On the Performance of AoA Estimation Algorithms in Cognitive Radio Networks

Tanuja Satish Dhope (Shendkar)*
Faculty of Electrical Engineering and Computing,
University of Zagreb, Croatia
tanuja__dhope@yahoo.com

Dina Simunic
IEEE Senior Member, Faculty of Electrical Engineering and Computing, University of Zagreb, Croatia
Dina.Simunic@ fer.hr

Abstract — **Bandwidth efficiency is very important parameter, because it relates to frequency spectrum, which is naturally, limited resource. The cognitive radio has been proposed as the future technology to meet the ever increasing demand of the radio spectrum by allocating the spectrum dynamically to allow unlicensed access on non-interfering basis. Conventional sensing methods usually relate to sensing the spectrum in three dimensions viz. frequency, time and space. However, there are other dimensions 'Angle' and 'Code' that need to be explored further for spectrum opportunity. This paper investigates the performance of Angle of Arrival (AoA) estimation algorithms like Bartlett's, Capon, MUSIC, Root-MUSIC and ESPRIT in cognitive radio networks. The results show that the performance of the algorithms improves with increasing number of array elements, increasing number of snapshots and increasing signal-to-noise ratio. This new approach of AoA estimation of licensed user, improves frequency reuse capability and increases channel capacity by multiplexing multiple users (licensed and unlicensed) into the same channel at the same time in the same geographical area by forming the beam of unlicensed user in the direction other than the licensed users' AoA direction.**

Keywords- ***AoA, Cognitive Radio, MUSIC, ESPRIT, Root-MUSIC.***

I. INTRODUCTION

Although spectrum is seen as a scarce natural resource, measurements show that often there are moments in time, and space where the spectrum is not being fully utilized by the services that have allocated it and therefore it is being used inefficiently[1]. ITU predicted that 1720 MHz spectrum will be required by year 2020 for new wireless technology. Many new wireless services/applications cannot be rolled out due to non-availability of spectrum [2], which demands dynamic allocation of spectrum instead of static [3]-[5].

Recently, there have been growing interests in cognitive radio (CR), where secondary opportunistic radio/unlicensed user (SU) exploits opportunistically spectrum "White Spaces", by means of knowledge of the environment and cognition capability, and adapts their radio parameters accordingly [3].Moreover, CRs must also be able to detect the arrival of other users/licensed users/primary users (PUs) in the band they are currently communicating in and perform spectrum mobility (change the channel) in order to minimize the possible interference in the network[6]-[9].

The conventional definition of the spectrum opportunity, which is often defined as "*a band of frequencies that are not being used by the PU of that band at a particular time in a particular geographic area*", only exploits three dimensions of the spectrum space: time, frequency, and space. However, there are other dimensions 'Angle' and 'Code' that need to be explored further for spectrum opportunity [10]**.** The angle dimension has not been exploited well enough for spectrum opportunity. It is assumed that the PUs and/or the SUs transmit in all the directions. But with the recent advances in multi-antenna technologies through beamforming, multiple users (PU and/or SU) can be multiplexed into the same channel at the same time in the same geographical area. Here angle dimension is different than geographical space dimension. In angle dimension, a PU and a SU can be in the same geographical area and share the same channel. Geographical space dimension refers to physical separation of radios in distance. This new dimension also creates new opportunities for spectral estimation where not only the frequency spectrum but also the angle of arrivals (AoAs) / Direction of Arrival (DoAs) needs to be estimated.

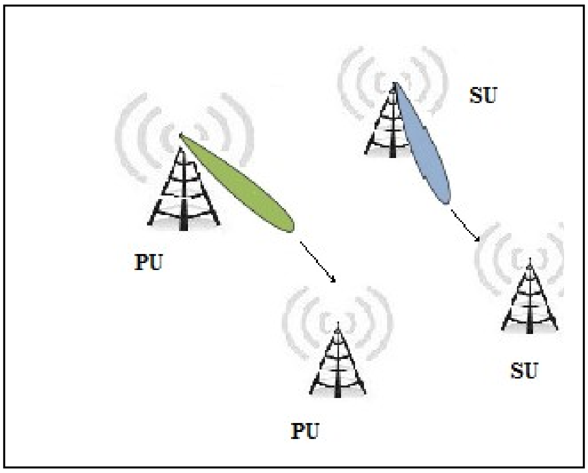

Figure 1. Spectrum Opportunities using 'Angle' dimensions (Direction of PU beam is sensed accordingly the direction of SU beam is decided)

In this paper, we have investigated the performance of five very popular AoA estimation algorithms: Bartlett, Capon, MUSIC, ESPRIT and root-MUSIC. In our investigation, we considered the following parameters: number of array elements, primary user spatial distribution, number of snapshots and Signal -to-Noise Ratio(SNR).

