Roll No. ..... Total Pages : 5

300205

May 2019

B.Tech. (CE/CSE/IT) 2nd Semester

MATHS-II

(Probability and Statistics)

(BSC106E)

Time : 3 Hours] [Max. Marks : 75

Instructions :

(i) It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.

(ii) Answer any four questions from Part-B in detail.

(iii) Different sub-parts of a question are to be attempted adjacent to each other.

PART–A

1. (a) A die is tossed thrice. Getting '5' or '6' on the die in a toss is taken as success. Find the mean of number of success. (1.5)

(b) Define Chebyshev's Inequality. (1.5)

(c) Write formula for mode and median of Normal Distribution. (1.5)

(d) A speaks truth 4 out of 5 times. A die is tossed. He reports that there is a six. What is the chance that actually there was six? (1.5)

(e) Find variance of Gamma Disribution. (1.5)

(f) Prove that (X,Y) possesses a bivariate normal distribution if every linear combination of X, Y is a normal variate. (1.5)

(g) Explain briefly Karl-Pearson's coefficient of correlation. (1.5)

(h) The mean weekly sales of soap bars in departmental stores was 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful ? (1.5)

(i) Define covariance of two random variables. (1.5)

(j) Define Null hypothesis and Confidence Limits. (1.5)

PART–B

2. (a) A random variable X has the following probability function :

Values of X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Find k, (ii) Evaluate $P(X < 6)$, $P(X \geq 6)$, and $P(0 < X < 5)$ (iii) If $P(X \leq a) > \frac{1}{2}$, find the minimum value of a, and (iv) Determine the distribution function of X. (8)

(b) Ten percent of the tools in a certain manufacturing process turn out to be defective . Find the probability that in a sample of 10 tools chosen at a random, exactly two will be defective by using the binomial distribution. (7)

3. (a) Let X be a continuous random variable with p.d.f.:

$$f(x) = \begin{cases} ax, & 0 \leq x \leq 1 \\ a, & 1 \leq x \leq 2 \\ -ax + 3a, & 2 \leq x \leq 3 \\ 0, & \text{elsewhere} \end{cases}$$

(i) Determine the constant a (ii) Compute $P(X \leq 1.5)$. (8)

(b) X be a normal variate with mean 30 and S.D. 5. Find the probabilities that

(i) $26 \leq X \leq 40$, (ii) $X \geq 45$. (iii) $|X - 30| > 5$. (7)

4. (a) Show that if X_1 and X_2 are standard normal variates with correlation coefficient ρ between them, then the correlation coefficient between X_1^2 and X_2^2 is given by ρ^2. (8)

(b) A and B are two weak students of statistics and their chances of solving a problem in statistics correctly are $\frac{1}{6}$ and $\frac{1}{8}$ respectively. If the probability of their making a common error is $\frac{1}{525}$ and they obtain the same answer, find the probability that their answer is correct. (8)

5. (a) For a distribution, the mean is 10, variance is 16, γ_1 is +1 and β_2 is 4. Obtain the first four moments about the origin, i.e., zero. Comment upon the nature of distribution. (8)

(b) Let (X, Y) have the joint p.d.f. given by :

$$f(x, y) = \begin{cases} 1, & \text{if } |y| < x, 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Show that the regression of Y on X is linear but regression of X on Y is not linear. (7)

6. (a) Find the least squares approximation of second degree for the discrete data :

X	–2	–1	0	1	2
Y	15	1	1	3	10

(8)

(b) The heights of six randomly chosen sailors are (in inches) : 63, 65, 68, 69, 71 and 72. Those of 10 randomly chosen soldiers are 61, 62, 65, 66, 69, 69, 70, 71, 72 and 73. Discuss the light that these data throw on the suggestion that sailors are on the average taller than soldiers. (7)

7. (a) A set of five similar coins is tossed 320 times and the result is

No. of heads :	0	1	2	3	4	5
Frequency :	6	27	72	112	71	32

Test the hypothesis (Chi-square test) that the data follow a binomial distribution. (8)

(b) A random sample of size 16 values from a normal population showed a mean of 53 and a sum of squares of deviation from the mean equals to 150. Can this sample be regarded as taken from the population having 56 as mean ? Obtain 95% and 99% confidence limits of the mean of the population. (7)

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300205

October, 2020

B.Tech. (CSE/CE/IT) II SEMESTER

Maths-II (Probability & Statistics) (BSC 106E)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*
4. *Use of Calculator is Allowed.*
5. *Normal distribution table is required*

