Q1 (10 points)

Oil is being pumped into a tank over a 12-hour period. The tank contains 120 gallons of oil initially (at time $t = 0$). The rate at which the oil is flowing into the tank at various times is modeled by $R(t) = 8.9 - 1.05t$ for $0 \le t \le 8$, where t is measured in hours and $R(t)$ is measured in gallons per hours.

(a) Using midpoints, estimate the gallons of oil in the tank at $t = 8$ hours using $n = 5$ subintervals.

(b) Graph an R vs t graph and use geometry to determine the true value of the oil in the tank at $t = 8$ hours. Is this the same as (a)? If so, why? Would this work for any other shape than a line?

a) $R(t) = 8.9 - 1.05t$

$\Delta t = \frac{(b-a)}{n} \quad \therefore \quad \frac{8-0}{5} = 1.6$

Since we know $n=5$ we use the ith midpoint formula $a + (i - 0.5) \times \Delta t$

a would be 0 as its the left endpoint of the entire interval

i is the subinterval so 1, 2, 3 ... n

There fore:

$t\_1 = 0 + (1 - 0.5) \times 1.6 = 0.8$

$t\_2 = 0 + (2 - 0.5) \times 1.6 = 2.4$

$t\_3 = 0 + (3 - 0.5) \times 1.6 = 4.0$

$t\_4 = 0 + (4 - 0.5) \times 1.6 = 5.6$

$t\_5 = 0 + (5 - 0.5) \times 1.6 = 7.2$

Now we evaluate the function at each point:

$R(t) = 8.9 - 1.05(t\_n)$

$t\_1 = 8.06$

$t\_2 = 6.38$

$t\_3 = 4.70$

$t\_4 = 3.02$

$t\_5 = 1.34$

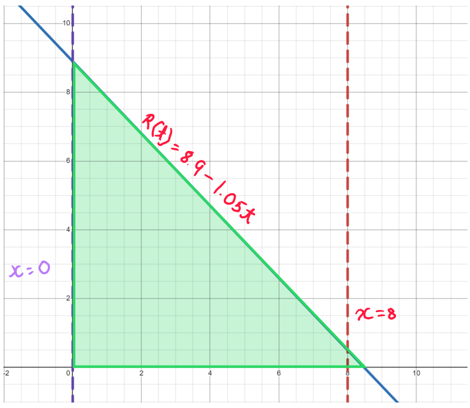
$\therefore$ Reimann Sum $= 1.6 \times (8.06 + 6.38 + 4.7 + 3.02 + 1.34)$

$= 1.6 \times 23.50$

≈ 37.60 gallons

The final answer then would be $120 + 37.6 = 157.6$ gallons. at $t = 8$ hours.

b)



Finding the area of the shaded region or the trapezoid:

Area of trapezoid $= (b1 + b2) \times \frac{h}{2}$

$b1 = R(0) = 8.9 - 1.05(0)$

$b1 = 8.9$

$b2 = R(8) = 8.9 - 1.05(8)$

$b2 = 0.5$

h = time which is given as 8

$\therefore$ Area of trapezoid $= (8.9 + 0.5) \times \frac{8}{2}$

$= 37.6$ gallons

Add this to the initial value of 120 gallons and we get 157.6 gallons.

Our answer found using the graph of R vs t is the same as the answer found in part a. Since we had a linear function, finding its area under the curve is relatively easy. Thus, for a non-linear function using the midpoint rule rather than the geometric method would be easier and likely more accurate. To summarise the method may work for shapes other than lines but it would be rather tedious to use