The organisation of paper is as follows: section II decribes various AoA estimatiom algorithms.Section III presents the simulation results based on different AoA estimation algorithms which is followed by conclusions in section IV.

II. AoA Estimation Algorithms

The purpose of AoA estimation algorithm is to use data received by the array to estimate the AoA of PU signal. As shown in Fig. 2**,** the received signals are first passed through a pre-filter of bandwidth B, matched to the bandwidth of PU signal. In source estimation block, each antenna system performs AoA estimation to find PU signal by calculating time delays between antenna elements [11]. Once the AoA of PU signals are estimated, then using adaptive algorithm which uses cost (error) function for calculating the optimum filter weights that maximize beam for SU toward the intended direction ;a direction other than PU signal direction while nulling beam pattern in the direction of PU signal which enables the SU to form a beam to the direction which is not coinciding with the direction of PU signal. That is each beam can be assigned to one SU, improving frequency reuse capability and increase in channel capacity.It is also possible to multiplexed multiple PUs and SUs into the same channel at the same time in the same geographical area.

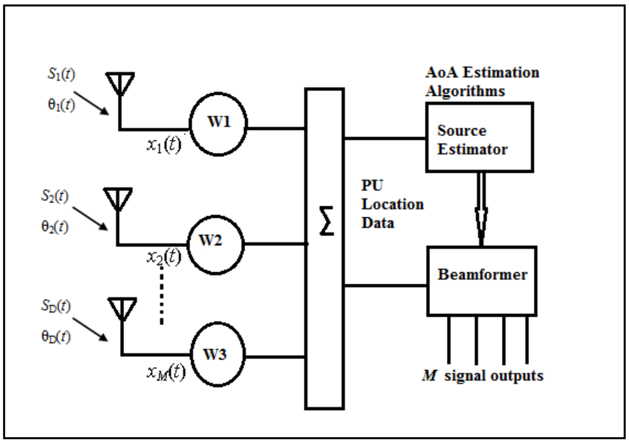

Figure 1. Beamforming at SU side with D arriving PU signals on M antenna elements

In practice we are interested in AoA of PU signal. The parameter estimation techniques are classified into two main categories [11]-[14], namely spectral-based and parametric approaches. Spectral-based approach forms some spectrum-like function of the parameters of interest, e.g., AoA. Locations of distinct separated highest peaks of the function are recorded as AoA estimates. The parametric approach requires simultaneous search for all parameters of interest and therefore often results in more accurate estimates at the at the expense of increased computational complexity.

A. System Model

An Uniform Linear Array (ULA) is considered, with D number of PU signals of frequncy f_0 arriving at M number of array elements of SU which are equally spaced at distance d between the elements.The channel noise for all channels is mutually non-coherent narrowband at f_0.

The steering vector of dimensions Mx1 corresponding to DoA at some angle θ is given by column vector :

$$\mathbf{v}(\theta) = [\ e^{(-j(m-1)2\pi d \sin(\theta)/\lambda)}]^{T} \qquad m = 1, 2, \ldots M \tag{1}$$

Where λ is wavelength, $f_0 = c / \lambda$ and c is the velocity of light.The columnwise combination of all D steering vectors is called array manifold matrix $\mathbf{V}$ of dimensions MxD given by $\mathbf{V}(\theta) = [\mathbf{v}(\theta_1) \colon \mathbf{v}(\theta_2) \colon \ldots\ldots\ldots. \ \colon \mathbf{v}(\theta_D)]$

The spatial correlation (covariance) matrix for the N number of snapshots is given by:

$$\mathbf{S_x} = \frac{1}{N}\sum_{t=1}^{N} \mathbf{x}(t)\mathbf{x}(t)^{\mathrm{H}} \tag{2}$$

where H denotes the Hermitian operator and $\mathbf{x}$ denotes denotes a vector of dimensions Mx1 consisting of received signals x_M. Substitution of (1) into (2) results in

$$\mathbf{S_x} = \frac{1}{N}\sum_{t=1}^{N} \mathbf{V}(\theta)\mathbf{s}(t)\mathbf{s}(t)^{\mathrm{H}}\, \mathbf{V}(\theta)^{\mathrm{H}} + \mathbf{n}(t)\mathbf{n}(t)^{\mathrm{H}} \tag{3}$$

$$\mathbf{S_x} = \mathbf{V}(\theta)\ \mathbf{S_s}\ \mathbf{V}(\theta)^{\mathrm{H}} + \sigma_{\mathrm{w}}^{2}\ \mathbf{I} \tag{4}$$

