

$$B = (l_1 \cos \theta_1, l_1 \sin \theta_1)$$

$$C = (l_1 \cos \theta_1 + l_2 \cos \theta_2, l_1 \sin \theta_1 + l_2 \sin \theta_2)$$

Now, we have information θ_1 and θ_2 can only move by 1°

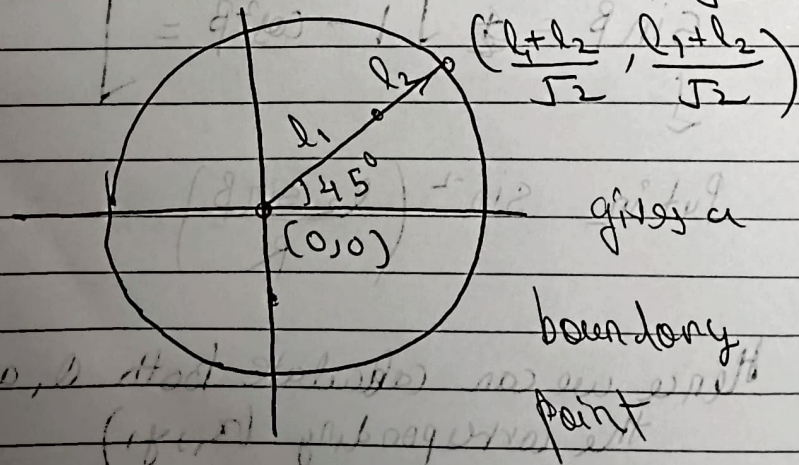
$$\text{Moves} = [(0, 1), (1, 0), (1, 1), (-1, 0), (0, -1)]$$

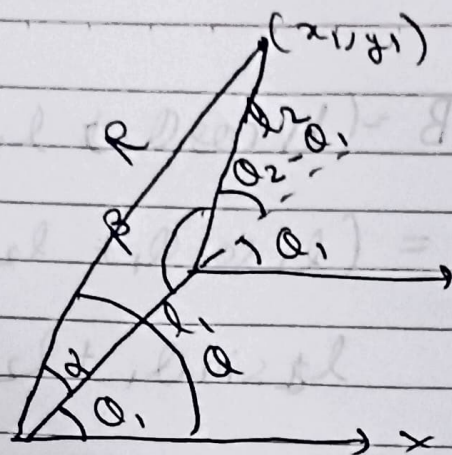
$$[(-1, -1), (1, -1), (-1, 1)]$$

8 possible moves

$$\theta_1, \theta_2 \in [0, 2\pi]$$

1. Some constraint is (x, y) cannot be greater than $(\frac{l_1 + l_2}{\sqrt{2}}, \frac{l_1 + l_2}{\sqrt{2}})$ and if $l_1 > l_2$ then a circle $(\frac{l_1 - l_2}{\sqrt{2}}, \frac{l_1 - l_2}{\sqrt{2}})$





$$\beta = \pi - (\alpha_2 - \alpha_1)$$

$$\alpha = \alpha_2 + \alpha_1$$

$$\cos \beta = \frac{R^2 + l_1^2 - l_2^2}{-2l_1 l_2}$$

$$\cos(\alpha_2 - \alpha_1) = \frac{R^2 - l_1^2 - l_2^2}{2l_1 l_2}$$

$$\alpha_2 = \alpha_1 + \cos^{-1} \left(\frac{R^2 - l_1^2 - l_2^2}{2l_1 l_2} \right)$$

By Sine Rule,

$$\frac{R}{\sin \beta} = \frac{l_2}{\sin \alpha} \Rightarrow \alpha = \sin^{-1} \left(\frac{l_2 \sin \beta}{R} \right)$$

$$\therefore \alpha_1 = \tan^{-1} \left(\frac{y_1}{x_1} \right) - \sin^{-1} \left(\frac{l_2 \sin \beta}{R} \right)$$

Now,

$$R^2 = x_1^2 + y_1^2$$

$$\sin \beta = \sqrt{1 - \cos^2 \beta} = \sqrt{\quad}$$

$$\text{Put in } \sin^{-1} \left(\frac{l_2 \sin \beta}{R} \right)$$

Hence we can calculate both α_1 and α_2 for the corresponding (x_1, y_1)

Now for (x_2, y_2) we can do the same

We get Q_{1f} and Q_{2f}

Hence we have to just move from Q_1 to Q_{1f}
and Q_2 to Q_{2f}

This makes the problem a simple problem of going from
 Q_1 to Q_{1f} and Q_2 to Q_{2f}

We have to move

Q_1 at 1° angle alone and Q_2 at 1° angle alone
independent of each other till Q_{1f} and Q_{2f}

Possible moves are

Successors

$(Q_1 - 1^\circ, Q_2), (Q_1 + 1^\circ, Q_2), (Q_1 + 1^\circ, Q_2 + 1^\circ)$
 $(Q_1 - 1^\circ, Q_2 + 1^\circ), (Q_1 + 1^\circ, Q_2 - 1^\circ), (Q_1 - 1^\circ, Q_2 - 1^\circ)$
 $(Q_1, Q_2 + 1^\circ), (Q_1, Q_2 - 1^\circ)$

and constraint of $Q_1 \rightarrow [0, 2\pi]$ and $Q_2 \rightarrow [0, 2\pi]$

till the goal state of Q_{1f} and Q_{2f}

Part A: (x_1, y_1) to (x_2, y_2) anywhere

Algo:-

Find θ_1, θ_2 from x_1 and y_1 with help of R, α, β and l_1, l_2

$R^2 = x^2 + y^2$ and also θ_1 and θ_2 from x_2, y_2

$$\theta_2 = \theta_1 + \cos^{-1} \left(\frac{R^2 - l_1^2 - l_2^2}{2l_1l_2} \right)$$

$$\theta_1 = \tan^{-1} \left(\frac{y_1}{x_1} \right) - \sin^{-1} \left(\frac{l_2 \sin \beta}{R} \right)$$

$$\cos \beta = \frac{R^2 - l_1^2 - l_2^2}{-2l_1l_2} \Rightarrow \sin \beta = \sqrt{1 - \cos^2 \beta}$$

Now,

Perform dfs for finding successor with the possible move and go closer to goal state

with the error of going to $\theta_{new} = \theta_1$ or $\theta_{new} = \theta_2$

⇒ This way we can find a optimal path by changing θ_1 and θ_2 till it reach θ_1 and θ_2

Part B: (x_1, y_1) to (x_2, y_2) in a straight line

Similar algorithm

Find θ_1, θ_2 and θ_{1f} and θ_{2f}

Now, perform dfs for finding successor with the possible move and go closer to goal state

but the constraint for lowest

$|\theta_{\text{new}} - \theta_1| + |\theta_{\text{new}} - \theta_2|$ and add a constraint to ensure that it lies on the line of (x_1, y_1) and (x_2, y_2)

As we can make new $x_{\text{new}}, y_{\text{new}}$

$$\Rightarrow x_{\text{new}} = l_1 \cos \theta_{1, \text{new}} + l_2 \cos \theta_{2, \text{new}}$$

$$y_{\text{new}} = l_1 \sin \theta_{1, \text{new}} + l_2 \sin \theta_{2, \text{new}}$$

↳

Make sure

$$\frac{x_{\text{new}} - x_1}{y_{\text{new}} - y_1} = \frac{x_2 - x_1}{y_2 - y_1}$$

↳ if true then add to the optimal path

This way we will achieve optimal path at the end of the iteration