Math 122 Chapter 1 Notes for Quiz

① $p \land q$ → p and q also known as CONJUNCTION

② $p \lor q$ → p or q also known as disjunction

③ $p \to q$ → if p then q = $p \to q$ is true when p is false

∴

p	q	$p \to q$
1	1	1
1	0	0
0	1	1
0	0	1

i.e. conditions ≥
both p and q are true = true
(p is true q is false = false)
p is false q is true = true
p is false q is also false = true
only remember this condition for false since rest are all true

④ $p \leftrightarrow q$ "p if and only if q"
= if p is true then q is true and if q is true then p is true
= so $p \leftrightarrow q$ is only true if p and q are both true or both are false so the truth table would be:

p	q	$p \leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

⑤ Statement that is always false is a contradiction
Statement that is always true is a tautology

⑥ Converse is e.g. $p \to q$ the converse would be $q \to p$

⑦ De Morgan's Law = $\neg(p \lor q) \Leftrightarrow \neg p \land \neg q$, $\neg(p \land q) \Leftrightarrow \neg p \lor \neg q$

⑧ Idempotence = $p \land p \Leftrightarrow p$, $p \lor p \Leftrightarrow p$
∴ essentially saying that the and/or of one truth value is logically equivallent to the value itself

⑨ Commutative = $p \land q \Leftrightarrow q \land p$, $p \lor q \Leftrightarrow q \lor p$
∴ this just means that p and q or p or q will be logically equivalent even if you switch their positions with eachother

⑩ Contra positive is when you take the og statement, flip the variables involved around and not all of them to get logically equivalent two statements so e.g. $p \to q$ contrapositive is $\neg q \to \neg p$

Proofs :

standard structure { → Theorem: universe + what is being proven
proof : working

Example:

Theorem: For all integers n, $n^2 - 4n + 5 > 0$

Proof : Let n be an integer

then $n^2 - 4n + 5$

$= n^2 - 4n + 4 + 1$

$= (n-2)^2 + 1$ → has to be ≥ 0

so $n^2 - 4n + 5 \geq 1$

$\therefore n^2 - 4n + 5 > 0$

Proof by Contradiction (※) :

→ Everything same only need to use " suppose for a contradiction "

Chapter 3: Sets

$\mathcal{U}$ = universe , S = Set , $\bar{S}$ = not in set but in universe

$\emptyset$ or $\{\}$ = empty set , $\subseteq$ = subset

Subset samples

$A = \{1, 2, 3, 4, 5\}$ $B = \{1, 2, 3\}$ $A \subseteq B$

Similarly if $A = \{\{1, 2\}, \{2, 1\}, \{1, 3\}, 3\}$ $B = \{\{1, 2\}\}$ $C = \{4\}$

then $A \subseteq B$ but $C \not\subseteq A$

MATH 122 QUIZ 2 NOTES + PRACTICE

Everything on previous page + below ↓

Quantifiers:

→ $\exists x, p(x)$ → $\exists x$ is an existentiam quantifier

statement translated → There exists x such that $p(x)$

→ $\forall x, p(x)$ → basically 'and' on steroids

For all x, $p(x)$ is true

Examples =

1) $p(x) \rightarrow x^2 + 3x + 2 < 0$ universe is R

$\exists x, p(x)$ will be true because any value between 1 and 2 for example will give less than 0

2) $\forall x, x^2 + 1 > 0$ Universe is R

If $x \in R$, then $x^2 \geq 0$ so its true as the square (x^2) means we only have positive values and +1 ensures its >0 all the time.

Compound Quantifiers:

→ $\exists x (\forall y \ldots)$ There exists an x such that for all y

→ $\forall x (\exists x \ldots)$ For all x, there exists a y such that

Negation of Quantifiers:

→ $\neg(\exists x, p(x)) \Leftrightarrow \forall x \neg p(x)$

→ $\neg(\forall x, p(x)) \Leftrightarrow \exists x \neg p(x)$

→ $\neg(\exists x (\forall y\ p(x,y))) \Leftrightarrow \forall y (\exists x \neg p(x,y))$

→ Argument Validation:

Premise 1

Premise 2

∴ outcome → must be true when premises are true for outcome to be valid

Weak Induction Template

Let $p(n)$ be the statement "___"

$p(1) \rightarrow$ base case

IH suppose $p(n)$

IS : work

Show $p(n+1)$

Hence by PMI ...

