Exercise #5

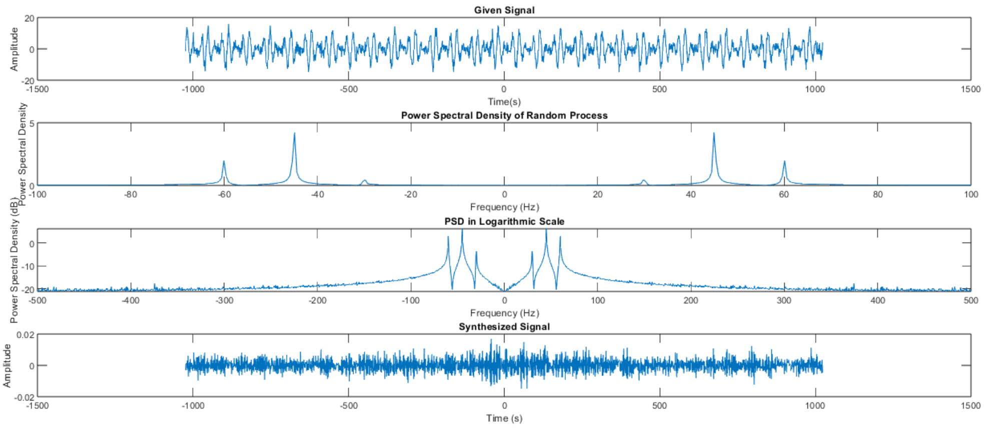
Task 5.1 Digital Signal Synthesis

Write a Matlab program. Synthesize the digital signal that you analyzed in task 4.1 based on the calculated PSD. The timeframe of the synthesized signal and the original signal should have the same length. Plot the signal.

```
clear all;
close all;
clc;
data = readmatrix('C:\Users\aggar\Downloads\testsignal.csv'); % Replace 'data.csv' with the
actual filename

% Access specific columns or rows
signal = data(1, :);
N = length(signal);
fs = 1000;

t = -N/2:N/2-1;

%To make the figure bigger and clear
figure1 = figure('Position', [30, 100 ,1500, 600]);

%Plot the signal
subplot(4,1,1);
plot(t, signal)
xlabel('Time(s)');
ylabel('Amplitude');
title('Given Signal');

%To find the minimum and maximum amplitude of the signal
max_value = max(signal);
min_value = min(signal);
disp(['Maximum value: ' num2str(max_value)]);
disp(['Minimum value: ' num2str(min_value)]);

%To find the PSD by using the ACF. So first find out the ACF by using Auto
%correlation function
ACF = xcorr(signal, 'biased');
PSD = abs(fftshift(fft(ACF, 2^nextpow2(N))))/N;

%Let's just guess what will be the frequency range
rangeOfPSD = length(PSD);
freq = (-fs/2):(fs/rangeOfPSD):(fs/2 - fs/rangeOfPSD);

subplot(4,1,2);
plot(freq, PSD)
xlabel('Frequency (Hz)')
ylabel('Power Spectral Density')
title('Power Spectral Density of Random Process')
xlim([-100, 100]);


subplot(4, 1, 3);
plot(freq, 10*log10(PSD));
ylabel('Power Spectral Density (dB)');
```

```
xlabel('Frequency (Hz)');
title('PSD in Logarithmic Scale');

% Synthesize the signal based on the PSD
synthesized_signal = ifft(sqrt(PSD) .* exp(1i*angle(fftshift(fft(ifftshift(signal),
2^nextpow2(N))))));

% Plotting the Synthesized Signal
subplot(4, 1, 4);
plot(t, real(synthesized_signal));
ylabel('Amplitude');
xlabel('Time (s)');
title('Synthesized Signal');
```

```
Maximum value: 15.963
Minimum value: -15.053
```



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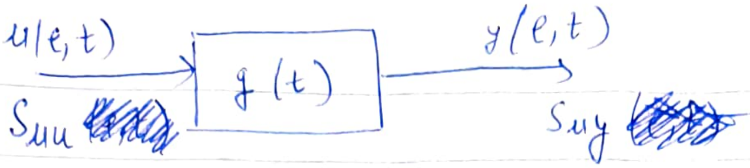
Task 5.2

a)



$$S_{uy} = \frac{S_1}{(1-J\omega b)(1+J\omega T_1)} \quad , \quad S_{yy} = \frac{S_1}{1+\omega^2 T_1^2}$$

