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Lab 2 Spectra of Gases and Solids

Objective

The objective of this lab is to investigate the principles of spectroscopy through the examination of the spectra from various gases and solid materials. The purpose is to understand how different elements and compounds emit or absorb light at specific wavelengths, resulting in distinct spectral lines. By analysing these spectra, insights into the composition and properties of the materials are obtained. Furthermore, the lab aims to demonstrate how these spectral characteristics can be applied in astronomical contexts to identify the chemical makeup of distant celestial bodies, thereby enhancing the understanding of the universe.

Introduction

The study of light and its interaction with matter is fundamental in understanding the physical properties of astronomical objects and various elements. Light exhibits a dual nature, behaving both as a particle and as a wave. In this experiment, we focus on the wave-like properties of light, characterised by its wavelength, frequency, and speed.

The wavelength of light determines its colour as perceived by the human eye. Visible light spans wavelengths approximately from 4000 angstroms to 7000 Å, with violet light at the shorter wavelength end (~4000 Å) and red light at the longer end (~7000 Å). Each colour corresponds to photons of specific energies; shorter wavelengths have higher energies.

Atoms and elements consist of electrons orbiting a nucleus in defined energy states. Transitions between these energy states involve the absorption or emission of photons with energies (and thus wavelengths) corresponding to the difference between these states. This quantum behaviour leads to unique spectral lines for each element.

Spectroscopy is the technique of analysing the spectrum of light from a source to determine its composition. There are three primary types of spectra:

1. **Continuous Spectrum**: Produced by hot, dense objects like incandescent bulbs or stars, this spectrum shows a smooth variation of intensity across a wide range of wavelengths without distinct features. An idealised continuous spectrum is described by blackbody radiation, where the peak wavelength shifts with temperature (Wien's Law), and the total emitted energy increases with temperature (Stefan-Boltzmann Law).

2. **Emission Line Spectrum**: When electrons in excited atoms of a hot, diffuse gas drop to lower energy states, they emit photons at specific wavelengths, producing bright lines against a darker background. Each line corresponds to a particular transition unique to an element.

3. **Absorption Line Spectrum**: Occurs when a continuous spectrum passes through a cooler, diffuse gas. Electrons absorb photons at specific wavelengths to move to higher energy states, resulting in dark lines superimposed on the continuous spectrum.

By analysing these spectra, astronomers can identify the elemental composition of astronomical objects without direct sampling. This is crucial for understanding stellar compositions, interstellar mediums, and even the atmospheres of exoplanets.

In this experiment, we utilise a diffraction grating—a device with many finely spaced lines—to disperse light into its component wavelengths. With 600 lines per millimetre, the grating effectively separates different wavelengths, allowing us to observe and analyse the spectra from various light sources. By calibrating our spectroscope using known emission lines from elements like helium, we can determine the wavelengths of unknown spectral lines, such as those from hydrogen.

<u>Procedure</u>

Part I: Observation of Different Types of Spectra

1. **Incandescent Light Bulb**:
   a. Observe the spectrum emitted by an incandescent bulb at two different power settings: low and high.
   b. Use a diffraction grating to view the spectrum and note the differences in intensity and colour distribution between the two power levels. (Image provided)
   c. Record observations, focusing on changes in brightness and the range of visible colours.
2. **Sunlight**:
   a. Examine the solar spectrum using the diffraction grating, directing sunlight through a narrow slit to isolate the spectrum. (Image Provided)
   b. Identify prominent absorption lines (Fraunhofer lines) within the continuous spectrum.
   c. Record the peak brightness (colour) and note the positions of several absorption lines.
3. **Fluorescent Lamp**:
   a. Observe the spectrum of a fluorescent lamp, which exhibits both emission lines and a continuous spectrum.
   b. Use the diffraction grating to separate and identify the discrete spectral lines superimposed on the continuum. (Image Provided)
   c. Record the wavelengths and colours of the prominent emission lines and discuss the presence of the continuous spectrum.

4. **Gas Discharge Tube**s:
   a. Analyse the emission spectra from gas discharge tubes containing different "mystery" elements.
   b. Use an online spectral line database or app to compare observed spectra with known emission lines of various elements.
   c. Identify each element by matching its spectral fingerprints and document the reasoning behind each identification.

Part II: Blackbody Continuum Spectra

Exploration Using Simulation:

- Access the PhET Blackbody Spectrum simulation online.
- Enable "Graph Values" and "Labels" for accurate readings.
- Adjust the temperature settings to simulate different stars:
  - Sun: 5800 K
  - Betelgeuse: 3500 K
  - Sirius A: 10,000 K
- For each star:
- Record the peak wavelength in angstroms by noting the wavelength at which the spectrum's intensity is highest.
- Determine the colour corresponding to the peak wavelength (e.g., red, green, blue, ultraviolet, infrared).
- Observe and note the overall colour of the star as depicted in the simulation.
- Compile the data into a table for comparison.

