

GATE 2022 BM-42

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Question: If

$$g(t) = \frac{df(t)}{dt}$$

$$F(s) = \frac{1+s}{s^2 + 12s + 32}$$

where $F(s)$ is the Laplace transform of the function $f(t)$, then what is the value of $g(t)$ at $t = 0$?
(GATE 2022 BM)

Solution:

Value	Parameter	Description
$g(t)$	$= \frac{df(t)}{dt}$	Derivative of $f(t)$ with respect to t
$F(s)$	$= \frac{1+s}{s^2+12s+32}$	Laplace transform of the function $f(t)$

TABLE 0

GIVEN PARAMETERS

$$F(s) = \frac{1+s}{(s+4)(s+8)} \quad (1)$$

Using partial fraction decomposition:

$$F(s) = \frac{A}{s+4} + \frac{B}{s+8} \quad (2)$$

$$1+s = A(s+8) + B(s+4) \quad (3)$$

$$1+s = (A+B)s + 8A + 4B \quad (4)$$

Comparing coefficients:

$$A+B=1 \quad (5)$$

$$8A+4B=1 \quad (6)$$

By Solving these equations, we will get

$$A = -\frac{3}{4} \quad (7)$$

$$B = \frac{7}{4} \quad (8)$$

$$F(s) = \frac{\frac{7}{4}}{s+8} - \frac{\frac{3}{4}}{s+4} \quad (9)$$

Inverse Laplace transform of $F(s)$

$$f(t) = \mathcal{L}^{-1}\left(\frac{\frac{7}{4}}{s+8}\right) - \mathcal{L}^{-1}\left(\frac{\frac{3}{4}}{s+4}\right) \quad (10)$$

$$= \frac{7}{4} \mathcal{L}^{-1}\left(\frac{1}{s+8}\right) - \frac{3}{4} \mathcal{L}^{-1}\left(\frac{1}{s+4}\right) \quad (11)$$

$$= \frac{7}{4} e^{-8t} - \frac{3}{4} e^{-4t} \quad (12)$$

$$g(t) = \frac{d}{dt} \left(\frac{7}{4} e^{-8t} - \frac{3}{4} e^{-4t} \right) \quad (13)$$

$$= \frac{7}{4} \cdot (-8) e^{-8t} - \frac{3}{4} \cdot (-4) e^{-4t} \quad (14)$$

$$= -14 e^{-8t} + 3 e^{-4t} \quad (15)$$

$$g(0) = -14 \times 1 + 3 \times 1 \quad (16)$$

$$= -14 + 3 = -11 \quad (17)$$

The value of $g(t)$ at $t = 0$ is -11