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NCERT Maths 10.5.3 Q14

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Question: Find the sum of odd numbers between 0 and 50.

Solution:

Symbol	Value	Description
x(0)	1	first term of AP
d	2	common difference
x(n)	(1+2n)u(n)	<i>n</i> -th term of AP

TABLE I Given Parameters

Last term of the given sequence is 49.

$$\therefore (2n+1) = 49$$
 (1)

$$2n = 48 \tag{2}$$

$$\implies n = 24$$
 (3)

Applying Z transform: From equation (??):

$$=\frac{1+z^{-1}}{(1-z^{-1})^2}, \quad |z|>|1| \tag{4}$$

For AP, the sum of first n+1 terms can be written as:

$$y(n) = x(n) * u(n)$$
(5)

$$Y(z) = X(z)U(z) \tag{6}$$

$$= \frac{1}{(1-z^{-1})^2} + \frac{2z^{-1}}{(1-z^{-1})^3}, \quad |z| > |1| \quad (7)$$

Using contour integration to find inverse Z transform:

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) z^{n-1} dz \tag{8}$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{1}{(1-z^{-1})^2} + \frac{2z^{-1}}{(1-z^{-1})^3} \right) z^{n-1} dz$$
 (9)

The sum of the terms of the sequence is computed using the residue theorem, expressed as R_i , which represents the residue of the Z-transform at z = 1 for the expression Y(z).

$$R_i = R_1 + R_2 (10)$$

 R_1 and R_2 are residues calculated at the poles of the Z-transform.

$$R_1 = \frac{1}{(2-1)!} \frac{d(z^{25})}{dz}|_{z=1}$$
 (11)

$$= 25 \tag{12}$$

$$R_2 = \frac{1}{(3-1)!} \frac{d^2(2z^{25})}{dz^2} \Big|_{z=1}$$
 (13)

$$= 25 \times 24 \tag{14}$$

$$=600$$
 (15)

$$R_i = 600 + 25 \tag{16}$$

(17)

Sum of the numbers is $R_i = 625$.

