Task 2.1 Ex-2

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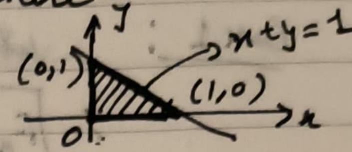
Matriculation No - 1428490

$$f_{xy}(x,y) = \begin{cases} 2 & \text{for } x \geq 0,\ y \geq 0 \text{ and } x+y \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$



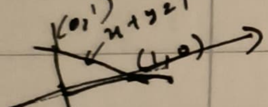
(i) $f_x(x) \stackrel{?}{=} \int_{-\infty}^{\infty} f_{xy}(x,y)\,dy$

$\because$ limits of dy varies from $y \geq 0$ to $y \leq 1-x$ for $f_{xy}(x,y) = 2$

$$\therefore f_x(x) = \int_0^{1-x} 2 \cdot dy \qquad \text{for } 0 \leq x \leq 1$$

$$= 2\,[y]_{y=0}^{y=1-x}$$

$$= 2(1-x)$$



$$\therefore \boxed{f_x(x) = \begin{cases} 2(1-x) & \text{for } 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}}$$

(ii) $f_y(y) = \int_{-\infty}^{\infty} f_{xy}(x,y)\,dx$

Limit of x varies from 0 to $1-y$ for $f_{xy}(x,y) = 2$

$$\therefore f_y(y) = \int_0^{1-y} 2 \cdot dx$$

$$= 2\,[x]_{x=0}^{x=1-y}$$

$$\boxed{f_y(y) = \begin{cases} 2(1-y) & \text{for } 0 \leq y \leq 1 \\ 0 & \text{elsewhere} \end{cases}}$$

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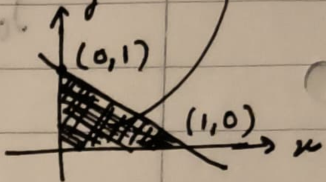
(iii) $F_{xy}(x,y) \rightarrow$ Joint cdf

<u>case 1:</u> $F_{xy}(x,y)$ for $x \le 0$ & $y \le 0$

$$F_{xy}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{xy}(u,v)\,du\,dv$$

$$\boxed{F_{xy}(x,y) = 0}$$

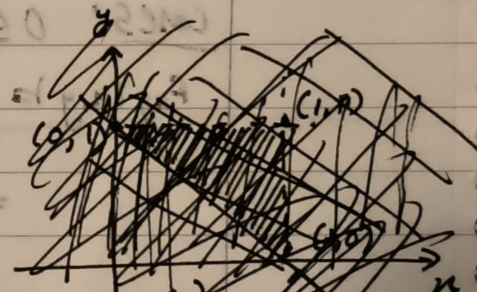
<u>case 2:</u> For $x \geq 0,\ y \geq 0$ & $x+y \leq 1$



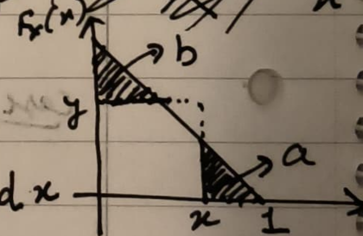
$$F_{xy}(x,y) = \int_0^x \int_0^y 2\,dy\,dx$$

