NCERT Maths 10.5.3 Q14

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Question: Find the sum of odd numbers between 0 and 50.

Solution:

Symbol	Value	Description
$x(0)$	1	first term of AP
d	2	common difference
$x(n)$	$(1 + 2n)u(n)$	n-th term of AP

TABLE I

GIVEN PARAMETERS

Last term of the given sequence is 49.

$$\therefore (2n + 1) = 49 \tag{1}$$

$$2n = 48 \tag{2}$$

$$\implies n = 24 \tag{3}$$

Applying Z transform: From equation (**??**):

$$= \frac{1 + z^{-1}}{(1 - z^{-1})^2}, \quad |z| > |1| \tag{4}$$

For AP, the sum of first n+1 terms can be written as :

$$y(n) = x(n) * u(n) \tag{5}$$

$$Y(z) = X(z)U(z) \tag{6}$$

$$= \frac{1}{(1 - z^{-1})^2} + \frac{2z^{-1}}{(1 - z^{-1})^3}, \quad |z| > |1| \tag{7}$$

Using contour integration to find inverse Z transform:

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z)z^{n-1}dz \tag{8}$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{1}{(1 - z^{-1})^2} + \frac{2z^{-1}}{(1 - z^{-1})^3} \right) z^{n-1} dz \tag{9}$$

The sum of the terms of the sequence is computed using the residue theorem, expressed as R_i, which represents the residue of the Z-transform at $z = 1$ for the expression $Y(z)$.

$$R_i = R_1 + R_2 \tag{10}$$

R_1 and R_2 are residues calculated at the poles of the Z-transform.

$$R_1 = \frac{1}{(2 - 1)!} \frac{d(z^{25})}{dz}|_{z=1} \tag{11}$$

$$= 25 \tag{12}$$

$$R_2 = \frac{1}{(3 - 1)!} \frac{d^2(2z^{25})}{dz^2}|_{z=1} \tag{13}$$

$$= 25 \times 24 \tag{14}$$
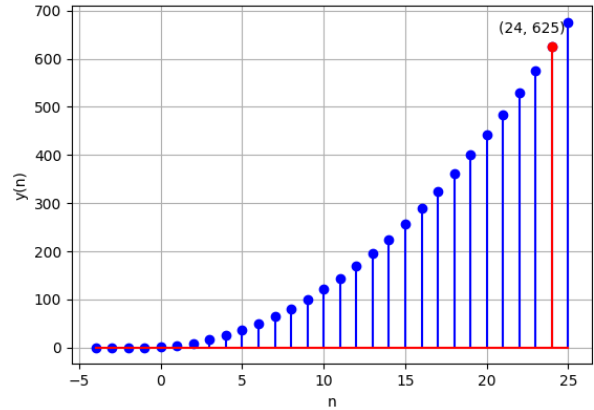
$$= 600 \tag{15}$$

$$R_i = 600 + 25 \tag{16}$$

$$\tag{17}$$

Sum of the numbers is $R_i = 625$.

Fig. 1. stem plots of y(n)