DEPARTMENT OF CSE
University Institute of Engineering and Technology
C.S.J.M UNIVERSITY KANPUR
Mathematics-III MTHS-201 [CSE]

Semester: 2023 -24 (Odd Semester) **YEAR: 2^{nd} (2K22)**

END SEMESTER EXAMINATION

Time: 3:00 hr Maximum **Marks: 50**

ALL QUESTIONS ARE COMPULSORY

SECTION-A 1×10=10

QUESTION -1

(a) Find the derivative of $f(z) = e^x(\cos y + i\sin y)$
(b) Evaluate $\oint_c (5z^4 - z^3 + 2)\, dz$ around the unit circle $|z| = 1$
(c) Discuss the singularity of $\frac{\cot \pi z}{(z-a)^2}$ at $z = \infty$
(d) Find the Fourier transform of $F(x) = e^{-|x|}$
(e) Write the inversion formula for Fourier sine transform
(f) Form partial differential equation by eliminating arbitrary constants of $x^2 + y^2 = (z-c)^2 \tan^2\alpha$
(g) Solve the following partial differential equation $yzp + zxq = xy$
(h) Evaluate $\oint_c \frac{e^{-z}}{z+1}\, dz$ *where C is* the circle $|z| = 2$
(i) Find the finite Fourier sine transform of $f(x) = x$
(j) Find the kind of singularity of the function $f(z) = \frac{1}{\sin z - \cos z}$ at $z = \frac{\pi}{4}$

SECTION-B [4×5=20]

QUESTION -2

Obtain the half- range sine series for $f(x) = lx - x^2$ in $(0, l)$ and hence, Evaluate

$$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots\dots\dots\dots\dots$$

QUESTION-3

Solve the following partial differential equation $(z^2 - 2yz - y^2)p + (xy + xz)q = xy - xz$

QUESTION -4

Find the finite Fourier cosine transform of $f(x) = e^{-x^2}$

QUESTION -5

Find the complete Integral of the partial differential equation $z = pq$

QUESTION- 6

Apply Calculus of residues, Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)^2}$ $a > 0$

PTO

SECTION-C

[10×2=20]

QUESTION -7

(a)Solve $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$ if $u(0,t) = 0 \quad u(x,0) = e^x \ , \ x > 0, \ u(x,t)$ is bounded

(b) If $u + v = \frac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$ and $f(z) = u + iv$ is an analytic function of$z = x + iy$, $find\ f)z)$ in terms of z

QUESTION -8

(a) Use Contour integration method , to Evaluate the following integral $\int_0^{2\pi} \frac{d\theta}{a + b\sin\theta}$ $where\ a > |b|$ Hence or otherwise evaluate $\int_0^{2\pi} \frac{d\theta}{1 - 2a\sin\theta + a^2}$ $\quad 0 < a < 1$

(b) Obtain the Fourier series of ${(\pi - x)}/{2}$ in the interval $(0,2\pi)$ & evaluate $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots \dots \dots$

Section B

Note: Each question is of 3 marks.

2. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{x^2+4}$ using contour integration.

3. Evaluate contour integral $\int_C \frac{sin\pi z + 2z}{(z+1)(z+4)}dz$ where C : $|z-1| = 3$

4. Expand f(z)= $\frac{1}{(z^2+2z-3)}$ in $0 <| z-1 |< 3$.

Section C

Note:Each question is of 6 marks.

5. Integrate f(z)= $\frac{1}{(z^2-1)^4}$ counter clockwise around the circle $|z-1| = 1$ using contour integration.

6. Evaluate $\int_0^{2\pi} \frac{d\theta}{5+3cos\theta}$ using contour integration.

Department of Computer Science & Engineering

UNIVERSITY INSTITUTE OF ENGINEERING AND TECHNOLOGY,CSJM UNIVERSITY,KANPUR

Engineering Mathematics-III(MTH-S201)

October'23(Odd Semester) Year:Second Year

Max. Marks: 30 Max.Time:1.5hrs

Section A

Note: All questions are compulsory. Each question is of 1 mark.

1. (a) Find Taylor series expansion of sin z about $\frac{\pi}{2}$.

   (b) If f(z)= $\frac{1}{(z-1)(z-2)}$ in $0 <| z-1 |< R$, find R.

   (c) Give an example of simple contour.

   (d) If radius of convergence of a series is infinity, what does it mean?

   (e) State Cauchy's integral theorem.

   (f) Find residue at z=0 of f(z)=$\frac{z+1}{z(z-2)}$

   (g) Evaluate $\int_C \frac{dz}{z-4}$ where C:$| z-1 |= 1$

   (h) Evaluate $\int_C \bar{z}\, dz$ where C is from z=0 to z=1 along y=x.

   (i) Find residue at z=0 of $\exp(1/z)$

4. Let $f(z)$ be Analytic function. Prove that if $Re f(z)$ is constant then $f(z)$ is constant.

Section C

Note:Each question is of 6 marks.

5. Find (i) harmonic conjugate of u $= e^x$ cos y .
   (ii) $f'(z)$.
6. Check continuity of f(z) at z $= 0$ given f(0) $= 0$ &
   $f(x) = \dfrac{Re(z)}{1+|z|^2}$ for $z \neq 0$. Is f(z) analytic? Justify.

Department of Computer Science & Engineering

UNIVERSITY INSTITUTE OF ENGINEERING AND TECHNOLOGY,CSJM UNIVERSITY,KA

Engineeting Mathematics-III(MTH-S201)

Semester:2023-2024(Odd Semester) **Year:Second**

Max. Marks: 30 Max.Time:1.5h

Section A

Note: All questions are compulsory. Each question is of 1 mark.

1. (a) "Every analytic function is differentiable".(True/False)
   (b) Write Cauchy Riemann equation in polar form.
   (c) Separate real and imaginary parts of sin(-1+2i).
   (d) Find Arg(z) where z= 3-3i.
   (e) Is f(z) = $\bar{z}$ analytic? Give reason.
   (f) Find modulus of e^z.
   (g) Is cos z a bounded function Justify.
   (h) Find value of ln(-1).
   (i) Express z= -3 in polar form.

Section B

Note: Each question is of 3 marks.

2. Find all solutions of $e^z = 4 - i\pi$
3. Find all roots of $z^2 + z + 1 - i = 0$.