Math & Stats Assistance Centre
MATH 101 Exam Review

Part I: Integration Techniques

Determine which technique(s) is appropriate for the following integrals. Then compute each integral.

1. $\displaystyle\int_0^2 \frac{1}{1+e^{-x}}dx$
2. $\displaystyle\int \arcsin(x)dx$
3. $\displaystyle\int_{-1}^1 \frac{1}{x^2}dx$
4. $\displaystyle\int \frac{5x^2+20x+6}{x^3+2x^2+x}dx$
5. $\displaystyle\int \sin^2(x)\cos^2(x)dx$
6. $\displaystyle\int_1^2 \frac{1}{x^2\sqrt{9-x^2}}dx$
7. $\displaystyle\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}}dx$
8. $\displaystyle\int \tan^4(x)dx$
9. $\displaystyle\int e^{5x}\cos(3x)dx$
10. $\displaystyle\int \frac{16}{(x^2+4)x^3}dx$
11. $\displaystyle\int_0^{\pi} \sin^3(x)\cos^8(x)dx$
12. $\displaystyle\int \frac{\sec^2(x)}{\tan^2(x)-\tan(x)}dx$
13. $\displaystyle\int \frac{x^3}{\sqrt{4+x^2}}dx$
14. $\displaystyle\int_1^4 \frac{\ln(x)}{x}dx$

Part II: Applications of Integration

1. Consider the the region in the first quadrant bounded by the curves $y=\sqrt{x}$ and $y=x-2$. For each of the solids of revolution described below, use either the washer or shell method to determine its volume (one method will be easier than the other in each case, so choose wisely).

   (a) The line $y=4$.

   (b) The line $x=0$.

2. Consider the region in the first quadrant bounded by the curves $y=x^3$ and $y=x$. Find the volume of the solid formed by revolving this region about:

   (a) The x-axis.

   (b) The line $x=-3$.

   (c) The y-axis.

   (d) The line $y=2$.

   (e) The solid whose base is that region, and whose cross-sections perpendicular to the x-axis are squares.

3. Find the volume of the solid of revolution formed by revolving the region bounded by the graphs of $y=x^2+1, y=0, x=0$ and $x=1$ about the y-axis.

4. A particle follows a path described by the curve $y = \sqrt{16 - x^2}$, How far does the particle travel between $x = -2\sqrt{2}$ and $x = -2$? Give an exact value

5. The town of Steacyville is in the midst of a flu epidemic. There were 500 people living in Steacyville when the town was quarantined so that no one could enter or leave. Initially, a family of 5 had contracted the flu, but after 4 days, 120 residents of the town had been infected. Assume that the rate at which the number of residents who had contracted the flu changes is proportional to the number of residents who had **not** yet been infected.

   (a) Set up and solve a differential equation for the number of residents who had contracted the flu. Use the initial conditions to solve for all of the constants.

   (b) How long would it take for half of the residents to contract the flu?

Part III: Complex numbers, Parametric equations, and Polar equations

1. Consider the circle centred at $(0, 0)$ with radius $2\ldots$

   (a) Describe this circle with an implicit formula in cartesian coordinates.

   (b) Find a parametrization for this circle that describes a motion that begins at $(2, 0)$ and traverses the circle once clockwise. Find a parametrization for this circle that describes a motion that begins at $(2, 0)$ and traverses the circle once counterclockwise.

   (c) Describe this circle with polar coordinates.

   (d) Suppose the circle is in the complex plane. Find an equation to describe that circle.

2. To multiply two complex numbers together, multiply their absolute values and add their arguments:

   (a) If z is a real and positive number, what is its argument? Use this to explain why a positive number multiplied by a positive number is another positive number.

   (b) If z is a real and negative number, what is its argument? Use this to explain why a negative number multiplied by a negative number is a positive number.

   (c) If $z = re^{i\theta}$ and $r > 1$, describe the path formed by $z, z^2, z^3, z^4, \ldots$.

   (d) If $z = re^{i\theta}$ and $r = 1$, describe the path formed by $z, z^2, z^3, z^4, \ldots$.

   (e) If $z = re^{i\theta}$ and $r < 1$, describe the path formed by $z, z^2, z^3, z^4, \ldots$.

3. Solve the equation $x^4 + 81 = 0$ (find all four complex solutions).

4. Find the area of one petal of the rose curve given by $r = 3\cos(3\theta)$.

5. Find the area of the region common to the two curves $r = -6\cos(\theta)$ and $r = 2 - 2\cos(\theta)$.

6. Find the length of the arc from $\theta = 0$ to $\theta = 2\pi$ for the polar graph $r = 3\sin(\theta)$.

7. Consider the curve $r = 4\sin(2\theta)$, $0 \leq \theta \leq \pi$.

   (a) Sketch the curve.

   (b) Find the slope for the line tangent to the curve at $\theta = \pi/4$.

(c) Set up, but do not evaluate, a definite integral representing the length of the curve on that interval.

(d) Find the area enclosed by the curve on that interval.

8. Consider the curve $x = \cos(t)$, $y = t + \sin(t)$, $0 \leq t \leq \pi$.

   (a) Sketch the curve. Be sure to indicate direction.

   (b) Find an equation for the line tangent to the curve at $t = \pi/2$.

   (c) Set up, but do not evaluate, a definite integral representing the length of the curve on that interval.

   (d) Find the area under the curve on that interval.

9. Set up, but do not evaluate, a definite integral that represents the length of the entire curve $r = 2 - 2\cos(\theta)$.

Part IV: Sequences and Series

1. Determine the convergence or divergence of the sequence with the given n^{th} term. If the sequence converges, find its limit.

   (a) $a_n = \dfrac{e^n}{3^n - 1}$

   (b) $b_n = \sin(n\pi)$

   (c) $c_n = \dfrac{e^n}{2^n + 1}$

   (d) $d_n = \ln\left(\dfrac{5n + 100}{6n - 100}\right)$

2. Evaluate the following sums:

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{100e^n}{\pi^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2 + 2n}$

3. Use an appropriate test to determine whether each of the following series converge absolutely, converge conditionally, or diverge:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{10n - 3}$

   (b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{n}{n^2 + 3}$

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 2^{n+1}}{3^n}$

   (e) $\displaystyle\sum_{n=1}^{\infty} 100\frac{7^n}{n!}$

   (f) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n + 1}$

4. Find the radius and interval of convergence for the following power series:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} x^n$

(b) $\sum_{n=1}^{\infty} 3^{n+1}(x-2)^n$

5. Using the series you know for $\frac{1}{1-x}$, e^x, $\sin(x)$ and $\cos(x)$, find power series for the following functions:

   (a) $f(x) = x^2 \cos(4x)$

   (b) $g(x) = \frac{3}{(1-x)^2}$

   (c) $h(x) = 2xe^{x^2}$ (this can be done 2 ways)

   (d) $q(x) = \ln(1-x^2)$