Part III: Spectrograph Calibration

1. **Measurement of Helium Spectrum**:
   a. Open the provided image of the helium spectrum in GIMP.
   b. Use the measurement tool to determine the pixel distances from the zeroth-order (central bright line) to each of the prominent helium emission lines.
   c. Record the pixel positions corresponding to each known helium line wavelength in a data table.
2. **Calibration Curve Construction**:
   a. Plot the known helium wavelengths (in angstroms) against the measured pixel positions to create a calibration curve.
   b. Use linear regression to determine the best-fit line, establishing the relationship between pixel position and wavelength.
   c. Calculate the slope (m) and y-intercept (b) of the line, applying the equation: $\lambda = m(x) + b$, where λ is the wavelength and x is the pixel position.
3. **Measurement of Hydrogen Spectrum**:
   a. Open the image of the hydrogen spectrum in GIMP.
   b. Measure the pixel distances from the zeroth-order line to each of the visible hydrogen Balmer lines.
   c. Record these measurements in a data table.

4. **Determination of Hydrogen Wavelengths**:
   a. Utilise the calibration equation derived from the helium spectrum to calculate the wavelengths of the hydrogen spectral lines based on their pixel positions.
   b. Record the calculated wavelengths and compare them with the known Balmer series wavelengths.
5. **Uncertainty Analysis**:
   a. Measure the width of the zeroth-order line in pixels and divide by two to estimate the uncertainty (x) in the pixel measurements.
   b. Propagate this uncertainty through the calibration equation to determine the uncertainty in the calculated wavelengths (λ).

Data Recording and Analysis

- Throughout the experiment, meticulously document all observations, measurements, and calculations.
- Include annotated images where applicable to illustrate spectral features and measurement points.
- Analyse the data to identify patterns and correlations between temperature, wavelength, and spectral characteristics.
- Discuss any discrepancies and potential sources of error in the measurements and calculations.

By conducting this experiment, we gain hands-on experience in spectroscopy techniques used by astronomers to decipher the elemental makeup and physical properties of celestial objects. Understanding these methods highlights the profound impact of spectroscopic analysis in astrophysics and the study of the universe.

Observations

Table 1: Blackbody properties of various sources

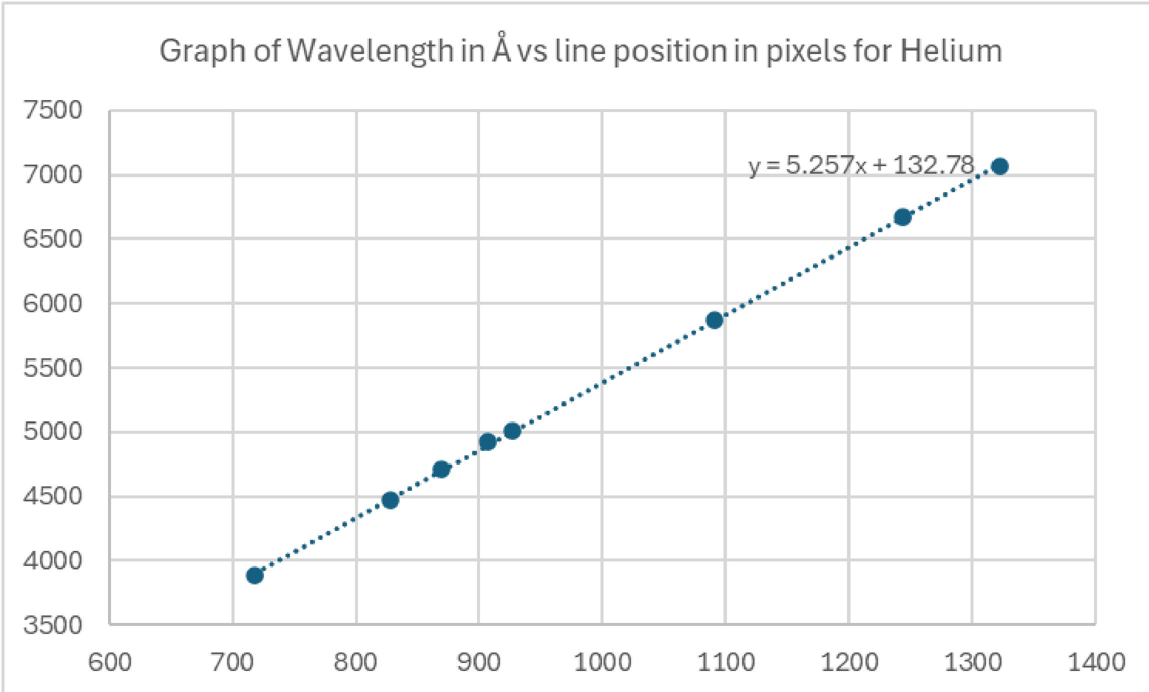
Source	**Temperature [K]**	**Peak Wavelength [Å]**	**Peak Colour**	**Overall Colour**
Sun	5800	5000	Between Light Blue & Green	#dffffd Baby Blue
Betelguese	3500	8280	IR	#ffb960 Saffron Mango
Sirius A	10000	2900	UV	#74adff Crystal Blue

