

# Sabancı University Faculty of Engineering and Natural Sciences

# CS301 – Algorithms

#### Homework 1

Due: March 6, 2024 @ 23.55 (upload to SUCourse)

#### PLEASE NOTE:

- Provide only the requested information and nothing more. Unreadable, unintelligible, and irrelevant answers will not be considered.
- Submit only a PDF file. (-20 pts penalty for any other format)
- Not every question of this homework will be graded. We will announce the question(s) that will be graded after the submission.
- You can collaborate with your TA/INSTRUCTOR ONLY and discuss the solutions of the problems. However, you have to write down the solutions on your own.
- Plagiarism will not be tolerated.

## Late Submission Policy:

- Your homework grade will be decided by multiplying what you normally get from your answers by a "submission time factor (STF)".
- If you submit on time (i.e. before the deadline), your STF is 1. So, you don't lose anything.
- If you submit late, you will lose 0.01 of your STF for every 5 mins of delay.
- We will not accept any homework later than 500 mins after the deadline.
- SUCourse's timestamp will be used for STF computation.
- If you submit multiple times, the last submission time will be used.



# Question 1

The recurrence relation of a recursive divide and conquer algorithm is given. Explain this recurrence, verbally, in terms of the size of each sub-problem, the cost of dividing the problem, and combining solutions.

$$T(n) = 3T(\frac{n}{4}) + 2n + n^3$$

# **ANSWER:**

Tan Vfoh Gelih 10:28285 tanufuk  $T(n) = 3T(\frac{n}{2}) + 2n + n^{3}$ Answer: > In terms of the size of each problem; The given recursion relation shows that we divide each problem of size a into 3 smaller subproblems of size n/4. This is expressed by the term -> The cost of dividing problem; In this recursion relationship, the cost associated with dividing the problem is 2n. This is linear cost associated with breaking the original problem into sub-problems - Combining solutions; The cost of combining solutions is n3. This is cubic cost and with merging or combining the solutions of the sub-problems. In divide and conquer algorithms, after doing the sub-problems, these solutions should be combined to get the solution of the original problem. Therefore, as a summary; 1 -> the size of each problem ( 1 -> input size) In -> cost of the dividing 13 -> cost of cambring

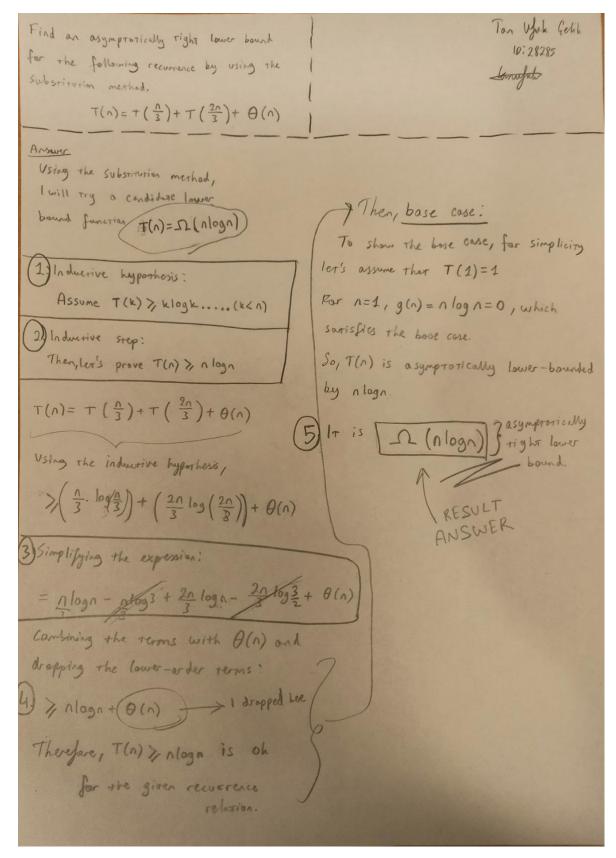


#### Question 2

Find an asymptotically tight lower bound for the following recurrence by using the substitution method.

$$T(n) = T(\frac{n}{3}) + T(\frac{2n}{3}) + \Theta(n)$$

## **ANSWER:**





# Question 3

For the following recurrences, either solve it by using the master method or show that it cannot be solved with the master method.

(a) 
$$T(n) = T(\frac{n}{2}) + \Theta(1)$$

#### **ANSWER:**

T(n)= 
$$T(\frac{n}{2})+\Theta(1)$$

Since the master theorem works with recurrences

of the form

In this case;  $a=1$ ,  $b=2$ ,  $f(n)=\Theta(1)$ 

when need  $\log_a^b$ , let's colculere it:

 $\log_a^b = \log_2^b = n^a = 1$ 

We need to campose  $\log_a^2$  with  $f(n)$ 
 $f(n)=\Theta(1)=\Theta(\log_a^2)$ 

It is case 2

 $O(n)=O(\log_a^2)$ 
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(b) 
$$T(n) = 3T(\frac{n}{4}) + n \lg n$$

#### **ANSWER:**

Tan Ufik Gelik

$$T(n) = 3T(\frac{n}{4}) + n\log n$$
 $a = 3$ ,  $b = 4$ ,  $f(n) = n\log n$ 
 $a > 1$ 

We need  $n\log^n n$ , let's calculate it:

 $n\log^n n = \log^1 n$ 

We need to compare  $n\log^n n + \log^n n$ 

We need to compare  $n\log^n n + \log^n n$ 

We need to compare  $n\log^n n + \log^n n$ 
 $f(n) = n\log n = \Omega(n\log^n n + \log^n n)$ 
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This finishes our necessory requirements and we have that

 $f(n) = O(f(n))$ 

Then,  $f(n) = O(n\log n)$