Q1 (10 points)

A cardboard box with a square base is to have a volume of 8 litres.

The cardboard for the box costs $\$0.01$ per squared centimeter of surface area, but the cardboard for the bottom is thicker, so it costs three times as much (just the bottom, not the top).

Find the dimensions of the box that will minimize the cost of the cardboard. You may round your answer to one decimal. Be sure to include units in your answer.

Give your answer as a written sentence in plain English that any ol' Joe off the street or in a Rona store can understand.

Our goal is to find dimensions of the box that are closest to 8 Litres while keeping costs minimal.

Volume of box (V) = Base Area × Height

Base area is a square Base Area = x^2

$\therefore\ 8000\,cm^3 = x^2 \times h$

Total Surface Area also need to be accounted for:

A_{total} = Base Area + (4 × Area of one side)

$\therefore A_{total} = x^2 + 4xh$

$\therefore$ Cost of cardboard = c_1 × (Area of sides) + c_2 × (Bottom Area)
where c_1 = cost for sides
and c_2 = bottom cost

$\therefore\ Cost = c_1 \times (x^2 + 4xh) + c_2 \times x^2$

$\therefore$ given that $c_2 = 3 \times c_1$, we get:

$C = c_1 \times (x^2 + 4xh) + 3c_1 \times x^2$

$C = c_1 \times (x^2 + 4xh + 3x^2)$

$C = c_1 \times (4xh + 4x^2)$

Since we know that $x^2 \times h = 8000$:

$h = \frac{8000}{x^2}$

Substituting: $C = c_1\left(4x\left(\frac{8000}{x^2}\right) + 4x^2\right)$

$\therefore C = c_1\left(\frac{32000}{x}\right) + 4x^2)$

Now we differentiate:

$$\frac{dC}{dx} = c_1 \times \left(\frac{-32000}{x^2} + 8x\right)$$

$$\therefore 0 = \frac{-32000c_1}{x^2} + 8c_1x$$

$$\therefore \frac{32000}{x^2} = 8x$$

$$\therefore 32000 = 8x^3$$

$$\therefore x^3 = 4000$$

$$\therefore x = \sqrt[3]{4000}$$

$$\therefore x \approx 15.9$$

Substituting x value back:

$h = \frac{8000}{(15.9)^2}$ $\therefore h \approx 31.6$

Dimensions: 15.9cm × 15.9cm × 31.6cm

Final Answer:

The dimensions for a cardboard box with volume 8 Litres that will minimize its cost are approximately 15.9cm × 15.9cm × 31.6cm.