Table 2: Helium spectrum wavelengths and line positions

Line No	**Line Position (*x* pix)**	**Wavelength λ (Å)**	**Description**
8	718	3889	Deep Violet
7	828	4471	Bright blue-violet
6	870	4713	Faint blue-violet
5	907	4922	Blue-green
4	927	5016	Blue-green
3	1091	5875	Yellow
2	1243	6678	Pale Red
1	1323	7065	Dark Red

Table 3: Hydrogen emission line measurements

Balmer Line	**Line Position (*x* pix)**	**Wavelength λ (Å)**	**Description**
Hγ	806	4369.92	Faint Blue-Violet
Hβ	904	4885.11	Blue
Hα	1228	6588.38	Bright Red

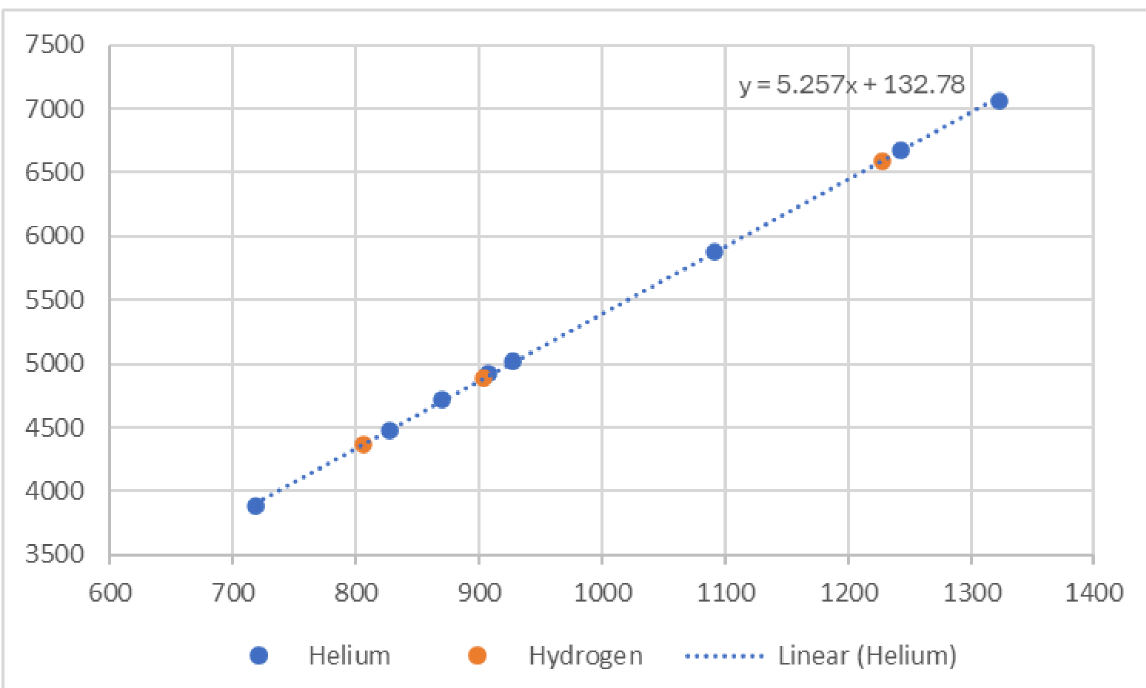


$$Line\ Equation\colon\ y\ =\ 5.257(x)\ +\ 132.78$$

Slope (m) = 5.257

Y-intercept (b) = 132.78

Graph of Wavelength in A vs line position in pixels for Helium + Hydrogen



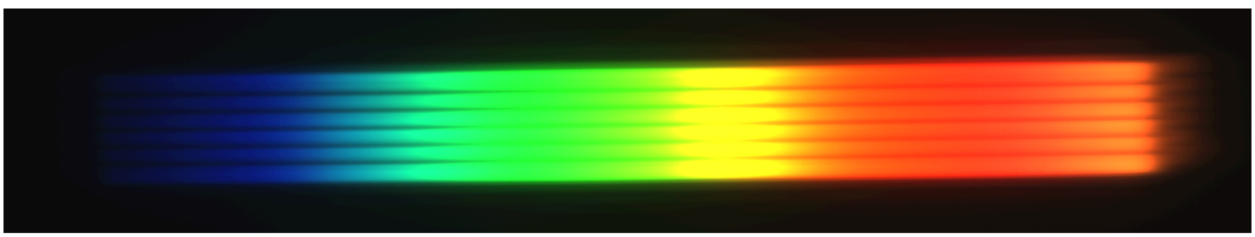
Calculating Uncertainty in the Hydrogen Lines

