NCERT Discrete - 11.5.9.2

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Question: The sum of three numbers in an arithmetic progression (AP) is 24 and the product of those three numbers is 440, find the values of the three numbers.

Solution: The following information is provided in the question:

Parameter	Value	Description
$x(n)$	$(x(0)+n\cdot d)\,u(n)$	$(n+1)$th term
d	3	common difference
$x(0)+x(1)+x(2)$	24	sum of the terms
$x(0)\cdot x(1)\cdot x(2)$	440	product of terms

TABLE 0
PARAMETERS

Let the three numbers in the arithmetic progression be denoted as $x(0)$, $x(1)$, and $x(2)$.
From Table 0

$$x(0)+x(1)+x(2)=24 \tag{1}$$
$$(x(1)-d)+x(1)+(x(1)+d)=24 \tag{2}$$
$$3x(1)=24 \tag{3}$$
$$\implies x(1)=8 \tag{4}$$

$$x(0)\cdot x(1)\cdot x(2)=440 \tag{5}$$
$$(8-d)\cdot(8)\cdot(8+d)=440 \tag{6}$$
$$(8-d)\cdot(8+d)=55 \tag{7}$$
$$64-d^2=55 \tag{8}$$
$$\implies d=3 \tag{9}$$
$$\implies x(0)=5 \tag{10}$$

$$x(n)=(5+3n)\,u(n) \tag{11}$$

From equation (**??**):

$$X(z)=\frac{5-8z^{-1}}{(1-z^{-1})^2};\quad |z|>|1| \tag{12}$$

Therefore, the required three numbers in AP are 5, 8, and 11.

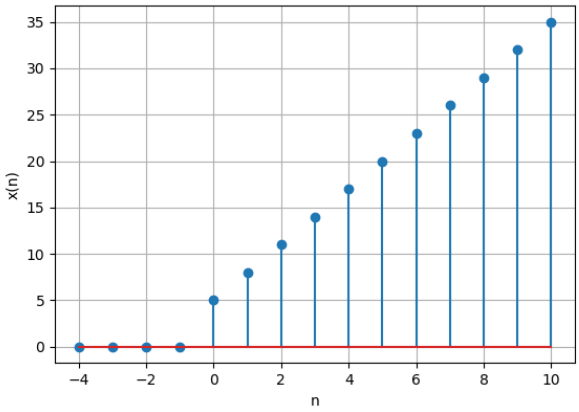

Fig. 0. stem plots of x(n)