Exercise #4

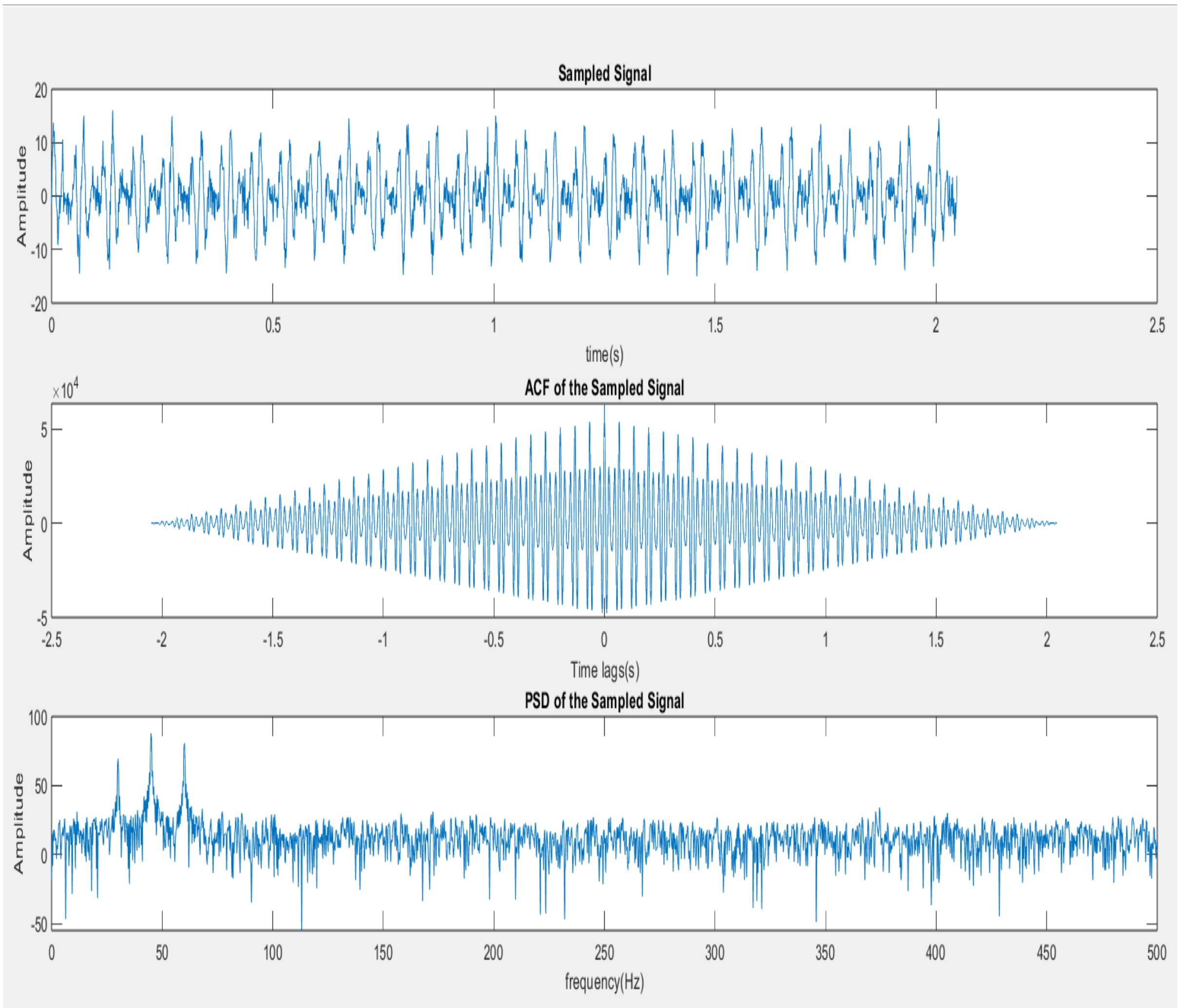
Task 4.1

Analyse the sampled time signal given in the CSV file “testsignal”. The only pre-knowledge that you’ve got is the sampling frequency which is 1 kHz. Use Matlab for reading the file, analyzing the data, and plotting the results. Discuss your results.

```
% Exercise 4
close all;
clear all;
clc;

% Read data from the csv file
data = readmatrix('C:\Users\aggar\Downloads\testsignal.csv');

Fs = 1000; % Sampling frequency
Ts = 1/Fs; % Sampling Period

N = length(data);   % Number of Sampling points
N1 = 2*N;           % Number of discrete points in FFT

t = (0:N-1)/Fs;

%figure1 = figure('Position', [200, 200, 2000, 2000]);
figure1 = figure('Position', [30, 100 ,1500, 600]);

% Plot the signal
subplot(3,1,1);
plot(t,data);
xlabel('time(s)');
ylabel('Amplitude');
title('Sampled Signal');

[auto_cor, lags] = xcorr(data); % Calculate ACF of data
size_of_ACF = length(auto_cor);
tau = lags/Fs;

% Plot the auto-correlation
subplot(3,1,2);
plot(tau,auto_cor);
xlabel('Time lags(s)');
ylabel('Amplitude');
title('ACF of the Sampled Signal');

% Calculate PSD
data_fft = fft(auto_cor);
data_fft = data_fft(1:N1/2);   % Take positive frequencies only
data_psd = (1/(Fs*N1))  * abs(data_fft).^2; % Calculate PSD
data_psd(2:end-1) = 2*data_psd(2:end-1);    % Multiply the amplitude of positive
frequencies by factor of 2
psd_freq = 0:Fs/length(auto_cor):Fs/2;
```

```
subplot(3,1,3);
plot(psd_freq, 10*log10(data_psd));
xlabel('frequency(Hz)');
ylabel('Amplitude');
