Math & Stats Assistance Centre
MATH 202 Exam Review Answer key

The following are brief answers to the MATH 202 Review Problems. If you have questions about how to solve any of the problems, please feel free to ask at the Math and Stats Assistance Centre.

1. Find the distance between the plane $6x + 2y - z = 1$ and the plane that passes through the points $(1, 2, 1), (0, 4, -1), (2, -5, -7)$.

   Answer: $\frac{8}{\sqrt{41}}$

2. Let Π_1 be a plane containing the points $P = (1, 0, 0)$, $Q = (0, 1, 0)$, and $R = (0, 0, 1)$, and let Π_2 be another plane containing P, Q, and $S = -R$. Find an equation for the line in which Π_1 and Π_2 intersect. What is the acute angle between Π_1 and Π_2? **Answer:** Equation: $\vec{r} = \vec{i} + t\langle 2, -2, 0\rangle;\ t \in \mathbb{R}$; Angle: $\arccos(1/3)$

3. Compute the following limit, or show that it does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{1 - e^{x^2+y^2}}{(x^2 + y^2)\ln(2 - x^2 - y^2)}$$

   Answer: $\frac{-1}{\ln(2)}$

4. Compute the given limit, or show that it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^3 - y^3}{x^3 + y^3}$$

   Answer: DNE

5. Find an equation of the line tangent to the curve of intersection of surfaces $xyz = 1$ and $x^2+2y^2+3z^2 = 6$ at the point $(1, 1, 1)$

   Answer: $(1, 1, 1) + t(2, -4, 2) = (1 + 2t, 1 - 4t, 1 + 2t)$ where t is any real number.

6. Find and classify all critical points of the function

$$f(x, y) = \frac{1}{x} + \frac{1}{y} + xy.$$

   Answer: Critical point is $(1, 1)$, which corresponds to a minimum.

7. Let the pressure P and temperature T at a point (x, y, z) be

$$P(x, y, z) = \frac{x^2 + 2y^2}{1 + z^2}, T(x, y, z) = 5 + xy - z^2$$

   (a) If the position of an airplane at time t is $(x(t), y(t), z(t)) = (2t, t^2 - 1, \cos(t))$, find $\frac{d}{dt}\left[(PT)^2\right]$ at time $t = 0$ as observed from the airplane.

   Answer: $g'(0) = -16$

(b) In which direction should a bird at the point $(0, -1, 1)$ fly if it wants to keep both P and T constant?

Answer: $\begin{bmatrix} \frac{-4}{\sqrt{21}} \\ \frac{-1}{\sqrt{21}} \\ \frac{2}{\sqrt{21}} \end{bmatrix}$

8. Solve the following initial value problem, given that b is a constant such that the differential equation is exact.

$$(6xy - y^3)dx + (4y + 3x^2 + bxy^2)dy = 0, y(0) = 3$$

Answer: $3x^2y - y^3x + 2y^2 = 18$

9. Consider the differential equation $y' = 3y^{2/3}, y(2) = 0$. Show using Picard's theorem, that we cannot guarantee a unique solution to this equation. Then, find two solutions to this equation.

Answer: One solution is $y' = 0$, and another is $y = (x + C)^3$.

10. Solve the following ODE's:

(a) $y' + 2xy = x, y(0) = 0$

Answer: $y = \frac{1}{2} - \frac{1}{2e^{x^2}}$

(b) A particular solution to $y'' - 2y' + y = \cos(t)e^t$

Answer: $y(t) = -\frac{1}{4}e^t\cos(2t)$.

11. (a) Find the Laplace transform of $\sin(2t)\cos(2t)$. **Answer:** $\frac{2}{s^2+16}$

(b) Find the inverse Laplace transform of $\frac{1}{s+4}$ **Answer:** e^{-4t}

12. Solve $x'' + x = \cos(t)$, $x'(0) = 0 = x(0) = 0$ using the method of the Laplace transform.

Answer: $\frac{1}{2}x\sin(x)$

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