instructions

This quiz consists of three questions, answer all of the questions. You may prioritize two of the three questions but try to complete all of the three. This is a closed book test, you must answer the questions independently. Any discussions or searching answers from internet are strictly prohibited. The duration of the quiz is 30 minutes. At the end of the quiz, submit your answers in a single pdf-file to Ms Kate Chow by email: pschow@se.cuhk.edu.hk.

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1. Consider the following minimization problem:

$$\begin{aligned} &\min_{x_1, x_2 \in \mathbb{R}} \; x_2 \\ &\text{subject to } \; 3x_2 - 3x_1^4 - 4x_1^3 + 12x_1^2 = 0. \end{aligned}$$

   (i) Write down the Lagrangian of this problem and find all KKT points.

   (ii) Find the global minimizer and the global minimum.

   (iii) Give a geometric interpretation of the problem. If a constraint $x_2 \geq -8$ is added to the problem, what is the global minimum?

2. Suppose that A is a real $n \times n$ symmetric matrix and two of its eigenvalues are equal. For any vector $v \in \mathbb{R}^n$, show that the vectors $v, Av, \ldots, A^{n-1}v$ are linearly dependent.

3. Suppose that $X_1, X_2, \ldots$ are independent and identically distributed random variables with mean μ and variance σ^2.

   (i) Show that

$$\lim_{n \to \infty} P\left(\frac{1}{n} \left| \sum_{i=1}^{n} X_i - n\mu \right| < \epsilon \right) = 1, \quad \text{for all } \epsilon > 0.$$

   (ii) Show that

$$P\left(\max_{1 \leq k \leq n} \left| \sum_{i=1}^{k} X_i - k\mu \right| > \epsilon \right) \leq \frac{n\sigma^2}{\epsilon^2}, \quad \text{for all } \epsilon > 0 \text{ and } n \geq 1.$$