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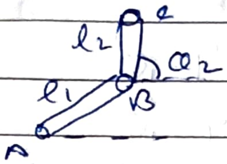
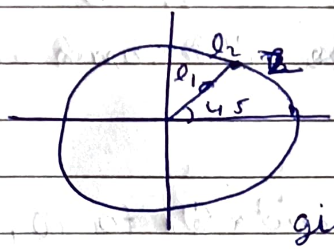
4 Quiz :- 1



$B = (l_1 \cos\theta + l_1 \sin\theta_1)$
$C = (l_1\cos\theta + l_2\cos\theta_2,\ l_1\sin\theta + l_2 \sin\theta_2)$

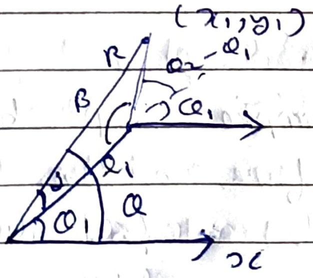
Now, we have θ_1 and θ_2 which move 1
Moves: [(0,1), (1,0), (1,1), (-1,0), (0,-1), (-1,-1), (1,-1), (-1,1)]
There are 8 possible moves

Some constraint is (x,y) cannot be greater than $\left(\frac{l_1+l_2}{\sqrt{2}}, \frac{l_1+l_2}{\sqrt{2}}\right)$ and if $l_1 > l_2$ then a circle $\left(\frac{l_1-l_2}{\sqrt{2}}, \frac{l_1-l_2}{\sqrt{2}}\right)$



$z = \left(\frac{l_1+l_2}{\sqrt{2}}, \frac{l_1+l_2}{\sqrt{2}}\right)$

gives a boundary point



$\beta = \pi - (\theta_2 - \theta_1)$
$\theta = \alpha + \theta_1$

$$\cos\beta = \frac{R^2 - l_1^2 - l_2^2}{-2 l_1 l_2}$$

$$\cos(\theta_2 - \theta_1) = \frac{R^2 - l_1^2 - l_2^2}{2 l_1 l_2}$$

$$\theta_2 = \theta_1 + \cos^{-2}\left(\frac{R^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

By Sine Rule,

$$\frac{R}{\sin\beta} = \frac{l_1}{\sin\alpha} \Rightarrow \alpha = \sin^{-2}\left(\frac{l_2 \sin\beta}{R}\right)$$

$$\therefore \theta_1 = \tan^{-2}\left(\frac{y_1}{x_1}\right) - \sin^{-2}\left(\frac{l_2 \sin\beta}{R}\right)$$

Now,

$$R^2 = x_1^2 + y_1^2$$

$$\sin\beta = \sqrt{1 - \cos^2\beta} \quad \therefore \sin^{-2}\left(\frac{l_2 \sin\beta}{R}\right)$$

Hence, we can calculate Both θ_1 and θ_2 for the corresponding (x_1, y_1)

Now for (x_2, y_2) we can do the same
We get θ_{1f} and θ_{2f}

Hence we have to just move θ_1 to θ_{1f} and θ_2 to θ_{2f}

This makes the problem simple probolm of going from θ_1 to θ_{1f} and θ_2 to θ_{2f}

We have to move
θ_1 at $1°$ angle alone and θ_2 at $1°$ angle alone independent of each other till θ_{1f} and θ_{2f}

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Possible moves are

$(\theta_1 - 1^\circ, \theta_2)$, $(\theta_1 + 1^\circ, \theta_2)$, $(\theta_1 + 1^\circ, \theta_2 + 1^\circ)$
$(\theta_1 - 1^\circ, \theta_2 + 1^\circ)$, $(\theta_2 + 1^\circ, \theta_2 - 1^\circ)$ $(\theta_1 - 1^\circ, \theta_2 - 1^\circ)$
$(\theta_1, \theta_2 + 1^\circ)$, $(\theta_1, \theta_2 - 1^\circ)$

and constraint of $\theta_1 \to [0, 2\pi]$ and $\theta_2 \to [0, 2\pi)$

till the goal state of θ_{1f} and θ_{2f}

·Part A: (x_1, y_1) to (x_2, y_2) any how

Algo: Find θ_1, θ_2 from x_1 and y_1 with help R, α, β and l_1, l_2

$R^2 = x^2 + y^2$ and also θ_{1f} and θ_{2f} from x_2, y_2

$$\theta_2 = \theta_1 + \cos^{-1}\left(\frac{R^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

$$\theta_1 = \tan^{-1}\left(\frac{y_1}{x_1}\right) - \sin^{-1}\left(\frac{l_2 \sin\beta}{R}\right)$$

$$\cos\beta = \frac{R^2 - l_1^2 - l_2^2}{-2 l_1 l_2} \Rightarrow \sin\beta = \sqrt{1 - \cos^2\beta}$$

Now, Perform dfs for finding Successor with the possible moves and go close to goal State
with the error of $\{\theta_{1new} - \theta_{1f} + \theta_{2new} - \theta_{2f}\}$

→ This way we can find a optimal path by changing to θ_1 and θ_2 till it reach θ_{1f} and θ_{2f}

Part B: (x_1, y_1) to (x_2, y_2) in a straight line

Similar algorithum
find θ_1, θ_2 and θ_{1f} and θ_{2f}

put the constraint for lowest
$|\theta_{1new} - \theta_{1f}| + |\theta_{2new} - \theta_{2f}|$ and add a constraint to ensure that it lies on the line of (x_1, y_1) and (x_2, y_2)

As we can make new x_{new}, y_{new}

$\Rightarrow x_{new} = l_1 \cos\theta_{1new} + l_2 \cos\theta_{2new}$
$y_{new} = l_1 \sin\theta_{1new} + l_2 \sin\theta_{2new}$

make sure $\dfrac{x_{new} - x_1}{y_{new} - y_1} = \dfrac{x_2 - x_1}{y_2 - y_1}$

∴ If true then add to the optimal path

This way we will achieve optimal path at the end of the iteration.