PART – C ($5 \times 12 = 60$ Marks)
Answer ALL Questions

28. a. Verify that the matrix $A=\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation and hence find A^4.

(OR)

b. Reduce the quadratic form $x_1^2+2x_2^2+x_3^2-2x_1x_2+2x_2x_3$ to the canonical form through an orthogonal transformation. Also find the rank, index, signature and nature of the quadratic form.

29. a. Expand $e^x \cos y$ in powers of x and y as far as the terms of the third degree.

(OR)

b. A rectangular box, open at the top, is to have a volume of 32cc. Find the dimensions of the box, that requires the least material for its construction.

30. a. Solve $\left(x^2D^2-xD+1\right)y=\left(\frac{\log x}{x}\right)^2$.

(OR)

b. Solve the equation $\frac{d^2y}{dx^2}+y=\tan x$, by the method of variation of parameters.

31. a. Find the equation of the circle of curvature of the parabola $y^2=12x$ at the point (3, 6).

(OR)

b. Find the evolute of the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

32.a.i. Test the convergence of the following series $\frac{3}{4}+\frac{3.4}{4.6}+\frac{3.4.5}{4.5.8}+\ldots\infty$.

ii. Examine the convergence of the following series $\frac{1}{2!}-\frac{2}{3!}+\frac{3}{4!}-\ldots$

(OR)

b. Test the converges of the following series $\frac{3}{7}x+\frac{3.6}{7.10}x^2+\frac{3.6.9}{7.10.13}x^3+\ldots\ (x>0)$.

* * * * *

Reg. No.														

B.Tech. DEGREE EXAMINATION, NOVEMBER 2019
First Semester

18MAB101T - CALCULUS AND LINEAR ALGEBRA
(For the candidates admitted during the academic year 2018 – 2019 onwards)

Note:
(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45[th] minute.
(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours — Max. Marks: 100

PART – A ($20 \times 1 = 20$ Marks)
Answer ALL Questions

1. If $A=\begin{bmatrix} 2 & 5 & 1 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$, then the Eigen values of A^{-1} are
   (A) $\frac{1}{2},\frac{1}{4},1$ (B) 1, 3, 4
   (C) 2, 4, 1 (D) $\frac{1}{3},\frac{1}{4},1$

2. The index of the canonical form $-y_1^2+y_2^2+4y_3^2$ is
   (A) 3 (B) 2
   (C) 1 (D) 0

3. The sum of the Eigen values of the matrix $A=\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is
   (A) 6 (B) 4
   (C) 5 (D) 2

4. The eigen vectors corresponding to the distinct eigen values of a real symmetric matrix are
   (A) Imaginary (B) Non-orthogonal
   (C) Real (D) Orthogonal

5. If $f(x, y)=0$ and y is an implicit function of x, then dy/dx is
   (A) $\frac{-\partial f/\partial x}{\partial f/\partial y}$ (B) $\frac{\partial f/\partial y}{\partial f/\partial x}$
   (C) $\frac{-\partial f/\partial y}{\partial f/\partial x}$ (D) $\frac{\partial f/\partial x}{\partial f/\partial y}$

6. If $J_1=J\left(\frac{x, y}{u, v}\right)$ and $J_2=J\left(\frac{u, v}{x, y}\right)$, then J_1J_2 is
   (A) 0 (B) 1
   (C) −1 (D) 2

7. u and v are functionally dependent if their Jacobian value is
(A) 0
(B) 1
(C) 2
(D) 3

8. The point at which there is no extreme value is
(A) Maximum point
(B) Minimum point
(C) Saddle point
(D) Stationary point

9. The complimentary function of $(D^2-2D+1)y=0$ is
(A) $C_1e^x+C_2e^{-x}$
(B) $(C_1+C_2x)e^x$
(C) $C_1e^{2x}+C_2e^{-2x}$
(D) $(C_1+C_2x)e^{-x}$

10. The particular integral of $(D^2+16)y=e^{-4x}$ is
(A) $\frac{x}{32}e^{-4x}$
(B) $\frac{1}{32}e^{-4x}$
(C) $\frac{1}{16}e^{-4x}$
(D) $\frac{x}{16}e^{-4x}$

11. The complementary function of $(x^2D^2-7xD+12)y=0$ is
(A) $\frac{c_1}{x^2}+\frac{c_2}{x^6}$
(B) $c_1x^2+\frac{c_2}{x^6}$
(C) $\frac{c_1}{x^2}+c_2x^6$
(D) $c_1x^2+c_2x^6$

12. In method of variation of parameters Complete Solution $=c_1f_1+c_2f_2$ + particular integral, where particular integral is
(A) $PI=Pf_1+Qf_2$
(B) $PI=Pf_2+Qf_1$
(C) $PI=PC_1+QC_2$
(D) $PI=PC_2+QC_1$

13. The curvature of a circle of radius 'r' is
(A) r
(B) $1/r$
(C) $1/r^2$
(D) r^2

14. The parametric equation of a parabola $y^2=4ax$ is
(A) $x=at^2, y=2at$
(B) $x=t, y=1/t$
(C) $x=at,\ y=2at$
(D) $x=2at, y=at^2$

15. The locus of centre of curvature is called
(A) Involute
(B) Evolute
(C) Radius of curvature
(D) Envelope

16. The radius of curvature at (3, 4) on the curve $x^2+y^2=25$ is
(A) 5
(B) 4
(C) 0
(D) 2

17. General term of $\frac{1}{3}, \frac{-2}{3}, \frac{3}{3^3}, \frac{-4}{3^4}, \ldots\ldots$ is
(A) $u_n=\frac{(-1)^{n+1}n}{3^n}$
(B) $u_n=\frac{(-1)^{n-1}n}{3^n}$
(C) $u_n=\frac{(-1)^{n-1}n+1}{3^n}$
(D) $u_n=\frac{(-1)^{n}n+1}{3^n}$

18. The series $\Sigma\frac{1}{n^p}$ is converges if
(A) $P=1$
(B) $P<1$
(C) $P>1$
(D) $P=0$

19. The series Σu_n of positive terms is convergent if
(A) $\lim_{n\to\infty}\frac{u_{n+1}}{u_n}<1$
(B) $\lim_{n\to\infty}\frac{u_n}{u_{n+1}}=1$
(C) $\lim_{n\to\infty}\frac{u_n}{u_{n+1}}<1$
(D) $\lim_{n\to\infty}\frac{u_{n+1}}{u_n}>1$

20. n[th] term of a series if in A.P.
(A) $t_n=a-(n-1)d$
(B) $t_n=a+(n+1)d$
(C) $t_n=a-(n+1)d$
(D) $t_n=a+(n-1)d$

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Verify that the sum of the Eigen values of A equals the trace of A and that their product equal $|A|$, for the matrix $A=\begin{bmatrix}1&0&0\\0&3&-1\\0&-1&3\end{bmatrix}$.

22. If $x=r\cos\theta, y=r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

23. Solve the equation $(D^2-4D+3)y=\sin 3x$.

24. Find the radius of curvature of the curve $r=a(1+\cos\theta)$ at the point $\theta=\pi/2$.

25. Test the converges of the following series
$\frac{1}{1.2}+\frac{2}{3.4}+\frac{3}{5.6}+\ldots\ldots\infty$.

26. Determine the nature of the following quadratic form without reducing it to canonical form:
$x_1^2+3x_2^2+6x_3^2+2x_1x_2+2x_2x_3+4x_3x_1$.

27. Find the envelope of the family of straight lines given by $y=mx\pm\sqrt{a^2m^2-b^2}$ where m is the parameter.

b. Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix}1 & 3 & 7\\ 4 & 2 & 3\\ 1 & 2 & 1\end{pmatrix}$ and hence find A^{-1}.

29. a. A rectangular box open at the top is to have volume of 32 cubic feet. Find the dimension in order that the total surface area is minimum.

(OR)

b.i. Find the Taylor's series expansion of $e^x \sin x$ near the point $\left(-1,\frac{\pi}{4}\right)$ upto 3rd degree terms.

ii. If $x=r\cos\theta, y=r\sin\theta$ verify that $\frac{\partial(x,y)}{\partial(r,\theta)}\times\frac{\partial(r,\theta)}{\partial(x,y)}=1$.

30. a. Solve the simultaneous equation $\frac{dx}{dt}+2y+\sin t=0$ and $\frac{dy}{dt}-2x-\cos t=0$.

(OR)

b. Solve $(D^2+4)y=4\tan 2x$, by using method of variation of parameters.

31. a. Find the equation of the circle of curvature of the parabola $y^2=12x$ at the point (3, 6)

(OR)

b. Find the evolute of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

32. a.i. Test the convergence of the series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\ldots\infty$.

ii. Test for convergence of the series $\Sigma(\log n)^{-2n}$.

(OR)

b. Test the convergence of the series $\Sigma\frac{1}{\sqrt{n+1}-1}$.

* * * * *

Reg. No.

B.Tech. DEGREE EXAMINATION, MAY 2019
First / Second Semester

18MAB101T – CALCULUS AND LINEAR ALGEBRA
(For the candidates admitted during the academic year 2018 – 2019 onwards)

Note:
(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The Eigen values of the matrix $\begin{pmatrix}1 & 2\\ 5 & 4\end{pmatrix}$ are
   (A) 1, 6
   (B) −1, 6
   (C) 1, −6
   (D) −1, −6

2. The inverse of the eigen values of the matrix $\begin{pmatrix}1 & 2\\ 0 & 3\end{pmatrix}$ is
   (A) $1,\frac{1}{2}$
   (B) 1, 2
   (C) $1,\frac{1}{3}$
   (D) 1, 3

3. The nature of the quadratic form, $x_1^2+2x_2^2+3x_3^2+2x_1x_2+2x_2x_3-2x_3x_1$ is
   (A) Indefinite
   (B) Positive definite
   (C) Negative definite
   (D) Negative semi definite

4. The signature of the quadratic form whose canonical form is $2y_1^2-y_2^2-y_3^2$
   (A) 1
   (B) −1
   (C) 0
   (D) 6

5. If u and v are functionally dependent then their Jacobian value is
   (A) Zero
   (B) One
   (C) Non-zero
   (D) > 0

6. If $rt-s^2<0$ at (a, b) then the point is
   (A) Maximum point
   (B) Minimum point
   (C) Saddle point
   (D) Zero

7. If $u=x^2+y^2+3xy$ then $\frac{\partial u}{\partial x}$ is
   (A) $2y+3x$
   (B) $3y$
   (C) $2x+3y$
   (D) $2x$

8. If $u=x^2, v=y^2$ then $\frac{\partial(u,v)}{\partial(x,y)}$ is

(A) $2xy$
(B) $4xy$
(C) $2y$
(D) xy

9. Complementary function of $(D^2-2D+1)y=0$ is

(A) $C_1e^{x}+C_2e^{-x}$
(B) $(C_1+C_2x)e^{x}$
(C) $C_1e^{2x}+C_2e^{-2x}$
(D) $(C_1+C_2x)e^{2x}$

10. Particular integral of $(D^2+9)y=e^{-2x}$ is

(A) $\frac{e^{-2x}}{15}$
(B) $\frac{e^{2x}}{15}$
(C) $\frac{e^{-2x}}{13}$
(D) $\frac{e^{2x}}{13}$

11. Solve of $(x^2D^2+xD+1)y=0$ is

(A) $Ae^{az}+Be^{bz}$
(B) $A\cos z+B\sin z$
(C) $(A+Bz)e^{z}$
(D) $(A+Bz)e^{-z}$

12. The complementary function of the second order differential equation having roots $\alpha+i\beta$ is

(A) $e^{-\alpha x}(c_1\cos\beta x+c_2\sin\beta x)$
(B) $c_1\cos\beta x+c_2\sin\beta x$
(C) $c_1\cos\beta x-c_2\sin\beta x$
(D) $e^{\alpha x}(c_1\cos\beta x+c_2\sin\beta x)$

13. The curvature of the straight line is

(A) 1
(B) -1
(C) 2
(D) 0

14. The radius of curvature of a curve $y=4\sin x$ at $x=\frac{\pi}{2}$ is

(A) 1/4
(B) $-1/4$
(C) 1/2
(D) $-1/2$

15. The evolute of a curve is the locus of ________.

(A) Centre of curvature
(B) Curvature
(C) Radius of curvature
(D) Line

16. The value of $\Gamma\left(\frac{1}{2}\right)$ is

(A) π
(B) $\pi/2$
(C) $\sqrt{\pi}$
(D) $\sqrt{\pi}/2$

17. The series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent, if

(A) $p=1$
(B) $p=0$
(C) $p>1$
(D) $p<1$

18. The series $\sum_{n=1}^{\infty}\frac{1}{n!}$ is

(A) Convergent
(B) Divergent
(C) Oscillating
(D) Monotonic

19. A sines Σu_n is said to be absolutely convergent if the series

(A) $\Sigma|u_n|$ is convergent
(B) Σu_n is divergent
(C) $\Sigma|u_n|$ is divergent
(D) Σu_n is convergent

20. The series $\Sigma\frac{n^3}{3^n}$ is

(A) Conditionally convergent
(B) Absolutely convergent
(C) Convergent
(D) Divergent

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Find the Eigen values and Eigen vectors of $\begin{bmatrix}1 & 1\\ 3 & -1\end{bmatrix}$.

22. Find $\frac{du}{dt}$ if $u=x^2+y^2+z^2$ where $x=e^t, y=e^t\sin t, z=e^t\cos t$.

23. Find the envelope of a family of straight lines $x\cos\alpha+y\sin\alpha=a\sec\alpha$, α is the parameter.

24. Show that $\Gamma(n+1)=n\Gamma(n)$.

25. Solve $(D^2+3D+2)y=\sin x$.

26. If $u=f(y-z,z-x,x-y)$,, prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$.

27. Test for convergence $u_n=\sqrt{\frac{3^n-1}{2^n+1}}$.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Deduce the quadratic form $6x_1^2+3x_2^2+3x_3^2-4x_1x_2-2x_2x_3+4x_3x_1$ to a canonical form and hence find rank, index and signature.

(OR)

27. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{(n+1)(2n+1)}$.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$ and hence find A^{-1}.

(OR)

b. Reduce the quadratic form $6x_1^2+3x_2^2+3x_3^2-4x_1x_2-2x_2x_3+4x_3x_1$ to canonical form by an orthogonal transformation. Also state its nature.

29. a. Explain $e^x \cos y$ in powers of x and y as far as the terms of the third degree.

(OR)

b. Find the volume of the largest rectangular parallelepiped that can be inserted in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

30. a. Solve $(D^2-7D+12)y=e^{5x}+\cos 2x$.

(OR)

b. Solve $(x^2D^2+xD-9)y=\frac{5}{x^2}$.

31. a. Find the equation of the circle of curvature of the parabola $y^2=12x$ at the point (3, 6).

(OR)

b. Find the evolute of the parabola $y^2=4ax$.

32. a.i. State the axioms of convergence of series by comparison test.

ii. Test the convergence of the series $\frac{3}{4}+\frac{3.4}{4.6}+\frac{3.4.5}{4.6.8}+\ldots\infty$.

(OR)

b.i. State the Cauchy's criterion of convergence of an infinite series.

ii. Test the convergence of the series $\sum_{n=1}^{\infty}\left(\sqrt{n^4+1}-\sqrt{n^4-1}\right)$.