$$\boxed{F_{xy}(x,y) = 2xy}$$

<u>case 3:</u> For $0 \le x \le 1,\ 0 \le y \le 1,\ x+y \geq 1$



$$F_{xy}(x,y) = 1 - \iint_a 2\,dx\,dy - \iint_b 2\,dy\,dx$$



$$= 1 - \int_y^1 \int_0^{1-y} 2\,dx\,dy - \int_x^1 \int_0^{1-x} 2\,dy\,dx$$

$$= 1 - \int_y^1 [2x]_0^{1-y}\,dy - \int_x^1 [2y]_0^{1-x}\,dx$$

$$= 1 - \int_y^1 [2-2y]\,dy - \int_x^1 [2-2x]\,dx$$

$$= 1 - \left[2y - y^2\right]_y^1 - \left[2x - x^2\right]_x^1$$

$$= 1 - (2-1-2y+y^2) - (2-1-2x+x^2)$$

$$= 1 - (1-2y+y^2) - (1-2x+x^2)$$

$$= 1 - (1-y)^2 - (1-x)^2$$

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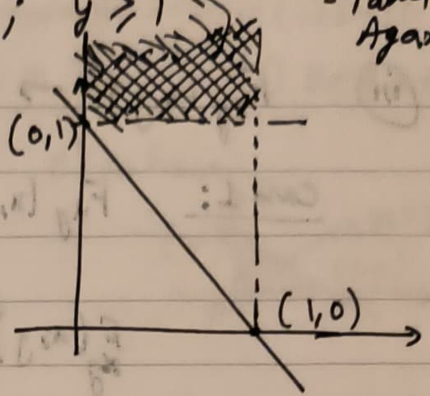
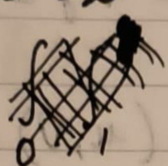
Case 4: For $0 \le x \le 1$; $y \ge 1$



$$F_{xy}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{xy}(x,y)\, dy\, dx$$



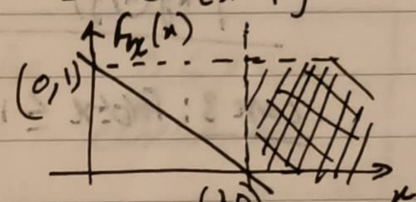
$$= \int_{0}^{x}\int_{0}^{1-x} 2\, dy\, dx$$

$$= \int_0^x [2y]_0^{1-x} dx = \int_0^x (2-2x)\, dx$$

$$= \left[2x - \frac{2x^2}{2}\right]_0^x$$

$$= 2x - x^2 = 1-(x-1)^2$$

Case 5: $0 \le y \le 1$; $x \ge 1$



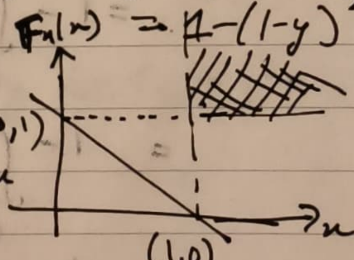
$$F_{xy}(x,y) = \int_0^{1-y}\int_0^{y} 2\, dy\, dx$$

$$= \int_0^{1-y} [2y]_0^{y} dx = [2yx]_0^{1-y}$$

$$= 2y(1-y) = 2y - y^2$$

$$F_x(x) = 1-(1-y)^2$$

Case 6: $y \ge 1$; $x \ge 1$



$$F_{xy}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{xy}(x,y)\, dy\, dx$$

$$\boxed{F_{xy}(x,y) = 1}$$

Conclusion

$$F_{xy}(x,y) = \begin{cases} 0 & \text{for } x \le 0 \,\&\, y \le 0 \\ & \text{for } x \le 0 \text{ or } y \le 0 \\ 2xy & \text{for } x \ge 0;\ y \ge 0;\ x+y \le 1 \\ 1-(1-y)^2-(1-x^2) & \text{for } 0 \le x \le 1;\ 0 \le y \le 1;\ x+y \ge 1 \\ 1-(1-x)^2 & \text{for } 0 \le x \le 1;\ y \ge 1 \\ 1-(1-y)^2 & \text{for } 0 \le y \le 1;\ x \ge 1 \\ 1 & \text{for } y \ge 1;\ x \ge 1 \end{cases}$$

(IV) $F_x(x) = F_{xy}(x, +\infty)$

$$F_x(x,y) = \begin{cases} 0 & \text{for } x \le 0 \\ 1-(1-x)^2 & \text{for } 0 \le x \le 1 \\ 1 & \text{for } x \ge 1 \end{cases}$$

(V) $F_y(y) = F_{xy}(+\infty, y)$

$$= \begin{cases} 0 & \text{for } y \\ 1-(1-y)^2 & \text{for } 0 \le y \le 1 \\ 1 & \text{for } y \ge 1 \end{cases}$$

