Set 4: Probability and Set Theory

Stat 260 A01: May 17, 2024

Definitions:

(a) An ***experiment*** is an activity with an observable or measurable outcome.

e.g. survey students for their birth month

(b) An ***outcome*** is one possible result of the experiment.

e.g. "March" or "December" etc

Outcomes must be mutually exclusive, essentially saying that no two outcomes can occur at the same time. March or December are outcomes however March AND December are two thus are not considered an outcome

(c) The ***sample space*** of an experiment is the <u>set</u> of all possible outcomes of the experiment.

capital → $S = \{\overbrace{\text{January}, \text{Feburary}, \dots, \text{December}}\}$ → they do not have an order inside brackets this is just by chance ordered

use curly brackets

seperated by commas

for any set, ignore duplicates e.g. {Jan, Jan, Feb} <=> {Jan, Feb}

(d) An ***event*** is a <u>subset</u> of the sample space.

$E_1 = \{\text{January}, \text{February}, \text{March}\}$ → subset of S → Event represented is that a student is born in Quarter 1 (Q1)

compound event = multiple outcomes

$E_2 = \{\text{July}\}$ → Represents the event that a student is born in july

simple event = one outcome

Constructing Probabilities

We have several options for constructing probabilities:

- **Classical Approach:** used when all outcomes are equally likely.

→ Theoretical or perfect world probability

The bars | | represent unique outcomes

$$\underbrace{P(E_1)}_{\text{Probability of } E_1 \text{ occuring}} = \frac{|E_1|}{|S|} = \frac{\text{num of unique outcomes in } E_1}{\text{num of unique outcomes in } S} = \frac{3}{12} = \frac{1}{4}$$

$$P(E_2) = \frac{|E_2|}{|S|} = \frac{1}{12}$$

- **Frequency Approach:** Counts the number of times an event occurs in many, many observations.

→ Experimental / observational Approach

$$P(A) = \lim_{n \to N} \frac{\text{num of occurences of } A}{n} \approx \frac{f}{n}$$

f → frequency

n → sample size

$$P(E_2) = \frac{7}{75}$$ → numbers taken in class for people born in july by total class size

In both models, the probability of an event is the sum of the probability of its outcomes (simple events).

$$P(E_1) = P(\{\text{Jan, Feb, March}\}) = P(\{\text{Jan}\}) + P(\{\text{Feb}\}) + P(\{\text{March}\})$$

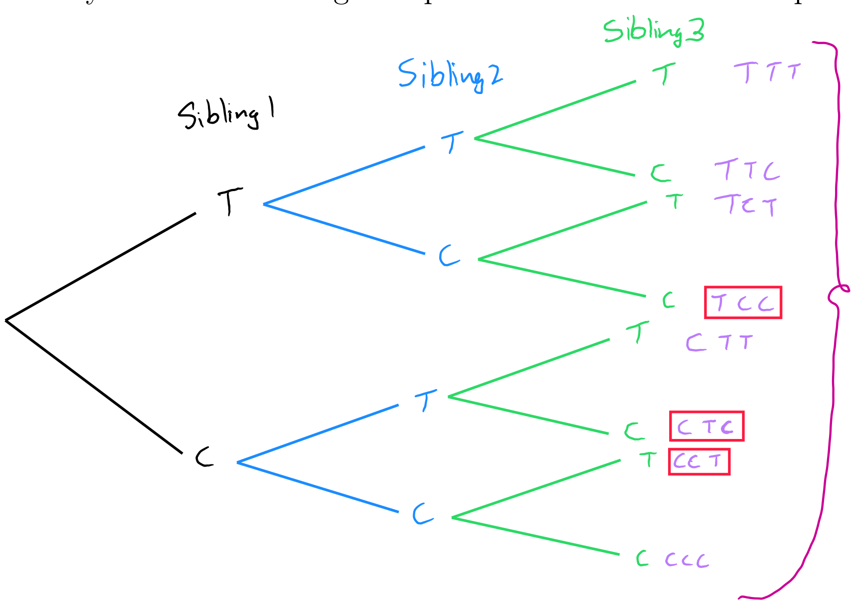
$$\frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$$ (Using classical approach)

Tree Diagrams

When an experiment is complex, use a systematic way to list possible outcomes.

Tree diagrams can be used to display outcomes when an experiment has a small number of branching stages.

Example 1: Three siblings register in a large-scale drug trial, wherein participants are split evenly into two groups: the Test Group (T) and the Control Group (C). What is the probability that exactly two of the siblings are placed in the Control Group?