title('PSD of the Sampled Signal');

% Display some basic statistics
mean_x = mean(data); % mean of signal
std_x = std(data); % standard deviation of signal
max_x = max(data); % maximum value of signal
min_x = min(data); % minimum value of signal
fprintf('Mean: %.2f\n',mean_x)
fprintf('Standard deviation: %.2f\n',std_x)
fprintf('Maximum: %.2f\n',max_x)
fprintf('Minimum: %.2f\n',min_x)
```

Matlab Code Result:




```
Mean: -0.01
Standard deviation: 5.57
Maximum: 15.96
Minimum: -15.05
```

Task 4.2

Given is the following random process with random noise added to it

$$x(\zeta, t) = \sin(2\pi f\, t) + \alpha \cdot \eta(\zeta, t)$$

where frequency f is 8 Hz, α is 0.05, and η(ζ, t) is Gaussian random noise.

a) Find the power spectral density of the random process x(ζ, t) using the Wiener Khintchine Theorem. Sketch the result.

Task 4.2

$$x(\zeta,t) = \sin(2\pi f_0 t) + \alpha \cdot n(\zeta,t)$$

Here considering $f_0 = 8$ Hz, $\alpha = 0.05$

a) PSD is given by the F.T of ACF of $x(\zeta,t)$

∴ ACF $R(\tau) = E[(\sin 2\pi f_0 t) + \alpha \cdot n(\zeta,t))(\sin(2\pi f_0(t+\tau)) + \alpha \cdot n(\zeta,t+\tau))]$

Using the properties of Expectation

$$R(\tau) = E[\sin(2\pi f_0 t) \cdot \sin(2\pi f_0 (t+\tau))] + \alpha^2 E[n(\zeta,t) \cdot n(\zeta,t+\tau)]$$

$$R(\tau) = \frac{1}{2}\cos(2\pi f_0 \tau) + \alpha^2 \sigma^2 \delta(\tau)$$

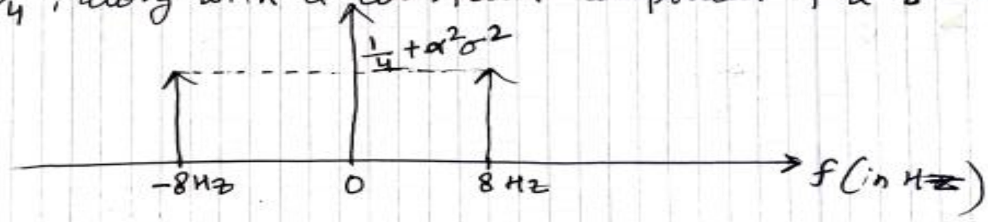
since $n(\zeta,t)$ is a guassian random noise, so its ACF is given by its variance(σ^2) multiplied by the dirac delta f^n

Now, F T of ACF will give the PSD.

