NCERT Maths 10.5.3 Q14

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Question: Find the sum of odd numbers between 0 and 50.

Solution:

Symbol	Value	Description
$x(0)$	1	first term of AP
d	2	common difference
$x(n)$	$(1+2n)u(n)$	n-th term of AP

TABLE 0
GIVEN PARAMETERS

Last term of the given sequence is 49.

$$x(n) = (2n+1)u(n) \tag{1}$$
$$\therefore (2n+1) = 49 \tag{2}$$
$$\implies n = 24 \tag{3}$$

Applying Z transform From equation (**??**):

$$X(z) = \frac{1+z^{-1}}{(1-z^{-1})^2}, \quad |z| > |1| \tag{4}$$

For AP, the sum of first n+1 terms can be written as

$$y(n) = x(n) * u(n) \tag{5}$$
$$Y(z) = X(z)U(z) \tag{6}$$
$$= \frac{1}{(1-z^{-1})^2} + \frac{2z^{-1}}{(1-z^{-1})^3}, \quad |z| > |1| \tag{7}$$

Using contour integration to find inverse Z transform

$$y(n) = \frac{1}{2\pi j}\oint_C Y(z)z^{n-1}dz \tag{8}$$
$$y(24) = \frac{1}{2\pi j}\int Y(z)z^{23}dz \tag{9}$$
$$= \frac{1}{2\pi j}\int \frac{1.z^{25}}{(z-1)^2}dz - \frac{1}{2\pi j}\int \frac{2.z^{25}}{(z-1)^3}dz \tag{10}$$
$$\because R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{11}$$

For R1 , $m = 2$, where m corresponds to number of repeated poles

$$R_1 = \frac{1}{1!}\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2.\frac{1.z^{25}}{(z-1)^2}\right) \tag{12}$$
$$= 25 \tag{13}$$

For R2 , $m = 3$

$$R_2 = \frac{1}{2!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3.\frac{2.z^{25}}{(z-1)^3}\right) \tag{14}$$
$$= 600 \tag{15}$$
$$\implies y(24) = R_1 + R_2 \tag{16}$$
$$= 625 \tag{17}$$

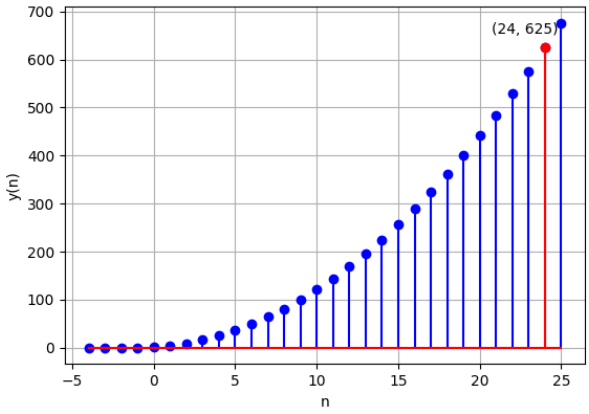

Fig. 0. stem plots of y(n)