PART-A

1. (a) Calculate the quartile coefficient of skewness from the given data :

Weight (lbs)	70-80	80-90	90-100	100-110
No. of persons		12	18	3542

Weight (lbs)	110-120	120-130	130-140	140-150
No. of persons		50	45	20 8

(1.5)

(b) Tbe first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Calculate the moments about the mean. Also evaluate $\beta_1\beta_2$. (1.5)

(c) If the events A and B are such that $P(A) \neq 0$, $P(B) \neq 0$ and A is independent of B, then prove that B is independent of A. (1.5)

(d) For a certain normal distribution, the first moment about 10 is 40 and the fourth moment about 50 is 48. What is arithmetic mean and standard deviation of the distribution ? (1.5)

(e) A speaks truth in 60% cases and B in 70% cases. In what percentage of cases are they likely to contradict each other? (1.5)

(f) If X represents the outcome when a pair of dice is thrown find the expectation E(X) and variance V(X). (1.5)

(g) During testing a sample of 300 chips 10 have been found to be defective. Can the manufacturers claim that 2% of the chips are defective may be accepted ? (1.5)

(h) Calculate the rank correlation coefficient from the following data showing ranks of 10 students in two subjects :

Maths	3	8	9	2	7
Physics	5	9	10	1	8
Maths	10	4	6	1	5
Physics	7	3	4	2	6

(1.5)

(i) If F is the pull required to lift a load W by means of pulley, fit a linear law $F = a + bW$ against the following data :

W	50	70	100	120
F	12	15	21	25

(1.5)

(j) Prove that the coefficient of correlation is independent of the change of scale and change of origin for the variables x and y. (1.5)

PART-B

2. (a) If $X_1, X_2, \ldots\ldots X_K$ are k independent Poisson variates with parameters $\lambda_1, \lambda_2, \ldots\ldots, \lambda_k$ respectively, prove that the conditional distribution :

$$P((X_1 \cap X_2 \cap \ldots\ldots\ldots\ldots \cap X_K \mid X),$$

where $X = X_1 + X_2 + \ldots\ldots + X_K$ is fixed, is multinomial. (7)

(b) State and prove Chebyshev inequality. (8)

II

3. (a) Show that the exponential distribution 'lacks memory', i.e., if X has an exponential distribution, then for every constant $a \geq 0$, one has $P(Y \leq x \mid X \geq a) = P(X \leq x)$ for all x, where $Y = X - a$. (8)

(b) The daily consumption of milk in a city, in excess of 20,000 litres, is approximately distributed as a Gamma variate with parameters $a = 1/10000$ and $\lambda = 2$. The city has a daily stock of 30,000 litres. What is the probability that the stock is insufficient on a particular day? (7)

4. Let X and Y have bivariate normal distribution with parameters :

$\mu_x = 5$, $\mu_y = 10$, $\sigma_x^2 = 1$, $\sigma_y^2 = 25$ and Corr $(X, Y) = \rho$.

(a) If $\rho > 0$, find ρ when $P(4 < Y < 16 \mid X = 5) = 0.954$.

(b) If $\rho = 0$, find $P(X + Y) \leq 16$. (15)

5. (a) Fit a binomial distribution to the following frequency distribution :

x	0	1	2	3	4	5	6
f	13	25	52	58	32	16	4

(8)

(b) Find the correlation coefficient between x and y for the given value. Find also the two regression lines.

x	1	2	3	4	5
y	10	12	16	28	25

x	6	7	8	9	10
y	36	41	49	40	50

(7)

6. (a) Determine the constants a, b and c by the method of least squares such that $y = ax^2 + bx + c$ fits the given data :

x	2	4	6	8	10
y	4.01	11.08	30.12	81.89	222.62

(8)

(b) A normally distributed population has mean 6.8 and standard deviation 1.5. A sample of size 400 has been 6.75. Is the difference between the population mean and the sample mean significant. (7)

7. (a) In a school the heights of six randomly choosen girls are 63, 65, 68, 69, 71 and 72 inches and those of nine randomly choosen boys 61, 62, 65, 66, 69, 70, 71, 72 and 73 inches. Test if the girls are taller than the boys. (8)

7. Let the function $f(z)$ be analytic inside a circle $|z-a| = R$, where R is the radius and a is the centre. Then $f(z)$ has the power series representation denotes factorial

$$f(z) = f(a) + (z-a)f'(a) + \frac{(z-a)^2}{2|} f''(a) + \ldots. + \frac{(z-a)^n}{n|} f^{(n)}(a) + \ldots., \; |z-a| < R,$$