$$\therefore \quad S(f) = \frac{1}{4}[\delta(f-f_0) + \delta(f+f_0)] + \alpha^2\sigma^2$$

$$S(f) = \frac{1}{4}[\delta(f-8) + \delta(f+8)] + \alpha^2\sigma^2$$

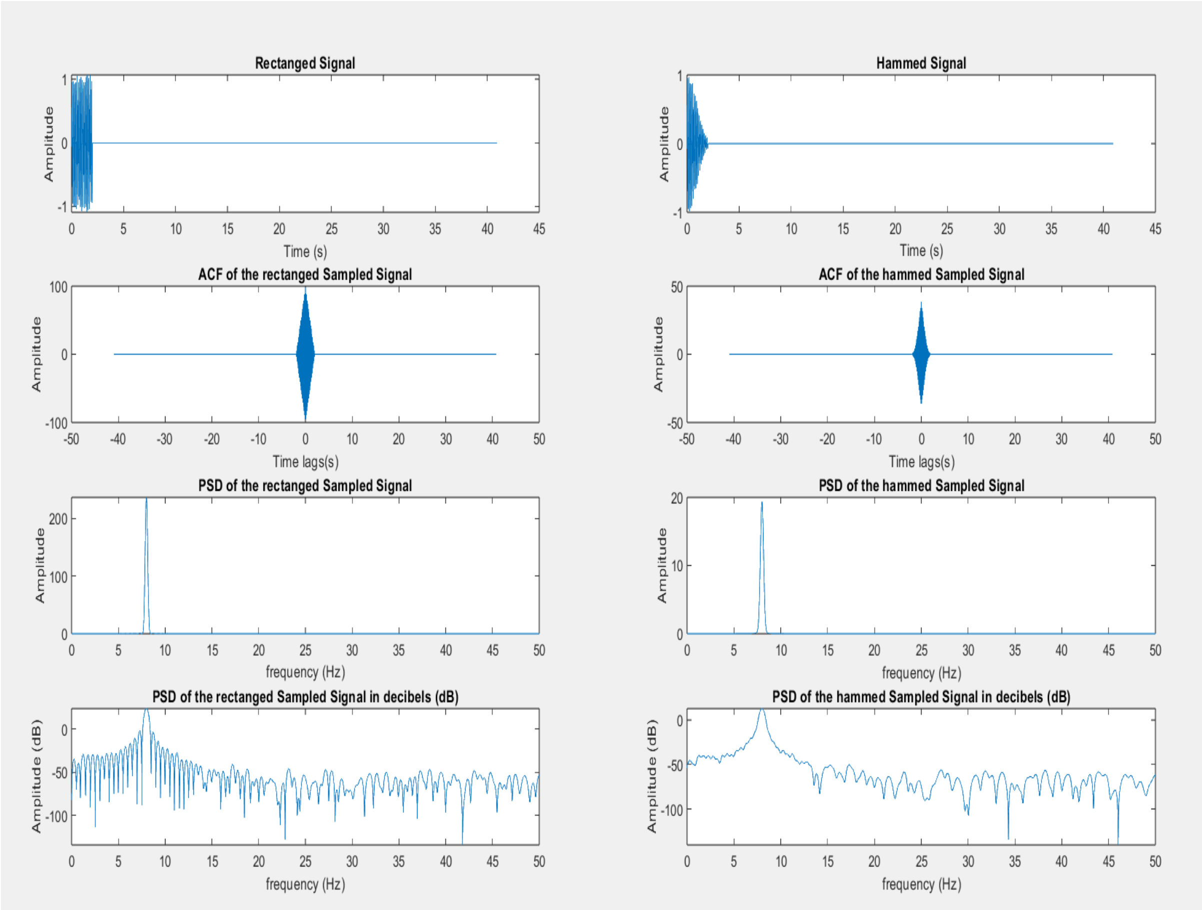
∴ The sketch of the PSD will show two spikes at frequencies −8 Hz and +8 Hz with a magnitude of $\frac{1}{4}$, along with a constant component of $\alpha^2\sigma^2$