Task 2.2

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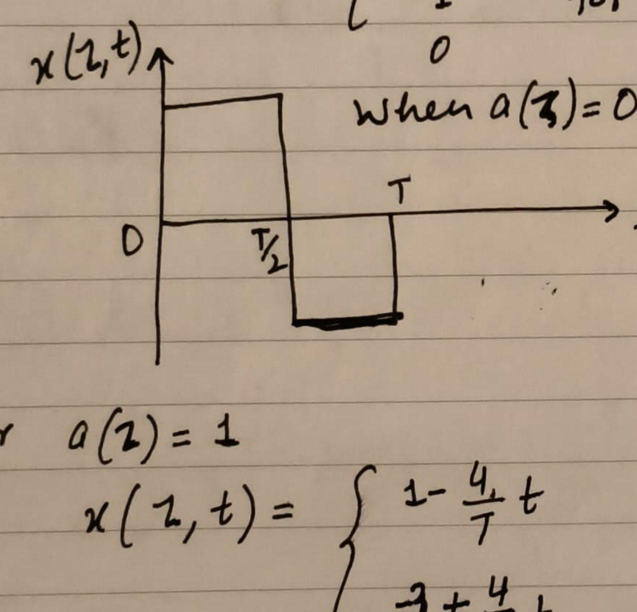
$$x(\zeta,t) = \begin{cases} 1-\frac{4}{T}t\, a(\zeta) & \text{for } 0 \le t < \frac{T}{2} \\ -1+\left(\frac{4}{T}t-2\right) a(\zeta) & \text{for } \frac{T}{2} \le t < T \\ 0 & \end{cases}$$

a) To sketch the distinct pattern f^n, we have to consider different cases based on the value of t and the random variable $a(\zeta)$

For $a(\zeta) = 0$

$$x(\zeta,t) = \begin{cases} 1 & \text{for } 0 \le t < \frac{T}{2} \\ -1 & \text{for } \frac{T}{2} \le t < T \\ 0 & \text{elsewhere} \end{cases}$$

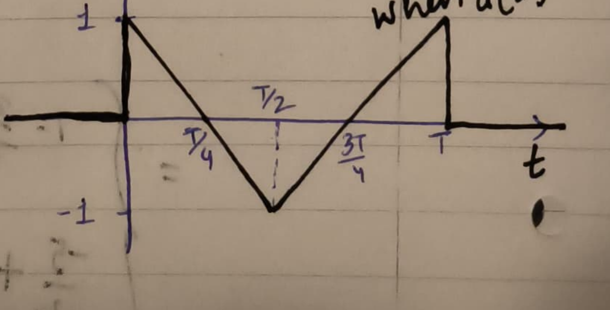


when $a(\zeta)=0$

For $a(\zeta) = 1$

$$x(\zeta,t) = \begin{cases} 1-\frac{4}{T}t & \text{for } 0 \le t < \frac{T}{2} \\ -3+\frac{4}{T}t & \text{for } \frac{T}{2} \le t < T \\ 0 & \text{elsewhere} \end{cases}$$

t	$x(\zeta,t)$
0	1
T/8	1/2
T/4	0
T/2	-1
3T/4	0
7T/8	1/2
$\lim_{t\to T}$	1



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b) $m_x^{(1)}(t) = \sum_{v=1}^{2} x(2,t)\, p_v$

$$= E[x(1,t)] = \begin{cases} E\left(1 - \frac{4}{T} t\, a(2)\right) & \text{for } 0 \le t < \frac{T}{2} \\ E\left(-1 + \left(\frac{4}{T} t - 2\right) a(2)\right) & \text{for } \frac{T}{2} \le t < T \end{cases}$$

For $a(2) = 0$, $P_0 = \frac{1}{4}$

and for $a(2) = 1$, $P_1 = \frac{3}{4}$

$\therefore\ m_x^{(1)}(t) = E[x(1,t)] = \sum_{v=0}^{1} x(1,t)\, p_v$

$$= P_0\, x(1,t) + P_1\, x(1,t)$$

$$m_x^{(1)}(t) = \begin{cases} \frac{1}{4}\left(1 - \frac{4}{T} t \cdot 0\right) + \frac{3}{4}\left(1 - \frac{4}{T} t \cdot 1\right) & \text{for } 0 \le t < \frac{T}{2} \\ \frac{1}{4}(-1) + \frac{3}{4}\left(-1 + \frac{4}{T} t - 2\right) & \text{for } \frac{T}{2} \le t < T \end{cases}$$

$$= \begin{cases} \frac{1}{4} + \frac{3}{4} - \frac{3}{T} t & \text{for } 0 \le t < \frac{T}{2} \\ -\frac{1}{4} - \frac{9}{4} + \frac{3}{T} t & \text{for } \frac{T}{2} \le t < T \end{cases}$$

$$= \begin{cases} 1 - \frac{3}{T} t & \text{for } 0 \le t < \frac{T}{2} \\ -\frac{5}{2} + \frac{3}{T} t & \text{for } \frac{T}{2} \le t < T \\ 0 & \text{elsewhere} \end{cases}$$

