Q1 (10 points)

The integrals

$$\int \frac{7\ln(x)}{x\left((\ln(x))^2+9\right)}dx \text{ and } \int \frac{7}{x\left((\ln(x))^2+9\right)}dx$$

while similar in appearance have solutions that look quite different.

(a) Evaluate the first integral.

(b) Evaluate the second integral.

a) Taking constant 7 out: $7 \cdot \int \frac{\ln(x)}{x((\ln(x))^2+9} dx$

Let $u = \ln(x)$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x} dx$ $\rightarrow 7 \cdot \int \frac{u}{\cancel{x}(u^2+9)} \cancel{x}\, du \rightarrow 7\int \frac{u}{u^2+9} du$

$dx = x\, du$

$\rightarrow$ Now we take u^2+9 as v

$\therefore \frac{dv}{du} = 2u$

$dv = 2u\, du$

$du = \frac{dv}{2u}$

$\rightarrow 7\int \frac{u}{v} \cdot \frac{dv}{2u} = 7\int \frac{1}{2} \cdot \frac{dv}{v} = \frac{7}{2}\int \frac{dv}{v}$

$= \frac{7}{2} \ln|v| + c$

$\therefore \frac{7}{2} \ln|u^2+9| + c$

$\therefore \frac{7}{2} \ln|(\ln(x))^2+9| + c$ $\rightarrow$ Final answer to part a)

b) Taking 7 out again $\rightarrow 7 \cdot \int \frac{1}{x((\ln(x))^2+9)} dx$

$\rightarrow u = \ln(x)$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x} dx$

$dx = x\, du$

$\rightarrow 7 \cdot \int \frac{1}{\cancel{x}(u^2+9)} \cancel{x}\, du \rightarrow 7\int \frac{1}{u^2+9} du$

$\rightarrow \int \frac{du}{a^2+u^2} = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + c$

$\rightarrow$ Applying the standard integral:

($a = 3$ since $3^2 = 9$)

$7 \cdot \int \frac{1}{3^2+u^2} du = \frac{7}{3} \arctan\left(\frac{u}{3}\right) + c$

Final Answer for part b) $\therefore \frac{7}{3} \arctan\left(\frac{\ln(x)}{3}\right) + c$