

1) Simple Harmonic Oscillator (Euler Method) [2 ½ marks]

1. Use the Euler method to evolve a simple harmonic oscillator with $m=1\text{kg}$ and $k=10\text{ kg/s}^2$, with $x(0) = 5\text{m}$, $v(0) = 0\text{ m/s}$, from $t=0$ to 10s using a time step of 0.01s .
2. Make a figure showing of x vs t and v vs t in two subplots.
3. Calculate the energy as a function of time using $E(t)=1/2 k x(t)^2 + 1/2 m v(t)^2$. Make a figure showing energy vs time.
4. By what fraction has the energy increased at the end of the simulation? Why?

2) Simple Harmonic Oscillator (Euler-Cromer Method) [1¼ marks]

1. Evolve the same harmonic oscillator as above, but with Euler-Cromer method. As before, use $m=1\text{kg}$, $k=10\text{ kg/s}^2$, $x(0) = 5\text{m}$, $v(0) = 0\text{ m/s}$. Use a time step of 0.01s and evolve for 10 s .
2. Make a figure showing x vs t and v vs t in two subplots.
3. Calculate the energy as a function of time using $E(t) = 1/2 k x(t)^2 + 1/2 m v(t)^2$. By what fraction does the energy deviate from the expected value?

3) Damped Harmonic Oscillator [1¼ marks]

1. Add a damping term to your code.
2. Calculate the value of c for critical damping: $c = 2 (k \cdot m)^{1/2}$.
3. Make a figure with 2 subplots showing positions vs time and velocity vs time for:
 - i. Critical damping
 - ii. 1/4 of critical damping
 - iii. four times critical damping