Strong Induction Template

Let $p(n)$ be the statement "____"

Base Check : base case (s)

$p(1), \ldots p(7)$ ↳ discuss this later

IH Let $n \geq 7$ Suppose $p(1) .. p(4)$ all true

IS work (using any/all of $p(1)$)

⋮

Show $p(n+1)$ true

Hence by PMI ...

eg of Strong Induction:

Theorem: You have a large supply of 3 ¢, 5 ¢ postage stamps. If $n \geq 8$, can make n ¢ out of 3 ¢ & 5 ¢ stamps

Proof:

Let $p(n)$ be the statement "$\exists a, b \geq 0$ such that $n = 3a + 5b$"

Base cases:

$p(8) = 3+5 = 8$ ✓

$p(9) = 3+3+3$ ✓

$p(10) = 5+5$ ✓

we do three cases because you can connect the rest n's eg 11 to any one of these base cases.

Ex2:

Theorem: Any $n \geq 8$ may be expressed as a sum of 3's and 5's

Proof: Let $p(n)$ be the statement "$\exists a, b \geq 0$ such that $n = 3a + 5b$".

$p(8)$: $8 = 3+5$ ✓

$p(9)$: $9 = 3+3+3$ ✓

$p(10)$: $10 = 5+5$ ✓

Greedy Algorithm for converting to base b.

n
- $n = 0$ expansion is 0
- n is positive
  1) Find largest k such that $b^k \le n$
  2) Apply division algorithm $n = d_k \times b^k + r$
  3) Substitute r, b^{k-1} in step 2, Repeat until $k = 0$

e.g. Convert 225_{10} to base 5

List powers of 5

$5^0 = 1$

$5^1 = 5$

$5^2 = 25$

$5^3 = 125$

$225 = 1 \times 125 + 100$

$= 1 \times 125 + 4 \times 25 + 0 \times 5 + 0 \times 1$

So $225_{10} = 1400_5$

e.g. Convert 5000_{10} to base 7

List powers of 7:

$7^0 = 1$

$7^1 = 7$

$7^2 = 49$

$7^3 = 343$

$7^4 = 2401$ → last one since 7^5 is greater than 5000

$5000 = 2 \times 7^4 + 198$

$= 2 \times 7^4 + 0 \times 7^3 + 198$

$= 2 \times 7^4 + 0 \times 7^3 + 4 \times 7^2 + 2$

$= 2 \times 7^4 + 0 \times 7^3 + 4 \times 7^2 + 2 \times 7^0$

$= (20402)_7$

base 16

eg. Convert 1147 to hexadecimal

$16^0 = 1$

$16^1 = 16$

$16^2 = 256$

$16^3 = 4096$

So $1147 = 4 \times 16^2 + 123$

$= 4 \times 16^2 + 7 \times 16^1 + 11$

$= 47B_{16}$

Conversion base 10 → base b ⇒ Greedy Algorithm

Conversion base b → base 10 ⇒ Just expand

$3A7D \to 3 \times 4096 + 10 \times 256 + 7 \times 16 + 13$

$= 12288 + 2560 + 112 + 13$

Convert 264_7 to base 3

base 7 → base 10 → base 3

$264_7 \to 2 \times 7^2 + 6 \times 7 + 4$

$= 98 + 42 + 4$

$= 144$

$3^0 = 1$

$3^1 = 3$

$3^2 = 9$

$3^3 = 27$

$3^4 = 81$

$144 \to 1 \times 81 + 63$

$144 \to 1 \times 3^4 + 2 \times 3^3 + 9$

$144 \to 1 \times 3^4 + 2 \times 3^3 + 1 \times 3^2 + 0 \times 3^1 + 0 \times 3^0$

$\to 12100_3$

Extended Euclidean Algorithm =

gcd(213, 96)

$\begin{pmatrix}1\\0\end{pmatrix}$ $\begin{pmatrix}0\\1\end{pmatrix}$ $\begin{pmatrix}1\\-2\end{pmatrix}$