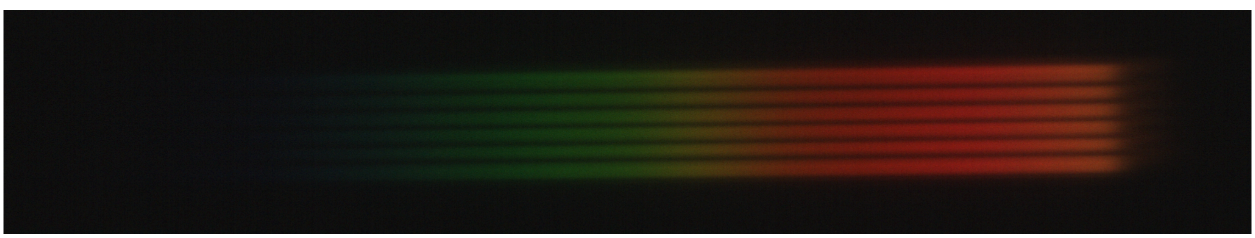
$$Sample\ Calculation\ for\ H\gamma\ =\ (\frac{(1111-1105)}{2})\ \times\ 5.257$$

Balmer Line	**Near Line Edge (pix)**	**Far Line Edge (pix)**	**Line Width (pix)**	**Half Width (pix)**	**Δλ (Å)**
Calculation	-	-	W = Far - Near	HW = W/2	m * HW
Hγ	1105	1111	6	3	15.771
Hβ	1200	1211	11	5.5	28.9135
Hα	1526	1537	11	5.5	28.9135

Answers

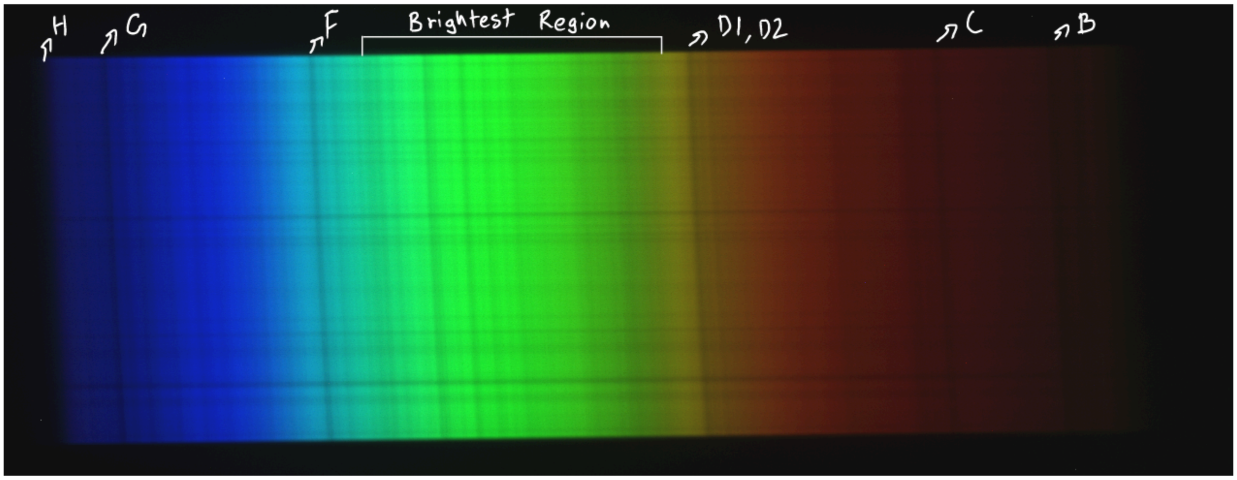
Incandescent Lamp:





