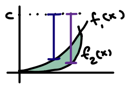
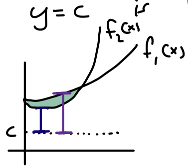
7.2: $V = \pi \int r_{out}^2 - r_{in}^2$: Disks method for Volume

- If we do it around $y=c$, we do it in terms of x:
  - if $y=c$ is higher up than our area, then radius has the form $r = c - f(x)$.



$r_{in} = c - f_1(x)$
$r_{out} = c - f_2(x)$

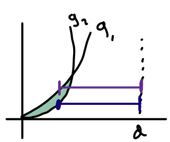
  - if $y=c$ is lower than our area, then radius has form $r = f(x) - c$



$r_{in} = f_2(x) - c$
$r_{out} = f_1(x) - c$

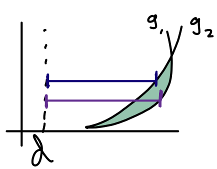
  - If $y=0$, then $r = f(x)$.

- If we do it around $x=d$, we do it in terms of y:
  - If $x=d$ is farther to the right than our area, $r = d - g(y)$.



$r_{out} = d - g_1(y)$
$r_{in} = d - g_2(y)$

  - If $x=d$ is more to the left than our area, $r = g(y) - d$

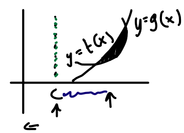


$r_{out} = g_1(y) - d$
$r_{in} = g_2(y) - d$

  - If $x=0$, then $r = g(y)$.

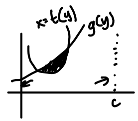
$V=2\pi\int rh$: Shells Method for Volume:

- If rotated around $x=\ldots$, do it in terms of x.
  - If around $x=0$ (y-axis), $r=x$.
  - If around $x=c$, where c is closer to y-axis than area:



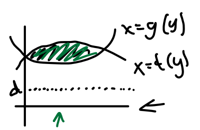
then $\to h=f(x)-g(x)$
$r=x-c$ (even if x is negative)

  - If around $x=c$, where c is farther to y-axis than area:



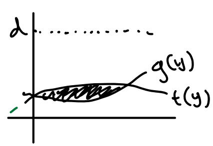
$h=g(y)-f(y)$.
$r=c-x$.

- If rotated around $y=\ldots$, do it in terms of y.
  - If around $y=0$ (x-axis), then $r=y$.
  - If around $y=d$, where d is closer to x-axis than area:



$h=f(y)-g(y)$.
$r=y-d$. (even if d negative)

  - If around $y=d$, where d is farther from x-axis than area:



$h=f(y)-g(y)$
$r=d-y$.

7.5: $2\pi \int r L$

- $y = f(x)$, $a \leq x \leq b$, around x-axis:

$$S = 2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2}\, dx$$

- $x = g(y)$, $c \leq y \leq d$, around x-axis:

$$S = 2\pi \int_c^d y \sqrt{1 + (g'(y))^2}\, dy$$

- $y = f(x)$, $a \leq x \leq b$, around y-axis:

$$S = 2\pi \int_a^b x \sqrt{1 + (f'(x))^2}\, dx$$

- $x = g(y)$, $c \leq y \leq d$, around y-axis:

$$S = 2\pi \int g(y) \sqrt{1 + (g'(y))^2}\, dy$$

- If we do it around another axis that is not the y or x-axis, we modify the radius (blue part), in a way very similar to how we do it in section 7.2 and 7.3. Look at pbs 23, 24 for an example.

8.2: Cheat Sheet

Harold's Series Convergence Tests Cheat Sheet

24 March 2016

1 Divergence or nth Term Test

Series: $\sum_{n=1}^{\infty} a_n$

Condition(s) of Convergence:
None. This test cannot be used to show convergence.

Condition(s) of Divergence:
$$\lim_{n\to\infty} a_n \neq 0$$

2 Geometric Series Test

Series: $\sum_{n=0}^{\infty} ar^n$

Condition of Convergence:
$$|r| < 1$$

Sum: $\mathbf{S} = \lim_{n\to\infty} \frac{a(1-r^n)}{1-r} = \frac{a}{1-r}$

Condition of Divergence:
$$|r| \geq 1$$

3 *p* - Series Test

Series: $\sum_{n=1}^{\infty} \frac{1}{n^p}$

Condition of Convergence:
$$p > 1$$

Condition of Divergence:
$$p \leq 1$$

4 Alternating Series Test

Series: $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$

Condition of Convergence:
$$0 < a_{n+1} \leq a_n$$
$$\lim_{n\to\infty} a_n = 0$$
or if $\sum_{n=0}^{\infty} |a_n|$ is convergent

Condition of Divergence:
None. This test cannot be used to show divergence.

* Remainder: $|R_n| \leq a_{n+1}$

5 Integral Test

Series: $\sum_{n=1}^{\infty} a_n$
when $a_n = f(n) \geq 0$
and $f(n)$ is continuous, positive and decreasing

Condition of Convergence:
$\int_1^{\infty} f(x)dx$ converges

Condition of Divergence:
$\int_1^{\infty} f(x)dx$ diverges

* Remainder: $0 < R_N \leq \int_N^{\infty} f(x)dx$

6 Ratio Test

Series: $\sum_{n=1}^{\infty} a_n$

Condition of Convergence:
$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$$

Condition of Divergence:
$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$$

* Test *inconclusive* if
$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$$

7 Root Test

Series: $\sum_{n=1}^{\infty} a_n$

Condition of Convergence:
$$\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$$

Condition of Divergence:
$$\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$$

* Test *inconclusive* if
$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$$

8 Direct Comparison Test

$(a_n, b_n > 0)$

Series: $\sum_{n=1}^{\infty} a_n$

Condition of Convergence:
$$0 < a_n \leq b_n$$
and $\sum_{n=0}^{\infty} b_n$ is absolutely convergent

Condition of Divergence:
$$0 < b_n \leq a_n$$
and $\sum_{n=0}^{\infty} b_n$ diverges

9 Limit Comparison Test

$(\{a_n\}, \{b_n\} > 0)$

Series: $\sum_{n=1}^{\infty} a_n$

Condition of Convergence:
$$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$$
and $\sum_{n=0}^{\infty} b_n$ converges

Condition of Divergence:
$$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$$
and $\sum_{n=0}^{\infty} b_n$ diverges

10 Telescoping Series Test

Series: $\sum_{n=1}^{\infty} (a_{n+1} - a_n)$

Condition of Convergence:
$$\lim_{n\to\infty} a_n = L$$

Condition of Divergence: None

NOTE:
1) May need to reformat with partial fraction expansion or log rules.
2) Expand first 5 terms. n=1,2,3,4,5.
3) Cancel duplicates.
4) Determine limit L by taking the limit as $n \to \infty$.
5) Sum: $S = a_1 - L$

NOTE: These tests prove convergence and divergence, not the actual limit L or sum $\mathbf{S}$.

Sequence: $\lim_{n\to\infty} a_n = L$
$(a_n, a_{n+1}, a_{n+2}, \ldots)$

Series: $\sum_{n=1}^{\infty} a_n = \mathbf{S}$
$(a_n + a_{n+1} + a_{n+2} + \cdots)$

• Remember ACV means $\sum|a_n|$ converges.
If a series is ACV, it is CCV too.
But if a series is CCV, it might or might not be ACV too, so always test $\sum|a_n|$ first.

Choosing a Convergence Test for Infinite Series

Courtesy David J. Manuel