STAT 260 Summer 2024: Written Assignment 7

Due: Upload your solutions to Crowdmark BEFORE 6pm (PT) Friday July 5.

You may upload and change your files at any point up until the due date of Friday July 5 at 6pm (PT).

A 2% per hour late penalty will be automatically applied within Crowdmark. The penalty is applied in such a way so that assignments submitted 6pm to 6:59pm will have 2% deducted, assignments submitted 7pm-7:59pm will have 4% deducted, etc.

Note that if you submit any portion of your assignment before the deadline, Crowdmark will NOT permit you to edit your submission (including make additional uploads) after the 6pm deadline passes. This means that if, for example, you upload only Question 1 before the deadline, you will not be able to upload Question 2 after the deadline. If you intend to submit late (with penalty) you must submit the entire assignment late.

Submission: Solutions are to be uploaded to Crowdmark. Here you will be asked to upload your solutions to each question separately. Your solution to Question 1 must be uploaded in the location for Question 1, your solution to Question 2 must be uploaded in the location for Question 2, etc. If your work is uploaded to the wrong location, the marker will not be able to grade it.

You may hand-write your solution on a piece of paper or tablet. If you wish to use this question sheet and write your solutions on the page, space has been provided below. One of the quickest ways to upload work is by accessing Crowdmark from within a web browser on a smartphone. In the area where you upload work, press the "+" button. This will give you the option of using a file already on your phone, or you can use the phone camera to photograph your work. If you complete your work on a tablet, save the file as a PDF or each question as a jpeg and drag/drop the file into the Crowdmark box. ***Photographs of laptop/tablet screens will not be graded***; take a proper screenshot.

Instructions: For full marks, your work must be neatly written, and contain enough detail that it is clear how you arrived at your solutions. ***You will be graded on correct notation.*** Messy, unclear, or poorly formatted work may receive deductions, or may not be graded at all. Only resources presented in lecture or linked to on the Stat 260 Brightspace page are permitted for use in solving these assignments; using outside editors/tutors, and/or software (include AIs) is strictly forbidden. Talking to your classmates about assigned work is a healthy practice that is encouraged. However, in the end, each person is expected to write their own solutions, in their own words, and in a way that reflects their own understanding.

Additional Instructions:

- *For each of the following questions, include the correct notation for the random variable that you are calculating in your solution, not just the numeric answer. For example: "$f(3) = P(X = 3) = 0.157$".*
- *For each new random variable that you use, include both its definition ("Let $X = \ldots$") and its distribution ("$X \sim DIST(??, ??)$").*
- *All calculations are to be completed using only your SHARP EL-510 calculator and the* STAT 260 Distribution Tables Booklet *on Brightspace. Do not use R or other software.*

1. **[6 marks]** The concentration on manganese (Mn) in southern sea otter (*Enhydra lutris nereis*) livers is approximately normally distributed, with a mean of 17 $\mu g/g$ dry wt (micrograms Mn per gram dry weight) and a standard deviation of 8 $\mu g/g$ dry wt.

   (a) Determine the probability that a random sea otter has a liver manganese concentration between 10 and 20 $\mu g/g$ dry wt.

$$P(10 < X < 20) = P\left(Z < \frac{20-17}{8}\right) - P\left(Z < \frac{10-17}{8}\right)$$

$$\frac{20-17}{8} = \frac{3}{8} = 0.375 \qquad \frac{10-17}{8} = \frac{-7}{8} = -0.875$$

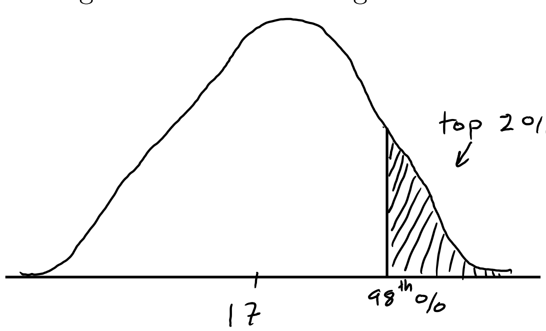
$$= P(Z < 0.375) - P(Z < -0.875)$$

$$= \left(\frac{0.6443 + 0.6480}{2}\right) - 0.1922 \rightarrow \text{from normal cdf table}$$

$$= 0.6462 - 0.1922$$

$$= 0.454$$

   (b) Determine the liver manganese concentration c (in $\mu g/g$ dry wt) that characterizes the top 2% of the population. That is, determine c such that only 2% of southern sea otters have a liver manganese concentration greater than c.



