Lab 5 - Kepler (Orbit of Mars)

Exploring the Night Sky (University of Victoria)

Lab 5: Kepler (Mars Orbit)

Objective:

This lab is based on Kepler's laws of planetary motion, which were initially derived from observations of the sun and planets. The aim of this lab is to help me become acquainted with the orbits of planets within our solar system and to gain insight into the seasonal variations affecting the apparent positions of the sun and planets in the night sky. Two coordinate systems, heliocentric and geocentric, will be utilized alongside elementary geometry to create a model illustrating the orbits of Earth and Mars.

Introduction:

1. **Kepler's Laws:**
   Johannes Kepler, a renowned astronomer of the 17th century, is famous for establishing three laws regarding the motion of planets. These laws were based on observations made by another astronomer, Tycho Brahe, who studied celestial bodies, particularly Mars, for many years. Brahe conducted his observations before the invention of the telescope, relying mainly on the naked eye or basic observational tools. Kepler's ability to derive his laws from these rough measurements showcases his remarkable scientific ingenuity. Alongside other influential astronomers like Copernicus and Galileo, Kepler played a pivotal role in shifting society's understanding from the geocentric model to the heliocentric model. Further information on Kepler's observation techniques can be found in reference link 1.

2. **Planetary Motion:**

   Today, we understand that the Earth orbits around the sun, while the sun remains relatively stationary. However, when observing the sun's position in the sky throughout the year, it seems to move due to the contrast with the fixed backdrop of stars. This apparent path followed by the sun is known as the ecliptic path, representing the projection of Earth's orbit into space. Early civilizations believed in a geocentric model of the universe, where everything revolved around the Earth, likely because of this apparent motion of the sun. Scientists also noticed that bright planets in the night sky move concerning the starry background, following a similar ecliptic path as the sun. This suggests that their orbits also lie approximately on the ecliptic. Although these orbits may seem circular, they are actually elliptical, with very slight eccentricities, except for Mercury, whose high eccentricity makes its orbit noticeably elliptical.

   In this lab, I worked primarily with the orbits of Earth and Mars. Using the same method Kepler used when deriving his three laws, I determined the orbit of Mars entirely based on measurements taken from Earth. Due to the nature of the ecliptic plane, or the fact that the planets' orbits lie in a flat line, I was able to make a roughly scaled map of a part of our solar system that showed the orbits of both Earth and Mars (see Figure). I used recorded measurements done by astronomers to pinpoint the exact location of the planet on the map. This involved drawing a straight line from Earth to the position the planet was observed in for both heliocentric and geocentric observations. Where the lines intersected was the location of the planet at that specific time.

3. **Eccentricities and Ellipses:**

   Planetary orbits take on elliptical shapes, akin to flattened circles where the radius varies around the perimeter. The eccentricity of an ellipse, measured as the longest distance from the centre to the edge, indicates how much the orbit deviates from a perfect circle. A low eccentricity means the orbit closely resembles a circle, while a high eccentricity signifies a more elongated shape. For example, if the longest radius of an ellipse extends from the centre to the edge at 0 degrees, it matches the radius at 180 degrees. However, the shortest radius spans from the centre to 90 degrees and 270 degrees.

   Most planets in our solar system follow orbits with very low eccentricities, almost resembling perfect circles. However, Mercury's orbit is an exception, displaying a notably higher eccentricity, causing it to appear more stretched out compared to other planets' orbits.

4. **Heliocentric Coordinates:**

   The term "heliocentric" originates from the Greek word for sun, "Helios," and refers to the concept that the sun is at the centre of our solar system. The heliocentric coordinate system is a three-dimensional framework used to measure radial distance, longitude, and latitude. Longitude represents east-west measurements and is gauged counterclockwise from 0 to 360 degrees. Latitudes denote north-south measurements, starting from 0 straight to the first point of Aries, 90 degrees at the northern pole, and -90 degrees at the southern pole.

5. **Geocentric Coordinates**:

   The term "geocentric" comes from the Greek word for Earth, "Geo," and refers to the belief that the Earth is at the centre of our solar system. The geocentric coordinate system shares similarities with the heliocentric system, particularly in how it measures latitude, starting from 0 degrees straight toward the first point of Aries.