* * * * *

Reg. No. | | | | | | | | | | | | | | |

B.Tech. DEGREE EXAMINATION, NOVEMBER 2018
First Semester

18MAB101T – CALCULUS AND LINEAR ALGEBRA
(For the candidates admitted during the academic year 2018 – 2019)

Note:
(i) **Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.**
(ii) **Part - B and Part - C should be answered in answer booklet.**

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The eigen values of A^2, given $A=\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$ are

(A) 3, 2, 5
(B) 9, 4, 25
(C) 3, 1, 4
(D) 9, 1, 16

2. The characteristics equation of the matrix $A=\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ is

(A) $\lambda^2-4\lambda-5=0$
(B) $\lambda^2+4\lambda-5=0$
(C) $\lambda^2+4\lambda+5=0$
(D) $\lambda^2+4\lambda-5=0$

3. The nature of the quadratic form, whose matrix is $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & 6 \end{pmatrix}$ is

(A) Indefinite
(B) Positive semi-definite
(C) Positive definite
(D) Negative definite

4. If the canonical form of a quadratic form is $-y_1^2+y_2^2+4y_3^2$, then the signature of QF is

(A) 0
(B) 3
(C) 2
(D) 1

5. If $u=xy^2+x^2y$, where $x=at^2$, $y=2at$, that $\frac{du}{dt}$

(A) $2a^2t^2(8+5t)$
(B) $2a^3t^3(8+5t)$
(C) $4a^3t^3(8+5t)$
(D) $4a^2t^2(8+5t)$

6. The stationary points of the function $3x^2-y^2+x^3$ are

(A) (0, 0), (–2, 0)
(B) (0, 0), (2, 0)
(C) (1, 1), (–2, 0)
(D) (1, 1) (2, 0)

7. The $x = r\cos\theta, y = r\sin\theta$, then the Jacobian of x, y with respect to r, θ $\left(i.e\ J\left(\frac{x,y}{r,\theta}\right)\right)$

(A) r
(B) $1/r$
(C) $2r$
(D) $2/r$

8. Maclaurin series for e^{x+y} is

(A) $1+(x+y)-\frac{(x+y)^3}{3!}+....$
(B) $1-(x-y)-\frac{(x-y)^3}{2!}-....$
(C) $1+(x-y)-\frac{(x-y)^3}{2!}+....$
(D) $1+(x+y)+\frac{(x+y)^3}{2!}+....$

9. The solution of $(D^2+4)y=0$ is

(A) $y=Ae^{2x}+Be^{2x}$
(B) $y=A\cos 2x+B\sin 2x$
(C) $y=Ae^{2x}+Be^{-2x}$
(D) $y=A\cos x+B\sin x$

10. The particular integral of the differential equation $(D^2-3D+2)y\ \cos x$ is

(A) $\frac{1}{10}(\cos x+3\sin x)$
(B) $\frac{1}{9}(\cos x-3\sin x)$
(C) $\frac{1}{10}(\cos x-3\sin x)$
(D) $\cos x-3\sin x$

11. The complementary function of $(D^2+4D+4)y=e^{-2x}$ is

(A) $CF=(Ax+B)e^{-2x}$
(B) $CF=(Ax+B)e^{2x}$
(C) $CF=A\cos 2x+B\sin 2x$
(D) $CF=Ae^{-2x}+Be^{-2x}$

12. The complementary function of $\left(x^2D^2+4xD+2\right)y=x^2+\frac{1}{x^2}$ is

(A) $CF=(Ax+B)e^{-2x}$
(B) $CF=(Ax-B)e^{-x}$
(C) $CF=Ae^{-x}+Be^{-2x}$
(D) $CF=\frac{A}{x}+\frac{B}{x^2}$

13. The radius of curvature of the curve $y=e^x$ at $x=0$ is

(A) $2\sqrt{2}$
(B) $\sqrt{2}$
(C) 2
(D) 4

14. The radius of curvature of a circle at any point is same as its

(A) Chord
(B) Radius
(C) Diameter
(D) Tangent

15. The value of $\Gamma\left(\frac{1}{2}\right)$ is .

(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{2}$
(C) π
(D) $\sqrt{\pi}$

16. When n is a +ve integer, $\Gamma(n+1)=$

(A) $(n+1)!$
(B) $n!$
(C) $(2n)!$
(D) $(n-1)!$

17. The series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent, if

(A) $p>1$
(B) $p<1$
(C) $p=1$
(D) $p=0$

18. The series $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+....$ is

(A) Convergent
(B) Oscillating
(C) Divergent
(D) Monotonic

19. As per D'Alembert's ratio test, if Σu_n is a series of positive terms and $\lim_{n\to\infty}\frac{u_{n+1}}{u_n}=l$, the series is convergent, if

(A) $l>1$
(B) $l=0$
(C) $l=1$
(D) $l<1$

20. The series $\Sigma\frac{1}{n}\sin\left(\frac{1}{x}\right)$ is

(A) Convergent
(B) Conditionally convergent
(C) Divergent
(D) Absolutely convergent

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Use Cayley-Hamilton theorem to find A^3, given that $A=\begin{pmatrix}-1 & 3\\ 2 & 4\end{pmatrix}$.

22. If $u=f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$

23. Find the Jacobian of y_1,y_2,y_3 with respect to x_1,x_2,x_3 if $y_1=\frac{x_2x_3}{x_1}, y_2=\frac{x_3x_1}{x_2}$ and $y_3=\frac{x_1x_2}{x_3}$.

24. Find the eigenvalues of the matrix $A=\begin{pmatrix}2 & -2 & 2\\ 1 & 1 & 1\\ 1 & 3 & -1\end{pmatrix}$.

25. Solve the equation $(D^2-4D+3)y=\sin 3x$.

26. Find the envelope of the family of straight lines given by $x\cos\alpha+y\sin\alpha=a\sec\alpha$.

27. Find the equation of the sphere whose centre is at $(-6,1,3)$ and radius 4.

PART – C ($5 \times 12 = 60$ Marks)
Answer ALL Questions

28. a. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$ and hence find A^{-1}.

(OR)

b. Reduce the $QF = 3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_3x_2 + 2x_1x_3$ to a diagonal canonical form and hence find its nature, rank, index and signature.

29. a. Expand $e^x \cos y$ in powers of x and y as far as the terms of the third degree.

(OR)

b. Find the volume of the largest rectangular parallopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

30. a. Solve $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\log x}{x^2}$.

(OR)

b. Solve $\frac{d^2y}{dx^2} + y = \sec x$ by the method of variation of parameters.

31. a. Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ of the curve $x^3 + y^3 = 3axy$.

(OR)

b. Find the equation of the circle of curvature the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$.

32. a. Find the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$ as a great circle.

(OR)

b. Find the equation of the right circular cylinder of radius 2 whose axis passes through (1, 2, 3) and has direction cosines proportional to (2, −3, 6).

*****

Reg. No.

B.Tech. DEGREE EXAMINATION, JUNE 2018
First / Second Semester

MA1001 – CALCULUS AND SOLID GEOMETRY
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
(i) **Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.**
(ii) **Part - B and Part - C should be answered in answer booklet.**

Time: Three Hours | Max. Marks: 100

PART – A ($20 \times 1 = 20$ Marks)
Answer ALL Questions

1. The matrix of the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_3x_1$ is

(A) $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 5 & 2 \\ 3 & 2 & 1 \end{pmatrix}$
(C) $\begin{pmatrix} 1 & 4 & 4 \\ 4 & 5 & 3 \\ 4 & 3 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 4 & 3 \\ 4 & 5 & 4 \\ 3 & 4 & 1 \end{pmatrix}$

2. The number of positive terms in the canonical form is called
(A) Signature (B) Index
(C) Quadratic form (D) Positive definite

3. Find the eigen values of A^2 if $A = \begin{pmatrix} 3 & 2 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$

(A) 6, 4, 10 (B) 9, 4, 25
(C) 9, 2, 5 (D) 3, 2, 5

4. Two eigen values of the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ are 1 and 2. Find the third eigen value.

(A) 3 (B) 6
(C) 2 (D) 1

5. If $z = x^2 + y^2 + 3xy$ then what is $\frac{\partial z}{\partial y}$?

(A) $2y+3x$ (B) $3y$
(C) $2x+3y$ (D) $2x$

6. $u=\sin^{-1}\left(\dfrac{x^2+y^2}{x-y}\right)$ is homogeneous function of degree.

(A) 2 (B) 3
(C) 1 (D) 4

7. If $f(x,y)$ is an implicit function then $\dfrac{dy}{dx}=?$

(A) $-f_y/f_x$ (B) $-f_x/f_y$
(C) f_y/f_x (D) f_x/f_y

8. If $rt-s^2<0$ at (a,b) then the point is

(A) Maximum point (B) Minimum point
(C) Saddle point (D) None of the above

9. Which of the following is the general solution to $\dfrac{d^2y}{dx^2}+3\dfrac{dy}{dx}-10y=0$?

(A) $y=Ae^{2x}+Be^{-5x}$ (B) $y=Ae^{-2x}+Be^{5x}$
(C) $y=Ae^{-2x}+Be^{-5x}$ (D) $y=Ae^{2x}+Be^{5x}$

10. Solution of $(D^2+4)y=0$ is

(A) $y=A\cos 2x+B\sin 2x$ (B) $y=Ae^{2x}+Be^{-2x}$
(C) $y=A\cos\sqrt{2}x+B\sin\sqrt{2}x$ (D) $y=(Ax+B)e^{2x}$

11. The PI of $(D^2+4)y=\sin 2x$ is

(A) $-\dfrac{x}{4}\cos 2x$ (B) $\dfrac{x}{4}\cos 2x$
(C) $\dfrac{x}{2}\cos 2x$ (D) $\dfrac{-x}{2}\cos 2x$

12. The equation $\left(a_0x^2D^2+a_1xD+a_2\right)y=Q(y)$ is called, where $a_0,a_1,a_2\in C$.

(A) Legendre's equation (B) Taylor's equation
(C) Clairaut's equation (D) Cauchy's equation

13. If the radius of curvature and curvature of a curve at any point are ρ and k respectively, then

(A) $\rho=\dfrac{-1}{k}$ (B) $\rho=k$
(C) $\rho=-k$ (D) $\rho=\dfrac{1}{k}$

14. The locus of centre of curvature is called

(A) Involute (B) Evolute
(C) Radius of curvature (D) Envelope

15. The envelope of the family of curves $A\alpha^2+B\alpha+C=0$ (α is parameter) is

(A) $B^2+4AC=0$ (B) $B^2-4AC=0$
(C) $B^2-AC=0$ (D) $B^2+AC=0$

16. The curvature of the straight line is

(A) 1 (B) 2
(C) −1 (D) 0

17. The direction ratios of the line joining $p(2,3,5)$ and $Q(-1,3,2)$ is

(A) $(-3,0,-3)$ (B) $(3,0,-3)$
(C) $(1,2,3)$ (D) $(-1,2,-3)$

18. The equation of the plane through $(1,2,3)$ and parallel to $3x+4y-5z=0$ is

(A) $3x+4y-5z-4=0$ (B) $3x+4y-5z+4=0$
(C) $3x+4y-5z=0$ (D) $3x+4y+5z=0$

19. Any two straight lines which are neither parallel nor intersects are called ______.

(A) Parallel lines (B) Coplanar
(C) Perpendicular lines (D) Skew lines

20. The equation of two parallel planes differ by ________ only

(A) x-axis (B) y-axis
(C) z-axis (D) Constant

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Find the eigen values and eigen vectors of $\begin{bmatrix}2 & -2 & 3\\ 1 & 1 & 1\\ 1 & 3 & -1\end{bmatrix}$.

22. Using Cayley-Hamilton theorem find the inverse of the matrix $\begin{bmatrix}2 & 1\\ 1 & -5\end{bmatrix}$.

23. If $u=\sin^{-1}\left(\dfrac{x^2+y^2}{x-y}\right)$, then show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\tan u$.

24. Find $\dfrac{dz}{dt}$, when $z=xy^2+x^2y, x=at^2; y=2at$.

25. Solve $\dfrac{d^2y}{dx^2}+6\dfrac{dy}{dx}+9y=3e^{4x}$.

26. Find the envelope of the family of straight lines $y=mx\pm\sqrt{a^2m^2+b^2}$.

b. Find the maximum and minimum distance of the point $(3,4,12)$ from the sphere $x^2+y^2+z^2=1$, by using Lagrange's method.

30. a. Solve by the method of variation of parameters $\frac{d^2y}{dx^2}+y=\operatorname{cosec} x$.

(OR)

b. Solve $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=x\log x$.

31. a. Find the circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $\left(\frac{a}{4},\frac{a}{4}\right)$.

(OR)

b. Find the evolute of the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$.

32. a. Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z=3$ and also find the point of contact.

(OR)

b. Find the equation of the right circular cylinder of radius 2 whose axis passes through (1, 2, 3) and has direction cosines proportional to (2, −3, 6).

* * * * *

Reg. No.														

B.Tech. DEGREE EXAMINATION, MAY 2018
First / Second Semester

15MA101 – CALCULUS AND SOLID GEOMETRY
(For the candidates admitted during the academic year 2017 – 2018 only)

Note:
(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The sum and product of eigen values of $A=\begin{bmatrix}2&1&1\\1&2&1\\0&0&1\end{bmatrix}$
   (A) 5, 3 (B) 3, 5
   (C) 2, 1 (D) 1, 2

2. If A is an orthogonal matrix then
   (A) $|A|=0$ (B) A is singular
   (C) $A^2=I$ (D) $A^T=A^{-1}$

3. The eigen values of A^2, if $A=\begin{bmatrix}3&2&4\\0&2&6\\0&0&5\end{bmatrix}$ is
   (A) 6, 4, 10 (B) 9, 4, 25
   (C) 9, 2, 5 (D) 3, 2, 5

4. The nature of quadratic form of $x_1^2+2x_2^2+3x_3^2+2x_1x_2+2x_2x_3-2x_3x_1$
   (A) Indefinite (B) Definite
   (C) Positive definite (D) Negative definite

