MATH 202 Final Exam
Supplementary Questions

December 2, 2024

The final exam will be cumulative. It will cover everything taught in the semester, and there will be an emphasis on the topics taught after Midterm 2. This document gives only questions on those new topics.

Question 1 (Second-order linear constant-coefficient differential equations by variation of parameters). Consider the differential equation $y'' - y = \dfrac{\mathrm{e}^x + \mathrm{e}^{-x}}{2}$ for $y = y(x)$.

(1) Find the complementary solutions y_c.

(2) Find the general solutions.

Question 2 (Second-order linear variable-coefficient equations). Solve the equation $y'' + \dfrac{6}{x}y' + \dfrac{4}{x^2}y = 0$ for $y = y(x)$ and $x > 0$.

For the next two questions, use the formulas

$$\mathscr{L}\{t^n\}(s) = \frac{n!}{s^{n+1}}, \quad \mathscr{L}\{\cos(\theta t)\}(s) = \frac{s}{s^2+\theta^2}, \quad \mathscr{L}\{\sin(\theta t)\}(s) = \frac{\theta}{s^2+\theta^2},$$

the derivative rule $\mathscr{L}\{f(t)t^n\}(s) = (-1)^n \dfrac{\mathrm{d}^n}{\mathrm{d}s^n}\mathscr{L}\{f(t)\}(s)$, or the translation rule $\mathscr{L}\{f(t)\}(s-a) = \mathscr{L}\{\mathrm{e}^{at}f(t)\}(s)$.

Question 3 (Derivation of Laplace transforms). Find the Laplace transform of the function $f(t) = \sin(2t) + \mathrm{e}^{10t}$.

Question 4 (Inverse Laplace transforms). Find the following inverse Laplace transform:

$$\mathscr{L}^{-1}\left\{\frac{8s+3}{(s-2)^2+16}\right\}(t).$$

Question 5 (Solving differential equations by Laplace transforms). Solve the following IVP by using Laplace transforms: $y = y(t)$ such that $y'' + y = \mathrm{e}^t$ subject to the initial conditions $y(0) = 0$ and $y'(0) = 1$.

Answers

Question 1. (1) $y_c = C_1 e^x + C_2 e^{-x}$. (2) $y = C_1 e^x + C_2 e^{-x} + \frac{1}{2} x \left(\frac{e^x - e^{-x}}{2} \right)$. $\square$

Question 2. $y = \frac{C_1}{x^4} + \frac{C_2}{x}$. $\square$

Question 3. $\frac{2}{s^2 + 4} + \frac{1}{s - 10}$. $\square$

Question 4. $e^{2t} \left(8 \cos(4t) + \frac{19}{4} \sin(4t) \right)$. $\square$

Question 5. $y = -\frac{1}{2} \cos t + \frac{1}{2} \sin t + \frac{e^t}{2}$. $\square$