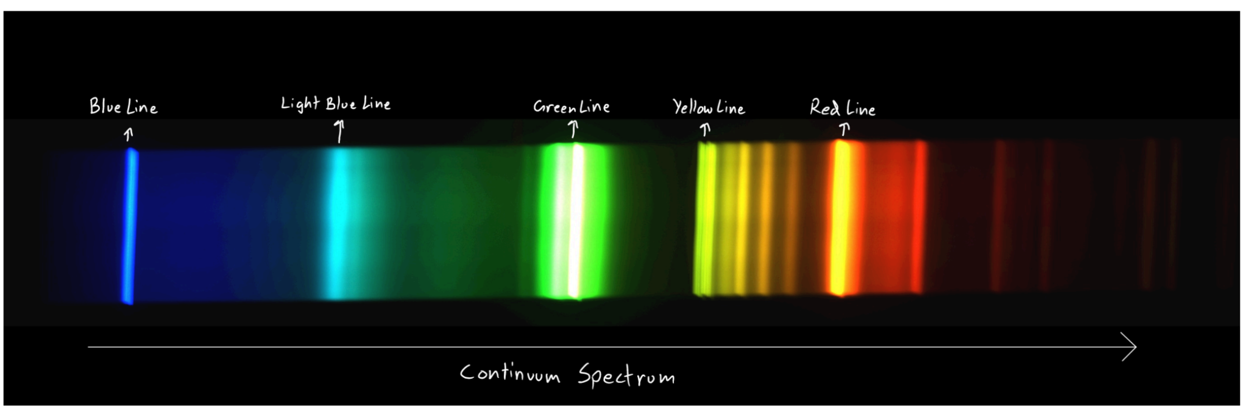
1. The high power incandescent spectrum (first image) is significantly brighter and more vivid compared to the low power spectrum (second image), displaying a broader range of colours, particularly more intense blues and greens. This difference in brightness and colour distribution is due to the higher temperature of the filament at increased power, which follows the principles of blackbody radiation. According to Wien's Law, as the filament heats up, the peak wavelength of emitted light shifts towards shorter wavelengths (bluer light), and the overall brightness increases with the fourth power of temperature as per the Stefan-Boltzmann Law. In contrast, the low power spectrum is dimmer and skewed towards reds and oranges, indicating a cooler filament with less energy and fewer high-energy blue wavelengths. This illustrates how temperature affects the visible spectrum emitted by incandescent light, with higher temperatures producing brighter and bluer light due to increased energy and shorter wavelengths.

Sunlight:



2. Annotations above.
3. In the solar spectrum image, the peak brightness is observed in the green region, indicating where the spectrum is most intense. Prominent absorption lines include H (Ca+) near 396.8 nm, G' (Hγ) at 434.0 nm, F (Hβ) at 486.1 nm, D1 and D2 (Na) around 589 nm, C (Hα) at 656.3 nm, and B (O2) at 686.7 nm. Comparing these lines to the Fraunhofer diagram, H (Ca+), G' (Hγ), F (Hβ), D1, D2 (Na), and C (Hα) are all solar in origin, reflecting elements like calcium, hydrogen, and sodium in the Sun's atmosphere. The B line, however, is terrestrial, caused by oxygen absorption in Earth's atmosphere. These absorption lines help identify the chemical composition of the Sun and the Earth's atmosphere.

Fluorescent Lamp:



4. Annotated Above.
5. Fluorescent lamps produce both emission and continuum spectra due to their construction. The gas inside the lamp, such as mercury vapour, is excited by an electrical current, causing electrons to jump to higher energy levels and emit light at specific wavelengths when they fall back. This results in the bright emission lines characteristic of the gas. Meanwhile, the fluorescent coating inside the tube absorbs some of the UV light emitted by the gas and re-emits it as visible light across a range of wavelengths, creating the continuum spectrum. This combination allows the lamp to produce light that is useful and pleasing for illumination.

Discharge Tube:

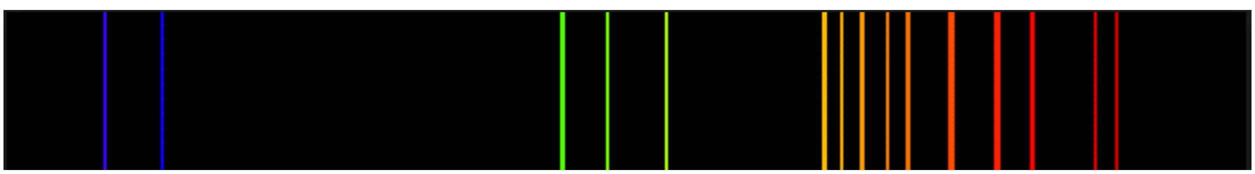
Spectrum 1: Sodium (Na)

The sodium spectrum is primarily characterised by two prominent bright yellow lines, known as the Sodium D-lines. Additionally, scattered blue lines to the left further support the identification of sodium.