5. If u and v are functional dependent then their Jacobian is
   (A) One (B) Non zero
   (C) Zero (D) Greater than zero

6. If $rt-s^2<0$ at (a,b) then the point is
   (A) Maximum point (B) Minimum point
   (C) Saddle point (D) Unique point

7. If $f(x,y)=e^{xy}$ then the value of $f_{yyy}(1,1)$ is
   (A) $-e$ (B) e^{-1}
   (C) e (D) $-e^{-1}$

8. If $u = x^2 + y^2 + 3xy$ then the value of $\frac{\partial u}{\partial x}$ is

(A) $2y+3x$ (B) $3y$

(C) $2x+3y$ (D) $2x$

9. Solution of $\left(D^2+4\right)y=0$ is

(A) $y = A\cos 2x + B\sin 2x$ (B) $y = Ae^{2x} + Be^{-2x}$

(C) $y = \left(Ax+B\right)e^{2x}$ (D) $y = \left(Ax+B\right)e^{-2x}$

10. If $y_1 = \cos ax, y_2 = \sin ax$ then the values of $y_1y_2' - y_2y_1'$

(A) $-a$ (B) 0

(C) 1 (D) a

11. Solution of $\left(x^2D^2 + xD + 1\right)y = 0$ is

(A) $Ae^z + Be^z$ (B) $A\cos z + B\sin z$

(C) $\left(Az+B\right)e^z$ (D) $\left(Az+B\right)e^{-z}$

12. P.I. ~~Prove that~~ $(D^2+a^2)y = \cos ax$

(A) $\frac{x}{2a}\sin ax$ (B) $\frac{-x}{2a}\sin ax$

(C) $\frac{x}{2a}\cos ax$ (D) $-\frac{x}{2a}\cos ax$

13. The curvature of the straight line is

(A) 1 (B) 2

(C) -1 (D) 0

14. A curve which touches each member of the family of curves is called ____ of that family.

(A) Evolute (B) Envelope

(C) Radius of curvature (D) Circle of curvature

15. Locus of centre of curvature is called ______

(A) Evolute (B) Involute

(C) Envelope (D) Normal

16. The curvature at any point of the circle is equal to __________ of its radius.

(A) Same (B) Square

(C) Reciprocal (D) Constant

17. Section of a sphere by a plane is

(A) Parabola (B) Ellipse

(C) Circle (D) Line

18. Semi vertical angle of the cone generated by revolving the lines $x+y=0, z=0$ about $x-$axis

(A) 30° (B) 90°

(C) 60° (D) 45°

19. The direction ratios of the line joining $P(2,3,5)$ and $Q(-1,3,2)$ is

(A) $(-3,0,-3)$ (B) $(3,0,-3)$

(C) $(3,0,3)$ (D) $(3,0,0)$

20. The centre of the sphere $(x-1)^2 + (y-2)^2 + (z-3)^2 = 16$

(A) $(1,2,3)$ (B) $(-1,2,-3)$

(C) $(1,2,-3)$ (D) $(1,-2,-3)$

PART – B ($5 \times 4 = 20$ Marks)

Answer ANY FIVE Questions

21. Find the Eigen vector of $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$.

22. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

23. Solve $\left(D^2+4\right)y = \sin 2x$.

24. Find the envelope of the straight line $x\cos\alpha + y\sin\alpha = a\sin\alpha\cos\alpha$, α is the parameter.

25. Find the equation of a sphere which has its centre at the point $(-1,2,3)$ and touches the plane $2x-y+2z=6$.

26. If $u = x^2 - y^2$, $v = 2xy$, find $J\left(\frac{u,v}{x,y}\right)$.

27. Find the equation of the cone whose vertex is at (1, 2, 3) and whose axis is $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{4}$ are semi vertical angle 30°.

PART – C ($5 \times 12 = 60$ Marks)

Answer ALL Questions

28. a. Verify Cayley – Hamilton theorem and hence find A^{-1} for

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

(OR)

b. Reduce by orthogonal reduction $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_3x_1$ to a canonical form and hence find its nature.

29. a. Expand $e^x \cos y$ is powers x, y upto 3rd degree terms.

(OR)

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ and hence find A^4.

(OR)

b. Reduce the quadratic form $2x_1^2+6x_2^2+2x_3^2+8x_1x_3$ to canonical form by orthogonal reduction. Find also the nature of the quadratic form.

29. a. Find the Taylor's series expansion of $x^2y^2+2x^2y+3xy^2$ near the point $(-2, 1)$, as far terms of the third degree.

(OR)

b. Find the shortest and the longest distances from the point $(1, 2, -1)$ to the sphere $x^2+y^2+z^2=24$.

30. a. Solve $(D^2+3D+2)y=x^2+\sin x$.

(OR)

b. Solve $\frac{d^2y}{dx^2}+y=cosecx$ by the method of variation of parameters.

31. a. Find the equation of the circle of curvature of the curve $x^3+y^3=3axy$ at the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$.

(OR)

b. Show that the evolute of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ is another cycloid.

32. a. Find the equation of the sphere that passes through the circle $x^2+y^2+z^2+2x+3y+z-2=0$ and $2x-y-3z-1=0$ and cuts orthogonally the sphere $x^2+y^2+z^2-3x+y-2=0$.

(OR)

b. Find the equation of the right circular cylinder, whose axis is $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{6}$ and of radius 2.

*****

B.Tech. DEGREE EXAMINATION, DECEMBER 2017
First Semester

MA1001 – CALCULUS AND SOLID GEOMETRY
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
(i) **Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.**
(ii) **Part - B and Part - C should be answered in answer booklet.**

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The sum of the eigen values of the inverse of $A=\begin{pmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{pmatrix}$ is

(A) 27/60
(B) 47/60
(C) 37/60
(D) 42/60

2. The product of the eigen values of the matrix $A=\begin{pmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & -3 \end{pmatrix}$ is

(A) –1
(B) –2
(C) 1
(D) 2

3. If the eigen values of the matrix of the quadratic form $2x_1^2+6x_2^2+2x_3^2+8x_1x_3$ are –2, 6, 6 then the nature of Q.F is

(A) Positive semi-definite
(B) Indefinite
(C) Definite
(D) Negative semi-definite

4. If $x^3+y^3=3ax^2y$, then $\frac{dy}{dx}$

(A) $\frac{x(2ay+x)}{y^2+ax^2}$
(B) $\frac{y^2-ax^2}{x(2ay-x)}$
(C) $\frac{x(2ay-x)}{y^2-ax^2}$
(D) $\frac{y^2+ax^2}{x(2ay+x)}$

5. If u and v are function of x and y, then $\frac{\partial(u,v)}{\partial(x,y)}\times\frac{\partial(x,y)}{\partial(u,v)}=$

(A) 1
(B) –1
(C) 0
(D) 2

6. If 'u' is a homogeneous function of degree 'n' then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$

(A) 1 (B) 0
(C) 2 (D) 3

Ans nu

7. If $AC-B^2<0$ at a pt (a, b), where A = f_{xx}, B = f_{xy}, C = f_{yy} of a function f(x, y), what can you say about the point (a, b)?
(A) Maximum point (B) Stationary point
(C) Saddle point (D) Minimum point

8. The eigen values of the matrix $A=\begin{pmatrix}4 & 1\\ 3 & 2\end{pmatrix}$ are

(A) 2, 4 (B) 1, 5
(C) −1, −5 (D) −2, −4

9. The Complementary Function (CF) of differential equation $(D^2-7D+12)y=0$ is
(A) $c_1e^{5x}+c_2e^{2x}$ (B) $c_1e^{-3x}+c_2e^{-4x}$
(C) $c_1e^{3x}+c_2e^{4x}$ (D) $c_1e^{-5x}+c_2e^{-2x}$

10. The particular integral of the differential equation $(D^2+6D+9)y=e^{4x}$ is
(A) $\frac{1}{49}e^{4x}$ (B) $\frac{4}{49}e^{4x}$
(C) $\frac{1}{49}e^{x}$ (D) $\frac{2}{49}e^{4x}$

11. The Complementary Function (CF) of the differential equation $(x^2D^2+4xD+2)y=x+\frac{1}{x}$

(A) $c_1e^{-z}+c_2e^{2z}$ (B) $c_1e^{z}+c_2e^{-2z}$
(C) $c_1e^{z}+c_2e^{2z}$ (D) $c_1e^{-z}+c_2e^{-2z}$

12. The Complementary Function (CF) of the differential equation $\frac{d^2y}{dx^2}+y=\sec x$ is

(A) $c_1\tan x+c_2\sec x$ (B) $c_1\cos x+c_2\sin x$
(C) $c_1\sin x+c_2\cos x$ (D) $c_1\sec x+c_2\operatorname{cosec}x$

13. The curvature of a straight line is
(A) One (B) Zero
(C) Two (D) Three

14. The equation of the circle of curvature, with centre of curvature at $(\bar{x},\bar{y})$ and radius of curvature ρ, is
(A) $(x-\bar{x})^2-(y-\bar{y})^2=\rho$ (B) $(x-\bar{x})^2+(y-\bar{y})^2=\rho$
(C) $(x-\bar{x})^2-(y-\bar{y})^2=\rho^2$ (D) $(x-\bar{x})^2+(y-\bar{y})^2=\rho^2$

15. If $A\alpha^2+B\alpha+C=0$ is the equation of a family of curves, the equation of the envelope of the family is
(A) $B^2-4AC=0$ (B) $A^2-4BC=0$
(C) $C^2-4AB=0$ (D) $B^2+4AC=0$

16. The radius of curvature 'ρ' for the curve $y=e^x$ at $x=0$, is
(A) 2 (B) $\sqrt{2}$
(C) $2\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

17. If two lines with direction cosines (l_1, m_1, n_1) and (l_2, m_2, n_2) are perpendicular, then
(A) $l_1l_2-m_1m_2-n_1n_2=0$ (B) $l_1l_2-m_1m_2-n_1n_2=1$
(C) $l_1l_2+m_1m_2+n_1n_2=0$ (D) $l_1l_2+m_1m_2+n_1n_2=\infty$

18. If a line makes angles α, β, γ with co-ordinate axes, then, $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=$
(A) ∞ (B) 0
(C) 1 (D) 2

19. The equation of a sphere, having the points (2, −3, 4) and (−1, 5, 7) as the end points of a diameter
(A) $x^2+y^2+z^2-x-2y-11z+11=0$ (B) $x^2+y^2+z^2+x+2y+11z+11=0$
(C) $x^2+y^2+z^2+2x-y+z-4=0$ (D) $x^2+y^2+z^2-2x-3y-6z+11=0$

20. The equation of the tangent plane at the point (1, −1, 2) to the sphere $x^2+y^2+z^2-2x+4y+6z-12=0$
(A) $y+3z+9=0$ (B) $x+y-9=0$
(C) $y+5z-9=0$ (D) $2y-5z+9=0$

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Find the eigen values and the corresponding eigen vectors of the matrix $A=\begin{pmatrix}5 & 4\\ 1 & 2\end{pmatrix}$.

22. If $u=2xy$, $v=x^2-y^2$, $x=r\cos\theta$ *and* $y=r\sin\theta$, compute $\frac{\partial(u,v)}{\partial(r,\theta)}$.

23. Find the minimum value of the function $f(x,y)=x^2+y^2+6x+12$.

24. Solve $(D^2+5D+4)y=e^{-x}\sin 2x$.

25. Find the envelope of a family of straight lines given by $x\cos\alpha+y\sin\alpha=a\sec\alpha$.

26. Find the equation of the sphere through the circle given by $x^2+y^2+z^2+3x+y+4z-3=0$ and $x^2+y^2+z^2+2x+3y+6=0$ and the point (1, −2, 3).

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Use Cayley-Hamilton theorem to find the value of the matrix given by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$ if the matrix $A=\begin{bmatrix}2&1&1\\0&1&0\\1&1&2\end{bmatrix}$.

(OR)

b. Reduce the quadratic form $10x_1^2+2x_2^2+5x_3^2+6x_2x_3-10x_3x_1-4x_1x_2$ to a canonical form by orthogonal reduction. Find also a set of non-zero values of x_1, x_2, x_3 which will make the quadratic form zero.

29.a.i. If $u=xyz,\ v=xy+yz+zx$ and $w=x+y+z$ find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$.

ii. Find the minimum values of $x^2+y^2+z^2$, when $ax+by+cz=p$.

(OR)

b.i. Expand $\dfrac{(x+h)(y+k)}{x+h+y+k}$ in a series of powers of h and k upto second degree terms.

ii. If $z=f(x,y)$ where $x=X\cos\alpha-Y\sin\alpha$ and $y=X\sin\alpha+Y\cos\alpha$ show that $\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial y^2}=\dfrac{\partial^2 z}{\partial X^2}+\dfrac{\partial^2 z}{\partial Y^2}$.

30. a. Solve (i) $(D^2+9)y=x^2+\cosh x$ (ii) $(x^2D^2+9xD+25)y=(\log x)^2$.

(OR)

b. Solve $(D+4)x+3y=t,\ 2x+(D+5)y=e^{2t}$ where $D\equiv\dfrac{d}{dt}$.

31. a. Find the equation of the circle of curvature $x^3+y^3=3axy$ at the point $\left(\dfrac{3a}{2},\dfrac{3a}{2}\right)$.

(OR)

b.i. Find the envelope of $x\cos\alpha+y\sin\alpha=a\sec\alpha$ where α is the parameter.

ii. Find the radius of curvature at the point (r, θ) on the curve $r^2\cos 2\theta=a^2$.

32. a. Find the equations of the spheres which pass through the circle $x^2+y^2+z^2=5$ and $x+2y+3z=3$ and touch the plane $4x+3y=15$.

(OR)

b. Find the equation of the cylinder whose generators are parallel to a line with line with direction ratios (1, –2, 3) and whose guiding curve is the ellipse $x^2+2y^2=1$ and $z=3$.

* * * * *

Reg. No.

B.Tech. DEGREE EXAMINATION, NOVEMBER 2017
First/ Second Semester
15MA101 – CALCULUS AND SOLID GEOMETRY
(For the candidates admitted during the academic year 2015 – 2016 onwards)

Note:
(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The quadratic form $x^2+2y^2+3z^2+2xy+2yz-2zx$ is
(A) Indefinite (B) Positive definite
(C) Positive semi definite (D) Negative definite

2. Two of the Eigen values of $\begin{bmatrix}3&-1&1\\-1&5&-1\\1&-1&3\end{bmatrix}$ are 3 and 6. Then the Eigen values of A^{-1} will be
(A) 2, 3, 6 (B) 0, 3, 6
(C) $\frac{1}{2},\frac{1}{3},6$ (D) $\frac{1}{2},\frac{1}{3}$, and 6

Ans $\frac{1}{2},\frac{1}{3},\frac{1}{6}$

3. The Eigen vectors corresponding to distinct Eigen values of a real symmetric matrix are____.
(A) Equal (B) Real
(C) Orthogonal (D) Distinct

4. The quadratic form corresponding to the matrix $\begin{bmatrix}2&1&-2\\1&2&-2\\-2&-2&3\end{bmatrix}$ is
(A) $2x_1^2+2x_2^2+3x_3^2+2x_1x_2-4x_2x_3-4x_3x_1$ (B) $x_1^2+x_2^2+x_3^3+x_1x_2$
(C) $x_1^2+x_2^2+x_3^2$ (D) $x_1^2+2x_2^2+2x_1x_2$

5. $\dfrac{du}{dx}$ when $u=\tan^{-1}\dfrac{y}{x},x^2+y^2=a^2$ is
(A) $-\dfrac{1}{x}$ (B) $-\dfrac{1}{y}$
(C) $\dfrac{1}{x}$ (D) $\dfrac{1}{y}$

Ans $-\dfrac{y}{a^2}$

6. If $x=r\cos\theta,\ y=r\sin\theta$ then $\dfrac{\partial(r,\theta)}{\partial(x,y)}$ is
(A) r (B) r^2
(C) –r (D) 1/r

7. Taylor expansion of e^{x+y} near origin is

(A) $1-(x+y)+\frac{x^2}{2}+....$

(B) $1+x+y+\frac{x^2}{2}+....$

(C) $1-x+y-\frac{x^2}{2}+....$

(D) $x+y+....$

8. The nature of the stationary point (1, 1) of the function f(x, y) if $fxx=6xy^3$, $fxy=9x^2y^2$, $fyy=6x^3y$ is

(A) Maximum point

(B) Minimum point

(C) Saddle point

(D) Doubtful case

9. The particular integral of $(D+1)^2 y=e^{-x}\cos x$, is

(A) $-e^{x}\cos x$

(B) $e^x \cos x$

(C) $-e^x \sin x$

(D) $e^x \sin x$

10. Solution of $x^2y''-xy'+y=0$ is

(A) $y=x(A\log x-B)$

(B) $y=A\log x+B$

(C) $y=\frac{1}{x}(A\log x+B)$

(D) $y=x(A\log x+B)$

11. Solving for x from $x'-y=t$ and $x+y'=1$, we get

(A) $x=A\cos t+B\sin t+2$

(B) $x=A\cos t$

(C) $x=A\sin t+2$

(D) $x=A\cos t+B\sin t$

12. The complimentary function of $x^2y''+4xy'+2y=x^2+\frac{1}{x^2}$

(A) $\frac{A}{x}-\frac{B}{x^2}$

(B) $\frac{A}{x}+\frac{B}{x^2}$

(C) $Ax+B$

(D) $\frac{A}{x}+B$

13. The envelope of the family $(x-\alpha)^2+y^2=2\alpha$, where α is the parameter is .

(A) $y^2=\alpha$

(B) $y=2\alpha$

(C) $y^2=-\alpha$

(D) $y^2=2\alpha$

Ans $y^2 = 1+2x$ No Ans.