Where σ_{w}^{2} is noise variance, $\mathbf{I}$ is an identity matrix of size MxM and $\boldsymbol{S_s}$ is received signal power matrix.

B. Beamforming Techniques

The principle behind beamforming technique is to "steer" the array in one direction at a time and measure output power. The steering locations that give maximum power yield AoA estimates. A number of sources will correspond to a number of peaks. The array response is steered by forming a linear combination of the sensor outputs[11]-[15].

The array output is :

$$y = \sum_{i=1}^{M} \mathbf{w}_i^{*}\ x_i \ = \mathbf{w}^{\mathrm{H}}\ \mathbf{x}(t) \tag{5}$$

where $\mathbf{w} = [\,\mathrm{w}_1,\ \mathrm{w}_1, \ldots\ldots, \mathrm{w}_M\,]^{\mathrm{T}}$ is a weighting vector, which determines the radiation pattern. Given samples $y(1), y(2), \ldots\ldots, y(N)$ and hence output power is:

$$\mathbf{Po/p(w)} = \frac{1}{N}\sum_{t=1}^{N} |\mathrm{y}(t)|^2 \ = \frac{1}{N}\sum_{t=1}^{N} \mathbf{w}^{\mathrm{H}}\, \mathbf{x}(t)\mathbf{x}(t)^{\mathrm{H}}\mathbf{w} \\ = \ \mathbf{w}^{\mathrm{H}}\ \mathbf{S_x}\ \mathbf{w} \tag{6}$$

Bartlett's [11] and Capon's methods are based on Beamforming techniques.

C. Bartlett's method

Bartlett's method is an extension of classical Fourier Transform based spectrum analysis. It maximises the power of beam forming output for a given input signal[11].
Bartlett's output spectrum is:

$$P_{bf} = \frac{\mathbf{v}(\theta)^{\mathrm{H}}\, \mathrm{S_x} \mathbf{v}(\theta)}{\mathbf{v}(\theta)^{\mathrm{H}}\, \mathbf{v}(\theta)} \tag{7}$$

D. Capon's method

Capon's method is also called Minimum Variance Distortionless Response algorithm (MVDR).The key is to minimize power contributed by noise and any signals coming from other direction than desired (PU signal direction) [11] [13]-[14].

$$\min_{\mathrm{w}} (\mathbf{w}^{\mathrm{H}}\, \mathbf{S_x}\, \mathbf{w}) \quad \text{Subject to } |\mathbf{w}^{\mathrm{H}}\mathbf{v}(\theta)| = 1 \tag{8}$$

The Capon's weight vector is found to be:

$$\mathbf{w}_{\text{capon}} = \frac{\mathbf{S_x^{-1}}\, \mathbf{v}(\theta)}{\mathbf{v}(\theta)^{\mathrm{H}}\, \mathbf{S_x^{-1}}\, \mathbf{v}(\theta)} \tag{9}$$

Thus Capon's output spectrum is:

$$P_{\text{capon}} = \frac{1}{\mathbf{v}(\theta)^{\mathrm{H}}\, \mathbf{S_x^{-1}}\, \mathbf{v}(\theta)} \tag{10}$$

E. Subspace Based Methods

In Subspace based method, the observed covariance matrix is decomposed into two orthogonal spaces: signal subspace and noise subspace. The AoA estimation is calculated from any one of the subspaces. The subspace based AoA estimation algorithm MUSIC and ESPRIT provide high resolution, they are more accurate and not limited to physical size of array aperture [2] [13]- [17].

