GATE 2022 BM-42

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Question: If

$$g(t) = \frac{df(t)}{dt} \tag{1}$$

$$F(s) = \frac{1+s}{s^2+12s+32} \tag{2}$$

where $F(s)$ is the Laplace transform of the function $f(t)$, then what is the value of $g(t)$ at $t = 0$?
(GATE 2022 BM)

Solution:

Value	Parameter	Description
$g(t)$	$\frac{df(t)}{dt}$	Derivative of $f(t)$ with respect to t
$F(s)$	$\frac{1+s}{s^2+12s+32}$	Laplace transform of the function $f(t)$

TABLE 0
Given Parameters

$$F(s) = \frac{1+s}{(s+4)(s+8)} \tag{3}$$

Using partial fraction decomposition:

$$F(s) = \frac{A}{s+4} + \frac{B}{s+8} \tag{4}$$

$$1+s = A(s+8) + B(s+4) \tag{5}$$

$$1+s = (A+B)s + 8A + 4B \tag{6}$$

Comparing coefficients:

$$A + B = 1 \tag{7}$$

$$8A + 4B = 1 \tag{8}$$

By solving these equations, we get

$$A = -\frac{3}{4} \tag{9}$$

$$B = \frac{7}{4} \tag{10}$$

$$F(s) = \frac{\frac{7}{4}}{s+8} - \frac{\frac{3}{4}}{s+4} \tag{11}$$

Inverse Laplace transform of $F(s)$

$$f(t) = \mathcal{L}^{-1}\left(\frac{\frac{7}{4}}{s+8}\right) - \mathcal{L}^{-1}\left(\frac{\frac{3}{4}}{s+4}\right) \tag{12}$$

$$= \frac{7}{4}\mathcal{L}^{-1}\left(\frac{1}{s+8}\right) - \frac{3}{4}\mathcal{L}^{-1}\left(\frac{1}{s+4}\right) \tag{13}$$

$$= \frac{7}{4}e^{-8t} - \frac{3}{4}e^{-4t} \tag{14}$$

$$g(t) = \frac{d}{dt}\left(\frac{7}{4}e^{-8t} - \frac{3}{4}e^{-4t}\right) \tag{15}$$

$$= \frac{7}{4}(-8)e^{-8t} - \frac{3}{4}(-4)e^{-4t} \tag{16}$$

$$= -14e^{-8t} + 3e^{-4t} \tag{17}$$

$$g(0) = -14 \times 1 + 3 \times 1 \tag{18}$$

$$= -14 + 3 = -11 \tag{19}$$

The value of $g(t)$ at $t = 0$ is -11

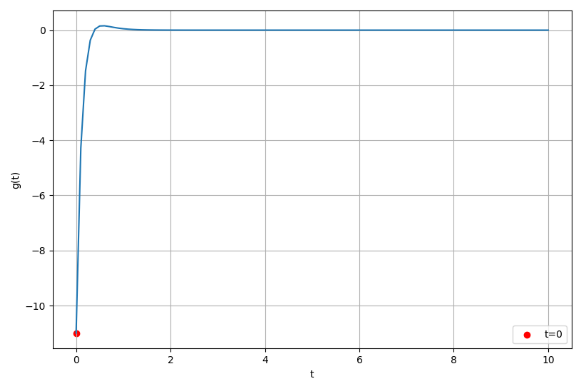

Fig. 0. Plot $g(t)$ vs t