

NCERT Maths 10.5.3 Q14

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Question: Find the sum of odd numbers between 0 and 50.

Solution:

Symbol	Value	Description
$x(0)$	1	first term of AP
d	2	common difference
$x(n)$	$(1 + 2n)u(n)$	n -th term of AP

TABLE 0
GIVEN PARAMETERS

Last term of the given sequence is 49.

$$x(n) = (2n + 1)u(n) \quad (1)$$

$$\therefore (2n + 1) = 49 \quad (2)$$

$$\Rightarrow n = 24 \quad (3)$$

Applying Z transform From equation (??):

$$X(z) = \frac{1 + z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (4)$$

For AP, the sum of first $n+1$ terms can be written as

$$y(n) = x(n) * u(n) \quad (5)$$

$$Y(z) = X(z)U(z) \quad (6)$$

$$= \frac{1}{(1 - z^{-1})^2} + \frac{2z^{-1}}{(1 - z^{-1})^3}, \quad |z| > 1 \quad (7)$$

Using contour integration to find inverse Z transform

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z)z^{n-1} dz \quad (8)$$

$$y(24) = \frac{1}{2\pi j} \int Y(z)z^{23} dz \quad (9)$$

$$= \frac{1}{2\pi j} \int \frac{1 \cdot z^{25}}{(z - 1)^2} dz - \frac{1}{2\pi j} \int \frac{2 \cdot z^{25}}{(z - 1)^3} dz \quad (10)$$

$$\therefore R = \frac{1}{(m - 1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z - a)^m f(z)) \quad (11)$$

For R_1 , $m = 2$, where m corresponds to number of repeated poles

$$R_1 = \frac{1}{1!} \lim_{z \rightarrow 1} \frac{d}{dz} \left((z - 1)^2 \cdot \frac{1 \cdot z^{25}}{(z - 1)^2} \right) \quad (12)$$

$$= 25 \quad (13)$$

For R_2 , $m = 3$

$$R_2 = \frac{1}{2!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z - 1)^3 \cdot \frac{2 \cdot z^{25}}{(z - 1)^3} \right) \quad (14)$$

$$= 600 \quad (15)$$

$$\Rightarrow y(24) = R_1 + R_2 \quad (16)$$

$$= 625 \quad (17)$$

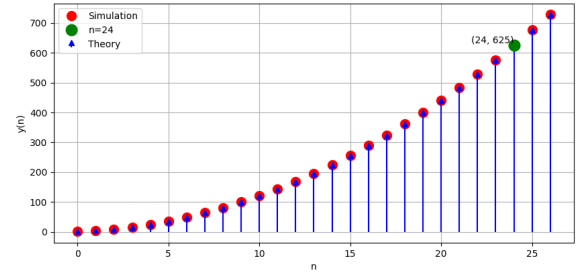


Fig. 0. Simulation vs Analysis of $y(n)$