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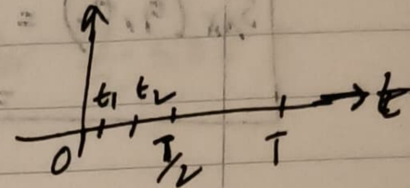
c) $$S_{xx}(t_1,t_2) = E\{x(\zeta,t_1)\,x(\zeta,t_2)\}$$
$$= \sum_{\nu=0}^{1} x_\nu(t_1)\,x_\nu(t_2)\,P_\nu$$

Case 1 → When t_1 and t_2 both are located in $0 \le t < \frac{T}{2}$

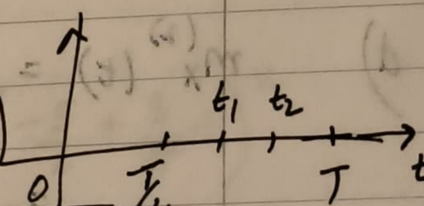


$$S_{xx}(t_1,t_2) = \left(1-\frac{4}{T}t_1\cdot 0\right)\left(1-\frac{4}{T}t_2\cdot 0\right)\left(\frac{1}{4}\right) + \left(1-\frac{4}{T}t_1\cdot 1\right)\left(1-\frac{4}{T}t_2\right)\cdot\frac{3}{4}$$

$$= (1)(1)\left(\frac{1}{4}\right) + \left(1-\frac{4}{T}t_1\right)\left(1-\frac{4}{T}t_2\right)\cdot\frac{3}{4}$$

$$\boxed{S_{xx}(t_1,t_2) = \frac{1}{4} + \frac{3}{4}\left(1-\frac{4}{T}t_1\right)\left(1-\frac{4}{T}t_2\right)}$$

Case 2 → When t_1 and t_2 both are located in $\frac{T}{2} \le t < T$



$$S_{xx}(t_1,t_2) = \frac{1}{4}(-1)(-1) + \frac{3}{4}\left(-3+\frac{4}{T}t_1\right)\left(-3+\frac{4}{T}t_2\right)$$

$$\boxed{S_{xx}(t_1,t_2) = \frac{1}{4} + \frac{3}{4}\left(-3+\frac{4}{T}t_1\right)\left(-3+\frac{4}{T}t_2\right)}$$

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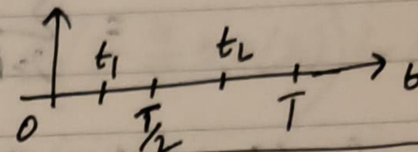
Case 3 → when t_1 lies between $0 \le t < \frac{T}{2}$

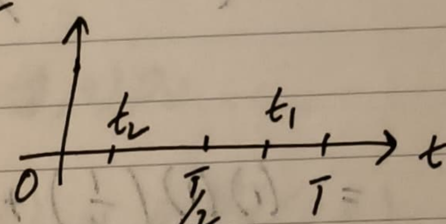
and t_2 lies between $\frac{T}{2} \le t < T$



$$\boxed{\begin{aligned} S_{XX}(t_1,t_2) &= \frac{1}{4}(1)(-1) + \frac{3}{4}\left(1-\frac{4}{T}t_1\right)\left(-3+\frac{4}{T}t_2\right) \\ &= -\frac{1}{4} + \frac{3}{4}\left(1-\frac{4}{T}t_1\right)\left(-3+\frac{4}{T}t_2\right) \end{aligned}}$$

