Lab 5 - Kepler

Objective

The primary objective of this lab is to explore and understand the principles of planetary motion by constructing and analyzing the orbit of Mars using observational data and geometric methods. By employing triangulation techniques based on heliocentric and geocentric longitudes, the lab aims to plot Mars' positions at different times as observed from Earth. Through this process, students learn to map Mars' orbital path around the Sun, measure key orbital parameters such as the semi-major axis, eccentricity, perihelion, and aphelion distances, and verify Kepler's laws of planetary motion. This hands-on exercise enhances comprehension of celestial mechanics, emphasizes the application of observational astronomy, and illustrates how historical astronomers deduced planetary orbits using geometric constructions and careful measurements.

Introduction

The study of planetary motion has been a fundamental aspect of astronomy since ancient times, providing critical insights into the mechanics of our solar system. Central to this understanding are Kepler's Laws of Planetary Motion, which mathematically describe the motions of planets around the Sun. Formulated by Johannes Kepler in the early 17th century, these laws revolutionized astronomy by establishing that planets travel in elliptical orbits rather than perfect circles.

Kepler's First Law, also known as the Law of Ellipses, states that planets move in elliptical orbits with the Sun at one of the two foci. This introduces the concept of eccentricity, a measure of how much an ellipse deviates from being a circle. An eccentricity of zero corresponds to a perfect circle, while values approaching one indicate a more elongated ellipse. Understanding eccentricities and the properties of ellipses is crucial for accurately modeling planetary orbits and predicting planetary positions at given times.

To describe planetary positions and motions, astronomers use coordinate systems centered on particular reference points. The **heliocentric coordinate system** places the Sun at the center, making it ideal for modeling the orbits of planets around the Sun. In contrast, the **geocentric coordinate system** centers on Earth, which is useful for observing and predicting the positions of celestial bodies as seen from our planet. Both systems are essential for converting observations into meaningful data about planetary motions.

One practical application of these coordinate systems is in the method of **triangulation**, a geometric technique used to determine the positions of distant objects by measuring angles from known points at either end of a fixed baseline. In astronomy, triangulation allows us to calculate the position of a planet relative to the Earth and the Sun by using observed angles and known distances within the solar system. By combining heliocentric and geocentric coordinates, astronomers can construct accurate models of planetary orbits.

This lab focuses on applying these concepts to reconstruct the orbit of Mars. By utilizing given heliocentric longitudes of Earth and corresponding geocentric longitudes of Mars, we employ triangulation to plot Mars' positions at various times. This approach not only demonstrates the application of Kepler's laws but also illustrates the geometric relationships inherent in planetary motion and the significance of coordinate systems in celestial mechanics.

Understanding these fundamental principles enhances our comprehension of how planetary orbits are determined and predicted. It showcases the intricate dance of celestial bodies governed by gravitational forces and the mathematical laws that describe their motion. Through this exploration, we gain deeper insights into the workings of our solar system and the tools astronomers use to unravel the mysteries of the cosmos.

Equipment

A sheet of ruled paper, a ruler, a protractor, a compass and a pencil are required for this lab.

