NCERT Physics 11.15 Q14

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Question: A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg, and its linear mass density is 4×10^{-2} kg/m. The length of the wire is 0.875 m. Determine the speed of a transverse wave on the string and the tension in the string.

Solution: The following information is provided in the question:

Parameters	Values
Mass per unit length	4.0×10^{-2} kg m^{-1}
Frequency	45 Hz
Mass	3.5×10^{-2} kg

TABLE I
PARAMETERS

To derive the expression for the speed of a transverse wave on a stretched string, let's start with Newton's second law applied to a small segment of the string. Consider a small element of the string of length Δx and mass Δm. The tension force T is acting in the positive y-direction, and the displacement of the string is in the y-direction.

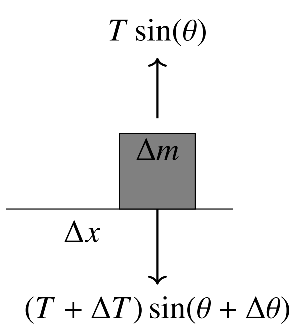

Fig. 1. FBD OF THE BODY

From the figure Fig. 1 the net force in the y-direction is given by

$$F_y = T\sin(\theta) - (T + \Delta T)\sin(\theta + \Delta\theta), \tag{1}$$

where θ is the angle of displacement.

Applying Newton's second law $F = ma$ to this element in the y-direction:

$$\Delta m \frac{\partial^2 y}{\partial t^2} = T\sin(\theta) - (T + \Delta T)\sin(\theta + \Delta\theta). \tag{2}$$

For small angles θ, $\sin(\theta) \approx \theta$, and we can simplify the expression:

$$\Delta m \frac{\partial^2 y}{\partial t^2} = T\theta - (T + \Delta T)(\theta + \Delta\theta). \tag{3}$$

Rearrange and divide by Δx to get the linear density μ:

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\theta - \frac{T}{\mu}\frac{\Delta T}{T}(\theta + \Delta\theta). \tag{4}$$

Now, take the limit as Δx approaches zero and replace θ with $\frac{\partial y}{\partial x}$:

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{\partial y}{\partial x} - \frac{T}{\mu}\frac{\partial}{\partial x}\left(\frac{\Delta T}{T}\frac{\partial y}{\partial x}\right). \tag{5}$$

Now, assume that the displacement is a sinusoidal wave of the form $y(x,t) = A\sin(kx - \omega t)$. Substitute this into the equation and solve for v:

$$v = \sqrt{\frac{T}{\mu}}. \tag{6}$$

This completes the derivation of the wave speed using first principles.

1. Find the wavelength (λ) for the fundamental mode:

$$\lambda = 2L \quad \Rightarrow \quad \lambda = 2 \times 0.875\,\text{m} = 1.75\,\text{m} \tag{7}$$

2. Use the frequency and wavelength to find the wave speed (v):

$$v = f \cdot \lambda \quad \Rightarrow \quad v = 45\,\text{Hz} \times 1.75\,\text{m} = 78.75\,\text{m/s} \tag{8}$$

3. Use the wave speed (v) to find the tension (T) using the wave equation:

$$T = \mu \cdot v^2 \quad \Rightarrow \quad T = (4 \times 10^{-2}\,\text{kg/m}) \times (78.75\,\text{m/s})^2 = 123.1875 \tag{9}$$

Therefore, the speed of the transverse wave on the string is 78.75 m/s, and the tension in the string is 123.1875 N.