Spectrum 2: Hydrogen (H)

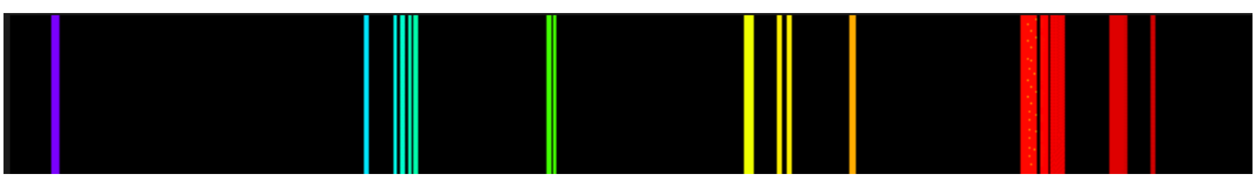
Hydrogen's spectrum is distinguished by a prominent red line, part of the Balmer series. This red line, evident on the right side, indicates transitions from higher energy levels to the second energy level. The presence of other lines in the series supports the identification of hydrogen.

Spectrum 3: Oxygen (O)



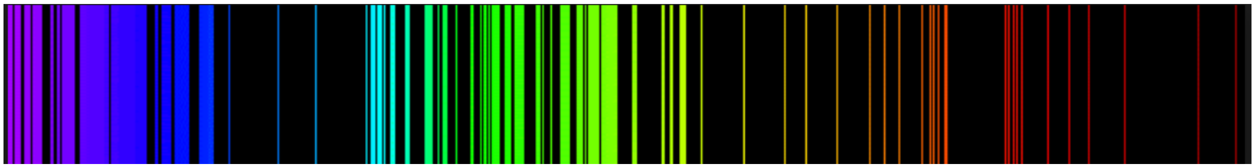
Oxygen's spectrum shows multiple lines in the green and red regions. These lines, primarily to the right, result from various electronic transitions within the oxygen atoms, confirming the presence of oxygen.

Spectrum 4: Nitrogen (N)



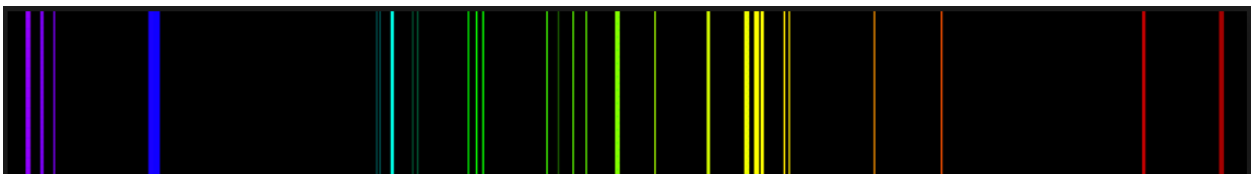
The nitrogen spectrum is marked by many lines, especially in the red and orange regions. This complex pattern, primarily spread across the right side, results from the diverse electronic transitions in nitrogen atoms.

Spectrum 5: Iron (Fe)



Iron's spectrum is characterised by a dense array of lines across the visible range, especially in the blue to green regions. This complexity arises from numerous electron transitions, confirming the presence of iron.

Spectrum 6: Mercury (Hg)



The mercury spectrum features strong lines in the blue and green regions, particularly noticeable towards the left. These lines confirm the element to be mercury.

Analysis

Spectral Features

1. In Part I, we observed different types of spectra from various light sources. Each spectral feature provides insights into the physical conditions of the emitting or absorbing material.
   a. Incandescent Lamp (Continuous Spectrum):
   The incandescent lamp produces a continuous spectrum due to the heating of its filament, typically made of tungsten, to high temperatures. As the filament's temperature increases, it emits thermal radiation across a wide range of wavelengths. This emission follows Planck's Law of blackbody radiation, where the intensity of emitted light depends on the temperature of the emitting object [1]. The continuous spectrum indicates that the source is a hot, dense object emitting photons due to thermal motions of particles, with higher temperatures resulting in increased intensity and a shift toward shorter wavelengths (bluer light), as described by Wien's Displacement Law.
   b. Sunlight (Absorption Line Spectrum):
   The solar spectrum displays a continuous spectrum with superimposed dark absorption lines, known as Fraunhofer lines. These lines occur when photons of specific energies are absorbed by atoms or ions in the cooler outer layers of the Sun or in Earth's atmosphere [2]. The presence of absorption lines indicates that light from a hot, dense source (the Sun's photosphere) passes through cooler, less dense gas layers where absorption occurs at characteristic wavelengths of elements such as hydrogen, calcium, and sodium. This combination reveals information about the Sun's composition and the intervening atmospheric conditions.

c. Fluorescent Lamp (Emission Lines and Continuum Spectrum):
The fluorescent lamp spectrum shows both bright emission lines and a continuous spectrum. Inside the lamp, an electric current excites mercury vapour, causing it to emit ultraviolet (UV) photons. These UV photons are absorbed by the fluorescent coating on the inner surface of the lamp, which then re-emits visible light across a range of wavelengths (the continuous spectrum) [3]. The emission lines correspond to the direct transitions of electrons in mercury atoms, emitting photons at specific wavelengths. This indicates that both the excitation of low-density gas (mercury vapour) and the fluorescence process contribute to the observed spectrum.

