Q1) Since $a|bc$ there exists an integer k such that $bc = ak$

$\gcd(a,b) = 1 \to$ co prime

since a, b are coprime:

$ax + by = 1$

$\to (ax + by) \times c = 1 \times c$

$\to a(cx) + b(cy) = c$

$\to b(cy)$ is a multiple of b and since $bc = ak$

$\therefore a(cx) + b(ak \cdot y) = c$

$\to$ since $b(ak+y)$ is a multiple of a we can say $a \cdot m$ for some integer m

$\therefore a(cx) + a \cdot m = c$

Since $a(cx+m) = c$ this implies $a|c$ $\square$

Q2) The elements are:

$\{(\emptyset, \{x\}), (\emptyset, \{y\}), (\emptyset, \{x,y\}), (\{x\}, \{x,y\}), (\{y\}, \{x,y\})\}$

Q3) a) The power set of S has $2^4 = 16$ elements

b)

Size	Binomial	Count
0	$\binom{4}{0}$	1
1	$\binom{4}{1}$	4
2	$\binom{4}{2}$	6
3	$\binom{4}{3}$	4
4	$\binom{4}{4}$	1

Alternatively you could list all elements and count them but this method is easier

c)

Size	Possibilities	Pair count
0	$(2^4-1) \times \text{count}(0) = 15 \times 1$	15
1	$(2^3-1) \times \text{count}(1) = 7 \times 4$	28
2	$(2^2-1) \times \text{count}(2) = 3 \times 6$	18
3	$(2^1-1) \times \text{count}(3) = 1 \times 4$	4
4	no B such that $A \subset B$	0
Total = 15+28+18+4 = 65		

Relation R has 65 elements.

Q4) Reflexivity:

$\to$ For xRx, $x \cdot x = 0$

This means $x^2 = 0 \to x = 0$

$\therefore xRx$ only holds if $x = 0$

i.e. xRx doesnt hold for all reals.

So the relation is not reflexive.

Symmetry:

If xRy, then $xy = 0$

For it to be symmetric, yRx, $yx = 0$.

Since $xy = yx$

Thus if $xy = 0$, $yx = 0$ automatically

$\therefore$ The relation is symmetric.

Transitivity:

Suppose xRy and yRz

This means $xy = 0$, $yz = 0$

if $x \cdot y = 0$ then $x = 0$ or $y = 0$

if $y = 0$ then $yz = 0$ holds for all z in $\mathbb{R}$

if $y = 0$ then xRz holds as $x \cdot z = 0$

and if $x = 0$ then xRz holds as we get $0 \cdot z = 0$

$\therefore xRy$ and yRz hold for both cases

$\therefore$ R is transitive

R is not reflexive
R is symmetric
R is transitive

Q5) Equivalence = Reflexive, Symmetric and Transitive

For any $(x,y) \in S$

$\to$ If $y = 0$ then $(x,y) R (x,y)$ as $y = 0$ and $t = 0$.

$\to$ If $y \neq 0$ then $(x,y) R (x,y)$ because $y \neq 0$ and $t \neq 0$ and $\frac{x}{y} = \frac{x}{y}$

So R is reflexive

Symmetric:

$\to$ If $y = 0$ and $t = 0$ then $(x,y) R (z,t)$ and $(z,t) R (x,y)$ both hold as the second component of both pairs is 0.

$\to$ If $y \neq 0$ and $t \neq 0$ and $\frac{x}{y} = \frac{z}{t}$, then $\frac{z}{t} = \frac{x}{y}$ holds so $(z,t) R (x,y)$

So R is symmetric

Transitivity:

$\to$ If $y = 0$, $t = 0$, and $v = 0$, then $(x,y) R (z,t)$ and $(z,t) R (u,v)$ imply $(x,y) R (u,v)$ as the second component of all pairs is 0.

$\to$ If $y \neq 0$, $t \neq 0$, and $v \neq 0$ and $\frac{x}{y} = \frac{z}{t}$ and $\frac{z}{t} = \frac{u}{v}$ then

$\frac{x}{y} = \frac{u}{v}$ so $(x,y) R (u,v)$

Thus R is Transitive

Since the relation on R is Reflexive, Symmetric and Transitive, R is an equivalence relation.

Q6) $xRy \longleftrightarrow |x-y| \leq 5$

Reflexive:

For xRx: $|x-x| = |0| = 0 \leq 5$ will always hold true.

$\therefore$ R is reflexive

Symmetry: R is symmetric if $xRy \to yRx$

For xRy: $|x-y| \leq 5 \to |y-x| = |x-y| \leq 5$

This condition is always true (by absolute value properties)

$\therefore$ R is symmetric

Transitive: xRy and $yRz \to xRz$

Consider xRz and yRz:

$|x-y| \leq 5$ and $|y-z| \leq 5$

check $\to |x-z| \leq 5$

$|x-z| \leq |x-y| + |y-z|$

given $|x-y| \leq 5$ and $|y-z| \leq 5$:

$|x-z| \leq 5+5 = 10$

$|x-z|$ could be greater than 10

$\therefore$ R is not transitive

R is Reflexive
R is Symmetric
R is not Transitive

Q7) For equivalence we need to check for Reflexivity, Symmetric and Transitivity all hold true.

Reflexive:

Proof $\to$ check if $a + a$ = even

$a + a = 2a$

$2a \to$ multiple of 2 so always even.

$a \sim a$ holds for all $a \in \mathbb{Z}$

$\therefore \sim$ is Reflexive

Symmetric:

Assume $a \sim b$, by definition this means $a+b$ is even

If $a+b$ is even $b+a$ is also even.

$\therefore b \sim a$ if $a \sim b$

$\therefore \sim$ is symmetric

Transitive:

Relation $\sim$ is transitive if for all $a, b, c \in \mathbb{Z}$ whenever $a \sim b$ and $b \sim c$ then $a \sim c$.

Proof:

Assume $a \sim b$ and $b \sim c$

By definition:

$a+b$ and $b+c$ is even

Since they're even $\to$

$a+b = 2m$, $b+c = 2n$ for some integers m and n.

$(a+b) + (b+c) = 2m + 2n$ $a + 2b + c = 2(m+n)$

$a+c = 2(m+n) - 2b$

$\to 2(m+n-b)$

Since $2(m+n-b)$ is even (multiple of 2) then $a+c$ is even.

$\therefore a \sim c$

$\therefore \sim$ is transitive

Since the relation $\sim$ on $\mathbb{Z}$ satisfies reflexivity, symmetry, transitivity, it is an equivalence relation.