14. The radius of curvature at (3, 4) on the curve $x^2+y^2=25$ is

(A) 5

(B) 4

(C) 0

(D) 2

15. Evolute of a curve is _______ of the normals of that curve.

(A) Involute

(B) Length

(C) Envelope

(D) End points

16. The parametric form of $xy=c^2$

(A) $x=ct, y=\frac{c}{t}$

(B) $x=ct, y=t$

(C) $x=\frac{c}{t}, y=t$

(D) $x=ct, y=\frac{1}{t}$

17. The centre of the sphere $7x^2+7y^2+7z^2-15x-25y-11z=0$ is

(A) $\left(\frac{15}{14},\frac{-25}{14},\frac{11}{14}\right)$

(B) $\left(\frac{15}{14},\frac{25}{14},\frac{-11}{14}\right)$

(C) $\left(\frac{15}{14},\frac{25}{14},\frac{11}{14}\right)$

(D) $\left(\frac{-15}{14},\frac{25}{14},\frac{11}{14}\right)$

18. Equation of cone with vertex at origin is

(A) Homogeneous equation of second degree in x, y, z

(B) Homogeneous equation of I degree

(C) Non homogenous equation

(D) Cannot be generalized

19. The right circular cylinder has its guiding curve

(A) Ellipse

(B) Circle

(C) Parabola

(D) Hyperbola

20. The spheres $x^2+y^2+z^2+6y+2z+8=0$ and $x^2+y^2+z^2+6x+8y+4z=0$ intersect at angle

(A) 2π

(B) $\frac{\pi}{3}$

No Ans.

(C) $\frac{\pi}{2}$

(D) π

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Find the Eigen vectors of $A=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$.

22. If $u=xy+yz+zx$ where $x=e^t$, $y=e^{-t}$ and $z=1/t$. Find $\frac{du}{dt}$.

23. Find the particular integral of $(D-3)^2y=xe^{-2x}$.

24. If the centre of curvature of a curve at a variable point 't' on it is $(2a+3at^2, -2at^3)$ find the evolute of the curve.

25. Find the equation of the core whose vertex is the origin and guiding curve is $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{1}=1, x+y+z=1$.

26. Find the possible extreme point of $f(x,y)=x^2+y^2+\frac{2}{x}+\frac{2}{y}$.

27. Find the nature, rank, index and signature of the quadratic form $x_1^2+2x_2^2-3x_3^2$.

b. Find the Eigen values and Eigen vectors of the matrix $A=\begin{bmatrix}2&0&1\\0&2&0\\1&0&2\end{bmatrix}$.

29. a. Expand $f(x,y)=e^{x}\cos y$ in powers of x and y upto second degree term, using Taylor's series.

(OR)

b. Find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

30. a. Solve the simultaneous differential equations $\frac{dx}{dt}+y=\sin t, \frac{dy}{dt}+x=\cos t$.

(OR)

b. Solve $\left(D^2+4D+3\right)y=5x^2+e^{2x}$.

31. a. Find the equation of circle of curvature of the curve $xy=c^2$ at (c,c).

(OR)

b. Show that the evolute of the curve $x=a(\cos\theta+\theta\sin\theta)$ $y=a(\sin\theta-\theta\cos\theta)$ is a circle.

32. a. Find the equation of the sphere having the circle $x^2+y^2+z^2+10y-4z-8=0, x+y+z=3$ as a great circle.

(OR)

b. Find the equation of the cone whose vertex is the point (1, 0, 1) and whose base is the curve $x^2+y^2=4, z=0$.

* * * * *

Reg. No.

B.Tech. DEGREE EXAMINATION, JUNE 2017
First / Second Semester

15MA101 – CALCULUS AND SOLID GEOMETRY
(For the candidates admitted during the academic year 2015-2016 onwards)

Note:
(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The matrix of the quadratic form $x_1^2+5x_2^2+x_3^2+2x_1x_2+2x_2x_3+6x_1x_3$ is

(A) $\begin{bmatrix}1&1&3\\1&5&1\\3&1&1\end{bmatrix}$ (B) $\begin{bmatrix}1&2&3\\1&5&2\\3&2&1\end{bmatrix}$

(C) $\begin{bmatrix}1&4&4\\4&5&3\\4&3&1\end{bmatrix}$ (D) $\begin{bmatrix}1&4&3\\4&5&4\\3&4&1\end{bmatrix}$

2. The Eigen values of an orthogonal matrix have the absolute value.
(A) 0 (B) 1
(C) 2 (D) ±1

3. If the sum of two Eigen values and trace of a 3×3 matrix A are equal, find the value of |A| is
(A) 0 (B) 1
(C) – (D) 2

4. The Eigen vectors corresponding to the distinct Eigen values of a real symmetric matrix are
(A) Imaginary (B) Non-orthogonal
(C) Real (D) Orthogonal

5. If $z=x^2+y^2+3xy$, then $\frac{\partial z}{\partial x}$ is
(A) 2x (B) 2y+3x
(C) 2x+3y (D) 3y

6. If $J_1=J\left(\frac{x,y}{u,v}\right)$ and $J_2=J\left(\frac{u,v}{x,y}\right)$, then J_1J_2 is
(A) 0 (B) 1
(C) −1 (D) 2

7. If $rt - s^2 < 0$ at (a,b), where $r = \frac{\partial^2 f}{\partial x^2}, t = \frac{\partial^2 f}{\partial x \partial y}, t = \frac{\partial^2 f}{\partial y^2}$, then the point (a, b) is

(A) Maximum point
(B) Minimum point
(C) Saddle point
(D) Extremum point

8. The degree of the homogenous function $f(x,y) = \frac{x^2 + y^2}{x - y}$ is

(A) 2
(B) 3
(C) −1
(D) 1

9. The particular integral of $(D^2 + 2D + 1)y = 7$ is

(A) 7
(B) −7
(C) 8
(D) 0

10. If $y_1 = \cos ax$ and $y_2 = \sin ax$, then $y_1 y_2' - y_2 y_1'$ is

(A) − a
(B) 0
(C) 1
(D) a

11. Solution of $(D^2 + 4)y = 0$ is

(A) $y = A\cos 2x + B\sin 2x$
(B) $y = Ae^{2x} + Be^{-2x}$
(C) $y = A\cos\sqrt{2}x + B\sin\sqrt{2}x$
(D) $y = (Ax + B)e^x$

12. Using the transformation $Z = \log x$, the differential equation $x^2 y'' - xy' + y = x^2$ converts into an equation with constant coefficient given by

(A) $(\theta^2 - 2\theta + 1)y = e^{2z}$
(B) $(\theta^2 - 2\theta + 1)y = e^z$
(C) $(\theta^2 + 2\theta + 1)y = e^{2z}$
(D) $(\theta^2 + 2\theta + 1)y = e^z$

13. The locus of center of curvature is called

(A) Involute
(B) Evolute
(C) Radius of curvature
(D) Envelope

14. The curvature of the straight line is

(A) 1
(B) 2
(C) −1
(D) 0

15. The radius of curvature in polar coordinates is

(A) $P = \frac{(r^2 + r'^2)^{3/2}}{r^2 - rr'' + 2r'^2}$
(B) $P = \frac{(r^2 - r'^2)^{3/2}}{r^2 - rr'' + 2r'^2}$
(C) $P = \frac{r^2 + r'^2}{r^2 - rr'' + 2r'^2}$
(D) $P = \frac{r^2 - r'^2}{r^2 - rr'' + 2r'^2}$

16. Evolute of a curve is envelope of ________ that curve.

(A) Tangent (C) Parallel
(B) Normal
(D) Locus

17. The centre of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 4z = 0$ is

(A) $(1, -2, 2)$
(B) $(-1, 2, -2)$
(C) $(1, 2, 2)$
(D) $(-1, -2, -2)$

18. Two spheres $x^2 + y^2 + z^2 + 2u_1 x + 2v_1 y + 2w_1 z + d_1 = 0$ and $x^2 + y^2 + z^2 + 2u_2 x + 2v_2 y + 2w_2 z + d_2 = 0$ cute orthogonally if

(A) $2u_1u_2 + 2v_1v_2 + 2w_1w_2 = d_1 + d_2$
(B) $u_1u_2 + v_1v_2 + w_1w_2 = d_1 + d_2$
(C) $u_1u_2 + v_1v_2 + w_1w_2 + d_1d_2 = 0$
(D) $2u_1u_2 + 2v_1v_2 + 2w_1w_2 - d_1d_2 = 0$

19. A surface generated by a straight line which is parallel to a fixed line that intersects a curve is known as

(A) Sphere
(B) Cone
(C) Cylinder
(D) Arch

20. A surface generated by a variable line which passes through a fixed point and intersects a given curve is

(A) Cylinder
(B) Cone
(C) Enveloping cylinder
(D) Sphere

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Find the sum and product of all Eigen values of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

22. Find the nature, rank, index and signature of the quadratic form $2x_1x_2 + 2x_2x_3 + 2x_1x_3$.

23. If $p = 3x + 2y - z, q = x - 2y + z, r = x + 2y - z$ find $\frac{\partial(p,q,r)}{\partial(x,y,z)}$.

24. Solve $(D^2 - 2D - 3)y = 5$.

25. Solve $(D^2 + 6D + 9)y = 0$

26. Find the envelope of the family of straight lines $y = mx + \frac{a}{m}$.

27. Find the centre and radius of the sphere $2x^2 + 2y^2 + 2z^2 - 6x + 8y - 8z - 1 = 0$

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & +1 & -1 \end{bmatrix}$ and hence find A^4.

(OR)

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a.i. Find Eigen values and Eigen vectors of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$.

ii. Find A^{-1} using Cayley-Hamilton theorem for the matrix $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$.

(OR)

b. Reduce the quadratic form $(2x_1^2 + 6x_2^2 + 2x_3^2 + 8x_1x_3)$ to canonical form by orthogonal reduction. Also find the rank, index, signature and nature of quadratic form.

29. a.i. If $u = \sin^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{1}{2}\tan u$.

ii. If $x = e^{u} + e^{-v}$ *and* $y = e^{-u} - e^{v}$ express $\left(x\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial y}\right)$ interms of $\dfrac{\partial z}{\partial u}$ *and* $\dfrac{\partial z}{\partial v}$.

(OR)

b. Using Lagrange's method, find the maximum and minimum distances of the point (3,4,12) from the unit sphere $x^2 + y^2 + z^2 = 1$

30. a. Solve $(x^2D^2 - 2xD - 4)y = 32(\log x)^2$.

(OR)

b. Solve by the method of variation of parameters, $\dfrac{d^2y}{dx^2} + y = \tan x$.

31. a. Find radius of curvature of the curve $xy^2 = a^3 - x^3$ *at* $(a,0)$.

(OR)

b. Find evolute of the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

32. a. Find the equation of right circular cone with vertex at (1, –2, 1), semi vertical angle 60° and the line $\dfrac{x-1}{3} = \dfrac{y+2}{-4} = \dfrac{z+1}{5}$ as it's axis.

(OR)

b. Find the equation of the right circular cylinder of radius 3 and axis $\dfrac{x-1}{2} = \dfrac{y-3}{2} = \dfrac{z-5}{-1}$.

* * * * *

Reg. No.														

B.Tech. DEGREE EXAMINATION, NOVEMBER 2016
First Semester

15MA101 – CALCULUS AND SOLID GEOMETRY
(For the candidates admitted during the academic year 2015 – 2016 onwards)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. If the sum of 2 Eigen values and trace of a 3×3 matrix a are equal, then |A| is
   (A) 0 (B) 1
   (C) –1 (D) 2

2. If $A = \begin{pmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{pmatrix}$, then Eigen values of A^{-1} are
   (A) $1, \frac{1}{3}, \frac{1}{4}$ (B) 1, 3, 4
   (C) $1^2, 3^2, 4^2$ (D) $1, \frac{1}{3^2}, \frac{1}{4^2}$

3. The Eigen values of orthogonal matrix are
   (A) 0 (B) ±1
   (C) ±2 (D) $\pm\frac{1}{2}$

4. The number of positive terms in the canonical form is called
   (A) Signature (B) Index
   (C) Semi definite (D) Positive definite

5. If f(x,y) = 0 is an implicit function of x and y , then $\dfrac{dy}{dx}$ is
   (A) $\dfrac{+fx}{fy}$ (B) $\dfrac{-fx}{fy}$
   (C) $\dfrac{fy}{fx}$ (D) $\dfrac{-fy}{fx}$

6. Disadvantages of Lagrange's method on maxima and minima is that generally
   (A) Only minimum can be determined (B) Only maximum can be determined
   (C) Extreme values obtained can't be classified as maximum or minimum (D) Both maximum and minimum can be determined

7. If $u = x^2 - y^2, v = 2xy$, then value of $J\left(\frac{u,v}{x,y}\right) \times J\left(\frac{x,y}{u,v}\right)$ is

(A) 0 (B) 1
(C) -1 (D) Not equal to zero

8. If $(rt-s^2)<0$ at a point (a,b) then (a,b) is
(A) Minimum point (B) Maximum point
(C) Saddle point (D) Extreme point

9. Solution of differential equation $(x^2D^2 + xD + 1)y = 0$ is
(A) $(Ae^x + Be^{-x})$ (B) $(A\cos x + B\sin x)$
(C) $(Ax+B)e^x$ (D) $(A\cos\log x + B\sin\log x)$

10. P.I of $(D^2+4)y = \sin 2x$ is
(A) $\frac{-x}{2}\cos 2x$ (B) $\frac{-x}{4}\sin 2x$
(C) $\frac{-x}{4}\cos 2x$ (D) $\frac{x}{4}\cos 2x$

11. The general solution of the differential equation $\left(\frac{d^2y}{dx^2} - y\right) = 0$ is
(A) $y = (Ax+B)e^x$ (B) $y = Ax + \frac{B}{x}$
(C) $y = A + Be^x$ (D) $y = Ae^x + Be^{-x}$

12. P.I of differential equation $(D^2+5)y = e^x$ is
(A) 0 (B) $\frac{2e^x}{3}$
(C) $\frac{e^x}{6}$ (D) $\frac{2e^x}{5}$

13. Curvature of the circle $x^2 + y^2 = a^2$ is
(A) 0 (B) a
(C) 1 (D) $\frac{1}{a}$

14. Locus of centre of curvature is called
(A) Involute (B) Evolute
(C) Envelope (D) Conic

15. Envelope of the curve $y = mx + \frac{a}{m}$ 'm' being parameter is
(A) $y^2 - 4ax = 0$ (B) $y^2 + 4ax = 0$
(C) $x^2 + y^2 = 1$ (D) $xy = c^2$

16. Evolute of the cycloid $x = a(\theta - \sin\theta), y = a(1-\cos\theta)$ is
(A) Astroid (B) Parabola
(C) Cycloid (D) Circle

17. Equation of cone having vertex at origin and passing through $x^2 + y^2 = 9, z = 3$ is
(A) $x^2 - y^2 + z^2 = 0$ (B) $x^2 + y^2 + z^2 = 0$
(C) $x^2 - y^2 = z^2$ (D) $x^2 + y^2 = z^2$

18. Equation of the sphere passing through 4 points (0,0,0), (a,0,0), (0,b,0) and (0,0,c) is
(A) $x^2 + y^2 + z^2 + ax + by + cz = 0$ (B) $x^2 + y^2 + z^2 - 2ax - 2by - 2cz = 0$
(C) $x^2 + y^2 + z^2 + 2ax + 2by + 2cz = 0$ (D) $x^2 + y^2 + z^2 - ax - by - cz = 0$

19. The intersection of a plane with right circular cone perpendicular to its axis is
(A) Parabola (B) Ellipse
(C) Circle (D) Hyperbola

20. If two spheres touch each other externally, then the distance between the centres of two spheres
(A) Equal to sum of the radii (B) Less than sum of the radii
(C) Greater than sum of the radii (D) Equal to sum of squares of the radii

PART – B ($5 \times 4 = 20$ Marks)

Answer ANY FIVE Questions

21. Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$.

