$$\int x\cos^2(3x)\,dx$$

Rewritten using: $\cos^2\theta = \frac{1+2\cos\theta}{2}$

$$\rightarrow \frac{1}{2}\,x\,(1+\cos(6x))\,dx$$

$$\rightarrow \frac{1}{2}\cdot\int x(1+\cos 6x)\,dx$$

$$\rightarrow \frac{1}{2}\int x\,dx + \frac{1}{2}\int (x\cos 6x)\,dx$$

$$\rightarrow \int x\,dx = \frac{x^2}{2} + C$$

$$\rightarrow \int \underset{u}{x}\cos\underset{dv}{(6x)} \qquad \text{L I A T E}$$

$$\rightarrow u = x \qquad dv = \cos(6x)$$
$$du = 1\,dx \qquad v = \frac{\sin(6x)}{6}$$

$$\rightarrow uv - v\int du$$

$$= x\left(\frac{\sin 6x}{6}\right) - \frac{\sin 6x}{6}\cdot 1\,dx$$

$$= x\left(\frac{\sin(6x)}{6}\right) + \frac{\cos 6x}{36} + C$$

$$\frac{1}{2}\int x\,dx + \frac{1}{2}\int x\cos 6x\,dx = \frac{1}{2}\left(\frac{x^2}{2}\right) + \left(x\left(\frac{\sin(6x)}{6}\right) + \frac{\cos 6x}{36}\right) + C$$

$$= \frac{x^2}{4} + \frac{x\sin(6x)}{12} + \frac{\cos(6x)}{72} + C$$

$$= \frac{1}{72}\left(18x^2 + 6x\sin(6x) + \cos(6x)\right) + C$$

= Final Answer