

# GATE 2022 BM-42

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**Question:** If

$$g(t) = \frac{df(t)}{dt} \quad (1)$$

$$F(s) = \frac{1+s}{s^2+12s+32} \quad (2)$$

where  $F(s)$  is the Laplace transform of the function  $f(t)$ , then what is the value of  $g(t)$  at  $t = 0$  ?  
(GATE 2022 BM)

**Solution:**

Value	Parameter	Description
$g(t)$	$\frac{df(t)}{dt}$	Derivative of $f(t)$ with respect to $t$
$F(s)$	$\frac{1+s}{s^2+12s+32}$	Laplace transform of the function $f(t)$

TABLE 0

GIVEN PARAMETERS

$$F(s) = \frac{1+s}{(s+4)(s+8)} \quad (3)$$

Using partial fraction decomposition:

$$F(s) = \frac{A}{s+4} + \frac{B}{s+8} \quad (4)$$

$$1+s = A(s+8) + B(s+4) \quad (5)$$

$$1+s = (A+B)s + 8A+4B \quad (6)$$

Comparing coefficients:

$$A+B=1 \quad (7)$$

$$8A+4B=1 \quad (8)$$

By solving these equations, we get

$$A = -\frac{3}{4} \quad (9)$$

$$B = \frac{7}{4} \quad (10)$$

$$F(s) = \frac{\frac{7}{4}}{s+8} - \frac{\frac{3}{4}}{s+4} \quad (11)$$

Inverse Laplace transform of  $F(s)$

$$f(t) = \mathcal{L}^{-1}\left(\frac{\frac{7}{4}}{s+8}\right) - \mathcal{L}^{-1}\left(\frac{\frac{3}{4}}{s+4}\right) \quad (12)$$

$$= \frac{7}{4}\mathcal{L}^{-1}\left(\frac{1}{s+8}\right) - \frac{3}{4}\mathcal{L}^{-1}\left(\frac{1}{s+4}\right) \quad (13)$$

$$= \frac{7}{4}e^{-8t} - \frac{3}{4}e^{-4t} \quad (14)$$

$$g(t) = \frac{d}{dt}\left(\frac{7}{4}e^{-8t} - \frac{3}{4}e^{-4t}\right) \quad (15)$$

$$= \frac{7}{4}(-8)e^{-8t} - \frac{3}{4}(-4)e^{-4t} \quad (16)$$

$$= -14e^{-8t} + 3e^{-4t} \quad (17)$$

$$g(0) = -14 \times 1 + 3 \times 1 \quad (18)$$

$$= -14 + 3 = -11 \quad (19)$$

The value of  $g(t)$  at  $t = 0$  is  $-11$

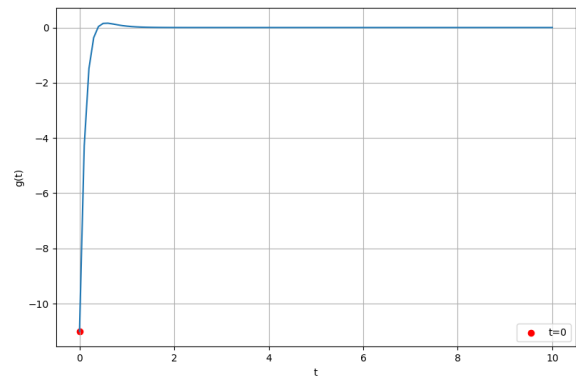


Fig. 0. Plot  $g(t)$  vs  $t$