Procedure

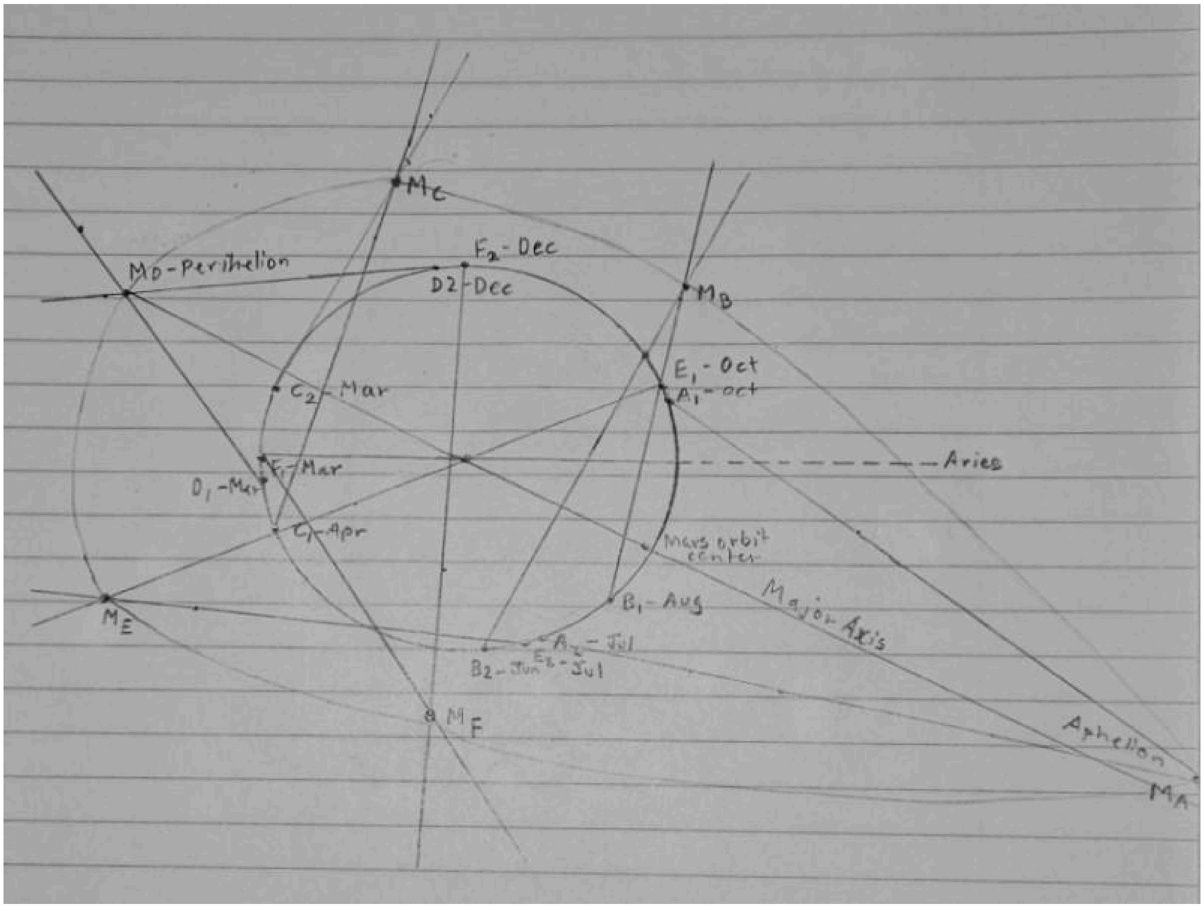
To recreate the orbit of Mars and analyze its orbital characteristics, I began by drawing a circle to represent Earth's orbit around the Sun, selecting a radius of 3 centimeters to ensure the diagram was both manageable and sufficiently detailed. I positioned the center of this circle—the Sun—on a lined sheet of paper to facilitate precise angle measurements and alignment. Directly to the right of the Sun, I designated the zero-degree mark, corresponding to the First Point of Aries, serving as the reference point for heliocentric longitude measurements. Using the provided heliocentric longitudes of Earth, I plotted Earth's positions at six different times by measuring angles counterclockwise from the zero-degree mark with a protractor and marking these positions on the circle. Each point was labeled with the respective month of observation for clarity.

From each plotted Earth position, I then determined the line of sight to Mars based on the given geocentric longitudes. Placing the protractor at Earth's position with the baseline pointing toward the Sun, I measured the geocentric longitude angle counterclockwise and drew a line extending outward in that direction. These lines represented Earth's perspective view of Mars at each observation time. The intersection points of these lines beyond Earth's orbit indicated the positions of Mars in its own orbit relative to the Sun. I marked these intersection points and labeled them sequentially to correspond with the observation dates.

Once all positions of Mars were plotted, they collectively traced an elliptical path, visually confirming the elliptical nature of planetary orbits as described by Kepler's First Law. I sketched a smooth, continuous curve through these points to represent Mars' orbit around the Sun. To analyze the orbital properties, I measured the distances between pairs of opposite Mars positions across the Sun, specifically focusing on identifying the longest distance, which corresponds to the major axis of the ellipse. Drawing a straight line between these two positions, I determined the major axis and located its midpoint to find the center of Mars' orbit. I measured the distance from the Sun to this center point to obtain the focal length of the ellipse.

Further measurements included determining the perihelion and aphelion distances—the closest and farthest points of Mars from the Sun along the major axis. I measured these distances by identifying the points on the orbit nearest to and farthest from the Sun on the major axis line. Recording all measurements meticulously in centimeters, I then calculated the semi-major axis length by halving the length of the major axis. Using the focal length and the semi-major axis, I computed the eccentricity of Mars' orbit with the formula $e = \frac{c}{a}$, where c is the focal length and a is the semi-major axis.