F. MUSIC algorithm

MUSIC stands for **MU**iltiple **SI**gnal **C**lassification, one of the high resolution subspace AoA algorithms, which gives an estimate of a number of arrived signals, hence their direction/angle of arrival [11][13-15][18]. Estimation of AoA is performed from one of subspaces either signal or noise, assuming that noise in each channel is highly uncorrelated. This makes the covariance matrix diagonal.
Writing the spatial covariance matrix in terms of eigenvalues and eigenvectors [11][13][14] gives

$$\mathbf{S_x} = \textstyle\sum_{i=1}^{M} T_i\ \boldsymbol{\varphi}_i\ \boldsymbol{\varphi}_i^{\mathrm{H}} \tag{11}$$

$$\mathbf{S_x} = \boldsymbol{\varphi}_i \beta\ \boldsymbol{\varphi}_i^{\mathrm{H}} \tag{12}$$

$$\beta = \mathrm{diag}[T_1, T_2, \ldots\ldots, T_M] \tag{13}$$

The noise subspace eigenvalues and eigenvectors are

$$T_i\ ,\ i = D+1, D+2, \ldots, M \tag{14}$$

$$\boldsymbol{\varphi}_i\ ,\ i = D+1, D+2, \ldots, M \tag{15}$$

The noise subspaces can be written in the form of M x $(M-D)$ matrix:

$$\boldsymbol{\vartheta}_N = [\boldsymbol{\varphi}_{D+1}, \boldsymbol{\varphi}_{D+2}, \ldots\ldots, \boldsymbol{\varphi}_M] \tag{16}$$

Equation (16) indicates that the desired value AoA of θ_1, θ_2,......, θ_D can be found out by finding a set of vectors that span $\boldsymbol{\vartheta}_N$ and projecting $\mathbf{v}(\theta)$ onto $\boldsymbol{\vartheta}_N$ for all values of θ and evaluating the D values of θ, where the projection is zero:

$$\left\|\mathbf{v}_i^{\mathrm{H}}\, \boldsymbol{\vartheta}_N\right\|^2 = 0 \qquad i = 1,2, \ldots\ldots, D \tag{17}$$

Thus, MUSIC Pseudospectrum is given as:

$$P_{\text{music}}\,(\theta) = \frac{1}{\text{abs}\,[\mathbf{v}(\theta)^{\mathrm{H}}\, \boldsymbol{\vartheta}_N\, \boldsymbol{\vartheta}_N^{\mathrm{H}}\, \mathbf{v}(\theta)\,]} \tag{18}$$

G. Root-MUSIC algorithm

Root-MUSIC is the polynomial version of MUSIC[12]. The array manifold matrix is expressed in polynomial form by evaluating at $z = \mathrm{e}^{\mathrm{j}\theta}$. If the eigendecomposition corresponds to the true spectral matrix, then MUSIC spectrum $P_{\text{music}}\,(\theta)$ becomes equivalent to the polynomial on the unit circle and peaks in the MUSIC spectrum exist as roots of polynomial lie close to the unit circle [5] [17-19]. That is $P_{\text{rmusic}}\,(z)|_{z=\mathrm{e}^{\mathrm{j}\theta}} = P_{\text{music}}\,(\theta)$. Ideally in absence of noise, the poles will lie exactly on the unit circle at the locations determined by AoA. Ultimately, we calculate the polynomial and select the J roots that are inside the unit circle. A pole of polynomial, $\mathrm{D}(z)\,|_{z=z_q} = |z_q| = |\,\exp(\mathrm{j}\arg(z_q)|$ will result in a peak in the MUSIC spectrum at

$$\theta = \sin^{-1}\{\lambda/2\pi d\}\arg[z_q] \qquad q = 1,2, \ldots\ldots, D \tag{19}$$

III. SIMULATION RESULTS

In order to evalute the performance of AoA estimation algorithms for PUs in the selected frequncy band, the simulations were run for three different sets of environment of PUs spatial distribution:

a) Three PUs with wide angular separation $\mathrm{E_W} = [0^0, 25^0, 55^0]$

b) Three PUs with narrow angular separation $\mathrm{E_N} = [-5^0, 10^0, 20^0]$ and

c) Four PUs with combination of wide and narrow separations $\mathrm{E_C} = [0^0\ 10^0\ 15^0\ 40^0]$

For analysis of the performance of these algorithms, regarding impact of number of array elements, snapshots and SNR parameters were set as follows:

1. Impact of number of array elements were considered for two environments (E_W and E_N) at SNR of 10dB with 200 snapshots.
2. Impact of number of snapshots was considered for environment E_N with 10 array elements at SNR of 10dB.
3. Impact of SNR was considered for environment E_C with 16 array elements and 200 snapshots.