Case 4 → When t_2 lies between $0 \le t < \frac{T}{2}$

and t_1 lies between $\frac{T}{2} \le t < T$



$$S_{XX}(t_1,t_2) = \frac{1}{4}(-1)(+1) + \frac{3}{4}\left(-3+\frac{4}{T}t_1\right)\left(1-\frac{4}{T}t\right)$$

$$\boxed{S_{XX}(t_1,t_2) = -\frac{1}{4} + \frac{3}{4}\left(-3+\frac{4}{T}t_1\right)\left(1-\frac{4}{T}t\right)}$$

d) $$m_X^{(2)}(t) = \sum_{\nu=0}^{1} x^2(\nu,t)\, p_\nu$$

$$= p_0 x^2(\nu,t) + p_1 x^2(\nu,t)$$

$$m_X^{(2)}(t) = \begin{cases} \frac{1}{4}(1)^2 + \frac{3}{4}\left(1-\frac{4}{T}t.1\right)^2 & \text{for } 0 \le t < \frac{T}{2} \\ \frac{1}{4}(-1)^2 + \frac{3}{4}\left(-1+\frac{4}{T}t-2\right)^2 & \text{for } \frac{T}{2} \le t < T \end{cases}$$

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$$m_x^{(2)}(t) = \begin{cases} \frac{1}{4} + \frac{3}{4}\left(1 + \frac{16}{T^2}t^2 - \frac{8}{T}t\right) & \text{for } 0 \le t < \frac{T}{2} \\ \frac{1}{4} + \frac{3}{4}\left(9 + \frac{16}{T^2}t^2 - \frac{24}{T}t\right) & \text{for } \frac{T}{2} \le t < T \end{cases}$$

$$m_x^{(2)}(t) = \begin{cases} 1 + \frac{12}{T^2}t^2 - \frac{6}{T}t & \text{for } 0 \le t < \frac{T}{2} \\ 7 + \frac{12}{T^2}t^2 - \frac{18}{T}t & \text{for } \frac{T}{2} \le t < T \end{cases}$$

Case 1: for $0 \le t < \frac{T}{2}$

$$\sigma_x^2(t) = m_x^{(2)}(t) - \left[m_x^{(1)}(t)\right]^2$$

$$= 1 + \frac{12}{T^2}t^2 - \frac{6}{T}t - \left(1 - \frac{3}{T}t\right)^2$$

$$\sigma_x^2(t) = \cancel{1} + \frac{12}{T^2}t^2 - \cancel{\frac{6}{T}t} - \cancel{1} - \frac{9}{T^2}t^2 + \cancel{\frac{6}{T}t}$$

$$\boxed{\sigma_x^2(t) = \frac{3}{T^2}t^2} \quad \text{for } 0 \le t < \frac{T}{2}$$

Case 2: for $\frac{T}{2} \le t < T$

$$\sigma_x^2(t) = m_x^{(2)}(t) - \left[m_x^{(1)}(t)\right]^2$$

$$= 7 + \frac{12}{T^2}t^2 - \frac{18}{T}t - \left(-\frac{5}{2} + \frac{3}{T}t\right)^2$$

$$= 7 + \frac{12}{T^2}t^2 - \frac{18}{T}t - \frac{25}{4} - \frac{9}{T^2}t^2 + \frac{15}{T}t$$

$$\boxed{\sigma_x^2(t) = \frac{3}{4} + \frac{3}{T^2}t^2 - \frac{3}{T}t} \quad \text{for } \frac{T}{2} \le t < T$$

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$$\text{variance} = \sigma^2(t) = \begin{cases} \dfrac{3t^2}{T^2} \; ; & 0 \le t < \dfrac{T}{2} \\ \dfrac{3}{4} + \dfrac{3}{T^2}t^2 - \dfrac{3t}{T} \; ; & \dfrac{T}{2} \le t < T \\ 0 \; ; & \text{otherwise} \end{cases}$$

