Math & Stats Assistance Centre

STAT 260 Exam Review

Here are some STAT 260 problems for practice only.

If you have questions about how to solve any of the problems please visit drop in to the Math & Stats Assistance Centre to speak to a tutor. Our exam period hours are posted on our webpage: uvic.ca/msac.

This is not a complete overview of final exam material. The Math & Stats Assistance Centre tutors are not involved in creating or evaluating your actual final exam, but we hope these materials will be useful to you!

Round all answers to 4 decimal places, unless the problem requires otherwise.

Part 1: Try this questions now, and the tutor will go over the solutions.

1. Consider the following data set.

   $$4, \quad 7, \quad 10, \quad 14, \quad 15, \quad 17, \quad 22, \quad 23, \quad 23$$

   What proportion of the scores are within 2 standard deviations of the mean?

Questions 2 and 3 refer to the following scenario: Alice, Bob, and Charlie are late for work independently with probabilities .25, .15, .10 respectively.

2. What is the probability that exactly two of Alice, Bob, and Charlie are late for work?
3. What is the probability that all of Alice, Bob, and Charlie are late on the same day exactly once in the next ten workdays?

Questions 4 and 5 refer to the following scenario: An electrian receives an average of 4 calls for assistance over the 48 hours period from Saturday to Sunday.

4. Given that they receive at least one call on Saturday, what is the probability they receive at most 4 calls on Saturday?
5. What is the probability that over a sample of 52 weekends, the average number of calls per weekend is more than 3.8?

Questions 6 and 7 refer to the following scenario: The amount of time (in seconds) for a certain chemical reaction to complete is a continuous random variable with pdf:

$$f(x) = \begin{cases} 2e^{-2x} & 0 \leq x < \infty \\ 0 & \text{otherwise.} \end{cases}$$

6. What is the probability that the chemical reaction takes at least 3 seconds?
7. Compute the expected chemical reaction time (in seconds).

8. Suppose X_1, X_2, X_3 is a random, independent sample of size $n = 3$ from a population distribution having unknown mean μ and unknown standard deviation σ. Consider the following estimators for μ:

$$A = \frac{1}{3}X_1 + \frac{1}{3}X_2 + \frac{1}{3}X_3 \qquad B = \frac{2}{9}X_1 + \frac{2}{9}X_2 + \frac{5}{9}X_3 \qquad C = 2X_1 - \frac{1}{2}X_2 - \frac{1}{2}X_3$$

Which of the following statements are true?

(i) The variance of A is less than the variance of B.

(ii) The expected value of C equals μ.

(iii) The variance of C equals $\frac{9}{2}\sigma^2$.

9. Suppose we want a 95% confidence interval for the difference between the mean gas prices in British Columbia and Ontario. We know that the standard deviations of gas prices are 22 cents and 14 cents respectively. What is the common sample size ($n_1 = n_2 = m$) we would need so that the 95% confidence interval for the true mean difference has a width of 4? You may assume that the estimated standard error follows the unpooled case and that the critical value is standard normally distributed.

10. LONG ANSWER QUESTION: Suppose that the average rent for a one bedroom apartment in Vancouver over a sample of 18 advertisements was $\$1500$ with a standard deviation of $\$200$ while the average rent for a one bedroom apartment in Victoria over a sample of 12 advertisements was $\$1400$ with a standard deviation of $\$160$. Let μ_1 and μ_2 be the mean rent in Vancouver and Victoria respectively. Assume the rents in each city are normally distributed.

(a) What are the null and alternative hypotheses if we wish to test the strength of the evidence that rent in Vancouver is different from the rent in Victoria?

(b) What is the formula for the appropriate test statistic? How is it distributed? If it is t-distributed, be sure to indicate the number of degrees of freedom.

(c) Find the observed value of the test statistic.

(d) Find the p-value. If your test statistic is z-distributed, this will be an exact value; if your test statistic is t-distributed, indicate the tightest possible bounds on the p-value.

(e) Is there sufficient evidence to reject the null hypothesis at the $\alpha = 0.05$ significance level? Justify your response by referencing the p-value. Interpret your decision in terms of the rent in Vancouver compared to Victoria (i.e. write your conclusion out in words).

Part 2: Try these questions later, answers will be posted on Brightspace and the Math & Stats Assistance Centre tutors would be happy to help you work through them.

Questions 11, 12, and 13 refer to the following scenario: A greenhouse performed an experiment where they planted the same seeds in three types of soil, and recorded the proportion of plants that grew to different heights below.

		Plant height		
		Short	Medium	Tall
	Soil A	.12	.13	.15
Soil Type	Soil B	.09	.11	.10
	Soil C	.09	.06	.15

11. What is the probability that a plant was "Tall" or "Soil C"?

12. If a plant was "Short", what is the probability that it was "Soil B"?

13. Which of the following statements are true?

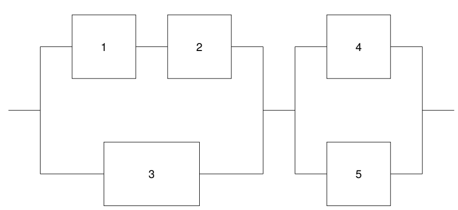
    (i) The events "Short" and "Soil A" are independent.

