Q1) Statement P "It is raining" Statement Q "The ground is wet"

"If it is raining then the ground is wet"
$P \to Q$ will always be true

"If the ground is wet, it is raining"
$Q \to P$ need not necessarily be true since the ground can be wet due to numerous other reasons such as a pipe leak for example

Q2) 1) $p \to \neg p$ can be replaced with $\neg p \vee \neg p$ $\therefore$ $p \wedge (\neg p \vee \neg p)$

2) Distributive Law: $p \wedge (\neg p \vee \neg p) = (p \wedge \neg p) \vee (p \wedge \neg p)$

3) As per law of contradiction, $p \wedge \neg p$ is always 0 or false

$\therefore (p \wedge \neg p) \vee (p \wedge \neg p) \Leftrightarrow 0 \vee 0$, law of disjunction $0 \vee 0 \Leftrightarrow 0$

Q3)

p	q	r	$p \to \neg q$	$r \to q$	$\neg(p \wedge r)$
1	1	1	0	1	0
1	1	0	0	1	1
1	0	1	1	0	0
1	0	0	1	1	1
0	1	1	1	1	1
0	1	0	1	1	1
0	0	1	1	0	1
0	0	0	1	1	1

Q4) Given premises: $p \to \neg q$
$r \to q$

Proof:

1) Assuming $p \wedge r$

2) Using above assumption, if p is true we use the given premise $p \to \neg q$ to derive $\neg q$ as true as well using modus ponens

3) Similarly if r is true, given premise $r \to q$ we use modus ponens to derive that q must be true given that r and $r \to q$ both are true

4) Now that we have q and $\neg q$ we know that this is not possible as it is a contradiction. Since the assumption $p \wedge r$ leads to a contradiction, we conclude that $\neg(p \wedge r)$ must be true therefore the argument is logically valid.

Q5) $n^3 + 2n + 6$ is a multiple of 3 for all integers n.

Universe $= n$

Case 1 $n = 3k$:

$(3k)^2 + 2(3k) + 6$

$\therefore 27k^3 + 6k + 6$

$\therefore 3(9k^3 + 2k + 2)$

Since we factored 3 out, this case clearly shows that the statement is a multiple of 3 given $n = 3k$.

Case 2 $n = 3k+1$:

$(3k+1)^3 + 2(3k+1) + 6$

$\therefore 27k^3 + 27k^2 + 9k + 1 + 6k + 2 + 6$

$\therefore 27k^3 + 27k^2 + 15k + 9$

Factoring 3 out: $3(9k^3 + 9k^2 + 5k + 3)$

this shows that for case 2, the statement is true.

Case 3 $n = 3k+2$:

$(3k+2)^3 + 2(3k+2) + 6$

$\therefore 27k^3 + 54k^2 + 36k + 8 + 6k + 4 + 6$

$\therefore 27k^3 + 54k^2 + 42k + 18$

Factoring 3 out: $3(9k^3 + 18k^2 + 14k + 6)$

In all three cases we show that $n^3 + 2n + 6$ is a multiple of 3. Therefore the statement $n^3 + 2n + 6$ is a multiple of 3 for all integers n.

Q6) We apply De Morgan's Law to the entire expression.

$\neg(p \wedge ((\neg q \vee r) \wedge (s \vee t))) \Rightarrow \neg p \vee \neg((\neg q \vee r) \wedge (s \vee \neg t))$

Step 2 is De Morgan's Law again:

$\neg p \vee \neg((\neg q \vee r) \wedge (s \vee \neg t)) \Rightarrow \neg p \vee (\neg(\neg q \vee r) \vee \neg(s \vee \neg t))$

Step 3 De Morgan's Law again:

$\neg p \vee (\neg(\neg q \vee r) \vee \neg(s \vee \neg t)) \Rightarrow \neg p \vee ((q \wedge \neg r) \vee (\neg s \wedge t))$

$\therefore \neg(p \wedge ((\neg q \vee r) \wedge (s \vee \neg t))$ is logically equivalent to $\neg p \vee ((q \wedge r) \vee (\neg s \wedge t))$.

Q7) part 1) $p, q, r, s, t, u, v, w, x, y, z$ = 11 statements

$\therefore$ 2^11 lines or 2048 lines should be there in the truth table

part 2) The first half or 1024 lines are where p is true or 1. Since 811 is within this half, p is true.

$\therefore$ we can find that

p	q	r	s	t	u	v	w	x	y	z
1	0	1	0	0	1	0	0	1	0	1
2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

Q8) Taking the following values for p, q, r, s:

p = True q = False r = False s = False

Premises:

$q \to p$ = False $\to$ True = True

$r \to s$ = False $\to$ False = True (free pass)

$p \vee s$ = True $\vee$ False = True

$\neg r \vee \neg s$ = True $\vee$ True = True

Conclusion $q \vee r$ = False $\vee$ False = False

The counterexample of $p = 1$, $q = 0$, $r = 0$, $s = 0$ makes all the premises true but conclusion false thus the argument is invalid as expected of the question