Question 1:

For this Poisson problem, we need to find $P(X \leq 25 \mid X \geq 20)$, where X follows a Poisson distribution with $\lambda = 7.5 \times 5 = 37.5$ particles.

```
> lambda <- 7.5 * 5  # Rate × time
>
> # Want: P(X ≤ 25 | X ≥ 20) = P(20 ≤ X ≤ 25) / P(X ≥ 20)
> p_20_to_25 <- sum(dpois(20:25, lambda))
> p_at_least_20 <- 1 - ppois(19, lambda)
> conditional_prob <- p_20_to_25 / p_at_least_20
> lambda
[1] 37.5
> p_20_to_25
[1] 0.01941556
> p_at_least_20
[1] 0.9993355
> conditional_prob
[1] 0.01942847
```

Question 2:

This is a binomial probability problem with n = 400, p = 0.028, and we need P(8 ≤ X ≤ 10).

```
> # Parameters
> n <- 400
> p <- 0.028
>
> # P(8 ≤ X ≤ 10)
> prob <- pbinom(10, n, p) - pbinom(7, n, p)
>
> # Display results
> prob
[1] 0.3070599
```

Question 3a:

For the normal distribution problem, we need P(29 < X < 40.3) where X follows N(28.3, 0.77^2).

```
> # Parameters
> mu <- 28.3
> sigma <- 0.77
> # P(29 < X < 40.3)
> prob <- pnorm(40.3, mean = mu, sd = sigma) - pnorm(29, mean = mu, sd = sigma)
> # Display results
> prob
[1] 0.1816511
```

Question 3b

For the conditional probability, we need P(29 < X < 40.3 | X ≥ 27).

```
> # P(29 < X < 40.3 | X ≥ 27) = P(29 < X < 40.3) / P(X ≥ 27)
> prob_29_to_40.3 <- pnorm(40.3, mean = mu, sd = sigma) - pnorm(29, mean = mu, sd =
sigma)
> prob_at_least_27 <- 1 - pnorm(27, mean = mu, sd = sigma)
> conditional_prob <- prob_29_to_40.3 / prob_at_least_27
> # Display results
> prob_29_to_40.3
[1] 0.1816511
> prob_at_least_27
[1] 0.9543243
> conditional_prob
[1] 0.1903452
```

Question 4

For the sum of 60 independent normal random variables, we use the properties of the normal distribution.

```
> # Parameters for a single day
> mu_day <- 12.3
> sigma_day <- 3.9
>
> # Parameters for sum of 60 days
> mu_sum <- 60 * mu_day
> sigma_sum <- sqrt(60) * sigma_day
>
> # P(750 < Sum < 800)
> prob <- pnorm(800, mean = mu_sum, sd = sigma_sum) - pnorm(750, mean = mu_sum, sd =
sigma_sum)
>
> # Display results
> mu_sum
[1] 738
> sigma_sum
[1] 30.20927
> prob
[1] 0.3255315
```

Question 5

This involves the distribution of the mean of binomial random variables.

```
> # Each day: X ~ Binomial(n=50, p=0.03)
> # Sample of 75 days: average follows normal approximation
> # Mean of X per day: 50 * 0.03 = 1.5
> # Variance of X per day: 50 * 0.03 * 0.97 = 1.455
> # Mean of the average of 75 days: 1.5
> # Variance of the average of 75 days: 1.455/75 = 0.0194
>
> mu_avg <- 50 * 0.03
> sigma_avg <- sqrt((50 * 0.03 * 0.97) / 75)
>
> # P(average ≤ 1)
> prob <- pnorm(1, mean = mu_avg, sd = sigma_avg)
>
> # Display results
> mu_avg
[1] 1.5
> sigma_avg
[1] 0.1392839
> prob
[1] 0.0001654717
```

Question 6a

For the sum of two independent normal random variables.

```
> # Parameters
> mu_L <- 1.2    # Laverne's mean time
> sigma_L <- 0.3  # Laverne's standard deviation
> mu_S <- 0.9    # Shirley's mean time
> sigma_S <- 0.2  # Shirley's standard deviation
>
> # Total time is normally distributed with:
> mu_total <- mu_L + mu_S
> sigma_total <- sqrt(sigma_L^2 + sigma_S^2)
>
> # P(total time ≤ 2)
> prob <- pnorm(2, mean = mu_total, sd = sigma_total)
>
> # Display results
> mu_total
[1] 2.1
> sigma_total
[1] 0.3605551
> prob
[1] 0.3907556
```

Question 6b

For the cost problem, we need to transform the normal distributions.

```
> # Cost parameters
> wage_L <- 25   # Laverne's hourly wage
> wage_S <- 40   # Shirley's hourly wage
>
> # Cost is normally distributed with:
> mu_cost <- wage_L * mu_L + wage_S * mu_S
> sigma_cost <- sqrt((wage_L * sigma_L)^2 + (wage_S * sigma_S)^2)
>
> # P(total cost ≤ 75)
> prob <- pnorm(75, mean = mu_cost, sd = sigma_cost)
>
> # Display results
> mu_cost
[1] 66
> sigma_cost
[1] 10.96586
> prob
[1] 0.7940998
```