d. Gas Discharge Tubes (Emission Line Spectrum):
Gas discharge tubes produce emission line spectra characterised by bright lines at specific wavelengths unique to the gas within the tube. When an electric current passes through the low-density gas, electrons are excited to higher energy levels. As they return to lower energy levels, they emit photons at wavelengths corresponding to the energy differences between these levels [4]. The resulting emission lines allow identification of the gas species and indicate that the emission originates from excited, low-density gas atoms or molecules.

2. Terrestrial absorption lines appear in the solar spectrum because sunlight passes through Earth's atmosphere before reaching the observer. Molecules and atoms in the atmosphere absorb specific wavelengths of light, creating additional absorption lines superimposed on the solar spectrum [5]. These lines are caused by:
    - Atmospheric Gases: Oxygen (O_2), nitrogen (N_2), water vapour (H_2O), and other gases absorb light at characteristic wavelengths due to electronic, vibrational, and rotational transitions
    - Absorption Processes: As photons travel through the atmosphere, they can be absorbed if their energies match the energy differences between molecular or atomic energy levels of atmospheric constituents.

The presence of terrestrial absorption lines in the solar spectrum indicates that Earth's atmosphere plays a significant role in shaping the observed spectrum. This effect must be considered when analysing astronomical spectra to differentiate between features originating from the source and those introduced by the observer's environment.

Blackbody Continuum Spectra

3. Betelgeuse is classified as a red supergiant star of spectral type M1-2 Ia-Iab [7]. It is a massive star nearing the end of its life, characterised by a cool surface temperature and a large radius. In the simulation, Betelgeuse's temperature was set at 3,500 K, resulting in a peak wavelength around 8280 Å (in the infrared region). The corresponding peak colour is therefore infrared, and visually, the star appears as a reddish-orange. The colour determined from the simulation matches the expected colour of Betelgeuse. Red supergiants like Betelgeuse emit most of their energy at longer wavelengths, beyond the visible spectrum, but due to the blackbody curve extending into the visible red region, they appear red to observers. This consistency confirms that the simulation's output aligns with Betelgeuse's observed properties.

4. Sirius A is classified as a spectral type A1V star [8]. It is a main-sequence star with a high surface temperature of approximately 9,940 K. In the simulation, setting the temperature to 10,000 K yielded a peak wavelength around 2900 Å, which is in the ultraviolet (UV) region. Despite the peak emission being in the UV, Sirius appears bluish-white to human observers. The colour determined from the simulation matches the expected colour based on Sirius's spectral type. A-type stars emit strongly in the blue and UV parts of the spectrum, and although the peak is not in the visible range, the significant emission in the blue end of the visible spectrum gives Sirius its characteristic bluish-white hue [9].
5. The Sun's effective surface temperature is approximately 5,800 K. Using Wien's Law:

   $$\lambda_{\max} = \frac{2.897 \times 10^7 \text{ K}}{T} = \frac{2.897 \times 10^7 \text{ K}}{5800\,\text{K}} \approx 4995$$

   This peak wavelength is in the blue-green part of the visible spectrum.

   In the solar spectrum image (Figure 2), the brightest intensity is observed around the green to yellow region, which aligns with the calculated peak wavelength. Although the Sun emits a continuous spectrum across the visible range, the human eye perceives it as white or slightly yellow due to the combined contributions of all visible wavelengths and atmospheric scattering [10].

   Therefore, the peak wavelength determined from the Sun's temperature agrees with the peak evident in the solar spectrum image.
6. We don't see green stars because the blackbody spectrum is continuous and broad, meaning stars emit light across a wide range of wavelengths. When a star's peak emission is in the green portion of the spectrum, it still emits significant amounts of red and blue light [11].
   Human colour perception integrates all the wavelengths emitted by the star. The combination of red, green, and blue light tends to produce a perception of white or slightly yellowish light. Additionally, the sensitivity of human vision peaks in the green region, but colour perception also depends on the relative intensities of other wavelengths present [12].
   Thus, stars with peak wavelengths in the green are perceived as white or yellowish-white rather than green due to the overlapping emissions across the visible spectrum and the way our eyes and brain interpret colours.

Spectral Calibration

7. We found earlier:
   Slope of calibration line (m): 5.257 Å/pix
   Uncertainty in pixel position (x): Calculated as half the width of the zeroth-order line.

