STAT 260 Summer 2024: Written Assignment 5

Due: Upload your solutions to Crowdmark BEFORE 6pm (PT) Friday June 21.

You may upload and change your files at any point up until the due date of Friday June 21 at 6pm (PT).

A 2% per hour late penalty will be automatically applied within Crowdmark. The penalty is applied in such a way so that assignments submitted 6pm to 6:59pm will have 2% deducted, assignments submitted 7pm-7:59pm will have 4% deducted, etc.

Note that if you submit any portion of your assignment before the deadline, Crowdmark will NOT permit you to edit your submission (including make additional uploads) after the 6pm deadline passes. This means that if, for example, you upload only Question 1 before the deadline, you will not be able to upload Question 2 after the deadline. If you intend to submit late (with penalty) you must submit the entire assignment late.

Submission: Solutions are to be uploaded to Crowdmark. Here you will be asked to upload your solutions to each question separately. Your solution to Question 1 must be uploaded in the location for Question 1, your solution to Question 2 must be uploaded in the location for Question 2, etc. If your work is uploaded to the wrong location, the marker will not be able to grade it.

You may hand-write your solution on a piece of paper or tablet. If you wish to use this question sheet and write your solutions on the page, space has been provided below. One of the quickest ways to upload work is by accessing Crowdmark from within a web browser on a smartphone. In the area where you upload work, press the "+" button. This will give you the option of using a file already on your phone, or you can use the phone camera to photograph your work. If you complete your work on a tablet, save the file as a PDF or each question as a jpeg and drag/drop the file into the Crowdmark box. ***Photographs of laptop/tablet screens will not be graded***; take a proper screenshot.

Instructions: For full marks, your work must be neatly written, and contain enough detail that it is clear how you arrived at your solutions. ***You will be graded on correct notation***. Messy, unclear, or poorly formatted work may receive deductions, or may not be graded at all. Only resources presented in lecture or linked to on the Stat 260 Brightspace page are permitted for use in solving these assignments; using outside editors/tutors, and/or software (include AIs) is strictly forbidden. Talking to your classmates about assigned work is a healthy practice that is encouraged. However, in the end, each person is expected to write their own solutions, in their own words, and in a way that reflects their own understanding.

Additional Instructions:

- *For each of the following questions, include the correct notation for the random variable that you are calculating in your solution, not just the numeric answer. For example: "$f(3) = P(X = 3) = 0.157$".*
- *For each new random variable that you use, include both its definition ("Let $X = \ldots$") and its distribution ("$X \sim DIST(??, ??)$").*
- *Include units where appropriate.*
- *All calculations are to be completed using only your SHARP EL-510 calculator and the* STAT 260 Distribution Tables Booklet *on Brightspace. Do not use R or other software.*

1. **[7 marks]** At a small farm, 80% of the chicken eggs produced are white, and 20% are brown. The eggs are packaged together at random in cartons of 1.5 dozen (18 eggs).

   (a) Suppose that through a hole in the carton you can see 2 brown eggs. What is the probability that the carton contains at least 5 brown eggs?

1a) P (white) = 0.8 → eggs are white

P(brown) = 0.2 → eggs are brown

18 eggs in 1 carton

Let X be no. of brown eggs in a carton
$X \sim \text{binomial}(n=18, p=0.2)$

We want to find $P(X \geq 5 \mid X \geq 2)$ $\quad P(A|B) = \frac{P(A \cap B)}{P(B)}$

$\therefore \frac{P((X \geq 5) \cap (X \geq 2))}{P(X \geq 2)} = \frac{P(X \geq 5)}{P(X \geq 2)}$

rewrite to use table → $= \frac{1 - P(X \leq 4)}{1 - P(X \leq 1)}$

from table → $\frac{1 - 0.7164}{1 - 0.0991} = \frac{0.2836}{0.9009} = 0.3147$

   (b) Although the eggs are mixed together at random in the cartons, the brown eggs are valued at 50 cents each, while the white eggs are valued at 40 cents each. Determine the expected value (in dollars) of the eggs in a mixed carton, and give its standard deviation.

