Set 25-27 Part B: Hypothesis Tests

Stat 260: July 19

Hypothesis Testing on p: → population proportion

- A value p_0 is proposed for p (the true population proportion)
- A study/experiment collects data that may support or refute this proposed value.

- **Requirements:**

  every confidence interval and hypothesis test

  (i) A random sample
  (ii) Large sample size ($n \geq 30$)
  (iii) $n\hat{p} \geq 5$, and $n(1-\hat{p}) \geq 5$.

  same as in the normal approx to binomial

- **Hypotheses:**

Right-Tailed:	Left-Tailed:	Two-Tailed:
$H_0 : p = p_0$ (or $H_0 : p \leq p_0$)	$H_0 : p = p_0$ (or $H_0 : p \geq p_0$)	$H_0 : p = p_0$
$H_a : p > p_0$	$H_a : p < p_0$	$H_a : p \neq p_0$

- **Test Statistic:**

  Never use t-distribution

  estimate

  parameter (from H_0)

$$Z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}}$$

standard error (never use $\hat{p}$ standard error for this)

- **p-values:**

  Same as in part a

  Right-tailed Test: p-value $= P(Z > z_{obs})$
  Left-tailed Test: p-value $= P(Z < z_{obs})$
  Two-tailed Test: p-value $= 2P(Z < -|z_{obs}|)$

- *For small sample, hypothesis tests for p use the binomial distribution, but this is beyond the scope of this course.*

$\hat{p} = \frac{75}{200}$

Example 1: Astigmatism is a common refractive error, in which the eye does not focus light evenly on the retina, causing blurred vision. In a sample of 200 random Canadians, it was found that 75 had some form of astigmatism. Is there evidence to suggest that more than 30% of Canadians have astigmatism? Report your conclusion, and state the estimated value of the parameter being tested and the (estimated) standard error.

p = Population of Canadians with astigmatism

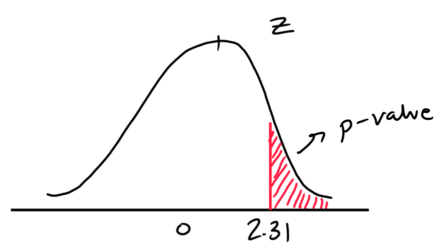
$H_0: p \leq 0.3 \quad H_1: p > 0.3$

Assumptions: $n = 200 \geq 30$

$n\hat{p} = (200)\left(\frac{75}{200}\right) = 75 \geq 5$ ✓, $n(1-\hat{p}) = (200)\left(\frac{125}{200}\right) = 125 \geq 5$

Test Statistic:

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$



Observed value of test statistic:

$$z_{obs} = \frac{\frac{75}{200}}{\sqrt{\frac{0.3(1-0.3)}{200}}} = 2.31$$

0.3 = p from hypothesis

always Standard Error

p-value $= P(Z > z_{obs}) = P(Z > 2.31)$

$= 1 - 0.9696$

$= 0.0104$

Conclusions:

Strong evidence against H_0

Strong evidence to suggest that the true proportion of Canadians with Astigmatism is not at most 30%.

- Estimated value of parameter: $\hat{p} = 75/200$
- Standard error $\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.3(1-0.3)}{200}}$

Extra Example 1: A 2015 study found that 90% of all seabirds had plastics in their stomachs. Suppose that a similar study is performed in 2020, which finds that in a sample of 350 Vancouver Island seabirds, 320 had plastics in their stomachs.

(a) Using the p-value approach, is there evidence to suggest that the proportion of Vancouver Island seabirds with plastics in their stomach is not 90%?

[Ans: $p - value = 0.3734$; no evidence against H_0]

(b) If we were testing the hypothesis at the level $\alpha = 0.05$, would we reject H_0?

[Ans: $p - value > 0.05$; Fail to reject H_0]

(c) Determine the 95% CI for p, the true population proportion of Vancouver Island seabirds with plastics in their stomach. Does this result agree with your solution in (b)?

[Ans: $[0.885, 0.944]$]

(d) Suppose the researchers receive funding to conduct yet another study in 2025. Use the data from the 2020 study to determine the sample size needed to estimate p with 96% confidence to within a 0.02 margin of error.

[Ans: $n = 824$]

Readings and Practice problems: See PDF on Brightspace for detailed instructions.