6. **Triangulation**:

   Kepler heavily relied on triangulation to determine the positions of Mars. Triangulation is a method that allows for relatively accurate measurements of distant objects. Essentially, measurements are taken from two different locations, and where these measurements intersect indicates the object's location. This presented a significant challenge for Kepler, as he had to wait for Earth to be in specific positions to take two separate measurements, while also considering Mars's movement. Understanding that Mars completes its orbit approximately every 687 days, Kepler spaced his measurements accordingly and used triangulation to pinpoint Mars's location. He repeated these observations throughout Mars's orbit, eventually constructing a comprehensive model of its orbit.

For this lab, I will utilize the heliocentric longitudes of Earth and the geocentric recorded positions of Mars obtained over numerous years. By triangulating these measurements, I will determine the orbit of Mars.

Equipment:

The only tools required for this lab are ruled paper and basic geometric drawing instruments, such as a compass, protractor, and ruler.

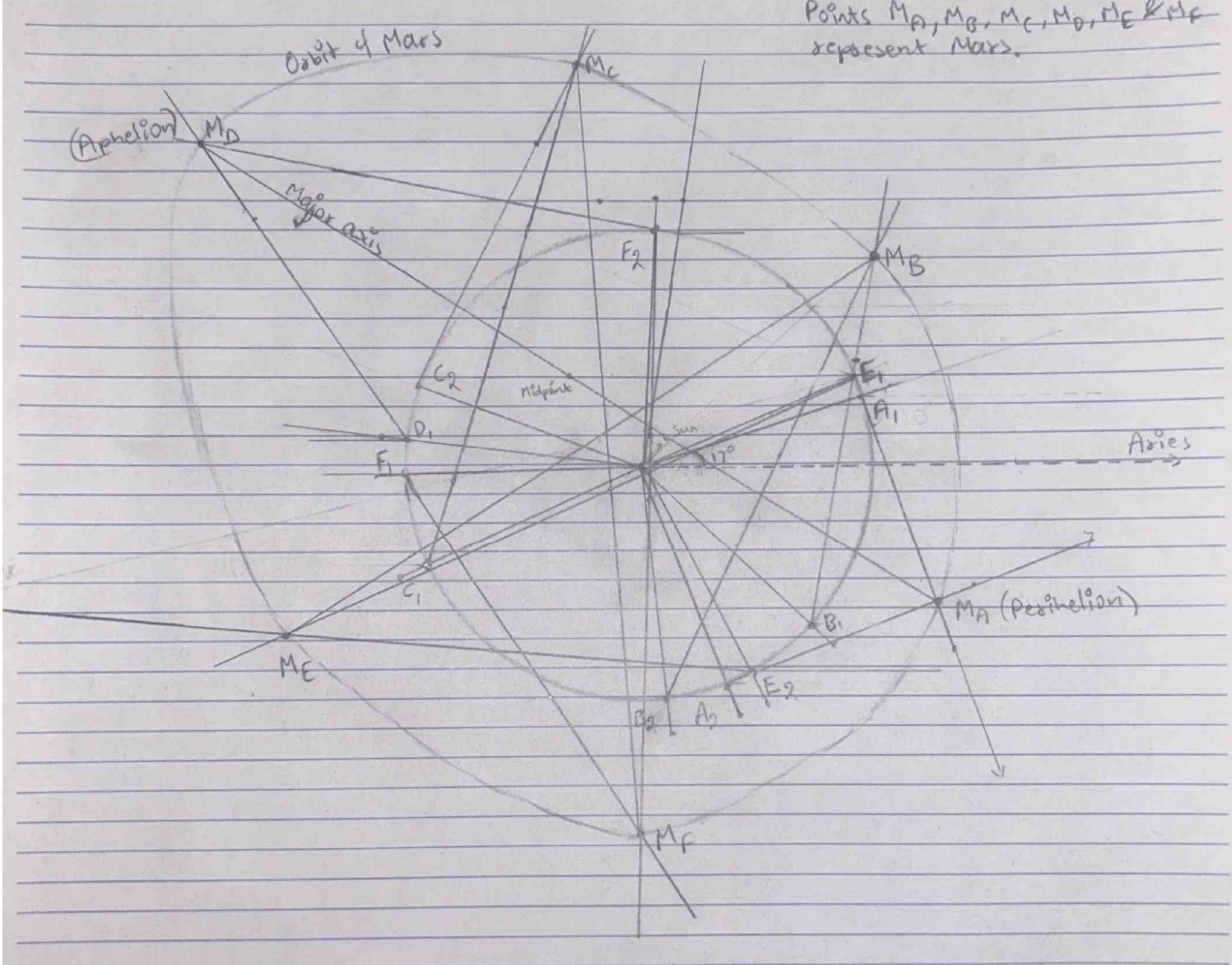
Procedure:

I followed a procedure similar to Kepler's, as outlined in Dr. Gingrech's lab exercise. On a lined paper, I drew a circle at the centre to represent Earth's orbit around the sun, with a radius of 5cm. The centre of the circle aligned with one of the lines, and I positioned the first point of Aries directly to the right. Utilizing heliocentric and geocentric measurements taken between 2001 and 2013 (listed in Table 1), I triangulated the location of Mars. Each pair of measurements, taken 687 days apart (or multiples thereof), allowed me to locate Mars accurately, as Earth's position changed while Mars remained in the same location.

For each pair of measurements, I first used the heliocentric longitudes of Earth to find its position on its orbit. Using a protractor, I measured the angle from the centre of Earth's orbit (representing the sun) counter-clockwise. Earth's position for that measurement was marked on the circle. Then, I drew a straight line to the right to represent the line to the first point of Aries (longitude 00). Next, using the geocentric angle measured counter-clockwise from the first point of Aries, I drew another straight line. Where these lines intersected was the position of Mars for that measurement pair. I repeated this process for each pair of points.

Once I completed this step, Mars's positions formed a smooth circular path. I then drew a curve enclosing these positions to represent Mars's orbit. Next, I measured the distances between opposite points of Mars's orbit. The longest measurement indicated the major axis of the orbit. I drew a line between these two positions and located the midpoint using a ruler, marking its length in centimetres. This measured distance from the sun to the midpoint was known as the focal length. Additionally, I measured the distance from the sun to each end of the major axis. The greater distance from the sun represents the aphelion, while the shorter distance represents the perihelion.

FIGURE : MAP OF MARS'S ORBIT



Point M_D to M_A is the major axis (Length = 15 cm)

Focal length = 2 cm

Length of Semi-Major Axis = 7.5 cm

Answers:

1. (1) Planets move in elliptical orbits with the sun as a focus
   (2) A planet covers the same area of space in the same amount of time no matter where it is in its orbit.
   (3) A planet's orbital period is proportional to the size of its orbit (its semi-major axis).
   (NASA, 2008)

2. First, we have to find the ratio,

$$ratio = \frac{measured\ distance}{actual\ distance}$$

$$ratio = \frac{5\,cm}{1\,AU}$$

$$\text{ratio} = 5\ \text{cm/AU}$$

Now, we have to calculate the distance in AU to each of the locations of Mars.