Fig. 3 shows the performance of five algorithms for different number of array elements for $E_W = [0^0, 25^0, 55^0]$ where the user spatial distribution is wide. Root-MUSIC showed significant down performance comparing to other four algorithms while MUSIC, Capon's and ESPRIT errors are less than 1^0 for 5 element array and above. Bartlett's error is nearly 4^0 for 5 array elements, reaching down to less than 1^0 for equal and more than 6 array elements. However, Root-MUSIC error is still above 25^0 coming down to 17^0 for 6 element arrays and reaching below 1^0 only for equal or more than 7 element arrays.

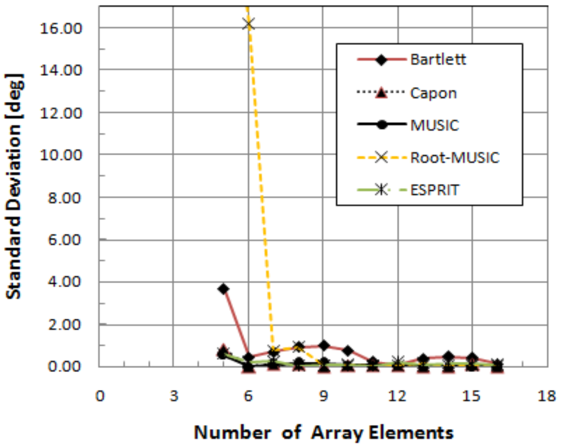

Figure 3. Performance analysis of AoA estimation algorithm as a function of array elements ($E_W = [0^0, 25^0, 55^0]$)

Fig. 4 shows the performance of algorithms for different number of array elements for $E_N = [-5^0, 10^0, 20^0]$. For the case of five element array, all algorithms except ESPRIT had significant AoA estimation errors, meaning that they are either around 23^0 (for MUSIC, Capon and Bartlett) or higher (41^0 for Root-MUSIC).The ESPRIT algorithm shows error less than 2^0. MUSIC and ESPRIT proved to be more robust giving error below 1^0 for equal to or more than 6 elements array .Root-MUSIC shows error below 1^0 for equal to or higher than 8 element arrays. Capon's and Bartlett's algorithms turned out to be the most sensitive to reduction in angular separation having an error of less than 1^0 only at equal to or more than 9 and 10 elements array, respectively. Thus, we can conclude that for environment with narrow angular separation Root-MUSIC algorithm sensitivity to angular separation is higher than for MUSIC algorithm, but better than for Capon and Bartlett's at 8 elements array.

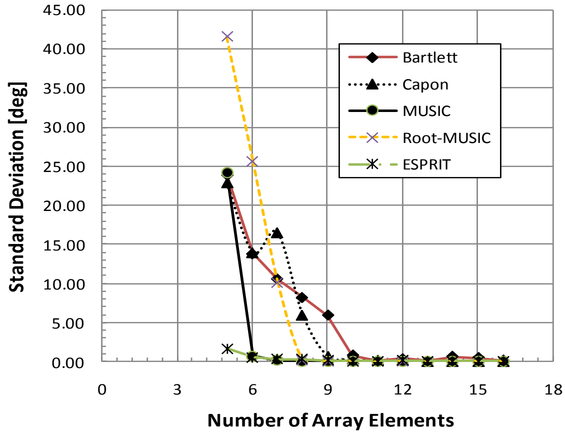

Figure 4. Performance analysis of AoA estimation algorithm as a function of array elements ($E_N = [-5^0, 10^0, 20^0]$)

Fig. 5 shows the performance of algorithms for different number of snapshots for $E_N = [-5^0, 10^0, 20^0]$. Below 40 snapshots all algorithms performed poorly. Root-MUSIC and Capon's show error more than 17^0 for snapshots of 10. Root-MUSIC shows less than 1^0 errors at 50 snapshots. All algorithms perform well for 150 snapshots onwards giving highest ranking to MUSIC, ESPRIT, Root-MUSIC and Capon's respectively. Taking the excess number of snapshots causes an error of 4^0 in Bartlett's AoA estimation (e.g., at 700 snapshots).