22. Find the radius of curvature for the curve $y = e^x$ where it crosses y axis.

23. Find Taylor's series expression of x^y near the point (1,1) upto second degree terms.

24. Determine whether the functions $u = \sin^{-1}x + \sin^{-1}y$ and $v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$ are functionally dependent.

25. Solve $(D^2+4)y = \cos^2 x$.

26. Find envelope of the family of straight lines $x\cos\alpha + y\sin\alpha = a\sec\alpha$, α being the parameter.

27. Find the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0, x + y + z = 3$ as a great circle.

29. a. i. Find maximum value of $x^2y^3z^4$ subject to the condition $x+y+z=9$.

ii. Find the Taylor's series expansion of $e^x\cos y$ at $(0,0)$ upto 3rd degree terms.

(OR)

b.i. Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid.

ii. If $u=f(x-y,y-z,z-x)$ find the value of $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$.

30. a. i. Find the radius of curvature of $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $\left(\frac{a}{4},\frac{a}{4}\right)$.

ii. Show that the Evolute of cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ is another cycloid.

(OR)

b. Find the Evolute of $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$.

31. a.i. Solve $\left(D^3-3D^2+4D-2\right)y=e^x+\cos x$.

ii. Solve $(x^2D^2+xD+1)y=4\sin(\log x)$.

(OR)

b. Solve the simultaneous equations

$Dx+2x-3y=5t$ given $x(0)=0$

$Dy-3x+2y=0$ $y(0)=-1$

32. a.i. Obtain the equation of sphere having the circle $x^2+y^2+z^2=9, x+y+z=9$ as a great circle.

ii. Show that the equation to the right circular cone whose vertex is Origin O, axis OZ and semi vertical angle α is $x^2+y^2=z^2\tan^2\alpha$.

(OR)

b. Find the equation of sphere that passes through the circle $x^2+y^2+z^2+x-3y+2z-1=0$, $2x+5y-z+7=0$ and cut orthogonally the sphere $x^2+y^2+z^2-3x+5y-7z-6=0$.

*****

Reg. No. | | | | | | | | | | | | | | |

B.Tech. DEGREE EXAMINATION, JUNE 2016
First Semester

MA1001 – CALCULUS AND SOLID GEOMETRY
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. Find Eigen values of A^{-1} if $A=\begin{pmatrix}2&5&-1\\0&3&2\\0&0&4\end{pmatrix}$

(A) 2, 3, 4
(B) $\frac{1}{2},\frac{1}{3},\frac{1}{4}$
(C) 2, 5, −1
(D) 0, 0, 0

2. Two Eigen values of $\begin{pmatrix}2&2&0\\2&1&1\\-7&2&-3\end{pmatrix}$ are −4 and 3. What is the third Eigen value?

(A) 1
(B) −1
(C) 2
(D) 3

3. If $x=r\cos\theta$; $y=r\sin\theta$. The value of $\frac{\partial(x,y)}{\partial(r,\theta)}=$

(A) r
(B) −r
(C) $\frac{1}{r}$
(D) r^2

4. If f(x,y) is a homogeneous function of degree 'n'. Then $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=$

(A) (n−1)f
(B) n(n−1)f
(C) n^2f
(D) n f

5. *Solve* $(D-1)^3y=0$

(A) $y=0$
(B) $y=Ae^x$
(C) $y=(A+Bx)e^x$
(D) $y=(A+Bx+Cx^2)e^x$

6. The particular Integral of $(D^2-4D-5)y=e^{2x}$.

(A) $-\frac{1}{9}e^{2x}$
(B) e^{2x}
(C) $\frac{1}{9}e^{2x}$
(D) $9e^{2x}$

7. Write the Equation of the circle of curvature

(A) $(x-\bar{x})^2+(y-\bar{y})^2=\rho^2$ (B) $(x+\bar{x})^2+(y+\bar{y})^2=\rho^2$

(C) $(x-\bar{x})^2+(y-\bar{y})^2=-\rho^2$ (D) $(x-\bar{x})^2+(y-\bar{y})^2=0$

8. The condition of orthogonality of two spheres is

(A) $2(u_1u_2+v_1v_2+w_1w_2)=d_1+d_2$ (B) $2(u_1u_2+v_1v_2-w_1w_2)=d_1+d_2$

(C) $2(u_1u_2+v_1v_2+w_1w_2)^2=d_1^2+d_2^2$ (D) $2(u_1u_2-v_1v_2+w_1w_2)=d_1-d_2$

9. Find the solution of $(D^2+1)y=\sin 2x$

(A) $y=(A\cos x+B\sin x)+\sin x$ (B) $y=(A\cos x+B\sin x)$

(C) $y=(A\cos x+B\sin x)-\dfrac{\sin 2x}{3}$ (D) $y=\sin x$

10. The inverse of Eigen values of $\begin{bmatrix}1 & 2\\5 & 4\end{bmatrix}$ are

(A) $-1, 6$ (B) $1, 6$

(C) $1, -6$ (D) $-1, -6$

11. The sum of squares of Eigen values of $\begin{pmatrix}1 & 7 & 5\\0 & 2 & 9\\0 & 0 & 5\end{pmatrix}$ is

(A) 25 (B) 32

(C) 30 (D) 2

12. Saddle points are_______.

(A) a minimum (B) a maximum

(C) neither maximum nor minimum (D) equal

13. The order and degree of $(D^2+3D+2)y=e^{-x}$

(A) 2, 1 (B) 1, 2

(C) 2, 2 (D) 1, 1

14. Evolute is the locus of______ of curvature

(A) Radius (B) Circle

(C) Centre (D) Reciprocal

15. The radius of curvature at any point (r,θ) for the curve $r=a\cos\theta$ is

(A) $\dfrac{a}{2}$ (B) $\dfrac{2}{a}$

(C) $\dfrac{3}{a}$ (D) $\dfrac{3}{2a}$

16. The radius of sphere $x^2+y^2+z^2-2y-4z=11$

(A) -4 (B) 4

(C) 3 (D) -3

17. The equation of sphere whose centre is (1, 0, 6) and radius 5 is

(A) $x^2+y^2+z^2-6x-6z-6=0$ (B) $2x^2+2y^2+2z^2-2x-2z-2=0$

(C) $x^2+y^2+z^2-2x-12z+12=0$ (D) $x^2+y^2+z^2-2x-2z-2=0$

18. The direction ratios of the line given by $x+y-z=1$ and $2x-y+7z+1=0$ is

(A) $(-1,-3,2)$ (B) $(1,3,-2)$

(C) $(-2,3,1)$ (D) $(2,-3,-1)$

19. The particular integral of $(D^2+2D+1)y=5$ is

(A) 0 (B) 5

(C) 2 (D) 1

20. The minimum value of $x^2+y^2+6x+12=0$

(A) $\dfrac{1}{2}$ (B) 2

(C) 1 (D) 3

PART – B (5 × 4 = 20 Marks)

Answer ANY FIVE Questions

21. Find Eigen values and Eigen vectors of $A=\begin{pmatrix}1 & 1\\3 & -1\end{pmatrix}$.

22. If $u=\tan^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\dfrac{1}{4}\sin 2u$.

23. Solve $(D^2+4D+4)y=e^{-2x}$.

24. Find the radius of curvature at the point $x=c$ on the curve $xy=c^2$.

25. Show that the spheres

$s_1=x^2+y^2+z^2+6y+2z+8=0$ and

$s_2=x^2+y^2+z^2+6x+8y+4z+20=0$

intersect at right angles.

26. Find the envelope of the family given by $x=my+\dfrac{1}{m}$; where m is a parameter.

27. Find the value of $\dfrac{du}{dt}$ given $u=y^2-4ax;\ x=at^2,\ y=2at$.

PART – C (5 × 12 = 60 Marks)

Answer ALL Questions

28. a. Reduce the Quadratic form to a canonical form for $Q=x^2+y^2+z^2-2xy+2yz$.

(OR)

b. Verify Cayley-Hamilton theorem for $A=\begin{pmatrix}1 & 2 & 3\\2 & 1 & -1\\1 & -1 & 1\end{pmatrix}$

29. a. Find the greatest and least distance of the point $(3, 4, 12)$ from the unit sphere $x^2+y^2+z^2=1$ whose centre is at the origin.

(OR)

b. i. Using Taylor's series, verify that, $\log(1+x+y) = (x+y) - \frac{1}{2}(x+y)^2 + \frac{1}{3}(x+y)^3 - \dots\dots$

ii. Find $\frac{du}{dx}$ if $u = \tan^{-1}(y/x)$, where $x^2+y^2=a^2$ by treating u as function of x and y only.

30.a.i. Solve $Dx + y = \sin t$ and $x + Dy = \cos t$ given that $x = 2$ and $y = 0$ when $t = 0$.

ii. Solve the equation $\frac{d^2y}{dx^2}+a^2y=\tan ax$ by the method of variation of parameters.

b. Solve $(x^2D^2 - xD + 1)y = \left(\frac{\log x}{x}\right)^2$

31.a. Show that the evaluate of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$ is another cycloid given by $x = a(\theta - \sin\theta)$ $y - 2a = a(1+\cos\theta)$

(OR)

b. Find the radius of curvature of the curve $xy = c^2$ at (c, c).

32. a. Find the equation of the sphere described on the line joining the points $(2, -1, 4)$ and $(-2, 2, -2)$ as diameter. Find also the area of the circle in which the sphere is cut by the plane $2x + 2y - z = 3$.

(OR)

b. Find the equation of the right circular cylinder whose axis is $\frac{x-2}{2}=\frac{y-1}{1}=\frac{z-0}{3}$ and which passes through the point $(0, 0, 3)$.

*****

Reg. No.

B.Tech. DEGREE EXAMINATION, MAY 2016
First Semester

15MA101 – CALCULUS AND SOLID GEOMETRY
(For the candidates admitted during the academic year 2015 – 2016)

Note:
(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45[th] minute.
(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. Find the eigen values of A^2 if
$$\begin{bmatrix} 3 & 2 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$
(A) 6, 4, 10 (B) 9, 4, 25
(C) 9, 2, 5 (D) 3, 2, 5

2. Find the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$
(A) Positive Definite (B) Negative Definite
(C) Indefinite (D) Positive semidefinite

3. If the sum of 2 eigen values and trace of a 3×3 matrix A are equal, then |A| is
(A) 0 (B) 1
(C) -1 (D) 2

4. Matrix of the Quadratic form $x^2 + xy$ is
(A) $\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$
(C) $\begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

5. If $u = ax^2 + 2hxy + by^2$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$
(A) u (B) 2u
(C) 3u (D) n(n−1)u

6. The stationary points of $x^2 + y^2 + 6x + 12$ are
(A) (−3, 0) (B) (0, 3)
(C) (0, −3) (D) (3, 0)

7. If $x^y = y^x$, then $\frac{dy}{dx}$ is
(A) Does not exists
(B) $(x \log y - y)\, y / x(y \log x - x)$
(C) $(x \log x - x) / (y \log y - y)$
(D) $(x \log x - y) / (y \log x - x)$

8. u and v are functionally dependent if their Jacobian value is
(A) 0
(B) 1
(C) Non zero
(D) >0

9. The particulars integral of $(D^2+4)y = e^{-x}$ is
(A) $e^{-x}/{-5}$
(B) $e^{-x}/5$
(C) $e^{-x}/3$
(D) $e^{-x}/9$

10. The complementary function of $(D^2+4)\, y = 0$ is
(A) $y = A\cos 2x + B\sin 2x$
(B) $y = A e^{2x} + B e^{-2x}$
(C) $y = A \cos\sqrt{2}\, x + B\sin\sqrt{2}\, x$
(D) $y = (Ax + B)\, e^{-2x}$

11. Using the transformation $z = \log x$, convert the differential equation $x^2y'' - xy' + y = x^2$ to a differential equation with constant coefficient
(A) $(\theta^2 - 2\theta + 1)\, y = e^{2z}$
(B) $(\theta^2 - 2\theta + 1)\, y = e^{z}$
(C) $(\theta^2 + 2\theta + 1)\, y = e^{2z}$
(D) $(\theta^2 + 2\theta + 1)\, y = e^{z}$

12. The particular integral of $(D-1)^2 y = e^x \sin x$ is
(A) $-e^x \cos x$
(B) $e^x \cos x$
(C) $-e^x \sin x$
(D) $-e^x \sin x$

13. The curvature of a straight line is
(A) 1
(B) 2
(C) −1
(D) 0

14. The radius of curvature of $x^2 + y^2 = 25$ at (4,3) is
(A) 5
(B) −5
(C) 25
(D) −25

15. The envelope of the family of straight line is $y = mx + \frac{a}{m}$
(A) $y^2 = 4ax$
(B) $x^2 = 4ay$
(C) $y^2 = -4ax$
(D) $x^2 = -4ay$

16. The locus of centre of curvature is
(A) Envelope
(B) Evolute
(C) Curvature
(D) Invalute

17. The tangent plane at $(1,2,0)$ to the sphere $3(x^2 + y^2 + z^2) + 8x + 12y + 16z - 47 = 0$
(A) $x - 2y + 3z - 10 = 0$
(B) $x + y + z - 9 = 0$
(C) $7x + 12y + 8z - 31 = 0$
(D) $7x + 12y + 8z + 31 = 0$

18. The centre and radius of sphere $2x^2 + 2y^2 + 2z^2 - 6x + 8y - 8z - 1 = 0$
(A) $(3,4,4), \sqrt{42}$
(B) $(-3,4,4), \sqrt{40}$
(C) $(-3,4,-4), \sqrt{42}$
(D) $(3,-4,4), \sqrt{42}$

19. The surface generated by a straight line which is always parallel to a given fixed line and intersects another curve is known as
(A) Cylinder
(B) Cone
(C) Sphere
(D) Right circular cone

20. The semivertical angle of the right circular cone having its vertex at the origin and passing through the circle $y^2 + z^2 = 25, x = 4$ is
(A) $\theta = \tan^{-1}(4/5)$
(B) $\theta = \tan^{-1}(5/4)$
(C) $\pi/4$
(D) $\pi/3$

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Show that the matrix $A = \frac{1}{3}\begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ is orthogonal.