$$S_{yy}(\omega) = H(J\omega)\, G^*(J\omega)\; S_{xx}(\omega)$$

$$S_{yy}(\omega) = \cancel{H(J\omega)}\, G^*(J\omega) \cdot \frac{S_{uy}(\omega)}{\cancel{H(J\omega)}}$$

$$G^*(J\omega) = \frac{S_{yy}(\omega)}{S_{uy}(\omega)} = \frac{\cancel{S_1}/(1+\omega^2 T_1^2)}{\cancel{S_1}/(1-J\omega b)(1+J\omega T_1)}$$

$$= \frac{(1-J\omega b)(1+J\omega T_1)}{1+\omega^2 T_1^2}$$

$$= \frac{(1-J\omega b)\cancel{(1+J\omega T_1)}}{(1-J\omega T_1)\cancel{(1+J\omega T_1)}}$$

$$G^*(J\omega) = \frac{1-J\omega b}{1-J\omega T_1}$$

$$G(J\omega) = \frac{1+J\omega b}{1+J\omega T_1}$$

b) $1 + jw T_1 = 0$] For casual system, we need to know the poles

$jwT_1 = -1$

$$w = \frac{-1}{jT_1} \Rightarrow \frac{j}{T_1}$$

$$\boxed{w = \frac{j}{T_1}}$$

So, T_1 must be greater than 0 for system to be casual.

$$\boxed{T_1 > 0}$$

c) $S_{uu}(\tau) = ?$

$$S_{uu} = \frac{S_{uy}}{G(jw)} = \frac{S_1 / (1-jwb)(1+jwT_1)}{(1+jwb) / (1+jwT_1)}$$

$$= \frac{S_1 \cancel{(1+jwT_1)}}{(1+jwb)(1-jwb)\cancel{(1+jwT_1)}}$$

$$S_{uu} = \frac{S_1}{1 + w^2 b^2}$$

$$S_{uu}(\tau) = \mathcal{F}^{-1}(S_{uu})$$

$$= \mathcal{F}^{-1}\left[\frac{S_1}{b^2\left(\frac{1}{b^2} + w^2\right)}\right]$$

$$\boxed{S_{uu}(\tau) = \frac{S_1}{2b} e^{-\left|\frac{\tau}{b}\right|}}$$

Task 5.3

$$S_{xx}(\tau) = ae^{-\alpha|\tau|} + b$$

Mean $= 0$, S.D $= 1$

$$y(\xi, t) = \begin{cases} 0 & \text{for } t \le t_0 \\ \int_{t_0}^{t} x(\xi, \lambda)\, d\lambda & \text{for } t > t_0 \end{cases}$$

a) $m_x^{(1)}(t) = \sqrt{\lim_{\tau \to \infty} S_{xx}(\tau)}$

$0 = \sqrt{\lim_{\tau \to \infty} ae^{-\alpha|\tau|} + b}$

$0 = \sqrt{0 + b}$

$\boxed{b = 0}$

$m_x^{(2)}(t) = \lim_{\tau \to 0} S_{xx}(\tau)$

$= \lim_{\tau \to 0} ae^{-\alpha|\tau|} + b$

$= a(1) + b$

$= a + 0 = a$

variance, $\sigma_x^2(t) = m_x^{(2)}(t) - \left(m_x^{(1)}(t)\right)^2$

$\sigma_x^2(t) = a - (0)^2$

$1^2 = a \Rightarrow \boxed{a = 1}$

$\therefore \; S_{XX}(\tau) = a e^{-\alpha|\tau|} + b$

Putting $\cancel{a} a = 1,\ b = 0$

$$\boxed{S_{XX}(\tau) = e^{-\alpha|\tau|}}$$

b) $S_{xy}(t_1, t_2) = E\{x(\zeta, t_1)\, y(\zeta, t_2)\}$

$$= E\left\{x(\zeta, t_1)\int_{t_0}^{t_2} x(\zeta, \lambda)\, d\lambda\right\}$$

$$= \int_{t_0}^{t_2} E\{x(\zeta, t_1).\, x(\zeta, \lambda)\, d\lambda\}$$

$$= \int_{t_0}^{t_2} S_{xx}(\lambda - t_1)\, d\lambda$$

$$S_{xy}(t_1, t_2) = \int_{t_0}^{t_2} e^{-\alpha|\lambda - t_1|}\, d\lambda$$

Case 1: $t_0 \le t_1 \le t_2$

$t_0 \quad t_1 \quad t_2$

$$S_{xy}(t_1, t_2) = \int_{t_0}^{t_1} \underset{\substack{\downarrow \\ (\text{Here } t_0 < \lambda < t_1)}}{e^{-\alpha|\lambda - t_1|}}\, d\lambda + \int_{t_1}^{t_2} \underset{\substack{\downarrow \\ (\text{Here } t_1 < \lambda < t_2)}}{e^{-\alpha|\lambda - t_1|}}\, d\lambda$$

