NCERT Discrete - 11.5.9.2

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Question: The sum of three numbers in an arithmetic progression (AP) is 24 and the product of those three numbers is 440, find the values of the three numbers.

Solution: The following information is provided in the question:

Parameter	Value	Description
d	3	common difference
x(n)	$(x(0)+n\cdot d)\,u(n)$	$(n+1)$th term
y(n)	24	sum of $(n+1)$ terms
p(n)	440	product of $(n+1)$ terms

TABLE 0
PARAMETERS

Let the three numbers in the arithmetic progression be denoted as $x(0)$, $x(1)$, and $x(2)$. Then,

$$y(2) = x(0) + x(1) + x(2) \tag{1}$$
$$= (x(1) - d) + x(1) + (x(1) + d) \tag{2}$$
$$\implies 3x(1) = 24 \tag{3}$$
$$x(1) = 8 \tag{4}$$
$$\text{From Table 0} \tag{5}$$
$$p(2) = x(0)\cdot x(1)\cdot x(2) \tag{6}$$
$$= 440 \tag{7}$$
$$(8)\cdot(8-d)\cdot(8+d) = 440 \tag{8}$$
$$d = 3 \tag{9}$$
$$x(0) = x(1) - d \tag{10}$$
$$\implies x(0) = 5 \tag{11}$$
$$x(n) = (x(0) + n\cdot d)\,u(n) \tag{12}$$
$$= (5 + 3n)\,u(n) \tag{13}$$
$$\text{From equation (??)} \tag{14}$$
$$X(z) = \frac{5 - 8z^{-1}}{(1 - z^{-1})^2}; \quad |z| > |1| \tag{15}$$

Therefore, the required three numbers in AP are 5, 8, and 11.

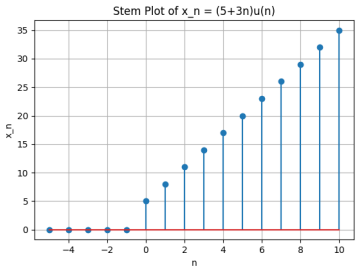

Fig. 0. stem plots of x(n)