Analog Assignment-1

Name -: Abburi Tanusha
Roll no -: EE23BTECH11201
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Question:

A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg, and its linear mass density is 4×10^{-2} kg/m. The length of the wire is 0.875 m. Determine the speed of a transverse wave on the string and the tension in the string.

Solution:

Given:

TABLE I
Input Parameters

Parameter	Value
Mass per unit length of the string	4.0×10^{-2} kg/m
Frequency of the vibrating string	45 Hz
Mass hanging from the string	3.5×10^{-2} kg

Derivation of Wave Equation:

1. Find the wavelength (λ) for the fundamental mode:

$$\lambda = 2L \quad \Rightarrow \quad \lambda = 2 \times 0.875\,\text{m} = 1.75\,\text{m} \tag{1}$$

2. Use the frequency and wavelength to find the wave speed (v):

$$v = f \cdot \lambda \quad \Rightarrow \quad v = 45\,\text{Hz} \times 1.75\,\text{m} = 78.75\,\text{m/s} \tag{2}$$

Derivation of Wave Equation:

The wave equation for transverse waves on a string is derive

$$\frac{\partial T}{\partial x}\frac{\partial^2 y}{\partial t^2} = T\frac{\partial^2 y}{\partial x^2}\mu - \frac{\partial}{\partial x}(T\frac{\partial^2 y}{\partial x^2}\mu)$$

After simplifying, the final wave equation is given by:

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2}$$

This equation relates the wave speed (v), tension (T), and lin

$$v^2 = \frac{T}{\mu} \tag{3}$$

3. Use the wave speed (v) to find the tension (T) using the wave equation:

$$T = \mu \cdot v^2 \quad \Rightarrow \quad T = (4\times10^{-2}\,\text{kg/m})\times(78.75\,\text{m/s})^2 = 123.1875 \tag{4}$$

Therefore, the speed of the transverse wave on the string is 78.75 m/s, and the tension in the string is 123.1875 N.