Using our earlier definition: $X \sim \text{Binomial}(n=18, p=0.2)$

Each brown egg is valued at 50 cents and Each white egg is valued at 40 cents.
Total Value (V) of the eggs in a carton is brown + white and since there are 18 eggs in total:

$V = 0.5X + 0.4(18 - X)$

Simplified: $V = 0.5X + 7.2 - 0.4X = 0.1X + 7.2$

$E(V) = E(0.1X + 7.2)$

Linearity of expectation: $0.1E(X) + 7.2$

Since X was defined above as a Binomial:

$E(X) = np = 18 \times 0.2 = 3.6$

Therefore:

$E(V) = 0.1 \times 3.6 + 7.2 = 0.36 + 7.2 = 7.56$

So the expected value of the eggs in a mixed carton is \$7.56

Standard Deviation Calculation -

$Var(X) = np(1-p) = 18 \times 0.2 \times 0.8 = 2.88$

Since $V = 0.1X + 7.2$, the variance of V is:

$Var(V) = (0.1)^2 \, Var(X) = 0.01 \times 2.88 = 0.0288$

$\sigma_V = \sqrt{0.0288} \approx 0.1697$

So, standard deviation of the value of eggs in a mixed carton is approximately \$0.1697.

So → Expected value of eggs in a mixed carton = \$7.56

→ The standard deviation of the value of eggs in a mixed carton is approximately \$0.1697

2. **[7 marks]** A small tourism information stand is open from 10am-2pm each day, during which visitors arrive according to a Poisson distribution, at an average rate of 4.5 visitors per hour.

(a) What is the probability that the stand sees at most 1 visitor in a 15 minute interval?

2a) Let X be the number of visitors in a 15 minute interval.
Avg rate per hour = 4.5 which we need to adjust for 15 minutes.

$\lambda = 4.5 \times \frac{15}{60} = 4.5 \times 0.25 = 1.125$ visitors per 15 minutes

$X \sim \text{Poisson}(\lambda = 1.125)$
We need to find $P(X \leq 1)$ (at most 1 visitor in 15 minutes)
Using pmf: $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$

For $k = 0$ and $k = 1$:

$P(X=0) = \frac{1.125^0 \cdot e^{-1.125}}{0!} = e^{-1.125} = 0.324652$

$P(X=1) = \frac{1.125^1 \cdot e^{-1.125}}{1!} = 1.125e^{-1.125} = 0.365234$

Thus, $P(X \leq 1) = P(X=0) + P(X=1) = 0.689886 \approx 0.69$

So finally the probability that the stand sees at most 1 visitor in a 15 minute interval is approximately: 0.69

(b) Ideally, the counsel that runs the stand would like to see between 10 to 25 (inclusive) visitors at the stand each (4 hour) day; less than 10 visitors suggests there is not enough interest to make operating the stand worthwhile, and more than 25 would make the stand too busy. On a random day, what is the probability that the total number of visitors falls outside that bound?

2b) Let X be a random variable representing the number of visitors at the stand in a day. $\lambda = 4.5 \times 4 = 18$
Desired range $10 \leq X \leq 25$

$X \sim \text{Poisson}(\lambda = 4 \times 4.5 = 18)$

$P(X < 10 \text{ or } X > 25) = P(X < 10) + P(X > 25)$

$= P(X \leq 9) + 1 - P(X \leq 25)$

from poisson cdf table → $= 0.0154 + 1 - 0.9554$

$= 0.06$

Probability that the total number of visitors exceeds the desired range is 0.06.

Question 2 continues on next page ...

(c) If we examine the visitor logs for 20 random days, what is the probability that on at least one of those days, the stand saw more than 30 visitors?

2c) we need p (Probability of more than 30 visitors on a day)

$p = P(x > 30) \iff 1 - P(x \leq 30)$

$P(x \leq 30) = 0.9967 \Rightarrow$ From poisson CDF Table

$\therefore P(x > 30) = 1 - 0.9967 = 0.0033$

P(atleast one day) $= 1 - P$ (none of the 20 days exceed 30 visitors)

$P(\text{none}) = (P(x \leq 30))^{20} = (0.9967)^{20} = 0.9360$

$\therefore P(\text{atleast one day}) = 1 - 0.9360$

$= 0.064$

Therefore probability that on atleast one of the 20 random days the stand sees more than 30 visitors is approximately - 0.064 or 6.4%.