Stat 260: only joint distributions

## Set 18: Joint Probability Distributions

Stat 260: June 26, 2024

So far, we have looked at univariate random variables. There are, however, many scenarios were several random variables interact with one another. For example, we might want to measure a subject's height X and weight Y: for this multivariate example, we would need jointly distributed random variables, and their multivariate distributions.

Let X and Y be discrete random variables. The joint probability mass function (joint **pmf**) f(x,y) is

$$f(x,y) = P(X = x \cap Y = y) = P(X = x, Y = y)$$

This joint pmf satisifies:

- (i)  $0 \le f(x,y) \le 1$ (ii)  $\underset{\text{ally}}{\le} y\left(\underset{\text{allx}}{\ge} f(x,y)\right) = 1$

**Example 1:** Suppose that the discrete random variables X and Y have the following joint pmf:

1

			Y		
	f(x,y)	3	6	9	
	-2	0.12	0.09	0.10	0.31
X	5	0.14	0.21	0.04	0.39
	7	0.06		0.13	0.30
marginal Grys		0.32	0.41	0.27	1
					marginal for 2L

check - sum all probabilities inside =1

(a) 
$$f(5,3) = P(x=5, y=3) = 0.14$$
 from table

(b) 
$$P(X = 5) = f(5, 3) + f(5, 6) + f(5, 9)$$
  
= 0.14 + 0.21 + 0.04 = 0.39

(c) 
$$P(Y \ge 6) = P(Y=6) + P(Y=q)$$
  
 $P(Y=6) = 0.09 + 0.21 + 0.11 = 0.41$   
 $P(Y=9) = 0.10 + 0.04 + 0.13 = 0.27$   
 $= 0.68$  or you could've done  $1 - P(Y=3)$ 

The marginal probability mass function distributions of discrete random variables

marginal for 
$$x \to f_X(x) = \sum_{\mathrm{all } y} f(x,y)$$
 and  $f_Y(y) = \sum_{\mathrm{all } x} f(x,y)$  for  $y$ 

## Example 1 Continued...

(d) Determine the marginal pmf of X

$$\frac{x}{f_{x}(x)} = \frac{5}{0.31} = \frac{7}{0.39} = \frac{7}{0.3}$$
 = Should sum to 1

(e) Find E[X]

$$E(x) = (-2)(0.31) + 5(0.39) + 7(0.3) = 3.43$$

Let X and Y be two random variables. The **conditional probability distribution** of the random variable Y given X = x is

$$f_{Y|X}(y|x) = f_{Y|X=x}(y) = \frac{f(x,y)}{f_X(x)} = f(y=y \mid x=x)$$

Likewise, the conditional distribution of X given that Y = y is

$$f_{X|Y}(x|y) = f_{X|Y=y}(x) = \frac{f(x,y)}{f_Y(y)}$$

Example 1 Continued...

(g) Determine 
$$f_{Y|X}(9|7) = P(y=9/X=7)$$

$$P(y=9/X=7)$$

P(x=7)

P(x=

The random variables X and Y are **independent** if for every (x, y) pair,

joint pmf 
$$\rightarrow f(x,y) = f_X(x)f_Y(y)$$
.  $\leftarrow$  marginals

## Example 1 Continued...

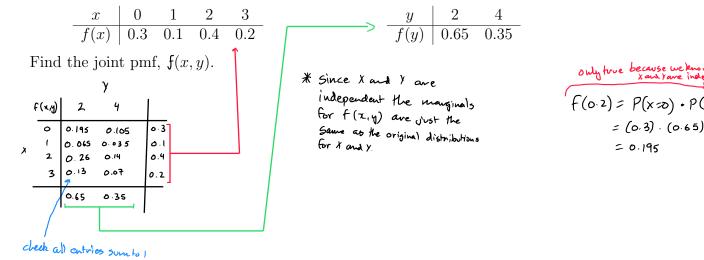
(f) Determine whether X and Y are independent.

$$f_{x}(5) \cdot f_{y}(3) = (0.39) \cdot (0.32) = 0.1248$$

**Note:** To determine whether the (discrete or continuous) random variables  $X_1, X_2, \ldots, X_n$  are independent, we would have to test whether every subset  $X_{i_1}, X_{i_2}, \ldots, X_{i_k}$  of variables (i.e. every pair, every triple, etc.) has the joint pmf of the subset equal to the product of the marginal pmf.

If the random variables 
$$X_1, X_2, \ldots, X_n$$
 are independent, then 
$$f(x_1, x_2, \ldots, x_n) = f_{X_1}(x_1) f_{X_2}(x_2) \ldots f_{X_n}(x_n)$$
 we can build the puf from the marginals

**Example 2:** Suppose that the discrete random variables X and Y are independent with pmfs:



Readings and Practice problems: See Set 19