

1. Let A and B be **independent** events, and let $P(A) = 0.4$ and $P(B) = 0.3$. What is the probability that either both of the events or none of the events will occur?

Questions 2 and 3 refer to the following setup:

A company has three departments: Marketing, Sales, and Customer Support. The company wants to determine the probability that an employee from any department will be promoted to a managerial position. The probability that an employee is from the Marketing department is 0.4, from Sales is 0.3, and from Customer Support is 0.3. The probability that an employee from Marketing will be promoted is 0.1, from Sales is 0.2, and from Customer Support is 0.05.

2. What is the probability that an employee will be promoted to a managerial position?
3. Suppose that we know the employee is promoted. What is the probability that this employee is from department of marketing?

4. The lifespan of a cell-phone component is known to be exponentially distributed, with a mean lifespan of 2 years. If a component has lasted at least 1.5 years, what is the probability it will last at least 3.5 years in total?

(Questions 5 and 6 refer to the following setup)

The following joint probability table $f(x, y)$ gives the relationship between the number of loaves of bread (X) and number of pastries (Y) sold by a bakery on a given hour.

$f(x, y)$		y	
		1	2
x	1	0.5	0.1
	2	0.3	0.1

5. Determine the probability that the bakery does not sell more pastries than loaves of bread in a given hour.

6. Find the covariance between X and Y .

7. Suppose a continuous random variable X with probability density function,

$$f(x) = \begin{cases} 1, & 1 \leq x \leq 2 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate $V(X)$.

Questions 8 and 9 refer to the following setup. Suppose that X is a discrete random variable that counts the number of clients served at a computer help-desk on a particular day. X has the following distribution:

x	0	1	2	3	4
$F(x)$	0.3	0.6	0.8	0.9	1

- 8 If it is known that there will be at least one client, what is the probability that there will be no more than three clients in total?

- 9 Find $V(4X - 3)$. You may use the fact that $\mu = 1.4$

- 10 It is known that 2% of all hard drives suffer catastrophic failure within the first three months of usage. Suppose that 500 hard drives are selected at random. Use an appropriate approximation to find the *approximate* probability that between thirteen and sixteen (inclusively) of these hard drives will suffer a catastrophic failure within the first three months of usage.

Questions 11 and 12 refer to the following setup. At a foundry, each steel ingot has a probability of 0.75 of being excellent in quality. This probability is independent of all other ingots.

- 11 Suppose that 15 ingots are selected at random. What is the probability that at least 10 but no more than 12 will be of excellent quality?

- 12 Suppose that 20 ingots are selected at random. What is the *exact* probability that exactly the expected number of excellent ingots will be selected? (Do **not** use an approximation.)

Questions 13, 14, and 15 refer to the following setup. The diameter of baseballs (in *mm*) made by a particular company is known to be normally distributed with a mean of 73 and a standard deviation of 1.1.

- 13 Calculate the probability that a randomly selected baseball will have a diameter which is larger than 75 mm ,
- 14 Find the value k , such that 95% of baseballs made by this company have a diameter **greater** than k .
- 15 Suppose that ten baseballs are selected at random. What is the probability that the average diameter will be less than 73.5 mm ?

Questions 16 and 17 refer to the following setup. A manufacturer finds that from a random sample of 300 shipments, 276 arrive on time at their destination. Let p be the population proportion of shipments that arrive on time at their destination.

- 16 Calculate a 95% confidence interval for p .
- 17 Using these observations as a pilot study, compute the sample size needed to estimate the proportion of shipments that arrive on time with a margin of error of 0.02, with 90% confidence.

Questions 18 and 19 refer to the following setup.

A company producing sprinkler systems for fire protection carried out a study in which they measured the temperature at which the sprinklers activated (in degrees Celsius). The following are their observations.

129.2 130.4 132.9 129.7 133.1 131.3

Let μ be the true mean temperature (in degrees Celsius) at which the sprinklers activate. Assume the temperatures are normally distributed among the population.

- 18 Construct a 90% confidence interval for μ .
- 19 Test the research hypothesis that the true mean activation temperature is greater than 130°C . Carry out this test at the significance level $\alpha = 0.1$.

20 In a random sample of 350 adults, it is found that 84 get some form of exercise on a daily basis. Let p be the population proportion of adults that get some form of exercise on a daily basis. Our research hypothesis is that more than 20% of all adults that get some form of exercise on a daily basis.

(a) Properly specify the null and alternative hypotheses. (Hypotheses in the form of sentences are unacceptable.)

(b) Compute the observed value of the test statistic.

(c) Specify the distribution (including the degrees of freedom if relevant) to be used for computing the p-value, and then find the p-value within the accuracy of the tables.

(d) What level of evidence do we have against the null hypothesis? (Very Strong/Strong/Moderate/Little to None)

(e) If we were testing our hypotheses at the level $\alpha = 0.01$, what would our conclusion be? In order to receive a mark, you must state how you made your decision.

- 21 A psychologist wants to test whether a meditation program improves stress levels. She measures the stress levels of 8 participants before and after following the meditation program for 8 weeks. The data below shows the stress level (on a scale of 1 to 10) of each participant before and after the program.

Participant	Before	After
1	8	7
2	7	4
3	6	4
4	9	6
5	5	4
6	8	4
7	7	4
8	6	4

The psychologist wants to test if there is a significant difference in stress levels after 8 weeks of meditation.

- 22 A researcher is studying the effectiveness of two different exercise programs for improving cardiovascular health. A study is conducted with 180 participants using Exercise Program A, and 120 participants using Exercise Program B. After the program, 135 participants using Exercise Program A showed improvement, while 85 participants using Exercise Program B showed improvement. Test whether there is a significant difference between the proportion of participants who show improvement with Exercise Program A and Exercise Program B at the 0.05 significance level.

- 23 In a study, we are comparing the lifespans of batteries made by two manufacturers. We have selected 12 batteries from Company A and 10 batteries from Company B. Let μ_1 and μ_2 be the population mean lifespan (in hours) for the batteries made by Company A and Company B (respectively). Assume that the battery lifespans for both manufacturers are normally distributed.

Company A	Company B
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$\bar{x}_1 = 60.5$	$\bar{x}_2 = 70.3$
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$s_1 = 5.2$	$s_2 = 8.5$
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$n_1 = 12$	$n_2 = 10$
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Test the research hypothesis that the batteries from two companies have different mean lifespans.

- 24 Suppose the tensile strengths of a particular type of steel used in construction are known to be normally distributed. If 25 samples of the steel are tested, and it is found that the average tensile strength is 520 MPa, with a standard deviation of 30 MPa. Let μ denote the true mean tensile strength of the steel. Should we conclude that the true mean tensile strength of the steel is greater than 500 MPa at the level $\alpha = 0.05$?