Sample Final 1

Instructions:

1. Questions 1 to 30 are short answer questions. On the real final exam, put your answer in the box provided. An example is given in question 1. **You must show some work for each question in order to receive any marks.**
2. Questions 31 and 32 are full-answer questions. For full-answer questions, marks will be deducted for incomplete or poorly presented solutions.
3. Space will be provided for you to work out your answers on the actual exam.

Duration: You should be able to complete this midterm within 3 hours. If you cannot, this means that more practice is still needed.

Questions 1, 2 and 3 refer to the following scenario: CM Company manufactures computer monitors at three different sites: Plant A, Plant B, Plant C. Monitors not requiring any warranty service are given a High Quality rating at the end of the warranty period, monitors requiring warranty service are given a Medium Quality rating, and monitors requiring replacement under warranty are given a Low Quality rating. The following table gives proportions of CM's total production for the various categories. Suppose that a CM Monitor is purchased at random from CM's inventory.

	Quality Rating		
	High	Medium	Low
Plant A	.13	.24	.03
Plant B	.09	.21	.05
Plant C	.08	.15	.02

1. What is the probability that the purchased monitor was manufactured by Plant C or will receive a High Quality rating?

Answer:

2. If the purchased monitor was manufactured by Plant A, what is the probability that it will earn a Low Quality rating?

3. Which of the following statements are true?
(i) The events "purchased monitor was manufactured by Plant A" and "purchased monitor was manufactured by Plant B" are independent.
(ii) The events "purchased monitor was manufactured by Plant A" and "purchased monitor was manufactured by Plant B" are mutually exclusive.
(iii) The events "purchased monitor will receive a High Quality rating" and "purchased monitor was manufactured by Plant C" are independent.

Questions 4, 5 and 6 refer to the following scenario: A manufacturer of a certain model of car has recalled all sold units to correct a brake defect. A second recall was necessary when a second brake defect was discovered. 55% of the owners of the affected cars responded to both recalls; 30% responded only to the first recall; 15% percent responded to neither recall. The probability that a malfunction in the brakes occurs during the warranty period after the second recall notice was issued is:

.1 for cars whose owners responded to both recalls,
.3 for cars whose owners responded only to the first recall,
.8 for cars whose owners responded to neither recall.

4. What is the probability that a car randomly chosen from all those sold, experiences a brake malfunction during the warranty period after the second recall?

5. If a car randomly chosen from all those sold experiences a brake malfunction during the warranty period after the second recall, what is the probability that the owner responded to neither recall?

6. For a randomly chosen owner, what is the expected value of the number X of recall notices the car owner responded to?

Questions 7, 8 and 9 refer to the following scenario: The route used by a certain motorist in commuting to work passes through three traffic lights. The probability that the commuter must stop at the first, second, third traffic light is .4, .5, .7, respectively. Assume the three events: commuter must stop at the first traffic light, commuter must stop at the second traffic light, commuter must stop at the third traffic light, are independent events.

7. What is the probability that on the way to work the commuter must stop all three traffic lights?

8. What is the probability that on the way to work the commuter must stop at exactly one of the three traffic lights?

9. What is the probability that the commuter will not have to stop at any of the three lights on exactly one of the next five trips to work? (i.e. What is the probability that exactly one of the next five trips to work will be stop-free?)

Questions 10 and 11 refer to the following scenario: Suppose that trees are randomly distributed over a certain forest region at the average rate of 60 trees per hectare.

10. What is the probability that in a certain one-quarter hectare plot there will be at most 10 trees?

11. Given that there are at most 20 trees in a certain one-quarter hectare plot, what is the probability that there are more than 10 trees in this plot?

12. Assume that 60% of all households in a certain large city have microwave ovens. What is the probability that 20 households chosen at random from this city will include at least 13 households having microwave ovens?

