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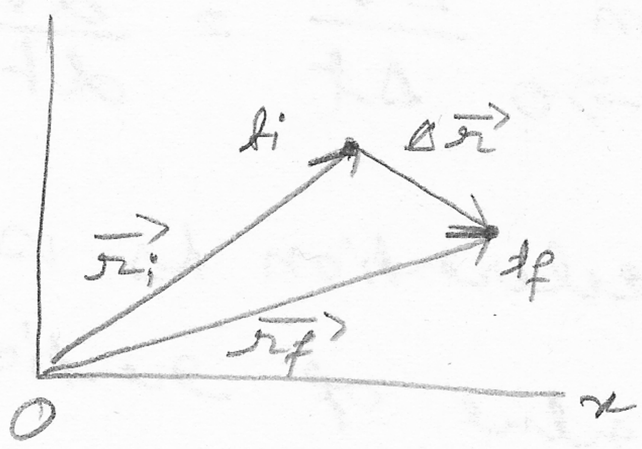
chapter outline - 4

[4.1] Motion in two dimentions

In two dimensions, we describe an objects position by it position vector $\vec{r}$.
We can define the displacement of the position vector $\vec{r}$ by $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$.



The average velocity of an object is the displacement of the object divided by the time it took for the displacement.

Average velocity equation → $\vec{V}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$

Instantaneous velocity equation →

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

The instantaneous velocity $\vec{v}$ is defined as the limit of the average velocity $\frac{\Delta\vec{r}}{\Delta t}$ as Δt approaches 0.

equation of average acceleration →

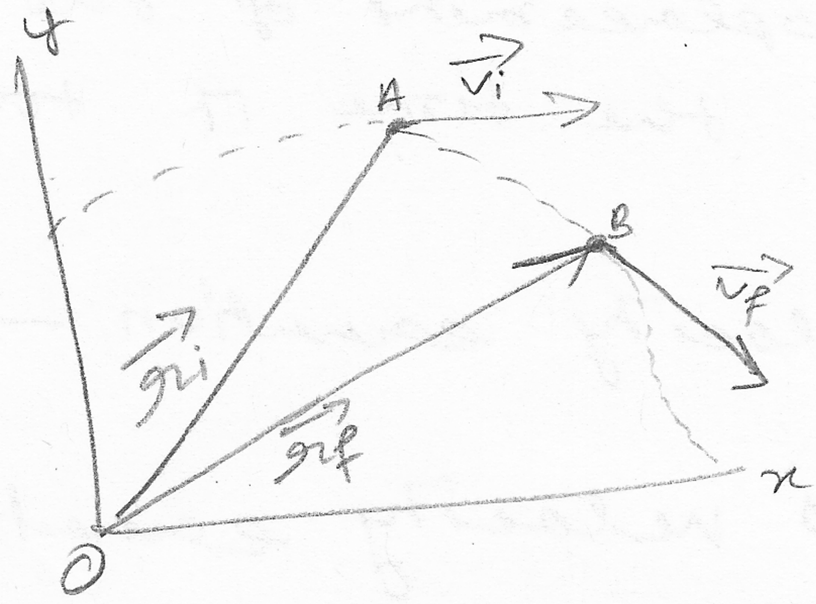
$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

The average acceleration of an object is the change of its instantaneous velocity vector $\Delta\vec{v}$ divided by the time interval Δt.

Equation of instantaneous acceleration →

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

Instantaneous acceleration is when we limit the value of ration $\frac{\Delta \vec{v}}{\Delta t}$ as Δt approaches 0.



4.2

We can describe two dimentional motion as the two separate motions on x and y axes. And they do not affect each other.