8 equally likely outcomes

Sample Space: {TTT, TTC, TCT, TCC, ...}

$$P(\text{exactly 2 in } C) = \frac{3}{8}$$

3 → num of outcomes

8 → sample space

Probability Rules

Example 2: Suppose that we want to examine the various blood types in humans:

$S = \{A^+, A^-, B^+, B^-, O^+, O^-, AB^+, AB^-\}$ → Not equally likely outcomes

$E = \{A^+, A^-\}$ ← event: has an A type blood

$F = \{A^+, B^+, O^+, AB^+\}$ ← event: has a positive blood type

Definitions:

(a) The ***complement*** of an event A is the event "A does not occur".

$\underbrace{\bar{E} = E' = E^c = \neg E}_{\text{ways to represent a complement event}} = \{B^+, B^-, O^+, O^-, AB^+, AB^-\}$ (preferrably dont use $\neg E$)

$\bar{F} = \{A^-, B^-, O^-, AB^-\}$

(b) The ***union*** of events A and B is the event that A occurs, B occurs, or they both occur.

$E \cup F = \{A^+, A^-, \cancel{A^+}, B^+, O^+, AB^+\}$

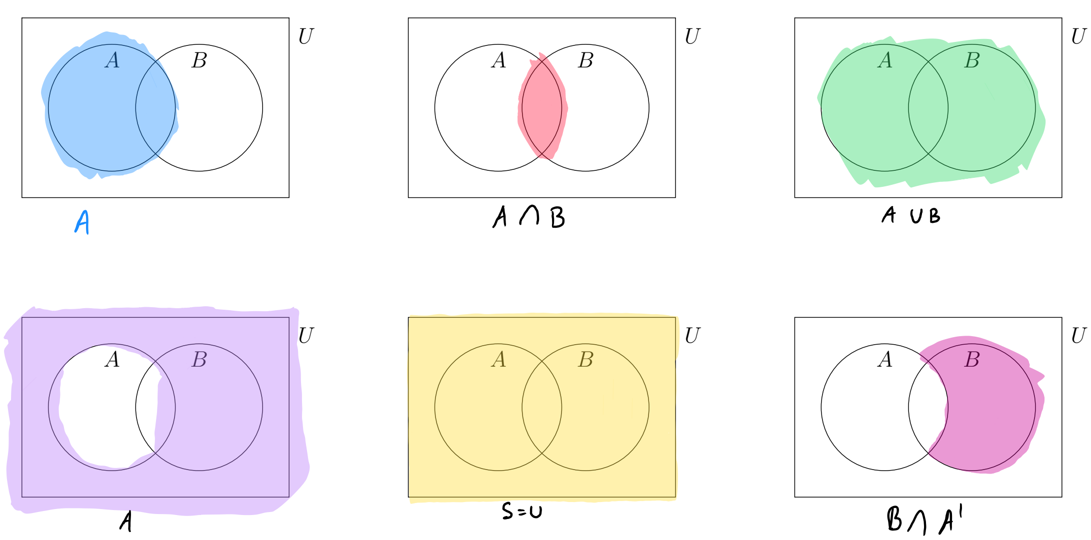
ignore cause repeat

(c) The ***intersection*** of events A and B is the event that both A and B occur.

common in both of them

$E \cap F = \{A^+\}$

Venn Diagrams

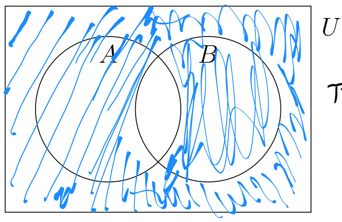


DeMorgan's Laws

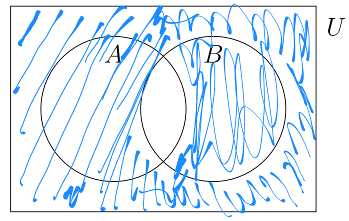
- $\overline{A \cup B}$

$\Updownarrow$

- $\overline{A \cap B}$



They're the same



Mutually Exclusive: Events A and B are ***mutually exclusive*** (or ***disjoint***) if

A and B do not occur simultaneously

$A \cap B = \emptyset$

$P(A \cap B) = 0$

$\emptyset$ is the empty set or the impossible event

$\emptyset = \{\}$ they both mean the same

$P(\emptyset) = 0$

Ex : we flip a coin once. $S = \{H, T\}$

let $A = \{H\}$ and $B = \{T\}$

$P(A \cap B) = 0$

Textbook Readings: Swartz 3.1, 3.2 [EPS 1.4]
Practice problems: Swartz 1.1, 1.3, 1.5, 1.7, 1.9, 1.11, 1.13