$n|$ denotes factorial n. (15)

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Sr. No...018202

September 2022

B. Tech.- II SEMESTER

Mathematics II (Probability and Statistics) (BSC-106E/BSCH-106E)

Time: 3 Hours **Max. Marks: 75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*
4. *The candidate is required to attempt the question paper in the language as per his/her medium of instruction.*

PART -A

Q1 (a) Define marginal probability for a discrete random variable. (1.5)

एक असतत यादृच्छिक चर के लिए सीमांत संभावना को परिभाषित करें।

(b) If X and Y are two mutually independent events, then find $Cov(X, Y)$. (1.5)

यदि X और Y दो पारस्परिक रूप से स्वतंत्र घटनाएँ हैं, तो $Cov(X, Y)$ ज्ञात कीजिये।

(c) If $Var(X) = 2$, then calculate $Var(3X - 2)$. (1.5)

यदि $Var(X) = 2$, तो $Var(3X - 2)$ की गणना करें।

(d) A continuous random variable X follows the probability law $f(x) = A(x + 1)$, (1.5)
$0 \leq x \leq 1$. Determine A.

एक निरंतर यादृच्छिक चर X संभाव्यता कानून $f(x) = A(x + 1), 0 \leq x \leq 1$ का अनुसरण करता है। तो A निर्धारित करें।

(e) Find the probability of drawing any one spade card from a full pack of cards. (1.5)

पत्तों के पूर्ण पैक से किसी एक हुकुम कार्ड को निकालने की प्रायिकता ज्ञात कीजिये।

(f) If two dice are thrown together, what is the probability that the sum is neither 7 nor 11? (1.5)

यदि दो पासों को एक साथ फेंक दिया जाता है, तो योग न तो 7 और न ही 11 होने की प्रायिकता क्या है?

(g) Find the mode of the discrete distribution $B\left(16, \frac{1}{4}\right)$. (1.5)

असतत वितरण $B\left(16, \frac{1}{4}\right)$ की विधा ज्ञात कीजिये।

(h) What is scatter diagram? (1.5)

स्कैटर आरेख क्या है?

(i) Define Type I error with example. (1.5)

उदाहरण के साथ प्रकार I त्रुटि परिभाषित करें।

P(A) + P(B) − P(A∩B)

(j) For a right-tailed large sample test, what is the critical value at 5% level of significance? (1.5)
एक दाएं पूंछ वाले बड़े नमूना परीक्षण के लिए, महत्व के 5% स्तर पर महत्वपूर्ण मूल्य क्या है?

PART –B

Q2 (a) In a college, 30% students fail in Physics, 25% fail in Mathematics and 10% students fail in both. One student is chosen at random. What is the probability that (a) he fails in Mathematics if he has already failed in Physics (b) he fails in Physics or Mathematics? (7)
एक कॉलेज में, 30% छात्र भौतिकी में अनुत्तीर्ण होते हैं, 25% गणित में अनुत्तीर्ण होते हैं और 10% छात्र दोनों में अनुत्तीर्ण होते हैं. एक छात्र को यादृच्छिक रूप से चुना जाता है। (a) गणित में असफल होने की प्रायिकता क्या है यदि वह भौतिकी में पहले से ही असफल हो चुका है (b) वह भौतिकी या गणित में विफल रहता है?

(b) Let X and Y be jointly distributed with the probability density function (8)

$$f_{X,Y}(x,y) = \begin{cases} 2-x-y, & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Are X and Y independent?
मान लीजिए कि X और Y को प्रायिकता घनत्व फ़ंक्शन के साथ संयुक्त रूप से वितरित किया जाता है

$$f_{X,Y}(x,y) = \begin{cases} 2-x-y, & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

क्या X और Y स्वतंत्र हैं?