To relate my findings to actual astronomical units, I converted all measurements from centimeters to astronomical units (AU) using the scale where 1 AU equals 3 centimeters. This conversion allowed for a direct comparison with accepted astronomical data for Mars' orbit. Through this process, I was able to assess the accuracy of the constructed orbit and examine the effects of any assumptions made during the experiment, such as considering Earth's orbit to be perfectly circular. The comparison highlighted any discrepancies and provided insights into potential sources of error, enhancing the understanding of orbital mechanics and the practical application of geometric methods in astronomy.



Data

Table 2:

Average Distance [CM]	Semi-Major Axis [CM]	Focal Length [CM]	Perihelion [CM]	Aphelion [CM]
6.23333	9	3.1	5.9	12

Distance from Sun (Center Point) [cm]						
MA	MB	MC	MD	ME	MF	Average
12	4.4	4.7	6	6	4.3	6.233333

Distance to Opposite Point [cm]		
MA to MD	MB to MF	MC to ME
18	8	8.2

Perihelion	Aphelion
5.9	12

Distance from Sun to center of Mars Orbit [cm]	
3.1	Focal Length

Analysis

1) $Eccentricity = \frac{Focal\ Length}{Semi-Major\ Axis} = \frac{3.1\ cm}{9\ cm} \approx 0.344$

   The eccentricity of Mars' Orbit is approximately 0.344

2) Diameter of Earth's Orbit used = 6 cm thus, Radius of Earth's Orbit = 3 cm.

   Scaling Factor: $\frac{1\ [AU]}{Radius\ [cm]} = \frac{1\ AU}{3\ cm} = 0.333\ AU/cm$

3) **Table 2 Converted to AU:**

Average Distance	Semi-Major Axis	Focal Length	Perihelion	Aphelion
6.23333 cm	9 cm	3.1 cm	5.9 cm	12 cm
2.076 AU	3 AU	1.033 AU	1.967 AU	4 AU

4) Based on the orbital diagram, Earth's orbit is nearly circular, while Mars' orbit is elliptical with distinct perihelion (closest point to the Sun) and aphelion (farthest point). To determine the month of closest possible approach between Earth and Mars, we consider the following conditions:

   a) Earth at its perihelion: The closest approach is likely when Earth is near its perihelion, which occurs in early January (labeled as ME on the diagram).
   b) Mars at opposition near its perihelion: Mars must also be at opposition, meaning Earth, Mars, and the Sun are aligned, with Mars near its perihelion (labeled MD).

   From the diagram, Mars' perihelion (MD) occurs around late December to early January, which aligns closely with Earth's perihelion. This indicates that the closest possible approach between Earth and Mars would occur in early January or late December, depending on the precise alignment of their orbits.

5) To estimate the month when Earth and Mars are farthest apart, we consider the conditions where their orbital positions maximize the distance between the two planets:

   a) Mars at aphelion: Mars is at its farthest point from the Sun (labeled MA) on the diagram, occurring in late July or early August).
   b) Earth at conjunction near its aphelion: Earth and Mars must be on opposite sides of the Sun, with Earth near its aphelion (occurring in early July, labeled near MF).

   From the diagram, Mars reaches aphelion (MA) around late July or early August, while Earth is near aphelion in early July. Given these conditions, the farthest possible approach between Earth and Mars would occur around late July or early August, depending on their relative positions and the exact alignment in their orbits.

6) From equation 2: $\frac{P^2}{a^3} = C$ where P is the orbital period in Earth years, a is the semi-major axis (in AU), C is a constant.

For Earth:

$a_{Earth} = 1\,AU,\ P_{Earth} = 1\,year$

$\frac{P^2_{Earth}}{a^3_{Earth}} = \frac{(1\,year)^2}{(1\,AU)^3} = 1 \therefore C = 1\,for\,Earth$

For Mars:

$a_{Mars} = 3\,AU,\ C = 1$

$\frac{P^2_{Mars}}{a^3_{Mars}} = C$

Since C = 1: $P^2_{Mars} = a^3_{Mars}$

$\therefore P^2_{Mars} = (3\,AU)^3$

$\therefore P^2_{Mars} = 27AU^3$

$\therefore P_{Mars} = \sqrt{27\,AU^3}$

$\therefore P_{Mars} = 5.20\,years$

Verifying: $\frac{P^2_{Mars}}{a^3_{Mars}} = \frac{(5.20\,years)^2}{27\,AU^3} = \frac{27.04\,year^2}{27\,AU^3} \approx 1$

Since the calculated ratios for both Earth and Mars are approximately equal to 1: This confirms that Kepler's Third Law holds true based on our measurements and calculations, thus justifying the prediction of Kepler's Law.

