Set 10: Variance

Stat 260 A01: May 23, 2024

The ***variance*** of a random variable X with probability distribution $f(x)$ is

$V(X) = Var(X) = \sigma_x^2 = E[(x-\mu_x)^2]$

$= \sum (x_i - \mu_x)^2 \cdot f(x)$ ← sum of squared differences

Computational Shortcut:

$\mu_x = E(x)$ constant

$$V(X) = E\left[(X-\mu_x)^2\right]$$

$$= E\left[x^2 - 2\mu_x X + \mu_x^2\right]$$

$$= E[x^2] + E[-2\mu_x X] + E[\mu_x^2]$$ → Expected value of a constant is the constant itself

$$= E[X^2] - 2\mu_x \underbrace{E[X]}_{\mu_x} + \mu_x^2$$

$$= E[x^2] - 2\mu_x\mu_x + \mu_x^2$$

$$= E[x^2] - 2\mu_x^2 + \mu_x^2$$

$$= E[x^2] - \mu_x^2 = E[x^2] - (E[x])^2$$

$V(X) = E(X^2) - E(X)^2$

The ***standard deviation*** of the random variable X is

$SD(x) = \sqrt{V(x)} = \sigma^2$

Example 1: Recall the example from Set 9, regarding the number of offspring of an ewe.

Number of offspring (x_i)	0	1	2	3
$f(x_i) = P(X = x_i) = p_i$	0.10	0.25	0.60	0.05

$V(X) = \sum (x_i - \mu_x)^2 \cdot f(x)$

$= (0-1.6)^2 \cdot (0.1) + (1-1.6)^2 \cdot (0.25) + (2-1.6)^2 \cdot (0.6) + (3-1.6)^2 (0.05)$

$= 0.54$ lambs^2

Shortcut: from set 9 $E[x^2] = 3.1$

$V(x) = E[x^2] - E[x]^2 = 3.1 - (1.6)^2$

$= 0.54$ lambs^2

$SD(x) = \sqrt{V(x)} = \sqrt{0.54} = 0.735$ lambs

Rules for Standard Deviation and Variance: For a constant c,

(i) $V[X + c] = V(x) + V(c) = V(x)$

(ii) $V[c] = 0$

(iii) $V[cX] = c^2 V(x)$

(iv) $SD[X + c] = \sqrt{V(x) + c} = \sqrt{V(x)} = SD(x)$

(v) $SD[c] = 0$

(vi) $SD[cX] = \sqrt{V(cx)} = \sqrt{c^2 V(x)} = |c| SD(x)$

$V(ax+b) = V(ax) + V(b)$ (when a const comes out you square it; const $= a^2 V(x)$)

warning $V(x+y) \neq V(x) + V(y)$

Example 1 Continued... Recall that a farmer will receive \$50 for each ewe and \$30 for each lamb.

$V(y)$ where y = amount of money earned from selling lewe and its lambs

$y = 50 + 30x$, $V(\text{const}) = 0$

$$V(y) = V(50 + 30x) = V(50) + V(30x)$$
$$= 0 + (30)^2 \cdot V(x)$$
$$= 30^2 \cdot (0.54)$$
$$= 486 \text{ dollars}^2$$

$$SD(y) = \sqrt{V(y)} = \sqrt{486} = 22.05\$$$

General Note:

$V(-2x) = (-2)^2 V(x)$

Readings: Swartz 4.2 [EPS discrete parts of 2.5 and 2.6]

Practice problems: 2.51, 2.53, 2.55, 2.63, 2.65, 2.75, 2.77 (Ignore the reference to Theorem 2.2. Find the variance of X using the computational form shown in class), 2.87, 2.9