Questions 13, 14 and 15 refer to the following scenario: The reaction time (in tenths of seconds) to a certain stimulus is a continuous random variable with pdf:

$$f(x) = \begin{cases} \frac{5-x}{8} & \text{if } 1 < x < 5 \\ 0 & \text{otherwise} \end{cases}$$

13. What is the probability that reaction time is at most 2 tenths of a second?

14. Compute the expected reaction time (in tenths of seconds).

15. Compute the 75th percentile of the distribution of reaction time (in tenths of seconds).

Questions 16, 17 and 18 refer to the following scenario: A certain company rates its product on reliability, X, and customer satisfaction, Y. Suppose the joint pmf of X and Y is given by the following table.

		y		
		-1	0	1
	0	.10	.03	.02
x	1	.06	.21	.33
	2	.04	.06	.15

16. Compute $P(X > Y)$.

17. Compute the covariance of X and Y.

18. Compute the variance of the total score $T = X + Y$.

19. Extensive experience with units of a certain type has suggested that the exponential distribution provides a good model for the time until failure. Suppose the mean time until failure is 4.8 years. What is the probability that a randomly selected unit of this type will last between 2.0 and 5.0 years?

Questions 20 and 21 refer to the following scenario: The breakdown voltage of a randomly chosen diode of a certain type is known to be normally distributed with mean value 35 volts and standard deviation 2 volts.

20. What value is such that only 3% of all diodes have breakdown voltages exceeding that value?

21. What is the probability that a randomly chosen diode will have breakdown voltage exceeding 36 volts?

22. Manufacture of a certain component requires two different machining operations. Machining time for each operation has a normal distribution, and the two times are independent of one another. The mean values are 30 minutes and 20 minutes, respectively, and the standard deviations are 3 minutes and 2 minutes, respectively. What is the probability that the total machining time for one of these components will be less than 45 minutes?

Questions 23 and 24 refer to the following scenario: A manufacturing process is designed to produce bolts with diameter = 12 mm. Once each day, a random sample of 36 bolts is selected and the diameters recorded. If the resulting sample mean is less than 11.9 mm or greater than 12.1 mm, the process is shut down for adjustment. The standard deviation for diameter is 0.3 mm.

23. What is the probability that the process will be shut down for adjustment on a day when the true process mean is actually 12 mm?

24. What is the probability that the process will not be shut down for adjustment on a day when the true process mean is 12.1 mm?

25. Suppose X_1, X_2, X_3 is a random sample of size $n = 3$ from a population distribution having unknown mean μ and unknown standard deviation σ. Consider the following three estimators for μ.

$$W = \frac{9}{16}X_1 + \frac{3}{16}X_2 + \frac{4}{16}X_3; \qquad Y = \frac{7}{16}X_1 + \frac{7}{16}X_2 + \frac{2}{16}X_3; \qquad T = X_1 + X_2 + X_3.$$

Which of the following statements are true?

i The expected value of T is μ.

ii The variance of Y is less than the variance of W, i.e. $Var(Y) < Var(W)$.

ii The variance of T is less than the variance of W, i.e. $Var(T) < Var(W)$.

Questions 26 and 27 refer to the following scenario: Six soil samples taken from a particular region were subjected to chemical analysis to determine the pH of each sample, yielding a sample mean pH of 5.9 and a sample standard deviation of .62. Assume the distribution of soil pH in this region is normal.

26. Compute the upper limit of a 95% confidence interval for the true mean pH of soil in this region.

27. How many observations would be needed to obtain a 95% confidence interval for true mean pH having length approximately .35?

28. In a batch chemical process, two catalysts are compared for their effect on the output of the process reaction. A sample of 12 batches was prepared using catalyst 1 and a sample of 10 batches was obtained using catalyst 2. The 12 batches for which catalyst 1 was used gave a sample mean yield of 85 with a sample standard deviation of 4, while the 10 batches for which

catalyst 2 was used gave a sample mean yield of 81 with a sample standard deviation of 5. Let μ_1 and μ_2 denote the true mean yields using catalysts 1 and 2, respectively. Compute the lower limit of a 90% confidence interval for $\mu_1 - \mu_2$, assuming that the data are normally distributed.