$< \frac{T}{2}$

$< T$

$\frac{T}{2}$

T

Task 2.3 Matlab Code:

```
clear all;
clc;
close all;
% Number of samples
N = 600;

% Given Frequency of the sinewave
f1=1;

% sampling frequency
FS = 100;

% time axis will begin from -1.5 to 1.5
t = (-N/2:N/2-1)*1/FS;
t1 = -3.5:0.01:3.5;

% Defining sine wave
y = sin(2*pi*f1*t1);

% Genrating the bigger plot for all examples
figurel= figure('Position',[200, 200, 1500, 1500]);

% Define the position 4 rows 2 columns position 1
subplot(4,2,1);

% Plot generated with respect to time domain and y is Sin wave.
plot(t1,y,'g','LineWidth',2);
title("Sine wave");
xlim([-3.5 3.5]);
xlabel ("Time in secs");
ylabel ("Amplitude");
grid;

% Define the position 4 rows 2 columns position 2.
subplot(4,2,2);

% Plot generated with respect to time domain and y is Sin wave
plot(t1, y, 'g', 'LineWidth', 2);
xlim([-3.5 3.5]);
title("Sine wave");
xlabel("Time in secs");
ylabel("Amplitude");
grid;

% Generates hamming window of length same as no of smaples as t.
ham = hamming(length(t));

% Define the position 4 rows 2 columns position 3
subplot(4,2,3);

% Plot generated W.r.t time domain and hamming window.
Ham_Data = [zeros(1,50), ham'];
Ham_Data=[Ham_Data zeros(1,50)];
Ham_data = Ham_Data';
t1(end) = [];

plot(t1, Ham_data-1, 'r','LineWidth', 2);
```

```
title("Hamming window");
xlim([-3.5 3.5]);
xlabel ("Time in secs");
ylabel( "Amplitude" );
grid;

% Generates rectangular window of length same as no of smaples as t
rect = rectwin(length(t));
Rect_Data = [zeros(1,50), rect'];
Rect_Data = [Rect_Data zeros(1,50)];

% Define the position 4 rows 2 columns position 4
subplot (4,2,4);

% Plot generated W.r.t time domain with rectangular window.
plot(t1, Rect_Data, 'r', 'LineWidth', 2);
xlim([-3.5 3.5]);
ylim([0 2]);
title("Rectangular Window");
xlabel("Time in secs");
ylabel("Amplitude");
grid;

% Using hamming window on sinosodal signal
y = sin(2*pi*f1*t);
hammedSignal = y.*ham';

hammedSignal_Data = [zeros(1,50), hammedSignal];
hammedSignal_Data = [hammedSignal_Data zeros(1,50)];

% Define the position 4 rows 2 columns position 5
subplot (4,2,5);

% Plot generated w.r.t time domain with Hammed Signal.
plot(t1, hammedSignal_Data, 'r', 'LineWidth', 2);
title("Hammed Signal");
xlim([-3.5 3.5]);
xlabel("Time in secs");
ylabel ("Amplitude");
grid;

% converted the row into col impulse repsponse
% Applying the window on whole signal
rectSignal = y.*rect';
rectSignal_Data = [zeros(1,50), rectSignal];
rectSignal_Data = [rectSignal_Data zeros(1,50)];

% Define the position 4 rows 2 columns position 6
subplot (4,2,6);

% Plot generated w.r.t time domain with rectanged Signal.
plot(t1, rectSignal_Data, 'r','LineWidth',2);
title("Rectanged signal");
xlim([-3.5 3.5]);
xlabel("Time in secs");
ylabel("Amplitude");
grid;

% To normalise the y axis we are passing the ‘biased’ on Hammed Signal.
```

```
[correlationOfHammedSignal, hammedLags] = xcorr (hammedSignal_Data, 'biased');

% To normalise the y axis we are passing the ‘biased’ on Rectanged Signal.
[correlationOfRectSignal, rectLags] = xcorr(rectSignal_Data, 'biased');

% To normalise the tau for Hamming Lags we have to multiply by 1/Fs
tauH = hammedLags*1/FS;

% To normalise the tau for we have to multiply by 1/Fs
tausR = rectLags*1/FS;

% Define the position 4 rows 2 columns position 7
subplot(4,2,7);

% Correlated hammed signal plotting with the same sin wave w.r.t tauH
plot(tauH, correlationOfHammedSignal, 'r', 'LineWidth',2);
title("Auto correlated Hammed signal");
xlim([-7 7]);
xlabel("\taus");
ylabel("x(t) * x(t)");

% Define the position 4 rows 2 columns position 8
subplot(4,2,8);

% Correlated rectanged signal plotting w.r.t tausR
plot(tausR, correlationOfRectSignal, 'r', 'LineWidth',2);
title("Auto correlated Rectaged signal");
xlim([-7 7]);
xlabel("\taus");
ylabel("x(t) * x(t)");
```

Output: