Set 5: Probability
Stat 260 A01: May 21, 2024

Three Axioms of Probability:

(i) For S, the sample space of an experiment, $P(S) = 1$

(ii) For any event A in the sample space, $0 \leq P(A) \leq 1$

(iii) If $A_1, A_2, A_3, \ldots$ are mutually exclusive events, then
$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + P(A_3) + P(A \cdots)$$

Example 1: On a single roll of a 6-sided die, what is the probability of a **2**, **4**, or **6** appearing?

$P(\text{roll } 2, 4, \text{ or } 6) \Rightarrow$ mutually exclusive as you can only get one number when rolling a die

$P(2) + P(4) + P(6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} \quad \frac{1}{2}$

Complement Rule: For any event A,

$P(\bar{A}) = 1 - P(A)$

Let $A =$ "roll a 2"

$P(\bar{A}) = \frac{5}{6}$ done by $1 - P(A)$

$\Rightarrow 1 - \frac{1}{6}$

$\Rightarrow \frac{5}{6}$

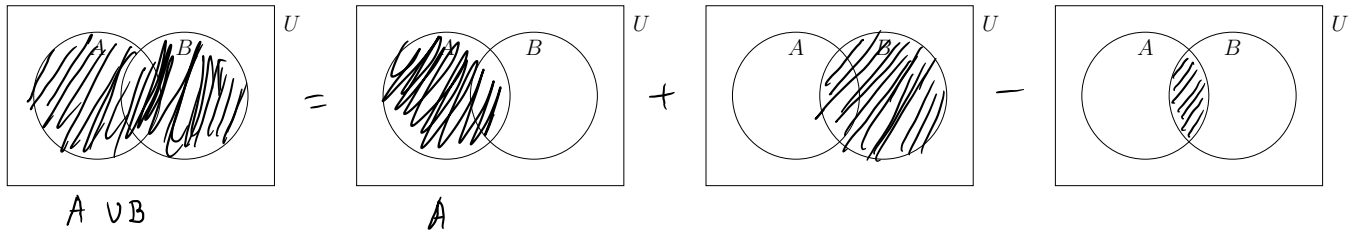$P(\phi) = 0$

$P(\text{roll an } 8 \text{ on a 6 sided die}) = P(\phi) = 0$

General Addition Rule for 2 Sets: Let A and B be events, then
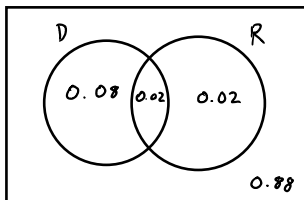
$$P(A \cup B) = P(A) + P(B) - P(A \wedge B)$$

$A \cup B$ = A + $-$

Example 2: 10% of UVic students have been attacked by deer, 4% have been attached by rabbits, and 2% have been attacked by both. What is the probability that a randomly selected student has been attacked by at least one of the two animals?

Let D = "has been attacked by a deer" and R = "attacked by a rabbit"

$$\text{want} \quad P(D \cup R) = P(D) + P(R) - P(D \cap R)$$
$$= 0.1 + 0.04 - 0.02$$
$$= 0.12 \approx 12\%$$

D R
0.08 0.02 0.02
0.88

General Addition Rule for 3 Sets: Let A, B, and C be events, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$
$$- P(A \wedge B) - P(B \wedge C) - P(A \wedge C)$$
$$+ P(A \wedge B \wedge C)$$

Extra Example 1: A veterinarian surveys their feline patients for allergies (A), cataracts (C), and diabetes (D). Among their 50 feline patients,

- 29 had allergies $P(A) = 29/50$
- 13 had cataracts $P(C) = 13/50$
- 17 had diabetes $P(D) = 17/50$
- 10 had allergies and cataracts $P(A \cap C) = 10/50$
- 7 had allergies and diabetes $P(A \cap D) = 7/50$
- 25 had diabetes or cataracts $P(D \cup C) = 25/50$
- 40 had at least one of the three conditions $P(A \cup C \cup D) = 40/50$
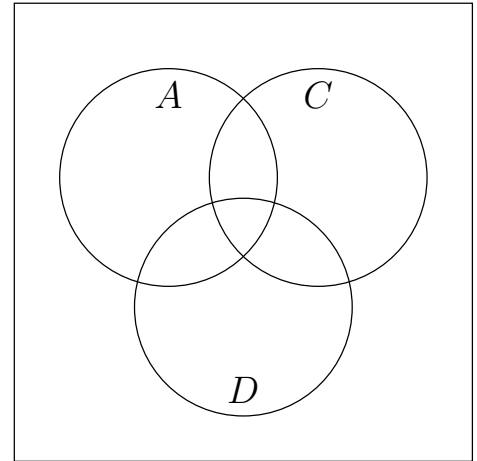
(a) What is the probability that a random cat from the clinic has all three of the conditions?

$P(A \cap C \cap D) =$

2 set general addition rule =
$P(D \cup C) = P(D) + P(C) - P(D \cap C)$

3 set general addition rule =
$P(A \cup C \cup D) = P(A) + P(C) + P(D) - P(A \cap D) - P(A \cap C) - P(C \cap D) + P(A \cap C \cap D)$

(b) What is the probability that a random cat from the clinic has exactly 2 of the 3 conditions?

Textbook Readings: Swartz 3.3 [EPS 1.5, 1.6, 1.7]
Practice problems: Swartz 1.39, 1.41, 1.43, 1.45, 1.47, 1.49, 1.53

L.Teshima 2024