22. If $u = 2xy$, $v = x^2 - y^2$, $x = r\cos\theta$, $y = r\sin\theta$. Compute $\frac{\partial(u,v)}{\partial(r,\theta)}$

23. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$

24. Solve $(D^2 + 4)\, y = x^4$.

25. Find the envelope of the family of line $\frac{x}{a} + \frac{y}{b} = 1$ where the parameters a and b are connected by the relation $a + b = c$.

26. Find the radius of curvature of the curve $r = a\sin\theta$, at the pole.

27. Prove that the two spheres $x^2 + y^2 + z^2 - 2x + 4y - 4z = 0$ and $x^2 + y^2 + z^2 + 10x + 2z + 10 = 0$ touch each other.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Verify that the matrix $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ satisfy its characteristic equation and hence find A^4.

(OR)

b. Reduce the Quadratic form $2x_1x_2 + 2x_2x_3 + 2x_1x_3$ to canonical form by orthogonal transformation and hence find its rank, index, signature and nature.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Diagnolize the matrix $A=\begin{pmatrix}3 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 3\end{pmatrix}$

(OR)

b. Verify Cayley Hamiltonian theorem for $A=\begin{pmatrix}1 & 2 & 3\\ 2 & -1 & 4\\ 3 & 1 & -1\end{pmatrix}$ and hence find A^4.

29.a.i Expand $\sin(xy)$ in powers of $(x-1)$ and $\left(y-\frac{\pi}{2}\right)$ upto second degree terms.

ii. If $u = xy+yz+zx$, $v = x^2+y^2+z^2$ and $w = x+y+z$, determine whether there is a functional relationship between u, v, w. If so, find it.

(OR)

b. i. Show that the function $f(x, y) = x^3+y^3-63(x+y)+12xy$ is maximum at $(-7, -7)$ and minimum at (3,3)

ii. Find the dimensions of the rectangular box without a top of maximum capacity, whose surface is 108 sq.cm.

30. a. Solve: $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=4\sin(\log x)$.

(OR)

b. Solve: $\frac{d^2y}{dx^2}+4y=\sec 2x$, by the method of variation of parameters.

31. a. Find the equation of the circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $\left(\frac{a}{4},\frac{a}{4}\right)$

(OR)

b. Find the evolute of the curve $x=a(\cos\theta+\theta\sin\theta)$, $y=a(\sin\theta-\theta\cos\theta)$.

32. a. Find the equation of the sphere that passes through the circle $x^2+y^2+z^2-2x+3y-4z+6=0$, $3x-4y+5z-15=0$ and cuts the sphere $x^2+y^2+z^2+2x+4y-6z+11=0$ orthogonally.

(OR)

b. i. Find the equation to the right circular cylinder of radius 3 whose axis passes through (1,–1,2) and has direction cosines proportional to 2, –1, 3.

ii. Find the equation of the right circular cone whose vertex is at (2, –3, 5) and axis makes equal angles with the coordinate axes, the semi-vertical angle is measured to be 30°.

* * * * *

Reg. No.										

B.Tech. DEGREE EXAMINATION, DECEMBER 2015
First Semester

MA1001 – CALCULUS AND SOLID GEOMETRY
(For the candidates admitted from the academic year 2013 – 2014)

Note:
(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45^{th} minute.
(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The Eigen values of A^3, if $A=\begin{pmatrix}1 & 2 & 3\\ 0 & 2 & -7\\ 0 & 0 & 3\end{pmatrix}$ are

(A) 1, 4, 9 (B) 1, 8, 27
(C) 1, 2, 3 (D) 1, 0, 0

2. The product of two Eigen values of $A=\begin{pmatrix}6 & -2 & 2\\ -2 & -3 & -1\\ -2 & -1 & 3\end{pmatrix}$ is 16. Then the third Eigen value is

(A) 2 (B) 1
(C) 3 (D) 0

3. Find the sum and product of the Eigen values of $\begin{pmatrix}7 & -2 & -2\\ -2 & 1 & 4\\ -2 & 4 & 1\end{pmatrix}$

(A) 7, 81 (B) 9, 81
(C) 14, 81 (D) 8, 80

4. The Eigen values of $A=\begin{pmatrix}5 & 4\\ 1 & 2\end{pmatrix}$ are

(A) 1, 6 (B) –1, 6
(C) –1, –6 (D) 1, –6

5. If $x=r\cos\theta$, $y=r\sin\theta$, the value of $\frac{\partial(x,y)}{\partial(r,\theta)}\times\frac{\partial(r,\theta)}{\partial(x,y)}=$

(A) 0 (B) –1
(C) 1 (D) 2

6. If u is a homogeneous function of degree 'n', then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$

(A) $-nu$ (B) n^2u
(C) $n(n-1)u$ (D) nu

7. Find $\frac{dy}{dx}$, if $x^3+y^3=3ax^2y$

(A) $\frac{x^2-2axy}{y^2-ax^2}$ (B) $\frac{xy-y^2}{y^2-ax^2}$
(C) $\frac{2ay-x^2}{y^2-ax^2}$ (D) $\frac{x(2ay-x)}{y^2-ax^2}$

8. If u and v are functions of r and s, where r and s are functions of x and y, then $\frac{\partial(u,v)}{\partial(x,y)}=$

(A) $\frac{\partial(u,v)}{\partial(r,s)}\times\frac{\partial(r,s)}{\partial(x,y)}$ (B) $\frac{\partial(r,s)}{\partial(u,v)}\times\frac{\partial(u,v)}{\partial(x,y)}$
(C) $\frac{\partial(u,v)}{\partial(x,y)}\times\frac{\partial(x,y)}{\partial(r,s)}$ (D) $\frac{\partial(x,y)}{\partial(r,s)}\times\frac{\partial(r,s)}{\partial(u,v)}$

9. The complementary function of $(D^2+6D+9)y=0$ is

(A) $(C_1+C_2x)e^{-3x}$ (B) $(C_1+C_2x)e^{3x}$
(C) $C_1e^{3x}+C_2e^{-3x}$ (D) $C_1e^{-3x}+C_2e^{3x}$

10. The particular integral of $(D^2+16)y=e^{-4x}$ is

(A) $\frac{1}{16}e^{4x}$ (B) $\frac{1}{16}e^{-4x}$
(C) $\frac{1}{32}e^{-4x}$ (D) $\frac{1}{32}e^{4x}$

11. The complementary function of $(x^2D^2+4xD+2)y=xlogx$ is

(A) $C_1e^{-2z}+C_2e^{z}$ (B) $C_1e^{z}+C_2e^{-z}$
(C) $C_1e^{z}+C_2e^{2z}$ (D) $C_1e^{-z}+C_2e^{-2z}$

12. The solution of the differential equation $(D^2+9)y=e^{-2x}$ is

(A) $C_1\cos 3x+C_2\sin x+\frac{e^{-2x}}{4}$ (B) $C_1\cos 3x+C_2\sin 3x+\frac{e^{-2x}}{13}$

(C) $C_1e^{3x}+C_2e^{-3x}+\frac{e^{-2x}}{13}$ (D) $(C_1+C_2x)e^{3x}+\frac{e^{-2x}}{13}$

13. The radius of curvature at any point (r, θ) for the curve $r=a\cos\theta$ is

(A) $\frac{a}{2}$ (B) a

(C) 2a (D) $\frac{a}{3}$

14. The radius of curvature of the curve $y=e^x$ at $x=0$ is
(A) $2\sqrt{2}$ (B) $\sqrt{2}$
(C) 2 (D) $2\sqrt{3}$

15. The envelope of the family of straight lines $y=mx+\frac{1}{m}$, is

(A) $y^2=3x$ (B) $y^2=x$
(C) $y^2=4x$ (D) $y^2=2x$

16. The equation of the envelope of the family $A\alpha^2+B\alpha+C=0$, where α is the parameter, is
(A) $A^2=BC=0$ (B) $B^2-AC=0$
(C) $B^2-4AC=0$ (D) $A^2-4BC=0$

17. The equation of the sphere, whose centre is at (–6, 1, 3) and radius 4 is
(A) $x^2+y^2+z^2+12x-2y-6z+30=0$ (B) $x^2+y^2+z^2-12x-2y+6z+30=0$
(C) $x^2+y^2+z^2+12x+2y+6z-30=0$ (D) $x^2+y^2+z^2-12x-2y-6z-30=0$

18. The equation of the sphere which is the join of the points (1, 2, 3) and (0, 4,–1) as diameter is
(A) $x^2+y^2+z^2-x+6y-2z-5=0$ (B) $x^2+y^2+z^2-x-6y+2z+5=0$
(C) $x^2+y^2+z^2+x+6y+2z+5=0$ (D) $x^2+y^2+z^2-x-6y-2z+5=0$

19. A surface generator by a line which intersects a fixed circle and is perpendicular to the plane of the circle is
(A) Right circular cylinder (B) Right circular cone
(C) Cone (D) Sphere

20. The equation of the cone with vertex of the origin and passing through $x^2+y^2=9, z=3$
(A) $x^2-y^2+z^2=0$ (B) $x^2-y^2=z^2$
(C) $x^2+y^2+z^2=0$ (D) $x^2+y^2=z^2$

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Find the rank, index, signature and nature of the quadratic form $2x_1^2+x_2^2+x_3^2+2x_1x_2-2x_1x_3-4x_2x_3$

22. If $u=f(r)$, $x=r\cos\theta$, $y=r\sin\theta$, prove that $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r)$.

23. Solve $(D^2+3D+2)y=\sin x$ where $D=\frac{d}{dx}$.

24. Find the radius of curvature of the curve $r=a(1+\cos\theta)$ at the point $\theta=\frac{\pi}{2}$.

25. Find the equation of the sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z=8$ is a great circle.

26. Find A^{-1} of the matrix $A=\begin{pmatrix}5 & 4\\1 & 2\end{pmatrix}$ using Cayley Hamilton theorem.

27. If $u=y+z, v=x+2z^2, w=x-4yz-2y^2$, find the Jacobian of u, v, w with respect to x, y, z.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Verify Cayley-Hamilton theorem for $A=\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 4\\ 3 & 1 & -1\end{bmatrix}$ and hence find A^{-1} and A^4.

(OR)

b. Reduce the quadratic form $Q=3x_1^2+5x_2^2+3x_3^2-2x_1x_2-2x_2x_3+2x_3x_1$ to a di
Cannonical form and hence find its rank, index and signature.

29. a. Find the extreme values of $sinx+siny+sin(x+y)$.

(OR)

b. Find the volume of the largest rectangular parallelopiped that can be inscribed
ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

30. a. Solve $(x+2)^2\frac{d^2y}{dx^2}-(x+2)\frac{dy}{dx}+y=3x+4$.

(OR)

b. Solve using variation of parameter method $\frac{d^2y}{dx^2}+4y=4\tan x$.

31. a. Find the equation of circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $\left(\frac{a}{4},\frac{a}{4}\right)$.

(OR)

b. Find the evolute of the curve $x = a\cos^3\theta, y = a\sin^3\theta$.

32.a.i Find the equation of the sphere having the circle $x^2+y^2+z^2-2x+4y-6z+7=0$ and $2x-y+$
for a great circle.

ii. Show that the sphere $x^2+y^2+z^2 = -9$ and $x^2+y^2+z^2-6x+13y-2z+9 = 0$ cut orthogonally

(OR)

b. i. Find the equation of the right circular cone whose vertex is at (2, –3, 5) axis makes angles with coordinate axis and the semi vertical angle is measured to be 30°.

ii. Find the equation of the right circular cylinder of radius 3 and axis $\frac{x-1}{2}=\frac{y-3}{2}=\frac{z-5}{-1}$

*****

Reg. No.

B.Tech. DEGREE EXAMINATION, DECEMBER 2015
First Semester

15MA101 – CALCULUS AND SOLID GEOMETRY
(For the candidates admitted during the academic year 2015 – 2016)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The Eigen values of the matrix $\begin{bmatrix}1 & 2\\ 5 & 4\end{bmatrix}$ are

(A) 1, 6
(B) –1, 6
(C) 1, –6
(D) –1, –6

2. If $A=\begin{bmatrix}3 & 5 & 3\\ 0 & 4 & 6\\ 0 & 0 & 1\end{bmatrix}$, then the Eigen values of A^{-1} are

(A) $1,\frac{1}{3},\frac{1}{4}$
(B) 1, 3, 4
(C) $1^2, 3^2, 4^2$
(D) $1,\frac{1}{3^2},\frac{1}{4^2}$

3. Two Eigen values of $A=\begin{bmatrix}3 & -1 & 1\\ -1 & 5 & -1\\ 1 & -1 & 3\end{bmatrix}$ are 3 and 6, then the third Eigen value is

(A) 1
(B) 2
(C) 3
(D) 4

4. The index of the canonical form $-y_1^2+y_2^2+4y_3^2$ is

(A) 3
(B) 2
(C) 1
(D) 0

5. If $f(x,y) = 0$ and y is an implicit function of x, then $\frac{dy}{dx}$ is

(A) $\frac{-\partial f/\partial x}{\partial f/\partial y}$
(B) $\frac{\partial f/\partial y}{\partial f/\partial x}$
(C) $\frac{-\partial f/\partial y}{\partial f/\partial x}$
(D) $\frac{\partial f/\partial x}{\partial f/\partial y}$

6. If $u = x^2-y^2$, $v = 2xy$, then the value of $\frac{\partial(u,v)}{\partial(x,y)}$ is

(A) $4(x^2-y^2)$
(B) x^2+y^2
(C) $4(x^2+y^2)$
(D) x^2-y^2

7. The point at which there is no extreme value is

(A) Maximum point
(B) Minimum point
(C) Saddle point
(D) Stationary point

8. If u is a homogeneous function of degree 'n' then by Euler's theorem, we have

(A) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu$
(B) $x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=nu$
(C) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=(n-1)u$
(D) $x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=(n-1)u$

9. The complimentary function of $(D^2-2D+1)y = 0$ is

(A) $C_1e^{x}+C_2e^{-x}$
(B) $(C_1+C_2x)e^{x}$
(C) $C_1e^{2x}+C_2e^{-2x}$
(D) $(C_1+C_2x)e^{-x}$

10. The roots of the auxiliary equation of $m^2-4=0$ are

(A) ± 2
(B) $\pm 2i$
(C) $\pm\sqrt{2}$
(D) $1\pm 2i$

11. If $1\pm 2i$ are the roots of a differential equation $f(D)y = 0$, then the complementary function is

(A) $Ae^{x}+Be^{-2x}$
(B) $e^{-2x}(A\cos x-B\sin x)$
(C) $e^{x}(A\cos 2x+B\sin 2x)$
(D) $e^{-x}(A\cos 2x+B\sin 2x)$

12. The particular integral of $(D^2+16)y=e^{-4x}$ is

(A) $\frac{x}{32}e^{-4x}$
(B) $\frac{1}{32}e^{-4x}$
(C) $\frac{1}{16}e^{-4x}$
(D) $\frac{x}{16}e^{-4x}$

13. The curvature of a circle of radius 'r' is

(A) r
(B) $\frac{1}{r}$
(C) $\frac{1}{r^2}$
(D) r^2

14. __________ is defined as the locus of centre of curvature.

