STAT 260 Summer 2024: Written Assignment 9

Due: Upload your solutions to Crowdmark BEFORE 6pm (PT) Friday July 26.

You may upload and change your files at any point up until the due date of Friday July 26 at 6pm (PT).

A 2% per hour late penalty will be automatically applied within Crowdmark. The penalty is applied in such a way so that assignments submitted 6pm to 6:59pm will have 2% deducted, assignments submitted 7pm-7:59pm will have 4% deducted, etc.

Note that if you submit any portion of your assignment before the deadline, Crowdmark will NOT permit you to edit your submission (including make additional uploads) after the 6pm deadline passes. This means that if, for example, you upload only Question 1 before the deadline, you will not be able to upload Question 2 after the deadline. If you intend to submit late (with penalty) you must submit the entire assignment late.

Submission: Solutions are to be uploaded to Crowdmark. Here you will be asked to upload your solutions to each question separately. Your solution to Question 1 must be uploaded in the location for Question 1, your solution to Question 2 must be uploaded in the location for Question 2, etc. If your work is uploaded to the wrong location, the marker will not be able to grade it.

You may hand-write your solution on a piece of paper or tablet. If you wish to use this question sheet and write your solutions on the page, space has been provided below. One of the quickest ways to upload work is by accessing Crowdmark from within a web browser on a smartphone. In the area where you upload work, press the "+" button. This will give you the option of using a file already on your phone, or you can use the phone camera to photograph your work. If you complete your work on a tablet, save the file as a PDF or each question as a jpeg and drag/drop the file into the Crowdmark box. ***Photographs of laptop/tablet screens will not be graded***; take a proper screenshot.

Instructions: For full marks, your work must be neatly written, and contain enough detail that it is clear how you arrived at your solutions. ***You will be graded on correct notation***. Messy, unclear, or poorly formatted work may receive deductions, or may not be graded at all. Only resources presented in lecture or linked to on the Stat 260 Brightspace page are permitted for use in solving these assignments; using outside editors/tutors, and/or software (include AIs) is strictly forbidden. Talking to your classmates about assigned work is a healthy practice that is encouraged. However, in the end, each person is expected to write their own solutions, in their own words, and in a way that reflects their own understanding.

Additional Instructions:

- *Each of the following calculations must follow the methods that were discussed in lecture. All choices for formulas, critical values, distributions, rounding rules, etc must the same as given in lecture.*
- *All calculations are to be completed using only your SHARP EL-510 calculator and the* STAT 260 Distribution Tables Booklet *on Brightspace. Do not use R or other software, and do not use other distribution tables from other books/online/etc.*
- *For all confidence intervals, give your answers in the interval* $[L_1, L_2]$ *form, rounded to 4 decimal positions.*
- *Don't forget to include units in your answers where appropriate.*

1. **[8 marks]** In order to plan out long-term ticket sales and staffing requirements, a local museum wants to determine the average time guests will spend exploring one of their new exhibits.

   (a) During the first few days of the exhibit's premiere, museum staff monitor 8 random guests and find that they spend the following times (in minutes) in the next exhibit:

   30 47 23 46 21 40 32 40

   Using this data, compute the 98.5% confidence interval for μ, the true mean time a visitor spends in the new exhibition.

$$\bar{x} = \frac{30+47+23+46+21+40+32+40}{8} = \frac{279}{8} = 34.875$$

$\bar{x} = 34.875 \rightarrow$ sample mean

$s_x = 9.92 \rightarrow$ Sample standard deviation

98.5% CI $\Rightarrow 1-0.985 = (0.015) \div 2 = 0.0075$

$v = n-1 \rightarrow v = 8-1 \therefore v=7$

$t_{(0.0075,7)} = 3.203 \rightarrow$ from t-distribution table

$$34.875 \pm (3.203) \cdot \frac{9.92}{\sqrt{8}} = 34.875 \pm 11.23 \text{ mins}$$

$$= [23.645 \text{ mins}, 46.105 \text{ mins}]$$

   (b) Suppose that several months later, once the initial novelty of a new exhibit has waned, the museum decides to repeat their study. If the museum staff want to estimate the true mean time that a visitor spends in the exhibit to within 3 minutes with 94% confidence, how many guests will they need to sample?

d

from part (a)

$S = 9.92$, $d=3$, 94% CI

94% CI $\rightarrow 1-\alpha = 0.94$

$\alpha = 0.06$

$$\frac{\alpha}{2} = \frac{0.06}{2} = 0.03$$

$z_{0.03} = -1.88 \rightarrow z_{0.03} = 1.88$

$$n = \left(z_{\alpha/2} \cdot \frac{s}{d}\right)^2$$

$$= \left(1.18 \cdot \frac{9.92}{3}\right)^2$$

$= 38.65 \approx 39$

$n = 39$ guests

(c) Several months later, and ignoring their findings in (b), the museum staff observe 140 guests, and find that they spend an average of 31 minutes (with standard deviation 8 minutes) in the new exhibit. Use this data to create a 80% confidence interval for μ, the true mean time a visitor spends in the new exhibition.

$\bar{x} = 31$, $s = 8$, $CI = 80\%$, $n = 140$

$n = 140 \geq 30$ so we will use $z_{\alpha/2}$ critical value

80% CI $\rightarrow 1 - \alpha = 0.80$

$\alpha = 0.20$

$\frac{\alpha}{2} = \frac{0.20}{2} = 0.10$

$z_{0.10} = -1.28 \rightarrow z_{0.10} = 1.28$

$\rightarrow 31 \pm \left(1.28 \cdot \frac{8}{\sqrt{140}}\right)$

$= [30.135 \text{ mins}, 31.865 \text{ mins}]$

2. **[6 marks]** A pharmaceutical company develops a new medication for the treatment of glaucoma. Before potentially taking their product to market, the company wants to determine the proportion of patients taking the medication that experience nausea as a side-effect.

(a) In an initial study, 75 glaucoma patients are given the medication, of which 12 report nausea as a side-effect. Use this data to calculate the 97.5% confidence interval for p, the true proportion of glaucoma patients taking that medication that experience nausea as a side-effect.

p = the population proportion of glaucoma patients that are taking the medication and experiencing side effects in the form of nausea

$\hat{p}$ = the sample proportion of glaucoma patients that are taking the medication and experiencing side effects in the form of nausea

$\hat{p} = \frac{12}{75} = 0.16$

$97.5\ CI \rightarrow 1-\alpha = 0.975$

$\alpha = 0.025$

$\frac{\alpha}{2} = \frac{0.025}{2} = 0.0125$

$z_{0.0125} = -2.24$

$= 2.24$

$= 0.16 \pm \left(2.24 \cdot \sqrt{\frac{(0.16)(1-0.16)}{75}}\right)$

$= 0.16 \pm (2.24 \cdot 0.0423)$

$= 0.16 \pm 0.095$

$= [0.065, 0.255]$

(b) Suppose that the pharmaceutical company plans to repeat their study on the medication's side-effects in 2 years. Using the current data (from part (a)) as a pilot study, determine the sample size needed to create a 96% confidence interval for p, the true proportion of the glaucoma medication users that experience nausea, to within 5%.

$96\%\ CI \rightarrow 1-\alpha = 0.96$

$\alpha = 0.04$

$\frac{\alpha}{2} = 0.02$

$\alpha = 0.05, \hat{p} = 0.16$

$z_{0.02} = -2.05 \rightarrow z_{0.02} = 2.05$

$n = \left(\frac{z_{\alpha/2}}{\alpha}\right)^2 \times \hat{p}(1-\hat{p})$

$n = \left(\frac{2.05}{0.05}\right)^2 \times 0.16(1-0.16)$

$n = 1681 \times 0.1344$

$n = 225.9264 \approx 226$

$n = 226$ patients