Set 7: Independence

Stat 260 A01: May 28, 2024

Recall: What is the probability of rolling a "3" on a standard 6-sided die?

P (roll 3) = 1/6

P (roll 3 | roll odd) = 1/3

P (roll 3 | its raining) = 1/6

useless info so it's only P(roll 3)

$P(A \cap B) = 0$ means mutually exclusive

independent and mutually exclusive ARE NOT THE SAME THING

Independence: Events A and B are **independent** if any of the following hold:

(i) $P(A|B) = P(A)$

(ii) $P(B|A) = P(B)$

(iii) $P(A \cap B) = P(A) \cdot P(B)$

test for independence using any one of these rules (if one holds they all hold)

only use if you know they are independent or to test for independence

If A and B are not independent, they are said to be **dependent**.

Say A and B are independent

$P(A|B) = P(A)$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$

always holds regardless of which A and B are independent

Example 1: Cornflower (or Bachelor's Button -*Centaurea cyanus*) is an invasive plant species to Vancouver Island. Suppose that a new herbicide is tested on 300 cornflower plants, and the percentage of plants with damage to their petals, stems, and roots is recorded. Of the treated plants, 20% of plants displayed damage to their petals damage (A), 80% had damage to their stems (B), and 5% had damage in their root systems (C). Moreover, 4% of plants had stem and root damage, while 85% of plants had damage to their petals or stems.

(a) Is having stem damage independent of having root damage?

$P(A) = 0.2$ $P(B) = 0.8$ $P(C) = 0.05$ $P(B \cap C) = 0.04$ $P(A \cup B) = 0.85$

Test: Does $P(B \cap C) = P(B) \cdot P(C)$

$P(B) \cdot P(C) = 0.8 \times 0.4$

$= 0.04 \Longleftrightarrow P(B \cap C)$ ∴ they are independent

$\Rightarrow$ B and C are independent

(b) Is having $\overbrace{\text{stem damage}}^{B}$ independent of having $\overbrace{\text{petal damage}}^{A}$?

Test : $P(A) \cdot P(B) = P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.85 = 0.2 + 0.8 - P(A \cap B)$

$\therefore P(A \cap B) = 0.15$

$P(A) \cdot P(B) = (0.2)(0.8) = 0.16$

Since $P(A) \cdot P(B) \neq P(A \cap B)$, they (A and B) are not independent

Independence of Three Events: Events A, B and C are **mutually independent** if and only if all of the following hold:

(i) $P(A \cap B \cap C) = P(A)P(B)P(C)$

(ii) $P(A \cap B) = P(A)P(B)$

(iii) $P(A \cap C) = P(A)P(C)$

(iv) $P(B \cap C) = P(B)P(C)$

must test all 4 conditions for mutual independence if a single equation fails, they are not mutualy independent

Back to corn flowers in ex 1

since $P(A \cap B) \neq P(A) \cdot P(B)$

A, B and C are not mutualy independent

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n Independent Events

If E_1, $E_2, E_3, \ldots, E_n$ are n independent events, then

$P(E_1 \cap E_2 \cap E_3 \cap E_n \ldots) = P(E_1)\,P(E_2)\,P(E_3) \ldots P(E_n)$

Not a test for independence but rather a result we can use if we already know they are independent.

Example 2: A hydrologist has four remote rainwater monitoring units. Each morning, each unit should send in a report of the previous day's collected rainfall. However, the units are old and sometimes fail to send their reports.

- Unit A fails to report on 3% of days (A).
- Unit B fails to report on 2% of days (B).
- Unit C fails to report on 6% of days (C).
- Unit D fails to report on 4% of days (D).

Assuming that each unit fails (or succeeds) to report independently of the others, what is the probability that on a random day:

(a) All four units successfully send their reports?

$P(A) = 0.03 \qquad P(A') = 0.97$

Want: $P(A' \cap B' \cap C' \cap D') = \underbrace{P(A') \cdot P(B') \cdot P(C') \cdot P(D')}$

$= (0.97)(0.98)(0.94)(0.96)$

$= 0.86$

Only allowed to do this because events are independent. Otherwise, we would need General Addition Rule.

(b) **At least one** unit fails to report?

$P(A \cup B \cup C \cup D) = 1 - P(\overline{A \cup B \cup C \cup D})$

$= 1 - P(A' \cap B' \cap C' \cap D')$ ← De Morgan's Law

$= 1 - 0.86$

$= 0.14$

Extra Example 1: A transit survey conducted at a university polled 1200 commuters for their position (student or faculty) and on how they arrived on campus that day.

	Car	City Bus	Bike	Walk	Total
Student	96	432	120	72	720
Faculty	120	288	48	24	480
Total	216	720	168	96	1200

(a) What is the probability that a randomly selected commuter from the survey was a student who walked to campus?

(b) If a randomly selected commuter from the survey was a student, what is the probability that they walked to campus?

(c) Which of the following statements are true?

(i) The events, “the commuter was a faculty member” and “the commuter walked” are independent.

(ii) The events, “the commuter was a faculty member” and “the commuter took the city bus” are independent.

(iii) The events, “the commuter was a faculty member” and “the commuter took the city bus” are mutually exclusive.

(iv) The events, “the commuter took biked” and “the commuter took the city bus” are mutually exclusive.

Textbook Readings: Swartz 3.4.1 (3.5, 3.6 are background and examples) [EPS 1.8]
Practice problems: Swartz 1.67, 1.69, 1.71