   From observations data:

   $$H\gamma = \left(\frac{(1111-1105)}{2}\right) \times 5.257 = 15.771$$

   $$H\beta = \left(\frac{(1211-1200)}{2}\right) \times 5.257 = 28.914$$

   $$H\alpha = \left(\frac{(1537-1526)}{2}\right) \times 5.257 = 28.914$$

Discussion

Do your values agree within error? How far off are they? What are some of the factors that may have affected your results?

Comparison of Calculated and Accepted Wavelengths:

Balmer Line	**Calculated λ**	**Accepted λ**	**Difference**	**Uncertainty Δλ**	**Within Error?**
Hγ	4369.92	4340	29.92	15.771	No
Hβ	4885.11	4861	24.11	28.9135	Yes
Hα	6588.38	6563	25.38	28.9135	Yes

- Hγ (H-gamma): The difference between the calculated and accepted wavelength is 29.92 Å, which exceeds the uncertainty of 15.771 Å. Therefore, it does not agree within error.
- Hβ (H-beta): The difference is 24.11 Å, which is less than the uncertainty of 28.9135 Å. It agrees within error.
- Hα (H-alpha): The difference is 25.38 Å, also less than the uncertainty of 28.9135 Å. It agrees within error.

Factors Affecting the Results:

1. Measurement Errors:
    - Pixel Positioning: Inaccuracies in measuring line positions due to the resolution of the image or difficulty in identifying the exact centre of spectral lines, especially for faint lines like Hγ.
    - Line Widths: Broad spectral lines can make precise determination of line centres challenging.
2. Calibration Errors:
    - Linear Fit Assumption: Assuming a linear relationship between wavelength and pixel position may introduce errors if the actual dispersion is slightly non-linear.
    - Reference Lines: Errors in measuring the helium reference lines can propagate into the calibration curve.
3. Instrumental Limitations:
    - Spectrograph Resolution: Limited spectral resolution may blur lines together or broaden them.
    - Detector Sensitivity: Variations in detector sensitivity across wavelengths can affect the measured intensities and perceived line positions.
4. Human Error:
    - Data Recording: Mistakes in recording measurements or calculations.

- Transcription Errors: Errors when transferring data from measurements to tables.

5. Environmental Factors:
   - Background Light: Ambient light interference may affect the clarity of spectral lines.

Applying what you have learned, how might spectroscopy be used to learn about extrasolar planet atmospheres?

Conditions under which you could observe such a spectrum:

- Transit Method: When an exoplanet passes in front of its host star (a transit), a fraction of the starlight passes through the planet's atmosphere before reaching the observer [15].
- Eclipse Observations: Observing the secondary eclipse, when the planet passes behind the star, allows for isolation of the planet's emitted or reflected light.
- High-Resolution Spectroscopy: Utilising sensitive instruments capable of detecting minute changes in the stellar spectrum caused by the planet.

Type of Spectrum:

- Absorption Spectrum: As starlight filters through the exoplanet's atmosphere during transit, atoms and molecules in the atmosphere absorb light at specific wavelengths, producing an absorption spectrum.
- Emission Spectrum: In some cases, thermal emission from the planet's atmosphere can be observed, providing an emission spectrum.

Challenges in This Type of Observation:

1. Signal-to-Noise Ratio:
   - The changes in the stellar spectrum caused by the exoplanet's atmosphere are extremely small (typically less than 0.01%), requiring very precise measurements.
2. Stellar Activity:
   - Variability in the host star's brightness or spectral features (e.g., star spots, flares) can mimic or obscure planetary atmospheric signals.
3. Instrumental Limitations:
   - Requires instruments with high spectral resolution and stability, often pushing the limits of current technology.
4. Atmospheric Interference:
   - For ground-based observations, Earth's atmosphere can introduce absorption features that must be carefully corrected using calibration with reference spectra.
5. Wavelength Coverage:

- Certain atmospheric constituents (e.g., water vapour, methane) have absorption features in the infrared, necessitating observations beyond the visible spectrum.

6. Time Constraints:
    - Transits are infrequent and of limited duration, providing narrow observation windows.

Despite these challenges, spectroscopy has successfully been used to detect atmospheric components such as water vapour, sodium, and molecules like carbon dioxide in exoplanet atmospheres, enhancing our understanding of their compositions and potential habitability.

Conclusion

In this experiment, we explored different types of spectra and their significance in understanding the physical properties of light sources. By analysing continuous, emission, and absorption spectra from various sources, we gained insights into the conditions of temperature and density that produce these spectral features. The calibration of a spectrograph using known helium emission lines allowed us to determine the wavelengths of hydrogen spectral lines and understand the challenges in precise spectral measurements. Furthermore, the application of spectroscopy to exoplanet atmospheres highlights the method's critical role in advancing astrophysical knowledge and the ongoing quest to detect signs of life beyond Earth.

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