29. To compare two brands of radial tires, 50 tires of each brand were tested and the following sample mean lifetimes (in kilometres) and sample standard deviations for each of the two brands were obtained:

Brand A	$m = 50$	$\bar{x} = 65,600$	$s_1 = 5000$
Brand B	$n = 50$	$\bar{y} = 61,500$	$s_2 = 7200$

Compute the upper limit of a 99% confidence interval for $\mu_1 - \mu_2$ where μ_1 is the true mean lifetimes of Brand A tires and μ_2 is the true mean lifetimes of Brand B tires.

30. The true proportion of defective memory chips produced by a certain company is p. Suppose a random sample of 400 memory chips are tested and 10 of them are found to be defective. Compute the lower limit of a 92% confidence interval for p.

31. LONG ANSWER QUESTION: Five samples of a ferrous-type substance are to be used to determine if there is a difference between a laboratory chemical analysis and an X-ray fluorescence analysis of the iron content. Each sample was split into two sub-samples and the two types of analysis were applied. Following are the data showing the iron content (in percent).

	Sample				
Analysis	1	2	3	4	5
Chemical	2.2	1.9	2.5	2.3	2.4
X-Ray	2.0	2.0	2.3	2.1	2.4

Do these data indicate that the two methods of analysis give, on average, a different result? Assume the relevant population distribution(s) is/are normal.

(a) Define the population parameter(s) of interest.
(b) State the null and alternative hypotheses in terms of the parameter.
(c) State the test statistic you will use. What distribution (including degrees of freedom, if appropriate) will you use to calculate the p-value.
(d) Find the observed value of the test statistic.
(e) Compute (or bracket) the p-value within the accuracy of the tables.
(f) What level of evidence against H_0 do you find?

32. LONG ANSWER QUESTION: A random sample of 10 times of first sprinkler activation for a series of tests with fire prevention sprinkler systems using an aqueous film-forming foam yielded a sample mean activation time of 24.3 seconds and a sample standard deviation of 5.1 seconds. The system has been designed so that the true mean activation time μ is at most 20 seconds. Do these data strongly contradict the validity of this specification? Before we ran the study, our belief was that the mean activation time was longer than 20 seconds. Assume the distribution of activation times is normal.

(a) Define the population parameter(s) of interest.

(b) State the null and alternative hypotheses in terms of the parameter.

(c) State the test statistic you will use. What distribution (including degrees of freedom, if appropriate) will you use to calculate the p-value.

(d) Find the observed value of the test statistic.

(e) Compute (or bracket) the p-value within the accuracy of the tables.

(f) What level of evidence against H_0 do you find?

Answers :

1. 0.47	2. 0.075	3. only ii
4. 0.265	5. 0.453	6. 1.40
7. 0.14	8. 0.36	9. 0.309
10. 0.118	11. 0.871	12. 0.416
13. 0.4375	14. 2.333	15. 3.0
16. 0.62	17. 0.16	18. 1.32
19. 0.306	20. 38.76	21. 0.3085
22. 0.0823	23. 0.0456	24. 0.50
25. only ii	26. 6.55	27. 49
28. 0.69	29. 7293	30. 0.01134

31.(a) μ_D = true mean difference (Chemical Analysis minus X-ray)
(b) $H_0 : \mu_D = 0$ and $H_1 : \mu_D \neq 0$
(c) $T = \frac{\bar{D}-0}{s_D/\sqrt{n_d}} \sim t_{(n_d-1)}$
(d) $t_{obs} = 1.58$
(e) $p - value = 2 * P(T_{(4)} \geq 1.58)$, $.10 < p - value < .20$
(f) There is no evidence against the null hypothesis.

32.(a) μ = true mean activation time
(b)$H_0 : \mu = 20$ (or $\mu \leq 20$), $H_1 : \mu > 20$.
(c) Test statistic $\dfrac{\overline{x} - \mu}{s/\sqrt{n}}$. The distribution is $t_{(9)}$.
(d) Observed test statistic 2.67.
(e) $p - value = P(T_{(9)} \geq 2.67)$; 0.01 <p-value< 0.015
(f) There is **strong** evidence against H_0.