An object's position vector moving on xy plane can be

described as $\vec{r} = x\hat{i} + y\hat{j}$ and the velocity of the object can be described by the following equation:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$= v_x\hat{i} + v_y\hat{j}$$

When an object moves along a path with constant acceleration we can obtain its final velocity by the following equation $\Rightarrow \vec{v}_f = \vec{v}_i + \vec{a}t$

equation of position vector when the acceleration is constant in two dimensions $\Rightarrow \vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$

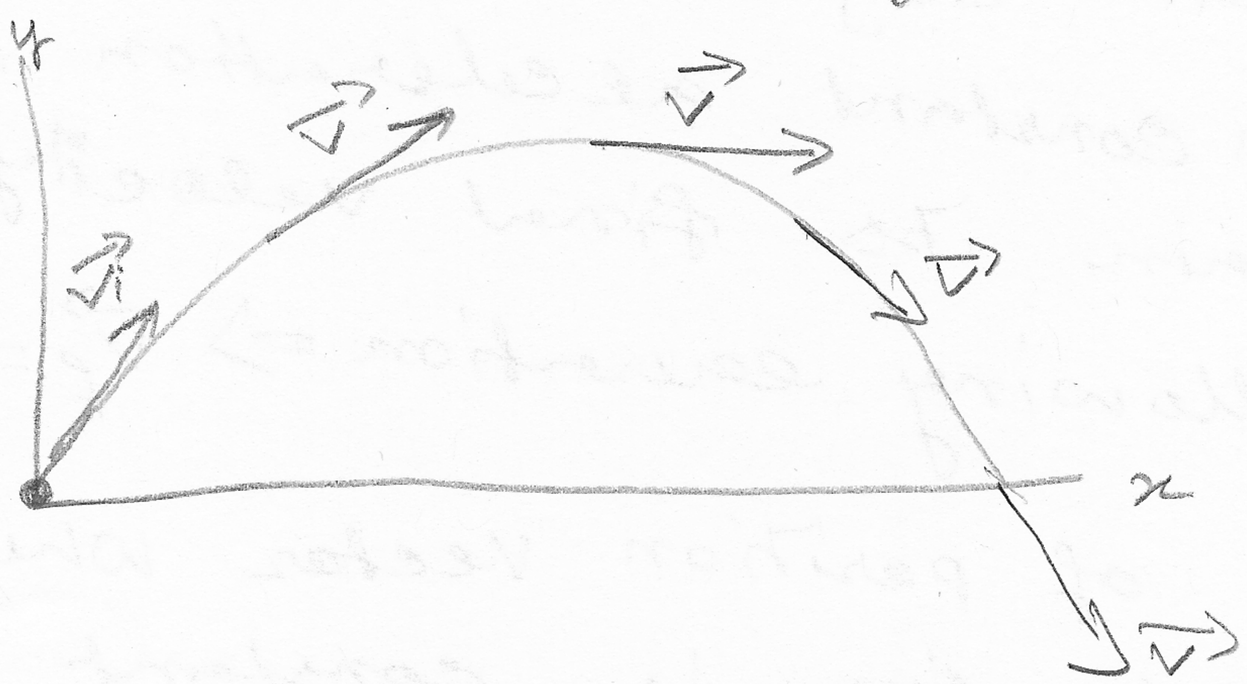
[4.3] Projectile Motion → When an object is in its free fall motion with a constant acceleration falling downward and the air resistence is negligible we call it projectile motion. The acceleration of Projectile motion is due to gravity.

$\vec{a} = \vec{g}$.

So, $\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{g}t^2$

The initial x component of velocity of projectile is $v_{xi} = v_i \cos\theta_i$

The initial y component of the velocity of projectile is $v_{yi} = v_i \sin\theta_i$



The object in the x direction with constant velocity have the following equation $\Rightarrow x_f = x_i + v_{xi} t$