$$\therefore \; = \int_{t_0}^{t_1} e^{-\alpha(t_1 - \lambda)}\, d\lambda + \int_{t_1}^{t_2} e^{-\alpha(\lambda - t_1)}\, d\lambda$$

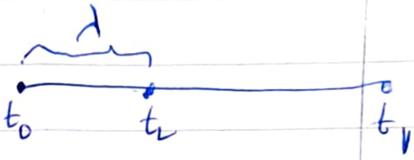
$$= \frac{1}{\alpha}\left[e^{-\alpha(t_1 - \lambda)}\right]_{t_0}^{t_1} - \frac{1}{\alpha}\left[e^{-\alpha(\lambda - t_1)}\right]_{t_1}^{t_2}$$

$$= \frac{1}{\alpha}\left[1 - e^{-\alpha(t_1 - t_0)} - e^{-\alpha(t_2 - t_1)} + 1\right]$$

$$\boxed{S_{xy}(t_1, t_2) = \frac{1}{\alpha}\left[2 - e^{-\alpha(t_1 - t_0)} - e^{-\alpha(t_2 - t_1)}\right]}$$

Case 2: $t_0 \le t_2 \le t_1$

Here $t_0 < \lambda < t_2$



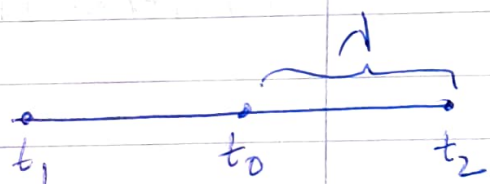
$$S_{xy}(t_1, t_2) = \int_{t_0}^{t_2} e^{-\alpha|\lambda - t_1|}\, d\lambda$$

$$= \int_{t_0}^{t_2} e^{-\alpha(t_1 - \lambda)}\, d\lambda$$

$$= \frac{1}{\alpha}\left[e^{-\alpha(t_1 - \lambda)}\right]_{t_0}^{t_2}$$

$$\boxed{S_{xy}(t_1, t_2) = \frac{1}{\alpha}\left[e^{-\alpha(t_1 - t_2)} - e^{-\alpha(t_1 - t_0)}\right]}$$

Case 3: $t_1 \le t_0 \le t_2$



$$S_{xy}(t_1, t_2) = \int_{t_0}^{t_2} e^{-\alpha|\lambda - t_1|}\, d\lambda$$

Here $\lambda > t_1$

$$= \int_{t_0}^{t_2} e^{-\alpha(\lambda - t_1)}\, d\lambda$$

$$= \frac{-1}{\alpha}\left[e^{-\alpha(\lambda - t_1)}\right]_{t_0}^{t_2}$$

$$= \frac{-1}{\alpha}\left[e^{-\alpha(t_2 - t_1)} - e^{-\alpha(t_0 - t_1)}\right]$$

$$\boxed{S_{xy}(t_1, t_2) = \frac{1}{\alpha}\left[e^{-\alpha(t_0 - t_1)} - e^{-\alpha(t_2 - t_1)}\right]}$$

Case 4: elsewhere $\Rightarrow S_{xy}(t_1, t_2) = 0$

$$\therefore \quad S_{xy}(t_1, t_2) = \begin{cases} \frac{1}{\alpha}\left(2 - e^{-\alpha(t_1 - t_0)} - e^{-\alpha(t_2 - t_1)}\right); & \text{for } t_0 \le t_1 \le t_2 \\ \frac{1}{\alpha}\left(e^{-\alpha(t_1 - t_2)} - e^{-\alpha(t_1 - t_0)}\right); & \text{for } t_0 \le t_2 \le t_1 \\ \frac{1}{\alpha}\left[e^{-\alpha(t_0 - t_1)} - e^{-\alpha(t_2 - t_1)}\right); & \text{for } t_1 \le t_0 \le t_2 \\ 0 & ; \text{elsewhere} \end{cases}$$

c) $y(\zeta, t)$ is not stationary process because of y depends on x and S_{xy} depends on time.