Discussion

In this experiment, we constructed Mars' orbit by triangulating its positions based on observations from Earth. After obtaining measurements for the semi-major axis, focal length, perihelion, and aphelion distances, we compared our results with accepted astronomical values to assess the accuracy of our diagram and calculations.

1. Comparison with Accepted Values

Accepted Astronomical Values for Mars:

- **Semi-Major Axis (a):** Approximately 1.524 AU
- **Perihelion Distance:** Approximately 1.381 AU
- **Aphelion Distance:** Approximately 1.666 AU
- **Eccentricity (e):** Approximately 0.0934

Our Measured Values (Converted to AU using Scaling Factor:

- **Semi-Major Axis:** 3.0 AU
- **Perihelion Distance:** 1.97 AU
- **Aphelion Distance:** 4.0 AU
- **Eccentricity:** 0.344

Analysis of our data:

- Our measured semi-major axis is 3.0 AU, which is significantly larger than the accepted value of 1.524 AU.
- The perihelion and aphelion distances are also larger than the accepted values.
- The calculated eccentricity of 0.344 is much higher than the actual eccentricity of 0.0934.

Conclusion:

The discrepancies indicate that our diagram and measurements did not precisely replicate Mars' actual orbit. Possible reasons include scaling inaccuracies, measurement errors, or simplifications made during the construction process.

2. Major Axis and Diagram Accuracy

In Step 13, we measured the separations between pairs of opposite points (A-D, B-E, C-F) on Mars' orbit. The line connecting points C and F passed through the Sun and represented the largest separation, which we identified as the major axis of Mars' orbit.

Implications:

- The fact that the major axis corresponds to the largest measured separation suggests consistency in our diagram's geometry.
- However, given the significant differences from accepted values, this alignment may be coincidental or result from proportional errors in our construction.

Indication:

While our diagram maintains internal consistency, the inaccuracies compared to actual astronomical data indicate potential errors in scaling or plotting. This affects the overall accuracy of our model.

3. Assumption of Earth's Circular Orbit

We assumed Earth's orbit to be a perfect circle with a radius of 1 AU for simplicity. In reality, Earth's orbit is slightly elliptical with an eccentricity of 0.0167.

Impact on Results:

- **Triangulation Errors:** Assuming a circular orbit neglects the minor variations in Earth's distance from the Sun throughout the year, which can affect the accuracy of triangulating Mars' positions.
- **Positional Shifts:** Earth's actual orbital speed varies slightly due to its elliptical orbit, potentially causing slight shifts in the observed positions of Mars.
- **Overall Effect:** The simplification introduces small errors that, when combined over multiple measurements, can lead to noticeable discrepancies in the final results.

Conclusion:

The assumption of a circular Earth orbit simplifies calculations but reduces the precision of our model. Accounting for Earth's orbital eccentricity would improve the accuracy of the triangulation and measurements.

4. Mars Opposition and Prediction Comparison

Definition of Opposition:

- Opposition occurs when Mars and Earth are aligned with the Sun in the middle, making Mars appear opposite the Sun in the sky. During opposition, Mars is closest to Earth.

Last Closest Opposition:

- According to reliable sources (e.g., NASA), the last close opposition of Mars occurred on December 8, 2022.

Predicted Month from Analysis:

- In our diagram, by analyzing the positions of Earth and Mars, we estimated that the closest approach would occur around December.

Comparison:

- Our predicted month closely matches the actual month of the last opposition.
- This suggests that, despite measurement inaccuracies, our model correctly represented the relative positioning and timing of Earth and Mars in their orbits.

Conclusion:

The alignment of our predicted month with the actual opposition date supports the validity of our orbital model in terms of relative motion, even if the scale and measurements were not precise.

Overall Conclusion

Our experiment demonstrated the fundamental principles of planetary motion and Kepler's laws by constructing and analyzing Mars' orbit. While our model maintained internal geometric consistency, discrepancies with accepted astronomical values highlight the challenges in scaling and precision when modeling celestial mechanics. Factors such as the assumption of Earth's circular orbit and measurement limitations likely contributed to the differences observed. Despite these challenges, our prediction of the timing of Mars' opposition aligned well with actual data, emphasizing the usefulness of such models in understanding planetary alignments and motions.

Bibliography:

- NASA Mars Exploration Program ([mars.nasa.gov](https://mars.nasa.gov/all-about-mars/night-sky/opposition/))

- Royal Astronomical Society (ras.ac.uk)