Evaluate the integral $\int \sin(\sqrt{x})\,dx$

$u = \sqrt{x} \Rightarrow u = x^{1/2}$

$$\frac{du}{dx} = \frac{1}{2} x^{-1/2}$$

$$dx = 2\sqrt{x}\,du$$

dx can be rewritten as:

$$dx = 2u\,du$$

$$\int \sin(\sqrt{x})\,dx = \int \sin(u) \cdot 2u\,du$$

$$= 2\int u \sin(u) \cdot du$$

Integration by parts:

$$\int u\,dv = uv - \int v\,du$$

Let $U = u$ and $dv = \sin u\,du$

then $dU = du$ and $v = -\cos(u)$

$\therefore$ applying all the values with IBP formula:

$$2\int u \sin(u)\,du = 2\left[-u\cos(u) - \int -\cos(u)\,du\right]$$

$$= 2\left[-u\cos(u) + \int \cos(u)\,du\right]$$

$$= 2\left[-u\cos(u) + \sin(u)\right]$$

Substituting $u = \sqrt{x}$ back:

$$2\left[-\sqrt{x}\cos(\sqrt{x}) + \sin(\sqrt{x})\right]$$

$$\therefore \int \sin(\sqrt{x})\,dx = 2\left[-\sqrt{x}\cos(\sqrt{x}) + \sin(\sqrt{x})\right] + C$$