Taylor series $f(x)$ centered at $x=a$ is given by:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$ where $f^{(n)}(a)$ is the n-th derivative of f evaluated at a

$f(x)$	$\frac{4}{1-x}$	$f(5) = \frac{4}{1-5} = \frac{4}{-4} = -1$
$f'(x)$	$\frac{4}{(1-x)^2}$	$f'(5) = \frac{4}{(1-5)^2} = \frac{4}{16} = \frac{1}{4}$
$f''(x)$	$\frac{8}{(1-x)^3}$	$f''(5) = \frac{8}{(1-5)^3} = \frac{8}{-64} = -\frac{1}{8}$
$f'''(x)$ ⋮	$\frac{64}{(1-x)^4}$	$f'''(5) = \frac{24}{(1-5)^4} = \frac{24}{256} = \frac{3}{32}$
$f^{(n)}(x)$	$\frac{4\cdot n!}{(1-x)^{n+1}}$	

So in general:

$$f^{(n)}(5) = \frac{4\cdot n!}{(1-5)^{n+1}} = \frac{4\cdot n!}{(-4)^{n+1}} = \frac{4\cdot n!}{(-1)^{n+1}\cdot 4^{n+1}} = \frac{n!}{(-1)^{n+1}\cdot 4^n}$$

Next Step: constructing taylor series:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(5)}{n!}(x-5)^n$$

Substituting from earlier calculations:

$$f(x) = \sum_{n=0}^{\infty} \frac{\frac{n!}{(-1)^{n+1}\cdot 4^n}}{n!}(x-5)^n$$

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{(-1)^{n+1}\cdot 4^n}(x-5)^n$$

Simplyfing:

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{4^n}(x-5)^n$$

$$\rightarrow f(x) = \sum_{n=0}^{\infty} (-1)^{n+1}\cdot \frac{1}{4^n}\cdot \frac{(x-5)^n}{4}$$

Final Answer:

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{4^n}(x-5)^n$$