(A) Involute
(B) Evolute
(C) Radius of curvature
(D) Envelope

15. The radius of curvature of the curve $y = 4\sin x$ at $x=\frac{\pi}{2}$ is

(A) 1/4
(B) −1/4
(C) 1/2
(D) −1 /2

16. The curvature of the straight line is

(A) 1
(B) 2
(C) 0
(D) −1

17. The radius of the sphere $x^2+y^2+z^2-2y-4z = 11$ is

(A) − 4
(B) 4
(C) 3
(D) −3

18. Two spheres cut each other orthogonally, if the tangent planes at a point of intersection

(A) $\frac{\pi}{2}$
(B) $\frac{\pi}{6}$
(C) $\frac{\pi}{3}$
(D) 2π

19. The centre of the sphere $x^2+y^2+z^2-2x+4y-4z = 0$ is

(A) $(-1, 2, -2)$
(B) $(-2, 4, -4)$
(C) $(2, -4, 4)$
(D) $(1, -2, 2)$

20. The section of a right circular cone by any plane perpendicular to its axis.

(A) Cone
(B) Circle
(C) Sphere
(D) Cylinder

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Two of the Eigen values of $A=\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ are 3 and 6. Find the Eigen values of A^{-1} and A^2.

22. Find the Taylor series expansion for x^y at (1, 1) upto second degree terms.

23. Solve: $\frac{d^2y}{dx^2}+4\frac{dy}{dx}-4y=e^{-2x}$.

24. Find radius of curvature at $\left(\frac{1}{4},\frac{1}{4}\right)$ on the curve $\sqrt{x}+\sqrt{y}=1$.

25. Find the envelope of the straight lines $y = mx + a/m$ where m is a parameter and 'a' is constant.

26. Find $\frac{du}{dt}$ of $u=\cosh\frac{x}{y}$ and $x=t^2$, $y=e^t$.

27. Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$.

29. a. Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

(OR)

b. Expand $e^x \cos y$ in powers of x and y in the neighbourhood of $\left(-1, \frac{\pi}{4}\right)$ upto 3rd degree terms.

30. a. Solve $y''+y=\tan x$, by the method of variation of parameter.

(OR)

b. Solve: $Dx+y=e^t,\ x-Dy=t$.

31. a. Show that evolute of the cycloid $x=a(\theta-\sin\theta),\ y=a(1-\cos\theta)$ is another cycloid.

(OR)

b. Find the equation of the circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$.

32. a. Show that the two spheres $x^2+y^2+z^2=25$ and $x^2+y^2+z^2-18x-24y-40z+225=0$ touch each other. Find their point of contact and also the equation of the common tangent plane.

(OR)

b. Find the equation of the right circular cone, whose vertex is at (2, –3, 5) and axis makes equal angles with the coordinate axes and semi vertical angle is 30°.

*****

Reg. No. | | | | | | | | | | | | | | |

B.Tech. DEGREE EXAMINATION, MAY 2015
First Semester

MA1001 – CALCULUS AND SOLID GEOMETRY
(For the candidates admitted from the academic year 2013 – 2014 onwards)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The Eigen values of $2A^{-1}$, if $A=\begin{bmatrix}2&0&0\\0&3&0\\0&0&4\end{bmatrix}$ are

(A) (0, 0 , 0) (B) (2, 3, 4)
(C) $\left(1, \frac{2}{3}, \frac{1}{2}\right)$ (D) $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)$

2. Two Eigen values of $A=\begin{bmatrix}3&-1&1\\-1&5&-1\\1&-1&3\end{bmatrix}$ are 3 and 6, then the third Eigen value is

(A) 1 (B) 2
(C) 3 (D) 4

3. The Eigen values of $\begin{pmatrix}1&2\\5&4\end{pmatrix}$ is

(A) (–1, 6) (B) (1, 6)
(C) (1,–6) (D) (–1,–6)

4. For a singular matrix of order 3, the product of the Eigen values is
(A) 0 (B) 1
(C) 3 (D) 2

5. If $J_1=\frac{\partial(x,y)}{\partial(r,\theta)},\ J_2=\frac{\partial(r,\theta)}{\partial(x,y)}$, then $J_1 J_2$ is

(A) 0 (B) 1
(C) –1 (D) 2

6. If $V=xy$, then $x\frac{\partial V}{\partial x}+y\frac{\partial V}{\partial y}$ is

(A) V (B) $2V$
(C) $3V$ (D) 0

7. The point at which there is no extremum value is
(A) Maximum point (B) Minimum point
(C) Saddle point (D) Stationary point

8. If $u = x+y$, where $x = t$, $y = e^t$, then $\frac{du}{dt}$ is
(A) $1+e^t$ (B) e^t
(C) 2 (D) 0

9. The particular integral of $(D^2+16)y = e^{-4x}$ is
(A) $\frac{x}{32}e^{-4x}$ (B) $\frac{1}{32}e^{-4x}$
(C) $\frac{1}{16}e^{-4x}$ (D) $\frac{x}{16}e^{-4x}$

10. The solution of $(D^3-D^2+D-1)y = 0$ is
(A) $y = Ae^x + B\cos x + C\sin x$ (B) $y = Ae^x + B\cos x - C\sin x$
(C) $y = Ae^{-x} + B\cos x + C\sin x$ (D) $y = Ae^x + B\cosh x + C\sinh x$

11. The particular integral of $(D^2+2)y = x^2$ is
(A) $\frac{x^2}{2}$ (B) $\frac{x^2-1}{2}$
(C) $\frac{x^2+1}{2}$ (D) $-\frac{x^2}{2}$

12. The complementary function of $(D^2-8D+15) = 0$ is
(A) $C_1 e^{-5x} + C_2 e^{-3x}$ (B) $C_1 e^{4x} + C_2 e^{-4x}$
(C) $C_1 e^{5x} + C_2 e^{3x}$ (D) $C_1 e^{2x} + C_2 e^{6x}$

13. The curvature of a circle of radius 'r' is
(A) r (B) $\frac{1}{r}$
(C) $\frac{1}{r^2}$ (D) r^2

14. The parametric equation of a ~~hyperbola~~ parabola is
(A) $x = at^2, y = 2at$ (B) $x = t,\ y = \frac{1}{t}$
(C) $x = at,\ y = 2at$ (D) $x = 2at, y = at^2$

15. The curve which touches every member of the family of curves is
(A) Evolute (B) Involute
(C) Envelope (D) Circle

16. The evolute of the parabola $y^2 = 4ax$ is
(A) $27ax^3 = 4(y-2a)^3$ (B) $x^2 = 4y$
(C) $27ay^2 = 4(x-2a)^3$ (D) $27ax^2 = 4(y-2a)^3$

17. The radius of the sphere, whose centre is (4, 4, −2) which passes through the origin is
(A) 2 (B) 4
(C) 6 (D) 8

18. The ~~length~~ equation of the tangent plane at the point (1,−1,2) to the sphere $x^2+y^2+z^2-2x+4y+6z-12 = 0$ is
(A) $x+y+z = 9$ (B) $y+5z = 9$
(C) $x+y-z = 9$ (D) $x+5z = 9$

19. The equation of the cone with vertex at the origin and passing through the curves $x^2+y^2 = 9$ and $z = 3$ is
(A) $x^2+y^2 = z^2$ (B) $x^2-y^2 = z^2$
(C) $x^2+y^2+z^2 = 0$ (D) $x^2+y^2+z^2 = 1$

20. The section of a right circular cone by any plane perpendicular to its axis is a
(A) Cone (B) Circle
(C) Sphere (D) Cylinder

PART – B ($5 \times 4 = 20$ Marks)
Answer ANY FIVE Questions

21. Find the Eigen values and Eigen vectors of $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

22. Find the envelope of $x\cos\alpha + y\sin\alpha = a\sec\alpha$, α is the parameter.

23. Find $\frac{du}{dt}$, if $u = x^2+y^2+z^2$, where $x = e^t, y = e^t\sin t, z = e^t\cos t$.

24. If $x = r\cos\theta, y = r\sin\theta$, verify that $\frac{\partial(x,y)}{\partial(r,\theta)} \times \frac{\partial(r,\theta)}{\partial(x,y)} = 1$.

25. Find the radius of curvature at $x = c$ on the curve $xy = c^2$.

26. Solve $(D^2-4D+4)y = \cos 2x$.

27. Find the equation of the sphere having its centre at (2, 3, 4) and passing through the point (1, 4, 9).

PART – C ($5 \times 12 = 60$ Marks)
Answer ALL Questions

28. a. Verify Cayley Hamilton theorem and hence find A^{-1}, where $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$.

(OR)

b. Reduce the quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ to a canonical form.

ii. The product of two Eigen values of the matrix $A=\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 16. Find the third Eigen value.

29. a. A rectangular box open at the top is to have volume of 32 cubic feet. Find the dimensions in order that the total surface area is minimum.

(OR)

b. i. Findthe expansion for $\cos x \cos y$ in power of x and y up to terms of 3^{rd} degree.

ii. If $u=\frac{yz}{x}, v=\frac{zx}{y}, w=\frac{xy}{z}$, find the Jacobian of u, v, w with respect to x, y, z.

30. a. Solve the simultaneous linear differential equations $\frac{dx}{dt}+7x-y=0; \frac{dy}{dt}+2x+5y=0$.

(OR)

b. Solve: $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=e^{-2x}+e^{3x}\sin x$

31. a. Find the evolute of the curve $x^{2/3}+y^{2/3}=a^{2/3}$.

(OR)

b. Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ of the curve $x^3+y^3=3axy$.

32.a.i Find the equation of the right circular cylinder whose axis is $x-2=z, y=0$ and passes through the point (3, 0, 0).

ii. Prove that the equation $7x^2+2y^2+2z^2+10zx+10xy+26x-2y+2z-17=0$ represents a cone.

(OR)

b. Show that the sphere $x^2+y^2+z^2+6y+2z+8=0$ and $x^2+y^2+z^2+6x+8y+4z+20=0$ intersect at right angles. Find their plane of intersection.

*****

Reg. No. | | | | | | | | | | | | | |

B.Tech. DEGREE EXAMINATION, DECEMBER 2014
First Semester

MA1001 – CALCULUS AND SOLID GEOMETRY
(For the candidates admitted from the academic year 2013 – 2014 onwards)

Note: (i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45^{th} minute
(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. If (2, 2, 3) are Eigen values of $A=\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$, then Eigen values of A^{-1} are

(A) $\left(\frac{1}{2},\frac{1}{2},\frac{1}{3}\right)$ (B) $\left(\frac{1}{2},2,3\right)$
(C) (3, –3, 7) (D) (2, 2, 3)

2. The Eigen values of $A=\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}$ are

(A) (5, –5) (B) (–5, –5)
(C) (5, 5) (D) (–5, 0)

3. The sum of the Eigen values of the identity matrix of order 2 is

(A) 0 (B) 1
(C) 2 (D) 3

4. If 2, 3 are Eigen values of the matrix $A=\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$, then the third Eigen value is

(A) 2 (B) 0
(C) 1 (D) 3

5. If $x=r\cos\theta, y=r\sin\theta$, then $\frac{\partial(x,y)}{\partial(r,\theta)}$ is

(A) 0 (B) 1
(C) r (D) $\frac{1}{r}$

6. If $u=x+y+z$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}$ is

(A) u (B) $3u$
(C) $2u$ (D) 0

Page - 24

7. The point (a,b) is called a stationary point, if at (a,b)

(A) $f_x = 0,\ f_y = 0$ (B) $f_{xx} = 0$

(C) $f_{yy} = 0$ (D) $f_{xy} = 0$

8. If $u = x^2,\ v = y^2$, then $\dfrac{\partial(u,v)}{\partial(x,y)}$ is

(A) $2xy$ (B) xy

(C) $6xy$ (D) $4xy$

9. The particular integral of $(D^2+2D+1)y = 5$ is

(A) 0 (B) 5

(C) 2 (D) 1

10. The solution of $(D^2-8D+15)y = 0$ is

(A) $C_1 e^{-5x}+C_2 e^{-3x}$ (B) $C_1 e^{4x}+C_2 e^{4x}$

(C) $C_1 e^{5x}+C_2 e^{3x}$ (D) $C_1 e^{2x}+C_2 e^{6x}$

11. The complementary function of 2nd order differential equation having roots $\alpha \pm i\beta$ is

(A) $e^{+\alpha x}[c_1 \cos\beta x+c_2 \sin\beta x]$ (B) $c_1 \cos\alpha x+c_2 \sin\beta x$

(C) $e^{\alpha x}[c_1 \cos\beta x+c_2 \sin\beta x]$ (D) $e^{\alpha x}[c_1 \cos\alpha x+c_2 \sin\beta x]$

12. The particular integral of $(D^2+9)y = e^{-2x}$ is

(A) $\dfrac{e^{-2x}}{15}$ (B) $\dfrac{e^{2x}}{13}$

(C) $\dfrac{e^{-2x}}{13}$ (D) $\dfrac{e^{-2x}}{14}$

13. The reciprocal of the curvature of the curve at any point 'P' is called

(A) Centre of curvature (B) Circle of curvature

(C) Radius of curvature (D) Chord of curvature

14. The envelope of the normal to the curve is the ____________ of a curve.

(A) Evolute (B) Involute

(C) Envelope (D) Circle

15. The parametric equation of a hyperbola is

(A) $x = ct,\ y = \dfrac{c}{t}$ (B) $x = a\cos\theta,\ y = b\sin\theta$

(C) $x = a\sec\theta,\ y = b\tan\theta$ (D) $x = a\sec\theta,\ y = a\tan 6$

16. The locus of the centre of curvature of the curve is

(A) Envelope (B) Evolute

(C) Involute (D) Circle

17. The radius of the sphere $x^2+y^2+z^2-2y-4z = 11$ is

(A) -4 (B) 4

(C) 3 (D) -3

18. Two spheres are said to cut each other orthogonally, if the tangent planes at a point of intersection are at

(A) Obtuse angle (B) Right angle

(C) Straight angle (D) Acute angle

19. The region of the straight circular cylinder whose axis is Z-axis and guiding curve is in a circle of radius 'a' in the XY-plane is

(A) $x^2+y^2 = a^2$ (B) $x^2+y^2 = 1$

(C) $x^2-y^2 = 1$ (D) $x+y = 9$

20. The section of a right circular cylinder by any plane perpendicular to the axis is a

(A) Cone (B) Rectangle

(C) Sphere (D) Circle

PART – B (5 × 4 = 20 Marks)

Answer ANY FIVE Questions

21. Find the Eigen values and Eigen vectors of $A = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$.

22. If $u = \tan^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y} = \dfrac{1}{4}\sin 2u$.

23. Solve $\dfrac{d^2y}{dx^2}+4\dfrac{dy}{dx}+4y = e^{-2x}$.

24. Find the radius of curvature at the point $x = c$ on the curve $xy = c^2$.

25. Find the envelope of the family given by $x = my+\dfrac{1}{m}$, where 'm' is a parameter.

26. Show that the spheres $x^2+y^2+z^2+6y+2z+8 = 0$ and $x^2+y^2+z^2+6x+8y+4z+20 = 0$ intersect at right angles.

27. Find the equation of the sphere on joining of $(1,-1,-1)$ and $(-3,4,5)$ as diameter.

PART – C (5 × 12 = 60 Marks)

Answer ALL Questions

28. a. Reduce the quadratic form $x^2+2y^2+z^2-2xy+2yz$ to a canonical form and hence find rank, index and signature.

