# MATH 202 Midterm 2
# Supplementary Questions

November 1, 2024

**Question 1.** Consider the DE: $\dfrac{\mathrm{d}p}{\mathrm{d}t} = -p^2 + 4p - 3$.

(1) If $p(0) = 4$, can $p(t)$ converge to 1 as $t \to \infty$?

(2) If $p(0) = 2$, find $\lim\limits_{t\to\infty} p(t)$ and $\lim\limits_{t\to-\infty} p(t)$.

**Question 2.** Consider the DE $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{9x}{y}$ without solving it.

(1) Show that the two lines $y = \pm 3x$ are solutions curves.

(2) Sketch a directional field by putting line segments at the following 4 points in the $x$-$y$ plane: $(1, 1)$, $(1, 2)$, $(2, 1)$, $(2, 2)$. Justify your sketch by calculating the slopes of the line segments.

**Question 3.** Consider the first-order DE: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \cos(x)y^2$.

(1) Determine *all* the types of first-order equation (separable, linear, exact, Bernoulli, or homogeneous) that this DE satisfies.

(2) Given the initial condition $y(0) = 1$, solve the initial value problem.

**Question 4.** Find the general solutions of the following DE:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y + \mathrm{e}^{3x}.$$

**Question 5.** Find the general solutions of the following DE:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{3t^2x + tx^2}{t^3 + t^2x}.$$

**Question 6.** Find $p(D)$ such that the following second-order equation can be represented in the form of $p(D)y = 0$:

$$2024y'' + 11y' + y = 0.$$

# Answers

**Question 1.** (1) The solution cannot converge to 1, since it is decreasing to the limit 3 as $t \to \infty$.

(2) $\lim_{t\to\infty} p(t) = 3$ and $\lim_{t\to-\infty} p(t) = 1$. ■

**Question 2.** (1) For $y = \pm 3x$, $\mathrm{d}y/\mathrm{d}x = \pm 3$ and $9x/y = 9x/(\pm 3x) = \pm 3$. Hence, $\mathrm{d}y/\mathrm{d}x = 9x/y$

(2) The slopes at $(1,1)$, $(1,2)$, $(2,1)$, $(2,2)$ are given by $9, 9/2, 18, 9$, respectively. ■

**Question 3.** The equation is separable and Bernoulli. The solution is given by $y(x) = \dfrac{1}{1-\sin(x)}$. ■

**Question 4.** $y(x) = C\mathrm{e}^x + \mathrm{e}^{3x}/2$. ■

**Question 5.** $t^3x + \frac{1}{2}t^2x^2 = C$. ■

**Question 6.** $p(D) = 2024D^2 + 11D + 1$. ■