213 ÷ 96 = 2 r 21

96 ÷ 21 = 4 r 12 $\begin{pmatrix}0\\-11\end{pmatrix}$

21 ÷ 12 = 1 r 9 $\begin{pmatrix}-9\\18\end{pmatrix}$

12 ÷ 9 = 1 r 3 → gcd

9 ÷ 3 = 3 r 0

$\begin{pmatrix}1\\0\end{pmatrix} - 2\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\-2\end{pmatrix}$

$\begin{pmatrix}0\\1\end{pmatrix} - 4\begin{pmatrix}1\\-2\end{pmatrix} = \begin{pmatrix}-4\\9\end{pmatrix}$

$\begin{pmatrix}1\\-2\end{pmatrix} - \begin{pmatrix}-4\\9\end{pmatrix} = \begin{pmatrix}5\\-11\end{pmatrix}$

$\begin{pmatrix}-4\\9\end{pmatrix} - \begin{pmatrix}5\\-11\end{pmatrix} = \begin{pmatrix}-9\\18\end{pmatrix}$

$\begin{pmatrix}5\\-11\end{pmatrix} - 3\begin{pmatrix}-9\\18\end{pmatrix} = \begin{pmatrix}32\\-65\end{pmatrix}$

213(32) + 96(-65) =

gcd(213, 96)

Now Repeat

$\begin{pmatrix}1\\0\end{pmatrix}$ $\begin{pmatrix}0\\1\end{pmatrix}$ $\begin{pmatrix}1\\-2\end{pmatrix}$

213 ÷ 96 = 2 r 21

$\begin{pmatrix}0\\1\end{pmatrix}$ $\begin{pmatrix}1\\-2\end{pmatrix}$ $\begin{pmatrix}-4\\9\end{pmatrix}$

96 ÷ 21 = 4 r 12

$\begin{pmatrix}1\\-2\end{pmatrix}$ $\begin{pmatrix}-4\\9\end{pmatrix}$ $\begin{pmatrix}5\\-11\end{pmatrix}$

21 ÷ 12 = 1 r 9

$\begin{pmatrix}-4\\9\end{pmatrix}$ $\begin{pmatrix}5\\-11\end{pmatrix}$ $\begin{pmatrix}-9\\20\end{pmatrix}$

12 ÷ 9 = 1 r 3

$\begin{pmatrix}5\\-11\end{pmatrix}$ $\begin{pmatrix}-9\\20\end{pmatrix}$

9 ÷ 3 = 3 r 0

$\begin{pmatrix}1\\0\end{pmatrix} - 2\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\-2\end{pmatrix}$

$\begin{pmatrix}0\\1\end{pmatrix} - 4\begin{pmatrix}1\\-2\end{pmatrix} = \begin{pmatrix}-4\\9\end{pmatrix}$

$\begin{pmatrix}1\\-2\end{pmatrix} - 1\begin{pmatrix}-4\\9\end{pmatrix} = \begin{pmatrix}5\\-11\end{pmatrix}$

$\begin{pmatrix}-4\\9\end{pmatrix} - 1\begin{pmatrix}5\\-11\end{pmatrix} = \begin{pmatrix}-9\\20\end{pmatrix}$

no need to calculate for r0 term

213(-9) + 96(20) =

262139 to base 16

122 Prac

gcd (8288, 15392) ← final question to do

gcd (42, 24) → 42 ÷ 24 = 1

$42 = 24 \cdot q + r$

1 r 18

24 ÷ 18   1 r $\boxed{6}$ ← gcd?

18 ÷ 6   3 r 0 ← stop when you get to zero

So for gcd (8288, 15392)

15392 ÷ 8288   1 r 7104

8288 ÷ 7104   1 r $\boxed{1184}$ ← gcd ✓

7104 ÷ 1184   6 r 0

gcd (12, 15) → 15 ÷ 12   1 r $\boxed{3}$ ← gcd

12 ÷ 3   4 r 0

gcd (9, 12, 21) → 21 ÷ 12   1 r 9

12 ÷ 9   1 r $\boxed{3}$ ← gcd → gcd = 3 for gcd (9, 12, 21)

9 ÷ 3   3 r 0

9 ÷ gcd(12, 21) → 9 ÷ 3   3 r 0

Extended Euclidean Algorithm: next page