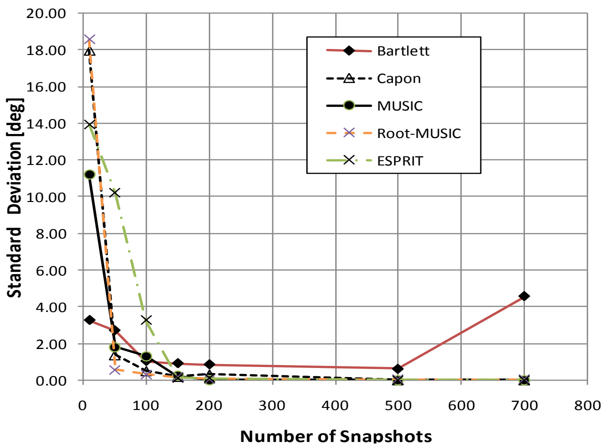

Figure 5. Performance analysis of AoA estimation algorithm as a function of snapshots ($E_N = [-5^0, 10^0, 20^0]$)

Fig. 6 analyses the performance of algorithm as a function of SNR with E_C where one separation was only 5^0. Bartlett's and Capon's algorithm fail to identify closely spaced signals even at high value of SNR (10dB), with error of 6^0 and 16^0 respectively. MUSIC, Root-MUSIC and ESPRIT show error below 1^0 at SNR of and higher to10dB. MUSIC outperformed the other two at SNR value of -6dB providing error less than 1^0 . At SNR of -10dB, MUSIC shows 2.5^0 errors, whereas Root-MUSIC performs better than ESPRIT and at SNR -5dB it shows error less than 1^0. ESPRIT performs well only at SNRs equal or higher than 10 dB.

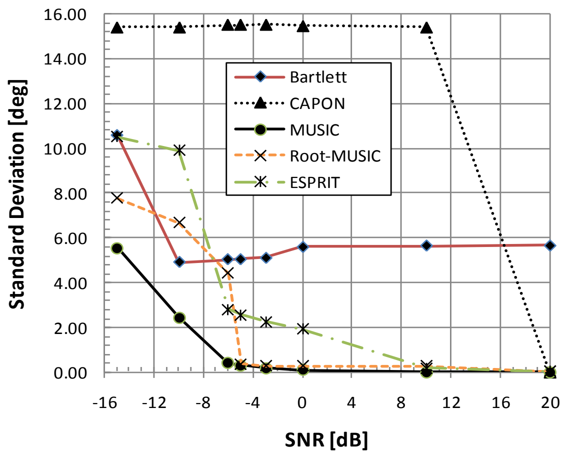

Figure 6.Performance analysis of AoA estimation algorithm as a function of SNR (E_C =[0^0 10^0 15^0 40^0])

IV. CONCLUSIONS

Performance of five AoA estimation algorithms (Bartlett, Capon, MUSIC, Root-MUSIC and ESPRIT) was analysed in the presence of the both: wide and narrow angular separation and combination of wide and narrow angular for PU signal. As expected, performance improvement was observed with increasing number of array elements, increasing angular separation between the PU signals and with increasing SNR for all algorithms. MUSIC algorithm showed superior performance in all aspects considered like number of array elements, number of snapshots , different values of SNR and for less angular seperation. Root-MUSIC and Bartlett's algorithms showed to be sensitive to small number of array elements typically for 5 array elements. Root-MUSIC and Bartlett's algorithm showed 25^0 and 4^0 AoA estimation error respectively for wide angular separation of PU signal (greater than 20^0). Capon's algorithm performance might be acceptable over Bartlett's if used in environments with wide (greater than 20^0) and narrow angular separation (equal to 10^0). For 5 elements array, ESPRIT shows good performance compared to other except MUSIC. Root-MUSIC outperformed MUSIC and ESPRIT for low value of snapshots (typically 50) giving AoA estimation error of 1^0. At smaller values of SNR, typically -15dB, the performance of Bartlett's and Capon's algorithms deteriorates significantly, compared to other algorithms. MUSIC, Root-MUSIC and ESPRIT show error below 1^0 at SNR of 10dB. MUSIC outperformed the other two, even though the SNR value is -6dB providing error less than 1^0. If one wants reliability and accuracy in AoA estimation for PU signals, the recommended choice would be MUSIC algorithm, as it showed superior performance in cases of smaller number of array elements, in poor SNR conditions, and in presence of narrow separation between PU signals equal to 5^0. This new approach of AoA estimation in cognitive radio network improves frequency reuse capability and increases channel capacity by multiplexing multiple users (licensed and unlicensed) into the same channel at the same time in the same geographical area, by forming the beam of unlicensed user in the direction other than the licensed users' AoA direction.

