NCERT Maths 10.5.3 Q14

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Question: Find the sum of odd numbers between 0 and 50.

Solution:

Symbol	Value	Description
$x(0)$	1	first term of AP
d	2	common difference
$x(n)$	$(1+2n)u(n)$	n-th term of AP

TABLE I
GIVEN PARAMETERS

$$x(n) = (1+2n)u(n) \tag{1}$$

Last term of the given sequence is 49.

$$(2n+1) = 49 \tag{2}$$
$$2n = 48 \tag{3}$$
$$n = 24 \tag{4}$$

Applying Z transform:

$$x(z) = \frac{1}{1-z^{-1}} + \frac{2z^{-1}}{(1-z^{-1})^2} \tag{5}$$
$$= \frac{1+z^{-1}}{(1-z^{-1})^2} \tag{6}$$

Region of Convergence or R.O.C :

$$|z| > 1 \tag{7}$$

For AP, the sum of first n+1 terms can be written as :

$$y(n) = x(n) * u(n) \tag{8}$$

Applying Z transform on both sides

$$Y(z) = X(z)U(z) \tag{9}$$
$$= \frac{1}{(1-z^{-1})^2} + \frac{2z^{-1}}{(1-z^{-1})^3} \tag{10}$$

Using contour integration to find inverse Z transform:

$$y(n) = \frac{1}{2\pi j}\oint_C Y(z)z^{n-1}dz \tag{11}$$
$$= \frac{1}{2\pi j}\oint_C \left[\frac{1}{(1-z^{-1})^2} + \frac{2z^{-1}}{(1-z^{-1})^3}\right] z^{n-1}\,dz \tag{12}$$

The sum of the terms of the sequence is computed using the residue theorem, expressed as R_i, which represents the residue of the Z-transform at $z = 1$ for the expression $Y(z)$.

$$R_i = R_1 + R_2 \tag{13}$$

R_1 and R_2 are residues calculated at the poles of the Z-transform.

$$R_1 = \frac{1}{(2-1)!}\left.\frac{d(z^{25})}{dz}\right|_{z=1} \tag{14}$$
$$= 25 \tag{15}$$

$$R_2 = \frac{1}{(3-1)!}\left.\frac{d^2(2z^{25})}{dz^2}\right|_{z=1} \tag{16}$$
$$= 25 \times 24 \tag{17}$$
$$= 600 \tag{18}$$

$$R_i = 600 + 25 \tag{19}$$
$$\tag{20}$$

Sum of the numbers is $R_i = 625$.