The object in the y direction with constant acceleration have the following equations $\Rightarrow$

$$v_{yf} = v_{yi} - gt \quad \text{---} \quad \text{(i)}$$

$$v_{yavg} = \frac{v_{yi} + v_{yf}}{2} \quad \text{---} \quad \text{(ii)}$$

$$y_f = y_i + \frac{1}{2}(v_{yi} + v_{yf})t \quad \text{---} \quad \text{(iii)}$$

$$y_f = y_i + v_{yi} t - \frac{1}{2}gt^2 \quad \text{---} \quad \text{(iv)}$$

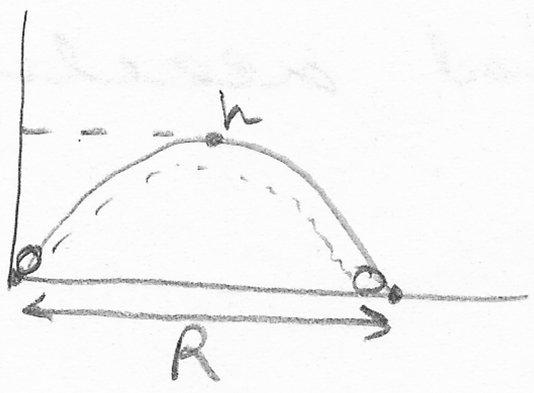
$$v_{yf}^2 = v_{yi}^2 - 2g(y_f - y_i) \quad \text{---} \quad \text{(v)}$$

under constant acceleration the object reaches its peack at time t. And we can solve t by the following equation $\Rightarrow$ $t = \frac{v_i \sin\theta_i}{g}$

we can also find the maximum height of the object by the following equation $h = \frac{v_i^2 \sin^2\theta}{2g}$

under constant acceleration we can find the Range of the object by the following equation $\rightarrow$ $\frac{2v_i^2 \sin\theta_i \cos\theta_i}{g}$

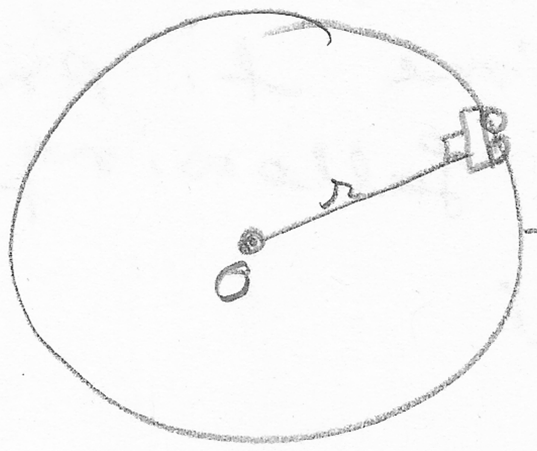
$R \Rightarrow \frac{v_i^2 \sin 2\theta_i}{g}$



4.4 uniform circular motion $\rightarrow$

Any object that moves in a circular path has circular motion. If the object moves with a constant speed we call it uniform circular motion.

Thus we define the centripetal acceleration equation $\rightarrow$ $a_c = \frac{v^2}{r}$



→ circular motion of a car

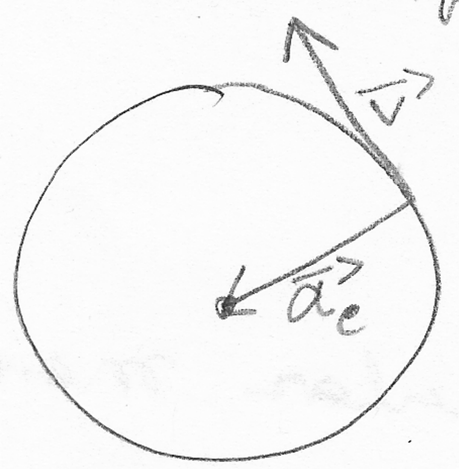
Equation of period = $T = \frac{2\pi r}{v}$

Equation of Angular speed = $\omega = \frac{2\pi}{T}$

Equations – (i) $v = r\omega$

(ii) $a_c = r\omega^2$

centripetal acceleration → when the direction of the acceleration vector is toward the center of the circle, it is called centripetal acceleration.



Angular speed → Angular speed is the rate at which an object changes its angle in radians in a given time period.

[4.6]

Relative velocity:

The relative velocity of an object A with respect to another object B is the velocity that object A would appear to have an observer situated on object B moving along with it.