Tutorial 1:

Find $f(x)$ and $g(x)$ so that neither $\lim_{x \to 2} f(x)$ nor $\lim_{x \to 2} g(x)$ exist but $\lim_{x \to 2} \frac{f(x)}{g(x)}$ does.

Note: We must make the denominator zero as anything divided by zero is undefined which is a requirement of the question

We must also ensure that denominators cancel each other out so that the quotient is defined

$$\lim_{x \to 2} f(x) \therefore \lim_{x \to 2} \left(\frac{1}{x-2}\right)$$

$$\therefore \frac{1}{2-2} \to \frac{1}{0} \Rightarrow \text{undefined div by zero}$$

$$\lim_{x \to 2} g(x) \to \lim_{x \to 2} \left(\frac{\sin(x)}{x-2}\right)$$

$$\to \frac{\sin(2)}{2-2} \Rightarrow \text{undefined div by zero}$$

Now we can focus on the quotient →

$$\lim_{x \to 2} \left(\frac{f(x)}{g(x)}\right) \Rightarrow \lim_{x \to 2} \left(\frac{\frac{1}{x-2}}{\frac{\sin(x)}{x-2}}\right)$$

$$\therefore \frac{1}{\cancel{x-2}} \times \frac{\cancel{x-2}}{\sin(x)}$$

$$\therefore \lim_{x \to 2} \left(\frac{1}{\sin(x)}\right)$$

$$\therefore \frac{1}{\sin(2)}$$

$$\therefore \approx 1.09975$$

Therefore $\lim_{x \to 2} \frac{f(x)}{g(x)}$ does exist and since $\lim_{x \to 2} (f(x))$ and $\lim_{x \to 2} (g(x))$ don't exist as proven above we have fulfilled all of the requirements for this question using $f(x) = \frac{1}{x-2}$ and $g(x) = \frac{\sin(x)}{x-2}$

P.S. Apologies for the poor hand writing