STAT 260 Summer 2024: Written Assignment 10

Due: Upload your solutions to Crowdmark BEFORE 6pm (PT) Friday August 2.

You may upload and change your files at any point up until the due date of Friday August 2 at 6pm (PT).

A 2% per hour late penalty will be automatically applied within Crowdmark. The penalty is applied in such a way so that assignments submitted 6pm to 6:59pm will have 2% deducted, assignments submitted 7pm-7:59pm will have 4% deducted, etc.

Note that if you submit any portion of your assignment before the deadline, Crowdmark will NOT permit you to edit your submission (including make additional uploads) after the 6pm deadline passes. This means that if, for example, you upload only Question 1 before the deadline, you will not be able to upload Question 2 after the deadline. If you intend to submit late (with penalty) you must submit the entire assignment late.

Submission: Solutions are to be uploaded to Crowdmark. Here you will be asked to upload your solutions to each question separately. Your solution to Question 1 must be uploaded in the location for Question 1, your solution to Question 2 must be uploaded in the location for Question 2, etc. If your work is uploaded to the wrong location, the marker will not be able to grade it.

You may hand-write your solution on a piece of paper or tablet. If you wish to use this question sheet and write your solutions on the page, space has been provided below. One of the quickest ways to upload work is by accessing Crowdmark from within a web browser on a smartphone. In the area where you upload work, press the "+" button. This will give you the option of using a file already on your phone, or you can use the phone camera to photograph your work. If you complete your work on a tablet, save the file as a PDF or each question as a jpeg and drag/drop the file into the Crowdmark box. ***Photographs of laptop/tablet screens will not be graded***; take a proper screenshot.

Instructions: For full marks, your work must be neatly written, and contain enough detail that it is clear how you arrived at your solutions. ***You will be graded on correct notation***. Messy, unclear, or poorly formatted work may receive deductions, or may not be graded at all. Only resources presented in lecture or linked to on the Stat 260 Brightspace page are permitted for use in solving these assignments; using outside editors/tutors, and/or software (include AIs) is strictly forbidden. Talking to your classmates about assigned work is a healthy practice that is encouraged. However, in the end, each person is expected to write their own solutions, in their own words, and in a way that reflects their own understanding.

Additional Instructions:

- *Each of the following calculations must follow the methods that were discussed in lecture. All choices for formulas, critical values, distributions, rounding rules, etc must the same as given in lecture.*
- *All calculations are to be completed using only your SHARP EL-510 calculator and the* STAT 260 Distribution Tables Booklet *on Brightspace. Do not use R or other software, and do not use other distribution tables from other books/online/etc.*
- *For all confidence intervals, give your answers in the interval* $[L_1, L_2]$ *form, rounded to 4 decimal positions.*
- *Don't forget to include units in your answers where appropriate.*

1. **[7 marks]** A potato chips factory receives large shipments of potatoes from farm distributors in semi-trailers. If the factory believes that more than 5% of the potatoes in a shipment are damaged (ex. bruised, scarred, etc), they will reject the entire trailer. A worker pulls a random sample of 125 potatoes from the trailer and finds that 8 are damaged. Is there sufficient evidence to suggest that the factory should reject the shipment? Perform the appropriate hypothesis test.

   (a) Using the correct notation, define the parameter of interest. Then, state the null and alternative hypotheses for this study in terms of the parameter.

   parameter of interest p = Population Proportion of damaged potatoes in the shipment.

   $H_0: p \leq 0.05$

   $H_1: p > 0.05$

   (b) Specify (with justification) the test statistic and state its distribution (Z or T, and degrees of freedom, if needed) it follows.

   $n = 125 \geq 30$ $\quad \hat{p} = \frac{8}{125}$

   $n\hat{p} = \cancel{125} \times \frac{8}{\cancel{125}} = 8 \geq 5$

   $n(1-\hat{p}) = \cancel{125} \times \frac{117}{\cancel{125}} = 117 \geq 5$

   $\therefore$ Test Statistic: $z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$

   (standard normal distribution (Z))

   (c) Compute the observed value of the test statistic.

   $n = 125,\ x = 8$

   $\hat{p} = \frac{8}{125} = 0.064$

   $p_0 = 0.05$

   Standard error(SE) $= \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.05 \times 0.95}{125}} = 0.0195$

   $Z_{obs} = \frac{\hat{p} - p_0}{SE} = \frac{0.064 - 0.05}{0.0195} = 0.7179 \approx 0.72$ $\quad \therefore Z_{obs} \approx 0.72$

   (d) Compute the p-value or provide a range of appropriate values for the p-value.

   $p = 1 - P(Z < Z_{obs})$

   p-value $= P(Z < 0.72)$

   p-value $= 1 - 0.7642$

   p-value $= 0.2358$

   (e) Using the p-value, determine the level of evidence against H_0. In a plain language sentence, state a conclusion for the study, and whether the factory should accept the shipment or not.

   Since the p-value (0.2389) is greater than the typical significance level, we do not have sufficient evidence to reject the null hypothesis.

   $\therefore$ Little to no evidence against H_0

   Conclusion:

   There is not enough evidence to suggest that the proportion of damaged potatoes is more than 5%. Therefore, the factory should accept the shipment.

2. **[7 marks]** Samples of purple sea stars (*Pisaster ochraceus*) are taken from the North and Southern ends of Vancouver Island, at Port Hardy and Sooke, respectively, and their diameters (in cm) are recorded as below:

Sooke:	23.4	27.0	19.8	22.8	19.2	23.4			$\rightarrow \bar{x}_s = 22.6$, $s_s = 2.831$, $n=6$
Port Hardy:	26.4	24.7	28.1	28.4	19.8	29.2	23.4	27.7	$\rightarrow \bar{x}_p = 25.96$, $s_p = 3.170$, $n=8$

Assuming that purple sea star diameters are normally distributed, is there sufficient evidence to suggest that the mean diameter of Sooke purple sea stars is different than Porty Hardy purple sea stars at the $\alpha = 0.10$ significance level? Perform the appropriate hypothesis test.

