Math 166

Area $\int_a^b y\ dx$ or $\int_a^b x\ dy$

Volume $\pi\int_a^b y^2\ dx$ or $\pi\int_a^b x^2\ dy$

Average for $f(x)$ over [a, b]: $\frac{1}{b-a}\int_a^b f(x)\,dx$

Fundamental theorem of calculus: $\frac{d}{dx}\left(\int_{g(x)}^{h(x)} f(t)\ dt\right) = f\big(h(x)\big)h'(x) - f\big(g(x)\big)g'(x)$

The reason $\frac{d}{dx}(\cosh x) = \sinh x$ and vice versa: $\int \frac{f'(x)}{g(x)}dx = \ln\big(f(x)\big) + C$

shell method: $V = \int_a^b 2\pi R(x) f(x) dx$; Remember: $\int_a^b 2\pi$(Radius)(Height)(Thickness)

Arc length: $L = \int_a^b \sqrt{1+(f'(x))^2}\ dx$ or $\int_c^d \sqrt{1+(f'(y))^2}\ dy$

Surface area: $SA = \int_a^b 2\pi r(x)\sqrt{1+(f'(x))^2}\ \ dx$; Remember: $\int_a^b 2\pi$ (Radius)(Length) dx

Work: $W = \int_a^b F(x)\ dx$

Pumping materials from a tank: $W = \int_a^b \rho$ (Distance)A(y) dy ; Remember: $\int_a^b \rho$ (Distance)(Area)(Thickness)

Fluid force: $F = \int_c^d \rho \cdot h(y) L(y) dy$; Remember: $F =$ (Density)(Depth)(Area)

1D Mass: $M = \int_a^b \delta(x)\ dx$; Remember: Mass = (Density)(Length)

2D Mass: $M = \int_a^b \delta(x)\big(f(x) - g(x)\big)dx$; Remember: $\int_a^b \rho$ (Height)(Width)

Moment-y: $M_y = \int_a^b x\ \delta(x)\big(f(x) - g(x)\big)dx$; Remember: $\int_a^b \rho$ (Location)(Height)(Width)

Moment-x: $M_x = \int_a^b \frac{1}{2}\delta(x)\ (f(x)^2 - g(x)^2)\ dx$

Coordinates: $(\bar{x}, \bar{y}) = (\frac{M_y}{M}, \frac{M_x}{M})$

Parametric curves

Parameterized curve: $(x(t), y(t))$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$$

$$\frac{d^2y}{dx^2} = \frac{x'(t)y''(t) - y'(t)x''(t)}{(x'(t))^3}$$

Length: $\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\ dt$

Area of parametric curves:

To the x-axis: $\int_a^b y(t)x'(t)dt$

To the y-axis: $\int_a^b x(t)y'(t)dt$

Surface area of parametric curves:

Basic formula: $SA = \int_a^b 2\pi R(t)\sqrt{\left((x'(t)\right)^2 + \left(y'(t)\right)^2}\ dt$; Remember: $SA = \int_a^b 2\pi$ (Radius)(Length)

Around x-axis: $SA = \int_a^b 2\pi y(t)\sqrt{\left((x'(t)\right)^2 + \left(y'(t)\right)^2}\ dt$

Around y-axis: $SA = \int_a^b 2\pi x(t)\sqrt{\left((x'(t)\right)^2 + \left(y'(t)\right)^2}\ dt$

Polar Curves

Area: $A = \int_\alpha^\beta \frac{1}{2}(f(\theta))^2 d\theta$ and $A = \int_\alpha^\beta \frac{1}{2}[(f(\theta))^2 - (g(\theta))^2]d\theta$

Length: $L = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}\ d\theta$

Techniques of integration

integration by parts: $\int u\,dv = uv - \int v\,du$

Numerical integration

$$\Delta x = \frac{b-a}{n}$$

Trapezoid: $\frac{1}{2}\Delta x(y_1 + 2y_2 + 2y_3 + 2y_4 + \cdots + 2y_{n-1} + y_n)$

Simpson: $\frac{1}{3}\Delta x(y_1 + 4y_2 + 2y_3 + 4y_4 + 2y_5 + \cdots + 4y_{n-1} + y_n)$ **Must have even slices.

Error analysis

Trapezoid: Error $|E_T| \leq \frac{M(b-a)^3}{12n^2}$; $|f''(x)| \leq M$ for $a \leq x \leq b$.

Simpson: Error $|E_S| \leq \frac{M(b-a)^5}{180n^4}$; $|f^{(4)}(x)| \leq M$ for $a \leq x \leq b$.