b) Write a Matlab program. Calculate and plot the PSD using the Wiener Khintchine Theorem. The random process is sampled at a sampling frequency of 100 Hz. The sampling buffer length is 4096. Use both, rectangular and Hamming windows. Plot the PSD with a linear and logarithmic scale. Compare the outcome of the PSD calculation concerning the window type.

```
% Task 4.2 b)
close all;
clc;


N = 4096;     % Number of samples
Fs = 100;    % Sampling Frequency
t = (0:N-1)/Fs;
f1 = 8;
a1 = 0.05;
t1 = (N/2:0:N-1/2)/Fs;

x = sin(2*pi*f1*t) + a1*randn(1,length(t));
N1 = 2*N;    % Number of discrete sampling points in FFT

window_length = 200;
rect_data = rectwin(window_length);
rect_data = [rect_data' zeros(1,N-window_length)];
rect_signal = x.*rect_data;
subplot(4,2,1);
plot(t, rect_signal);
title('Rectanged Signal');
xlabel('Time (s)');
ylabel('Amplitude');

hamm_data = hamming(2*window_length);
hamm_data = hamm_data(window_length+1:end);
hamm_data = [hamm_data' zeros(1, N-window_length)];
hamm_data = hamm_data';
hamm_signal = x.*hamm_data';
subplot(4,2,2);
plot(t, hamm_signal);
title('Hammed Signal');
xlabel('Time (s)');
ylabel('Amplitude');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Rectanged Signal
[rect_auto_cor,rect_lags] = xcorr(rect_signal);
size_of_rect_ACF = length(rect_auto_cor);
rect_tau = rect_lags/Fs;
subplot(4,2,3);
plot(rect_tau,rect_auto_cor);
xlabel('Time lags(s)');
ylabel('Amplitude');
title('ACF of the rectanged Sampled Signal');

rect_data_fft = fft(rect_auto_cor);
rect_data_fft = rect_data_fft(1:N1/2);    % Take positive frequencies only
rect_data_psd = (1/(Fs*N1))  * abs(rect_data_fft).^2; % Calculate PSD
```

```
rect_data_psd(2:end-1) = 2*rect_data_psd(2:end-1);    % Multiply the amplitude of
positive frequencies by factor of 2
rect_psd_freq = 0:Fs/length(rect_auto_cor):Fs/2;

subplot(4,2,5);
plot(rect_psd_freq, rect_data_psd);
xlabel('frequency (Hz)');
ylabel('Amplitude');
title('PSD of the rectanged Sampled Signal')

subplot(4,2,7);
plot(rect_psd_freq, 10*log10(rect_data_psd));
xlabel('frequency (Hz)');
ylabel('Amplitude (dB)');
title('PSD of the rectanged Sampled Signal in decibels (dB)')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Hammed Signal
[hamm_auto_cor,hamm_lags] = xcorr(hamm_signal);
size_of_hamm_ACF = length(hamm_auto_cor);
hamm_tau = hamm_lags/Fs;
subplot(4,2,4);
plot(hamm_tau,hamm_auto_cor);
xlabel('Time lags(s)');
ylabel('Amplitude');
title('ACF of the hammed Sampled Signal');

hamm_data_fft = fft(hamm_auto_cor);
hamm_data_fft = hamm_data_fft(1:N1/2);   % Take positive frequencies only
hamm_data_psd = (1/(Fs*N1))  * abs(hamm_data_fft).^2; % Calculate PSD
hamm_data_psd(2:end-1) = 2*hamm_data_psd(2:end-1);    % Multiply the amplitude of
positive frequencies by factor of 2
hamm_psd_freq = 0:Fs/length(hamm_auto_cor):Fs/2;

subplot(4,2,6);
plot(hamm_psd_freq, hamm_data_psd);
xlabel('frequency (Hz)');
ylabel('Amplitude');
title('PSD of the hammed Sampled Signal')

subplot(4,2,8);
plot(hamm_psd_freq, 10*log10(hamm_data_psd));
xlabel('frequency (Hz)');
ylabel('Amplitude (dB)');
title('PSD of the hammed Sampled Signal in decibels (dB)')
```

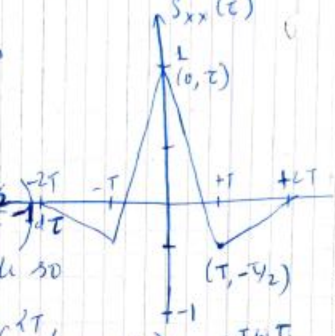


c) In both the PSD in Part a and b, we get the spike or max. amplitude at 8Hz frequency. So the PSD is part a and part b is almost similar.