(a) Using the correct notation, define the parameter of interest. Then, state the null and alternative hypotheses for this study in terms of the parameter.

parameters of interest: μ_s mean diameter of purple sea stars at Sooke

μ_p mean diameter of purple sea stars at Port Hardy

$H_0: \mu_s - \mu_p = 0$

$H_1: \mu_s - \mu_p \neq 0$

(b) Specify (with justification) the test statistic and state its distribution (Z or T, and degrees of freedom, if needed) it follows.

Test statistic: t-test $\rightarrow$ small sample size ($n_s, n_p < 30$)

Rule of Thumb $\rightarrow$ $\frac{s_{big}}{s_{small}} = \frac{3.170}{2.831} = 1.119 < 1.25 \rightarrow$ pooled t-test $\sigma_s = \sigma_p$ (assume)

$$T = \frac{(\bar{x}_s - \bar{x}_p) - \overbrace{(\mu_s - \mu_p)_0}^{H_0}}{\sqrt{\frac{(n_p-1)s_p^2 + (n_s-1)s_s^2}{n_p+n_s-2}\left(\frac{1}{n_p}+\frac{1}{n_s}\right)}}$$

$\nu = n_s + n_p - 2$

$\nu = 6+8-2$

$\nu = 12$

(c) Compute the observed value of the test statistic.

$$t_{obs} = \frac{(22.6 - 25.9625) - 0}{\sqrt{\frac{(8-1)(3.170^2) + (6-1)(2.831^2)}{8+6-2}\left(\frac{1}{6}+\frac{1}{8}\right)}}$$

$t_{obs} = -2.052$

(d) Compute the p-value or provide a range of appropriate values for the p-value.

$p\text{-value} = 2P(T_{12} < -|t_{obs}|)$

$= 2P(T_{12} < -2.052$

$= 2P(T_{12} > 2.052)$

$\rightarrow 0.025 < 2\ p\text{-value} < 0.05$

$\rightarrow 0.05 < p\text{-value} < 0.10$

(e) Using the p-value, state your the conclusion of the hypothesis test at the $\alpha = 0.10$ significance level. Then, give your conclusion as a plain language sentence that a non-statisican could understand.

Conclusion: $p\ (0.05, 0.10) < \alpha = 0.10$, we reject H_0

Plain Language Conclusion:

There is sufficient evidence to suggest that the mean diameter of purple sea stars at Sooke is different from those at Port Hardy.

3. **[6 marks]** Polymethylmethacrylate (PMMA) is a bone cement used in the surgical fixation of artificial joints. Surgical standard 20g packets of PMMA with and without 1 ml of contaminating blood mixed-in were extruded into cylindrical molds and allowed to set. After curing, the samples were tested for their flexural strength, and the results were recorded below:

Uncontaminated (PMMA without blood): $n_1 = 27, \quad \bar{x}_1 = 67.7$ MPa, $\quad s_1 = 12.3$ MPa

Contaminated (PMMA with blood): $n_2 = 16, \quad \bar{x}_2 = 59.9$ MPa, $\quad s_2 = 8.0$ MPa

It is reasonable to assume that the flexural strengths are approximately normally distributed.

Let μ_1 be the mean flexural strength of the uncontaminated PMMA in MPa.
Let μ_2 be the mean flexural strength of the contaminated PMMA in MPa.

(a) Calculate the 98% confidence interval for the true mean difference (Uncontaminated - Contaminated) in flexural strengths of the uncontaminated and the contaminated PMMA samples. Make sure to explicitly state which distribution (and degrees of freedom, if applicable) you used, and to justify your selection.

$(n_1, n_2 < 30)$, σ_1 and σ_2 are unknown → T-test

small samples

$\alpha = 0.02$

$\alpha/2 = 0.01$

Rule of Thumb → $\dfrac{s_{big}}{s_{small}} = \dfrac{12.3}{8.0} = 1.5375 > 1.4$ so we assume $\sigma_1 \neq \sigma_2$

Unpooled t-test

$$r = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = \frac{\left(\frac{12.3^2}{27} + \frac{8^2}{16}\right)^2}{\frac{\left(\frac{12.3^2}{27}\right)^2}{27-1} + \frac{\left(\frac{8^2}{16}\right)^2}{16-1}} = 40.55$$

$r = 40 \quad t_{40,0.01} = 2.423$

$$98\%\ CI: (67.7 - 59.9) \pm 2.423 \times \left(\sqrt{\frac{(12.3)^2}{27} + \frac{(8)^2}{16}}\right)$$

$$= 7.8 \pm 7.5087$$

$[0.2913 \text{ MPa}, 15.3087 \text{ MPa}]$ → Final CI

(b) Suppose that researchers want to repeat this study in future. Using the data above as a pilot study, determine the common sample size needed to construct a 90% confidence interval for the true mean differences in the flexural strengths, to within 2 MPa. Be sure to explicitly state how many of each sample type are required.

$\alpha = 0.1$

$\frac{\alpha}{2} = 0.05$

$$z_{\alpha/2} = z_{0.05} = \frac{-1.64 + (-1.65)}{2} = -1.645$$

$$n = \left(\frac{z_{\alpha/2}}{d}\right)^2 (s_1^2 + s_2^2)$$

$$= \left(\frac{-1.645}{2}\right)^2 (12.3^2 + 8^2)$$

$$= 145.64$$

So $n = 146$