

Set 24: Confidence Intervals for a Binomial Proportions

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Consider the following common Binomial Distribution problem:

Suppose that 10% of all raccoons suffer from hypertension. In a collection of 20 random raccoons, what is the probability that exactly 3 suffer from hypertension?

The **sample proportion** \hat{p} is

$$\hat{p} = \frac{\# \text{ of objects in a sample with a trait}}{\text{sample size}} =$$

- \hat{p} is a _____ for p , the population proportion.

Example 1: What proportion of people are left-handed?

Theorem: For large sample sizes, the sample proportion $\hat{p} = \frac{X}{n}$:

- is approximately normally distributed,
- has expected value p and variance $p(1 - p)/n$.

The **confidence interval** for p , the true population proportion is:

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$[L_1, L_2] = \left[\hat{p} - z_{\alpha/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}, \hat{p} + z_{\alpha/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \right]$$

Example 2: In a survey of the eating habits of 300 random Canadians, 28 responded that they followed a vegetarian diet. Find the 95% confidence interval for p , the true population proportion of Canadians following a vegetarian diet.

Extra Example 1: Suppose that researchers want to determine the population proportion of North Pacific right whales (*E. japonica*) that bear strike scars from ship propellers. In their study, they observe 45 North Pacific right whales, of which 7 have ship propeller scars. Determine the 90% confidence interval for p , the true population proportion of North Pacific right whales with propeller scars.

Ans: [0.067, 0.244]

Extra Example 2: It is known that 7.5% of Canadians suffer from at least one food allergy. In an elementary school with 250 students, what is the probability that at least 10% of students have food allergies?

Ans: 0.0668

Sample Size for Estimating p

The sample size need to construct a $(1 - \alpha)100\%$ for p within margin of error d is give by:

Case 1: When a point estimate \hat{p} is known:

Case 2: When no estimate \hat{p} is given:

Example 3: You create the premier goat-management app. Before gaining investors, you need to determine what proportion of smartphone-users have goats. How many smartphone owners must you survey if you want to be 99% confident that the sample proportion is within 0.02 of the true population proportion p if:

(a) there is no information available to estimate p ?

(b) we find from a previous study that 8% of smartphone-users own goats?

Readings: Swartz 6.1.2 [EPS 5.10] **Practice problems:** EPS 5.39, 5.41, 5.43, 5.45, 5.47, 5.49

Devore 7ed: Read pg 265 - 267 [we use the simpler formula]. Problems §7.2: 19, 21, 23, 25