Q1 (10 points)

The Cosine Fresnel Function is an integral function commonly used in optics and geometry. It is given by

$$C(x) = \int_0^x \cos(t^2)\,dt$$

This function has no elementary antiderivative (i.e. no alternative algebraic expression).

Answer the following.

(a) Consider the function $h(x)$ defined by $h(x) = e^{-x^2} C(x^3)$. Determine the derivative $h'(x)$.

(b) Estimate the area under the curve $y = C(x)$ on the interval $[-1, 1]$ by using 4 subintervals and Simpson's Rule. Round your answer to five decimals.

a) $h'(x)$ is $\frac{d}{dx}[u(x)v(x)] = u'(x)\,v(x) + u(x)\,v'(x)$

let $u(x) = e^{-x^2}$ and $v(x) = C(x^3)$

$u'(x) = 2x\,e^{-x^2}$ and $v'(x)$: chain rule below

$v'(x) \downarrow$

since $C(x) \Rightarrow \int_0^x \cos(t^2)dt$ we take $f(g(x))$ as $C(g(x))$

where $g(x) = x^3$

Step1: finding $\frac{dg}{dx} \rightarrow 3x^2$

Step2: finding $\frac{df}{dg} \rightarrow \cos(g^2)$

Substitute $g(x)$ back $\rightarrow \cos(x^6)$

Step3: Chain rule $\downarrow$

$$\frac{dv}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx} = \cos(x^6) \cdot 3x^2$$

Therefore: $v'(x) = 3x^2 \cos(6x)$

Since we have $u(x)$, $u'(x)$, $v(x)$ and $v'(x)$ we use product rule to find $h'(x)$

$\therefore h'(x) = -2x e^{-x^2} C(x^3) + e^{-x^2} 3x^2 \cos(x^6)$

b) interval $[-1, 1]$ and subintervals 4

$\therefore \frac{b-a}{n} = \frac{1-(-1)}{4} = \frac{2}{4} = 0.5$

given $\Delta x = 0.5$ we can calculate rest of the values.

$x_0 = a = -1$ (starting point) $= C(1) = \cos(1) \approx 0.54030$

$x_1 = x_0 + 0.5 = -1 + 0.5 = -0.5 = C(-0.5) = \cos(1/4) \approx 0.96891$

$x_2 = x_1 + 0.5 = -0.5 + 0.5 = 0 = C(0) = \cos(0) \approx 1.00000$

$x_3 = x_2 + 0.5 = 0.5 = C(0.5) = \cos(1/4) \approx 0.96891$

$x_4 = x_3 + 0.5 = 0.5 + 0.5 = 1$ (ending point) $= C(1) = \cos(1) \approx 0.54030$

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$$

$= \cos(1) + 4\cos(1/4) + 2\cos(0) + 4\cos(1/4) + \cos(1)$

$\approx 0.54030 + 3.87565 + 2 + 3.87565 + 0.54030$

≈ 1.80532