We want to find c such that $P(x \geq c) = 0.02$ or $P(x \leq c) = 0.98$

$$P(x \leq c) = P\left(\frac{x-\mu}{\sigma} \leq \frac{c-17}{8}\right) = 0.98$$

$z_{0.98} \approx 2.05 \rightarrow$ From normal cdf table

$$\frac{c-17}{8} = 2.05$$

$$c - 17 = 16.4$$

$$c = 16.4 + 17$$

$$c = 33.4 \ \mu g/g$$

2. **[4 marks]** The survival time (in minutes) of aphids exposed to a certain pesticide are known to be gamma distributed with $\alpha = 3$ and $\beta = 6$. Suppose that a random sample of 10 aphids are exposed to the pesticide. What is the probability that at least 2 of the aphids survive longer than 30 minutes?

Let x be the survival time of an aphid:

$X \sim \text{Gamma}(\alpha=3, \beta=6)$

$P(x > 30) = 1 - F(30)$

$$F(30) = \int_0^{30} \frac{x^{\alpha-1} \cdot e^{-x/\beta}}{\beta^\alpha \cdot \Gamma(\alpha)}\, dx$$

$$= \int_0^{30} \frac{x^2 \cdot e^{-x/6}}{6^3 \cdot 2!}$$

$$= \frac{1}{432} \int_0^{30} \underset{u}{x^2} \cdot \underset{dv}{e^{-x/6}}\, dx$$

$$= \frac{1}{432} \left[-6e^{-\frac{x}{6}} x^2 - \int \underset{dv}{-12e^{-x/6}}\, \underset{u}{x} \right]_0^{30}$$

$$= \frac{1}{432} \left[-6e^{-x/6} x^2 - \left(-12\left(-6e^{-x/6} x - 36e^{-x/6}\right)\right) \right]_0^{30}$$

$$= \frac{1}{432} \left[-6e^{-x/6} x^2 + 12\left(-6e^{-x/6} x - 36e^{-x/6}\right) \right]_0^{30}$$

$$= \frac{1}{432} \left[-\frac{7992}{e^5} + 432 \right]$$

$$= \frac{-7992 + 432e^5}{432e^5} \approx 0.87534$$

$P(x > 30) = 1 - 0.87534$
$= 0.12466$

Let Y be the number of aphids that survive longer than 30 minutes.

$X \sim \text{Binom}(n=10, p=0.12466)$

$P(x \geq 2) = 1 - P(x < 2) = 1 - [P(x=0) + P(x=1)]$

$P(x=0) = \binom{10}{0} (0.12466)^0 (0.87534)^{10} = 0.264$

$P(x=1) = \binom{10}{1} (0.12466)^1 (0.87534)^{10} = 0.376$

$P(x \geq 2) = 1 - [0.264 + 0.376]$

$= 1 - 0.64$

$= 0.36$

3. [**3 marks**] At a certain bank branch, clients request to enter the safety deposit box room according to a Poisson process, where the mean time between client entries is 1.25 hours. Suppose that it has been at least 40 minutes since the last client entered. What is the probability the total time between the previous client and the next client exceeds 1.5 hours?

Mean time between entries $= 1.25$ hours

$\lambda = 1/1.25 = 0.8$

Let X be the time between entries:

$X \sim \text{Exponential}(\lambda = 0.8)$

40 mins $= 2/3$ hours

As $X > 2/3$, we need:

memoryless

$P(X > 1.5 \mid X > 2/3) = P(X > 1.5 - 2/3)$

$= P(X > 5/6)$

$P(X > 5/6) = e^{-\lambda b}$

$= e^{-0.8(5/6)}$

$= e^{-2/3} \approx 0.5134$