    (ii) The events "Medium" and "Soil B" are mutually exclusive.

    (iii) The events "Tall" and "Soil C" are independent.

14. Let A and B be two events such that $P(A|B) = \frac{3}{7}, P(B|A) = \frac{9}{32}$, and $P(A' \cap B') = \frac{3}{25}$. Determine the exact value of $P(A \cap B)$.

Questions 15 and 16 refer to the following scenario: Consider the system of components connected as in the following diagram.



The subsystem of components 1, 2, and 3 works if either 1 and 2 both work or 3 works. The subsystem consisting of components 4 and 5 works if 4 or 5 works. The whole system works if both subsystems work. Components 1 through 5 work independently with probabilities 0.9, 0.9, 0.7, 0.8, 0.8 respectively.

15. What is the probability that the system does not work?

16. Given that the system works, what is the probability that component 1 works?

Questions 17, 18, and 19 refer to the following scenario: The **cumulative** distribution of a discrete random variable is as follows.

x	0	1	2	3	4
$F(x) = P(X \leq x)$	0.3	0.55	0.75	0.9	1

17. Compute $P(X \leq 3 \mid X \geq 1)$.

18. Find $E(-4X + 1)$.

19. Find $\sigma_{(-4X+1)}$.

Questions 20, 21, and 22 refer to the following scenario: Bank customers are asked to rate their teller on a scale from 0 to 2 on quality of service X and speed of service Y. Suppose the joint pmf of X and Y is given by the following table:

		y			
		0	1	2	
	0	.18	.05	.21	
x	1	.03	.02	.02	
	2	.11	.04	.34	

20. What is the probability that $X = Y$?

21. Compute the covariance of X and Y.

22. Compute the variance of the difference $D = X - Y$.

23. Prof. Goodtime would like to give a gold star to students in the highest 8% of the grade distribution of her stats class. If grades are normally distributed with a mean of 70 and a standard devation of 12, determine the lowest grade a student needs to receive a gold star.

24. A sample of 12 dryers used an average of 3.5kWh of electricity, with a standard deviation of 0.8kWh. Assuming that electricity usage is approximately normally distributed, find the upper limit of an 80% confidence interval for the true mean electricity usage.

25. A halogen lightbulb has exponentially distributed lifetime with mean $\mu = \dfrac{1}{\lambda} = 3000$ hours. If a lightbulb has already been used for 2000 hours, what is the probability it will last at least another 2000 hours?

26. We want to construct a 94% confidence interval for the proportion of people who drink tea so that it has a width of 0.06. Assuming no prior knowledge of the proportion, what sample size would we need?

27. The following data on vacation days taken by Victoria and Vancouver employees was collected.

Victoria	$n_1 = 8$	$\overline{x_1} = 9.7$	$s_1 = 3.7$
Vancouver	$n_2 = 17$	$\overline{x_2} = 6.6$	$s_2 = 1.9$

We assume that the number of vacation days taken is normally distributed for Victoria and Vancouver employees. Compute the lower limit of a 90% confidence interval for the true mean difference between vacation days taken by Victoria and Vancouver employees.

28. In the 2020 New Brunswick provincial election, the Liberal party won 62% of the vote in majority francophone (French-speaking) ridings, while they won 20% of the vote in majority anglophone (English-speaking) ridings. A new poll found that 73 out of a sample of 100 voters in majority francophone ridings were planning to vote Liberal, while 27 out of a sample of 150 voters in majority anglophone ridings were planning to vote Liberal. What is the observed test statistic you would use to determine whether the difference between the proportion of Liberal support in majority francophone riding and majority anglophone ridings has increased since the 2020 election? You may assume it is an unpooled test.

29. LONG ANSWER QUESTION: We are interested in determining whether the proportion of people who have brown eyes is greater than 60%. In a random sample of 72 people, 46 had brown eyes.

    (a) Define the population parameter of interest.

    (b) State the null and alternative hypotheses in terms of the parameter.

    (c) What is the formula for the appropriate test statistic? How is it distributed? If it is t-distributed, be sure to indicate the number of degrees of freedom.

    (d) Compute the observed value of the test statistic.

    (e) Find the p-value. If your test statistic is z-distributed, this will be an exact value; if your test statistic is t-distributed, indicate the tightest possible bounds on the p-value.

    (f) Construct a 93% confidence interval for the true proportion of people with brown eyes.

Find more resources online at www.uvic.ca/msac
Please report any errors directly to the Math & Stats Assistance Centre at msacpc@uvic.ca, so we can update this for future students.

This resource was developed by the Math & Stats Assistance Centre at UVic to be freely distributed to students in STAT 260. These materials are not to be used for commercial purposes; if you purchased this through a third party then please let us know.