Q3 (a) In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0.4096 and 0.2048, respectively. Find the parameter p for the distribution. (7)
5 स्वतंत्र परीक्षणों से युक्त द्विपद वितरण में, 1 और 2 सफलताओं की संभावनाएं क्रमशः 0.4096 और 0.2048 हैं। वितरण के लिए पैरामीटर p ढूंढें।

(b) If X has a Poisson distribution such that $P(X=2) = \frac{2}{3}P(X=1)$, evaluate (8)
(a) $P(X=0)$ and (b) $P(X=3)$.
यदि X में पॉइसन वितरण है जैसे कि $P(X=2) = \frac{2}{3}P(X=1)$, मूल्यांकन करें
(a) $P(X=0)$ और (b) $P(X=3)$।

Q4 (a) Determine the moment generating function of Gamma distribution. (7)
गामा वितरण के मोमेंट जनरेटिंग फ़ंक्शन को निर्धारित करें।

(b) The mean-yield for a one-acre plot is 662 kilos with s.d. 32 kilos. Assuming normal distributions, how many one-acre plots in a batch of 1000 plots would you expect to have yield (a) over 700 kilos and (b) below 650 kilos? (Given that the $P(0 < Z < z) = 0.3830$ when $z = 1.19$ and $P(0 < Z < z) = 0.1430$ when $z = 0.38$). (8)
एक एकड़ के भूखंड के लिए औसत उपज 662 किलो है जिसमें s.d. 32 किंलो है। सामान्य वितरण को मानते हुए, 1000 भूखंडों के बैच में आप कितने एकड़ के भूखंडों की उपज (a)

700 किलो से अधिक और (b) 650 किलो से कम होने की उम्मीद करेंगे? (यह देखते हुए कि $P(0 < Z < z) = 0.3830$ जब $z = 1.19$ और $P(0 < Z < z) = 0.1430$ जब $z = 0.38$)।

Q5 (a) A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes by using Chebychev's inequality. (7)
एक सममित पासे को 600 बार फेंका जाता है। चेबीचेव की असमानता का उपयोग करके 80 से 120 छक्के प्राप्त करने की प्रायिकता के लिए निचली सीमा ज्ञात कीजिये।

(b) Fit a straight line to the following data (8)

x	0	1	2	3	4
y	1.0	1.8	3.3	4.5	6.3

Hence find the difference between the actual value and the obtained value of y when $x = 3$.

निम्न डेटा में कोई सीधी रेखा फ़िट करें

x	0	1	2	3	4
y	1.0	1.8	3.3	4.5	6.3

साथ ही $x = 3$ होने पर वास्तविक मान और y के प्राप्त मान के बीच का अंतर ज्ञात कीजिये।

Q6 (a) The means of two single large samples of 1000 and 2000 members are 67.5 inches and 68.0 inches, respectively. Can the samples be regarded as drawn from the same population of s.d. 2.5 inches? (Test at 5% level). (7)
1000 और 2000 सदस्यों के दो एकल बड़े नमूनों के साधन क्रमशः 67.5 इंच और 68.0 इंच हैं। क्या नमूनों को s.d. 2.5 इंच की समान आबादी से खींचा गया माना जा सकता है? (5% स्तर पर टेस्ट)।

(b) In a sample of 1000 people in Haryana, 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this state? (Test at 1% level). (8)
हरियाणा में 1000 लोगों के सैंपल में 540 चावल खाने वाले हैं और बाकी गेहूं खाने वाले हैं। क्या हम मान सकते हैं कि चावल और गेहूं दोनों इस राज्य में समान रूप से लोकप्रिय हैं? (1% स्तर पर परीक्षण)।

Q7 (a) A die is thrown 60 times with following results: (7)

Face	1	2	3	4	5	6
Frequency	8	7	12	8	14	11

Test at 5% level if the die is unbiased, assuming that $\chi^2_{0.05}$ for 5 d. f. is 11.1.
एक पासे को निम्नलिखित परिणामों के साथ 60 बार फेंक दिया जाता है:

Face	1	2	3	4	5	6
Frequency	8	7	12	8	14	11

5% स्तर पर परीक्षण करें यदि पासा निष्पक्ष है, यह मानते हुए कि $\chi^2_{0.05}$ का मान 11.1 है 5 d.f. के लिए।

(b) The mean weekly sale of soap bars in a shop was 146.3 bars per store. After an advertising campaign, the mean weekly sales in 22 stores for a week changed to 153.7 and showed a standard deviation of 17.2. Is the advertising campaign successful? [Given that tabulated value of $t_{0.05}$ for 21 d. f. is 1.72]. (8)

एक दुकान में साबुन सलाखों की औसत साप्ताहिक बिक्री प्रति स्टोर 146.3 बार थी। एक विज्ञापन अभियान के बाद, एक सप्ताह के लिए 22 दुकानों में औसत साप्ताहिक बिक्री 153.7 हो गई और 17.2 का मानक विचलन दिखाया गया। क्या विज्ञापन अभियान सफल है? [यह देखते हुए कि $t_{0.05}$ का सारणीबद्ध मान 1.72 है 21 d.f. के लिए।