MATH 202 Midterm 1 Supplementary Questions

October 3, 2024

Question 1. Find the combination of inequalities to describe the following solid in 3D: the solid ball of radius 4 centered at (0,0,0) with the part below the plane z=2 (excluding the plane) removed.

Question 2. Let $\vec{u} = \vec{i} + 2\vec{j} + \vec{k}$ and $\vec{v} = \vec{k}$ be vectors in 3D. Find the length of $\vec{u} + \vec{v}$.

Question 3. Let $L: \vec{r}(t)$ be the line satisfying the following two conditions: (1) It is perpendicular to both $\vec{u} = \langle 1, 1, 1 \rangle$ and $\vec{v} = \langle 1, 2, 1 \rangle$, and (2) it passes through (2, -1, -1). Find $\vec{r}(t)$.

Question 4. Find the domain and the range of the function

$$z = f(x, y) = \sqrt{x + y - 2}.$$

Question 5. Find $\lim_{(x,y)\to(0,0)} \frac{3x^3}{2x^2+2y^2}$.

Question 6. Let x = st, $y = s^2t$, and w = f(x, y) be such that $\partial w/\partial x = e^y$ and $\partial w/\partial y = xe^y$. Find $\partial w/\partial s$ and $\partial w/\partial t$.

Question 7. Find parametric equations for the line in 3D that is perpendicular to the graph of the equation $x^2 + 2y^2 + 3z^2 = 6$ at the point (1, 1, 1).

Answers

Question 1. $x^2 + y^2 + z^2 \le 4^2$ and $z \ge 2$.

Question 2. 3.

Question 3. $\vec{r}(t) = \langle 2, -1, -1 \rangle + t \langle -1, 0, 1 \rangle, -\infty < t < \infty.$

Question 4. The range is $z \ge 0$. The domain is $x + y \ge 2$.

Question 5. 0.

Question 6. $\partial w/\partial s = te^{s^2t} + 2s^2t^2e^{s^2t}, \ \partial w/\partial t = se^{s^2t} + s^3te^{s^2t}.$

Question 7. $x = 1 + 2t, y = 1 + 4t, z = 1 + 6t, -\infty < t < \infty.$