## STAT 260 Summer 2024: Written Assignment 4

Due: Upload your solutions to Crowdmark BEFORE 6pm (PT) Friday June 14.

You may upload and change your files at any point up until the due date of Friday June 14 at 6pm (PT).

A 2% per hour late penalty will be automatically applied within Crowdmark. The penalty is applied in such a way so that assignments submitted 6pm to 6:59pm will have 2% deducted, assignments submitted 7pm-7:59pm will have 4% deducted, etc.

Note that if you submit any portion of your assignment before the deadline, Crowdmark will NOT permit you to edit your submission (including make additional uploads) after the 6pm deadline passes. This means that if, for example, you upload only Question 1 before the deadline, you will not be able to upload Question 2 after the deadline. If you intend to submit late (with penalty) you must submit the entire assignment late.

**Submission:** Solutions are to be uploaded to Crowdmark. Here you will be asked to upload your solutions to each question separately. Your solution to Question 1 must be uploaded in the location for Question 1, your solution to Question 2 must be uploaded in the location for Question 2, etc. If your work is uploaded to the wrong location, the marker will not be able to grade it.

You may hand-write your solution on a piece of paper or tablet. If you wish to use this question sheet and write your solutions on the page, space has been provided below. One of the quickest ways to upload work is by accessing Crowdmark from within a web browser on a smartphone. In the area where you upload work, press the "+" button. This will give you the option of using a file already on your phone, or you can use the phone camera to photograph your work. If you complete your work on a tablet, save the file as a PDF or each question as a jpeg and drag/drop the file into the Crowdmark box. ***Photographs of laptop/tablet screens will not be graded***; take a proper screenshot.

**Instructions:** For full marks, your work must be neatly written, and contain enough detail that it is clear how you arrived at your solutions. ***You will be graded on correct notation***. Messy, unclear, or poorly formatted work may receive deductions, or may not be graded at all. Only resources presented in lecture or linked to on the Stat 260 Brightspace page are permitted for use in solving these assignments; using outside editors/tutors, and/or software (include AIs) is strictly forbidden. Talking to your classmates about assigned work is a healthy practice that is encouraged. However, in the end, each person is expected to write their own solutions, in their own words, and in a way that reflects their own understanding.

### Additional Instructions:

*For each of the following questions, include the correct notation for the random variable that you are calculating in your solution, not just the numeric answer. For example: "$f(3) = P(X = 3) = 0.157$". Don't forget to include units where appropriate.*

1. **[7 marks]** A small ferry can carry up to 8 cars across a lake. The number of cars on board at a sailing $X$ is given by the following cumulative probability distribution function (CDF). Note that the ferry keeps to a tight schedule and will still sail even with 0 cars on board, as it needs to reach the other side of the lake for the return trip.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $F(x)$ | 0.05 | 0.14 | 0.22 | 0.38 | 0.67 | 0.84 | 0.92 | 0.96 | 1 |

*Answer the following questions using* ***only*** *the CDF. No marks will be given for making/using a pmf.*

(a) What is the probability that a sailing has 2-6 cars on it (that is, at least 2 and at most 6 cars)?

$F(6) - F(1) = 0.92 - 0.14$

$= 0.78$

(b) Suppose that the ferry allows for optional advanced reservations. If this morning's sailing has 5 reservations (and assuming none of the reservations cancel), what is the probability that there are an even number of cars on the sailing?

Since 5 are confirmed even no's above 5 are 6 and 8

$\therefore P(x=6)$ and $P(x=8)$

$\rightarrow P(x=6) = F(6) - F(5) = 0.92 - 0.84 = 0.08$

$\rightarrow P(x=8) = F(8) - F(7) = 1 - 0.96 = 0.04$

So $P(x=6) + P(x=8) = 0.08 + 0.04$

$= 0.12$

(c) If we look back at the ferry's records for 10 random sailings, what is the probability that at least one of those 10 sailings had no cars on it? *Assume that the numbers of cars on random sailings are independent of one another.*

$= 1 - (1 - P(x=0))^{10}$

$= 1 - (1 - 0.05)^{10}$

$= 1 - 0.95^{10}$

$\approx 0.4013$

2. [**7 marks**] A locksmith offers smart-lock installations on doors. For a commercial building, the number of exterior doors $X$ that the locksmith is hired to install smart-locks on is given by the CDF below:

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $F(x)$ | 0.14 | 0.68 | 0.86 | 0.93 | 0.98 | 1 |

(a) What is the expected value and variance of $X$? *Include the appropriate units in your solution.*

| $P(x=1)$ | $P(x=2)$ | $P(x=3)$ | $P(x=4)$ | $P(x=5)$ | $P(x=6)$ |
|---|---|---|---|---|---|
| 0.14 | 0.54 | 0.18 | 0.07 | 0.05 | 0.02 |

$E(x) = 1(0.14) + 2(0.54) + 3(0.18) + 4(0.07) + 5(0.05) + 6(0.02)$

$= 0.14 + 1.08 + 0.54 + 0.28 + 0.25 + 0.12$

$= 2.41$ exterior doors

Variance $V(X) = E(x^2) - E(x)^2$

| $E(1^2)$ | $E(2^2)$ | $E(3^2)$ | $E(4^2)$ | $E(5^2)$ | $E(6^2)$ |
|---|---|---|---|---|---|
| 0.14 | 2.16 | 1.62 | 1.12 | 1.25 | 0.72 |

$E(x^2) = 0.14 + 2.16 + 1.62 + 1.12 + 1.25 + 0.72 = 7.01$

$V(x) = 7.01 - (2.41)^2$

$V(x) = 7.01 - 5.8081$

$V(x) = 1.2019$ external doors$^2$

(b) The locksmith charges an initial \$100 consultation fee for each commercial building that they work on. However, the locksmith reduces the fee by \$15 for each smart-lock that they install at that building. So, if they install 3 smart-locks, the fee is reduced down to \$55. Let the random variable $Y$ be the adjusted consultation fee that the locksmith charges for each commercial building. Determine the expected value, variance, and standard deviation of $Y$.
*Include the appropriate units in your solution.*

$E(Y) = E(100 - 15x) = 100 - 15E(x)$

$= 100 - 15(2.41)$

$= \$63.85$

$Var(Y) = Var(100 - 15x) = (-15)^2 \times Var(x)$

$= 225 \times 1.2019$

$= 270.4275$ dollars$^2$

$SD(Y) = \sqrt{Var(Y)}$

$= \$16.45$