(OR)

b. i. Given the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix}$, find the value of $A^8-5A^7+7A^6-3A^5+A^4-5A^3-8A^2+2A-I$.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Verify Cayley Hamiltonian theorem and hence find the inverse for the matrix

$$A=\begin{pmatrix}1 & 0 & -2\\ 2 & 2 & 4\\ 0 & 0 & 2\end{pmatrix}$$

(OR)

b. Reduce the quadratic form $3x_1^2+5x_2^2+3x_3^2-2x_2x_3+2x_3x_1-2x_1x_2$ to canonical form by orthogonal reduction

29. a. Examine the maximum and minimum values of $f(x,y) = x^3+y^3-3xy$.

(OR)

b. A rectangular box open at the top is to have a volume of 32CC. Find the dimensions of the box that requires the least material for its construction.

30. a. Solve by the method of variation of parameters $\dfrac{d^2y}{dx^2}+y=cosecx$.

(OR)

b. Solve the Euler's equation $x^2\dfrac{d^2y}{dx^2}-7x\dfrac{dy}{dx}+12y=x^2$.

31. a. Find the equation of the circle of curvature of the parabola $y^2 = 12x$ at (3,6).

(OR)

b. Find the equation of the evolute of the curve $x=a(\cos t+t\sin t)$, $y=a(\sin t-t\cos t)$

32. a. Show that the plane $2x-2y+z+12 = 0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z = 3$ and also find the point of contact.

(OR)

b. Find the equation of the right circular cylinder whose axis is $\dfrac{x-1}{2}=\dfrac{y}{3}=\dfrac{z-3}{1}$ and radius 2.

* * * * *

Reg. No.

B.Tech. DEGREE EXAMINATION, MAY 2014
First Semester

MA1001 – CALCULUS AND SOLID GEOMETRY
(For the candidates admitted from the academic year 2013 – 2014)

Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
Part - B and Part - C should be answered in answer booklet.

hree Hours　　　　Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

The sum of the Eigen values of $A=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$ is

(A) 2　　(B) 4
(C) −3　　(D) 0

A homogeneous polynomial of the ________ degree in any number of variables is called a quadratic form.

(A) First　　(B) Second
(C) Third　　(D) Fourth

The index of the canonical form $-y_1^2+y_2^2+4y_3^2$ is

(A) 3　　(B) 2
(C) 1　　(D) 0

If $A=\begin{pmatrix}3 & 5 & 3\\ 0 & 4 & 6\\ 0 & 0 & 1\end{pmatrix}$, the Eigen values of A^{-1} are

(A) 1, 1/3, 1/4　　(B) 1, 3, 4
(C) $1^2, 3^2, 4^2$　　(D) $1, 1/3^2, 1/4^2$

If $u=\dfrac{x^3+y^3}{x^2+y^2}$ then $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}$ is

(A) 2u　　(B) 3u
(C) 4u　　(D) 6u

6. If $f(x,y) = 0$ and y is an implicit function of x then $\dfrac{dy}{dx}$ is

(A) $\dfrac{-\partial f/\partial x}{\partial f/\partial y}$　　(B) $\dfrac{\partial f/\partial y}{\partial f/\partial x}$
(C) $\dfrac{-\partial f/\partial y}{\partial f/\partial x}$　　(D) $\dfrac{\partial f/\partial x}{\partial f/\partial y}$

7. $u = x^2-y^2$, $v = 2xy$, then the value of $\frac{\partial(u,v)}{\partial(x,y)}$ is

(A) $4(x^2-y^2)$ (B) x^2+y^2
(C) $4(x^2+y^2)$ (D) x^2-y^2

8. $f(x,y) = \frac{(x^2+y^2)}{(x+y)}$ is a homogeneous function of degree

(A) 2 (B) 1
(C) 0 (D) 3

9. The roots of the auxiliary equation of $(m^2-4) = 0$ are

(A) ± 2 (B) $\pm 2i$
(C) $\pm \sqrt{2}$ (D) $1 \pm 2i$

10. The particular integral of $(D+1)y = e^x$ is

(A) $-\frac{1}{2}e^x$ (B) $\frac{e^x}{2}$
(C) e^{-x} (D) 0

11. If $1\pm2i$ are the roots of a differential equation $f(D)y = 0$ then the complementary function (C.F) is

(A) Ae^t+Be^{-2t} (B) $e^{-2x}(A\cos x-B\sin x)$
(C) $e^x(A\cos2x+B\sin2x)$ (D) Ae^t+Be^{2t}

12. $\frac{1}{(D^2+a^2)}\cos ax$ is

(A) $\frac{x}{2a}\sin ax$ (B) $\frac{-x}{2a}\sin ax$
(C) $\frac{x}{2a}\cos ax$ (D) $\frac{-x}{2a}\cos ax$

13. If the radius of curvature and curvature of a curve at any point are ρ and k respectively, then

(A) $\rho = -1/k$ (B) $\rho = k$
(C) $\rho = -k$ (D) $\rho = 1/k$

14. ____________ is defined as the locus of centre of curvature.

(A) Involute (B) Evolute
(C) Radius of curvature (D) Envelope

15. The envelope of the family of curves $A\alpha^2+B\alpha+C = 0$ is ____________ (α is the parameter).

(A) $B^2+4AC = 0$ (B) $B^2-AC = 0$
(C) $B^2+AC = 0$ (D) $B^2-4AC = 0$

16. The radius of curvature of a curve in polar coordinates is

(A) $\frac{\left(r^2+r'^2\right)^{3/2}}{\left(r^2-rr''+2r'^2\right)}$ (B) $\frac{\left(r^2-r'^2\right)^{3/2}}{\left(r^2-rr''+2r'^2\right)}$

(C) $\frac{\left(r^2-r'^2\right)^{3/2}}{\left(r^2+rr''+2r'^2\right)}$ (D) $\frac{\left(r^2+r'^2\right)^{2/3}}{\left(r^2-rr''+2r'^2\right)}$

17. The distance of the point $(-1,2,3)$ from the plane $2x-y+2z = 6$ is ____________.

(A) 2/3 (B) $2/\sqrt{3}$
(C) 4/3 (D) $4/\sqrt{3}$

18. Equation of the tangent plane to the sphere $x^2+y^2+z^2-2x-4y+2z-3 = 0$ at $(-1,4,-2)$ is

(A) $2x-2y+z+12 = 0$ (B) $2x+2y+2z+12 = 0$
(C) $2x+2y+2z-12 = 0$ (D) $x+y+z+12 = 0$

19. The centre of the sphere $x^2+y^2+z^2-2x+4y-4z = 0$ is

(A) $(-1,2,-2)$ (B) $(-2,4,-4)$
(C) $(2,-4,4)$ (D) $(1,-2,2)$

20. A surface generated by a line which intersects a fixed circle and is perpendicular to the plane of the circle is ____________.

(A) Sphere (B) Right circular cylinder
(C) Cone (D) Right circular cone

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. If λ is an Eigen value of A, prove that

(i) $\frac{1}{\lambda}$ is an Eigen value of A^{-1}.

(ii) λ^2 is an Eigen value of A^2.

22. Verify Euler's theorem for the function $u = x^2+y^2+2xy$.

23. Solve $(D^2-4D+4)y = \cos 2x$.

24. Find the radius of curvature at $x = c$ on the curve $xy = c^2$.

25. Find the equation of the sphere which has its centre at the point $(-1, 2, 3)$ and touches the plane $2x-y+2z = 6$.

26. Obtain the Maclaurin's series of $e^x\cos y$ upto second degree terms.

27. Find the envelope of the family of straight lines $y = mx + a/m$, where 'm' is theparameter.

b. Reduce the quadratic form $6x_1^2+3x_2^2+3x_3^2-4x_1x_2-2x_2x_3+4x_3x_1$ to a canonical form and hence find rank, index and signature.

29. a. Expand $e^x \sin y$ in powers of x and y near the point $\left(-1,\frac{\pi}{4}\right)$ as far as the terms of the third degree.

(OR)

b. Find the volume of the largest parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

30. a. Solve $(D^2+4)y = 4\tan 2x$ using the method of variation of parameters.

(OR)

b. Solve $(x+2)^2\frac{d^2y}{dx^2}-(x+2)\frac{dy}{dx}+y=3x+4$

31. a. Find the equation of the circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $\left(\frac{a}{4},\frac{a}{4}\right)$.

(OR)

b. Find the evolute of the curve $x = a\cos^3\theta,\ y = a\sin^3\theta$.

32. a. Find the equation of the sphere passing through (–1, 6, 6), (0, 7, 10), (–4, 9, 6) and having its centre on the plane $2x+2y-z=4$.

(OR)

b. i. Find the equation of the right circular cone whose vertex is at the origin, whose axis is the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and whose semi-vertical angle is 30°.

ii. Find the equation of the right circular cylinder of radius 2 whose axis passes through (1,2,3) and has direction cosines proportional to (2,–3,6).

* * * * *

Reg. No.

B.Tech. DEGREE EXAMINATION, NOVEMBER 2013
First Semester

MA1001 – CALCULUS AND SOLID GEOMETRY
(For the candidates admitted during the academic year 2013 - 2014)

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The Eigen values of the matrix $\begin{bmatrix}1 & 2\\ 5 & 4\end{bmatrix}$ are.
   (A) 1,6 (B) –1,6
   (C) 1, –6 (D) –1, –6

2. Two of the Eigen values of a 3×3 matrix, whose determinant is equal to 4 are –1 and 2, then the third Eigen value is
   (A) 2 (B) –2
   (C) 1 (D) –1

3. The inverse of the Eigen values of the matrix $\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$ is
   (A) 1, 1/2 (B) 1, 2
   (C) 1, 1/3 (D) 1, 3

4. The nature of the quadratic form $2x^2+3y^2+2z^2+2xy$ is
   (A) Indefinite (B) +ve definite
   (C) Positive semi definite (D) Negative definite

5. If $u=x^2,\ v=y^2$, then $\frac{\partial(u,v)}{\partial(x,y)}$ is
   (A) 2xy (B) 4xy
   (C) 6xy (D) xy

6. If u is a homogenous function of degree 'n' then by Euler's theorem, we have
   (A) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu$ (B) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=(n-1)u$
   (C) $x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=nu$ (D) $x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=(n-1)u$

7. If $u=\frac{x^3+y^3}{x^2+y^2}$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is
   (A) 2u (B) 3u
   (C) 4u (D) 6u

No Answer
Ans: u

8. Differentiation of implicit function f(x,y) = 0 is

(A) $\frac{dy}{dx}=\frac{fx}{fy}$ (B) $\frac{dy}{dx}=-\frac{fx}{fy}$

(C) $\frac{dy}{dx}=-\frac{fy}{fx}$ (D) $\frac{dy}{dx}=\frac{fy}{fx}$

9. The complementary function of $(D^2-2D+1)y=0$ is

(A) $C_1e^{x}+C_1e^{-x}$ (B) $(C_1+C_2x)e^{x}$

(C) $C_1e^{-2x}+C_2e^{2x}$ (D) $(C_1+C_2x)e^{-x}$

10. The solution of $x^2\frac{d^2y}{dx^2}-7x\frac{dy}{dx}+12y=0$ is

(A) $C_1e^{-2z}+C_2e^{6z}$ (B) $C_1e^{2z}+C_2e^{-6z}$ No Answer

(C) $C_1e^{2z}+C_2e^{6z}$ (D) $C_1e^{-2z}+C_2e^{-6z}$

11. The particular integral of $(D^2+9)y=e^{-2x}$ is

(A) $\frac{e^{-2x}}{15}$ (B) $\frac{e^{2x}}{13}$

(C) $\frac{e^{-2x}}{13}$ (D) $\frac{e^{-2x}}{14}$

12. The complementary function of the second order differential equation having roots $\alpha \pm i\beta$ is

(A) $e^{-\alpha x}(C_1\cos\beta x+C_2\sin\beta x)$ (B) $e^{0x}(C_1\cos\beta x+C_2\sin\beta x)$

(C) $e^{\beta x}(C_1\cos\alpha x+C_2\sin\alpha x)$ (D) $e^{\alpha x}(C_1\cos\beta x+C_2\sin\beta x)$

13. The curvature of the straight line is

(A) 1 (B) 2

(C) −1 (D) 0

14. The radius of curvature of the curve y = 4 sinx at $x=\frac{\pi}{2}$ is

(A) 1/4 (B) − 1/4

(C) 1/2 (D) − 1/2

15. The envelope of $ty-x=at^2$, t is the parameter is

(A) $x^2=4ay$ (B) $y^2=4ax$

(C) $x^2+y^2=1$ (D) $x^2-y^2=1$

16. The locus of centre of curvature is called

(A) Curvature (B) Radius of curvature

(C) Evolute (D) Envelope

17. The radius of the sphere $x^2+y^2+z^2-2y-4z=11$ is

(A) −4 (B) 4

(C) 3 (D) −3

18. The equation of the cone with vertex at the origin and passing through the curves $x^2+y^2=9$, $z=3$ is

(A) $x^2+y^2=z^2$ (B) $x^2-y^2=z^2$

(C) $x^2+y^2=-z^2$ (D) $x^2-y^2=-z^2$

19. The equation of the sphere having points (−4,5,1) and (4,1,7) as ends of its diameter is

(A) $x^2+y^2+z^2-6y-8z+4=0$ (B) $x^2+y^2+z^2-6y+8z+4=0$

(C) $x^2+y^2+z^2-6y-8z-4=0$ (D) $x^2+y^2+z^2+6y-8z+4=0$

cylinder

20. A surface whose generator touches a given surface and is directed in a given direction is called

(A) Cylinder (B) Enveloping cylinder

(C) Cone (D) Right circular cone

PART – B (5 × 4 = 20 Marks)

Answer ANY FIVE Questions

21. Find the Eigen values and Eigen vectors of $A=\begin{bmatrix}2&-2&3\\1&1&1\\1&3&-1\end{bmatrix}$.

22. If $u=\sin^{-1}\left(\frac{x^2+y^2}{x-y}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$.

23. Solve $(D^2+3D+2)y=\sin x$.

24. Find the envelope of $x\cos\alpha+y\sin\alpha=a\sec\alpha$. (α is a parameter)

25. Find the equation of the right circular cone with origin as vertex and the axes of coordinates as its three generators.

26. Solve: $(D^2+2D+1)y=e^{-x}+3$

27. If $u=f(y-z,\ z-x,\ x-y)$, prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$

PART – C (5 × 12 = 60 Marks)

28. a. Verify Cayley Hamilton theorem and hence find A^{-1} for $A=\begin{bmatrix}1&2&3\\2&-1&4\\3&1&-1\end{bmatrix}$.

(OR)