Task 4.3

Task 4.3

$$S_{xx}(\omega) = \int_{-\infty}^{\infty} S_{xx}(\tau)\, e^{-j\omega\tau}\, d\tau$$



A/c to the property of PSd →

$$S_{xx}(\omega) = S_{xx}(-\omega)$$

so $\int_0^T \left(\frac{-\frac{\tau}{2}-\tau}{T}+1\right)d\tau + \int_T^{2T}\left(\frac{0+\frac{\tau}{2}}{2T-T}-1\right)d\tau$

similarly will be on negative side so

$$= 2\left[\int_0^T \left(1-\frac{3\tau}{2T}\right)e^{-j\omega\tau}\,d\tau + \int_T^{2T}\left(-1+\frac{\tau}{2T}\right)e^{-j\omega\tau}\,d\tau\right]$$

$$\Rightarrow 2\left[\int_0^T \cos\omega\tau\, d\tau - \frac{3}{2T}\int \tau\cos\omega\tau\,d\tau - \int_T^{2T}\cos\omega\tau\,d\tau + \frac{1}{2T}\int_T^{2T}\tau\cos\omega\tau\,d\tau\right]$$

$$\Rightarrow 2\left[\frac{\sin\omega\tau}{\omega}\Big|_0^T - \frac{3}{2T}\left(\frac{T\sin\omega T}{\omega}+\frac{\cos\omega T}{\omega^2} - 0 - \frac{\cos\omega T}{\omega^2}\right) - \left(\frac{\sin\omega 2T}{\omega}-\frac{\sin\omega T}{\omega}\right) + \frac{1}{2T}\left(\frac{2T\sin 2\omega T}{\omega} + \frac{\cos\omega 2T}{\omega} - \frac{T\sin\omega T}{\omega} - \frac{\cos\omega T}{\omega^2}\right)\right]$$

$$\Rightarrow 2\left[\frac{\sin\omega T}{\omega} - \frac{3T\sin\omega T}{2\omega T} - \frac{3\cos\omega T}{2T\omega^2} + \frac{3}{2T\omega^2} - \frac{\sin\omega 2T}{\omega} + \frac{\sin\omega T}{\omega} + \frac{\sin 2\omega T}{\omega} + \frac{\cos\omega 2T}{2T\omega^2} - \frac{\sin\omega T}{2\omega} - \frac{\cos\omega T}{2T\omega^2}\right]$$

$$\Rightarrow \frac{2}{\omega}\left[\cancel{\sin\omega T} - \frac{3}{2}\sin\omega T - \frac{3}{2T\omega}\cos\omega T + \frac{3}{2T\omega} - \sin\omega T + \sin\omega T + \sin 2\omega T + \frac{\cos\omega 2T}{2T\omega} - \frac{\sin\omega T}{2} - \frac{\cos\omega T}{2T\omega}\right]$$

$$\Rightarrow \frac{2}{\omega}\left[\frac{3}{2T\omega} - \frac{2\cos\omega T}{T\omega} + \frac{\cos 2T}{2\omega T}\right]$$

$$\Rightarrow \frac{2}{\omega^2}\left[\frac{3}{2T} - \frac{2\cos\omega T}{T} + \frac{\cos\omega 2T}{2T}\right]$$

$$\Rightarrow \frac{2}{\omega^2 T}\left[\frac{3}{2} - 2\cos\omega T + \frac{\cos 2\omega T}{2}\right] \quad \text{---} \quad (1)$$

from identification →

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\& \ \sin^2 x = 1 - \cos^2 x$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\Rightarrow \frac{2}{\omega^2 T}\left[\frac{3}{2} - 2\cos\omega T + \cos^2\omega T - \frac{1}{2}\right]$$

$$\Rightarrow \frac{2}{\omega^2 T}\left[1 - 2\cos\omega T + \cos^2\omega T\right]$$

$$\boxed{S_{XX}(\tau) = \frac{2}{\omega^2 T}\left[1 - \cos\omega T\right]^2}$$

To sketch →

ω	S_{XX}
0	0
π/T	$8T/\pi^2$
$2\pi/T$	0
$3\pi/T$	$8T/9\pi^2$
$4\pi/T$	0
$5\pi/T$	$8T/25\pi^2$
$6\pi/T$	0