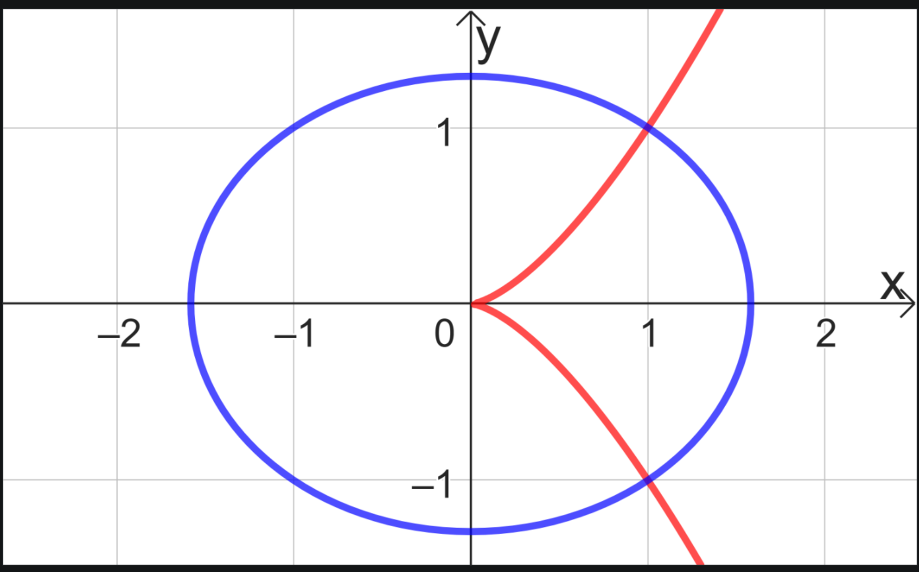
Q1 (10 points)

Consider the curves $y^2 = x^3$ and $2x^2 + 3y^2 = 5$ which are graphed below.



(a) Find the equation of the line tangent to $2x^2 + 3y^2 = 5$ at $(1, -1)$.

Write your tangent line in the form $y = mx + b$ and simplify the appropriate values (without approximating).

(b) Find the equation of the line tangent to $y^2 = x^3$ at $(1, -1)$.

Write your tangent line in the form $y = mx + b$ and simplify the appropriate values (without approximating).

(c) What is the geometric relationship between the tangent lines obtained in (a) and (b)? Justify your answer using the values of the slopes in parts (a) and (b).

a) $2x^2 + 3y^2 = 5$

derivative → $4x + 6y = 0$

→ $\frac{4\cancel{x}}{6y}$ $\frac{2x}{3y}$

→ $m = \frac{2(1)}{3(-1)}$

→ $m = -\frac{2}{3}$

∴ $y = mx + c$

→ $-1 = -\frac{2}{3}(1) + c$

→ $c = -\frac{5}{3}$

→ $y = \frac{2}{3}x - \frac{5}{3}$ at $(1, -1)$

b) $y^2 = x^3$

∴ $2y\frac{dy}{dx} = 3x^2$

$\frac{dy}{dx} = \frac{3x^2}{2y}$

$m = \frac{3(1)^2}{2(-1)}$

$m = -\frac{3}{2}$

∴ $y = mx + c$

$c = -\frac{3}{2}(1) + 1$

$c = \frac{1}{2}$

∴ $y = -\frac{3}{2}x + \frac{1}{2}$ at point $(1, -1)$

c) The x-intercept is the same for the tangent lines found in parts a and b. Their slopes are inverse of each other as seen below

a) $\frac{2}{3}$ b) $-\frac{3}{2}$

To confirm our answers and whether they intercept or not we take the product of the slopes which is -1 thus we can confirm that the tangent lines intersect at 90° at point $(1, -1)$ which can be seen in the graph as well.