Set 11: Binomial Distribution

Stat 260 A01: June 5, 2023

Recall: A ***Random Variable*** X is a function that assigns a numeric value to each event in a sample space S.

Discrete RV's
- Table of pmfs
- Poisson
- Binomial

Continous RV's
- Normal Distribution
- T- distribution
- Uniform dist
- Exponential

Examples

X counts the King in a hand of cards → discrete

y measures the time it takes to boil a kettle → continous

Binomial Experiment (or Trial): An experiment consisting of a **fixed** number of trials, each with the following properties:

(i) Each trial has 2 outcomes: "success" and "failure"

(ii) Each trial is independent of all other trials.

(iii) For each trial, the probability of a success is p, and the probability of failure is $q = 1-p$.

The ***Binomial Random Variable:***

X counts the number of successes in n trials.

It has **parameters**: n (# of trials), p (prob of success)

Example 1: For each of the following experiments, determine if they are Binomial or not.

(i) Flip a coin and observe whether it is Heads ~~or Tails~~ → Binomial, $n=1$, $p=1/2$

(ii) Roll a 6-sided die and record its face-up value. → Not Binomial (6 outcomes not 2)

(iii) Play the lottery and observe whether you win or not. → Binomial, $n=1$, $p=$ some crazy unlikely number

(iv) Play the lottery and observe how much money you win. → Not Binomial (too many outcomes)

Example 2: About 10% of humans are left-handed. What is the probability that (exactly) one of three friends is left-handed?

trials = observe weather left handed or not

"success" = person is left handed
"failure" = person is not left handed

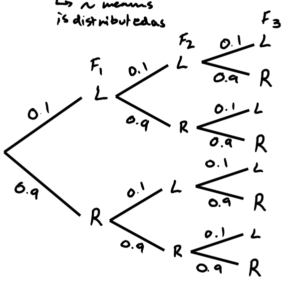
→ dont have to write necessarily

Everything below is required for tests hw etc...

Let X = be the num of left handed friends.

$X \sim \text{Binomial}(n = 3, p = 0.1)$

$\sim$ means is distributed as



$$P(X=1) = P(LRR) + P(RLR) + P(RRL)$$

$$= (0.1)(0.9)(0.9) + (0.9)(0.1)(0.9) + (0.9)(0.9)(0.1)$$

$$= 3(0.1)^1(0.9)^2$$

# of arrangements of 1 L's 2R's →

$$= \binom{3}{1}(0.1)(0.9)^2$$

Notation: The number of ways that x objects can be selected from a collection of n objects is

$$\binom{n}{x} = {}_nC_x = C(n, x) = \frac{n!}{x!\,(n-x)!}$$

Binomial coefficient

This is also the number of ways we can arrange x objects of Type 1 and $n-x$ objects of Type 2 in a row.

Example 2 Continued...
Complete the pmf for X, the number of left-handed individuals from a group of three.

x	0	1	2	3
$P(X=x)=f(x)$		0.243)		0.001

$P(X=0) = P(LLL) = (0.1)^3 = 0.001$

$P(X=2) = 1 -$ sum of rest $= 0.027$ or

$\rightarrow P(LLR) + P(LRL) + P(RLL) = \binom{3}{2}(0.1)^2(0.9)^1 = 0.027$

$P(X=1) =$ from above

$P(X=0) = P(RRR) = (0.9)^3 = 0.729$

Factorials can be done on gdc using nCr $\rightarrow$ 2nd func + num -

Binomial Density Function: In a binomial experiment, the binomial random variable X counts the number of "successes" out of n trials, where the probability of each success is p.

$P(X=k) = \binom{n}{k} p^k q^{n-k}$

k = successes

$n-k$ = failures

Example 3: Approximately 40% of people globally have blood Type O. Suppose that a sample of 20 random subjects are tested for their blood type.

(a) What is the probability exactly 15 of the 20 are Type O?

Let X = num of ppl with Type O blood (out of 20)

$X \sim$ Binomial $(n=20, p=0.04)$

$P(X=15) = \binom{20}{15}(0.4)^{15}(1-0.4)^{20-15}$

$\rightarrow = 0.00129$

(b) What is the probability at most 15 of the 20 are Type O?

$P(X \leq 15) = P(X=0) + P(X=1) + P(X=2) \ldots + P(X=15)$ correct but tm work

Binomial Cumulative Distribution Function for a binomial random variable X is:

$$P(X \leq x) = F(x) = \sum_{t \leq x} P(X=t)$$

$$= \sum_{t \leq x} \binom{n}{t} p^{t} (1-p)^{n-t} \quad \text{For } x = 0, 1, 2 \ldots n$$

Example 3 Continued...

$P(X \leq 15) = 0.9997$ (From Binomial Table)

(c) What is the probability that more than 12 of the 20 are Type O?

$P(X > 12) = 1 - P(X \leq 11)$ → transform into $X \leq$ something form to use values from the table

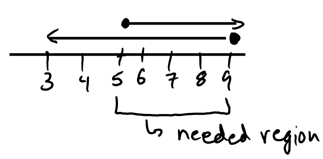
$= 1 - 0.9790$

$= 0.0210$

(d) Suppose that at least 5 of the subjects are known to be Type O. What is the probability that less than 9 of subjects are Type O?

$$P(X < 9 \mid X \geq 5) = \frac{P((X<9) \cap (X \geq 5))}{P(X \geq 5)}$$

$$= \frac{P(5 \leq X < 9)}{P(X \geq 5)} = \frac{P(X \leq 8) - P(X \leq 4)}{1 - P(X \leq 4)}$$



$$= \frac{0.5956 - 0.0510}{1 - 0.0510}$$

$= 0.5739$

(e) Out of the 20, how many subjects do we expect to have Type O?

We could make a pmf table but 21 calculations is too work

For a **binomial random variable** X:

$$E[X] = np = \mu_X \qquad V[X] = \sigma_X^2 = np(1-p) \qquad SD[X] = \sigma_X = \sqrt{np(1-p)}$$

Example 3 Continued...

$E(x) = (20)(0.40) = 8$

$V(x) = np(1-p) = 20\,(0.40)(0.60) = 4.8$ people

$SD(x) = \sqrt{V(x)} = \sqrt{4.8} = 2.19$ people

Readings: Swartz 4.3 [EPS 3.1]
Practice problems: Swartz 3.5, 3.7, 3.9, 3.11, 3.17, 3.19, 3.5