GATE 2022 BM-42

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Question: If

$$g(t) = \frac{df(t)}{dt} \tag{1}$$

$$F(s) = \frac{1+s}{s^2+12s+32} \tag{2}$$

where $F(s)$ is the Laplace transform of the function $f(t)$, then what is the value of $g(t)$ at $t = 0$?
(GATE BM 2022)

Solution:

Value	Parameter	Description
$g(t)$	$\frac{df(t)}{dt}$	Derivative of $f(t)$ with respect to t
$F(s)$	$\frac{1+s}{s^2+12s+32}$	Laplace transform of the function $f(t)$

TABLE 0
GIVEN PARAMETERS

Using Initial value Theorem

$$f(0) = \lim_{s\to\infty} sF(s) \tag{3}$$

$$= \lim_{s\to\infty} \frac{s(s+1)}{s^2+12s+32} \tag{4}$$

$$= 1 \tag{5}$$

$$G(s) = sF(s) - f(0) \tag{6}$$

$$= \frac{s(s+1)}{s^2+12s+32} - 1 \tag{7}$$

$$= \frac{s^2+s-(s^2+12s+32)}{s^2+12s+32} \tag{8}$$

$$G(s) = \frac{-11s-32}{s^2+12s+32}; \quad Re(s) > -4 \tag{9}$$

Using Partial fraction decomposition

$$G(s) = \frac{A}{s+4} + \frac{B}{s+8} \tag{10}$$

$$-11s - 32 = A(s+8) + B(s+4) \tag{11}$$

$$-11s - 32 = (A+B)s + (8A+4B) \tag{12}$$

Equating coefficients:

$$-11 = A + B \tag{13}$$

$$-32 = 8A + 4B \tag{14}$$

By solving these equations , we get

$$A = 3 \tag{15}$$

$$B = -14 \tag{16}$$

$$G(s) = \frac{3}{s+4} - \frac{14}{s+8} \tag{17}$$

Inverse Laplace transform of $G(s)$

$$g(t) = \mathcal{L}^{-1}\left(\frac{3}{s+4}\right) - \mathcal{L}^{-1}\left(\frac{14}{s+8}\right) \tag{18}$$

$$= 3e^{-4t} - 14e^{-8t} \tag{19}$$

$$g(0) = 3e^{-4\cdot 0} - 14e^{-8\cdot 0} \tag{20}$$

$$= 3 - 14 \tag{21}$$

$$= -11 \tag{22}$$

Verifying g(0) by Initial value theorem

$$g(0) = \lim_{s\to\infty} sG(s) \tag{23}$$

$$= \lim_{s\to\infty} s \cdot \frac{-11s-32}{s^2+12s+32} \tag{24}$$

$$= \lim_{s\to\infty} \frac{-11 - \frac{32}{s}}{1 + \frac{12}{s} + \frac{32}{s^2}} \tag{25}$$

$$= \lim_{s\to\infty} \frac{-11}{1} \tag{26}$$

$$= -11 \tag{27}$$

The value of $g(t)$ at $t = 0$ is -11.

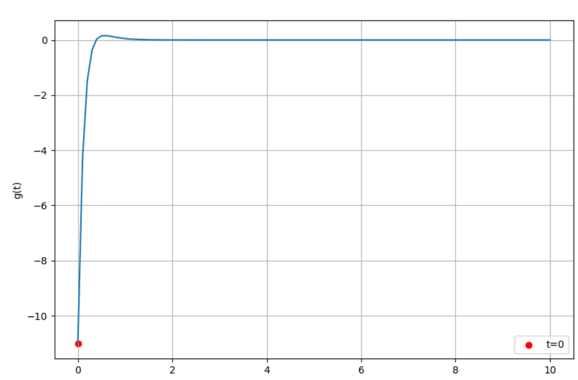

Fig. 0. Plot $g(t)$ vs t