A) Distance between Sun and Mars 'A' Position

$$ratio = \frac{measured\ distance}{actual\ distance}$$

$$5\,cm/AU = \frac{5.5\,cm}{actual\ distance}$$

$$actual\ distance = \frac{5.5\,cm}{5\,cm/AU}$$

$$actual\ distance = 1.1\ AU$$

B) Distance between Sun and Mars 'B' Position

$$ratio = \frac{measured\ distance}{actual\ distance}$$

$$5\,cm/AU = \frac{4.8\,cm}{actual\ distance}$$

$$actual\ distance = \frac{4.8\,cm}{5\,cm/AU}$$

$$actual\ distance = 0.96\ AU$$

C) Distance between Sun and Mars 'C' Position

$$ratio = \frac{measured\ distance}{actual\ distance}$$

$$5\ cm/AU = \frac{6.6\ cm}{actual\ distance}$$

$$actual\ distance = \frac{6.6\ cm}{5\ cm/AU}$$

$$actual\ distance = 1.32\ AU$$

D) Distance between Sun and Mars 'D' Position

$$ratio = \frac{measured\ distance}{actual\ distance}$$

$$5\ cm/AU = \frac{9.5\ cm}{actual\ distance}$$

$$actual\ distance = \frac{9.5\ cm}{5\ cm/AU}$$

$$actual\ distance = 1.9\ AU$$

E) Distance between Sun and Mars 'E' Position

$$ratio = \frac{measured\ distance}{actual\ distance}$$

$$5\ cm/AU = \frac{7.4\ cm}{actual\ distance}$$

$$actual\ distance = \frac{7.4\ cm}{5\ cm/AU}$$

$$actual\ distance = 1.48\ AU$$

F) Distance between Sun and Mars 'F' Position

$$ratio = \frac{measured\,distance}{actual\,distance}$$

$$5\,cm/AU = \frac{6.8\,cm}{actual\,distance}$$

$$actual\,distance = \frac{6.8\,cm}{5\,cm/AU}$$

$$actual\,distance = 1.36\,AU$$

$$Average = \frac{1.1+0.96+1.32+1.9+1.48+1.36}{6} = 1.35\,AU$$

3. Eccentricity:

$$eccentricity = \frac{focal\,length}{semi-major\,axis}$$

$$eccentricity = \frac{2\,cm}{7.5\,cm} eccentricity = 0.266$$

4. Aphelion:

$$distance = \frac{9.5\,cm}{5\,cm/AU} distance = 1.9\,AU$$

Perihelion:

$$distance = \frac{5.5\,cm}{5\,cm/AU} distance = 0.96\,AU$$

5. Mars and Earth approach each other closely approximately every two years due to their orbits in the same direction, with Mars orbiting around the sun at a faster pace. This alignment occurs because Mars completes its orbit about twice as fast as Earth.

6. Given that Earth is slightly ahead of Mars at point A_1, and considering Mars's slower orbital speed, their closest approach likely occurred slightly before this measurement was taken. Since the measurement was recorded in October, the closest approach probably occurred around August or September.

7. Since Mars and Earth are closest every two years, they would theoretically be farthest apart during the same months in the odd years. For instance, if they were closest in 2019 and 2021, they would be farthest apart in 2020. Consequently, they would reach their farthest point during August or September of the odd years.

Discussion:

This lab involved several calculations, and I'd like to discuss how my results compared to accepted values. Firstly, I calculated the average distance from the sun to Mars as 1.35 AU, whereas the actual average distance is 1.524 AU. This discrepancy may be due to potential errors in recording measurements, such as error in ruler markings, as well as the need to round measurements to two significant digits, which could have influenced the calculations.

Next, I determined the eccentricity of Mars's orbit to be 0.226, which is relatively high for a planet's orbit. This value deviates from the actual eccentricity of Mars, which is 0.093. Again, measurement issues may have contributed to this discrepancy.

Lastly, I calculated the aphelion and perihelion distances of Mars's orbit as 1.9 AU and 0.96 AU, respectively, whereas the accepted values are 1.64 AU and 1.35 AU. These calculations were also slightly off, possibly due to the assumption of a perfect circular orbit for Earth instead of considering its elliptical shape with low eccentricity.

Reflecting on my prediction of the month of closest approach between Earth and Mars, I anticipated it to occur in August or September. However, the last closest approach happened on October 6, 2020. Although my prediction was close regarding the month, I underestimated the frequency of these events. I anticipated them to occur every two years, given Mars's roughly twice-as-long orbit compared to Earth's. Further research revealed that the next closest approach is not expected until 2035, indicating a difference of 15 years. This discrepancy highlights the importance of considering various factors when making predictions in astronomy.

References:

1. NASA. (2008, June 26). Kepler Laws and Orbits. Retrieved from https://science.nasa.gov/resource/orbits-and-keplers-laws/

2. Gohd, C. (2020, October 6). Mars at its closest to Earth until 2035. Retrieved from https://www.space.com/mars-closest-to-earth-until-2035

Orbits of planets travel in ellipse shapes. An ellipse is similar to a circle, but the radius changes along the perimeter. The eccentricity of an ellipse is measured using the longest measurement from the center to the outside edge. A low eccentricity means there is little variation in the radius as you course the perimeter. On the other hand, a high eccentricity

will have a very large radius at one point (and its opposite point), and a small radius at the quarter-way point (and its opposite). To reiterate, if the longest
radius of an ellipse is from the center to the edge at 0 degrees, the radius will match from the center to the edge at 180 degrees. In this example, the shortest radius will go from the center to 90 degrees, and
the center to 270 degrees.

Most planets have an orbit that follows an ellipse with a very low eccentricity. In fact, they can almost be considered circle orbit paths. The exception to this rule is Mercury. Mercury’s orbit has a very high
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