REFERNCES

[1] FCC, "Spectrum policy task force report," in Proceedings of the Federal Communications Commission (FCC '02), Washington, DC, USA, Nov. 2002.

[2] T. Dhope, D. Simunic and M. Djurek, "Application of DOA Estimation Algorithms in Smart Antenna Systems," Studies in informatics and Control, vol.19, Issue 4, pp.445-452, Dec. 2010

[3] K-C. Chen and R. Prasad, "Cognitive radio networks," John Wiley & Sons Ltd, 2009.

[4] T. Dhope, D. Simunic and R.Prasad, "TVWS opportunities and regulation:-empowering rural India,"14th International Symposium on WPMC '11, Brest, France, pp.201-205, Oct.2011.

[5] T. Dhope, D. Simunic and R.Prasad, "TVWS opportunities and regulatory aspects in India," 14th International Symposium on WPMC '11, Brest, France, pp. 566-570, Oct.2011.

[6] T. Dhope, D. Simunic and A.Kerner, "Analyzing the Performance of Spectrum Sensing Algorithms for IEEE 802.11 af Standard in Cognitive Radio Network*, Studies in informatics and Control, vol.21 , Issue 1 , pp.93-100 , Mar. 2012

[7] T. Dhope, D. Simunic ,"Spectrum Sensing Algorithm for Cognitive Radio Networks for Dynamic Spectrum Access for IEEE 802.11 af standard",IJRRWSN,Vol.2.,No.1,Mar. 2012(in publication)

[8] T. Dhope, D. Simunic, "Performance Analysis of Covariance Based Detection in Cognitive Radio,"35th Jubilee International Convention MIPRO 2012 ,Opatija, Croatia, May 2012 (accepted)

[9] T. Dhope, D. Simunic, "Hybrid Detection Method for Spectrum Sensing in Cognitive Radio," 35th Jubilee International Convention MIPRO 2012. Opatija, Croatia, May 2012(accepted)

[10] Tevfik Y¨ucek and H¨useyin Arslan, "A Survey of Spectrum Sensing Algorithms for Cognitive Radio Applications,"IEEE COMMUNICATIONS SURVEYS & TUTORIALS, VOL. 11, NO. 1, FIRST QUARTER 2009,pp-116-130,2009.

[11] H. Trees, Optimum Array Processing, Detection, Estimation and Modulation, Part IV,John Wiley and Sons, New York , 2002.

[12] B.D. Rao and K.V.S. Hari," Performance analysis of ROOT-MUSIC," IEEE Transactions on Acoustics, Speech and Signal Processing, Vol. 37, pp. 1939-1949, Dec. 1989.

[13] R.J. Weber and Y. Huang,"Analysis for Capon and MUSIC DOA Estimation Algorithms," Antennas and Propagation Society International Symposium, 2009. APSURSI '09,pp. 1–4, June 2009.

[14] M. Jalali, M. Moghaddasi and A. Habibzadeh, "Comparing accuracy for ML, MUSIC, ROOT-MUSIC and spatially smoothed algorithms for 2 users," Microwave Symposium (MMS), 2009 Mediterrannean IEEE conference, pp. 1–5, Nov.2009.

[15] S.W. Varade and K.D.Kulat,"Robust Algorithms for DOA Estimation and Adaptive Beamforming for Smart Antenna Application," 2nd international conference on Emerging Trends in Engineering and Technology (ICETET), 2009 , pp. 1195–1200, Dec.2009.

[16] Z.I. Khan, R.A. Awang A.A Sulaiman, M.H. Jusoh, N.H Baba ,M.M.D. Kamal, and N.I. Khan," Performance analysis for Estimation of signal Parameters via Rotational Invariance Technique (ESPRIT) in estimating Direction of Arrival for linear array antenna," IEEE International Conference on RF and Microwave Conference, 2008, pp. 530–533, Dec. 2008.

[17] F.A Leon De and S.J.J Marciano, "Application of MUSIC, ESPRIT and SAGE Algorithms for Narrowband Signal Detection and Localization, "TENCON, IEEE Region 10 Conference, pp. 1-4, Nov. 2006.

[18] W.X. Sheng, J.Zhou, D.G.Fang and Y.C. Gu, "Super Resolution DOA Estimation in Switched Beam Smart Antenna, Antennas," Propagation and EM Theory, 2000. Proceedings. ISAPE 5th International Symposium, pp. 603-606, 2000.