Given parametric Equations:

$x = 2\cos(t)$ where $0 \le t \le 2\pi$
$y = \sin(t)$

a) We want to find distance squared from any point $(x(t), y(t))$ on the ellipse to the point $\left(\frac{3}{4}, 0\right)$

distance squared between two points general form:

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$$d^2(t) = \left(\frac{3}{4} - 2\cos(t)\right)^2 + \left(0 - \sin(t)\right)^2$$

$$\rightarrow d^2(t) = \left(\frac{3}{4} - 2\cos(t)\right)^2 + \sin^2(t)$$

b) Finding critical points of $d(t)$

$$\left(\frac{3}{4} - 2\cos(t)\right)^2 + \sin^2(t)$$

Expanded:

$$d(t) = \frac{9}{16} - 3\cos(t) + 4\cos^2(t) + \sin^2(t)$$

$$d'(t) = 3\sin(t) - 8\cos(t)\sin(t) + 2\sin(t)\cos(t)$$
$$= 3\sin(t) - 6\cos(t)\sin(t)$$
$$= 3\sin(t)(1 - 2\cos(t))$$

Now we set $d'(t)$ to 0:

$3\sin(t)(1 - 2\cos(t)) = 0$

$\sin(t) = 0$ when

$t = 0, \pi, 2\pi$

$\cos(t) = 0$ when

$t = \frac{\pi}{3}$ or $\frac{5\pi}{3}$

$\therefore$ critical points are $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

c) To determine which critical points minimize the distance we need to evaluat $d(t)$ at each critical point:

$$d(0) = d(2\pi) = \left(\frac{3}{4} - 2\right)^2 + 0^2 = \frac{25}{16} = 1.5625$$

$$d(\pi) = \left(\frac{3}{4} + 2\right)^2 + 0^2 = \frac{121}{16} = 7.5625$$

$$d\left(\frac{\pi}{3}\right) = d\left(\frac{5\pi}{3}\right) = \left(\frac{3}{4} - 1\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{13}{16} = 0.8125$$

The minimum value occurs at $t = \frac{\pi}{3}$ and $t = \frac{5\pi}{3}$

To find corresponding points on the ellipse:

At $t = \frac{\pi}{3} = 2\cos\left(\frac{\pi}{3}\right) = 1$ $y = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

At $t = \frac{5\pi}{3} = 2\cos\left(\frac{5\pi}{3}\right) = 1$ $y = \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$\therefore$ The points on the ellipse closest to $\left(\frac{3}{4}, 0\right)$ are $\left(1, \frac{\sqrt{3}}{2}\right)$ and $\left(1, -\frac{\sqrt{3}}{2}\right)$